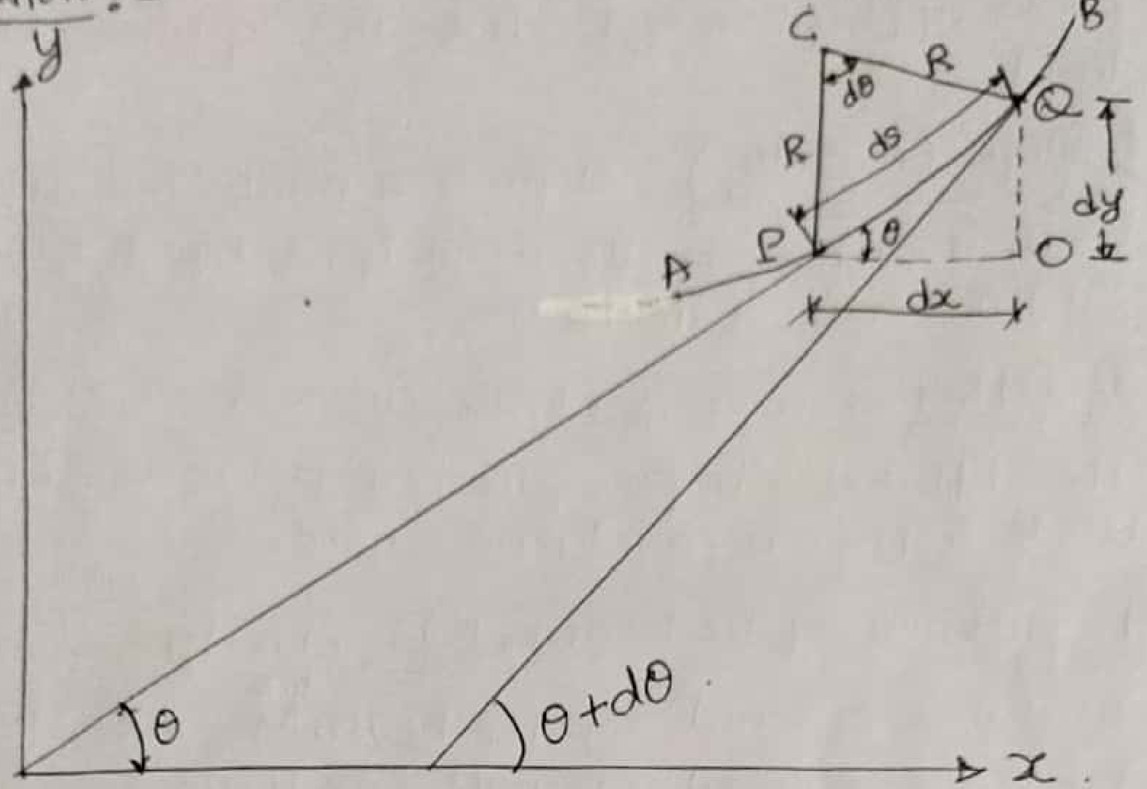
Derive the differential eqn of deflected curve for the beam. OR Derive moment curvature equation OR Derive differential equation of flexure.

Solution :-



Let,

AB → deflected shape of the beam.

PQ → small portion of beam

ds → Length of small portion of beam.

R → Radius of curvature of an arc PQ.

P → point on the curve where tangent make an angle θ with x-axis

Q → point on the curve where tangent make an angle θ+dθ with x-axis

C → Centre of curvature where normal drawn from P & Q will meet.

$\boxed{\therefore PC = QC = R}$

P & Q → very close to each other ∴ bending is very small.

$\angle PCQ = d\theta$

$PQ = ds = R\,d\theta$

$\therefore R = \frac{ds}{d\theta}$

$R \doteq \frac{dx}{d\theta}$ $(\because ds \simeq dx)$

& $\boxed{\frac{1}{R} = \frac{d\theta}{dx}}$ —— ①

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Treating PQO as a Δ^{le}.

$$\tan\theta = \frac{dy}{dx}$$

Since 'θ' is very small

$$\boxed{\theta = \frac{dy}{dx}} \quad \text{--- (2)}$$

Differentiate eqn (2) w.r.t "x"

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2}$$

W.K.T. $\boxed{\frac{1}{R} = \frac{d^2y}{dx^2}}$ --- (3)

From bending eqn of the beam, W.K.T

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{1}{R} \times E$$

$$\frac{M}{EI} = \frac{1}{R} \quad \text{or} \quad \boxed{\frac{1}{R} = \frac{M}{EI}} \quad \text{--- (4)}$$

By comparing eqn (3) & (4)

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \quad \text{or}$$

$$\boxed{M = EI\frac{d^2y}{dx^2}} \quad \#$$

Where, EI $\rightarrow$ Flexural Rigidity of the beam.

E $\rightarrow$ young's modulus

I $\rightarrow$ Moment of Inertia.

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Deflection of Beams.

→ Moment area method.

→ Conjugate beam method.

→ Double integration and Macaulay's method.

[major object to find slope θ and deflection δ]

→ methods available to find deflection of beams :-

moment area method.

conjugate Beam method.

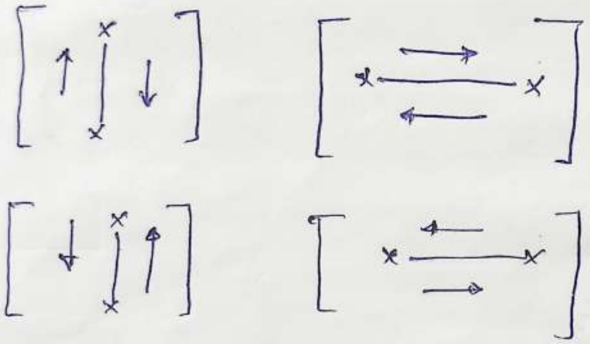
Double Integration and Macaulay's method.

Types of Beams considered are cantilever beam and simply supported beam.

Sign conventions.

Shear Force ↑ → (+ve) ↓ ← (-ve).

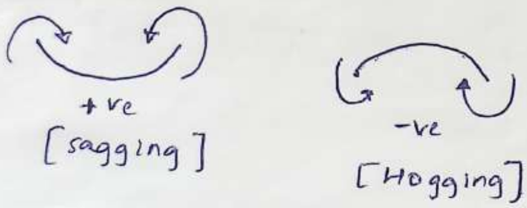
Moments:



(+ve).

(-ve).

Bending moments.



Moment Area method.

Theorem 1. [slope]: The change in the slope b/w two points on a straight member under flexure is equal to area of bending moment diagram b/w these 2 points.

ie. $\theta = \dfrac{\text{Area of Bending moment diagram b/n 2 points}}{EI}$.

$$\boxed{\theta = \frac{\text{Area of BMD.}}{EI}} \qquad \boxed{\theta = \frac{\text{Area of M}}{EI}}$$

EI: Flexural bending resistance against bending.

Theorem 2: [Deflection]:

Deflection at a point in a beam in the direction perpendicular to its original straight line measured from tangent to the elastic curve at another point is given by moment of area of BMD about the point where deflection is required.

$$\delta = \frac{\text{moment of Area of BMD}}{EI}$$

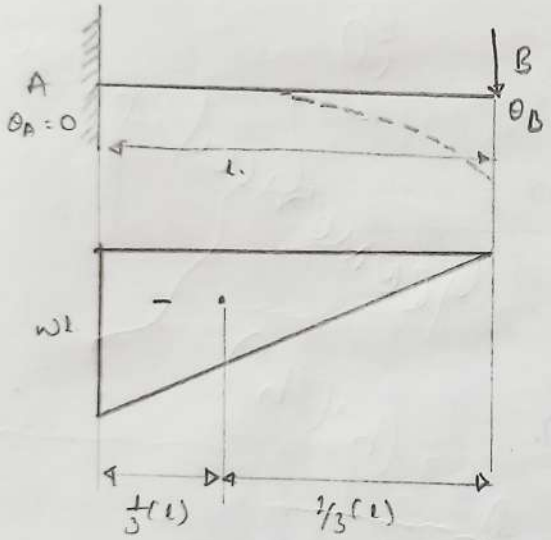
$$\boxed{\Delta = \frac{\text{Area of BMD} \cdot \bar{x}}{EI}}$$ [b/n centroided and point where defn is req]

where $\bar{x}$ = centroidal distance to the point where deflection is required.

Note: If one support is fixed slope θ equals 0 and deflection Δ equals to 0.

Maximum deflection is at loading point.

1. Determine the slope and deflection of cantilever beam of length (L) with a point load w at free end.



$\theta_A = 0$.

$$\theta_B = \frac{\text{Area of BMD}}{EI} \text{ in b/n B and A}$$

$$= \frac{\frac{1}{2}(l)(-wl)}{EI}$$

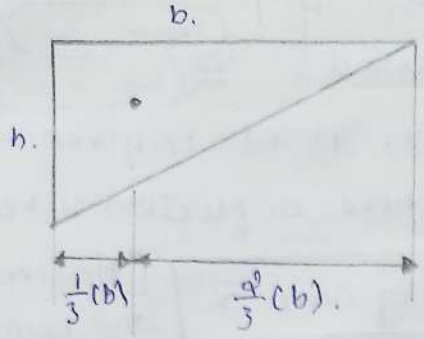
$$\boxed{\theta_B = \frac{-wl^2}{2EI}}$$

$$\theta_B = \frac{\text{Area of BMD}}{EI} \cdot \bar{x} \quad \text{(from centroid to B)}$$

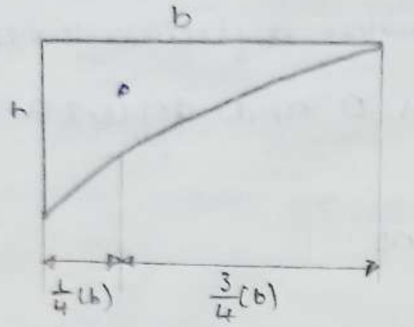
$$\delta_{max} = \frac{-wl^2}{\cancel{2}EI}\left(\frac{\cancel{2}}{3}l\right).$$

$$\boxed{\delta_{max} = \frac{-wl^3}{3EI}}$$

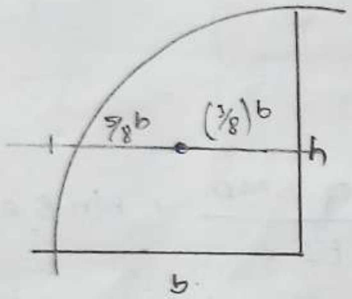
Geometrical properties:



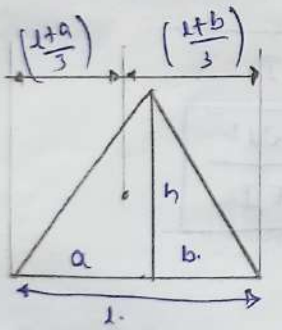
$$\text{Area} = \tfrac{1}{2}(b)(h)$$



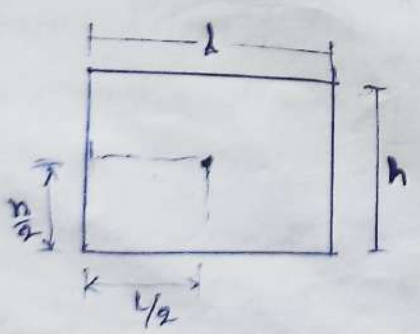
$$A = \tfrac{1}{3} b(h).$$



$$A = \frac{2}{3}(b)(h).$$

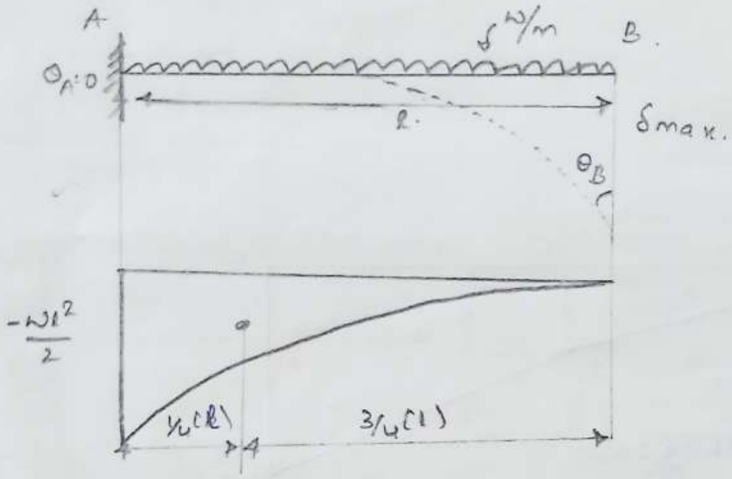


$$A = \frac{1}{2}(l) \times (h).$$



$$A = l \times h$$

Find the slope and deflection of cantilever beam with length(l), with UDL w/unit length acting throughout the length. Find max slope and deflection.



$\theta_{max} = \theta_B$.

$\delta_{max} = \delta_B$.

max slope $= \theta_{max} = \theta_B$.

$\theta_B = \frac{\text{Area of BMD}}{EI}$

[In b/n A & B]

$= \theta_B = \frac{1}{3}(l)(wl^2/2) = \frac{-wl^3}{6EI}$ $\boxed{\theta_B = \frac{-wl^3}{6EI}}$.

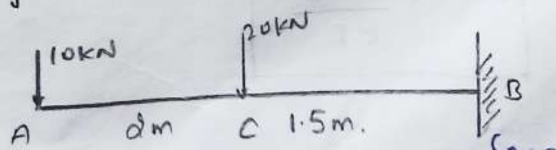
max deflection (δ_{max}). [$\delta_{max} = \delta_B$].

$\delta_B = \frac{\text{Area of BMD}}{EI} \cdot \bar{x}$ [From centroid to point B].

$= \left[\frac{-wl^3}{6EI}\right] \cdot \left(\frac{3}{4}l\right)$.

$\boxed{\delta_B = \frac{-wl^4}{8EI}}$.

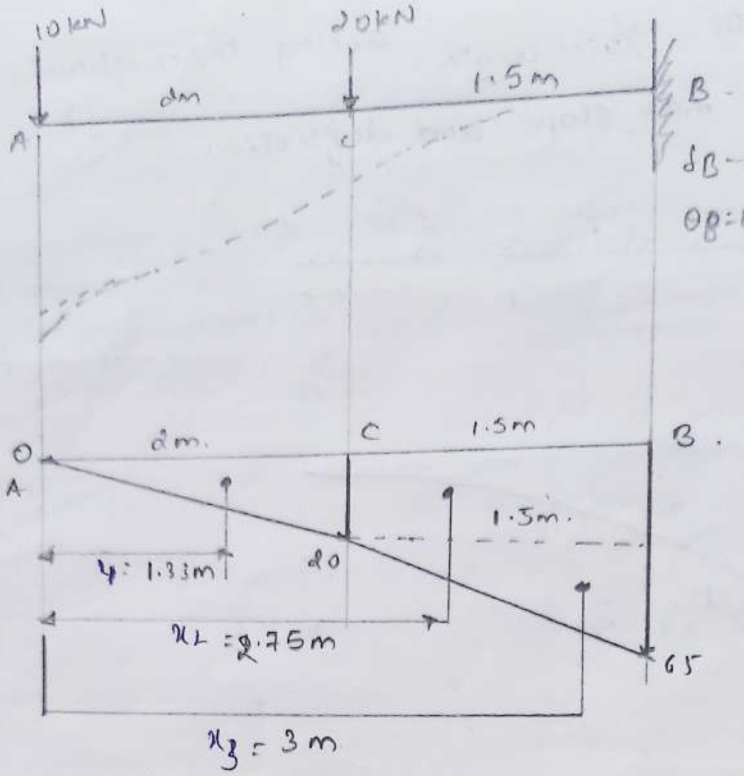
Calculate the slope and deflection at free end using moment area method for the figure shown.



$\delta_B = 0$

$\theta_B = 0$.

$\delta_B = 0$ $\theta_B = 0$



$(BM)_B = -(20 \times 1.5) + (-10 \times 3.5)$

$= -65 kN.$

$BM)_C = -10 \times 2. = -20 kN.$

$BM)_A = 0.$ BM at free end 0.

$x_1 = 2/3(2) = 1.33 m.$

$x_2 = \frac{1.5}{2} + 2 = 2.75m.$

$x_3 = \frac{2}{3}(1.5) + 2 = 3m$

<u>max slope (θ_A)</u>

$$\theta_A = \frac{\text{Area of BMD.}}{EI.} \quad [\text{In b/n } A \& B].$$

$$= \frac{\left[\frac{1}{2}[2 \times 20] + [1.5 \times 20] + \left[\frac{1}{2} \times 1.5 \times 45\right]\right]}{EI.} = 83.75$$

$$\boxed{\theta_A = \frac{-(83.75)}{EI}}$$

Not req to this plm. $\theta_C = \frac{\text{Area of BMD}}{EI}$ (In b/n A & C). $= \frac{-[\frac{1}{2}(2 \times 20)] + ((1.5 \times 20)(\frac{1}{2} \times 1.5 \times 45)}{EI.}$

$= \theta_C =$ ~~(Ag~~

$$\boxed{\theta_C: \frac{-63.75}{EI}}$$

maximum deflection (δ_A).

$$\delta_A = \frac{\text{Area of BMD}}{EI}(\bar{x}). \quad \text{(Towards A)}.$$

$$= -\left(\frac{183.75}{EI}\right.$$

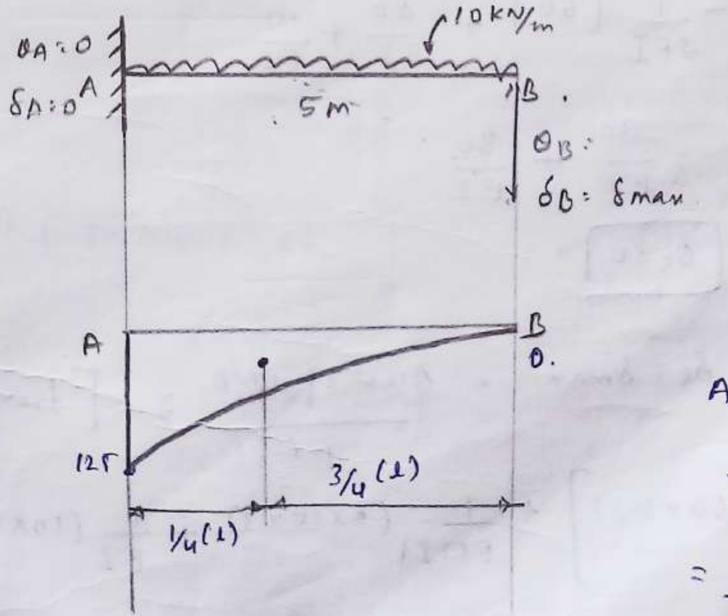
$$= \frac{-1}{EI}\left[\left(\tfrac{1}{2}\times 2\times 20\right)\overset{x_1}{(1.33)} + (1.5\times 20)\overset{x_2}{(2.75)} + \left(\tfrac{1}{2}\times 1.5\times 45\right)\overset{x_3}{(3)}\right]$$

$$\boxed{\delta_A = -\frac{210.35}{EI}.}$$

$$\delta_C = \frac{-1}{EI}\left[(1.5\times 20)(1.5/2) + \tfrac{1}{2}(1.5)(45)\left(\tfrac{2}{3}(1.5)\right)\right]$$

$$\boxed{\delta_C = -\frac{56.25}{EI}.}$$

Find max. slope and deflection for the fig shown using moment area method.



$\frac{wl^4}{8EI}$. $\frac{wl^3}{6EI}$.

$\delta_A = 10\times 5\times 5/2$
$= 125.$

Area of BMD =
$\frac{1}{3}(b)(h)$.

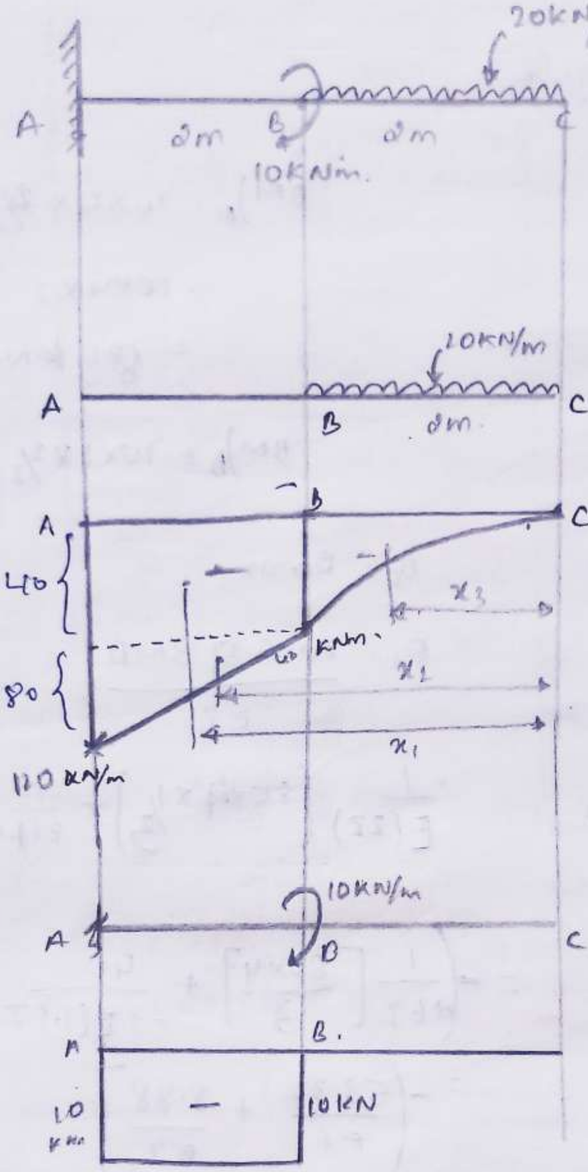
$= \frac{1}{3}(125)(5)$.

$-$ Area $= 208.33$

$$\theta_B = \frac{\text{Area of BMD}}{EI} = \frac{-208.33}{EI}. \quad \text{[In b/n A \& B]}.$$

$$\theta_B = \delta_{max} = \delta_B = \left(\frac{\text{Area of BMD}}{EI}\right)(\bar{x}) = \frac{-208.33}{EI}\left[\tfrac{3}{4}(5)\right] = \frac{-781.23}{EI}$$

Calculate slope and deflection for the figure shown. Consider constant EI.



$BM_B = 20 \times 2 \times 2/2 = 40$ KNm

$BM_B = 20 \times 2 \left(\frac{2}{2} + 2\right)$

$= 120$ KNm.

$x_1 = \left(\frac{2}{2} + 2\right) = 3m.$

$x_2 = \left(\frac{2}{3} \times 2\right) + 2 = 3.33m$

$x_3 = \frac{3}{4}(2) = 1.5m.$

$x_4 = \frac{2}{2} + 2 = 3m.$

Slope at c . $\theta_c = \theta_{max}$.

$\theta_c = \frac{\text{Area of BMD}}{EI}$ [In b/w A & c].

$= \frac{-1}{EI}\left[(2 \times 40) + \frac{1}{2}(80 \times 2) + \frac{1}{3}(40 \times 2) + (10 \times 2)\right].$

$$\boxed{\theta_c = \frac{-206.66}{EI}}$$

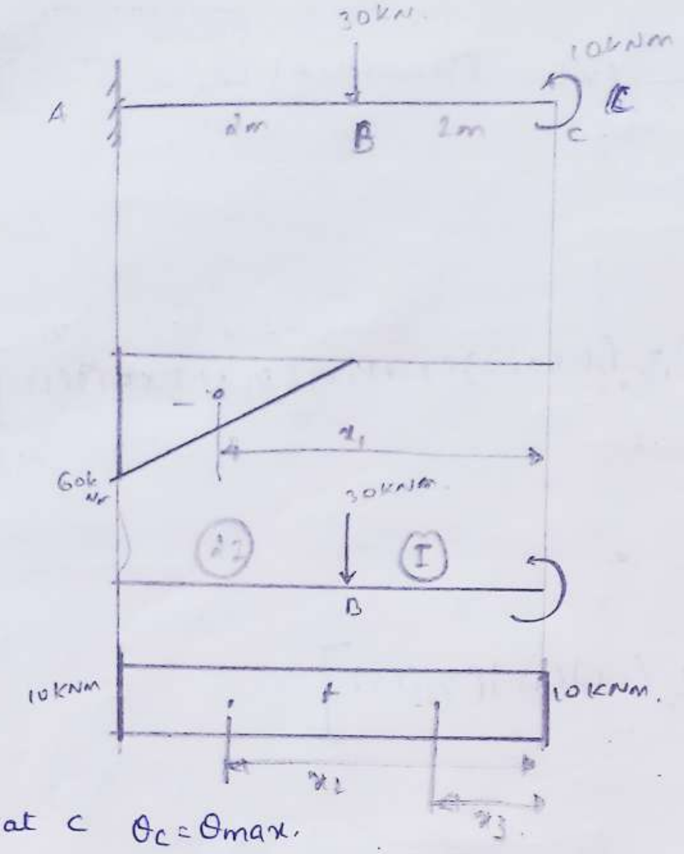
Deflection at c.

$\delta_c = \delta_{max}$. [from

$= -\frac{1}{EI}\left[(40 \times 2 \times 3) + (80 \times 3.33) + \left(\frac{40 \times 2 \times 1.5}{3}\right) + (20 \times 3)\right]$

$$= \boxed{\delta_c = \frac{-606.4}{EI}}$$

Determine the slope and deflection for the figure shown below.



$BM)_A = -10 + 30(2)$.

$BM)_A = 50$ kN.

$BM)_B = -10$ kNm.

$BM)_A = 0$.

$x_1 = \frac{2}{3}(2) + 2 = 3.33$ m.

$x_2 = \frac{2}{2} + 2 = 3$m.

$x_3 = 2/2 = 1$m.

Slope at C $\theta_C = \theta_{max}$.

$= \frac{\text{Area of BMD}}{EI}$ (In. b/n A&C)

$= \cancel{\text{rex}} \; -\frac{1}{E(2I)}\left[\frac{1}{2}(2)(6)\right] \cancel{*} \frac{1}{EI}[10 \times 2] + \frac{1}{E(2I)}[10 \times 2]$.

$= -\frac{1}{2EI}[60] + \frac{20}{EI} + \frac{10}{EI}$.

$= \frac{-30}{EI} + \frac{30}{EI}$.

$\boxed{\theta_C = 0}$

Deflection at C.

$\delta_C = \delta_{max} = \frac{\text{Area of BMD}}{EI} \cdot \bar{x}$ [from centroid to C].

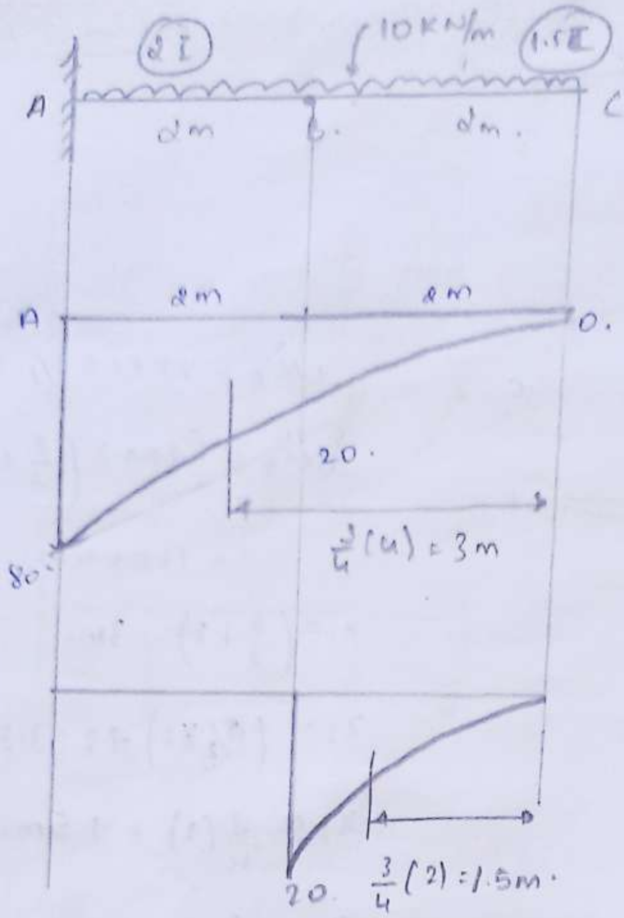
$= -\frac{1}{E(2I)}\left[\frac{1}{2} \times 2 \times 60 \times 3.33\right] + \frac{1}{E(2I)}(2 \times 10 \times 3) + \frac{1}{EI}(10 \times 2 \times 1)$

$= \frac{-199.8}{2EI} + \frac{30}{EI} + \frac{20}{EI}$.

$= \frac{-99.9}{EI} + \frac{50}{EI}$.

$\boxed{\delta_C = \frac{-49.9}{EI}}$

Determine the slope and deflection of figure shown.



$BM)_A = 10\times 4\times \frac{4}{2}$

$= 10\times 4\times 2$

$= 80$ kNm.

$BM)_B = 10\times 2\times \frac{2}{2} = 20$ kNm.

$\theta_C = \theta_{max}$.

$\theta_C = \frac{\text{Area of BMD}}{EI}$

$= \frac{1}{E(2I)}\left(80\times 4\times \frac{1}{3}\right) + \frac{1}{E(1.5I)}\left[\frac{1}{3}\times 20(2)\right]$

$= -\left(\frac{1}{2EI}\left[\frac{80\times 4}{3}\right] + \frac{40}{3E(1.5I)}\right)$

$= -\left(\frac{53.33}{EI} + \frac{8.88}{EI}\right)$

$\boxed{\theta_C = \frac{-62.21}{EI}}$

$\delta_C = \delta_{max}$.

$= -\frac{1}{2EI}\left(\frac{80\times 4}{\cancel{3}}\times \cancel{3}\right) - \frac{1}{E(1.5I)}\left(\frac{20\times 2}{\cancel{3}}\times \frac{\cancel{3}}{4}(2)\right)$

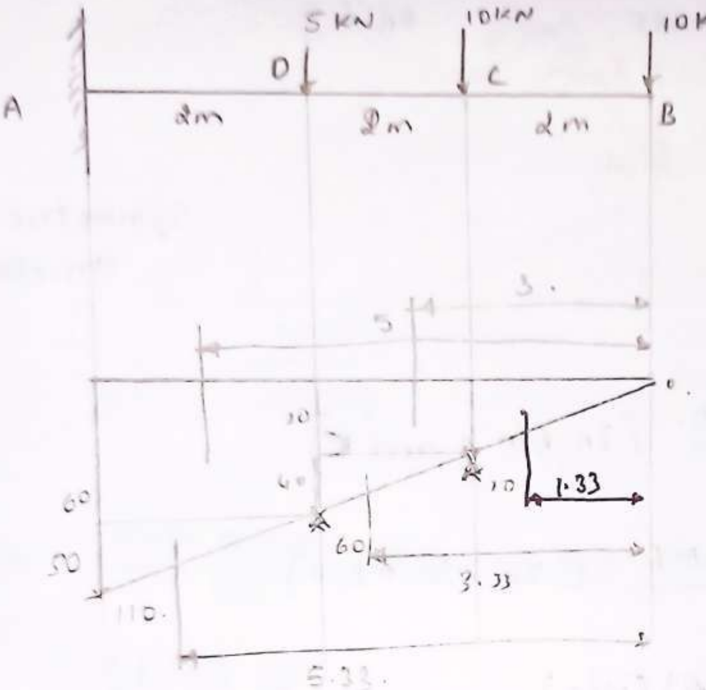
$= \frac{-160}{EI} - \frac{13.33}{EI} = -\frac{173.33}{EI}$

$\boxed{\delta_{max} = \frac{-173.33}{EI}}$

Determine the slope and deflection at point B of cantilever beam shown in fig by using moment Area method.

$E = 2\times10^5\ N/mm^2$ and $I = 2\times10^8\ mm^4$.

$$I = \frac{2\times10^8}{(10^3)^4}\ m^4 = 2\times10^{-4}\ m^4$$



$$E = \frac{2\times10^2\times10^3\ N}{(10^{-3})^2}$$

$$E = \frac{2\times10^2}{10^{-6}}\ kN$$

$$\boxed{E = 2\times10^{+8}\ kN/m^2}$$

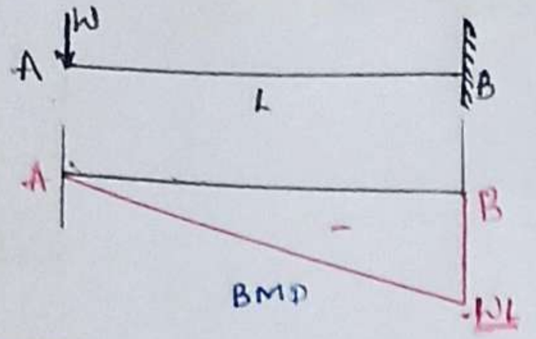
$$BM_A = (5\times2)+(10\times4)+(10\times6) = -110\ kN/m.$$

$$\theta_D = -\frac{1}{EI}\left[(60\times2)+\left(\frac{1}{2}\times50\times2\right)+(20\times2)+\left(\frac{1}{2}\times40\times2\right)+\left(\frac{1}{2}\times20\times2\right)\right]$$

$$= \frac{-270}{EI} = -6.75\times10^{-3}\ rad$$

$$\delta_{max} = \delta_B = \frac{-1}{EI}\left[(60\times2\times5)+(50\times5.33)+(40\times3)+(40\times3.33)+(20\times1.33)\right]$$

$$= \boxed{\delta_B = -0.0286\ m}$$

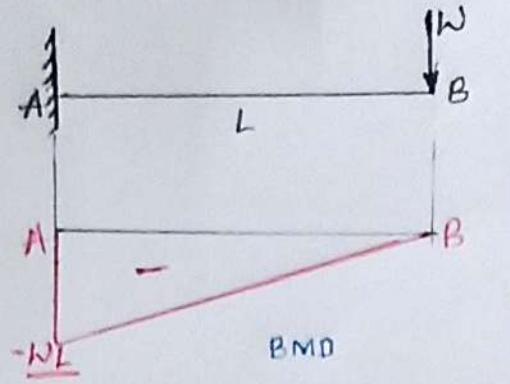


$BM_A = 0$

$\because$ free end

$BM_B = -W \times L$

$= -WL$

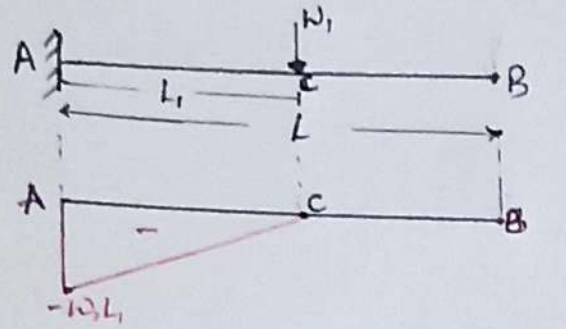


$BM_A = -W \times L$

$BM_A = -WL$

$BM_B = 0$

$\because$ Free end.

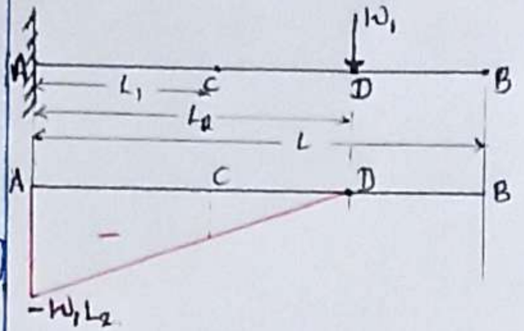


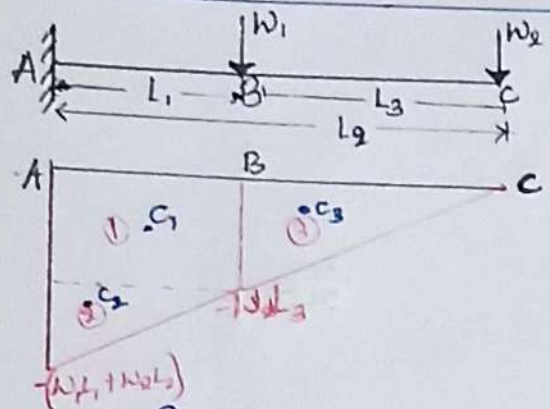
$BM_A = -W_1 \times L_1$

$BM_A = -W_1 L_1$

$BM_C = W_1 \times 0 = 0$

$BM_B = 0$ ($\because$ FE)





$BM_C = 0$

$BM_B = -W_2 L_3$

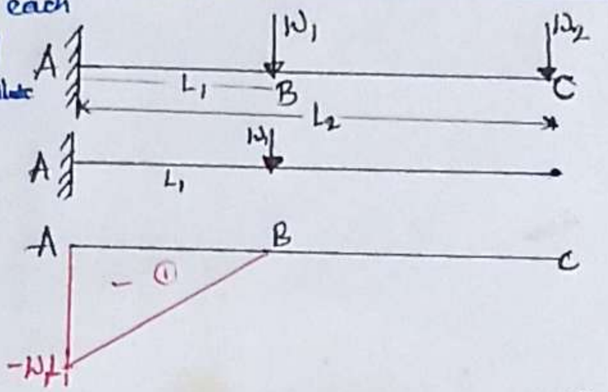
$BM_A = -W_1 L_1 - W_2 L_2$

$= -[W_1 L_1 + W_2 L_2]$

OR

Considering each load separately to calculate BM.

OR



→ Considering only 1st load

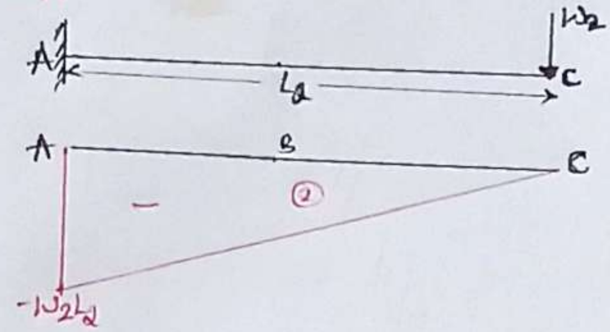
$BM_A = -W_1 L_1$

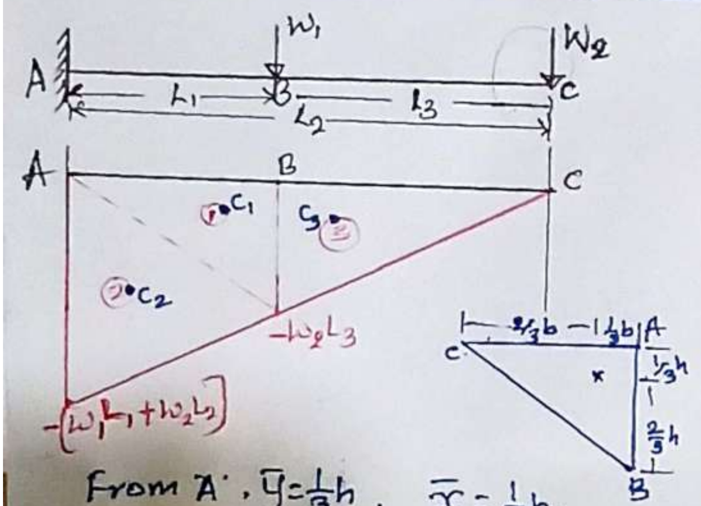
$BM_B = 0$

→ Considering only 2nd Load

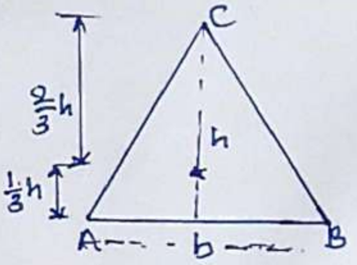
$BM_A = -W_2 L_2$

$BM_C = 0$





From 'A', $\bar{y} = \frac{1}{3}h$, $\bar{x} = \frac{1}{3}b$.



$A = \frac{1}{2}bh$

$\bar{x} = \frac{b}{2}$

$\bar{y} = \frac{1}{3}h$ from base 'AB'

$\bar{y} = \frac{2}{3}h$ from crown 'C'

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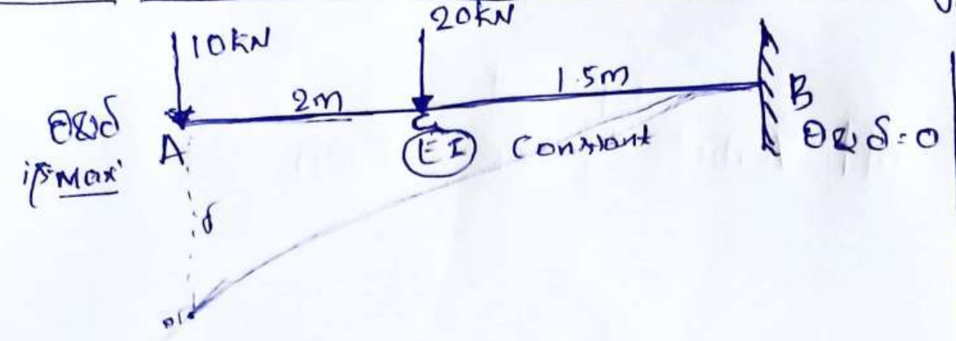
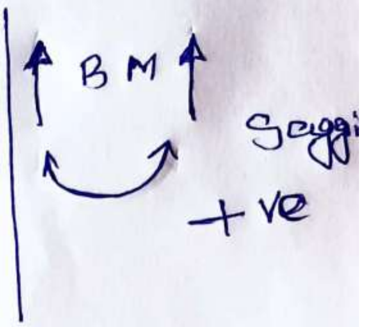
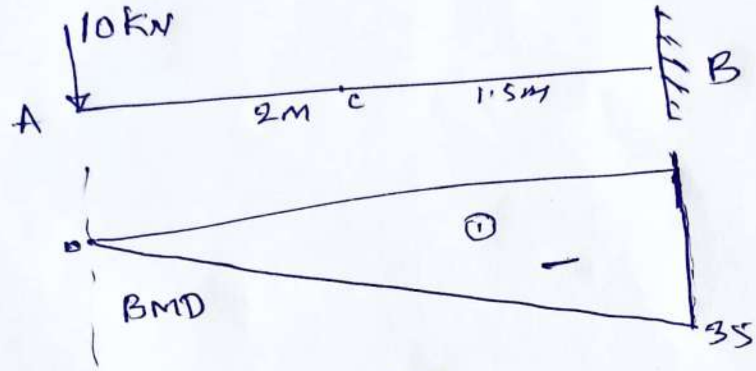
Determine Max Slope & Deflection for given figure.





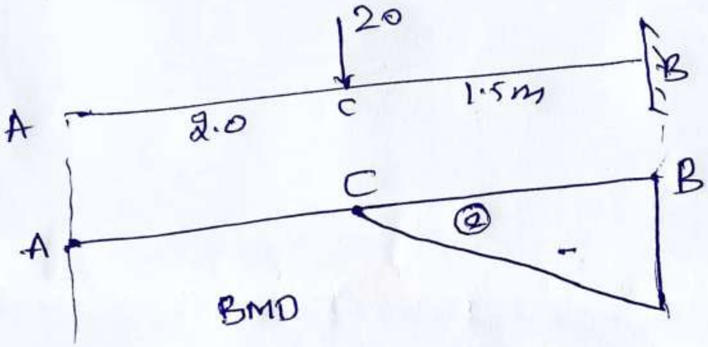


$BM_A = 0$

$BM_B = -10 \times 3.5$

$= -35\ kN\text{-}m$

Don't take @ 'C'



$BM_C = 0$

$BM_B = 20 \times 1.5$

$BM_B = 30\ kN\text{-}m$

Don't take at 'A'

∴ δ & θ should calculate by using ① BMD ②

$$\theta_{max} = \theta_A = \frac{\text{Area of BMD}}{EI} = \frac{-\frac{1}{2} \times 3.5 \times 35}{EI} - \frac{\frac{1}{2} \times 1.5 \times 30}{EI}$$

$$\theta_A = \frac{-61.25}{EI} - \frac{22.50}{EI} = \frac{-83.75}{EI} = \theta_{max}$$

$$\delta_{max} = \delta_A = \frac{\text{Area of BMD}}{EI} \times \bar{x}$$

$$\delta_A = \frac{-1}{EI}\left[\frac{1}{2} \times 3.5 \times 35 \times \left(\frac{2}{3} \times 3.5\right)\right] - \frac{1}{EI}\left[\frac{1}{2} \times 1.5 \times 30 \times \left(\frac{2}{3} \times 1.5 + 2\right)\right]$$

$$= -\frac{1}{EI}\left[142.71 + 67.5\right]$$

$$\delta_A = \frac{-210.21}{EI} = \delta_{max}$$

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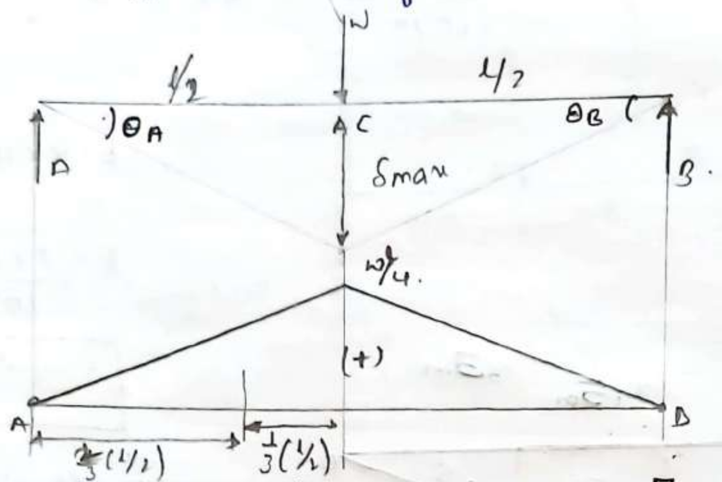
Simply Supported Beam

Q. Calculate slope at support and deflection at mid-span. for the figure shown.
Calculate for a SSB for span L with a point load acting at centre. Find slope at supports and deflection at loading point.



Symmetric structure

slope at A = $\frac{\text{Area of BMD.}}{EI.}$ [In b/n A and B].

$$\theta_A = \theta_B = \frac{\text{Area of BMD}}{EI} \quad [\text{In b/n A \& C}]$$

$$\theta_A = \theta_B = \frac{\frac{1}{2}\left(l/2\right)\left(wl/4\right)}{EI.}$$

$$\boxed{\theta_A = \theta_B = \frac{wl^2}{16EI}}$$

Note:

Deflection δ is considered as $\frac{\text{Area of BMD.}}{EI} \cdot \bar{x}$.

where.

$\bar{x}$ = is considered from centroid to the support point.

$\delta_C = \delta_{max} = \frac{\text{Area of BMD}}{EI} \cdot \bar{x}$ (from centroid to support).

$$= \frac{\frac{1}{2}\left(l/2\right)\left(\frac{wl}{4}\right)}{EI}\left(\tfrac{2}{3}\, l/2\right). = \frac{wl^3}{48EI}$$

$$\boxed{\delta = \frac{wl^3}{48EI}}$$

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A simply supported beam of span [l] loaded with UDL throughout. Determine the slope at support and deflection at mid-span.

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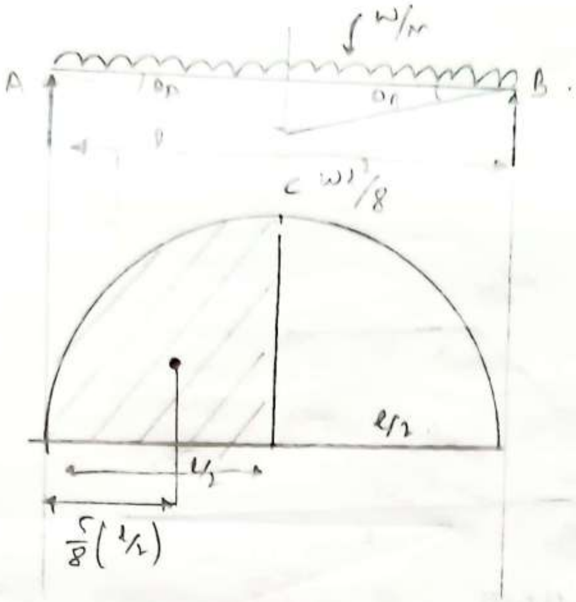


$$\theta_A = \theta_B = \frac{\text{Area of BMD}}{EI} \text{ (b/n A \& C) (or) (b/n B and C).}$$

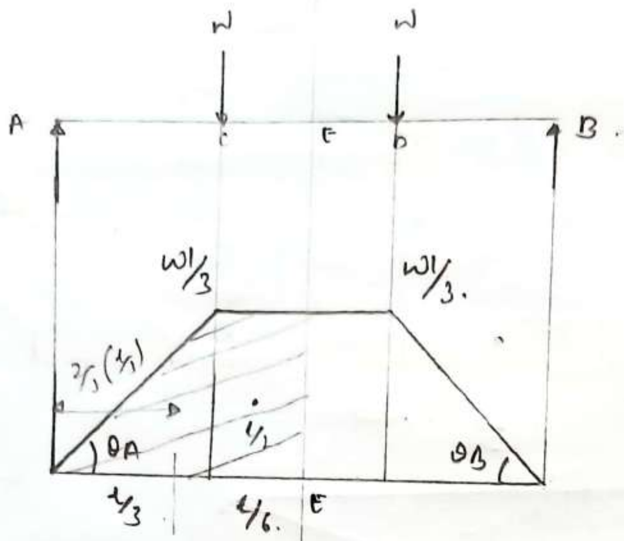
$$= \frac{\frac{2}{3}\left(l/2\right)\frac{wl^2}{8}}{EI}$$

$$\boxed{\theta_A = \theta_B = \frac{wl^3}{24EI}}$$

$$\delta_c = \delta_{max} = \left[\left(\frac{2}{3} \times \frac{l}{2} \times \frac{wl^2}{8}\right)\left(\frac{5}{8} \times \frac{l}{2}\right)\right]\frac{1}{EI} = \frac{\text{Area of BMD}}{EI} \cdot \bar{x}$$

$$\boxed{\delta_{max} = \frac{5wl^4}{384EI}}$$

A simply supported beam subjected to a point loads at 1/3 points. Calculate maximum slope and deflection.



$\frac{l}{2} - \frac{l}{3} = \frac{3l-2l}{6}$

$= \frac{l}{6}$

$x_1 = \frac{2}{3}\left(\frac{l}{3}\right) = \frac{2l}{9}$

$x_2 = \left(\frac{l}{6}\right)\frac{1}{2} + \frac{l}{3}$

$= \frac{l}{12} + \frac{l}{3}$

$= \frac{3l+12l}{36}$

$= \frac{15l}{36}$

$= \frac{5l}{12}$

$\theta_A = \theta_B = \frac{\text{Area of BMD}}{EI}$ [In b/n A & E].

$= \frac{\left[\frac{1}{2} \times \frac{l}{3} \times \frac{wl}{3}\right] + \left[\frac{l}{6} \times \frac{wl}{3}\right]}{EI}$.

$= \frac{\left(\frac{wl^2}{18} + \frac{wl^2}{18}\right)}{EI} = \frac{wl^2}{9EI}$.

$$\boxed{\theta_A = \theta_B = \frac{wl^2}{9EI}}$$

$\delta_c = \delta_{max} = \frac{\text{Area of BMD}}{EI}(\bar{x})$.

$= \frac{\left[\frac{1}{2} \times \frac{l}{3} \times \frac{wl}{3} \times \frac{2l}{9}\right] + \left[\frac{l}{6} \times \frac{wl}{3} \times \frac{5l}{12}\right]}{EI}$.

$$\boxed{\delta_{max} = \frac{23wl^3}{648EI}}$$

$\frac{\left(\frac{wl^3}{81} + \frac{5wl^3}{216}\right)}{EI}$. $= \frac{216wl^3 + 405wl^3}{17496EI}$.

$= \frac{621wl^3}{17496EI} = \frac{207}{5832}\frac{wl^3}{EI}$

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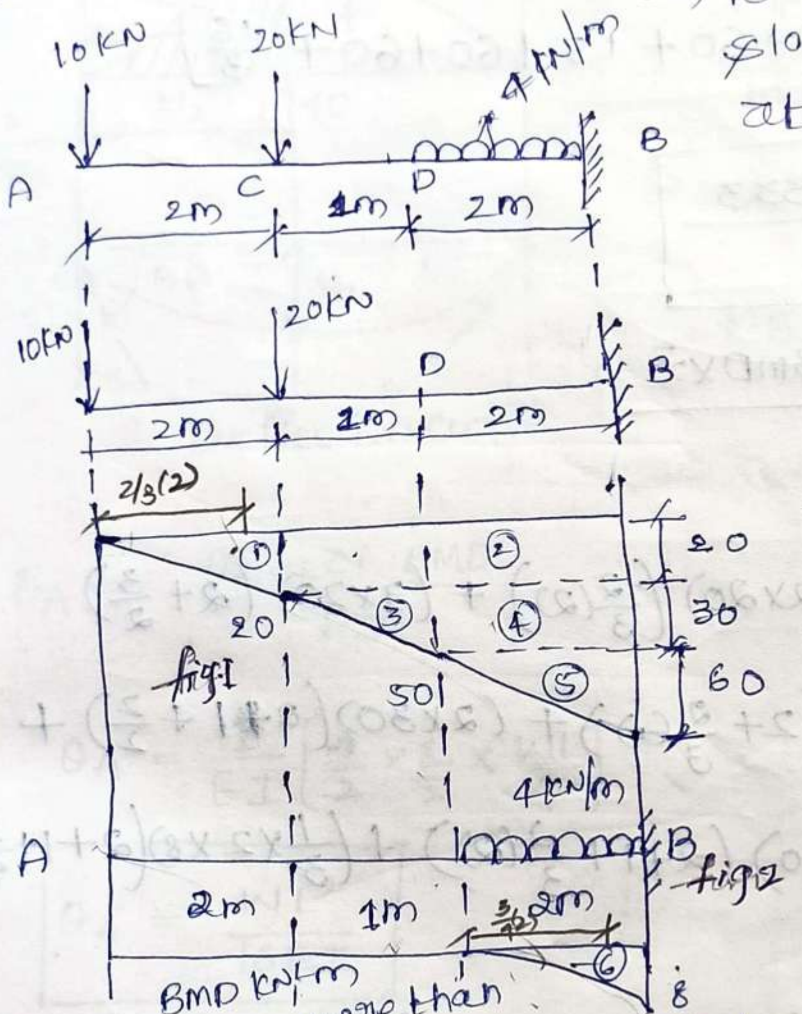
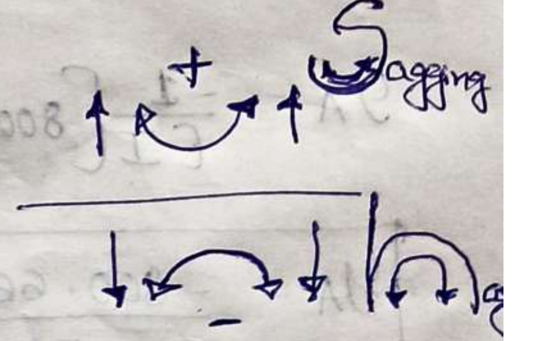
3/5/18

(4) → To determine the slope and Deflection at free End



Note :- whenever more than one type of loading is given seperate the diagrams

To draw BMD :-



for fig 1

$M_A = 0$

$M_C = -10 \times 2 = -20$ KN-m

$M_D = -10 \times 3 - 20 \times 1 = -50$ KN-m

$M_B = -10 \times 5 - 20 \times 3 = -110$ KN-m

for fig 2

$M_A = 0$

$M_C = 0$

$M_D = 0$

$M_B = -(4 \times 2)\left(\frac{2}{2}\right) = -8$ KN-m

$$\theta_A = \frac{\text{Area of BMD}}{EI}$$

$$\theta_A = \frac{-1}{EI}\left[\left(\frac{1}{\cancel{2}}\times\cancel{2}\times20\right)+(3\times20)+\left(\frac{1}{2}\times1\times30\right)+(2\times30)+\left(\frac{1}{\cancel{2}}\times\cancel{2}\times60\right)+\left(\frac{1}{3}\times2\times8\right)\right]$$

$$\theta_A = \frac{-1}{EI}\left[20+60+15+60+60+\frac{16}{3}\right]$$

$$\boxed{\theta_A = \frac{-220.3333}{EI}}$$

$$y_A = \frac{\text{Area of BMD}\times\bar{x}}{EI}$$

$$y_A = \frac{-1}{EI}\left[\left(\frac{1}{2}\times2\times20\right)\left(\frac{2}{3}(2)\right)+(3\times20)\left(2+\frac{3}{2}\right)+\left(\frac{1}{2}\times1\times30\right)\left(2+\frac{2}{3}(1)\right)+(2\times30)\left(2+1+\frac{2}{2}\right)+\left(\frac{1}{2}\times2\times60\right)\left(2+1+\frac{2}{3}(2)\right)+\left(\frac{1}{3}\times2\times8\right)\left(2+1+\frac{3}{4}(2)\right)\right]$$

$$y_A = \frac{-1}{EI}\left[20(1.3333)+60(3.5)+15(2.6667)+60(4)+60(4.3333)+\frac{16}{3}(4.5)\right]$$

$$y_A = \frac{-1}{EI}\left[800.6645\right]$$

$$\boxed{y_A = \frac{-800.6645}{EI}}$$

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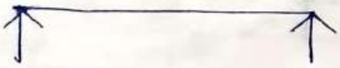
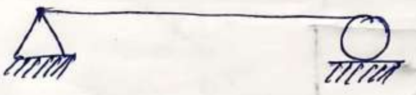
Conjugate Beam Method :-

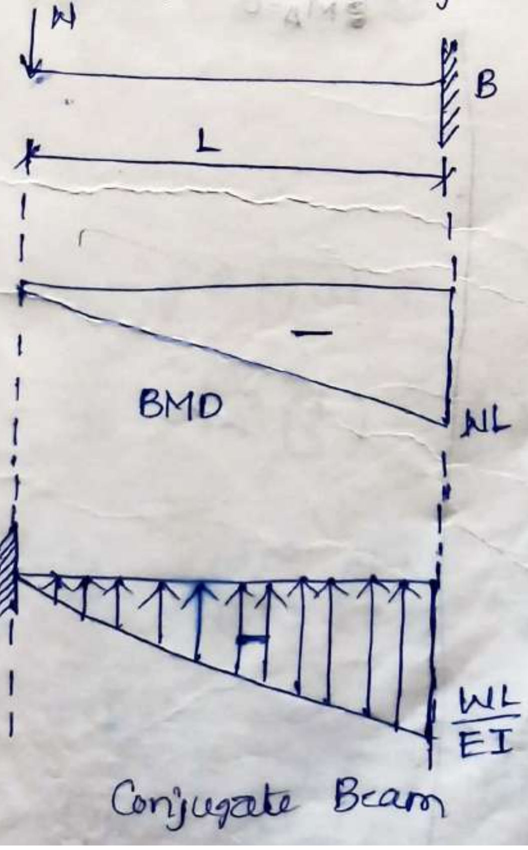
Theorem 1:- The rotation (or) slope at a point is equal to the shear force in the Conjugate Beam

Theorem 2:- The Deflection in a Beam is equal to the bending moment in the Conjugate Beam.

Sl.no	Real Beam	Conjugate Beam
1		
2		
3		
4		

(i) Cantilever beam subjected to point load.

⇒



Conjugate Beam

$M_A = 0$

$M_B = -W \times L = -WL$

$\theta_A = \frac{1}{2} \times L \times \frac{-WL}{EI}$

$$\boxed{\theta_A = \frac{-WL^2}{2EI}}$$

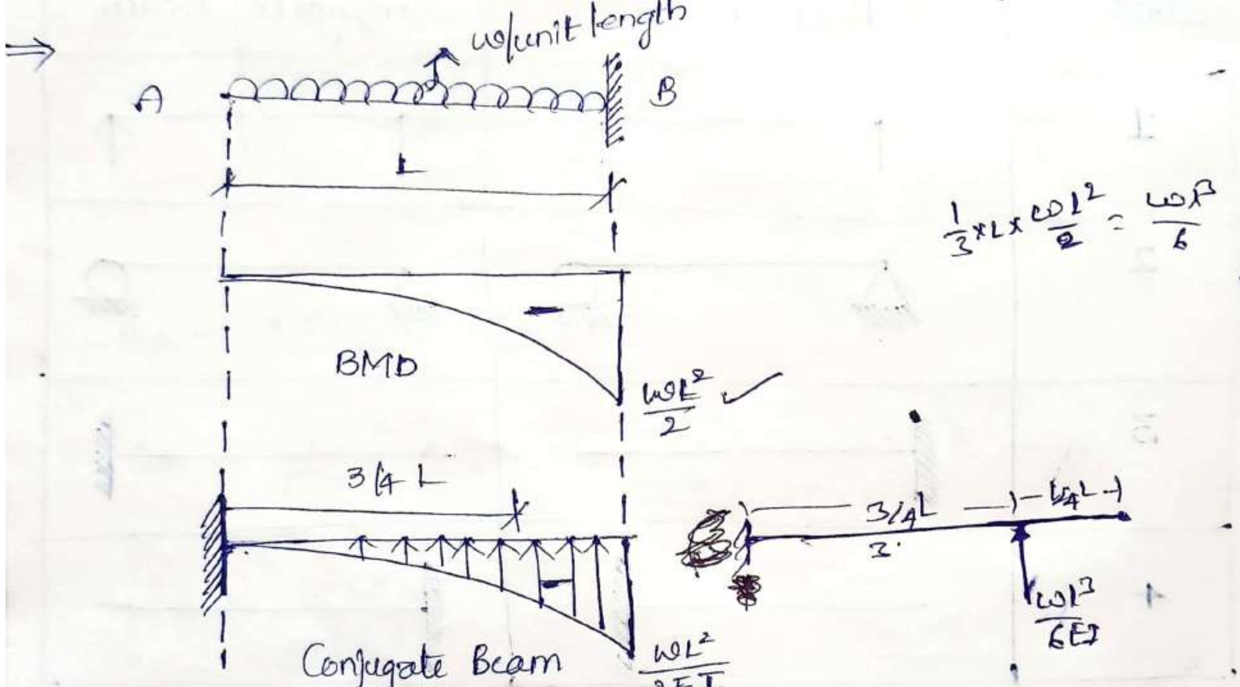
Consider a cantilever beam of span 'L' subjected to point load at free End

$$y_A = \left[\frac{1}{2} \times l \times -\frac{Wl}{EI}\right] \times \frac{2}{3}(L)$$

$$\boxed{y_A = -\frac{Wl^3}{3EI}}$$

(2) Cantilever Beam Subjected to UDL throughout

⇒



$$\frac{1}{3} \times L \times \frac{wl^2}{2} = \frac{wl^3}{6}$$

$$\boxed{\begin{array}{l} M_A = 0 \\ M_B = -w \times L \times \frac{L}{2} = -\frac{wL^2}{2} \end{array}}$$

$M_A = 0$

$$\cancel{\theta_A = \frac{1}{2} \times L \times \frac{-wL}{EI}}$$

$$\boxed{\cancel{\theta_A = -\frac{wl^2}{2EI}}}$$

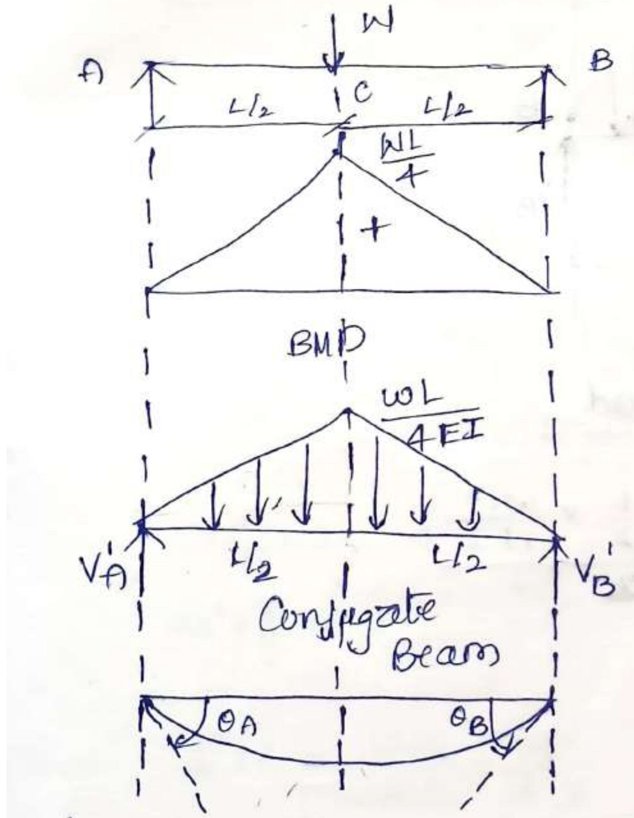
$$\theta_A = \frac{1}{3} \times L \times -\frac{wL^2}{2EI}$$

$$\boxed{\theta_A = -\frac{wL^3}{6EI}}$$

$$y_A = \left[\frac{1}{3} \times L \times -\frac{wL^2}{2EI}\right] \times \frac{3}{4} L$$

$$\boxed{y_A = \frac{-wL^4}{8EI}}$$

(3) Calculate the slope at support and Deflection at centre for a simply supported Beam subjected to point load at centre.



$$V_A' = V_B' = \frac{\text{Total load}}{2}$$

$$V_A' = V_B' = \frac{\frac{1}{2} \times L \times \frac{wL}{4EI}}{2}$$

$$V_A' = V_B' = \frac{wL^2}{16EI}$$

θ_A = Shear force at A

$$\boxed{\theta_A = +V_A' = \frac{wL^2}{16EI}}$$

~~θ_B = Shear force at B~~

y_C = B.M at C

$$y_C = V_A'\left(\frac{L}{2}\right) - \left(\frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI}\right)\left(\frac{1}{3}\left(\frac{L}{2}\right)\right)$$

$$y_C = \frac{wL^2}{16EI}\left(\frac{L}{2}\right) - \frac{wL^3}{96EI}$$

$$y_C = \frac{wL^3}{32EI} - \frac{wL^3}{96EI}$$

$$\boxed{y_C = \frac{wL^3}{48EI}}$$

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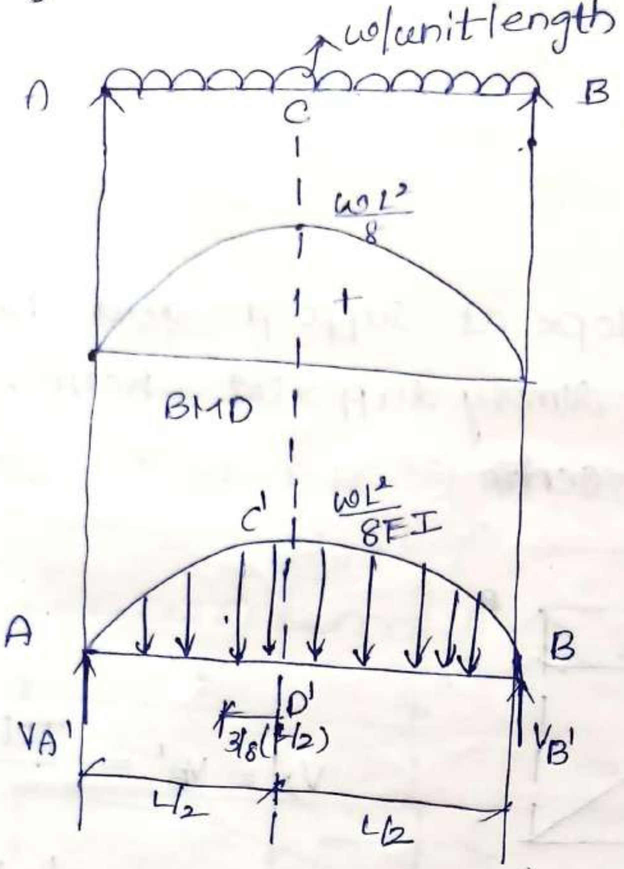
④ Simply Supported Beam Subjected to UDL



$$V_A' = V_B' = \frac{\text{Total load}}{2}$$

$$\Rightarrow V_A' = V_B' = \frac{\cancel{2}\left(\frac{\cancel{2}}{3} \times \frac{L}{\cancel{2}} \times \frac{wL^2}{8EI}\right)}{\cancel{2}}$$

$$V_A' = V_B' = \frac{wL^3}{24EI}$$

θ_A = SF at A

$$\theta_A = +V_A' = \frac{wL^3}{24EI}$$

y_C = BM at C

$$y_C = V_A' \times \frac{L}{2} - \left(\frac{\cancel{2}}{3} \times \frac{L}{\cancel{2}} \times \frac{WL^2}{8EI}\right)\left(\frac{3}{8}\left(\frac{L}{2}\right)\right)$$

$$y_C = \frac{wL^3}{24EI}\left(L/2\right) - \frac{wL^3}{24EI}\left(\frac{3L}{16}\right)$$

$$y_C = \frac{wL^4}{48EI} - \frac{wL^4 3}{384EI} \Rightarrow \boxed{y_C = \frac{5wL^4}{384EI}}$$

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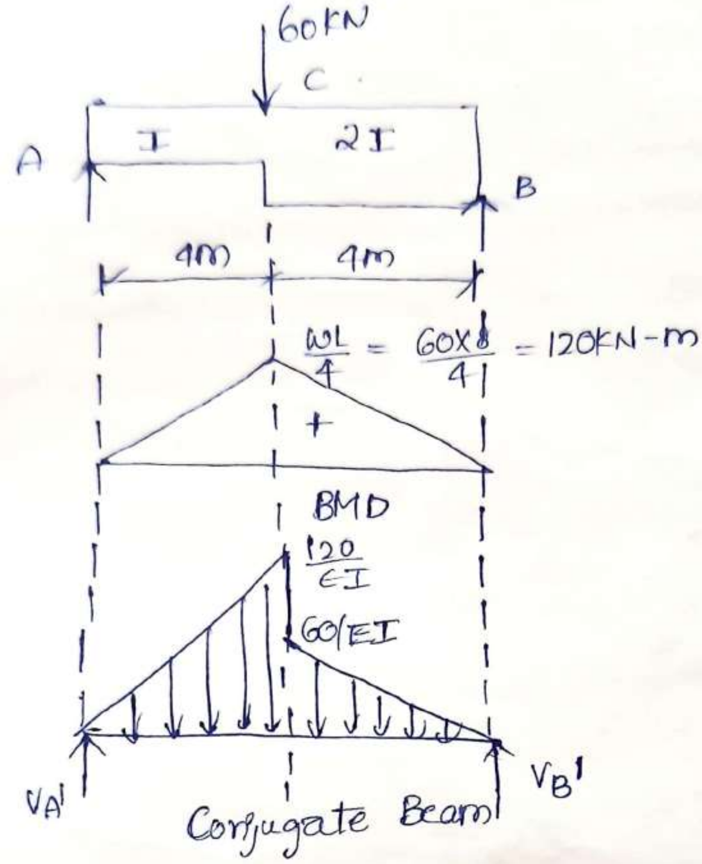
(5) using Conjugate Beam method Find the Deflection at point 'C' and slope at point 'A' for the simply supported beam shown in figure



$\Sigma M_B' = 0$

$$V_A' \times 8 - \left(\frac{1}{2}\times 4\times \frac{120}{EI}\right)\left(4+\frac{1}{3}(4)\right) - \left(\frac{1}{2}\times 4\times\frac{60}{EI}\right)\left(\frac{2(4)}{3}\right) = 0$$

$$V_A' \times 8 = \frac{1280}{EI} + \frac{320}{EI} \qquad \boxed{V_A' = \frac{200}{EI}} = \theta_A$$

$\Sigma V = 0$

$$V_A' - \left(\frac{1}{2}\times 4 \times \frac{120}{EI}\right) - \left(\frac{1}{2}\times 4\times\frac{60}{EI}\right) + V_B' = 0$$

$$\frac{200}{EI} - \frac{240}{EI} - \frac{120}{EI} + V_B' = 0$$

$$\boxed{V_B' = \frac{160}{EI}} = \theta_B$$

$$V_C' = -\frac{40}{EI} = \theta_C$$

$$\delta_C = V_A' \times 4 - \left(\frac{1}{2}\times 4\times\frac{120}{EI}\right)\left(\frac{1}{3}\times 4\right) = \frac{480}{EI}$$

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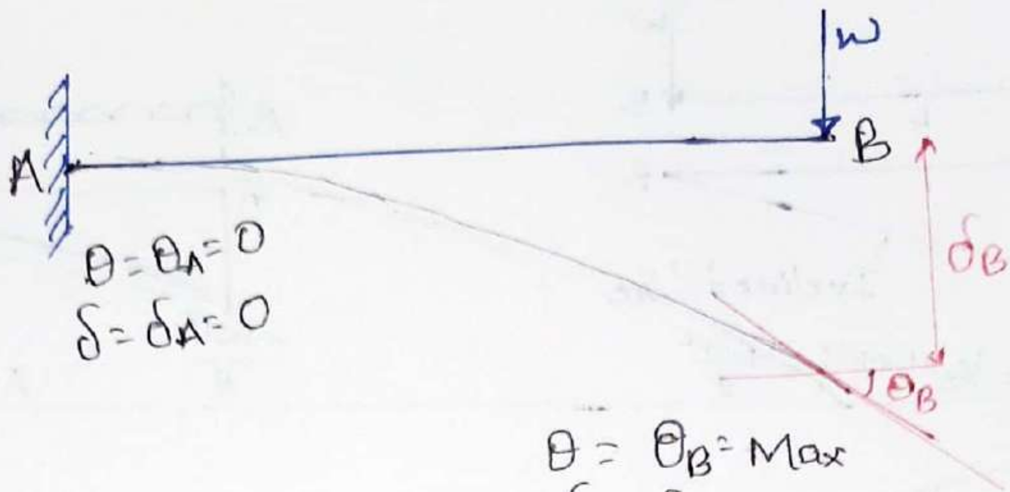

$\theta = \theta_B = \text{Max}$

$\delta = \delta_B = \text{Max.}$

Note: *) BM at freeend is Zero

*) Slope & deflection is Zero at fixed Support.

*) Slope & deflection is Maximum at free ends.

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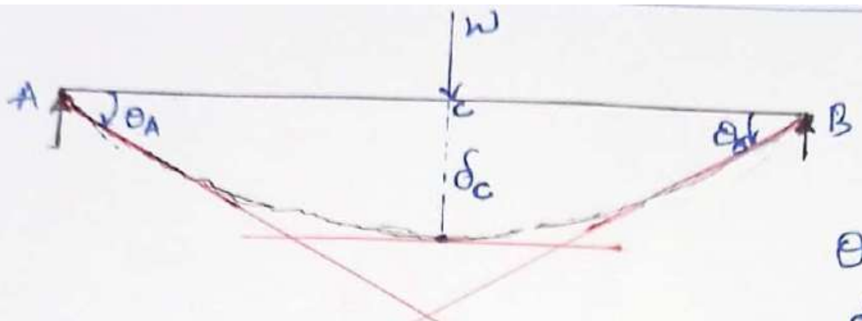

$\theta = \theta_A = \text{Max}$
$\delta = \delta_A = 0$

$\theta = \theta_B = \text{Max}$
$\delta = \delta_B = 0$

$\theta = \theta_C = 0$
$\delta = \delta_C = \text{Max}.$

Note :*) where Max Deflection there zero is slope in simply supported beam.

*) Moment is Zero [ie other than fixed Supports] in Simple, Roller & hinge Supports.

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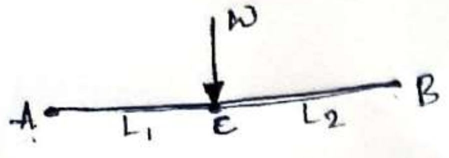


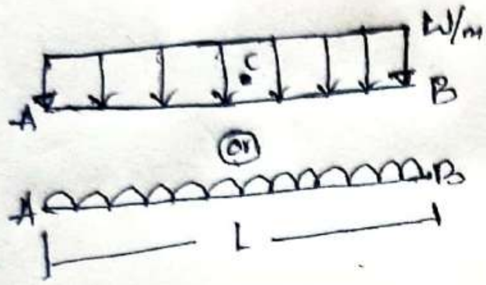
Load $= W =$ Force.

$M_a = WL_1$ = Moment

$M_b = WL_2$

$\bar{x}$ from A $= L_1$

$\bar{x}$ from B $= L_2$



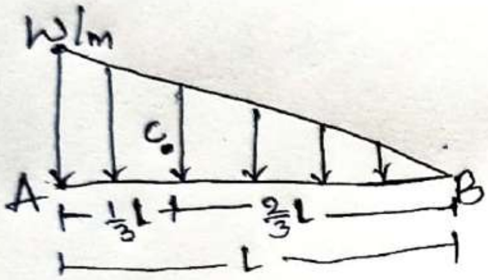
load $= WL$

$M_a = (WL)\frac{L}{2} = \frac{WL^2}{2}$

$M_b = \frac{WL^2}{2}$

$\bar{x}_a = \frac{L}{2}$

$\bar{x}_b = \frac{L}{2}$



Load $= \frac{1}{2} \times L \times W$

Force $= \frac{WL}{2}$

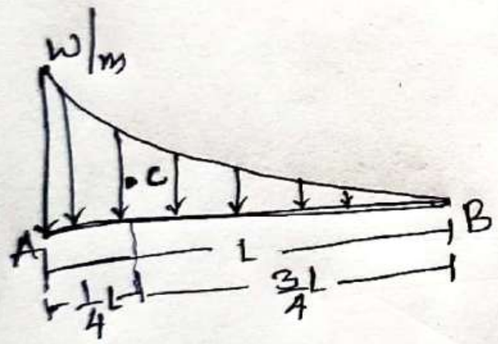
Moment@A $= \frac{WL}{2}\left(\frac{1}{3}L\right)$

$M_a = \frac{WL^2}{6}$

$M_b = \frac{WL}{2}\left(\frac{2}{3}L\right) = \frac{WL^2}{3}$

$\bar{x}_a = \frac{1}{3}L$

$\bar{x}_b = \frac{2}{3}L$



Load $=$ Force $= \frac{1}{3} \times L \times W$

Force $= \frac{WL}{3}$

Moment $=$ Force $\times \perp^{lar}$ distance

$M_a = \frac{WL}{3} \times \left(\frac{1}{4}L\right)$

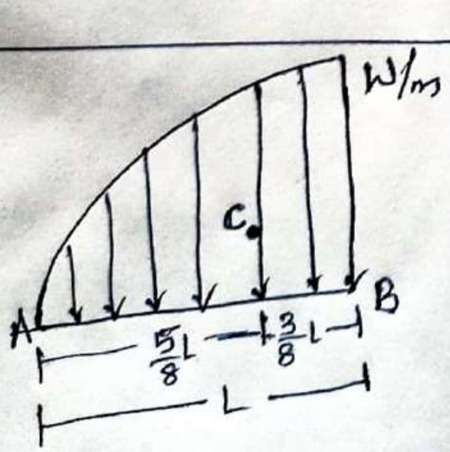
$= \frac{WL^2}{12}$

$M_b = \frac{WL}{3} \times \left(\frac{3}{4}L\right)$

$= \frac{WL^2}{4}$

$\bar{x}_a = \frac{1}{4}L$

$\bar{x}_b = \frac{3}{4}L$



Force $= \frac{2}{3} \times L \times W$

$F = \frac{2WL}{3}$

Moment $= F \times \perp^{lar} d$

$M_a = \frac{2WL}{3} \times \frac{5}{8}L$

$M_a = \frac{10WL^2}{24} = \frac{5WL^2}{12}$

$M_b = \frac{2WL}{3} \times \frac{3}{8}L = \frac{WL^2}{4}$

$\bar{x}_a = \frac{5}{8}L$

$\bar{x}_b = \frac{3}{8}L$

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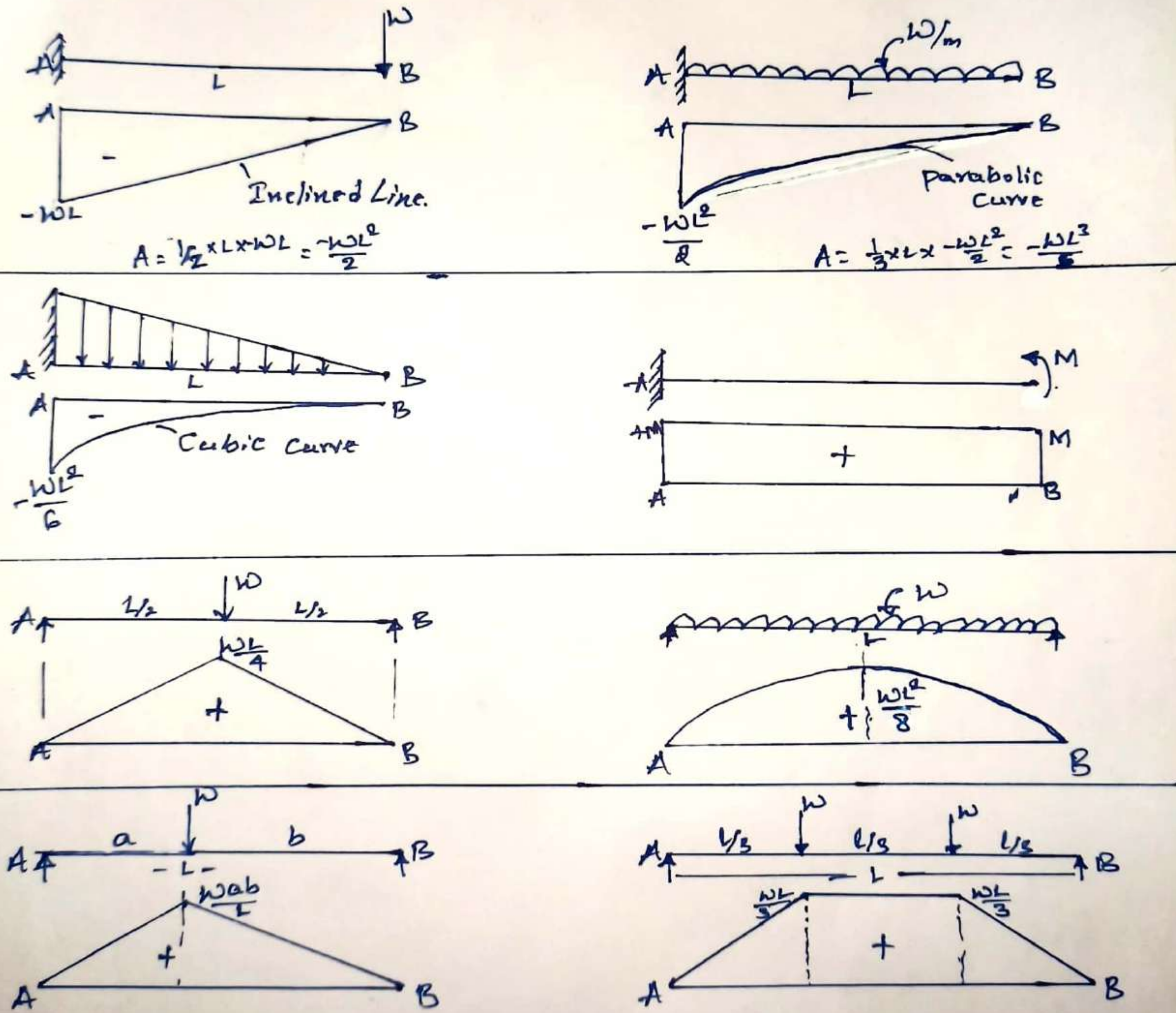

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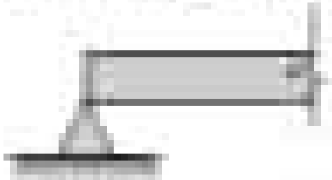
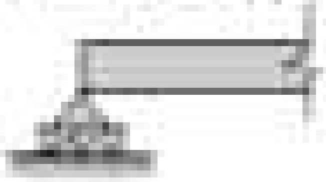
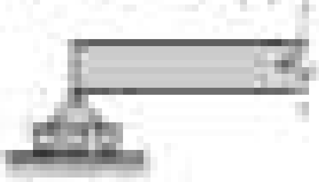
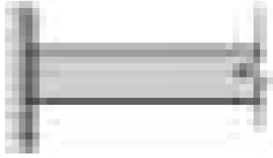
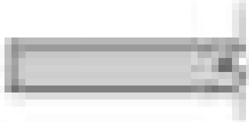
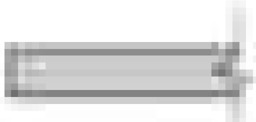
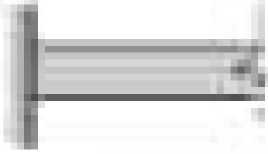
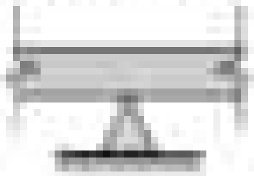
Real Beam Support	Conjugate Beam Support
Hinged Support 	Hinged Support 
Roller Support 	Roller Support 
Fixed Support 	Free End 
Free End 	Fixed Support 
Interior Support 	Internal Hinge 
Internal Hinge 	Interior Support 

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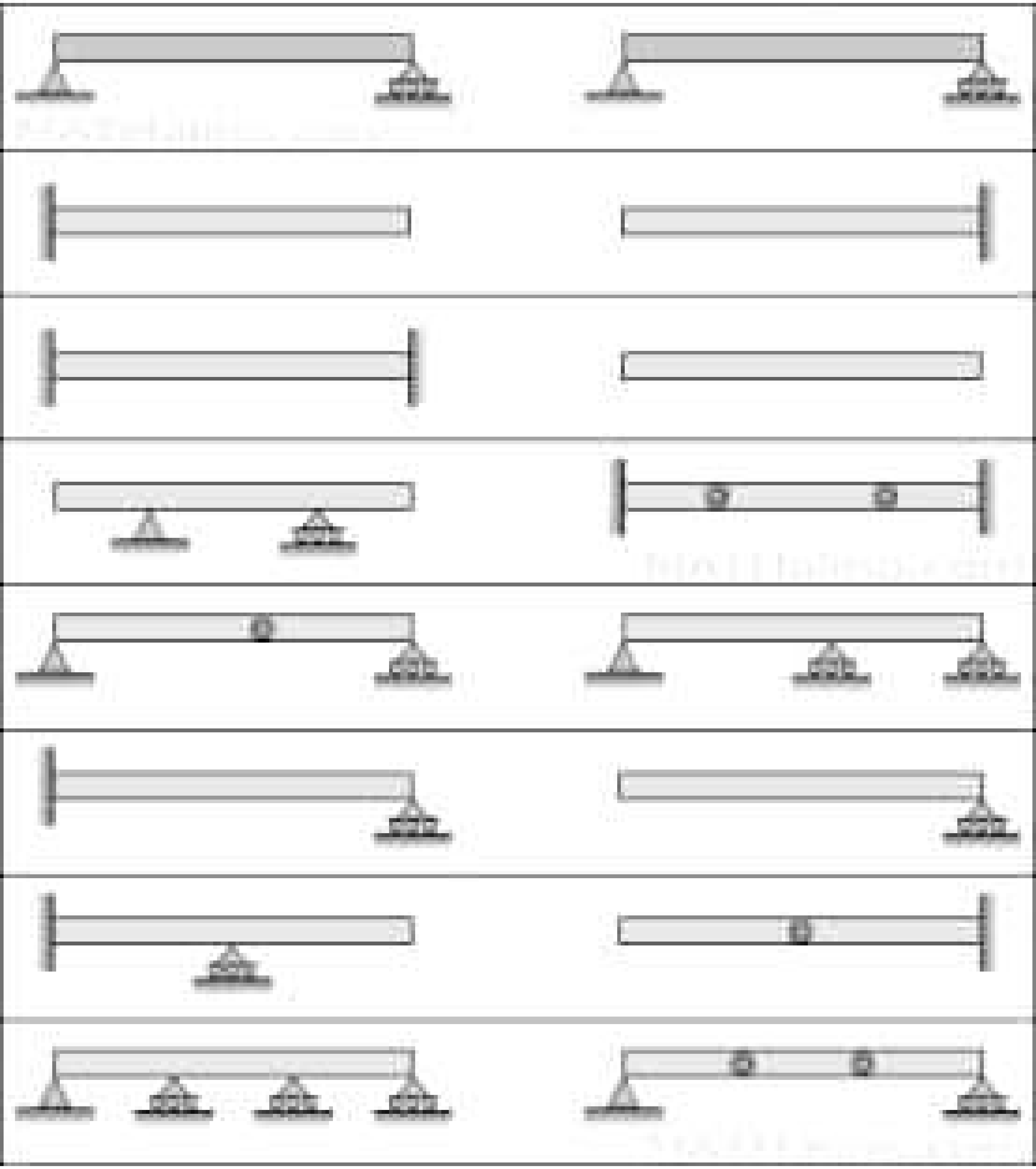
Real Beam	Conjugate Beam



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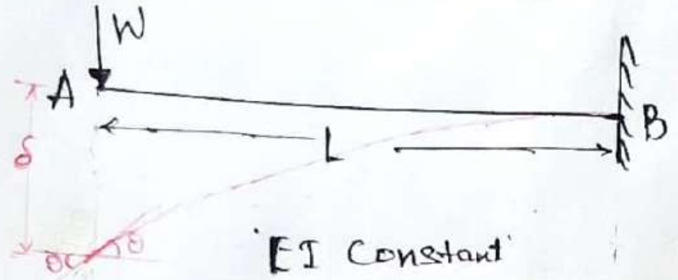
1) Cantilever beam :



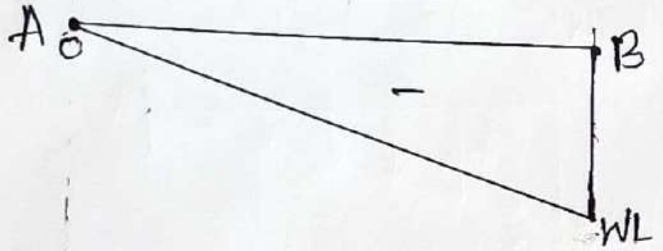
Real beam,

In this $\theta \,\&\, \delta = 0$ at B [∵ It is fixed end]

ie $\theta_B = \delta_B = 0$.

$\theta \,\&\, \delta$ will be Maximum at 'A'

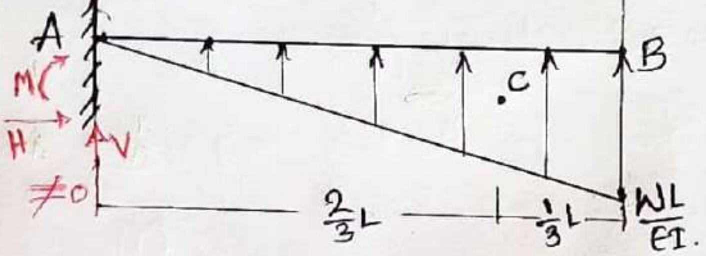
$\theta_A = \theta_{Max}$, $\delta_A = \delta_{max.} = ?$



BMD

$BM_A = 0$ [∵ BM is in free end]

$BM_B = -WL$



Conjugate beam

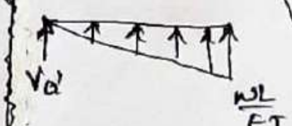
$\frac{M}{EI}$ diagram.

In Conjugate beam method.

Real Beam ⟶ Conjugate beam.

T_1:- Slop at 'A' in Real beam = Shear force at A in Conjugate beam.

$\theta_A = V_a'$



$\Sigma V = 0$

$V_a' + \frac{1}{2} \times L \times \frac{WL}{EI} = 0$

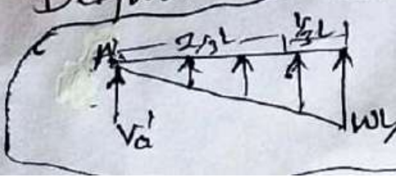
$V_a' = -\frac{WL^2}{2EI}$

T_2 :- Deflection at 'A' in Real beam = Bending Moment at 'A' in Conjugate beam.

$\delta_A = BM_A$

Slope @ A = SF @ A = $V_a' = -\frac{1}{2} \times L \times \underbrace{\frac{WL}{EI}}_{\text{Force}} = \boxed{-\frac{WL^2}{2EI} = \theta_A}$ (radian)

Deflection 'A' = BM @ A = $M_a' = \underbrace{\left(\frac{1}{2} \times L \times \frac{WL}{EI}\right)}_{\text{Force}} \times \underbrace{\left(\frac{2}{3}L\right)}_{\perp^{lar}\text{ distance}} = \boxed{-\frac{WL^3}{3EI} = \delta_A}$ [mm]



BM_A $d = \frac{2}{3}L$

$F = \frac{1}{2} \times L \times \frac{WL}{EI}$

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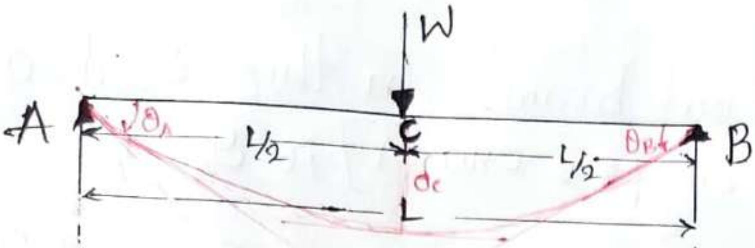
2) Simply Supported beam



Real beam

In this $\theta = 0$ at C

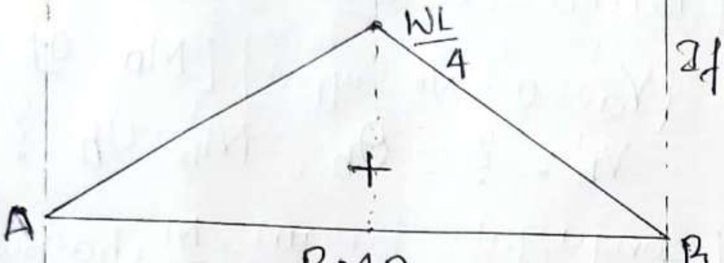
$\delta = 0$ at A & B

[∵ In simply supported beam. If it is θ is Maximum, δ is Zero]

ie $\theta_{max} = \delta = 0$

(or)

$\delta_{max} = \theta = 0.$

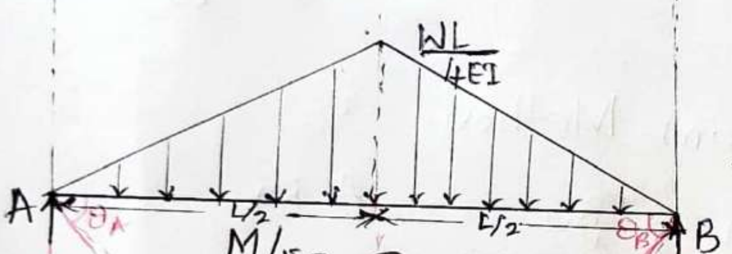


BMD

*) $\theta_A = \theta_B = \theta_{Maximum}$

∴ δ_A & $\delta_B = 0.$

*) $\delta_C = \delta_{Max}$. ∴ $\theta_C = 0.$



M/EI Diagram

Conjugate beam

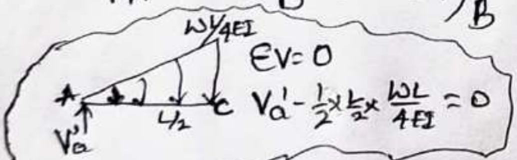
θ_A is Slope from A to C, θ_B is Slope from B to C

In Conjugate beam Method

upto C

$$\text{Slope } \theta_A = \text{SF at A} = SF)_A = V_a' = \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI} = \boxed{\frac{WL^2}{16EI} = \theta_A}$$

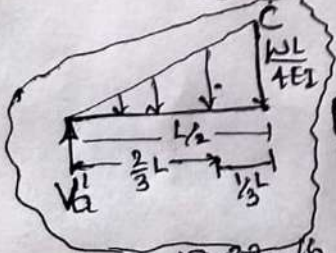
$$|||^{ly} \quad \theta_B = SF)_B = V_b' = \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI} = \boxed{\frac{WL^2}{16EI} = \theta_B}$$



[Also we can say that $\theta_A = \theta_B$. Because Symmentric beam]

Deflection at C = BM at C = BM_C | BM_A & $BM_B = 0$ ∵ Simple Supports

$\delta_C = BM_C$ ⇒ Considering half portion ie CA (or) CB



$$BM_C = V_a' \times \frac{L}{2} - \left(\frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI}\right)\left(\frac{1}{3} \cdot \frac{L}{2}\right)$$

$$BM_C = \frac{WL^2}{16EI} \cdot \frac{L}{2} - \frac{WL^2}{16EI} \cdot \frac{L}{6} = \frac{WL^3}{32EI} - \frac{WL^3}{96EI} = \frac{WL^3}{48EI}$$

$$\frac{1}{32} - \frac{1}{48} = \frac{48-32}{1536} = \frac{16}{}$$

$$\therefore \boxed{\delta_C = \frac{WL^3}{48EI}}$$

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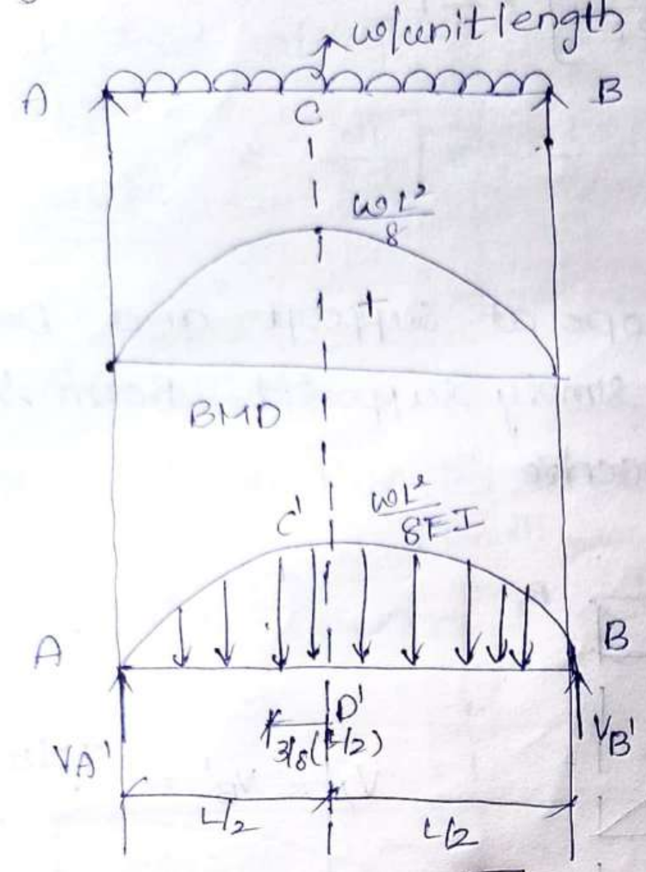
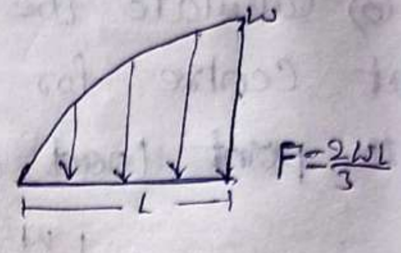
Simply Supported Beam Subjected to UDL





$\Sigma V = 0$

$$V_A' + V_B' - 2\left(\frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8EI}\right) = 0$$

$\Sigma M_B = 0$

$$V_A' \times L - 2\left(\frac{2}{3} \cdot \frac{L}{2} \cdot \frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = 0$$

$$V_A' L - \frac{wL^4}{24EI} = 0$$

$$\boxed{V_A' = \frac{wL^3}{24EI}}$$

$$\therefore \boxed{V_B' = \frac{wL^3}{24EI}}$$

Because Symmetric Beam

$$V_A' = V_B'$$

Or

$$V_A' = V_B' = \frac{\text{Total load}}{2}$$

$$\Rightarrow V_A' = V_B' = \frac{\cancel{2}\left(\frac{\cancel{2}}{3} \times \frac{L}{\cancel{2}} \times \frac{wL^2}{8EI}\right)}{\cancel{2}}$$

$$V_A' = V_B' = \frac{wL^3}{24EI}$$

θ_A = SF at A

$$\theta_A = +V_A' = \frac{wL^3}{24EI}$$

y_C = BM at C

$$y_C = V_A' \times \frac{L}{2} - \left(\frac{\cancel{2}}{3} \times \frac{L}{\cancel{2}} \times \frac{wL^2}{8EI}\right)\left(\frac{3}{8}\left(\frac{L}{2}\right)\right)$$

$$y_C = \frac{wL^3}{24EI}\left(L/2\right) - \frac{wL^3}{24EI}\left(\frac{3L}{16}\right)$$

$$y_C = \frac{wL^4}{48EI} - \frac{wL^4 3}{384EI} \Rightarrow \boxed{y_C = \frac{5wL^4}{384EI}}$$

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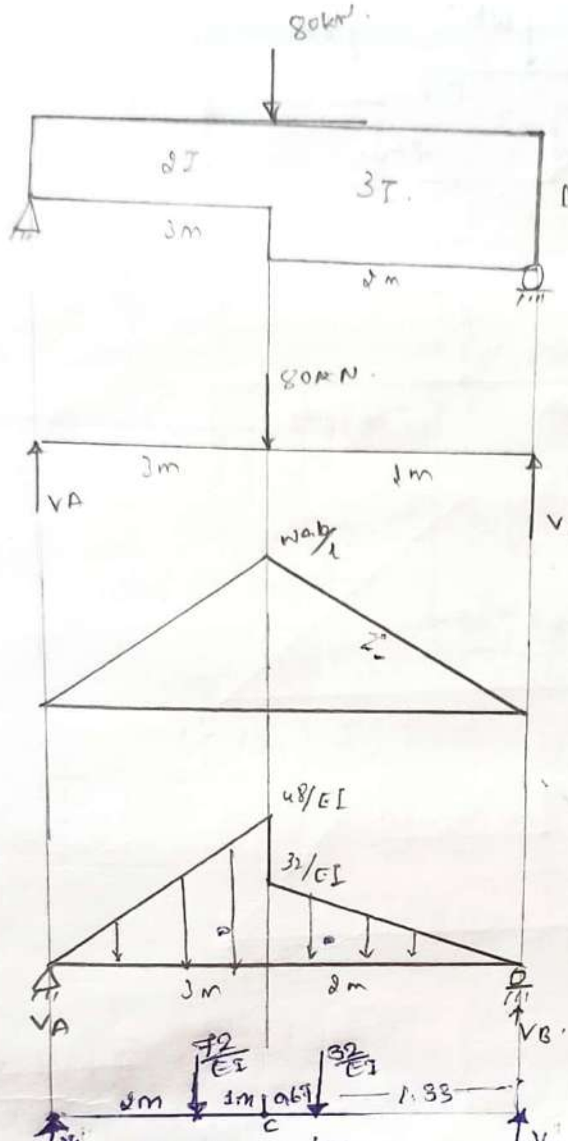
Determine slope at A deflection at C : for SSB.



$\frac{1}{2} \times 48 \times 3 = 72$

$= \frac{1}{2} \times 2 \times 32 = 32$

Reaction:- $\Sigma V = 0$. $V_A + V_B = \frac{104}{EI}$

$\Sigma M_A = 0$ $V_B = \frac{52.27}{EI}$ $V_A = \frac{51.73}{EI}$.

$\theta_A = SF)_A$ in conj beam $= V_A = \frac{51.73}{EI}$.

$\theta_B = SF)_B$ in conj beam $V_B = -\frac{52.27}{EI}$.

$\delta_C = BM)_C$ in con. beam

$= (V_A \times 3) - \left(\frac{1}{2} \times 3 \times \frac{48}{EI}\right)\left(\frac{1}{3} \times 3\right)$

$\boxed{\delta_C = \frac{83.19}{EI}}$

Point loads.

$\Sigma V = 0$, $V_A + V_B = \frac{72}{EI} + \frac{32}{EI}$

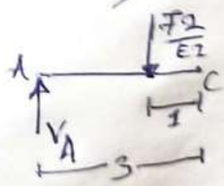
$V_A + V_B = \frac{104}{EI}$

$\Sigma M_B = 0$. $V_A \times 5 - \frac{72}{EI} \times 3 - \frac{32}{EI} \times 1.33 = 0$

$\therefore V_A = \frac{51.73}{EI}$

$\therefore V_B = \frac{52.27}{EI}$

BM_C



$BM_C = V_A \times 3 - \frac{72}{EI} \times 1$

$= \frac{51.73 \times 3}{EI} - \frac{72}{EI}$

$\boxed{BM_C = \frac{83.19}{EI}}$

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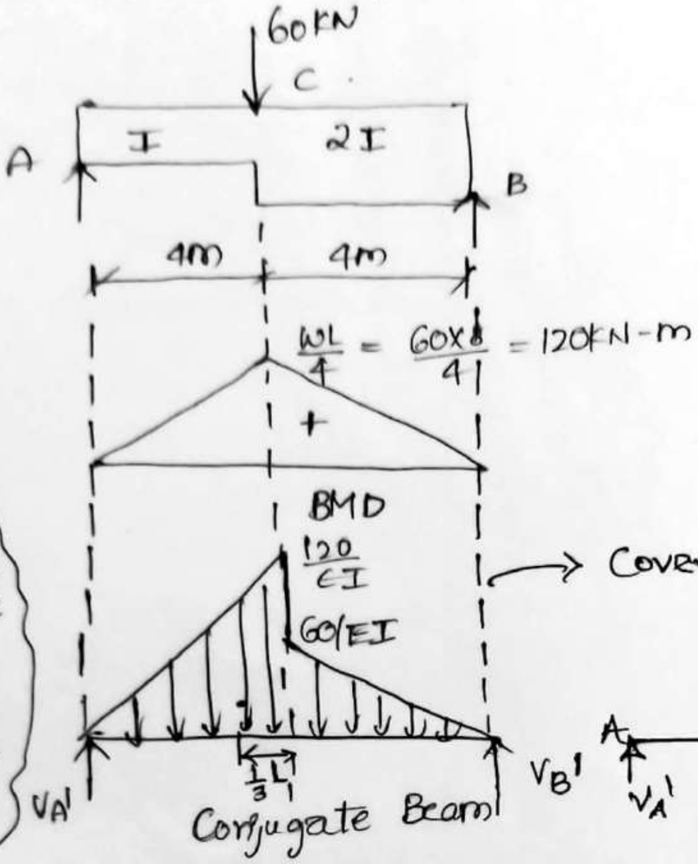
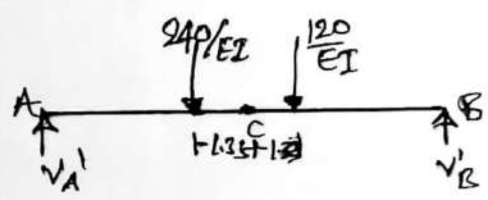
5) using Conjugate Beam method Find the Deflection at point 'C' and slope at point 'A' for the Simply supported beam shown in figure



$= \frac{1}{2} \times 4 \times \frac{120}{EI}$
$= \frac{240}{EI}$
$= \frac{1}{2} \times 4 \times \frac{60}{EI}$
$= \frac{120}{EI}$

→ Coverted into point load.



$\Sigma M_B' = 0$

$V_A' \times 8 - \left(\frac{1}{2} \times 4 \times \frac{120}{EI}\right)\left(4 + \frac{1}{3}(4)\right) - \left(\frac{1}{2} \times 4 \times \frac{60}{EI}\right)\left(\frac{2}{3}(4)\right) = 0$

$V_A' \times 8 = \frac{1280}{EI} + \frac{320}{EI}$ $\boxed{V_A' = \frac{200}{EI}} = \theta_A$

$\Sigma V = 0$

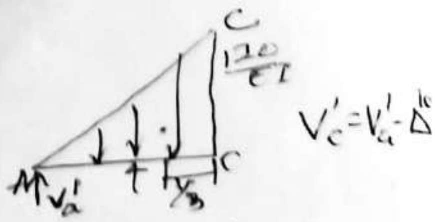
$V_A' - \left(\frac{1}{2} \times 4 \times \frac{120}{EI}\right) - \left(\frac{1}{2} \times 4 \times \frac{60}{EI}\right) + V_B' = 0$

$\frac{200}{EI} - \frac{240}{EI} - \frac{120}{EI} + V_B' = 0$

$\boxed{V_B' = \frac{160}{EI}} = \theta_B$

* Not Required

$\boxed{V_C' = -\frac{40}{EI}} = \theta_C$



$BM_C = \delta_C = V_A' \times 4 - \left(\frac{1}{2} \times 4 \times \frac{120}{EI}\right)\left(\frac{1}{3} \times 4\right) = \frac{480}{EI}$

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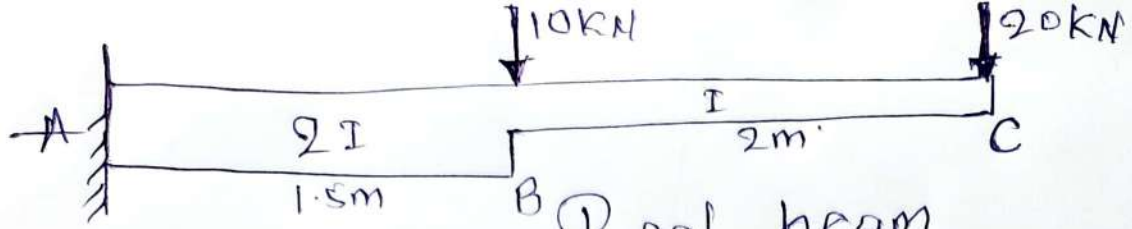


Real beam

from fig. θ_A & $\delta_A = 0$. δ_C & θ_C = Max.

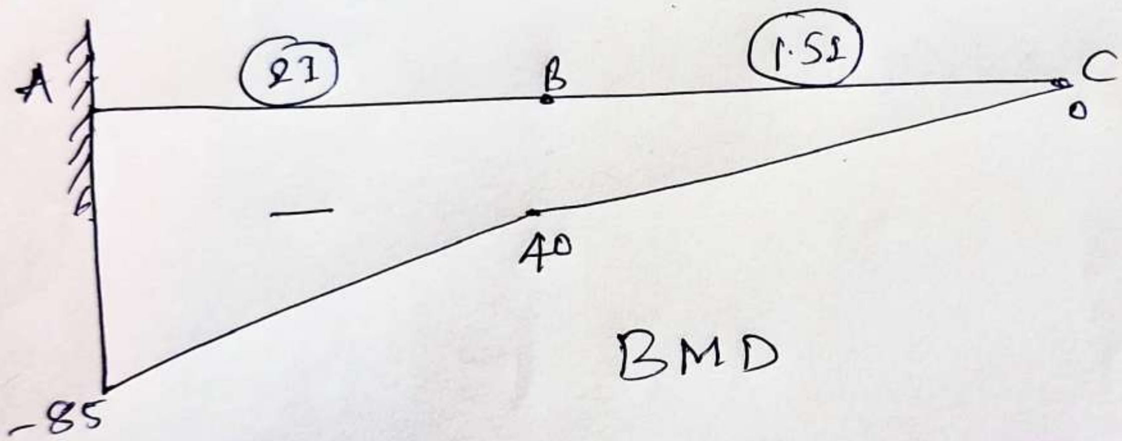
BM values.

$BM_C = \underline{0}$ [∵ free End]

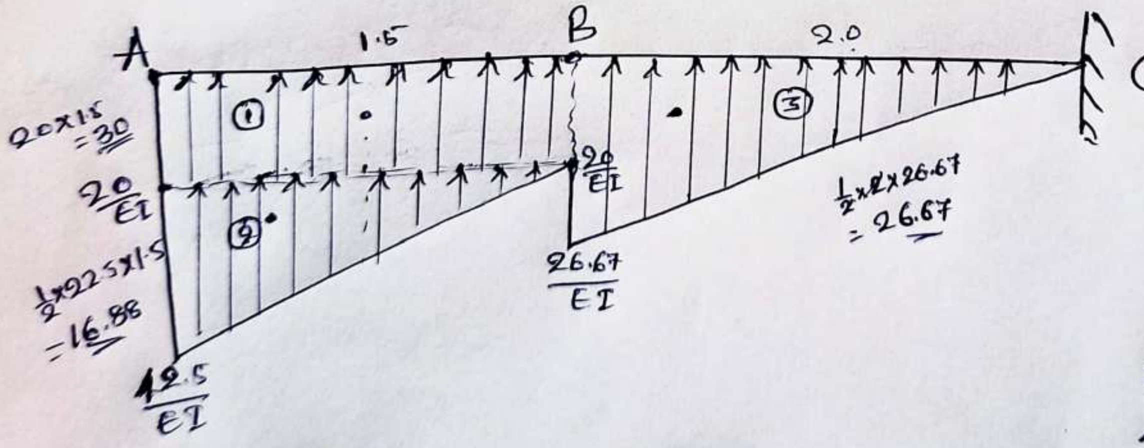
$BM_B = (-20 \times 2) = \underline{-40}$ kN-m (hogging)

$BM_A = (-20 \times 3.5) + (-10 \times 1.5)$.

$BM_A = -70 - 15 = \underline{-85}$ kN-m (hogg)



BMD



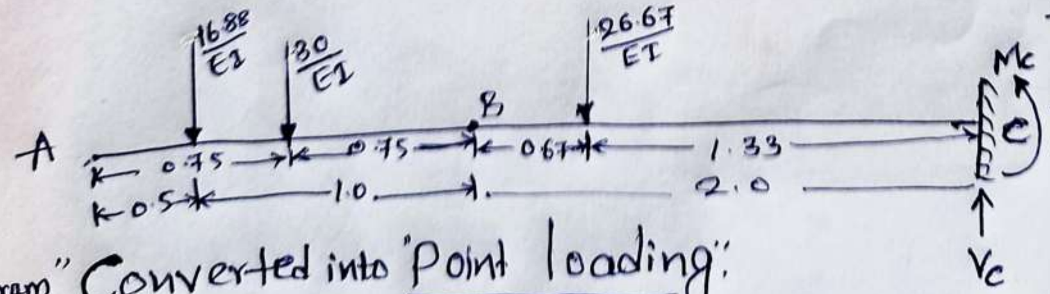
Conjugate Beam

$\frac{M}{EI}$ diagram

$\frac{85}{2EI} = \frac{42.5}{EI}$

$\frac{40}{2EI} = \frac{20}{EI}$

$\frac{40}{1.5EI} = \frac{26.67}{EI}$



"$\frac{M}{EI}$ diagram" Converted into "Point loading".

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Need to determine θ & δ at free end.

$\Sigma V = 0$ [Consider A to C]

$$V_C - \frac{16.88}{EI} - \frac{30}{EI} - \frac{26.67}{EI} = 0$$

$$\boxed{V_C = \frac{73.55}{EI}} = \theta_C$$

$\Sigma M_C = 0$

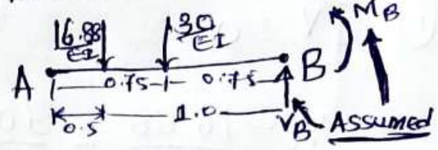
$$-\left(\frac{16.88}{EI} \times 3\right) - \left(\frac{30}{EI} \times 2.75\right) - \left(\frac{26.67}{EI} \times 1.33\right) + M_C = 0$$

$$M_C = \frac{50.64}{EI} + \frac{82.5}{EI} + \frac{35.47}{EI}$$

$$\boxed{M_C = \frac{168.61}{EI}} = \delta_C$$

(*) Rarecase asking at intermedi

Consider A to B

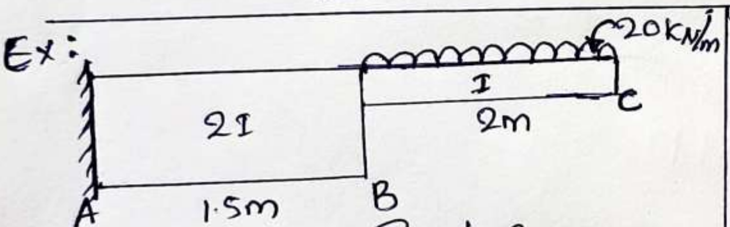


$$V_B = \frac{46.88}{EI} = \theta_B$$

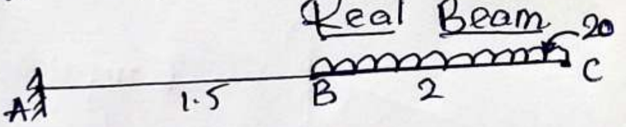
$$M_B = \frac{16.88}{EI} \times 1 + \frac{30}{EI} \times 0.75 = \frac{39.38}{EI}$$

$$\therefore \delta_B = \frac{39.38}{EI}$$

Ex:

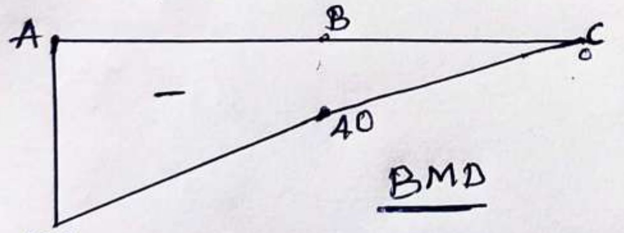


Real Beam

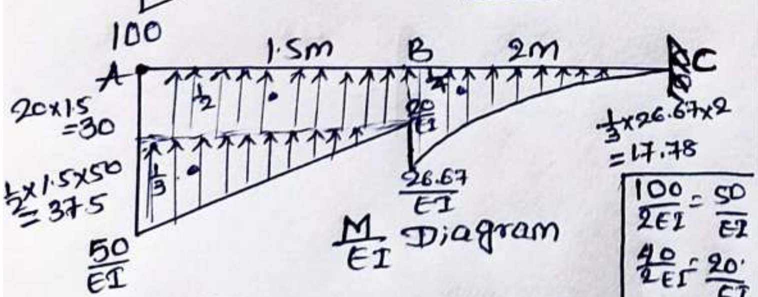


$BM_C = 0$, $BM_B = -20 \times 2 \times \frac{2}{2} = -40\,kN\text{-}m$

$BM_A = (20 \times 2)(1.5 + 1) = -100\,kN\text{-}m$



BMD



$\frac{M}{EI}$ Diagram

$$\frac{100}{2EI} = \frac{50}{EI}$$
$$\frac{40}{2EI} = \frac{20}{EI}$$
$$\frac{40}{1.5E} = \frac{26.67}{EI}$$