

Module 5.

①

Digital Electronic Fundamentals:

Difference between analog & digital signals

Number System - Binary Hexadecimal

Conversion - Decimal to Binary

Hexadecimal to decimal and vice-versa

Boolean algebra

Basic and universal gates

Half and Full adder

Multiplexer,

Decoder

SR & JK Flipflops.

Shift Register, 3-bit Ripple Counter

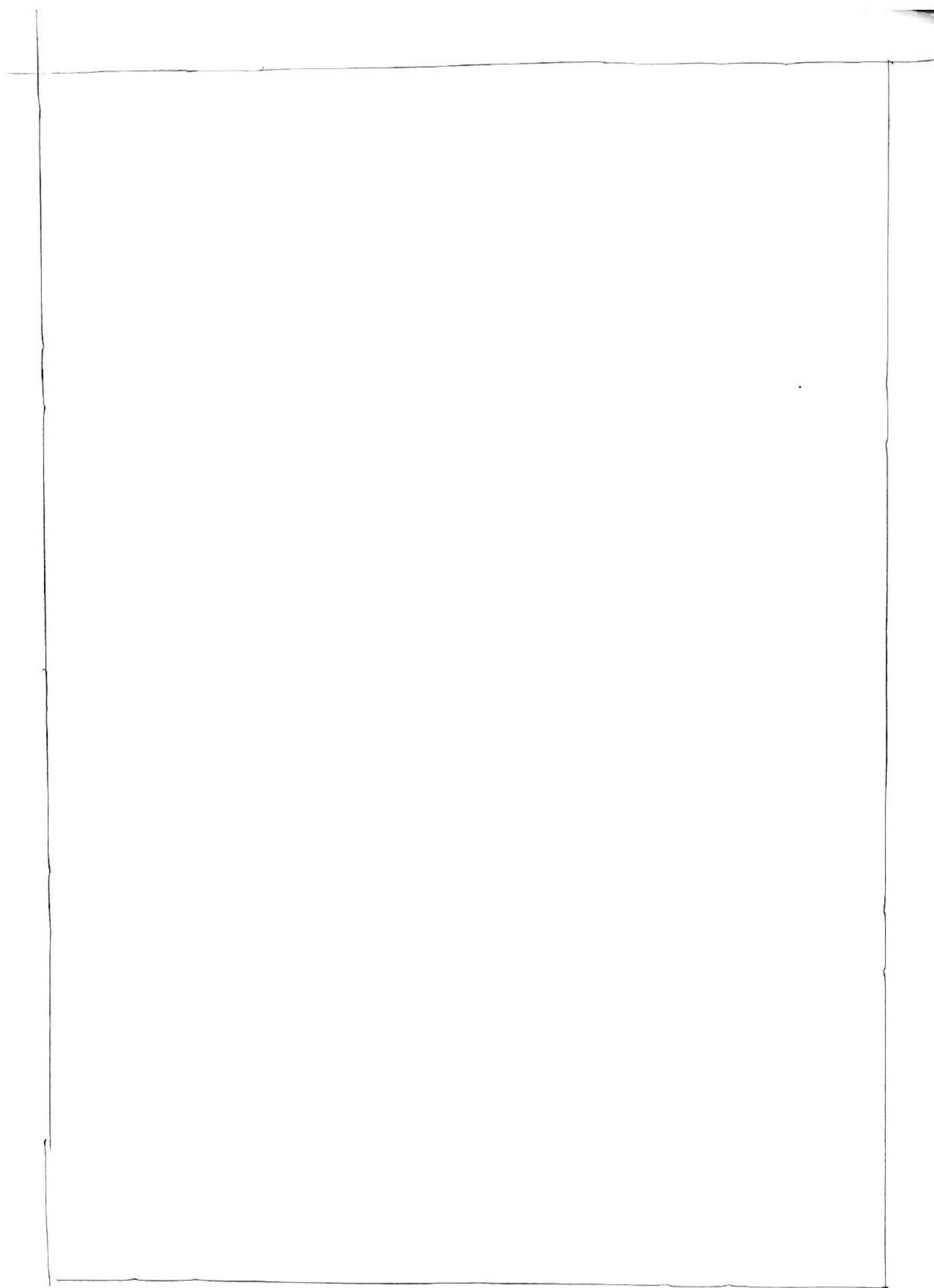
(refer 10.1 - 10.7 of Text 1)

Basic Communication system

Principle of operations of mobile phone

(refer 18.2 and 18.18 of Text 1).

RBT Levels: L1 & L2



Difference between Analog signals & Digital signals

Analog signal: An analog signal is a continuous signal they have infinite number of possible values.

Example sine wave. voice signal. video signal.

Digital signal: - These are the discrete & non continuous signal has finite number of possible values. Example Computer Data. Data storage on memory. It has two possible values binary '1' and '0'.

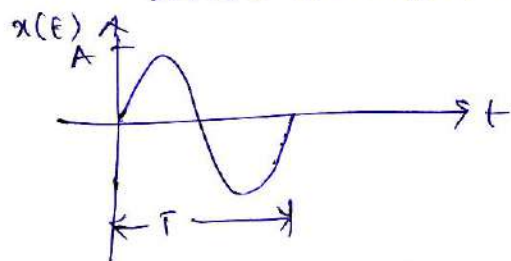
Analog signal.

1. An Analog signal is a signal that can have one of ~~an~~ infinite number of possible values.

2. Analog signal is a continuous signal which represents physical measurements.

3. Denoted by sine wave.

$$x(t) = A \sin(2\pi ft + \phi)$$



4. Uses continuous range of values to represent information.

5. Analog signals are more affected by noise or distortion during transmission.

6. They are best suited for transmission of audio & video.

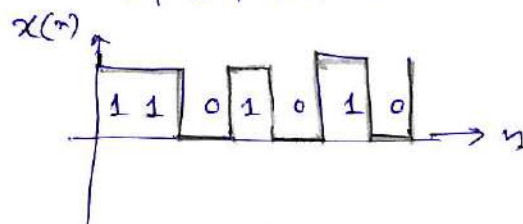
Digital signal.

A digital signal is a signal that can have one of finite set of possible values at any time.

Digital signals are non-continuous & discrete in nature.

Denoted by square wave.

1 1 0 1 0 1 0



uses discrete or discontinuous values to represent information.

Digital signals are less immune to noise or distortion during transmission.

They are best suited for transmission of computer data.

Introduction to Number System.

Number system is the Basis for Counting various items. The representation of Numbers in a specific way is called Number system. We already know the familiar decimal number system with its 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Modern Computers communicate and operate with binary numbers which use only the digits 0 & 1.

The representation of Decimal number, in binary format takes more digits. For large decimal number, the binary string is large. and $\therefore$ They do not like working with binary numbers. This fact gives rise to three new number systems namely: Octal, Hexadecimal and Binary coded decimal.

Different number systems

The different number systems are

- Decimal system
- Hexadecimal number system
- Binary number system
- Octal number system.

Decimal Number System.

Decimal numbers can be expressed in terms of units, tens, hundreds, thousands & so on. Decimal number 5678.9 can be written

$$\text{as } 5000 + 600 + 70 + 8 + 0.9 = 5678.9$$

The base of the Decimal number system is 10, d, or D. It is written as a subscript. The position of a digit with reference to the decimal point determines its value/weight. In above example the left most digit, which has greatest weight is called most significant digit (MSD). The right most digit, has least weight is called least significant digit (LSD).

$$(1) \quad \quad \quad 5 \quad 6 \quad 7 \quad 8 \quad . \quad 9$$

$$\text{In powers of } 10 \quad 5 \times 10^3 \quad 6 \times 10^2 \quad 7 \times 10^1 \quad 8 \times 10^0 \quad . \quad 9 \times 10^{-1}$$

Any number in the decimal system is represented by using the symbols.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Another eg: (5689.723)₁₀

Number	5	6	8	9	.	7	2	3
Position	Digit ₃	Digit ₂	Digit ₁	Digit ₀	.	Digit ₋₁	Digit ₋₂	Digit ₋₃
Weightage	10 ³	10 ²	10 ¹	10 ⁰		10 ⁻¹	10 ⁻²	10 ⁻³

MSD. LSD.

Explain Binary number system.

Binary system with two digits is a base-two system.

The binary digits (bits) are 0 & 1.

Each symbol that appears in the number is called a Bit.

Each bit has a weightage which depends on the position of the bit in the number.

• The right most bit is called least significant bit. weightage is 2⁰. The weightage of bit one left hand side to the least significant bit is 2¹. Thus as we move to the left the weightage of the bit increases by the multiple of 2.

Eg.

Number	1	0	1	1
Position	bit ₃	bit ₂	bit ₁	bit ₀
Weightage	2 ³	2 ²	2 ¹	2 ⁰

$$1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0 = 8 + 2 + 1 = \underline{(12)}_{10}$$

for fraction.

Number	1	1	0	0	.	1	1
Position	bit ₃	bit ₂	bit ₁	bit ₀	.	bit ₋₁	bit ₋₂
Weightage	2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²

$$1 \times 2^3 + 1 \times 2^2 + 1 \times 2^{-1} + 1 \times 2^{-2} = 8 + 4 + 0.5 + 0.25 = \underline{(12.75)}_d = \underline{(1100.11)}_2$$

Octal number system:-

Any number in the number system is represented by using the symbols.

0, 1, 2, 3, 4, 5, 6, 7.

Consider Example of representing 2356 in octal representation.

- Each symbol in the number is called as Digit.
- Each digit has a weightage which depends on the position of the digit in the number.
- The right most digit is called Least Significant digit. The weightage of this digit is 8^0 .
- The weightage of the digit on the left hand side of the LSD is 8^1 .
- Thus as we move to the left weightage of the digit increase by the multiple of 8.

Eg.

Number	2	3	4	1
position	Digit 3	Digit 2	Digit 1	Digit 0
weightage	8^3	8^2	8^1	8^0

The value of digit 2 is.

$$= 3 \times 8^2 = 3 \times 64 = 192.$$

Number	1	2	3	0	4	3	1
position	Digit 7	D-1	D-2	D-3	Digit -1	-2	-3
weightage	8^2	8^1	8^0	•	8^{-1}	8^{-2}	8^{-3}

The value of Digit 1 is

$$= 2 \times 8^1 = 16.$$

Digit 2 is

$$1 \times 8^2 = 64.$$

$$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 4 \times 8^{-1} + 3 \times 8^{-2} + 1 \times 8^{-3} = (83.548)_{10}$$

Hexadecimal Number System

The hexadecimal number system has a base of 16 having 16 characters. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

For Example. 32A.1B can be represented in power of 16 as shown below.

3	2	A	.	1	B
Digit 2	Digit 1	Digit 0	.	Digit -1	Digit -2
16^2	16^1	16^0	.	16^{-1}	16^{-2}

$$N = 3 \times 16^2 + 2 \times 16^1 + A \times 16 + 1 \times 16^{-1} + B \times 16^{-2}$$

$$= 3 \times 256 + 2 \times 16 + 10 \times 16 + 1 \times 16^{-1} + 11 \times 16^{-2}$$

$$= (960.1054688)_{10} = (960.1054688)_{10}$$

Each symbol that appears in the number is called a digit. Each digit has a weightage which depends on the position of the digit in the number.

Number	2	3	5	6
position	Digit 3	Digit 2	Digit 1	Digit 0
weightage	16^3	16^2	16^1	16^0

$$2 \times 16^3 + 3 \times 16^2 + 5 \times 16^1 + 6 \times 16^0$$

The value of digit 2 is. $= 3 \times 16^2$

$$= 3 \times 256$$

$$= 768$$

$$N = 2 \times 16^3 + 3 \times 16^2 + 5 \times 16^1 + 6 \times 16^0$$

N	0	.	1	2	3
P	16^0	.	16^{-1}	16^{-2}	16^{-3}
W	16^0	.	16^{-1}	16^{-2}	16^{-3}

04

$$= (0.07104)_{10}$$

Relationship between decimal, binary, octal & hexadecimal no.

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

What do you mean by bits, bytes, nibbles, word, double word.

if3	b ₃₁	b ₃₀	b ₂₉	b ₂₈	b ₂₇	b ₂₆	b ₂₅	b ₂₄	b ₂₃	b ₂₂	b ₂₁	b ₂₀	b ₁₉	b ₁₈	b ₁₇	b ₁₆	b ₁₅	b ₁₄	b ₁₃	b ₁₂	b ₁₁	b ₁₀	b ₉	b ₈	b ₇	b ₆	b ₅	b ₄	b ₃	b ₂	b ₁	b ₀
	Nibble 7				Nibble 6				Nibble 5				Nibble 4				Nibble 3				Nibble 2				Nibble 1				Nibble 0			
	Byte 3								Byte 2								Byte 1								Byte 0							
	Word 1																Word 0															
	Double word																															

A number can be represented by the equation
general equation.

$$N = d_n \times r^n + d_{n-1} \times r^{n-1} + \dots + d_2 \times r^2 + d_1 \times r^1 + d_0 \times r^0$$

N → value of the entire number.

d_n → is the value of the nth digit from the decimal point, and.

r → is the radix or base.

Number of digital (n+1); (0, 1, 2, ... n)

Conversion between Decimal, Octal, Hexadecimal and Binary Numbers:-

general:- Conversion from any number system to Decimal.

step 1:- Identify the weightage of each digit in the number system.

step 2:- multiply each digit by the weightage of the digit.

step 3:- add all the products.

Binary to Decimal Conversion.

① Convert the given binary number 11010 into decimal.

position.	Bit 4.	Bit 3	Bit 2	Bit 1	Bit 0
given Number :	1	1	0	1	0
Weightage :	2^4	2^3	2^2	2^1	2^0

$$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 16 + 8 + 0 + 2 + 0 = 26$$

$$\therefore (11010)_2 = (26)_{10}$$

②. $(1101101)_2$ to decimal.

position :	Bit 6	Bit 5	Bit 4	Bit 3	Bit 2	Bit 1	Bit 0
given No. :	1	1	0	1	1	0	1
weightage :	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$= 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 64 + 32 + 0 + 8 + 4 + 0 + 1$$

$$= (109)_{10}$$

Converting Binary Number 1011.110 to Decimal.

position	Bit 3	Bit 2	Bit 1	Bit 0	Bit -1	Bit -2	Bit -3
Binary No.	1	0	1	1	1	1	0
Weightage	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

$$= (1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) + (1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3}).$$

$$= (8 + 0 + 2 + 1) + (0.5 + 0.25 + 0)$$

$$= (11.75)_{10}$$

Convert given octal Number 212 into decimal.

position	Digit 2	Digit 1	Digit 0
given No.	2	1	2
weightage	8^2	8^1	8^0

$$N = 2 \times 8^2 + 1 \times 8^1 + 2 \times 8^0.$$

$$= 2 \times 64 + 1 \times 8 + 2 \times 1$$

$$= 128 + 10 = (138)_{10}.$$

Convert $(235.67)_8 = (?)_{10}$

position	Digit 2	Digit 1	Digit 0	Digit -1	Digit -2
Number	2	3	5	6	7
Weightage	8^2	8^1	8^0	8^{-1}	8^{-2}

$$= 2 \times 8^2 + 3 \times 8^1 + 5 \times 8^0 + 6 \times 8^{-1} + 7 \times 8^{-2}$$

$$= 2 \times 64 + 24 + 5 + 0.75 + 0.109375$$

$$= (157.8594)_{10}$$

Convert given Hexadecimal No. to Decimal.

(i). $(285.A9)_{16} = (?)_{10}$

position	Digit 2	Digit 1	Digit 0	.	Digit -1	Digit -2
No.	2	8	5	.	A	9
Weightage	16^2	16^1	16^0	.	16^{-1}	16^{-2}

$$= 2 \times 16^2 + 8 \times 16^1 + 5 \times 16^0 + (10 \times 16^{-1}) + (9 \times 16^{-2})$$

$$= 512 + 128 + 5 + 0.625 + 0.03516$$

$$= (645.66016)_{10}$$

(ii). $(235)_{16} = (?)_{10}$

position	Digit 2	Digit 1	Digit 0
Number	2	3	5
Weightage	16^2	16^1	16^0

$$= 2 \times 16^2 + 3 \times 16^1 + 5 \times 16^0$$

$$= 2 \times 256 + 3 \times 16 + 5 \times 1$$

$$= (565)_{10}$$

Case 2:- Conversion of number in decimal Number system to any other Number System.

Integer part Conversion [Successive division for Integer part]

- perform repeated division of given decimal value by the base of number system into which the conversion is required.
- After each division, the remainder is taken as one digit / bit in destination number system.
- For the next division, take the quotient of the previous division as the dividend.

For fractional part Conversion [Successive multiplication for fractional part]

perform the repeated multiplication of the fractional part of the decimal number with the base of the number system into which the conversion required.

- after each conversion, the integer part is taken as one digit/bit in the destination number system.
- For the next multiplication, take only previous fractional part
- repeat above steps till fractional part of the previous multiplication becomes '0'. (or until desired value we get.)

Decimal to Binary:-

① $(25.625)_{10} = (?)_2$

Integer

$$\begin{array}{r}
 2 \overline{) 25} \\
 \underline{2 12} - 1 \\
 2 \overline{) 6} - 0 \\
 2 \overline{) 3} - 0 \\
 \underline{1} - 1
 \end{array}$$

MSB ↑ LSB

fractional.

$$\begin{array}{r}
 0.625 \times 2 \\
 \hline
 1 \leftarrow 1.25 \\
 0.25 \times 2 \\
 \hline
 0 \leftarrow 0.5 \\
 0.5 \times 2 \\
 \hline
 1 \leftarrow 1.0
 \end{array}$$

$= (11001.101)_2$

② $(264.325)_{10} = (?)_2$

$$\begin{array}{r}
 2 \overline{) 264} \\
 \underline{2 132} - 0 \\
 2 \overline{) 66} - 0 \\
 2 \overline{) 33} - 0 \\
 2 \overline{) 16} - 1 \\
 2 \overline{) 8} - 0 \\
 2 \overline{) 4} - 0 \\
 \underline{1} - 0
 \end{array}$$

MSB ↑ LSB

$(264.325)_{10} = (10001000.0101001)_2$

$$\begin{array}{r}
 0.325 \times 2 \\
 \hline
 0.65 \times 2 \rightarrow 0 \\
 \hline
 1.3 \rightarrow 1 \\
 0.3 \times 2 \\
 \hline
 0.6 \rightarrow 0 \\
 \hline
 0.6 \times 2 \\
 \hline
 1.2 \rightarrow 1 \\
 0.2 \times 2 \\
 \hline
 0.4 \times 2 - 0 \\
 \hline
 0.8 \times 2 - 0 \\
 \hline
 1.6 - 1
 \end{array}$$

Convert the Decimal number 75.62 into octal.

① Approximate the result to first four places.
 $(75.62)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 75 \text{ Remainder.} \\ \hline 8 & 9 - 3 \uparrow \text{LSD} \\ 8 & 1 - 1 \\ & 0 - 1 \text{ MSD} \end{array}$$

$$(113.4753)_8$$

$$\begin{array}{r} 0.62 \times 8 \\ \hline 4.96 \rightarrow 4 \\ \downarrow \\ 0.96 \times 8 \\ \hline 7.68 \rightarrow 7 \\ \downarrow \\ 0.68 \times 8 \\ \hline 5.44 \rightarrow 5 \\ \downarrow \\ 0.44 \times 8 \\ \hline 3.52 \rightarrow 3 \end{array}$$

②. $(384.832)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 384 \\ \hline 8 & 48 - 0 \\ & 6 - 0 \end{array}$$

$$(600.6517)_8$$

$$\begin{array}{r} 0.832 \times 8 \\ \hline 6.656 \rightarrow 6 \\ \downarrow \\ 0.656 \times 8 \\ \hline 5.248 \rightarrow 5 \\ \downarrow \\ 0.248 \times 8 \\ \hline 1.984 \rightarrow 1 \\ \downarrow \\ 0.984 \times 8 \\ \hline 7.872 \rightarrow 7 \end{array}$$

solve $(658.825)_{10} = (?)_8$

②. $(12.125)_{10} = (?)_2$

Convert 5386.345 decimal into hexadecimal

Conversion of Integer part by successive division method.

$$\begin{array}{r} 16 \overline{) 5386} \\ 16 \overline{) 336} - A \\ 16 \overline{) 21} - 0 \\ \underline{ 5} \end{array}$$

$$(150A.5851\dots)_{16}$$

$$0.345 \times 16$$

$$5.52 \rightarrow 5.$$

↓

$$0.52 \times 16$$

$$8.32 \rightarrow 8$$

↓

$$0.32 \times 16$$

$$5.12 \rightarrow 5$$

$$0.12 \times 16$$

$$1.92 \rightarrow 1$$

Convert $(196.284)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{) 196} \\ 16 \overline{) 12} - 4 \\ \underline{ 0} - 0 \end{array}$$

$$(C4)_{16}$$

$$(C4.48B)_{16}$$

$$0.284 \times 16$$

$$4.544 \rightarrow 4$$

$$0.544 \times 16$$

$$8.704 \rightarrow 8$$

↓

$$0.704$$

$$11.264 \rightarrow 11$$

Conversion from Binary to Octal

Step 1: Arrange group of 3 bits starting from LSB for Integer part & MSB for fractional part, by adding 0's at the end, if required.

Step 2: Write equivalent octal No. for each group of 3-bits.

Convert 10101101.0111 to octal equivalent

Adding to make a group of 3 bits. $(010 \ 101 \ 101 \ . \ 011 \ 100) \rightarrow$ Adding zero's to make 2. a group of 3 bits.

$$(2 \ 5 \ 5 \ . \ 3 \ 4)_8$$

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7
1000	10

Octal to Binary Conversion:

Each octal no. is converted individually to its binary equivalent to get binary conversion.

Convert $(125.62)_8$ to binary.

Step 1: Write equivalent 3-bit binary no. for each octal digit.

Step 2: Remove any leading or trailing zeros.

$(1 \ 2 \ 5 \ . \ 6 \ 2)_8$

$(\boxed{00} \ 010 \ 101 \ . \ 110 \ 01\boxed{0})_2$

Trailing zero.

Leading zero.

$= (1010101.11001)_2$

Conversion from binary into Hexadecimal the following table is used.

Convert 1101101.101011 into Hexadecimal

$\boxed{0}110 \ 1101 \ . \ 1010 \ 11\boxed{0}$

$(6 \ 0 \ . \ AE)_16$

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F
16	10000	10
17	10001	11

Convert binary to ~~octal~~ Hexadecimal

$(1101101110.1001100)_2 = (\)_{16}$

$0011 \ 0110 \ 1110 \ . \ 1001 \ 1010$

$(3 \ 6 \ E \ . \ 9 \ A)_{16}$

HEXADECIMAL to Binary

Convert $(8A9.BA)_{16} = (?)_2$

Step 1 $1000 \ 1010 \ 1001 \ . \ 1011 \ 1010$

Step 2 $(100010101001.10111010)_2$

Convert given Hexa to binary.

$$(345AB)_{16} = (?)_2.$$

$$0011 \ 0100 \ 10101 \cdot 1010 \ 1011 \\ (1101000101 \cdot 10101011)_2.$$

Octal to Hexadecimal :-

1. Convert octal no. into binary

2. from binary no. convert to Hexadecimal

Convert $(61525)_8$ to its Hexadecimal equivalent.

$$\text{Step 1: } (110 \ 001 \ 101 \cdot 010 \ 101 \cdot)_8$$

$$0001 \ 1000 \ 1101 \cdot 0101 \ 0100 \cdot$$

$$(1 \ 8 \ D \cdot 54)_{16}.$$

Convert Hexadecimal to octal Conversion.

Convert Hexa to binary

Convert binary no. to its octal equivalent.

Convert $(BC66AF)_{16}$ to octal.

$$B \ C \ 6 \ 6 \cdot \ A \ F$$

$$\text{Step 1: } 1011 \ 1100 \ 0110 \ 0110 \cdot 1010 \ 1111$$

$$\text{Step 2: } 001011 \ 110001 \ 100110 \cdot 101011 \ 1100$$

$$\text{Step 3: } (13 \ 6 \ 1 \ 4 \ 6 \cdot 536)_8.$$

Binary representation of decimal numbers from 0 to 15. is shown below in table. Binary digits in short form we call as bits. These numbers can take only one of the two values.

The primary advantage of using the binary number system as opposed to the 10 discrete line method in decimal number system is that it minimises the number of lines required to two.

Binary numbers are used in digital systems.

A logic 1 can be represented by a saturated transistor, a light turned ON, a relay energised or a magnet magnetised in a particular direction.

A '0' can be represented as a cut-off transistor, a light turned off, a de-energised relay, or the magnet magnetised in the opposite direction.

Binary.	Decimal.
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

Converting Decimal to Binary Decimal to Binary Conversion.

n	power of 2	
	2^n	2^{-n}
0	1	1.0
1	2	0.5
2	4	0.25
3	8	0.125
4	16	0.0625
5	32	0.03125
6	64	0.015625
7	128	0.0078125
8	256	0.00390625
9	512	0.001953125
10	1024	0.0009765625
11	2048	0.00048828125
12	4096	0.000244140625
13	8192	0.0001220703125
14	16384	0.00006103515625
15	32768	0.000030517578125
16	65536	0.0000152587890625

Convert $(69)_{10}$ to Binary

From Table 64 is largest number without exceeding 65

$69 > 64$:

$$64 = 1000000_2, 69 - 64 = 5.$$

From table 4 is largest number without exceeding 5

$$4 = 100_2 \quad 5 - 4 = 1 \text{ one is the}$$

largest number in the table.

$$1 = 1_2 \quad \begin{array}{r} 1000000 \\ 100 \\ 1 \end{array}$$

$$(69)_{10} = (1000101)_2$$

Division method

$$(69)_{10} = (?)_2$$

$$\begin{array}{r} 2 \overline{) 69} \\ 2 \overline{) 34} - 1 \uparrow \\ 2 \overline{) 17} - 0 \\ 2 \overline{) 8} - 1 \\ 2 \overline{) 4} - 0 \\ 2 \overline{) 2} - 0 \\ 1 - 0 \end{array}$$

$$(69)_{10} = (1000101)_2$$

Binary Addition and Subtraction.

Similar to Decimal addition.

Binary addition.

The simple addition consists of Four possible elementary operations, namely.

$$0+0=0$$

$$0+1=1$$

$$1+0=1$$

$$1+1=10_2$$

In above case The first three operation produces a sum whose length is one digit, but when the last operation is performed sum is two digits.

Higher significant bit of the result is called Carry.

Lower significant bit is called Sum.

This operation is known as Half addition.

Full addition:-

The operation which performs addition of three bits (two significant bits) and one Carry (previous) is called a Full addition.

Table shows the truth table for Full addition.

A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

① Add 10111_2 and 10111_2

$$\begin{array}{r}
 \text{Carry} \quad 1 \ 1 \ 1 \ 1 \ 1 \\
 \text{Number 1} \quad 1 \ 0 \ 1 \ 1 \ 1 \ 1 \\
 \text{Number 2} \quad \quad 1 \ 0 \ 1 \ 1 \ 1 \\
 \hline
 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0_2
 \end{array}$$

②. ADD. 1111 and 1111

$$\begin{array}{r}
 \text{Carry} \quad 1 \ 1 \\
 \text{Number 1} \quad 1 \ 1 \ 1 \ 1 \\
 \text{Number 2} \quad 1 \ 1 \ 1 \ 1 \\
 \hline
 1 \ 1 \ 1 \ 1 \ 0_2
 \end{array}$$

Binary subtraction :-

① Subtract 1110 from 10000

$$\begin{array}{r}
 \text{Borrow} \quad \downarrow \\
 10000 \\
 \quad 1110 \\
 \hline
 00010_2
 \end{array}$$

②. Subtract 10101 from 101010 .

$$\begin{array}{r}
 \text{Borrow} \\
 101010 \\
 \quad 10101 \\
 \hline
 010101_2
 \end{array}$$

Binary multiplication.

① Multiply 1111_2 by 101_2 .

$$\begin{array}{r}
 1111 \times 101 \\
 \hline
 1111 \\
 0000 \\
 1111 \\
 \hline
 1001011_2
 \end{array}$$

Result is 8bit.

$$= (01001011)_2$$

② Divide

110110_2 by 110_2 .

$$\begin{array}{r}
 1001 \\
 110 \overline{) 110110} \\
 \underline{110} \downarrow \downarrow \downarrow \\
 000110 \\
 \underline{110} \\
 0
 \end{array}$$

These two are called paper methods.

Computer uses different multiplication methods.

And different Division method such one Long division method is Explained Here

Computer Division method.

It is successive subtraction.

8 bit dividend is to be divided by the divisor in 4-bit representation.

dividend = 00110110, divisor = 0110.

The Quotient will be formed in the right half of the MQ register and the remainder in the left half.

① MQ register = 00110110 D register = 0110.

② The divisor is then subtracted from the dividend.

if MSB = 0, then +ve number.

MSB = 1 then negative number.

③ If the result is positive then the error has occurred for the quotient would be greater than four bits.

MQ 00110110
Subtract D 0110

MQ 11010110
Add D 0110

MQ 100110110

Shift MQ left 01101100

Subtract D 0110
MQ 00001100

Add 1 to quotient
MQ 00011011

Shift MQ left 00011010

Subtract D 0110
10111010

Add D 0110
00011010

Result is negative, the division is valid.

Result is positive

Result is negative

put a zero Quotient

$$\begin{array}{r}
 \text{MQ} - 0001\ 1010 \\
 \text{Shift MQ left} \quad 0011\ 0100 \\
 \text{Subtract D} \quad 0110 \\
 \hline
 1101\ 0100 \quad \text{Result is negative.}
 \end{array}$$

$$\begin{array}{r}
 \text{Add D} \quad 0110 \\
 \boxed{1} \quad 0011\ 0100 \\
 \text{Shift MQ left} \quad 0110\ 1000 \\
 \text{Subtract D} \quad 0110 \\
 \hline
 0000\ 1000 \quad \text{Result is positive.}
 \end{array}$$

$$\begin{array}{r}
 \text{Add 1 to Quotient} \quad 1 \\
 \hline
 0000\ 1001 \quad \text{Final answer.}
 \end{array}$$

Remainder Quotient.

If the result is negative then the quotient will be four or less bits that is sufficiently small to be contained in a 4-bit register.

The MQ register is next shifted left one bit & the divisor subtracted from it.

If the result is positive, 1 is added to the LSB (least significant bit) of the MQ register, whereas quotient is accumulated.

If it's negative, the divisor is added to MQ & the MQ shifted left one bit.

This effectively puts a '0' in the bit of quotient.

The process is continued until the MQ register has been shifted left four bits the number of bits in the D register. The remainder is then in the left half of the MQ register & the quotient in the right half.

Complement of Binary numbers:-

In Digital computer arithmetic process is done not only with positive numbers; we also do it with -ve (negative) number.

Processor manages the signed numbers and unsigned numbers. The 1's & 2's Complement are useful in this case.

What is 1's Complement?

1's complement of a binary number is just an inversion of individual bit's i.e. 1 to 0, & 0 to 1.

1's complement of $(1010101)_2$ is.

$$= (0101010)_2.$$

2's complement

The 2's complement of a binary number is addition of 1 to the 1's complement of that binary number.

Eg. 2's complement of $(10101011)_2$.

1's complement 01010100 .

ADD 1 to LSB.

$$(01010101)_2$$

It also holds to decimal binary number.

Find 2's complement of 0.1110100 .

First 1's complement 0.0001011

ADD 1 to LSB.

$$(0.0001100)_2$$

2's Complement of Floating point Number

Eg. ①. (010111.1100) Compute 2's Complement.

Step 1's Complement 101000.0011

$$\begin{array}{r} \text{ADD } 1 \\ \hline (101000.0011)_2 \end{array}$$

Difference of Two positive Binary Number (m-n).

Consider two binary numbers m & n . To subtract n from m . we can follow these steps.

1. ADD 2's Complement of n to m .

2. Neglect ^(or delete) the Carry at the left end, if Carry is present.

3. The result is $(m-n)$.

4. If sum has no carry at the left end, take 2's Complement of the sum. Attach a negative sign. The result is $(m-n)$.

Example :-

given $m = 11010110$ $n = 01000101$.

Find $(m-n)$ & $(n-m)$.

① 2's Complement of n .

$n = 01000101$

1's comp. 10111010 .

$$\begin{array}{r} \text{ADD } 1 \\ \hline 10111011 \end{array}$$

$m = 11010110$.

10111011

$$\begin{array}{r} \hline \boxed{1}10010001 \end{array}$$

$$(m-n) = (10010001)_2$$

⑥ $n-m$

2's complement of m .

$$m = 11010110.$$

1's comp. 00101001

$$\begin{array}{r} \text{Add } 1' \\ 00101001 \\ \hline 00101010 \end{array}$$

$$n = 01000101$$

$$00101010$$

No carry. 01101111

1's comp. 10010000.

$$\begin{array}{r} 10010000 \\ \hline 10010001 \end{array}$$

$$(n-m) = -10010001$$

Binary subtraction using 2's complement method.

$$(28)_{10} - (19)_{10}$$

Binary number $(28)_{10}$

$$011100$$

$(19)_{10}$

$$010011$$

Let 2's Complement of 19

$$010011$$

$$101100$$

$$\begin{array}{r} 101100 \\ \hline 101101 \end{array}$$

Carry

$$111$$

$$28 \quad 011100$$

$$19 \quad 101101$$

$$\begin{array}{r} 011100 \\ 101101 \\ \hline 1001001 \end{array}$$

Delete

$$\text{Result} = (001001)_2 = (9)_{10}.$$

$$(19)_{10} - (28)_{10}$$

$$19 = 010011$$

$$28 = 011100$$

$$\begin{array}{r} 2's \text{ complement of } 28 \Rightarrow 011100 \\ 100011 \\ \hline 1 \\ \hline 100100 \end{array}$$

$$\begin{array}{r} 19 \\ 2's, 28 \\ \hline 010011 \\ 100100 \\ \hline 110111 \end{array} \rightarrow \text{NO carry.}$$

Result is -ve.

Then 2's complement of Result:

$$\begin{array}{r} \rightarrow 001000 \\ \hline 001001 \end{array} \quad \text{The result} = -(\underline{\underline{001001}})_2$$

Subtract using 2's Complement

(Q). $4 - 9$

$$4 = 0100$$

$$9 = 1001 \rightarrow 2's \text{ complement}$$

$$\begin{array}{r} 0110 \\ 1 \\ \hline 0111 \end{array}$$

$$\begin{array}{r} 0100 \\ 0111 \\ \hline 1011 \end{array}$$

NO carry.
result -ve.
take 2's complement

$$\begin{array}{r} 0100 \\ 1 \\ \hline 10101 \end{array}$$

$$-(0101)_2 = (-5)_{10}$$

(Q). $8 - 2$

$$1000 \rightarrow 8$$

$$0010 \rightarrow 2$$

Take 2's complement of 2.

$$\begin{array}{r} 1101 \\ 1 \\ \hline 1110 \end{array}$$

$$1000$$

$$1110$$

$$\begin{array}{r} 110110 \\ \hline \end{array}$$

neglect

result is +ve.

$$\boxed{\text{Result} = 0110_2} \quad (6)_{10}$$

Computer Division method.

It uses successive subtraction.

dividend is 00110110.

divisor is 0110.

The Answer is . 0000 1001
8 bit Number. .

①. Add $(28)_{10}$ and $(15)_{10}$ Converting them into binary

$$(28)_{10} = 11100$$

$$(15)_{10} = 1111$$

Carry 1 1

1 1 1 0 0

1 1 1 1

$$\underline{\hspace{2cm}} \\ (101011)_2$$

28

+ 15

43

$$\begin{array}{r} 2 \overline{) 28} \\ 2 \overline{) 14} - 0 \\ 2 \overline{) 7} - 0 \\ 2 \overline{) 3} - 1 \\ \hline 1 - 1 \end{array}$$

Subtraction using 1's Complement Method.

operation $m - n$ using 1's Complement.

1. Take 1's Complement of n

2. Result $\leftarrow m + 1$'s Complement of n .

3. If carry is generated, result is +ve, add in the true form. Add carry to the result to get the final result.

4. If carry is not generated then the result is -ve and in the 1's Complement-form.

perform: ① $(28)_{10} - (19)_{10}$ using 1's Complement

$(19)_{10}$ Binary 010011
1's 101100.

$(28)_{10}$ Binary 011100.

Carry 111
28 011100
19 + 101100

1001000
1

001001 → Result $(9)_{10}$.

② $(19) - (28)_{10}$.

Binary equivalent of 19 = 010011

Binary equivalent of 28 = 011100

Take 1's Complement of 011100

100011 → 1's comp 28.

Carry 11
010011
100011

110110
001001

No carry, Negative Result
Take 1's complement

$-(001001)_2 = (-9)_{10}$ Result.

Boolean Algebra Theorems

In 1854 George Boole invented a systematic treatment of logic for algebraic system.

Boolean algebra is a system of mathematical logic. It differs from both ordinary algebra and the binary number system.

Variable:- The symbol which represents an arbitrary elements of an Boolean Algebra is known as variable. Variables are called biliteral, these can take on two values, binary 1 or 0. It may be any alphabet.

Constant:- The constant value may be '1' or 0. For example in expression $y = A + 1$ here A is a variable, 1 is constant.

Complement: A complement of a variable is represented by a bar over the letter.

Example: Complement of $A = \bar{A}$.

A prime symbol ' is also used instead of bar.

Literal:- Each occurrence of a variable in Boolean function either in a complemented or an uncomplemented form is called a literal.

Boolean function:-

Boolean expressions are constructed by combining the Boolean constants and variables with the Boolean operations. These Boolean expressions are called Boolean functions.

Example. (1). $f(A, B, C) = (A + \bar{B}) \cdot C$
 $f = AB + BC + CA$

Boolean Laws.

AND Law - Multiplication Symbol ($\cdot$)

$$Y = A \text{ AND } B \Rightarrow Y = A \cdot B.$$

In AND operation, the output is 1 only if both A and B are 1; otherwise it is zero.

Truth Table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR Law - Addition symbol (+).

$$Y = A \text{ OR } B \Rightarrow Y = A + B.$$

In the OR operation, the result is '1' if either A or B or both are 1.

Truth Table.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT - Complement symbol (Super -, Super')

$$Y = \bar{A}$$

$$Y = A'$$

In NOT operation, the output is the result of input

Truth table

A	Y
0	1
1	0

Special Case;

AND operation : $A \cdot 0 = 0$ $A \cdot 1 = A$, $A \cdot A = A$ $A \cdot \bar{A} = 0$

OR operation $A + 0 = A$ $A + 1 = 1$ $A + A = A$ $A + \bar{A} = 1$

NOT operation $\overline{\bar{A}} = A$

$$\overline{\overline{A}} = A$$

$$\overline{\overline{\overline{A}}} = A.$$

Identity Law: $1 \cdot A = A$ $0 + A = A$

(16)

Laws of Boolean Algebra

Three Basic algebra laws.

- Commutative Law.
- Associative Law.
- Distributive Law.

Idempotent law

Idempotent Law is shown below.

ent $A + A = A$

$A \cdot A = A$

Redundance law (absorption law)

$A + AB = A$

$A(A+B) = A$

Inverse law Null Law

$A + \bar{A} = 1$

$0 \cdot A = 0$ $1 + A = 1$

$A \cdot \bar{A} = 0$

Commutative law

Law 1: $A + B = B + A$. This states that the order in which the variables are ORed makes no difference in the output.

Proof. Law 1.

$A + B = B + A$

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

B	A	B + A
0	0	0
0	1	1
1	0	1
1	1	1

Law 2

$AB = BA$

The commutative law of multiplication states that the order in which the variables are ANDed makes no difference in the output.

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

B	A	BA
0	0	0
0	1	0
1	0	0
1	1	1

Associative Law

This law states that in the ORing of several variables the result is the same regardless of the grouping of the variables.

Law 1: $A + (B + C) = (A + B) + C$

Law 2: - $(AB)C = A(BC)$. The associative law of multiplication states that it makes no difference in what order the variable are grouped when ANDing several variables.

Distributive Law:-

Law: $A(B+C) = AB+AC$

The distributive law states that ORing several variables and ANDing the result with a single variable is equivalent to ANDing the result with a single variable with each of the several variables & then ORing the products.

Some identities of Boolean algebra

Name	AND form	OR form.
① Identity law	$1 \cdot A = A$	$0 + A = A$
② Null Law	$0 \cdot A = 0$	$1 + A = 1$
③ Idempotent law	$A \cdot A = A$	$A + A = A$
④ Inverse law	$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$
⑤ Commutative law	$A \cdot B = B \cdot A$	$A + B = B + A$
⑥ Associative law	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$	$(A + B) + C = A + (B + C)$
⑦ Distributive law	$A + BC = (A + B) \cdot (A + C)$	$A(B + C) = A \cdot B + A \cdot C$
⑧ Absorption law	$A \cdot (A + B) = A$	$A + AB = A$
⑨ Demorgan's law	$\overline{A \cdot B} = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A} \cdot \bar{B}$

Proof. Distributive law.

$$\begin{aligned}
 (A+B)(A+C) &= A \cdot A + AB + AC + BC \\
 &= A + AB + AC + BC \\
 &= A[1 + (B+C)] + BC \\
 &= A + BC \quad \text{as } 1 + (B+C) = 1.
 \end{aligned}$$

Absorption law

$$\begin{aligned}
 A(A+B) &= AA + AB \\
 &= A + AB \\
 A(1+B) &= \underline{A}
 \end{aligned}$$

Demorgan's Theorem :-

$$\boxed{\overline{A+B} = \overline{A} \cdot \overline{B} \text{ and } \overline{A \cdot B} = \overline{A} + \overline{B}}$$

It states that any logical binary expressions remain unchanged if we.

1. Change all variables to their complements.
2. Change all AND operations to ORs.
3. Change all OR operations to ANDs and
4. Take the complement of the entire expression.

Truth table to prove Demorgan's Theorems.

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A+B}$	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0
1	0	0	1	1	1	0	0
1	1	0	0	0	0	1	1

Solve :-

$$\text{Ques } x + \overline{x} \cdot y = x + y$$

$$1. x + \overline{x} \cdot y$$

$$x \cdot (1+y) + \overline{x}y$$

$$x + xy + \overline{x}y$$

$$x + y(x + \overline{x})$$

$$x + y$$

prove that.

$$x \cdot y + y \cdot z + \overline{y} \cdot z = x \cdot y + z$$

Consider LHS.

$$x \cdot y + y \cdot z + \overline{y} \cdot z = xy + z(y + \overline{y})$$

$$= xy + z \cdot 1$$

$$= xy + z \cdot \text{RHS}$$

prove that

$$x + y \cdot z = (x + y)(x + z)$$

$$x + yz$$

$$1 \cdot x + yz$$

$$x \cdot (1+y) + yz$$

$$x + xy + yz$$

$$x \cdot 1 + xy + yz$$

$$x(1+z) + xy + yz$$

$$= x \cdot 1 + xz + xy + yz$$

$$= x \cdot x + xy + xz + yz$$

$$= x(x+y) + z(x+y)$$

$$= (x+z)(x+y)$$

Digital circuits

Digital circuit are divided into two major categories.

- Combinational circuits
- Sequential circuits.

Combinational circuits are circuits where output depends on the present input only.

The sequential circuit produces the output on the basis of both present & previous inputs. Sequential ckt's have memory.

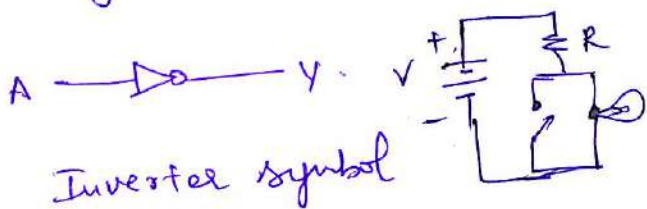
Logic gates: are the building blocks of digital circuits. A logic gate is an electronic device with a single output having one or many inputs. The output is controlled by the input. gates used to create digital ckt's & complex IC. Complex Integrated circuits used to perform several functions

Eg. Microprocessor & Microcontroller.

① Not gate:-

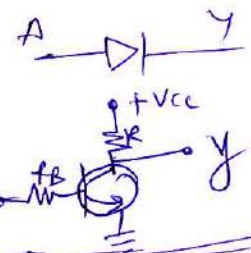
It is also known as an inverter, because it changes the input to its opposite.

A Low voltage input (0) is converted to a high voltage output; a high voltage (1) is converted to low voltage (0).



Input A	Output Y
0	1
1	0

Truth table

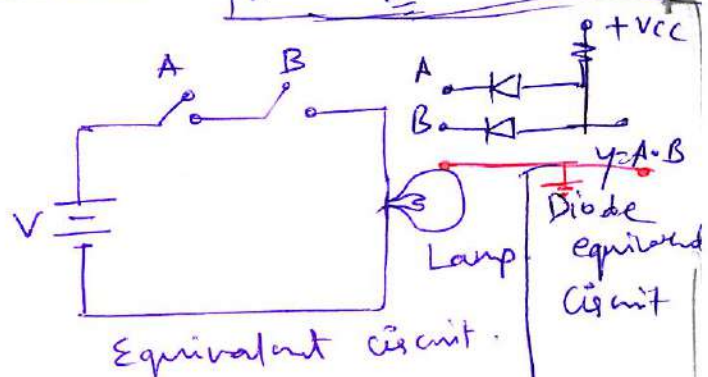


② AND gate:-



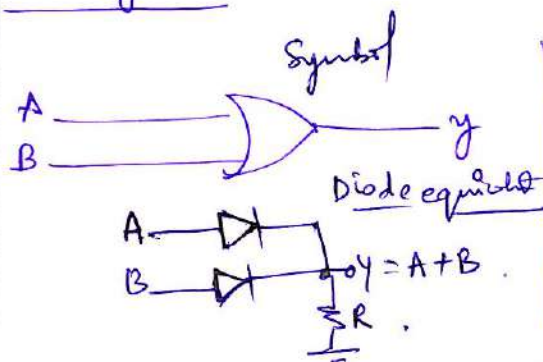
Input A	Input B	Output Y
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

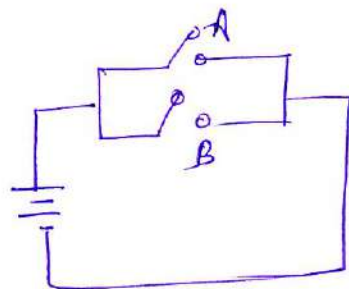


The output of AND gate is 1, only when all its inputs are also '1'. otherwise, its output will be '0'.

OR gate



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

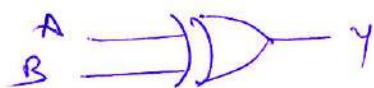


Equivalent circuit

The output of OR gate will be '0' when all its inputs are also '0'. otherwise its output will be '1'.

XOR gate:-

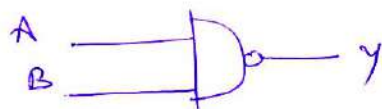
XOR stands for Exclusive OR. An XOR gate compares two values & if they are different. So its output will only be at '0' when all its inputs have the same value. otherwise, its output will be '1'.



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

NAND gate:-

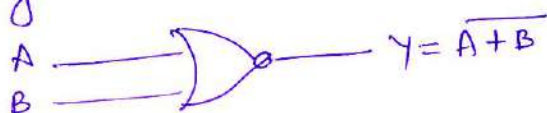
This is an AND gate with the output inverted. The output is high when either of inputs A or B is high. low if neither is high.



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

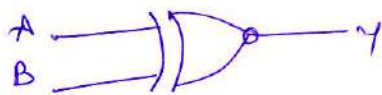
NOR gate:-

The OR gate with the output inverted is called NOR operation. The output is high only when neither A nor B are high.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR gate: - It is the complement of XOR gate.
 Its output is 1 when both inputs are equal i.e. (0,0) or (1,1).
 XNOR operation is represented as $Y = A \odot B$.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Non-Inverter or buffer

The value entered on its input will be found on its output.
 A typical appⁿ for a buffer is to increase the fan-out of a given logic gate.

Fan out is the maximum ^{number of} gates ~~can~~ a given Integrated Ckt is capable of being connected to.



A	Y
0	0
1	1

Algebraic simplification.

Factorize the following equations.

- (a) $Y = A\bar{B} + AB$
- (b) $Y = AB + AC + BD + CD$
- (c) $Y = (B + CA)(C + \bar{A}B)$
- (d) $Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD$

(a) $Y = A\bar{B} + AB$
 $= A(\bar{B} + B)$
 $= A \cdot 1$
 $= A$

(b) $Y = AB + AC + BD + CD$
 $= A(B + C) + D(B + C)$
 $= (A + D)(B + C)$

(c) $Y = (B + CA)(C + \bar{A}B)$
 $= BC + CCA + \bar{A}BB + \bar{A}ABC$
 $= BC + CA + \bar{A}B + 0$; $A\bar{A} = 0$ $B \cdot B = B$ $CC = C$
 $= BC + \bar{A}B + AC$

$$(d) Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD$$

$$= \bar{A}\bar{B}\bar{C}(\bar{D}+D) + \bar{A}\bar{B}C(\bar{D}+D)$$

$$= \bar{A}\bar{B}\bar{C} \cdot 1 + \bar{A}\bar{B}C \cdot 1$$

$$= \bar{B}\bar{C}(\bar{A}+A)$$

$$= \bar{B}\bar{C} \cdot 1$$

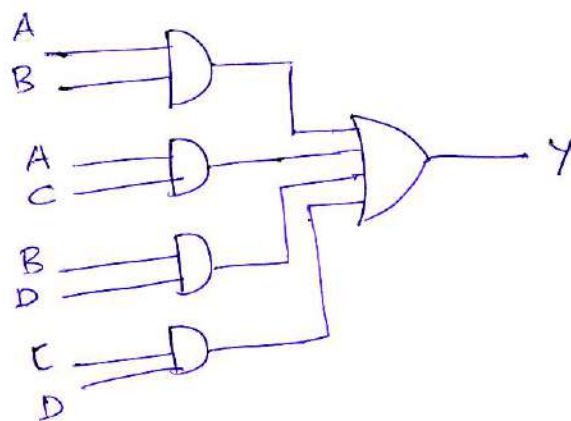
$$= \bar{B}\bar{C}$$

Compare the AND/OR gate realisation of the Boolean function of Example given below & its reduced form.

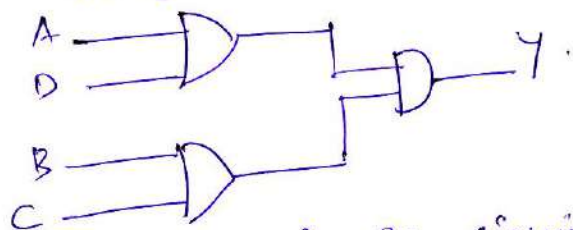
$$Y = AB + AC + BD + CD$$

$$= A(B+C) + D(B+C)$$

$$= (A+D)(B+C)$$

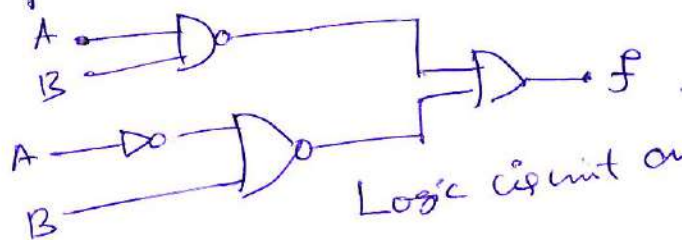


The simplified expression consists of only one AND and two OR gate.



4 AND gates & 1 OR gate necessary implement above expression.

Analyze the logic circuit of figure below and show that it can be replaced by a single NAND gate.



Logic circuit analysis.

$$f = AB + A + B$$

$$= (\bar{A} + \bar{B}) + AB$$

$$= \bar{A} + [\bar{B}(1+A)] = \bar{A} + \bar{B} \cdot 1 = \bar{A} + \bar{B} = \overline{A \cdot B} = \text{Demorgan's law}$$

Hence the above circuit is simply equivalent to NAND gate.

Sum-of product (SOP)

It is widely used for the Canonical form i.e. a OR (disjunction) of minterms. Its De Morgan dual is a product of sum (POS).

POS for the Canonical form is a Conjunction (AND) of maxterms.

Minterms are called products because they are the logical AND of a set of variables.

Maxterms are called the sums because they are the logical OR of set of variable.

Example of SOP.

$$F = AB + \bar{C}\bar{D} + \bar{B}C$$

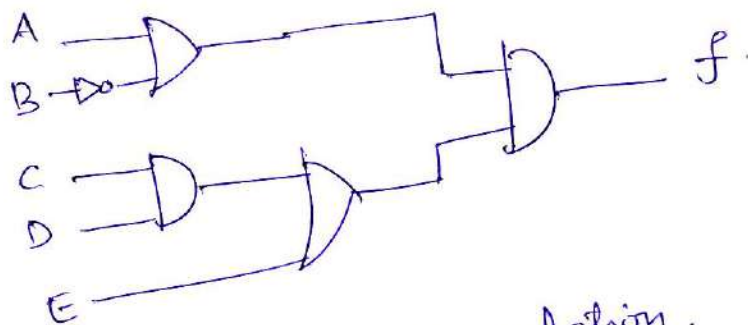
$$Y = A\bar{B} + \bar{A}B$$

POS

$$F = (A+B)(\bar{B}+C)(\bar{C}+D+E).$$

$$Y = (A+B)(C+D).$$

- ①. For the Boolean function $f = (A+\bar{B})(CD+E)$. Obtain the AND-OR Implementation.



AND-OR Implementation.

Simplify and realize using basic gates.

$$Y = A + \bar{A}B + ABC\bar{C}$$

$$= A(1 + B\bar{C}) + \bar{A}B$$

$$= A \cdot 1 + \bar{A}B$$

$$= A + \bar{A}B$$

$$= (A + \bar{A})(A + B)$$

$$= 1 \cdot (A + B)$$

$$Y = \underline{A + B}$$

$$\left| \begin{array}{l} A + BC = (A + B)(A + C) \\ A + \bar{A} = 1 \end{array} \right.$$



Simplify -

$$Y = A + \bar{A}B + ABC + A\bar{C} + AB$$

$$= A \cdot A + \bar{A}B + ABC + A\bar{C} + 0 + AB$$

$$= A \cdot A + \bar{A}B + ABC + A\bar{C} + A\bar{A} + AB$$

$$= \overset{\checkmark}{A \cdot A} + \bar{A}B + A\bar{A} + \overset{\checkmark}{AB} + ABC + A\bar{C}$$

$$= A(A + B) + (\bar{A}(B + A) + ABC + A\bar{C})$$

$$= (A + \bar{A})(A + B) + ABC + A\bar{C}$$

$$= 1 \cdot (A + B) + A(B\bar{C} + \bar{C})$$

$$=$$

$$= 1 \cdot (A + B) + A[(B + \bar{C})(C + \bar{C})]$$

$$= (A + B) + A(B + \bar{C})$$

$$= A + A(B + \bar{C}) + B$$

$$= A[1 + B + \bar{C}] + B$$

$$= A + B$$

Simplify

$$(i) Y = \bar{A}C + \bar{A}\bar{C}$$

$$= \bar{A}C + \bar{A} + \bar{C}$$

$$= \bar{A}(C+1) + \bar{C}$$

$$Y = \bar{A} + \bar{C}$$

$$(ii) Y = \overline{AB + \bar{A}B + A}$$

$$Y = \overline{AB} \cdot \overline{\bar{A}B} \cdot \bar{A}$$

$$= (\bar{A} + \bar{B}) \cdot AB \cdot \bar{A}$$

$$= 0$$

$$(\bar{A} \cdot A = 0)$$

$$(iii) Y = \overline{(\bar{A} + C)(B + \bar{D})}$$

$$= \overline{(\bar{A} + C)(B + \bar{D})}$$

$$= \overline{(\bar{A} + C)} + \overline{(B + \bar{D})}$$

$$= (A \cdot \bar{C}) + \bar{B}D$$

$$\boxed{Y = A\bar{C} + \bar{B}D}$$

Combinational Circuit

These are the circuits whose output is dependent only on the state of its inputs. The output is a pure function of the present input. Does not depend on past values.

Examples of these circuits are

Half adder

Full adder

Multiplexer

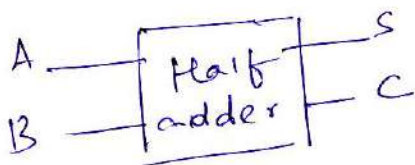
Decoder

The following example will be further discussed.

Half adder

It is a simple functional digital circuit built from two logic gates. The half adder adds two one-bit binary numbers (A and B). The output is the sum of the two bits (S) and the carry (C).

The truth table & schematic diagram is shown in figure below



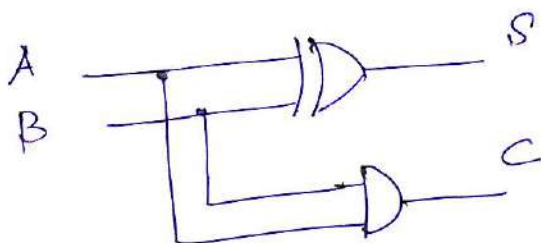
Logic symbol

$$S = \bar{A}B + A\bar{B} = A \oplus B$$

$$C = AB$$

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

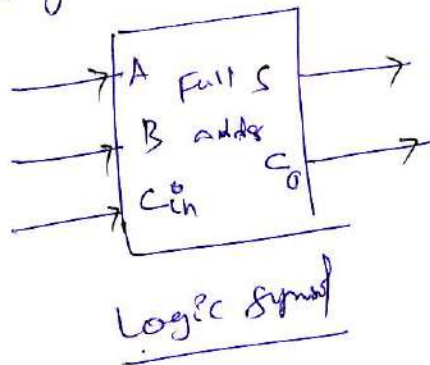
Truth table.



Full adder

The full adder circuit adds three one-bit binary numbers (A, B, C), and outputs two one-bit binary numbers, a sum (S) and a carry (C_o).

The Full adder is simply two half adders joined by an OR gate. The Logic symbol, Truth table & Circuit diagram is shown below.



A	B	C _{in}	S	C _o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

From Truth Table

$$S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$= \bar{A}\bar{B}C + ABC + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$= (\bar{A}\bar{B} + AB)C + (\bar{A}B + A\bar{B})\bar{C}$$

$$= (\overline{A \oplus B})C + (A \oplus B)\bar{C}$$

$$= \text{let } x = A \oplus B$$

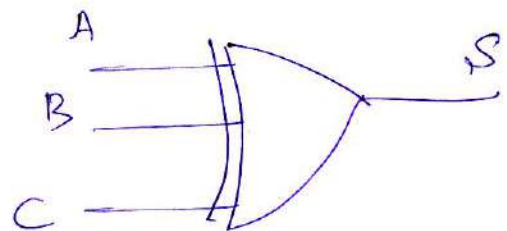
$$= \bar{x}C + x\bar{C}$$

$$= x \oplus C$$

$$S = \underline{\underline{A \oplus B \oplus C}}$$

sum using logic gates is shown in fig

Table: Truth Table



$$S = A \oplus B \oplus C$$

$$\text{Carry} = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC + ABC + ABC$$

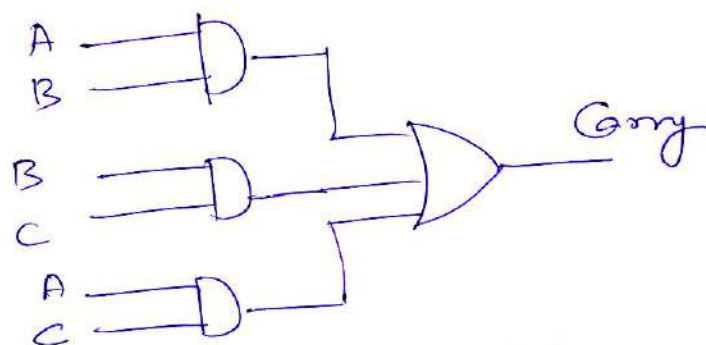
$$= \bar{A}BC + ABC + A\bar{B}C + ABC + AB\bar{C} + ABC$$

$$= (\bar{A}+A)BC + (\bar{B}+B)AC + (\bar{C}+C)AB$$

$$= BC + AC + AB$$

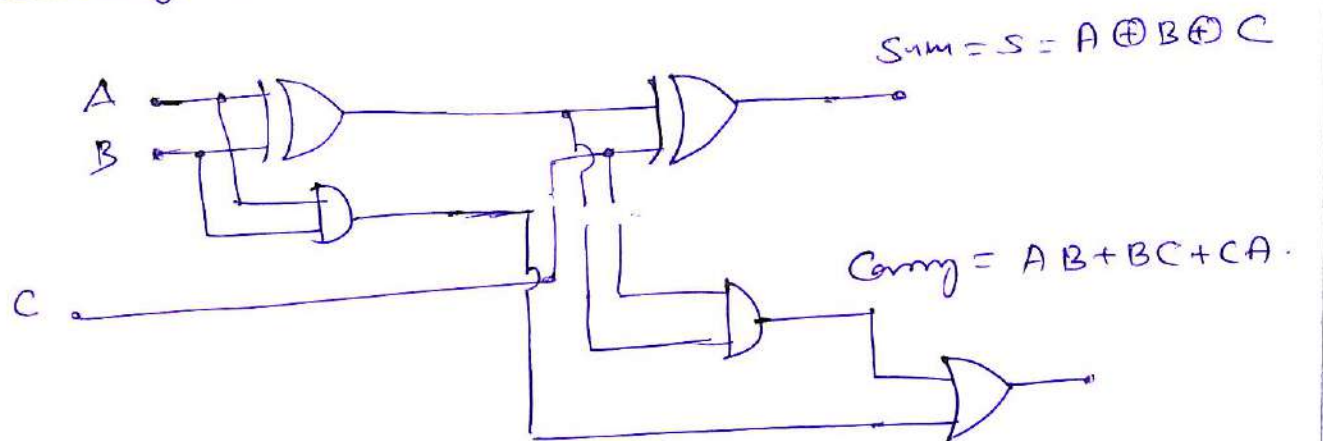
$$\text{Carry} = AB + AC + BC$$

It can be implemented by three AND gate & one OR gates.



Carry of Full adder

Realizing Full adder using Half adder & OR gates.



$$\text{Carry} = AB + C(A \oplus B)$$

$$= AB(C + \bar{C}) + C(\bar{A}B + A\bar{B})$$

$$\text{Carry} = ABC + AB\bar{C} + \bar{A}BC + A\bar{B}C$$

Ripple - carry - adder

A ripple carry adder is a logic circuit in which the carry-out of each full adder is the carry-in of the succeeding next most significant full adder. It is called a ripple carry adder because each carry bit gets rippled into the next stage.

The full adder is a component in a cascade of adders, which adds 8, 16, 32 bit etc binary numbers.

A n-bit adder can be designed by connecting the carry out and carry in lines of n-full adders.

Fig shows the 4-bit adder. This adder is called ripple - carry adder. Further, four 4-bit adders can be connected to form a 16-bit adders etc.

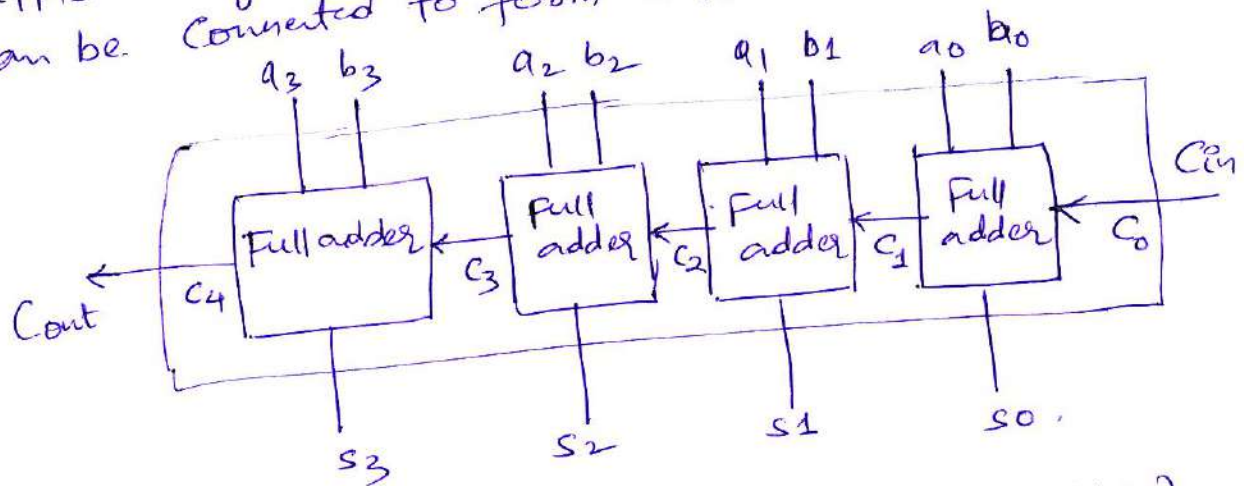


Fig: 4-bit adder (ripple carry adder)

Example ①.

$$\begin{array}{r}
 0 = C_{in} \\
 1111 = A \\
 0001 = B \\
 \hline
 1000 \\
 \text{Cout } s_3 s_2 s_1 s_0
 \end{array}$$

Example ②

$$\begin{array}{r}
 C_{in} = 0 \quad 1 \\
 A = 1111 \\
 B = 1111 \\
 \hline
 11111 \\
 \text{Cout } s_3 s_2 s_1 s_0
 \end{array}$$

Multiplexers

It is a logical circuit that selects one of several analog or digital input signals and forwards the selected input into a single line.

Multiplex means many to one. Digital multiplexers provide the digital equivalent of an analog selector switch.

A digital multiplexer connects one of 2^n inputs to a single output line, so that the logical value of the input selected is transferred to the output with the help of select lines.

The objective of a multiplexer is to select one signal from a group of 2^n inputs, to be an output on a single output line.

Block diagram or black box representation of multiplexer (mux) is shown below in figure.

The data lines $D_0, D_1, D_2, D_3, D_4, D_5, D_6, D_7$ are the data input lines & F is the output line. Lines A, B , and C are called the select lines.

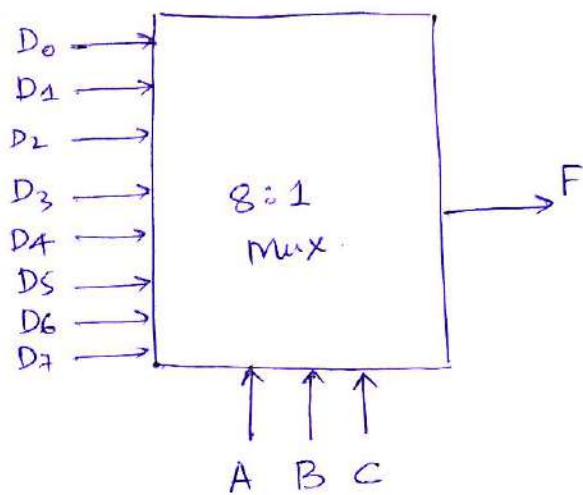
The select lines are interpreted as a three-bit binary number, which is used to choose one of the D -lines to be output on the line F .

The $2^3 = 8$ input lines selected with 3 select lines & one data line is output at one time based on select lines.

Implementation:

mux is designed with a regular pattern of AND and OR gates as shown in figure.

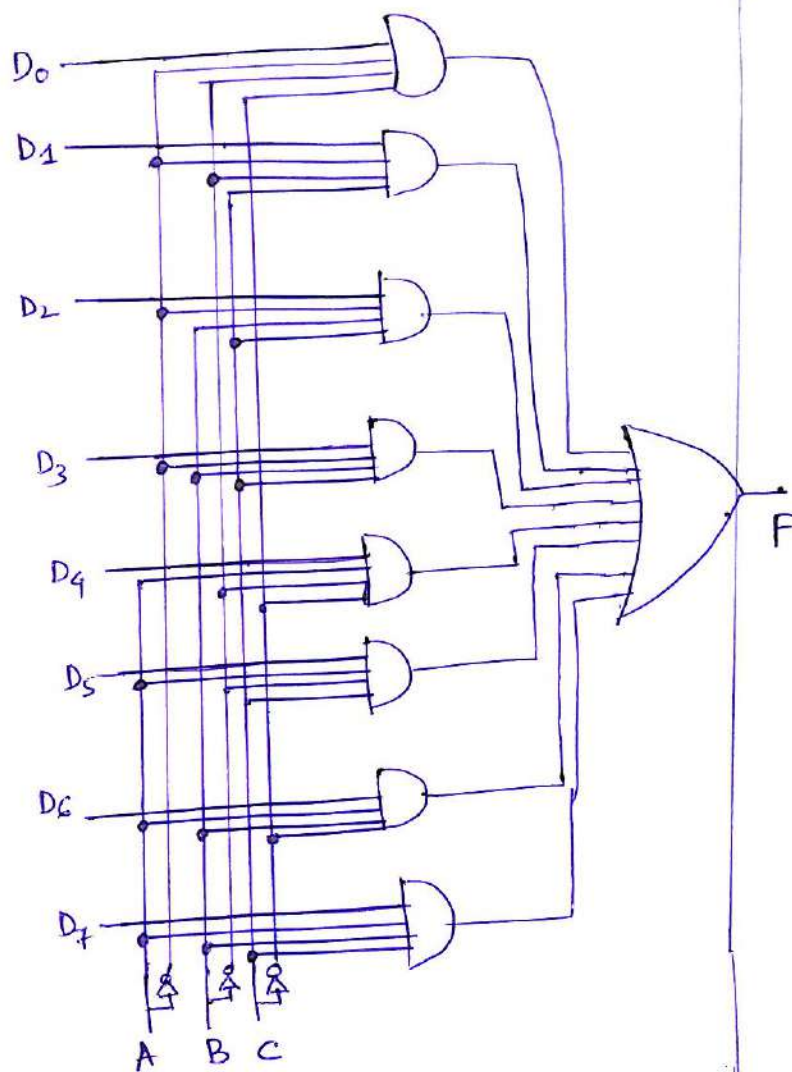
Truth table is shown below.



8-to-1 Multiplexer

Select lines			Output
A	B	C	F
0	0	0	D_0
0	0	1	D_1
0	1	0	D_2
0	1	1	D_3
1	0	0	D_4
1	0	1	D_5
1	1	0	D_6
1	1	1	D_7

Truth Table



Implementation of an 8-to-1 Multiplexer.

For Example,
Suppose we want to output $Y = D_5$, then the three inputs to the corresponding AND gate should be 101. Then

$$\left. \begin{array}{l} \text{if } D_5 = 0, F = 0 \\ D_5 = 1, F = 1 \end{array} \right\} F = D_5$$

The corresponding select inputs are
 $5 \rightarrow 101$ or $A\bar{B}C$

Application :-

Choose one of several registers to be used as ALU (Arithmetic Logic unit) input. A 2^n -to-1 multiplexer can be used to implement an arbitrary Boolean function of n variables, by associating each input line of the multiplexer with a row of the truth table for the function.

Decoder :-

A decoder is a multiple-input, multiple output logic circuits which converts coded inputs into coded outputs, where the input and the output codes are different. The input code has lesser bits than the output code. A binary decoder generally has n inputs and 2^n outputs. The binary decoder activates one of the 2^n outputs based on the input applied.

The objective of the decoder is to decode an n -bit binary number, producing a signal on one of 2^n output lines. Figure shows an 3-to-8 decoder.

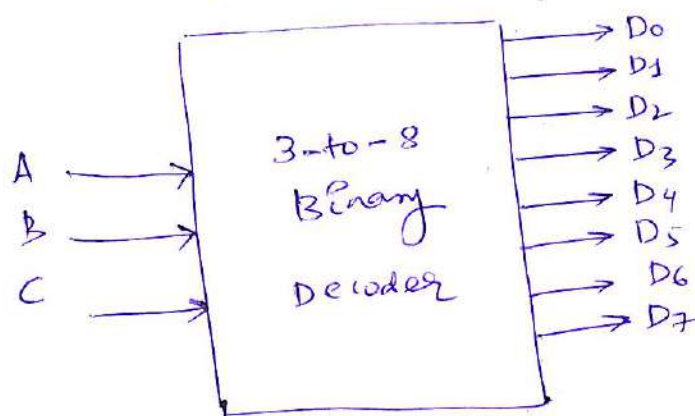


Fig: 3-to-8 decoder

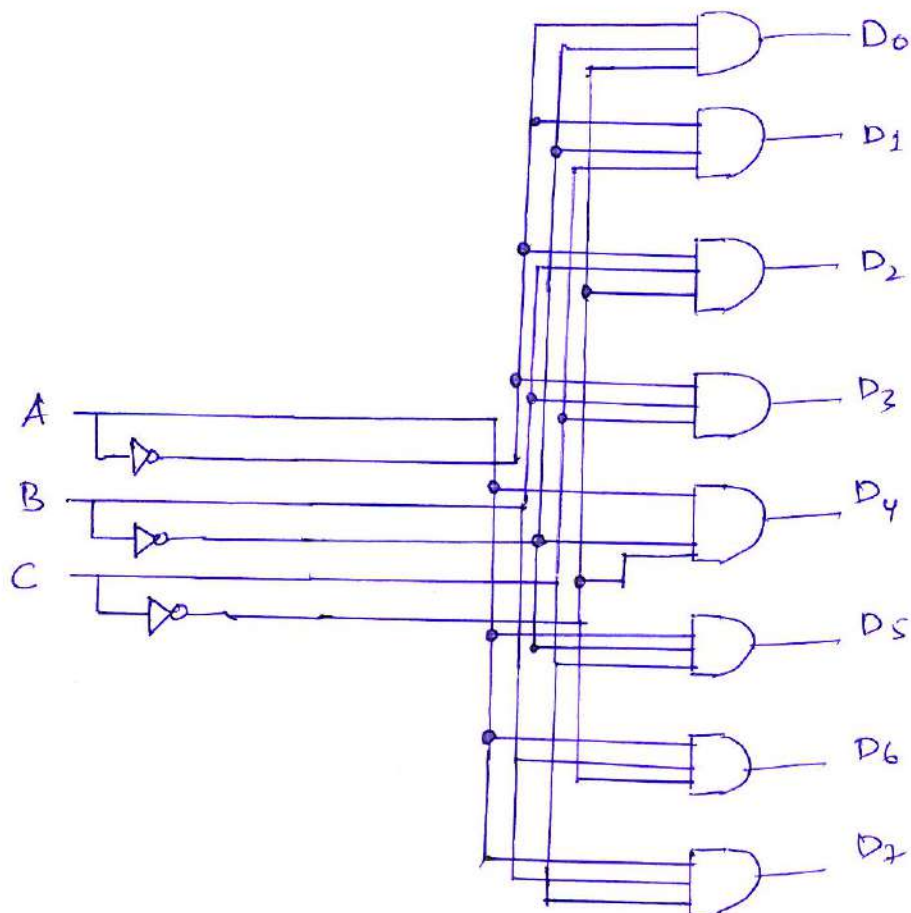
Implementation :-

A decoder is often implemented with an additional input called an "enable" line. When the line is enabled, the circuit is a decoder. When it is disabled, all the outputs are '0'.

The same circuit can be used as a de-multiplexer, which directs a single data input line to one of 2^n output lines, depending on the values of n select lines.

Figure shows one 3-to-8 decoder implementation.

A	B	C	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



3-to-8 decoder.

SR FlipFlop / SR Latch.

The simplest memory circuit and Example of Sequential circuit is SR FlipFlop / SR Latch.

A FlipFlop is an electronic circuit which has memory. It is used to store one bit memory. If its output is 1/0, it remains same unless input changes on change in input, its output changes (Flips) to 0/1 and then remains constant.

A FlipFlop is a basic element of all sequential system.

Operation :-

A NOR gate produces output 1 only when both inputs are 0, otherwise the output is 1.

Latch is circuit that has two stable states & can be used to store state information.

It holds one bit of data. Latches are example of sequential circuits and are memory elements.

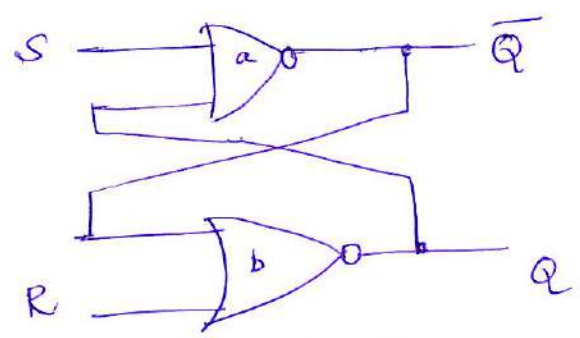
SR latch using NOR gate is explained below.

S means set state output $Q = 1$ and $\bar{Q} = 0$

R means Reset state output $Q = 0$ and $\bar{Q} = 1$.

Previous values are very important here.

Assume SR latch / FlipFlop is in set state.



SR FlipFlop.

Inputs		Present state Q_n	next state Q_{n+1}	
S	R			
0	0	0	0	No change
0	0	1	1	
0	1	0	0	reset
0	1	1	0	
1	0	0	1	set
1	0	1	1	
1	1			Not allowed

Case ①. Let SR FF be in set state, we make $S=0$ and $R=0$. The input to 'a' NOR are $(0,1)$; So $\bar{Q}=0$, & inputs to 'b' NOR $(0,0)$; So $Q=1$. Therefore SR FlipFlop remains in set state i.e no state change.

It similarly shows that when the SR FF in reset state we make $S=0, R=0$ it remains in reset. hence this state is called No change & inputs are $S=0, R=0$.

Case ②.
If output is in set state ($Q=1, \bar{Q}=0$) we input $S=0, R=1$, gate 'b' conducts first $(1,0)$ results in $Q=0$. hence the output changes from set to reset state. But if the output was in reset state ($Q=0, \bar{Q}=1$) and $S=0, R=1$, gate 'b' conducts first produces the output $Q=0$. hence the result is for $S=0, R=1$. $Q=0$. hence ^{this} state is called Reset state.

Case ③. If the output is in reset state ($Q=0, \bar{Q}=1$) & $S=1, R=0$, then gate 'a' conducts first & produces output $\bar{Q}=0$ later gate 'b' produces output $Q=1$. hence for $S=1, R=0$, the value of $Q=1$ (set). if the output is in set state, ($Q=1, \bar{Q}=0$) & $S=1$ and $R=0$, then also the output $Q=1, \bar{Q}=0$. hence this state output is called as set state.

Case ④
Both $S=1$ and $R=1$ are not permitted as this would cause a conflict, & outcome would be uncertain.
The above operations of SR FlipFlop are given in table.

Clocked SR Flip Flop.

With in a digital Computer, a clock is used to synchronise changes in the contents of memory elements. A clock is a signal which oscillates between 0 and 1 as shown in figure.

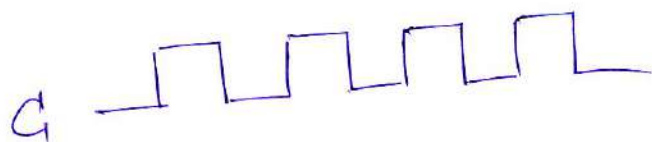


Fig: clock signal.

The clock can be applied to a latch so that change in the latch's value can only occur when the clock is in the '1' (high) state.

The clocked SR latch/FF is shown below.

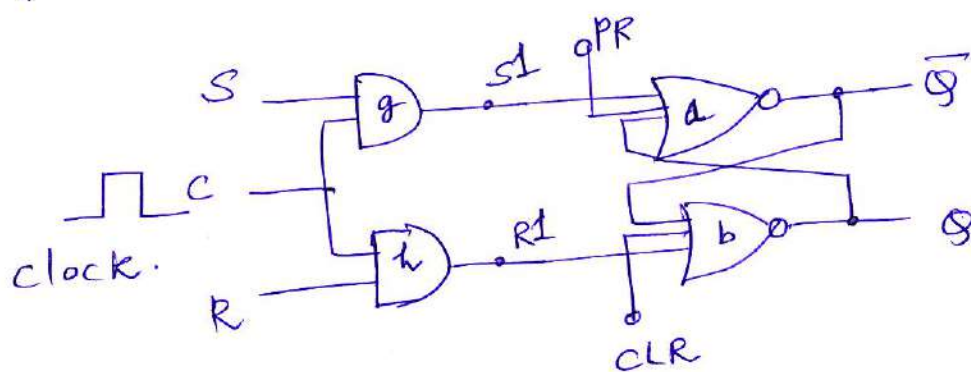


Fig. Clocked SR Flip Flop.

When $C=0$, the output of both AND gate g and h are zero. So Q cannot change, the latch is non-operational.

When $C=1$, output of gate g is 1 then $S1=S$ & output of gate h is 1. $R1=R$, the latch is now operational. The operational procedure is same as discussed in previous section.

The term Level triggered is applied to latches to indicate that its ability to change value depends on the level (low or high) of the clock signal.

The term edge triggered is used with clocked FlipFlop we have positive edge triggering and Negative edge triggering also.

positive edge triggering



The state change from Zero to One i.e. Low to High.

Negative edge triggering



The output changes when there is a clock signal change from high to low.

preset and clear.

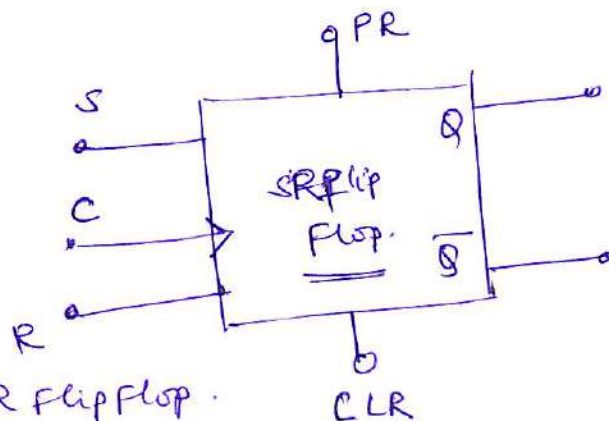
In figure one additional input is provided to each NOR gate; CLR on Q side NOR and PR on $\bar{Q}$ side NOR. When CLR = 0, PR = 0, the circuit behaves as an SR FlipFlop. The block representation is shown below.

In absence of a clock pulse (C=0), the output of NOR gates are zero. making CLR = 1 & PR = 0, results in $Q = 0$, $\bar{Q} = 1$ thus by clearing the FlipFlop.

For PR = 1 and CLR = 0, the FF produces $Q = 1$, $\bar{Q} = 0$, thus by presetting the FlipFlop.

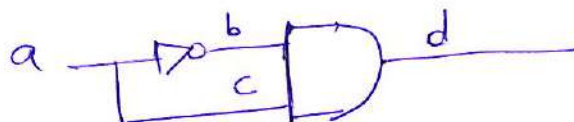
These are asynchronous inputs as their action does not need a clock pulse.

Symbol of SR FlipFlop.



FlipFlops.

FlipFlop are used for more precise synchronization. They are edge-triggered. That means they can change with change in the value of the clock signals; that is on its rising or falling edge. This is done by the use of a small circuit, called a pulse generator, as shown in below figure.



pulse generator (edge triggering).
When 'a' is connected to a clock signal, a short pulse will be generated on 'd', whenever 'a' makes a transition from 0 to 1 and 1 to 0.

There are four commonly used flip-flop.

D-FlipFlop

T-FlipFlop

RS FlipFlop and

JK FlipFlop.

The behaviour of each of them can be described by a state table. The state table shows how the values of the flip-flop change in response to its inputs.

State table of SR FlipFlop / RS FlipFlop.

RS (Reset-Set)		
R	S	Q _{next}
0	0	Q
0	1	1
1	0	0
1	1	undefined

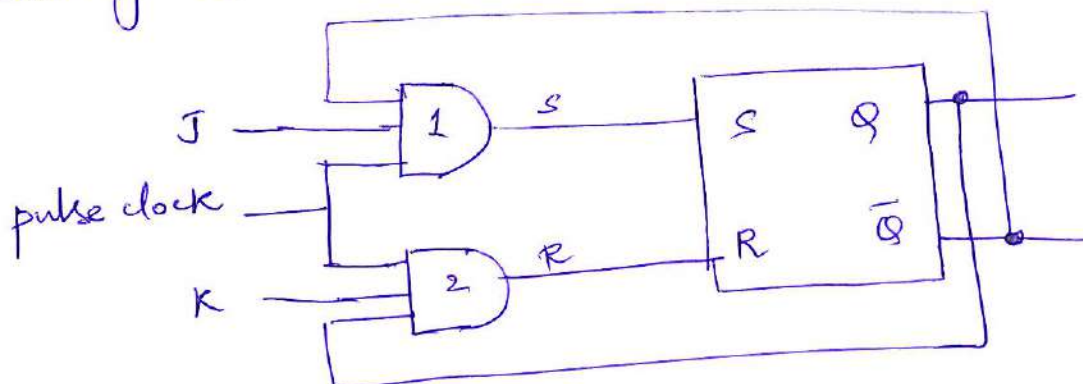
D (Data)	
D	Q_{next}
0	0
1	1

T (Toggle)	
T	Q_{next}
0	Q
1	$\bar{Q}$ (Toggle)

JK FlipFlop		
J	K	Q_{next}
0	0	Q
0	1	0
1	0	1
1	1	$\bar{Q}$

JK FlipFlop.

The undefined state of SR FlipFlop is overcome by converting SR FlipFlop to JK FlipFlop by feeding $\bar{Q}$ to the upper AND & Q to lower AND gate.



JK FlipFlop.

The SR FlipFlop determines the output of JK FlipFlop. The values of S, R are governed by the input J, K and Q .

The outputs for various combinations of inputs (J, K). When a clock pulse is applied & explained.

clock pulse may be positive edge Triggered [PT] or Negative edge triggered).

(i)

(a) when $J=0, K=0, Q=0$ & $\bar{Q}=1$

Then $S=0, R=0$ produced by two AND gates, these are inputs to SR Flipflop. The output of SRFF is $Q_{next}=0, \bar{Q}_{next}=1$. hence this state is No change.

(b) when $J=0, K=0, Q=1$ & $\bar{Q}=0$

Here also $S=0, R=0 \Rightarrow Q_{next}=1, \bar{Q}_{next}=0$, No change

(ii) (a) when $J=0, K=1, Q=0$ and $\bar{Q}=1$

First AND gate produces $S=0$, and second AND gate also produces $R=0$. hence $Q_{next}=0, \bar{Q}_{next}=1$. (No change)

(b) when $J=0, K=1, Q=1$ and $\bar{Q}=0$.

AND gate 1 produces $S=0$,
AND gate 2 produces $R=1$

when $S=0, R=1$ then Q_{next} changes to 0 & $\bar{Q}_{next}=1$.
hence this state is called Reset state.

(iii)

(a) when $J=1, K=0, Q=0$ and $\bar{Q}=1$

First AND gate produces $S=1$ and AND gate 2 produces $R=0$, then the output of SR latch $Q_{next}=1$ (set state) and $\bar{Q}_{next}=0$.

(b) when $J=1, K=0, Q=1$ and $\bar{Q}=0$

AND gates produces $S=0, R=0$. hence this state is No change because SR latch produces output as $Q_{next}=1$ & $\bar{Q}_{next}=0$.

all these are shown in table.

Now consider $(1,1) = (J,K)$ inputs which were entries not permitted in an SR-FlipFlop.

(IV)

② $J=1, K=1, Q=0 \Rightarrow S=1, R=0, Q_{next}=1, \bar{Q}$ set state

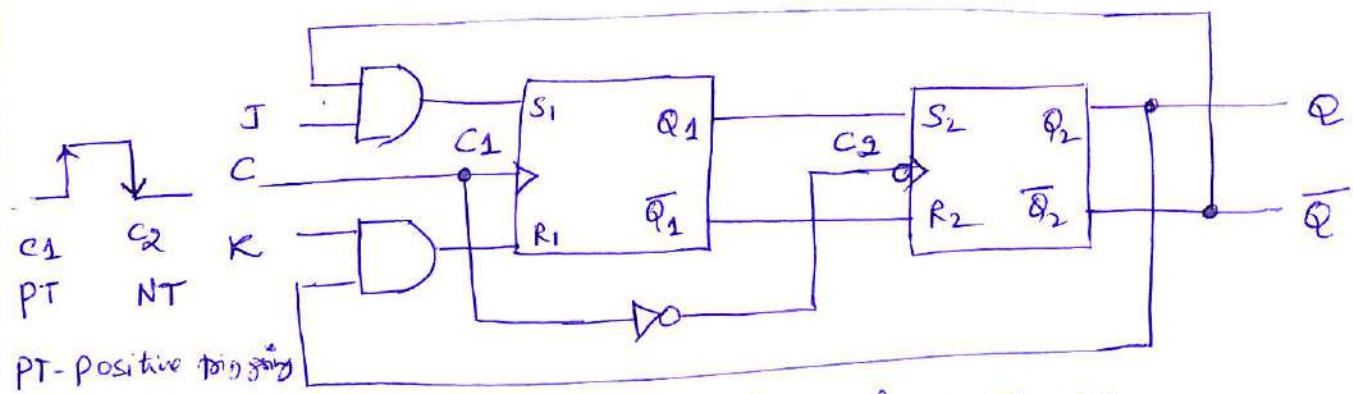
③ $J=1, K=1, Q=1 \Rightarrow S=0, R=1, Q_{next}=0, \bar{Q}$ reset state

This state is called Toggle state because the output toggles i.e. the state changes from 0 to 1, or 1 to 0.

JK master slave FlipFlop.

The toggle state of JK FlipFlop causes the problem. If any clock pulse is too long, the output state will change more than once and the final state of the FlipFlop will be indeterminate. To avoid this problem, a JKFF is constructed with two SR FlipFlops in master slave connection as shown in figure.

In JKFF output is feedback to the input, therefore any changes in the output results in corresponding changes in the input due to this in the positive half of the clock pulse when J & K are both high (1) the output toggles continuously. This condition is known as race around condition.



PT - Positive triggering
NT - Negative triggering

Fig: JK master slave FlipFlop.

operation of JK master slave flip flop.

In master slave JKFF : By virtue of master to slave connection, $S_2 = Q_1$, $R_2 = \overline{Q_1}$.

It means that input to the slave is always 0/1 or 1/0.
 $\therefore$ As pulse C goes low, $C_2 = 1$ and slave transfers the output of the master to the system output $Q_2 = Q_1$.

Consider when $J=1, K=0$ and $Q=0$. Then

$S_1 = 1, R_1 = 0$ when clock pulse $C_1 = C = 1$ then
 $Q_1 = 1$ (set) = S_2 . It is connected to slave at
 negative triggering i.e. $C=0, C_2=1$ the output $Q_2 = Q = 1$ (set).

$$\begin{aligned} \text{(i)} \quad J=1, K=0, Q=0 &\Rightarrow S_1=1, R_1=0 \xrightarrow{C=C_1=1} Q_1=1=S_2 \xrightarrow{C=0, C_2=1} Q_2=Q=1 \\ J=1, K=0, Q=1 &\Rightarrow S_1=0, R_1=0 \xrightarrow{C=C_1=1} Q_1=1=S_2 \xrightarrow{C=0, C_2=1} Q_2=Q=1 \end{aligned}$$

(ii) $J=0, K=0, Q=0$. Set
 master & slave flip flop remains in previous state here
 this state is called Nochange state Nochange

(iii) $J=0, K=1, Q=1$ $\Rightarrow S_1=0, R_1=1 \xrightarrow{C=C_1=1} Q_1=0 \Rightarrow S_2 \xrightarrow{C=0, C_2=1} Q_2=Q=0$.
 $J=0, K=1, Q=0 \Rightarrow S_1=0, R_1=0 \xrightarrow{C=C_1=1} Q_1=0 \Rightarrow S_2 \xrightarrow{C=0, C_2=1} Q_2=Q=0$.
 This state is called Reset state. Reset state

(iv) For $J=1, K=1$ toggling should happen at output without any indeterminate solution.

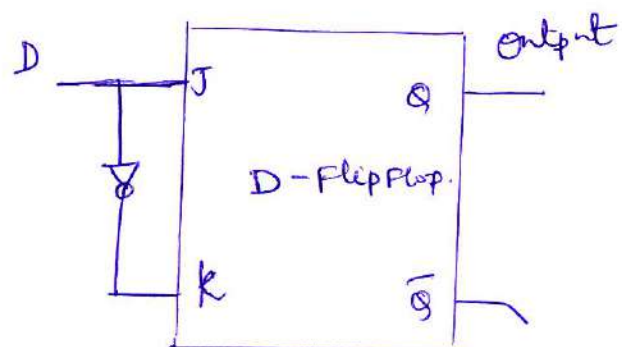
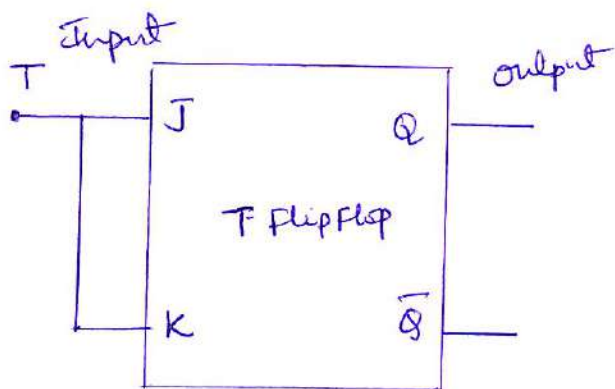
$$\begin{aligned} J=1, K=1, Q=1 &\Rightarrow S_1=0, R_1=1 \xrightarrow{C=C_1=1} Q_1=0 \Rightarrow S_2=0, R_2=1 \xrightarrow{C=0, C_2=1} Q_2=Q=\overline{Q} \\ J=1, K=1, Q=0 &\Rightarrow S_1=1, R_1=0 \xrightarrow{C=C_1=1} Q_1=1 \Rightarrow S_2=1, R_2=0 \xrightarrow{C=0, C_2=1} Q_2=Q=\overline{Q} \end{aligned}$$

Thus confirms the toggle operation of the JK master-slave flip-flop. The master FF operates when $C=1$ & slave operates when $C=0$. When slave operates master is inactive and unwanted toggling cannot occur. Hence JK master slave avoids race-around condition of JK flip flop.

The sequential action of slave (after master has become inoperative) and C goes low, the JK master-slave FF is also called as trailing edge triggered Flip Flop.

T-(Toggle) Flip Flop

In this Flip Flop J and K inputs are tied together. When $T=0$ with the application of the clock pulse the output Q remains in the same state. When $T=1$ output toggles from 0 to 1 or 1 to 0 state with every clock pulse.



D-(Data) Flip Flop

D Flip Flop also called Data flip-flop has one input D. With the application of each clock pulse the data (0 or 1) is transferred to the output Q. The modified JKFF as a FF is shown above. also Refer Truth table in previous pages.

Shift register

The shift register is a sequential circuit that can be used for the storage or the transfer of binary data.

A shift register is a digital circuit constructed by cascade of flip-flops, sharing the same clock, in which the output of each flip-flop is connected to the data input of the next flip-flop in the chain, results in shifts of data.

In Figure, shift register consists of two RS flip-flops when a clock pulse occurs, the value of the left flip-flop is copied to the right flip-flop. This is known as a shift register.

Flip-flops are used within a CPU (central processing unit) to implement register. A register is an ordered group of n -flip-flops.

For example, in the Intel architecture, EAX is a 32-bit register. It can hold a 32-bit binary register.

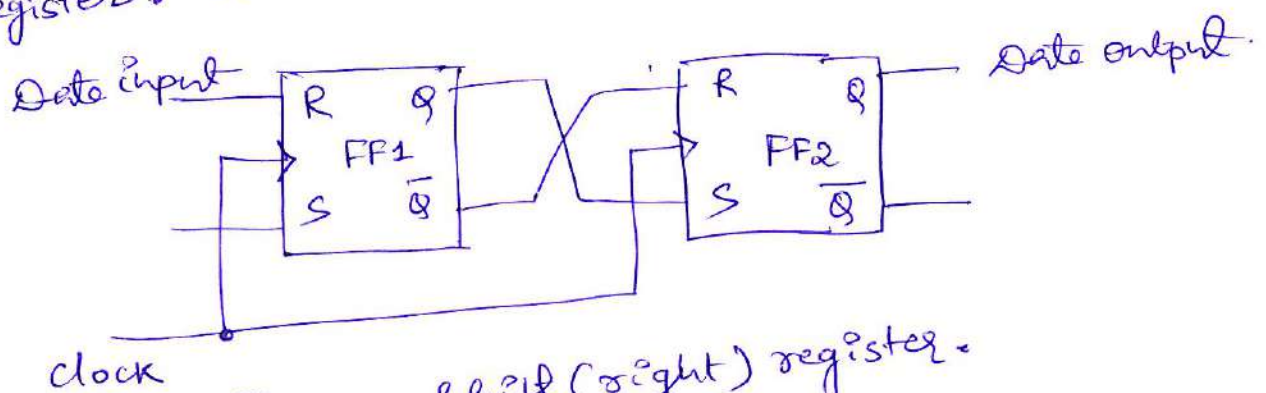


Fig. Shift (right) register.

Simple shift register, the data string presented at data in is shifted right one stage each time. The bit from FF1 is shifted into FF2 input and upon clock pulse shifted at output of FF2.

The binary information in a register can be moved from stage to stage within the register or into or out of register upon application of clock pulses.

This type of movement or shifting is essential for certain arithmetic and logic operations used in processors. Shift register applications involves storage and transfer of data in a digital system.

Figure - show a Four bit shift register employing D as flipflops, which trailing edge triggered indicated by a small circle at the back of the clock triangle.

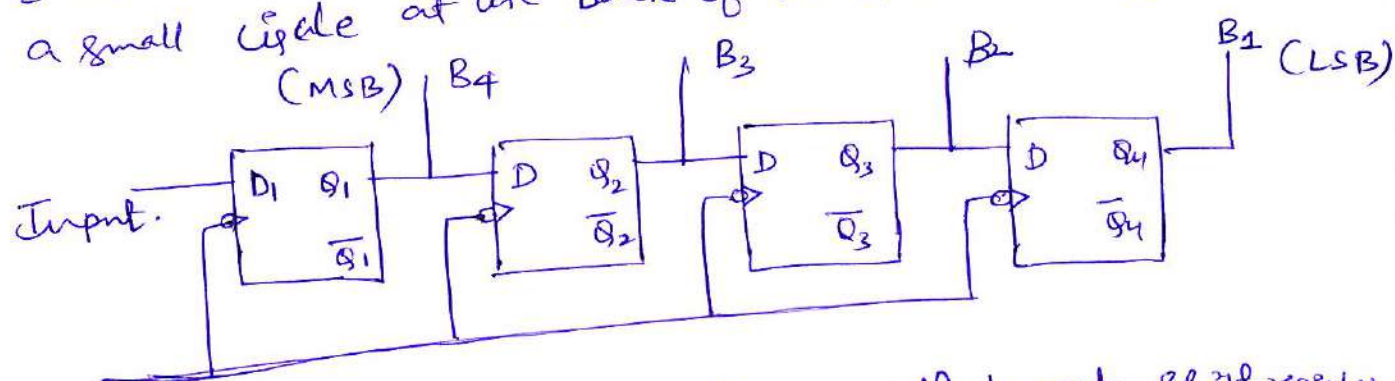


Figure: Serial input - parallel out shift register

Let the input sequence be $A_4 A_3 A_2 A_1$ and initial state of the register be $Q_1 = Q_2 = Q_3 = Q_4 = 0$.

Before applying clock pulse, the four data (D) bits are.

$$D_1 = A_1, D_2 = Q_1 = 0, D_3 = Q_2 = 0, \text{ and } D_4 = Q_3 = 0.$$

After the first pulse arrives, data (D) in all the flip-flop shifts to Qs and the state of register read from left to right becomes $A_1 0 0 0$. After the second pulse, the state of the register becomes $A_2 A_1 0 0$ and finally at the end of fourth pulse it is $A_4 A_3 A_2 A_1$. The register state can be read simultaneously at B_4, B_3, B_2, B_1 . This is called serial-in parallel-out shift register. After four more clock pulses, the output can be read serially at B₁.

The truth Table is shown below.

Serial Input Data	Shift pulses	Q ₁	Q ₂	Q ₃	Q ₄
—	—	x	x	x	x
0	T ₁	0	x	x	x
1	T ₂	1	0	x	x
0	T ₃	0	1	0	x
1	T ₄	1	0	1	0
x	T ₅	x	1	0	1
x	T ₆	x	x	1	0
x	T ₇	x	x	x	1
x	T ₈	x	x	x	x

Binary Counter :-

A Counter is a sequential circuit that counts the number of input pulses.

A Counter that counts in terms of binary is called a binary counter. The count output of n-bit binary counter is 2^n states.

Therefore, it can count from 0 to $(2^n - 1)$. The number of states of counter is called as its modulus (m).

For n-bit counter, $m \leq 2^n$.

Based on the concept of applying inputs to FF to start counting we have two types of Counter.

- Asynchronous Counter

- Synchronous Counter.

In Asynchronous Counter, the FF are clocked sequentially while in a Synchronous Counter, they are clocked simultaneously. In Asynchronous Counter, time delay of each FF get added & this count action is slower than in Synchronous Counter.

Asynchronous Counters (ripple counters)

T FlipFlops are used in these counters. A 3 bit binary ripple counter is shown in figure below.

The small circles at the back of the clock input stand for the fact that these FF are triggered by trailing edge of the input pulse (1 going 0).

For Toggling, all T inputs are kept high (1). From left to right, the first T-FlipFlop receives pulses from the counter. The other two receive pulses from the output of the preceding T-FlipFlop.

The pulses kind of ripple through & hence, the name ripple counter. 3-bit ripple counter is presented in table. At 8th pulses, the counter resets to 0.

The Timing diagram of the counter is shown below in figure. It is observed that pulse frequency gets divided by 2 at each stage.

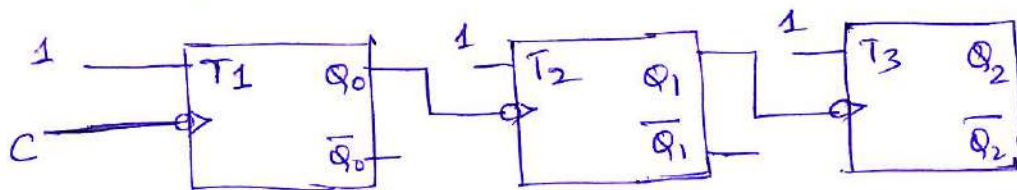


Fig:- 3 bit (modulo 8) ripple counter.

Truth table of 3-bit ripple counter

Q_2	Q_1	Q_0	Input pulse Count
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7
0	0	0	8

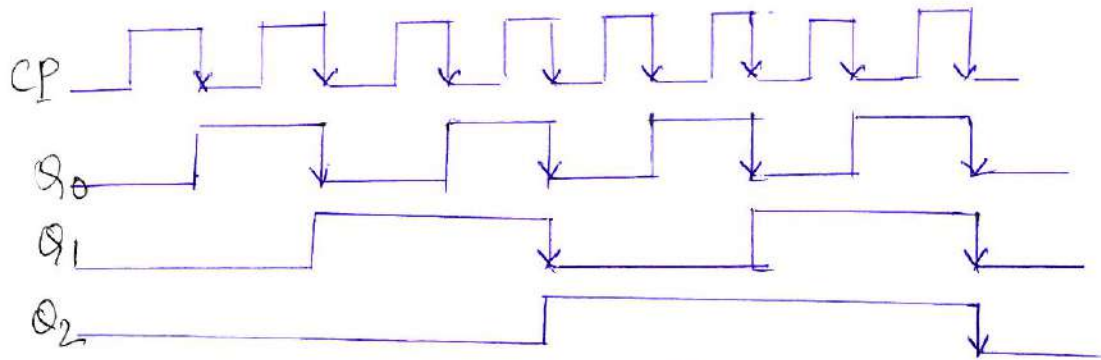
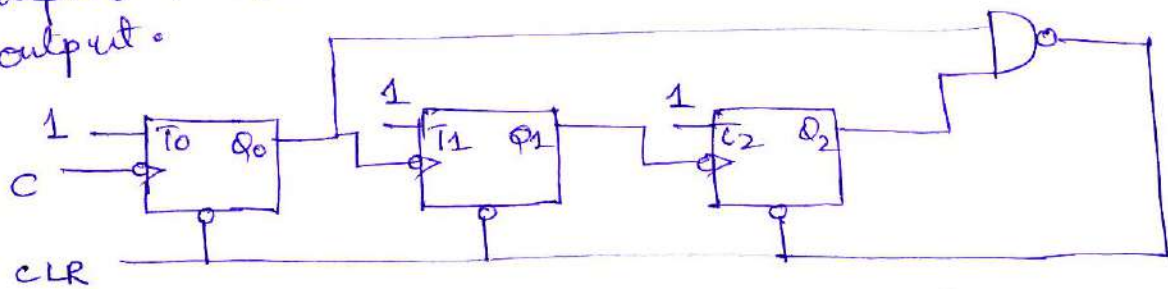


Fig. Timing diagram of 3-bit ripple counter.

Modulo-5 ripple counter

It is possible to design modulo-5 ripple counter.

As $2^3 = 8 > 5$ we need 3 T-FlipFlops. The counter circuit is shown in figure. where the CLR (clear) terminals are also shown. As the pulse count reaches $5 \rightarrow 101$. The counter must reset to zero (000). As output 1's combination is unique to count 5, these are fed to a NAND gate, which produces 0 output to clear the three flipflops & produces 000 as output.



modulo-5 ripple counter.

C	Q ₂	Q ₁	Q ₀
↓	0	0	0
↓	0	0	1
↓	0	1	0
↓	0	1	1
↓	1	0	0
↓	0	0	0

Synchronous Counter

In an asynchronous counter, as the pulse ripples through all the flip-flops, this time delay adds up.

In synchronous counter, the clock pulse is applied to all the flip-flop simultaneously and so the time delay is that of one counter. Counters are fast in operation.

For modulo -16 Synchronous Counter, Four Flip-Flop are needed. Constructed by using T-FlipFlop.

The Truth table for 4-bit Synchronous Counter is shown below.

Truth Table.

Input clock.	Q ₃	Q ₂	Q ₁	Q ₀	T ₃	T ₂	T ₁	T ₀
0	0	0	0	0	0	0	0	1
1	0	0	0	1	0	0	1	1
2	0	0	1	0	0	0	0	1
3	0	0	1	1	0	1	1	1
4	0	1	0	0	0	0	0	1
5	0	1	0	1	0	0	1	1
6	0	1	1	0	0	0	0	1
7	0	1	1	1	1	1	1	1
8	1	0	0	0	0	0	0	1
9	1	0	0	1	0	0	1	1
10	1	0	1	0	0	0	0	1
11	1	0	1	1	0	1	1	1
12	1	1	0	0	0	0	0	1
13	1	1	0	1	0	0	1	1
14	1	1	1	0	0	0	0	1
15	1	1	1	1	1	1	1	1
0	0	0	0	0				

From Table we find that

Q_0 continuously toggles, therefore

$$T_0 = 1$$

Q_1 toggles when $Q_0 = 1$, therefore

$$T_1 = Q_0$$

Q_2 toggles when $Q_0 = Q_1 = 1$ therefore

$$T_2 = Q_1 Q_0$$

Q_3 toggles when $Q_0 = Q_1 = Q_2 = 1$ therefore

$$T_3 = Q_2 Q_1 Q_0$$

The corresponding $T = 1$ are indicated in the same table. To generate T_2 & T_3 , two AND gates are needed. To meet these requirements, the circuit diagram of the synchronous counter is drawn in figure.

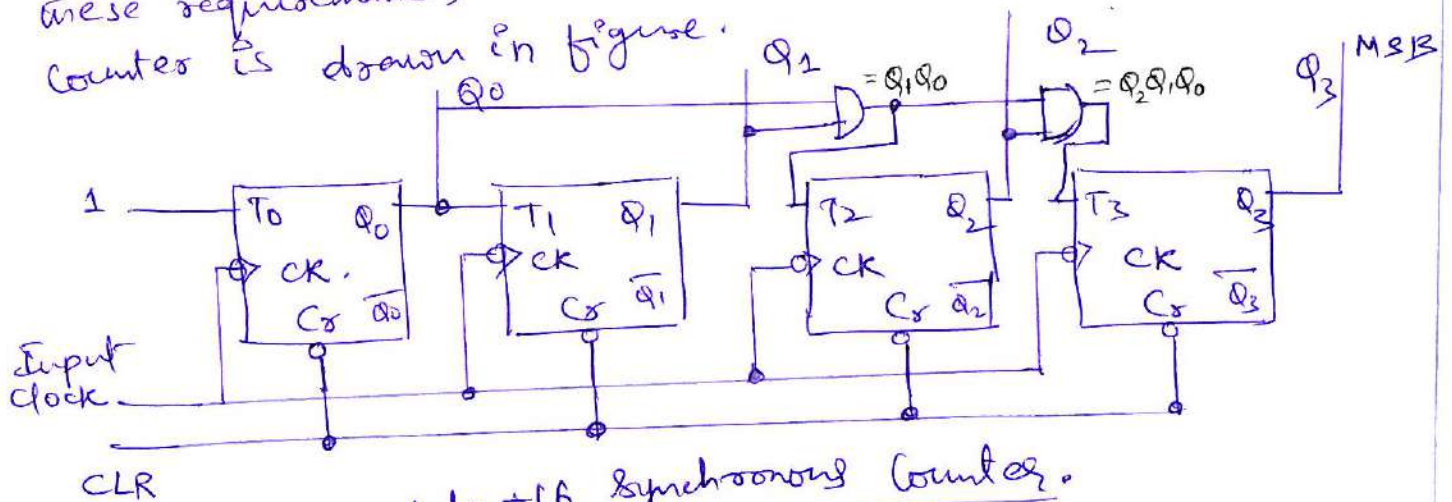
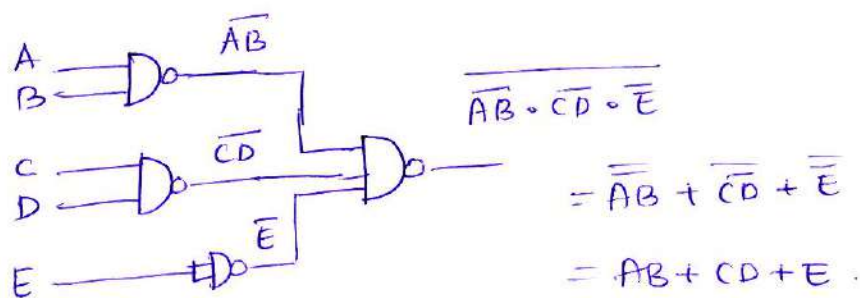


Fig. Modulo-16 Synchronous Counter.

- (1) Consider the Boolean expression $Y = AB + CD + E$ Implement it using NAND gates.

$$Y = AB + CD + E$$

$$\begin{aligned}\overline{Y} &= \overline{AB + CD + E} \\ &= \overline{AB} \cdot \overline{CD} \cdot \overline{E}\end{aligned}$$



- (2) Realise the following logic function using NAND gates only

$$Y = AB + \overline{A}\overline{B} + \overline{A}B$$

$$\overline{Y} = \overline{AB + \overline{A}\overline{B} + \overline{A}B}$$

$$= \overline{AB + \overline{A}\overline{B} + \overline{A}B}$$

$$= (\overline{AB + \overline{A}\overline{B}}) \cdot \overline{\overline{A}B}$$

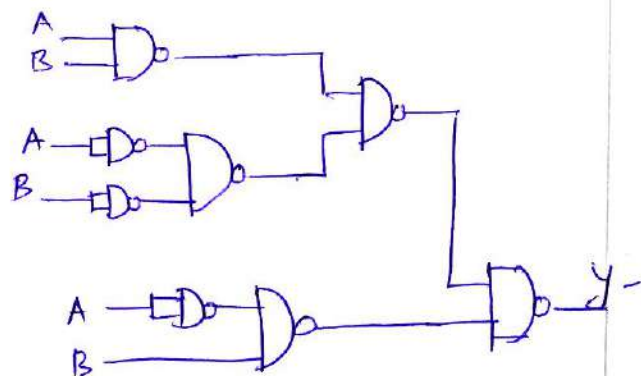
$$= (\overline{AB + \overline{A}\overline{B}}) \cdot \overline{\overline{A}B}$$

$$= (\overline{AB + \overline{A}\overline{B}}) + \overline{\overline{A}B}$$

$$Y = \overline{AB} \cdot \overline{\overline{A}\overline{B}} + \overline{\overline{A}B}$$

$$\overline{Y} = \overline{\overline{AB} \cdot \overline{\overline{A}\overline{B}} + \overline{\overline{A}B}}$$

$$Y = \overline{\overline{AB} \cdot \overline{\overline{A}\overline{B}} \cdot \overline{\overline{A}B}}$$

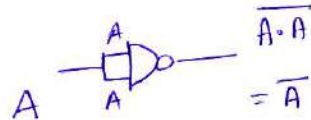


Realize Basic gate using NAND gate

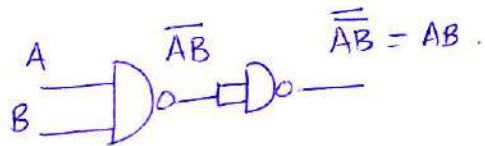
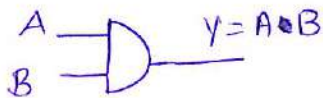
NOT gate



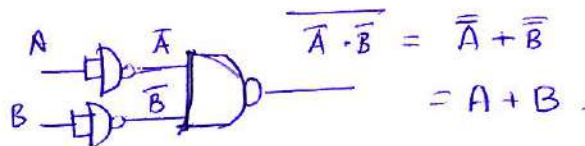
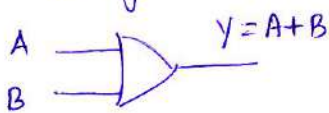
only NAND realization



AND gate



OR gate

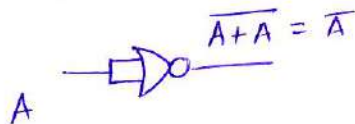


Realize Basic gate using NOR gate

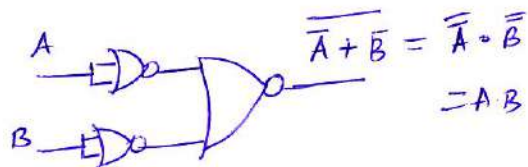
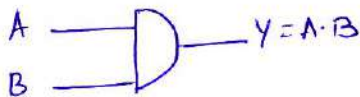
NOT gate



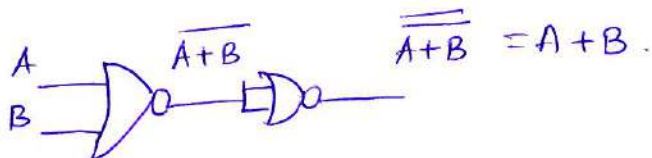
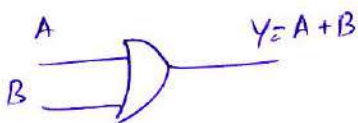
only NOR realization



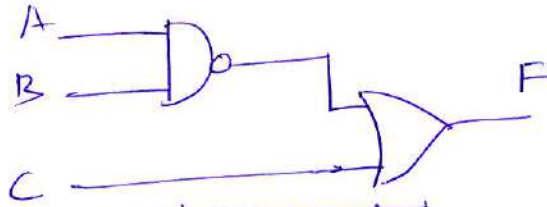
AND gate



OR gate



Write the truth table for the given circuit



Inputs A B C	output $\overline{AB}$	output $\overline{AB} + C = F$
0 0 0	1	1
0 0 1	1	1
0 1 0	1	1
0 1 1	1	1
1 0 0	1	1
1 0 1	1	1
1 1 0	0	0
1 1 1	0	1

Simplify the Boolean equation & then draw its logic circuit with appropriate gates.

$$Y = (\overline{A} + B + C)(A + B + \overline{C})$$

$$= \overline{A}A + AB + AC + \overline{A}B + B \cdot B + BC + \overline{A}\overline{C} + B\overline{C} + \overline{C}C$$

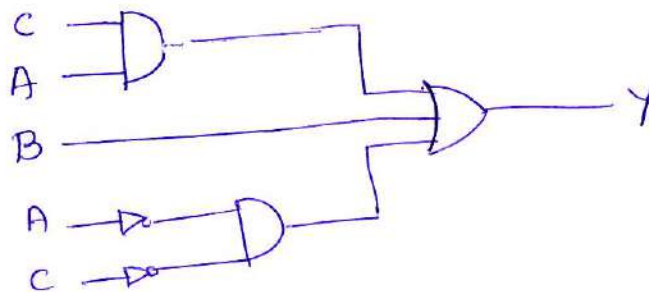
$$= AB + AC + \overline{A}B + B + BC + \overline{A}\overline{C} + B\overline{C}$$

$$= B + AB + \overline{A}B + BC + B\overline{C} + AC + \overline{A}\overline{C}$$

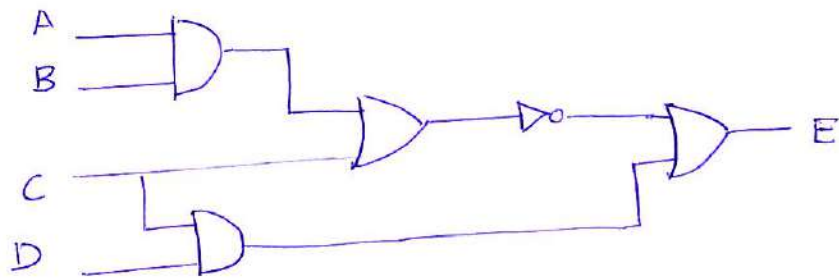
$$= B(1 + A + \overline{A} + C + \overline{C}) + AC + \overline{A}\overline{C}$$

$$= B \cdot 1 + AC + \overline{A}\overline{C}$$

$$= B + AC + \overline{A}\overline{C}$$

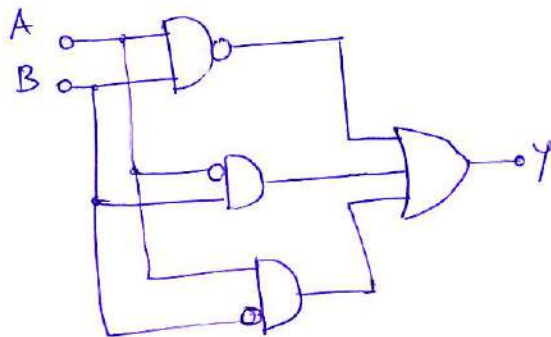


19. Write the Boolean expression for the given circuit in Fig. E=?



$$E = \overline{(\overline{AB} + C)} + CD$$

For the Logic circuit of Fig 10-32 write the Boolean expression for Y in simple form



$$Y = \overline{AB} + \overline{A} \cdot B + A \overline{B}$$

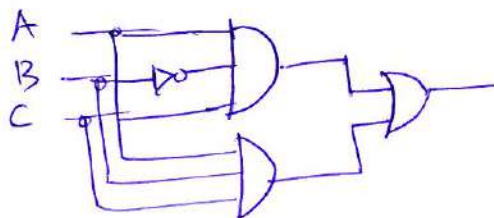
$$= \overline{A} + \overline{B} + \overline{A} B + A \overline{B}$$

$$= \overline{A} (1+B) + B (1+A)$$

$$= \overline{A} + \overline{B}$$

$$\boxed{Y = \overline{A \cdot B}}$$

Draw the Logic circuit for $Y = A \overline{B} C + A B \overline{C}$ & then simplify and draw the simplified circuit

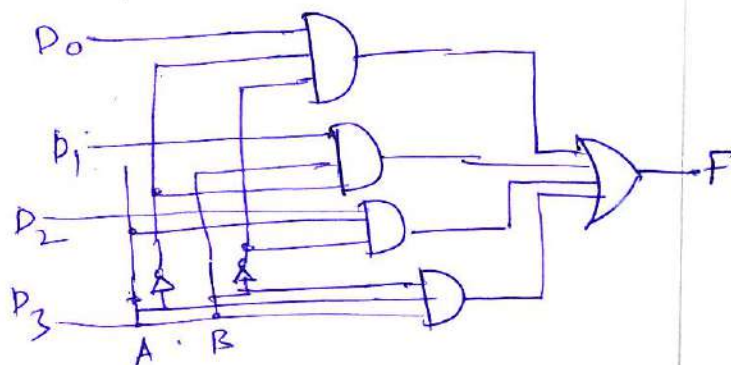
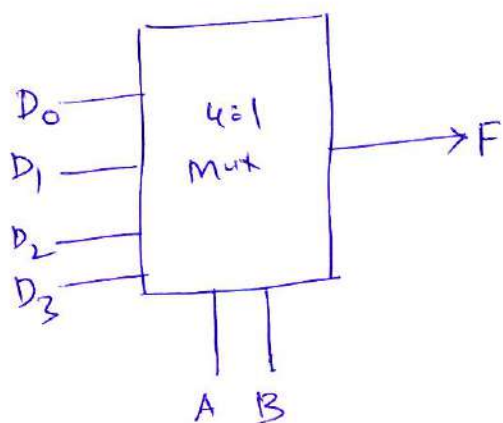


$$Y = A \overline{B} C + A B \overline{C}$$

$$Y = A C (\overline{B} + B) = A C$$

$$\boxed{A \text{ AND } C \rightarrow Y = A C}$$

Implement the 4:1 multiplexer using basic gates.



4:1 multiplexers

Implementation of an 4:1 mux

6. Subtract the following into decimal numbers in 8 bit 2's Complement form.

$$+45 - 56$$

$$(110001)_2 - (111000)_2$$

$m - n$

$$m = (00110001)_2$$

$$n = (00111000)_2$$

$$n = 00111000$$

2's complement of $n \Rightarrow 1's n = 11000111$

$$\begin{array}{r} 11000111 \\ + 1 \\ \hline 11001000 \end{array}$$

$$m = 00110001$$

$$\begin{array}{r} 2's n = 11001000 \\ + 11111001 \\ \hline \end{array}$$

$\rightarrow$ No Carry, result is negative.

$$\begin{array}{r} 00000110 \\ + 1 \\ \hline - (0000111)_2 \end{array} \quad (-7)_{10}$$

$$\begin{array}{r} 2 \overline{) 45} \\ \underline{24} - 1 \\ 2 \overline{) 12} - 0 \\ \underline{6} - 0 \\ 2 \overline{) 3} - 0 \\ \underline{1} - 1 \end{array}$$

$$\begin{array}{r} 2 \overline{) 56} \\ \underline{28} - 0 \\ 2 \overline{) 14} - 0 \\ \underline{7} - 0 \\ 2 \overline{) 3} - 1 \\ \underline{2} - 1 \end{array}$$

perform the indicated operations in binary.

a) $\Rightarrow (32)_8 + (73)_8$

$$\begin{array}{r} 32 = 011\ 010 \\ \quad 111\ 011 \\ \hline 1010\ 101 \\ (125)_8 \end{array}$$

b) $(175)_8 - (114)_8$

$$\begin{array}{r} 175 = 001\ 111\ 101 \\ 114 = 001\ 001\ 100 \\ \hline 0001\ 1000\ 1 \\ (061)_8 \end{array}$$

c) $(7E)_{16} + (AD)_{16}$

$$\begin{array}{r} (7E)_{16} = 0111\ 1110 \\ (AD)_{16} = 1010\ 1101 \\ \hline 1001\ 0101 \\ (12B)_{16} \end{array}$$

d) $(BC)_{16} - (84)_{16}$

$$\begin{array}{r} (BC)_{16} = 1011\ 1100 \\ (84)_{16} = 1000\ 0100 \\ \hline 0011\ 1000 \\ (38)_{16} \end{array}$$

Basic Communication System.

communication is application of Electrical Technology. we use satellite television, fax machine cellular phone etc. Communication systems include the generation, storage and transmission of information. The basic elements of Communication system are transmitter, receiver and communication channels.

Elements of Communication system

Communication systems are used to transfer information from a generation point to the place where it is needed/processed.

- The information at the generation point is not in the form that can travel long distance through the channel. we use a device called modulator cum transmitter to modulate data for transmission.
- In the receiving end, the information must undergo the reverse process such as demodulation/decoding.
- The main elements are.

- ① Source
- ② Input transducer
- ③ Modulator & Transmitter
- ④ Communication channel
- ⑤ Demodulator and receiver
- ⑥ Output transducer.
- ⑦ Destination.

The source is mostly analog & sometimes it may be digital. Fig shows basic elements of a communication system.

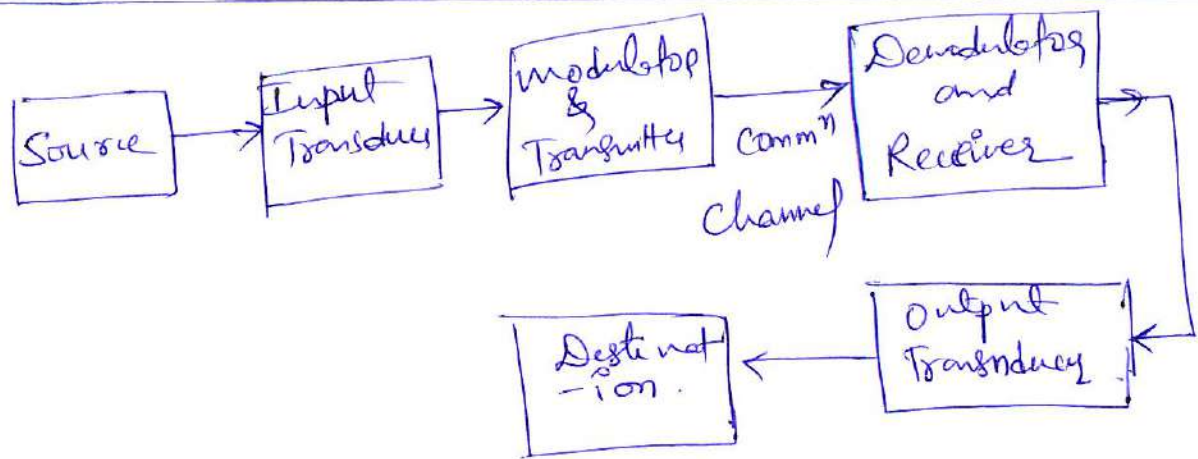


Figure 1 : Communication System.

Source is the information we are sending the example of sources are analog audio, video, signals from measuring devices in the field, & digital data.

Audio signal $\rightarrow$ range 20Hz to 20KHz.

Video signal $\rightarrow$ range from dc to 4.2MHz. Binary signal 1's & 0's.

Input transducer converts one form of signal into another. audio signal is converted to electrical signal using mic or microphone.

Communication channel can be a pair of conductors, optical fiber or just free space.

The signal can be directly sent through a twisted pair telephone cable. But a radio link through free space cannot be used directly for audio signals; instead an antenna of great height would be required.

A high frequency carrier signal is used to carry the message signal after performing modulation. At the receiver the information can be demodulated. The message is called modulating signal.

The output transducer converts electrical signal back to audio signal (original message) Example is speaker.

Principle of operation of mobile phone

Most commonly used device in communication is a mobile phone. It is also called as cellular phone.

A cellular/mobile system provides standard telephone operation by full duplex (two way radio at remote locations).

It provides a wireless connection to the public switched telephone network (PSTN) from any user location within the radio range of the system.

The main purpose of the cellular/mobile system is that it divides the entire geographical area into many small areas called cells and they will be served by transmitter and receiver as shown in figure (1).

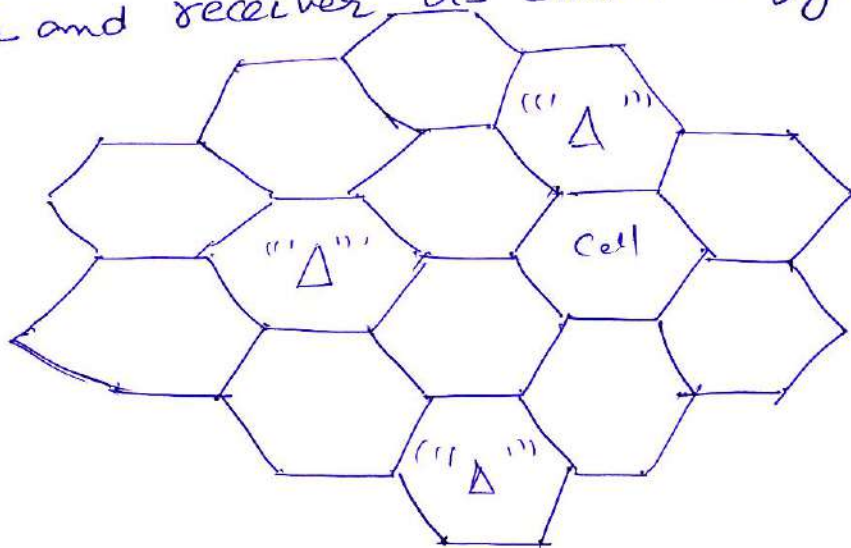


Fig (1). Cell Area in hexagon shape

A single cell covers several square kilometers contains its own receivers and low power transmitter. The cell shape is hexagon. If any shape such as triangle, square and circle considered results in overlap of areas.

Basic Cellular System Consists of

- mobile stations
- Base stations
- Mobile switching centre. [MSC]

The mobile switching centre is also known as mobile telephone switching office (MTSO). The MTSO controls the cells and provides the interface between each cell and main telephone office.

Here each mobile communicates via radio with one of the base stations and may be handed off (switched from one cell to another) to any other base station throughout the duration of the call.

The mobile station commonly consists of

- ① A Transceiver
- ② An antenna
- ③ Control unit

The base station consists of several transmitters and receivers which simultaneously handle full duplex communication and generally have towers which support several transmitting and receiving antennas.

The base station serves as a bridge between all mobile users in the cell and connects the simultaneous mobile calls via telephone lines or microwave link to the MSC.

The MSC coordinates the activities of all the base stations and connects the entire cellular system to the PSTN. Most of the cellular system also provides a service known as roaming.

The cellular system operates in the range 800-900 MHz. newer digital cellular system operate at 1.7-1.8 GHz band & have greater capacity. Total of 832 channels can be used in the cell.

The block diagram shown in figure 2 shows the working of mobile networks through GSM.

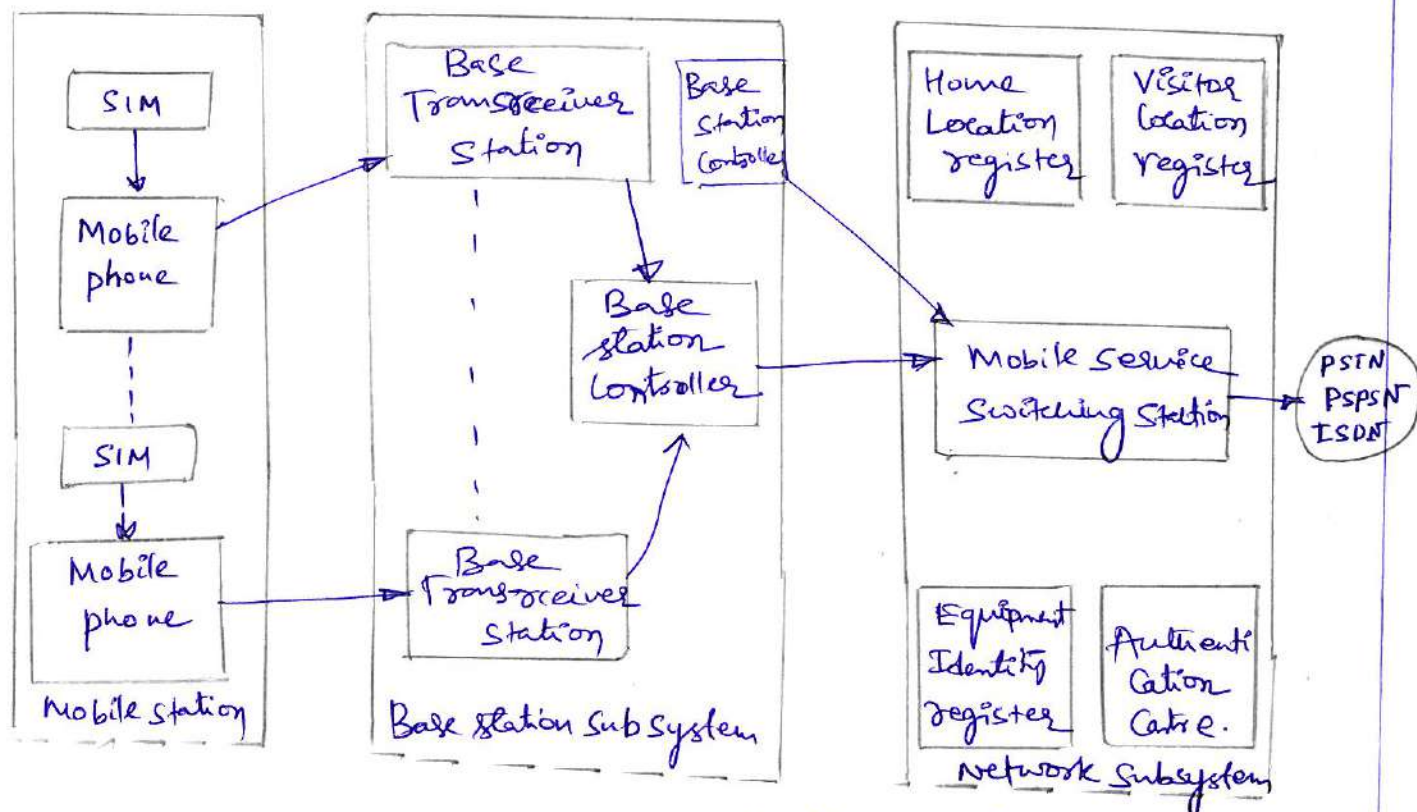


Fig 2 Block diagram of GSM system.

The mobile station

It consists of mobile phone (terminal) and a smart card called Subscriber Identity module [SIM]. By inserting SIM card into mobile phone, the user is able to receive calls at that terminal, make calls from that terminal, and receive other subscribed services. The mobile equipment is uniquely identified by the International Mobile Equipment Identity (IMEI). The SIM card contains the International mobile subscriber Identity (IMSI) used to identify the subscriber to the system, a secret key for authentication, & other information.

Mobile switching centre or mobile service switching station
This is control component of network subsystem. It acts like a normal switching node of the PSTN or ISDN. It provides all the functionality needed to handle a mobile subscriber, such as registration, authentication, location updating, hand overs, & call routing to a roaming subscriber. The signalling system No. 7 is used for roaming signalling in ISDN.

The Home Location Register (HLR) and Visitor Location Register (VLR). together with MSC, provides the all-routing and roaming capabilities of GSM. HLR contains the administrative information of each subscriber & current location of mobile.

The equipment Identity Register (EIR) is a database that contains a list of valid mobile equipment on the network, where each mobile station is identified by its International mobile equipment Identity (IMEI).

An Authentication Centre is a protected database that stores a copy of the secret key stored in each subscriber's SIM card, which is used for authentication & encryption over the radio channel.

Base station subsystem :-

It is composed of two parts, the Base Transceiver Station (BTS) and Base Station Controller (BSC).

The BTS houses a radio transceiver that defines a cell and handles the radio-link protocol with the mobile station. In large urban area more number of BTS will be deployed.

Base Station Controller manages the radio resources for one or more BTS's. It handles radio-channel setup, frequency hopping, and hand overs. BSC is the connection between the mobile station and the mobile service switching centre (MSC).

Cellular Telephone unit

The block diagram of a cellular mobile radio unit. The unit consists following blocks.

1. Control unit
2. Logic unit
3. Receiver
4. Frequency synthesizer
5. Transmitter.

Control unit: It is a set of speaker & microphone with touch tone dialing facility and it stores the memory like numbers and dialing features.

Logic unit: - It is the microprocessor controlled master control unit for cellular radio. It controls the complete operation of Mobile Telephone switching office (MTSO) and mobile unit.

Receiver: - The cellular receiver consists of RF amplifier, FM demodulator and filters. An RF amplifier boosts the level of received cell site signal. Received signal is monitored by MTSO. If there is a weak signal in the present cell then mobile unit is shifted to other site where the signal is strong.

Frequency Synthesizer: - It is used to generate various signals required for transmitter and receiver. It acts as a signal generator. When a mobile unit initiates a call, MTSO identifies the user and assigns a frequency channel which is not used by any other mobile in the cell. MTSO sends a unique code for setting channel frequencies.

Transmitter: - A low power FM transmitter operating at a frequency range of 825 to 845 MHz. There is a 66630 KHz transmit channel.

The Transmitter produces a deviation of $\pm 12 \text{ KHz}$. The modulated output is translated up to final transmitter frequency with the help of mixer, one second input to the transmitter comes from frequency synthesizer. The unique feature of high power translator is that it is controllable by cell site and MTSO. [Mobile Telephone Switching office].

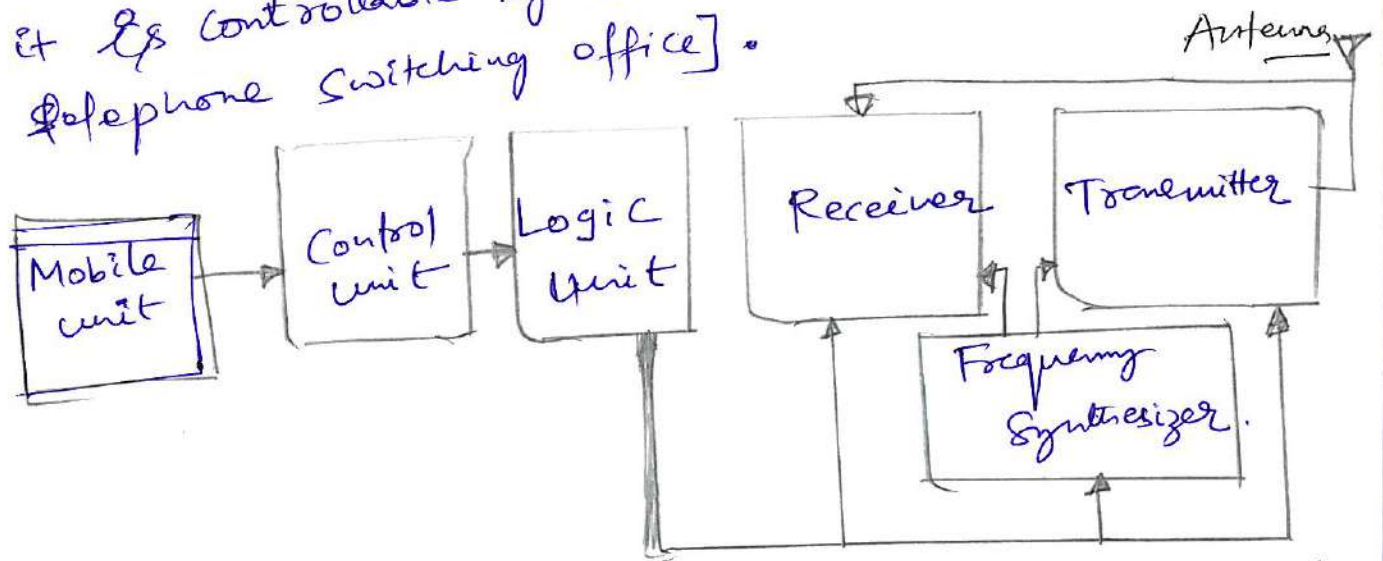


Fig ③: Block diagram of cellular mobile radio network