MODULE – 4

What is Dictionary?

Definition: A dictionary is a collection of pairs (key, item) where

- The first component of the pair is the key
- The second component of the pair is the item which represent associated information with respect to key.
- No two pairs have the same key in the dictionary i.e., key is always unique.
- For example,

Key	Item		
9845070827	Padma Reddy	No. 256,	2nd main, Bengaluru.
9900170827	Padma Reddy	No. 256,	2nd main, Bengaluru.
888888888	Mithil	No. 256,	2nd main, Bengaluru.
999999999	Monalika	No. 256,	2nd main, Bengaluru.
(Mobile Number)	(Name with postal address)		

- In the above dictionary, telephone number is considered as the *key* and name of the person along with postal address is considered as the item associated with key.

What is ADT Dictionary?

Definition: An Abstract data type Dictionary in short called ADT Dictionary is defined as

- Collection of n pairs (key, item) where each pair has a key along with associated item and
- Set of various operations to be performed on those items (objects).
- The operations specified may be insert, delete, display dictionary contents, search etc.
- ADT Dictionary is
  - Objects : Collection of n pairs where each pair has a key and an associated item such as
    - key : element to be searched.
    - item : information such as name, address etc.
    - d : Dictionary
    - n : Number of pairs (key, item)
  - Functions :

void	insert (item, key, d)	::	Insert the *item* with *key* into dictionary *d*
void	is_empty (d, n)	::	if n == 0 return TRUE else return FALSE
void	delete_item (key, d)	::	If *key* is present in dictionary *d*, delete the entry otherwise display "key not found"
void	search (key, d)	::	if *key* is present in dictionary return the corresponding item else return 0

Note: ADT dictionary can be efficiently implemented using binary search tree.

What is Binary search tree?

Definition: A binary search tree is a binary tree which is either empty or non-empty. If it is not empty, it must satisfy the following properties:

- Each node has exactly one key and all keys must be different.
- For each node say x in the tree, all the the keys in the left subtree must be less than key (x)
- For each node say x in the tree, all the the keys in the right subtree must be greater than key (x)
- For example, all the binary trees shown below are binary search trees.

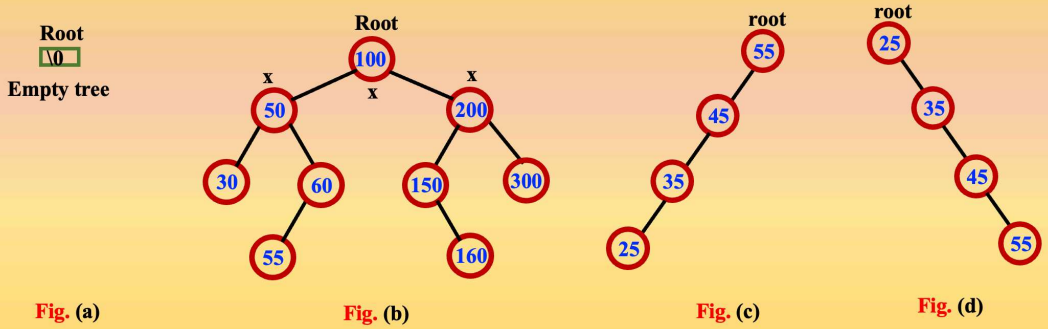
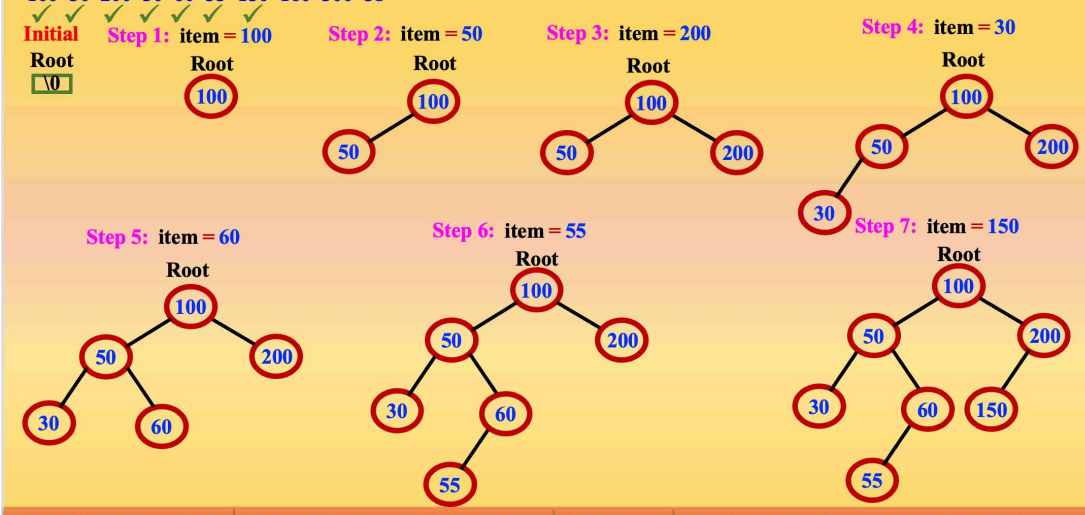


Fig. (a) Fig. (b) Fig. (c) Fig. (d)

How to create a binary search tree for the given the items?

Problem : Create a binary search tree for the following items:

100 50 200 30 60 55 150 160 300 58

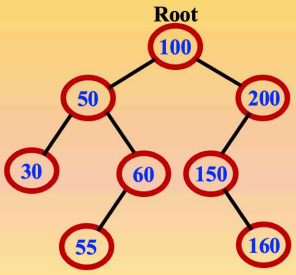


How to create a binary search tree for the given the items?

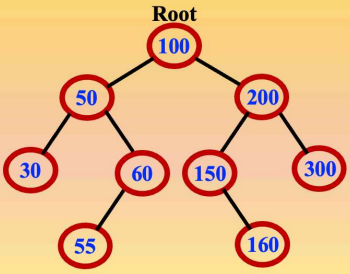
Problem : Create a binary search tree for the following items:

100 50 200 30 60 55 150 160 300 58

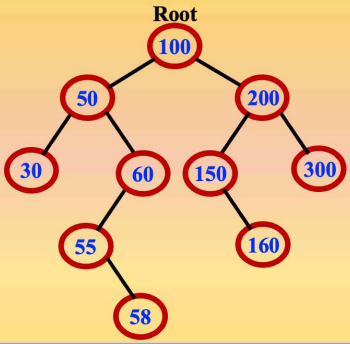
Step 8: item = 160



Step 9: item = 300

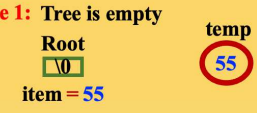


Step 10: item = 58

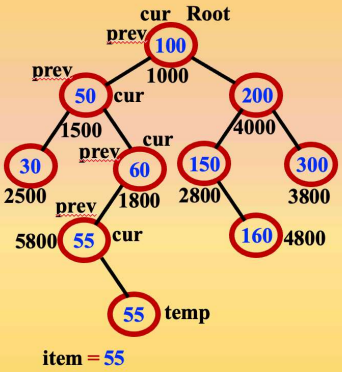


How to create a binary search tree with duplicates?

Case 1: Tree is empty



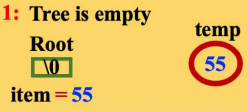
Case 2: Tree is existing



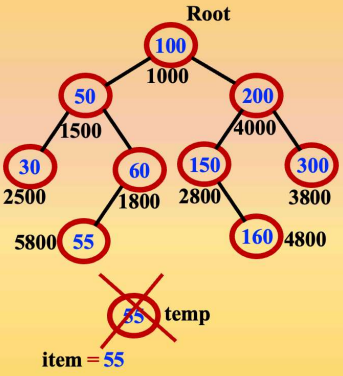

```
NODE  insert  ( int  item,  NODE  root )
{
     NODE   temp,  prev,  cur ;

     // create a node
     temp  =  getnode ( );
     temp –> info  =  item;
     temp –> llink = temp –> rlink = NULL;

     // Insert a node for the first time
     if ( root  ==  NULL )  return   temp;

     // Find the appropriate place to insert
     cur  =  root;
     while ( cur  != NULL )
     {
          prev  =  cur;
          if ( item  <  cur –> info )
               cur  =  cur –>llink ;
          else
               cur  =  cur –>rlink;
     }

     // Insert the item at appropriate place
     if ( item  <  prev –> info )
          prev –>llink  =  temp ;
     else
          prev –>  rlink  =  temp ;

     return   root;
}
```

How to create a binary search tree without duplicates?

Case 1: Tree is empty



Case 2: Tree is existing




```
NODE  insert  ( int  item,  NODE  root )
{
     NODE   temp,  prev,  cur ;

     temp  =  getnode ( );
     temp –> info  =  item;
     temp –> llink = temp –> rlink = NULL;

     if ( root  ==  NULL )  return   temp;

     cur  =  root;
     while (  cur  != NULL )
     {
          prev   =  cur;
          if ( item == cur –> info ) {
               printf ( "Duplicate item " );
               free (temp);   return root;
          }
          if ( item <  cur –> info )
               cur  =  cur –>llink ;
          else
               cur  =  cur –>rlink;
     }
     if ( item <  prev –> info )
          prev –>llink  =  temp ;
     else
          prev  –>  rlink  =  temp ;

     return   root;
}
```



```
int  search  ( int  item,  NODE  root )
{
        NODE   cur ;

        if ( root  ==  NULL )  return   0;

        cur   =  root;
        while (  cur  != NULL )
        {
              if ( item == cur –> info ) return  1;
              if ( item <  cur –> info )
                    cur  =  cur –>llink ;
              else
                    cur  =  cur –>rlink;
        }

        return  0;
}
```

C program to create a BST, traverse the tree and search for an item

```
#include <stdio.h>
#include <stdlib.h>

struct   node
{
        int                info;
        struct   node    * llink;
        struct   node    * rlink;
};

typedef   struct   node *  NODE;

NODE   getnode   ( ) ;

NODE   insert ( int   item ,  NODE  root ) ;
int      search ( int   item ,  NODE  root ) ;
void     preorder   ( NODE  root ) ;
void     inorder    ( NODE  root ) ;
void     postorder ( NODE  first ) ;
```

```
void   main  ( )
{
    int          choice,   item;
    NODE      root = NULL;
    for   ( ;; )
    {
        printf ( " 1:Insert       2:Preorder   3:Inorder: " );
        printf ( " 4:Postorder   5:Search     6:Exit : "  );
        scanf ( " %d ",  &choice );
        switch (   choice )
        {
            case 1  :  printf (" Enter the item : ");      scanf ( " %d ",  &item );
                       root  =  insert ( item, root ) ;
                       break;
            case 2  :  if (root  ==  NULL)  {
                          printf ("Tree is empty\n" ) ; break;
                       }
                       printf ("Preorder :" ) ;     preorder (root );   break;
            case 5  :  printf (" Enter the item : ");      scanf ( " %d ",  &item );
                       flag  =  search ( item, root ) ;
                       if (flag  ==  1 )
                          printf (" Item found \n " );
                       else
                          printf (" Item  not found \n " );
                       break;
            default:   exit ( 0 );
        }
    }
}
```

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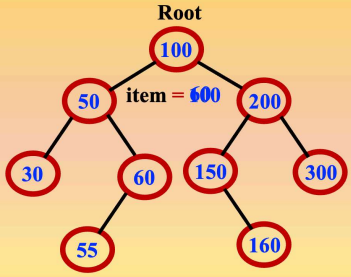
How to search for an item in a binary search tree (recursive procedure)?

```
NODE   search   ( int item, NODE   root )
{
    if ( root  ==NULL )  return  NULL;
    if ( item  ==  root–>info ) return root;
    if ( item  <   root–>info )
        return     search  ( item,   root –>llink ) ;
    return     search ( item,   root –>rlink ) ;
}
```

Case 1: Tree is empty

Case 2: Tree is existing



How to count the number of nodes in a tree?

```
int   count = 0;

void count_node ( NODE   root )
{
    if ( root  ==NULL ) return;
    count_node  ( root –>llink ) ;
    count++;
    count_node  ( root –>rlink ) ;
}
```

How to count the number of leaves in a tree?

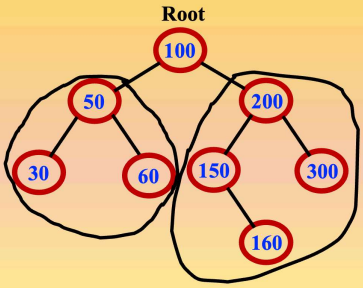
```
int   count = 0;

void count_leaf   ( NODE   root )
{
    if ( root  ==NULL ) return;
    count_leaf    ( root –>llink ) ;
    if ( root–>llink  ==  NULL  &&   root–>rlink ==  NULL ) count++;
    count_leaf    ( root –>rlink ) ;
}
```

How to find the height of the tree?

```
int max ( int a, int b)
{
    return a > b ? a : b
}

int height ( NODE root )
{
     if ( root == NULL ) return 0;
     return 1 + max ( height ( root -> llink ) , height ( root -> rlink) ) ;
}
```

Recursive definition to find height of the tree

$$H(root) = \begin{cases} 0 & \text{if root} == \backslash 0 \\ 1 + \max (H (root -> llink) , H (root -> rlink)) & \text{otherwise} \end{cases}$$



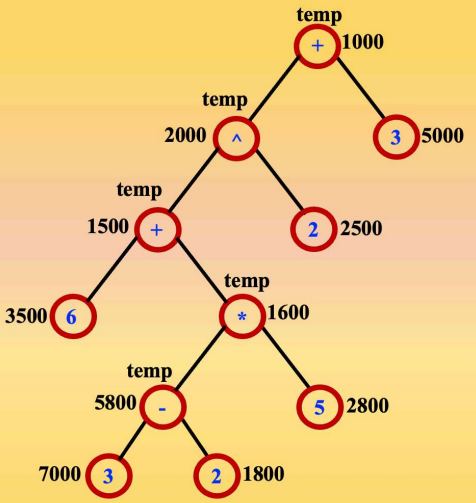
1 + MAX (Height = 2 , Height = 3)

How to create an expression tree?

Infix: (6 + (3 - 2) * 5) ^ 2 + 3

Postfix:

6	3	2	-	5	*	+	2	^	3	+	\0		

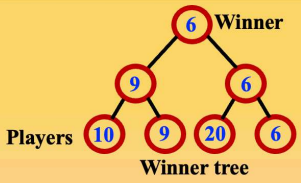



```
NODE create_expression_tree (char  postfix [] )
{
     NODE  temp, stack [ 20 ];
     int   i,  top = -1;

     for ( i = 0;  postfix [ i ]  !=  '\0';  i++ )
     {
          temp  =  getnode ( );
          temp -> info  =  postfix [ i ];
          temp -> llink = temp -> rlink = NULL;

          if (  isalnum ( temp -> info ) )
               stack [++top ] =  temp;
          else
          {
               temp -> rlink =  stack [top-- ];
               temp -> llink  =  stack [top-- ];
               stack [++top ] =  temp;
          }
     }
     return   s [top-- ];
}
```

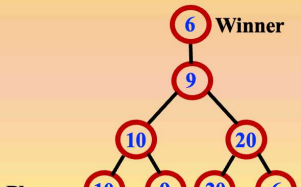
What is s selection/tournament tree?

Definition: A selection tree or tournament tree is a tree data structure which is used to select a winner in a knockout tournament.

- The leaves of the tree represent players entering the tournament
- Each internal node represent a winner or a loser in the match.
- If the internal nodes in a tree represent winners the tree is called winner tree.
- If the internal nodes in a tree represent losers the tree is called loser tree.
- The two types of selection trees are:
  - Winner tree
  - Loser tree



Winner tree



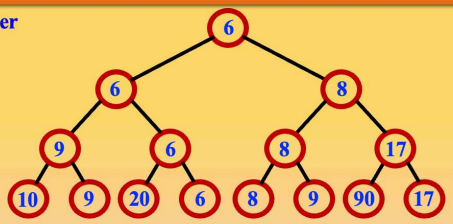
Loser tree

Note: Using selection trees we can sort the elements in ascending/descending order

What is winner tree?

Definition: A selection tree where each internal node represents the winner is called winner tree. A winner tree is a complete binary tree with n-leaf nodes and n - 1 internal nodes where

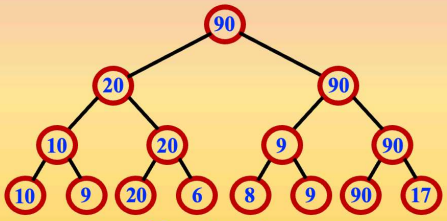
- Each internal node records the winner of the match.
- A winner tree can be either a min winner tree or max winner tree.
- To determine the winner of the match, we assume that each player is associated with a value.
  - In a min winner tree, each internal node represents the smaller of its two children i.e., the player with the smaller value wins.
  - In a max winner tree, each node represents the larger of its two children i.e., the player with the larger value wins.
- The root node represents the smallest node in min winner tree.
- The root node represents the largest node in max winner tree.



Min winner tree

Number of leaves = 8 = n

Number of internal nodes = 7 = n - 1

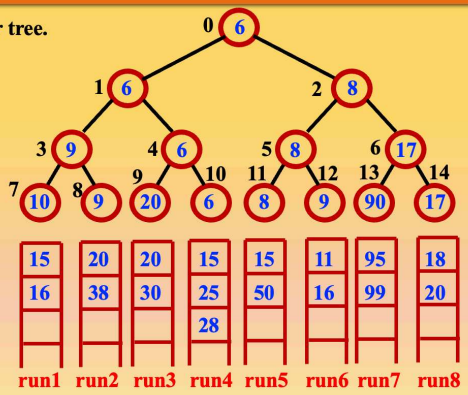


Max winner tree

Note: If two value are same, the left child will be the winner.

What is the procedure to construct min winner tree?

- A set of records arranged in ascending order is input to get a winner tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Between every two players, select the smaller item and elevate to parent position.
- Each non-leaf node in the tree represent the winner in the tournament and the root node contains the smallest item indicating winner of the tournament.
- A winner tree may be represented using sequential representation (array representation) for binary trees
- The number above the node indicates the position of that node in array representation.
- The root node contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.



Output: 6

What is the procedure to construct min winner tree?

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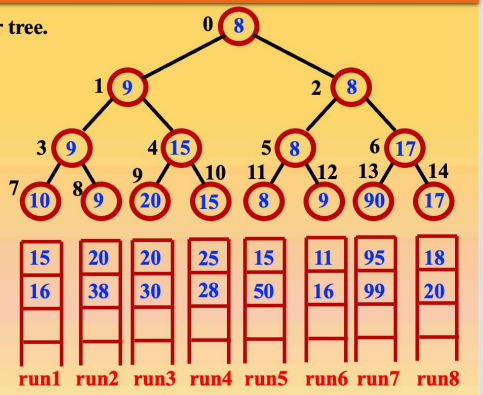

Output: 6 8

- Now, reconstruct the winner tree by playing the tournament along the path from root node

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What is the procedure to construct min winner tree?

- A set of records arranged in ascending order is input to get a winner tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
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- The number above the node indicates the position of that node in array representation.
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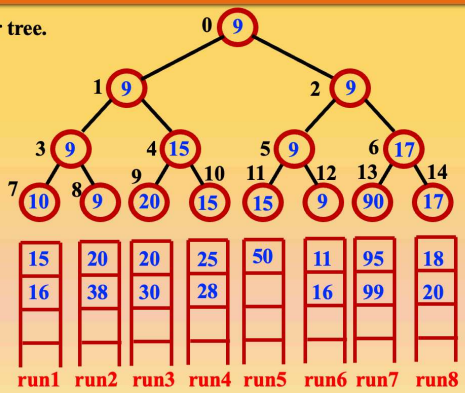
- Now, reconstruct the winner tree by playing the tournament along the path from root node



Output: 6 8 9

What is the procedure to construct min winner tree?

- A set of records arranged in ascending order is input to get a winner tree.
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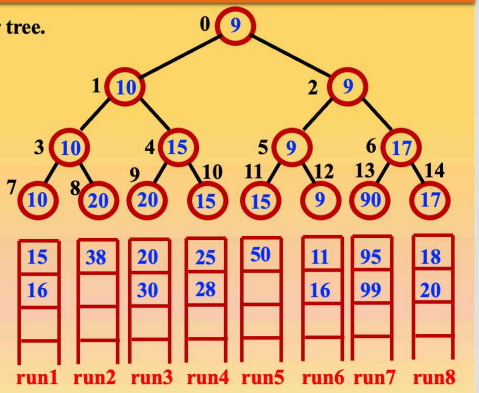
- Now, reconstruct the winner tree by playing the tournament along the path from root node



Output: 6 8 9 9

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What is the procedure to construct min winner tree?

- A set of records arranged in ascending order is input to get a winner tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Between every two players, select the smaller item and elevate to parent position.
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- A winner tree may be represented using sequential representation (array representation) for binary trees
- The number above the node indicates the position of that node in array representation.
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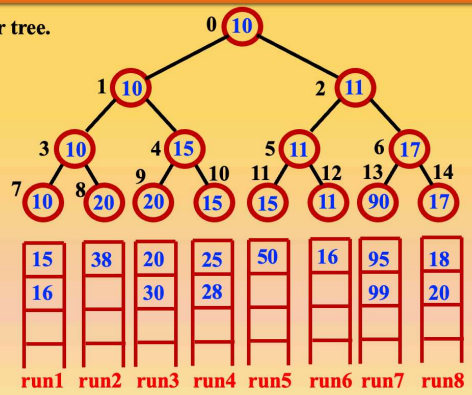
- After removing the smallest node, obtain the next node from corresponding run.
- Now, reconstruct the winner tree by playing the tournament along the path from root node



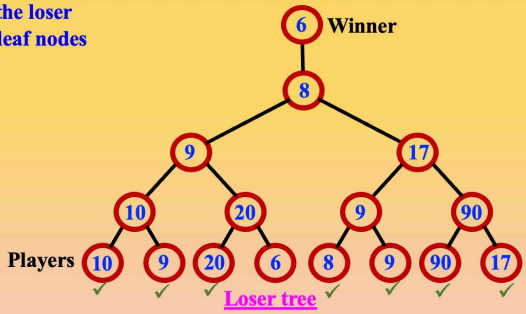
Output: 6 8 9 9 10 and so on.

tournament tree (Winner tree) sort

What is loser tree?

Definition: A selection tree where each internal node represents the loser is called loser tree. A loser tree is a complete binary tree with n-leaf nodes and n - 1 internal nodes where

- Each internal node records the loser of the match.
- To determine the loser of the match, we assume that each player is associated with a value.
- In a loser tree, each internal node represents the loser. i.e., we select a player who loses the match.
- The final player who has not lost any match is the winner of the tournament and it is written right above the root node.

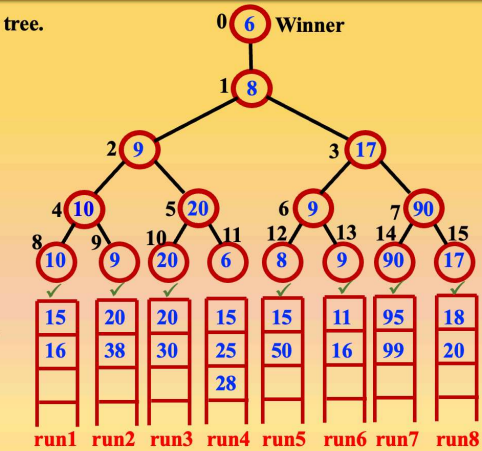


Number of leaves = 8 = n

Number of internal nodes = 7 = n - 1

What is the procedure to construct a loser tree?

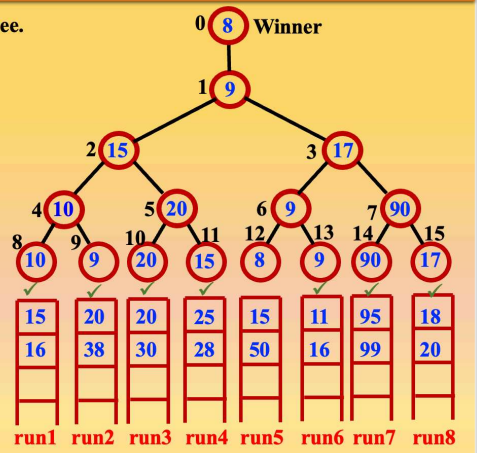
- A set of records arranged in ascending order is input to get a loser tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Among the players, select a player who lost the match and elevate to parent position.
- Each non-leaf node in the tree represent the loser in the tournament and a person who has not lost is the winner of the tournament.
- The winner is indicated by parent of root node and contains the smallest item.
- The loser tree may be represented using sequential representation (array representation) for binary trees
- The number by the side of the node indicates the position of that node in array representation.
- The parent of root contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.



Output: 6

What is the procedure to construct a loser tree?

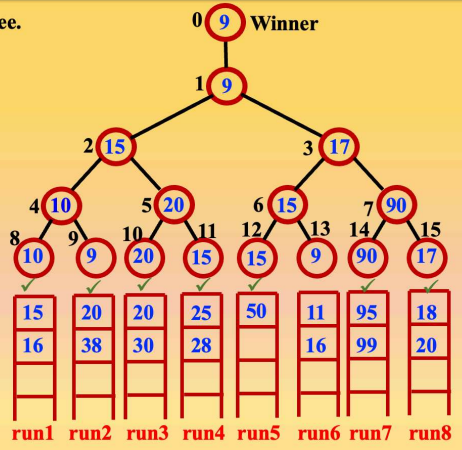
- A set of records arranged in ascending order is input to get a loser tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Among the players, select a player who lost the match and elevate to parent position.
- Each non-leaf node in the tree represent the loser in the tournament and a person who has not lost is the winner of the tournament.
- The winner is indicated by parent of root node and contains the smallest item.
- The loser tree may be represented using sequential representation (array representation) for binary trees
- The number by the side of the node indicates the position of that node in array representation.
- The parent of root contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.



Output: 6 8

What is the procedure to construct a loser tree?

- A set of records arranged in ascending order is input to get a loser tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Among the players, select a player who lost the match and elevate to parent position.
- Each non-leaf node in the tree represent the loser in the tournament and a person who has not lost is the winner of the tournament.
- The winner is indicated by parent of root node and contains the smallest item.
- The loser tree may be represented using sequential representation (array representation) for binary trees
- The number by the side of the node indicates the position of that node in array representation.
- The parent of root contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.



Output: 6 8 9

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What is the procedure to construct a loser tree?

- A set of records arranged in ascending order is input to get a loser tree.
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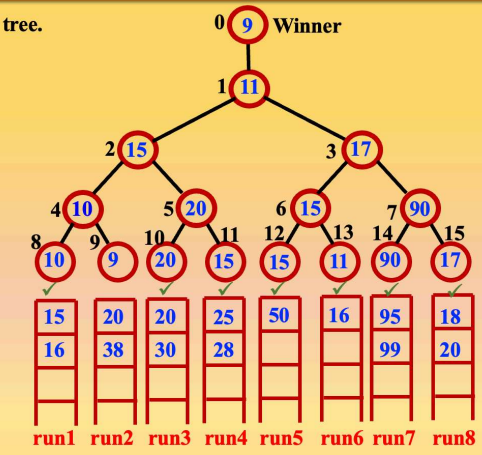

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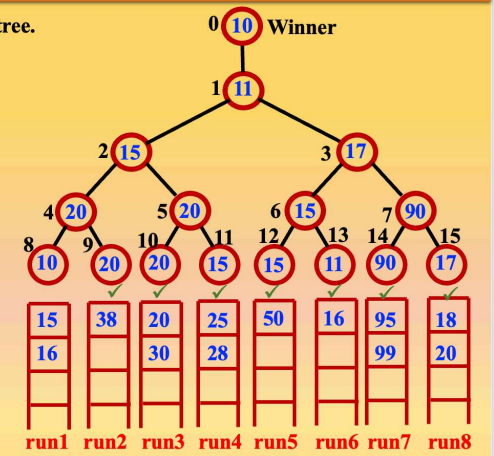

Output: 6 8 9 9 10

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What is the procedure to construct a loser tree?

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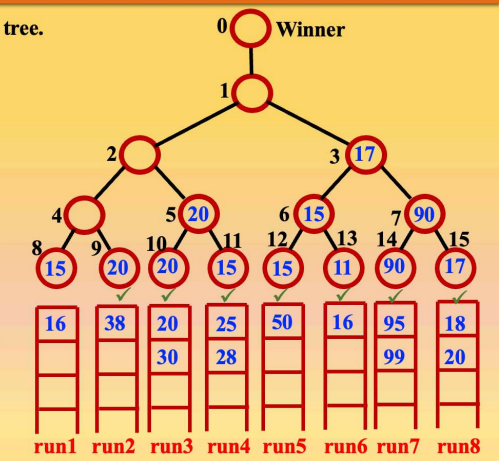


Output: 6 8 9 9 10 and so on.

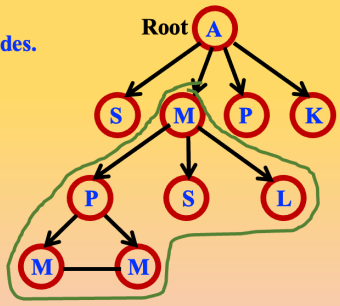
tournament tree (Loser tree) sort

What is a tree?

❖ Tree is a non-linear data structure where nodes are linked to each other in parent-child relationship such that there is only one path between any given two nodes.
- There is a special node called root node for which there is no parent.
- Remaining nodes are partitioned into subtrees

❖ Tree is also defined as acyclic directed graph

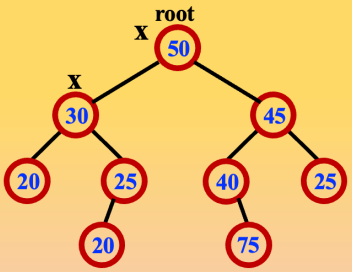
Ex : In the tree shown in figure on right hand side:
- The tree has 10 nodes: A, S, M, P, K, P, S, L, M, M
- Node A is root node and it is written at the top.
- Nodes S, M, P, K are children of node A and hence there are four subtrees identified by S, M, P, K



Note: An m-ary tree is defined as a tree where each node has maximum of m children.
- If m = 2, the tree is 2-ary tree or bin-ary tree.
- If m = 3, the tree is 3-ary tree or tern-ary tree.
- If m = 4, the tree is 4-ary tree or quad-ary tree.

What are basic tree terminologies?

Root node: A node in a tree which has no parent is called root node.
- ❖ Root node is the topmost node in a tree.
- ❖ Using root node, any node in the tree can be accessed.
- ❖ There is only one root node in a tree.
- ❖ In the given tree node containing item 50 is the root node.



Descendants: The nodes that are reachable from node x while moving downwards are called descendants of node x.
- ❖ All the nodes below 30 i.e, 20, 25 and 20 are descendants of 30.
- ❖ All the nodes below 45 i.e, 40, 25 and 75 are descendants of 45.

Left descendants: The nodes that are reachable from left side of node x while moving downwards are called left descendants of node x.
- ❖ The nodes 30, 20, 25 and 20 are left descendants of 50.
- ❖ The nodes 40 and 75 are left descendants of 45.

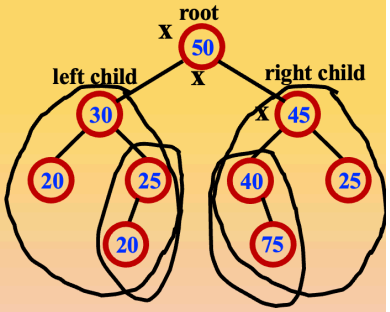
Right descendants: The nodes that are reachable from right side of node x while moving downwards are called right descendants of node x.
- ❖ The nodes 45, 40, 25 and 75 are right descendants of 50.
- ❖ The nodes 25 and 20 are right descendants of 30.

What are basic tree terminologies?

Left subtree: All the nodes that are all left descendants of node x form the left subtree of x.
- ❖ The nodes 30, 20, 25 and 20 together form the left subtree of node 50.
- ❖ The nodes 40 and 75 together form the left subtree of node 45.

Right subtree: All the nodes that are all right descendants of node x form the right subtree of x.
- ❖ The nodes 45, 40, 25 and 75 together form the right subtree of node 50.
- ❖ The nodes 25 and 20 together form the right subtree of node 30.

Child: A node which is the first descendant of a given node x is the child of node x.
- ❖ A node which is the first left descendant of a node is called left child.
- ❖ A node which is the first right descendant of a node is called right child.



What are basic tree terminologies?

Left subtree: All the nodes that are all left descendants of node x form the left subtree of x.

- The nodes 30, 20, 25 and 20 together form the left subtree of node 50.
- The nodes 40 and 75 together form the left subtree of node 45.

Right subtree: All the nodes that are all right descendants of node x form the right subtree of x.

- The nodes 45, 40, 25 and 75 together form the right subtree of node 50.
- The nodes 25 and 20 together form the right subtree of node 30.

Child: A node which is the first descendant of a given node x is the child of node x.

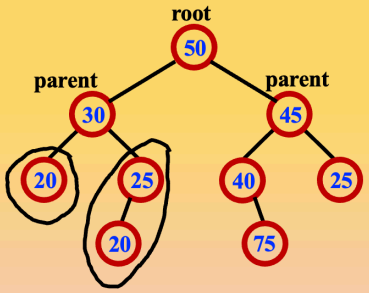
- A node which is the first left descendant of a node is called left child.
- A node which is the first right descendant of a node is called right child.

Parent: A node having left subtree or right subtree or both is said to be a parent.

- The node 30 is the parent of nodes 20 and 25
- The node 45 is the parent of nodes 40 and 25

Siblings: The nodes having the same parent are called siblings.

- The nodes 20 and 25 are siblings.
- The nodes 40 and 25 are siblings.



What are basic tree terminologies?

Ancestors: The nodes that are reachable from node x while moving upwards are called ancestors of node x.

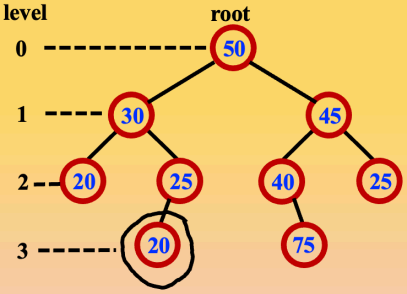
- All the nodes above 20 i.e, 25, 30 and 50 are ancestors of 20.
- All the nodes above 75 i.e, 40, 45 and 50 are ancestors of 75.

Leaf / external node: A node having empty left child and empty right child is called a leaf node or a terminal node or an external node.

- The nodes 20, 20, 75 and 25 are all leaf nodes.

Internal nodes: The nodes except leaf nodes are called internal nodes.

- The nodes 25, 40, 30, 45 and 50 are all internal nodes.



Level: The total number of edges from root to a node is called level of a node or depth of a node

- The total number of edges from 75 to root = 3. So, level of 75 is 3.

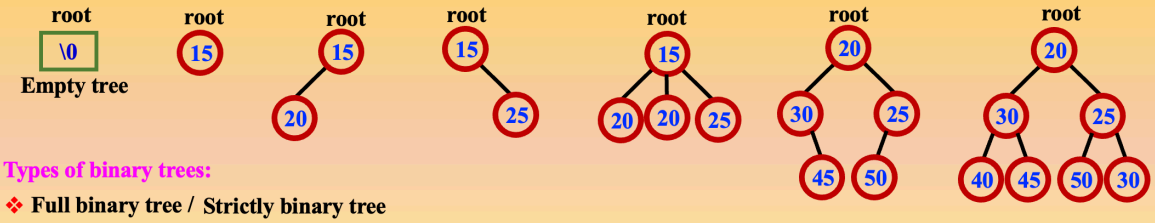
Height: The total number of nodes from a farthest leaf node to root is called height of a tree or depth of a tree.

- The total number of nodes from 75 to root = 4. So, height of tree is 4.
- The height of the tree = Number of levels = 4.

What is a binary tree? What are the different types of binary trees?

Definition: An m-ary tree where m = 2 is called 2-ary tree or bin-ary tree or binary tree.

- In other words, a tree where each node in the tree has maximum of two children is called binary tree.
- Each node in a binary tree can have either 0, 1 or 2 children but, a node can not have more than two children.
- An empty tree can also be considered as binary tree.
- For example, the binary trees are shown below:



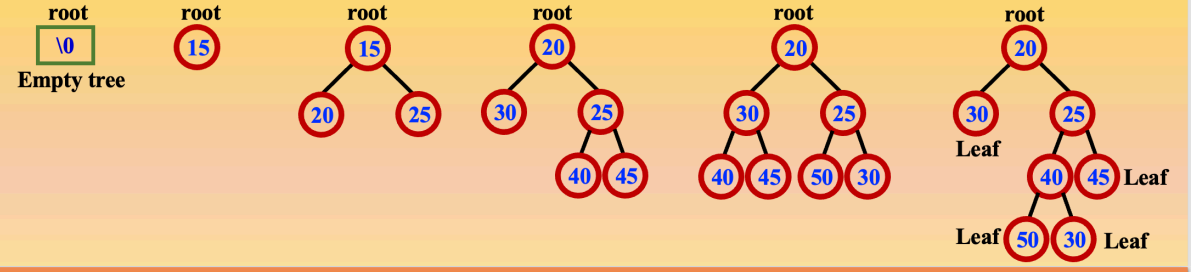
Types of binary trees:

- Full binary tree / Strictly binary tree
- Complete binary tree
- Almost complete binary tree
- Binary search tree
- AVL trees
- B-trees
- RED-BLACK-trees

What is a Full binary tree?

Definition: A binary tree where each node has either 0 or 2 children is called full binary tree or strictly binary tree.
In other words, a binary tree in which all the nodes have two children except the leaf nodes is called full binary tree.

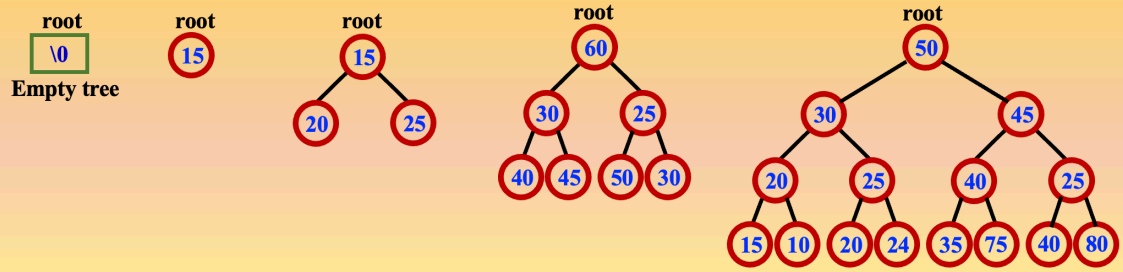
- An empty tree can also be considered as full binary tree.
- For example, all following binary trees are full binary trees:



What is a complete binary tree?

Definition: A binary tree where each node has either 0 or 2 children (full binary tree or strictly binary tree) and all the leaf nodes are at the same level is called complete binary tree.

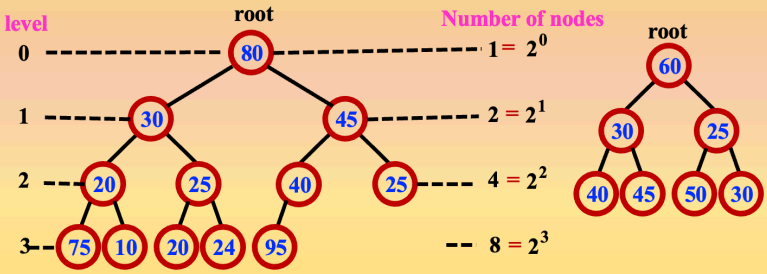
- An empty tree is considered as complete binary tree.
- At any level i in a complete binary tree the number of nodes = 2^i
- For example, all following binary trees are complete binary trees:



What is an almost complete binary tree?

Definition: A binary tree is an almost complete binary tree with the following properties:

- If i is the level of the tree, the number of nodes in i^{th} level must be 2^i
- If number of nodes in i^{th} level $< 2^i$ then the number of nodes in $(i-1)^{th}$ level must be 2^{i-1} and all the nodes i^{th} level must be filled from left to right only.
- A node in an almost complete binary tree cannot have right child without having left child. But, a node can have only left child.



How to represent a tree?

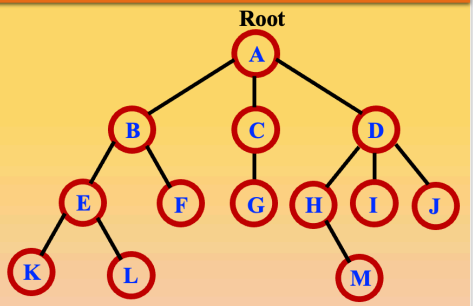
A tree can be represented using three different ways:

- List representation
- Left-child Right sibling representation
- Left child – Right child (Degree 2) representation

What is list representation of a tree?

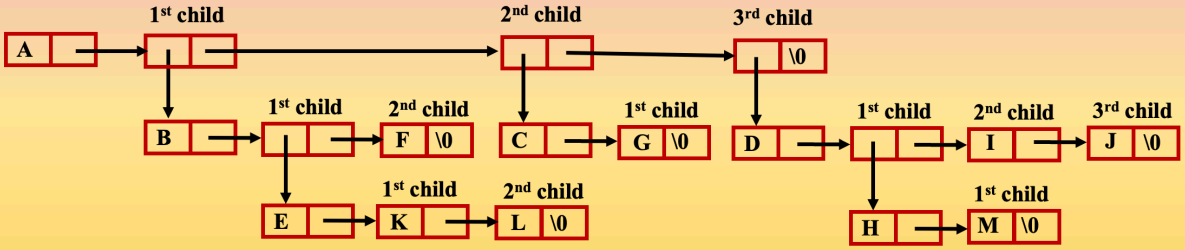
A tree can be represented using list as shown below:

- The root node comes first.
- It is immediately followed by a list of subtrees of that node
- It is recursively repeated for each subtree.



Observe the following points:

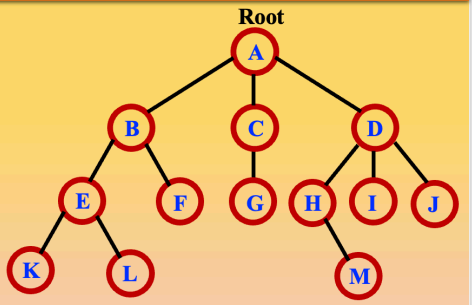
- There are 3 children for node A in the tree. So, there are 3 nodes to the right of A in list representation.
- A's first child is B, 2nd child is C and 3rd child is D and they are shown using down links.

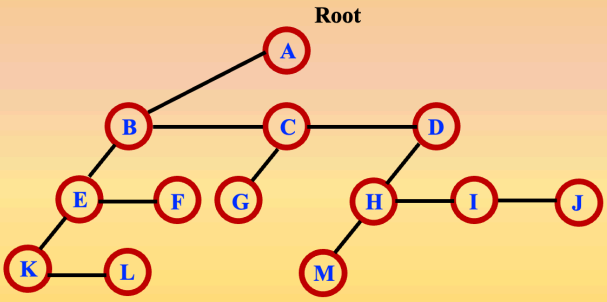


How to represent a tree using left-child right sibling representation?

A left child right sibling representation of a tree can be obtained as shown below:

- The root node comes first.
- The left pointer of a node in the tree will be the left child in this representation
- The remaining children of a node in the tree (siblings) can be inserted horizontally to the left child in the representation.





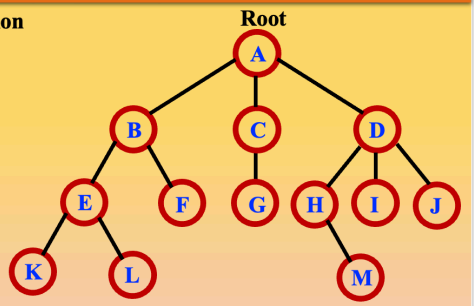
Observe the following points:

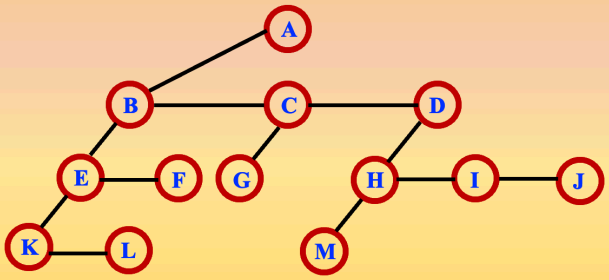
- A's left child is B in the tree. So, A's left child is B in the representation.
- A's remaining children such as C and D in the tree are inserted horizontally to node B in the representation.

How to represent a tree in Left child – right child (degree 2) representation?

A tree can be represented as left child – right child or degree 2 representation as shown below:

- Obtain the left-child right sibling representation.
- Rotate the horizontal lines clockwise by 45 degrees.

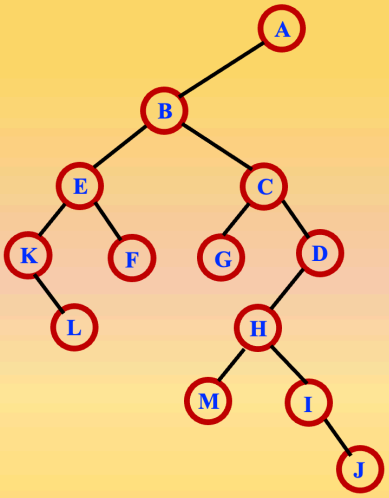


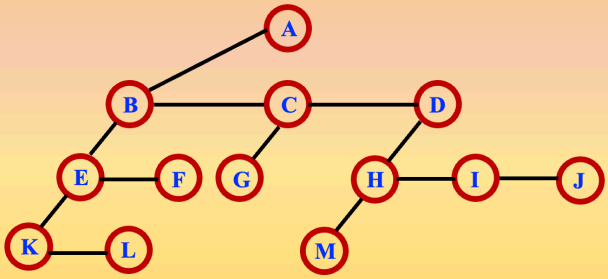


How to represent a tree in Left child – right child (degree 2) representation?

A tree can be represented as left child – right child or degree 2 representation as shown below:

- Obtain the left-child right sibling representation.
- Rotate the horizontal lines clockwise by 45 degrees.





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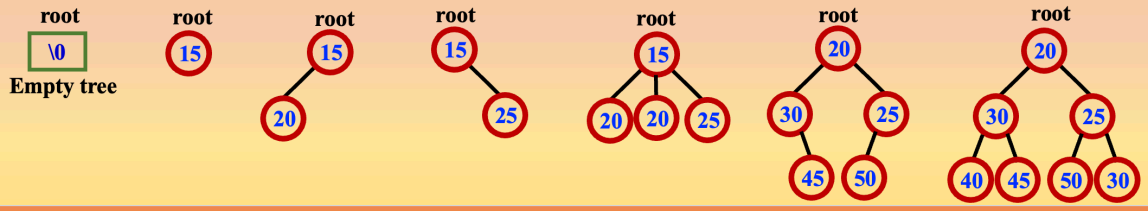
What is a binary tree?

Definition: An m-ary tree where m = 2 is called 2-ary tree or bin-ary tree or binary tree.

- In other words, a tree which has finite set of nodes that is either empty or consists of a root node and each node in the tree has maximum of two children i.e., left subtree and right subtree is called binary tree.
  - Root : A node without a parent is called root node. It is the first node in the tree.
  - Left subtree : A tree connected to left side of a node is called left subtree.
  - Right subtree : A tree connected to right side of a node is called right subtree.
- Each node in a binary tree can have either 0, 1 or 2 children but, a node can not have more than two children.
- An empty tree can also be considered as binary tree.
- For example, the binary trees are shown below:



What is skewed binary tree?

Definition: A skewed binary tree is a binary tree where all the nodes are inserted towards one side only.

- If all the nodes are inserted towards left subtree, the binary tree is said to be skewed towards left.
- If all the nodes are inserted towards right subtree, the binary tree is said to be skewed towards right.
- For example,

Left skewed binary tree

Right skewed binary tree

What is ADT binary tree?

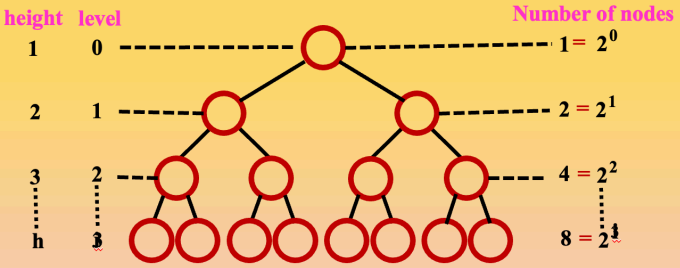
Definition: An Abstract data type binary tree in short called ADT binary tree is defined as

- Set of items (objects) along with type of each item to be stored in the tree and
- Set of various operations to be performed on those items (objects).
- The operations specified may be insert, delete, display tree contents, compare two trees etc.
- ADT Binary_Tree is
  - Objects : Finite set of nodes either empty or consisting of a node, left subtree and right subtree
  - Functions :
    - item : element to be inserted.
    - root : the root node of the binary tree.

NODE	insert (item, root)	::	Inserts an item into tree and returns the address of the root node
NODE	delete_item (item, root)	::	Deletes an item from the tree if found otherwise display "Item not found"
void	preorder (root)	::	Display tree in preorder if tree is not empty
void	inorder (root)	::	Display tree in inorder if tree is not empty
void	postorder (root)	::	Display tree in postorder if tree is not empty
int	count_nodes (root)	::	Returns number of nodes in the tree
int	height (root)	::	Returns height of the tree

How to find number of nodes at level i and height h?

The height of the tree can be computed as shown below:

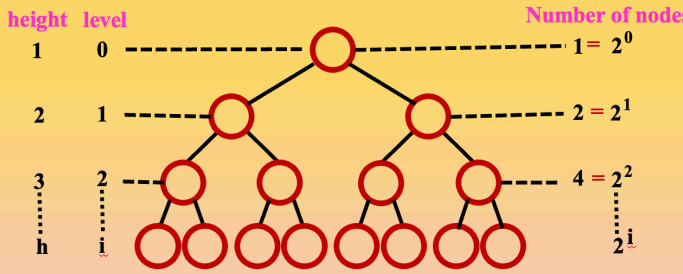


Maximum number of nodes at level i = 2^i

Maximum number of nodes at height h = 2^i

How to find total number of nodes at height h (or depth h)

The height of the tree can be computed as shown below:

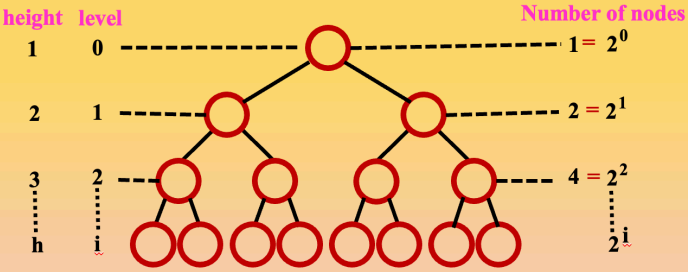


Total number of nodes $= 2^0 + 2^1 + 2^2 + 2^3 + \ldots\ldots 2^i$ G.P $= \frac{a\,(r^n - 1)}{r - 1}$

$$n = \frac{1\,(2^{i+1} - 1)}{2 - 1} = 2^{i+1} - 1$$

Total number of nodes $= 2^{i+1} - 1$

How to find height (or depth) of the tree?



Total number of nodes $= 2^0 + 2^1 + 2^2 + 2^3 + \ldots\ldots 2^i$

$$n = \frac{1\,(2^{i+1} - 1)}{2 - 1} = 2^{i+1} - 1$$

Total number of nodes $= 2^{i+1} - 1$

Height/Depth of the tree = h = max. level + 1 = i + 1

$n = 2^{i+1} - 1 = 2^h - 1$

$2^h = n + 1$

Taking log on both sides,

$\log(2^h) = \log(n + 1)$

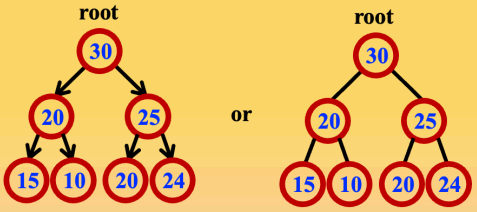
$h \log_2(2) = \log_2(n + 1)$

$h = \log(n)$

So, height of the tree $h = \log_2(n)$

In the above tree black nodes are dummy nodes. They are not present.

What is linked representation of a binary tree?




```
typedef struct node * NODE;

struct node
{
    int     info;
    NODE    llink;
    NODE    rlink;
};

struct node *root;
        OR
NODE root;
```

How to traverse the binary tree?

The tree can be traversed using following traversal methods:

- Preorder
  1. Visit the node
  2. Recursively traverse left subtree in Preorder
  3. Recursively traverse right subtree in Preorder
- Inorder
  1. Recursively traverse left subtree in Inorder
  2. Visit the node
  3. Recursively traverse right subtree in Inorder
- Postorder
  1. Recursively traverse left subtree in Postorder
  2. Recursively traverse right subtree in Postorder
  3. Visit the node

Linked representation

```
void preorder ( NODE root )
{
    if ( root == NULL) return;
    printf (" %d ", root -> info);
    preorder (root -> llink);
    preorder (root -> rlink);
}

void inorder ( NODE root )
{
    if ( root == NULL) return;
    inorder (root -> llink);
    printf (" %d ", root -> info);
    inorder (root -> rlink);
}

void postorder ( NODE root )
{
    if ( root == NULL) return;
    postorder (root -> llink);
    postorder (root -> rlink);
    printf (" %d ", root -> info);
}
```


How to traverse the binary tree?

The tree can be traversed using following traversal methods:

- Preorder
  1. Visit the node
  2. Recursively traverse left subtree in Preorder
  3. Recursively traverse right subtree in Preorder
- Inorder
  1. Recursively traverse left subtree in Inorder
  2. Visit the node
  3. Recursively traverse right subtree in Inorder
- Postorder
  1. Recursively traverse left subtree in Postorder
  2. Recursively traverse right subtree in Postorder
  3. Visit the node

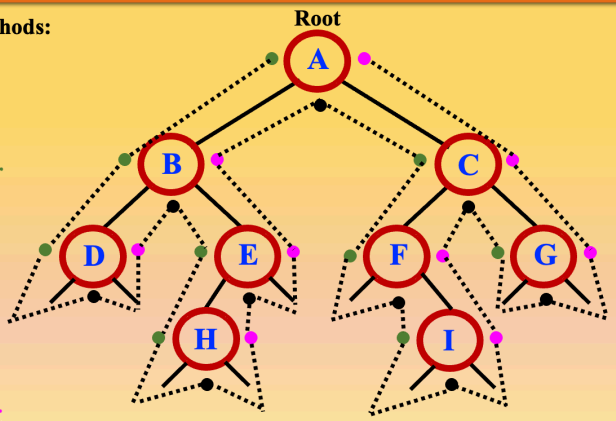
Array representation

```
void preorder ( int root[], int i )
{
    if ( root [ i ] == 0 ) return;
    printf (" %d ", root [ i ] );
    preorder (root, 2 * i + 1 );
    preorder (root, 2 * i + 2 );
}

void inorder ( int root[], int i )
{
    if ( root [ i ] == 0 ) return;
    inorder (root, 2 * i + 1 );
    printf (" %d ", root [ i ] );
    inorder (root, 2 * i + 2 );
}

void postorder ( int root[], int i )
{
    if ( root [ i ] == 0 ) return;
    postorder (root, 2 * i + 1 );
    postorder (root, 2 * i + 2 );
    printf (" %d ", root [ i ] );
}
```

How to traverse the binary tree?

The tree can be traversed using following traversal methods:

- Preorder
  1. Visit the node
  2. Recursively traverse left subtree in Preorder
  3. Recursively traverse right subtree in Preorder
- Inorder
  1. Recursively traverse left subtree in Inorder
  2. Visit the node
  3. Recursively traverse right subtree in Inorder
- Postorder
  1. Recursively traverse left subtree in Postorder
  2. Recursively traverse right subtree in Postorder
  3. Visit the node

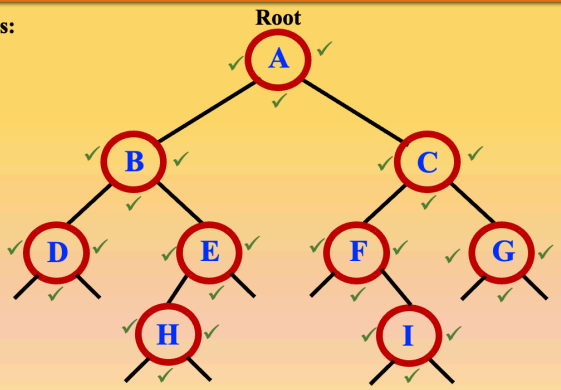


1. Preorder : A B D E H C F I G
2. Inorder : D B H E A F I C G
3. Postorder : D H E B I F G C A

How to traverse the binary tree?

The tree can be traversed using following traversal methods:

- Preorder
  1. Visit the node
  2. Recursively traverse left subtree in Preorder
  3. Recursively traverse right subtree in Preorder
- Inorder
  1. Recursively traverse left subtree in Inorder
  2. Visit the node
  3. Recursively traverse right subtree in Inorder
- Postorder
  1. Recursively traverse left subtree in Postorder
  2. Recursively traverse right subtree in Postorder
  3. Visit the node



1. Preorder : A B D E H C F I G
2. Inorder : D B H E A F I C G
3. Postorder : D H E B I F G C A

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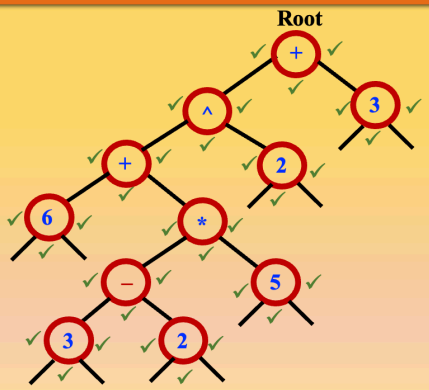
How to traverse the binary tree?

The tree can be traversed using following traversal methods:

- Preorder
  1. Visit the node
  2. Recursively traverse left subtree in Preorder
  3. Recursively traverse right subtree in Preorder
- Inorder
  1. Recursively traverse left subtree in Inorder
  2. Visit the node
  3. Recursively traverse right subtree in Inorder
- Postorder
  1. Recursively traverse left subtree in Postorder
  2. Recursively traverse right subtree in Postorder
  3. Visit the node



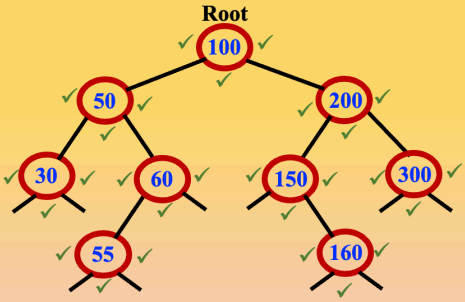
Infix : (6 + (3 – 2) * 5) ^ 2 + 3

1. Preorder : + ^ + 6 * - 3 2 5 2 3
2. Inorder : 6 + 3 - 2 * 5 ^ 2 + 3
3. Postorder : 6 3 2 - 5 * + 2 ^ 3 +

How to traverse the binary tree?

The tree can be traversed using following traversal methods:

- Preorder
  1. Visit the node
  2. Recursively traverse left subtree in Preorder
  3. Recursively traverse right subtree in Preorder
- Inorder
  1. Recursively traverse left subtree in Inorder
  2. Visit the node
  3. Recursively traverse right subtree in Inorder
- Postorder
  1. Recursively traverse left subtree in Postorder
  2. Recursively traverse right subtree in Postorder
  3. Visit the node



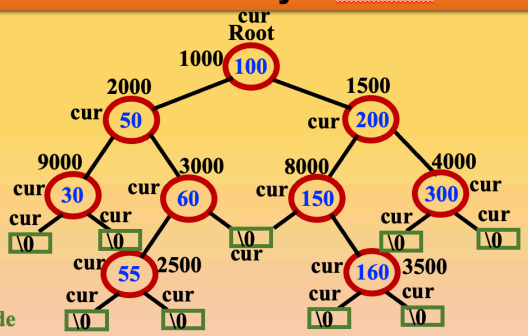
Tree sort

1. Preorder : 100 50 30 60 55 200 150 160 300
2. Inorder : 30 50 55 60 100 150 160 200 300
3. Postorder : 30 55 60 50 160 150 300 200 100

How to design a program to traverse the tree iteratively in inorder?

```
void inorder ( NODE root )
{
    NODE  cur,  stack[20] ;
    int       top  =  -1;

    if ( root == NULL)
    {
        printf ("List is empty\n");
        return;
    }

    cur  =  root;
    for ( ;; )
    {
        while  (cur  !=  NULL)          // Reach leftmost node
        {
            s [++ top ] = cur;          // Push the node
            cur  =  cur -> llink;       // Traverse left subtree
        }

        if  (top  ==  -1) return;       // All nodes have been visited

        cur  =  s [ top -- ];           // Remove the node from stack
        printf (" %d ",  cur -> info);  // Visit the node
        cur  =  cur -> rlink;           // Traverse right subtree
    }
}
```

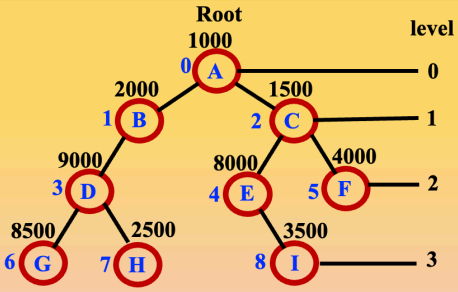
Stack: 4000, 3500, 8000, 1500, 2500, 3000, 9000, 2000, 1000



(2) Inorder : 30 50 55 60 100 150 160 200 300

What is level order traversal of a binary tree?

Definition: The nodes in a tree are numbered starting with root on level 0, continuing with the nodes on level 1, level 2 and so on. Nodes on any level are numbered from left to right. Visiting the nodes using the ordering suggested by node numbering is called level order traversal of a tree.

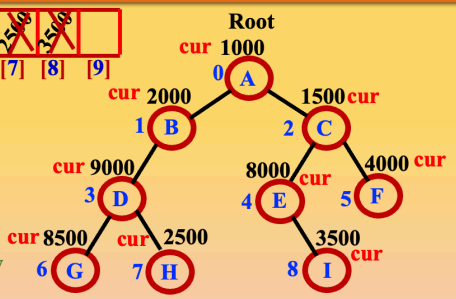


Level order : A B C D E F G H I

How to design a function for level order traversal of a binary tree?

```
void level_order ( NODE root )
{
    NODE    cur,  q[20] ;
    int       front,  rear;

    if (root  ==  NULL)
    {
        printf ("Tree is empty\n") ;
        return;
    }
    front  =  0,   rear  =  -1;
    q [ ++ rear ]  =  root;          // Insert root into queue
    while  (front  <=  rear )        // As long as q is not empty
    {
        cur  =  queue [front++];     // Delete from queue

        printf (" %d ",  cur -> info);   // Visit the node

        if  (cur -> llink  !=  NULL)
            q [ ++ rear ] =  cur -> llink;   // Insert left child into q

        if  (cur -> rlink  !=  NULL)
            q [ ++ rear ] =  cur -> rlink;   // Insert right child into q
    }
}   printf ( " \n ");
```

q: [0] 1000, [1] 2000, [2] 1500, [3] 9000, [4] 8000, [5] 4000, [6] 8500, [7] 2500, [8] 3500, [9]

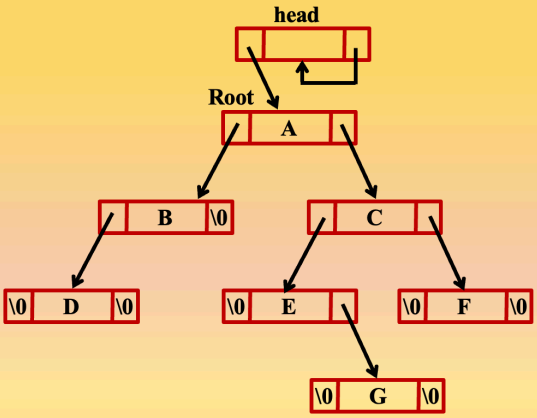


Level order : A B C D E F G H I

What are the disadvantages of binary trees?

Observe the following points:

- Total number of nodes = 7 = n
- Number of actual addresses = 6 = n - 1
- Number of NULL links = 8 = n + 1
- Total number of links = 14 = 2 n
- NULL links are more than the actual addresses
- There are n + 1 NULL links out of 2n total links



Disadvantages of binary trees

- Wasting memory simply by storing \0 characters
- Traversing a tree uses implicit stack in case of recursive traversal and uses explicit stack in case of iterative traversal. So, most of the time is spend in push and pop operations.
- Traversing a binary tree is time consuming.

Note: All the above disadvantages can be overcome using threaded binary trees.

What is threaded binary tree?

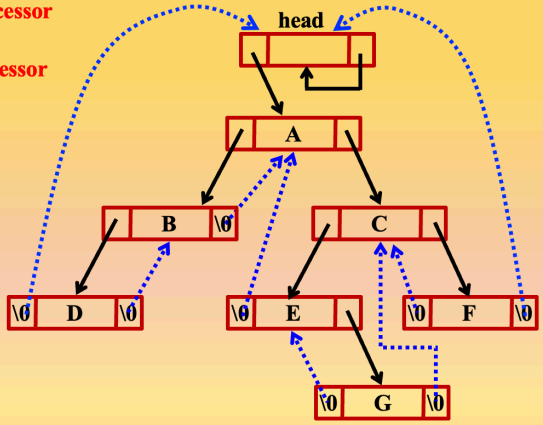
Definition: In a threaded binary tree, all the NULL links are replaced by actual addresses called threads.

- If left link of a node is NULL, replace it with its inorder predecessor if exists. Otherwise, replace it with address of header node.
- If right link of a node is NULL, replace it with its inorder successor if exists. Otherwise, replace it with address of header node.
- A binary tree where all the NULL links are replaced by actual addresses (either inorder predecessor or inorder successor or header node) is called a threaded binary tree.
- The structure can be defined as shown below:

```
struct node
{
        int             info;

        struct node     *llink;

        struct node     *rlink;
};
typedef struct node * NODE;
```

- The header node can be declared as shown below:

```
struct node * head ;
        OR
NODE head ;
```



Inorder traversal: D B A E G C F

How to represent a threaded binary tree in memory?

To represent a threaded binary tree in memory

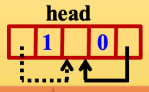
- We must be able to distinguish between normal link and a thread.
- It is done by adding two additional fields to the node structure :

lthread : 1 – llink is a thread (denoted by dotted arrow)
0 – llink is normal link (denoted by black arrow)

rthread : 1 – rlink is a thread (denoted by dotted arrow)
0 – rlink is normal link (denoted by black arrow)

```
struct   node
{
        int             info;
        short int       lthread;
        struct  node    *llink;
        short int       rthread;
        struct  node    *rlink;
};
typedef struct node * NODE ;
```

- An empty tree can be represented as shown below:



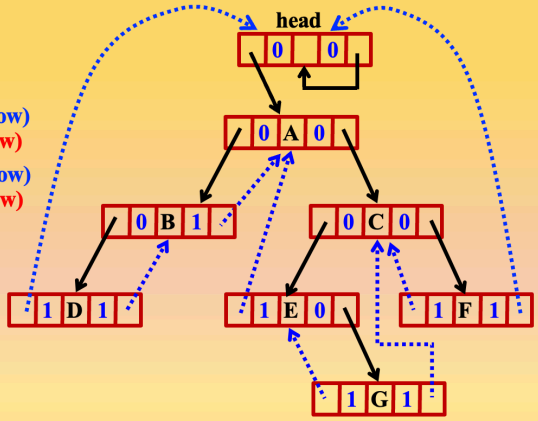


Fig. Memory representation of a threaded binary tree

Inorder traversal: D B A E G C F

How to traverse a threaded binary tree?

The function to find inorder successor can be written as shown below:

```
NODE  inorder_successor ( NODE  x )
{
      NODE     cur ;
      cur  =  x–> rlink;   // Get the address of right node
      if ( x –> rthread == 1) return  cur;
      // Keep moving left till you get thread in left link
      while ( cur  –> lthread  ==  0) cur  =  cur  –> llink;
      return  cur;
}
```

The function to traverse the tree in inorder is shown below:

```
void  inorder ( NODE  head )
{
        NODE  cur;
        cur  =  head;
        for ( ;; )
        {
            cur  =  inorder_successor ( cur );
            if ( cur  ==  head ) return;
            printf (“%d  ”,  cur –>info );
        }
}
```

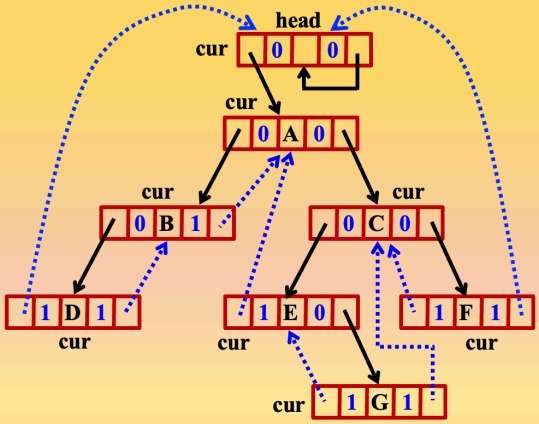


Fig. Memory representation of a threaded binary tree

Inorder traversal: D B A E G C F
✓ ✓ ✓ ✓ ✓ ✓ ✓

MODULE – 4

What is Dictionary?

Definition: A dictionary is a collection of pairs (key, item) where

- The first component of the pair is the key
- The second component of the pair is the item which represent associated information with respect to key.
- No two pairs have the same key in the dictionary i.e., key is always unique.
- For example,

Key	Item
9845070827	Padma Reddy No. 256, 2nd main, Bengaluru.
9900170827	Padma Reddy No. 256, 2nd main, Bengaluru.
888888888	Mithil No. 256, 2nd main, Bengaluru.
999999999	Monalika No. 256, 2nd main, Bengaluru.
(Mobile Number)	(Name with postal address)

- In the above dictionary, telephone number is considered as the *key* and name of the person along with postal address is considered as the item associated with key.

What is ADT Dictionary?

Definition: An Abstract data type Dictionary in short called ADT Dictionary is defined as

- Collection of n pairs (key, item) where each pair has a key along with associated item and
- Set of various operations to be performed on those items (objects).
- The operations specified may be insert, delete, display dictionary contents, search etc.
- ADT Dictionary is
  - Objects : Collection of n pairs where each pair has a key and an associated item such as
    - key : element to be searched.
    - item : information such as name, address etc.
    - d : Dictionary
    - n : Number of pairs (key, item)
  - Functions :

void	insert (item, key, d)	::	Insert the *item* with *key* into dictionary *d*
void	is_empty (d, n)	::	if n == 0 return TRUE else return FALSE
void	delete_item (key, d)	::	If *key* is present in dictionary *d*, delete the entry otherwise display "key not found"
void	search (key, d)	::	if *key* is present in dictionary return the corresponding item else return 0

Note: ADT dictionary can be efficiently implemented using binary search tree.

What is Binary search tree?

Definition: A binary search tree is a binary tree which is either empty or non-empty. If it is not empty, it must satisfy the following properties:

- Each node has exactly one key and all keys must be different.
- For each node say x in the tree, all the the keys in the left subtree must be less than key (x)
- For each node say x in the tree, all the the keys in the right subtree must be greater than key (x)
- For example, all the binary trees shown below are binary search trees.

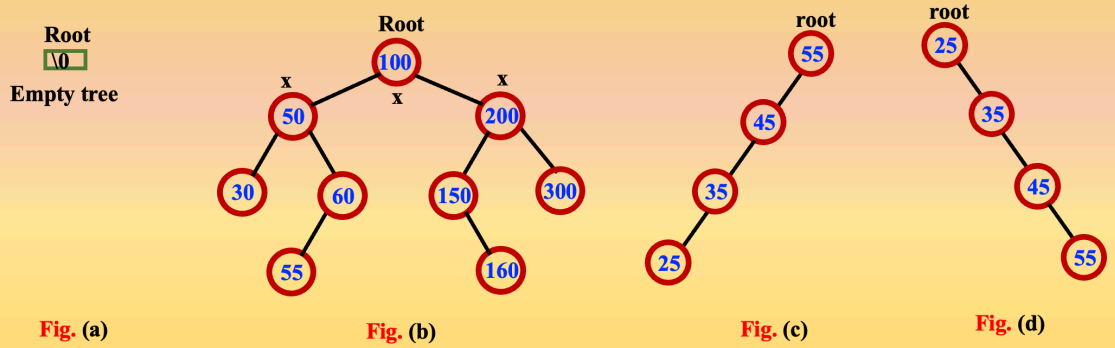
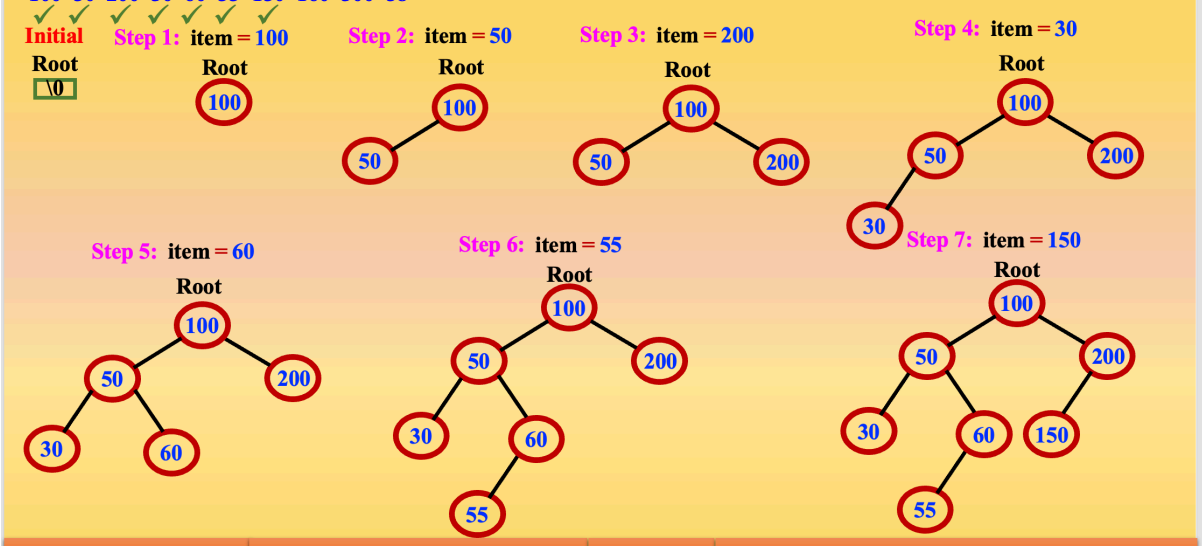


Fig. (a) Fig. (b) Fig. (c) Fig. (d)

How to create a binary search tree for the given the items?

Problem : Create a binary search tree for the following items:

100 50 200 30 60 55 150 160 300 58

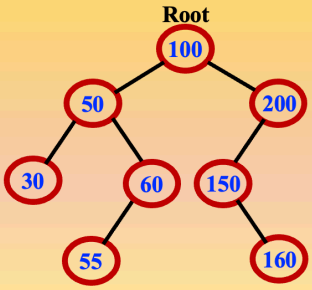


How to create a binary search tree for the given the items?

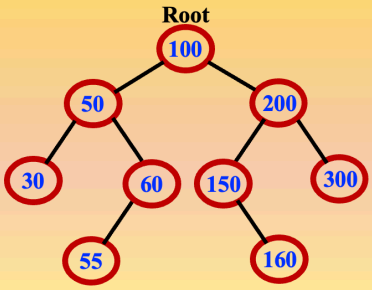
Problem : Create a binary search tree for the following items:

100 ✓ 50 ✓ 200 ✓ 30 ✓ 60 ✓ 55 ✓ 150 ✓ 160 ✓ 300 ✓ 58 ✓

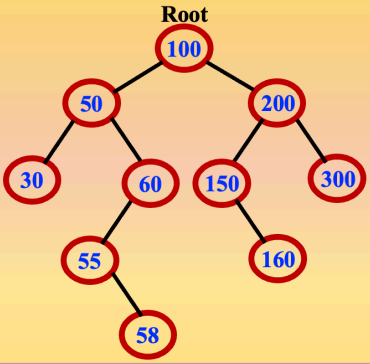
Step 8: item = 160



Step 9: item = 300



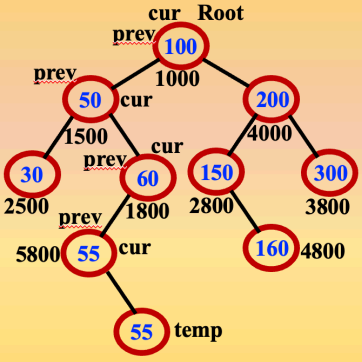
Step 10: item = 58



How to create a binary search tree with duplicates?

Case 1: Tree is empty

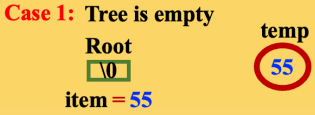
Case 2: Tree is existing



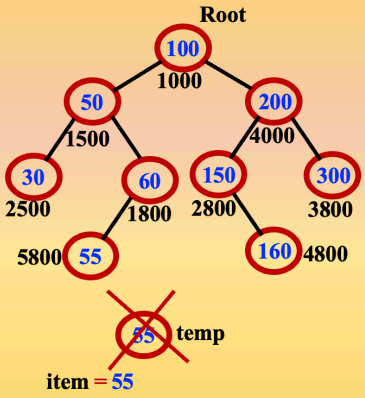

```
NODE  insert  ( int item,  NODE  root )
{
    NODE  temp,  prev,  cur ;

    // create a node
    temp  =  getnode ( );
    temp –> info  =  item;
    temp –> llink = temp –> rlink = NULL;

    // Insert a node for the first time
    if ( root  ==  NULL )  return  temp;

    // Find the appropriate place to insert
    cur  =  root;
    while ( cur  != NULL )
    {
          prev  =  cur;
          if ( item <  cur –> info )
                cur  =  cur –>llink ;
          else
                cur  =  cur –>rlink;
    }

    // Insert the item at appropriate place
    if ( item <  prev –> info )
          prev –>llink  =  temp ;
    else
          prev –>  rlink  =  temp ;

    return  root;
}
```

How to create a binary search tree without duplicates?

Case 1: Tree is empty



Case 2: Tree is existing




```
NODE insert ( int item, NODE root )
{
    NODE temp, prev, cur ;

    temp = getnode ( );
    temp -> info = item;
    temp -> llink = temp -> rlink = NULL;

    if ( root == NULL ) return temp;

    cur = root;
    while ( cur != NULL )
    {
        prev = cur;
        if ( item == cur -> info ) {
            printf ( "Duplicate item " );
            free (temp);  return root;
        }
        if ( item < cur -> info )
            cur = cur -> llink ;
        else
            cur = cur -> rlink;
    }
    if ( item < prev -> info )
        prev -> llink = temp ;
    else
        prev -> rlink = temp ;

    return root;
}
```

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```
int search ( int item, NODE root )
{
    NODE cur ;

    if ( root == NULL ) return 0;

    cur = root;
    while ( cur != NULL )
    {
        if ( item == cur -> info ) return 1;
        if ( item < cur -> info )
            cur = cur -> llink ;
        else
            cur = cur -> rlink;
    }

    return 0;
}
```

C program to create a BST, traverse the tree and search for an item

```
#include <stdio.h>
#include <stdlib.h>

struct   node
{
       int                info;
       struct   node   * llink;
       struct   node   * rlink;
};

typedef   struct   node *   NODE;

NODE   getnode   ( ) ;

NODE   insert ( int  item ,  NODE  root ) ;
int      search ( int  item ,  NODE  root ) ;
void     preorder   ( NODE  root ) ;
void     inorder    ( NODE  root ) ;
void     postorder ( NODE  first ) ;
```

```
void   main  ( )
{
    int          choice,   item;
    NODE      root = NULL;
    for  ( ;; )
    {
        printf (  " 1:Insert       2:Preorder   3:Inorder: " );
        printf (  " 4:Postorder   5:Search     6:Exit : "   );
        scanf (  " %d ",  &choice );
        switch (   choice )
        {
            case 1  :  printf (" Enter the item : " );      scanf ( " %d  ",  &item );
                       root  =  insert ( item,  root ) ;
                       break;
            case 2  :  if (root  ==  NULL)  {
                          printf ( "Tree is empty\n" ) ; break;
                       }
                       printf ( "Preorder :" ) ;     preorder ( root );   break;
            case 5  :  printf (" Enter the item : " );      scanf ( " %d  ",  &item );
                       flag  =  search ( item,  root ) ;
                       if  (flag  ==  1 )
                          printf (" Item found \n " );
                       else
                          printf (" Item  not found \n " );
                       break;
            default:   exit ( 0 );
        }
    }
}
```

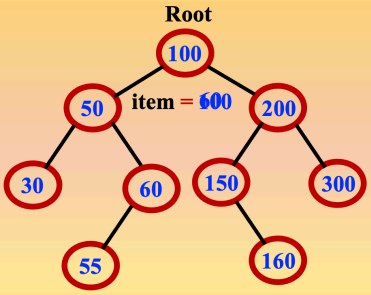

How to search for an item in a binary search tree (recursive procedure)?

```
NODE   search   ( int item,  NODE   root )
{
    if ( root  ==  NULL )  return  NULL;
    if ( item  ==  root->info )  return  root;
    if ( item  <  root->info )
       return     search  ( item,   root->llink ) ;
    return     search ( item,   root->rlink ) ;
}
```

Case 1: Tree is empty

Root → \0 item = 100

Case 2: Tree is existing



How to count the number of nodes in a tree?

```
int   count = 0;

void  count_node ( NODE   root )
{
    if ( root  ==  NULL )  return;
    count_node  ( root->llink ) ;
    count++;
    count_node  ( root->rlink ) ;
}
```

How to count the number of leaves in a tree?

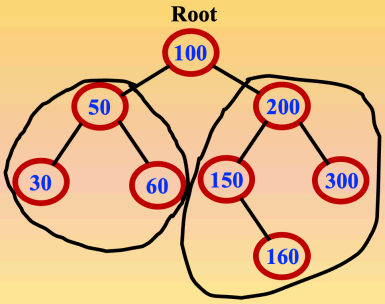
```
int   count = 0;

void  count_leaf   ( NODE   root )
{
    if ( root  ==  NULL )  return;
    count_leaf    ( root->llink ) ;
    if ( root->llink  ==  NULL  &&  root->rlink  ==  NULL )  count++;
    count_leaf    ( root->rlink ) ;
}
```

How to find the height of the tree?

```
int max ( int a, int b)
{
    return a > b ? a : b
}

int height ( NODE root )
{
     if ( root == NULL ) return 0;
     return 1 + max ( height ( root -> llink ) , height ( root -> rlink) ) ;
}
```

Recursive definition to find height of the tree

$$H(root) = \begin{cases} 0 & \text{if root} == \backslash 0 \\ 1 + \max (H (root \rightarrow llink) , H (root \rightarrow rlink)) & \text{otherwise} \end{cases}$$



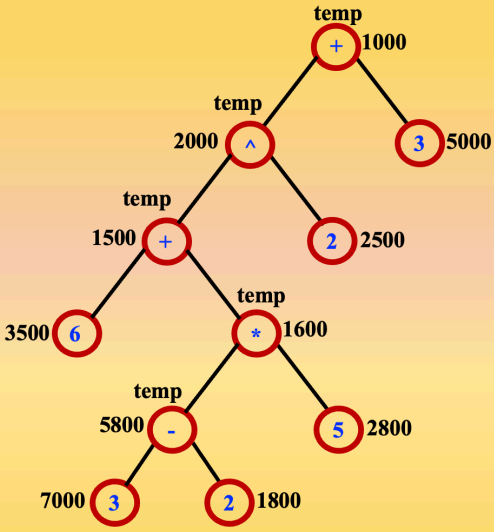
1 + MAX (Height = 2 , Height = 3)

How to create an expression tree?

Infix: (6 + (3 - 2) * 5) ^ 2 + 3

Postfix:

6	3	2	-	5	*	+	2	^	3	+	\0		

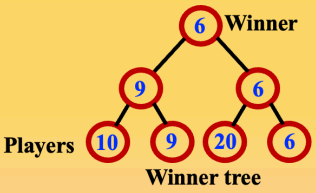



```
NODE create_expression_tree (char  postfix [] )
{
      NODE  temp, stack [ 20 ];
      int   i,  top = -1;

      for ( i = 0;  postfix [ i ]  !=  '\0';  i++ )
      {
            temp  =  getnode ( );
            temp -> info =  postfix [ i ];
            temp -> llink = temp -> rlink = NULL;

            if (  isalnum ( temp -> info ) )
                  stack [++top ] =  temp;
            else
            {
                  temp -> rlink =  stack [top-- ];
                  temp -> llink  =  stack [top-- ];
                  stack [++top ] =  temp;
            }
      }
      return   s [top-- ];
}
```

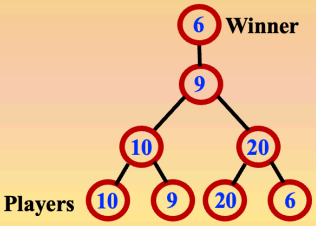
What is s selection/tournament tree?

Definition: A selection tree or tournament tree is a tree data structure which is used to select a winner in a knockout tournament.

- The leaves of the tree represent players entering the tournament
- Each internal node represent a winner or a loser in the match.
- If the internal nodes in a tree represent winners the tree is called winner tree.
- If the internal nodes in a tree represent losers the tree is called loser tree.
- The two types of selection trees are:
  - Winner tree
  - Loser tree



Winner tree



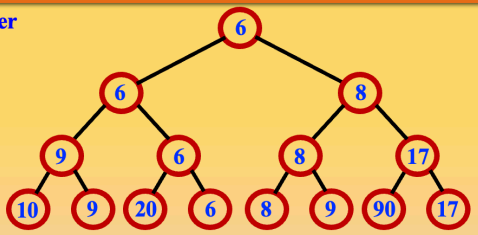
Loser tree

Note: Using selection trees we can sort the elements in ascending/descending order

What is winner tree?

Definition: A selection tree where each internal node represents the winner is called winner tree. A winner tree is a complete binary tree with n-leaf nodes and n - 1 internal nodes where

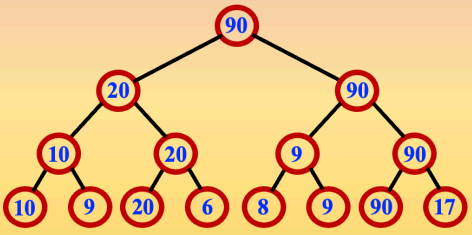
- Each internal node records the winner of the match.
- A winner tree can be either a min winner tree or max winner tree.
- To determine the winner of the match, we assume that each player is associated with a value.
  - In a min winner tree, each internal node represents the smaller of its two children i.e., the player with the smaller value wins.
  - In a max winner tree, each node represents the larger of its two children i.e., the player with the larger value wins.
- The root node represents the smallest node in min winner tree.
- The root node represents the largest node in max winner tree.



Min winner tree

Number of leaves = 8 = n

Number of internal nodes = 7 = n - 1

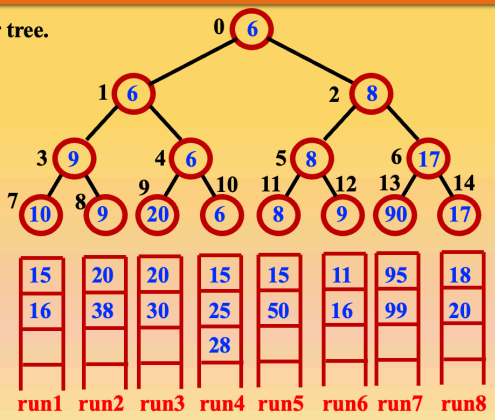


Max winner tree

Note: If two value are same, the left child will be the winner.

What is the procedure to construct min winner tree?

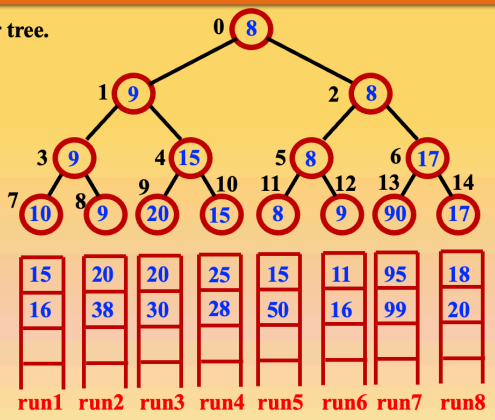
- A set of records arranged in ascending order is input to get a winner tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Between every two players, select the smaller item and elevate to parent position.
- Each non-leaf node in the tree represent the winner in the tournament and the root node contains the smallest item indicating winner of the tournament.
- A winner tree may be represented using sequential representation (array representation) for binary trees
- The number above the node indicates the position of that node in array representation.
- The root node contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.



Output: 6

What is the procedure to construct min winner tree?

- A set of records arranged in ascending order is input to get a winner tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Between every two players, select the smaller item and elevate to parent position.
- Each non-leaf node in the tree represent the winner in the tournament and the root node contains the smallest item indicating winner of the tournament.
- A winner tree may be represented using sequential representation (array representation) for binary trees
- The number above the node indicates the position of that node in array representation.
- The root node contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.
- Now, reconstruct the winner tree by playing the tournament along the path from root node



Output: 6 8

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What is the procedure to construct min winner tree?

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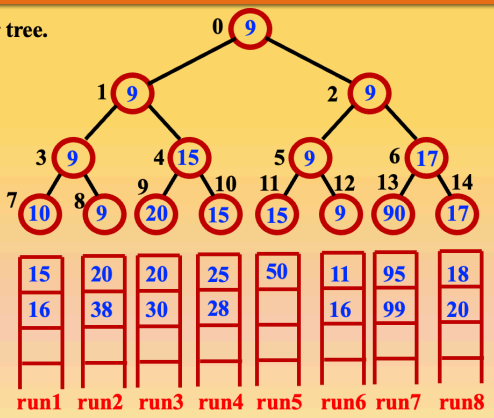

Output: 6 8 9

What is the procedure to construct min winner tree?

- A set of records arranged in ascending order is input to get a winner tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Between every two players, select the smaller item and elevate to parent position.
- Each non-leaf node in the tree represent the winner in the tournament and the root node contains the smallest item indicating winner of the tournament.
- A winner tree may be represented using sequential representation (array representation) for binary trees
- The number above the node indicates the position of that node in array representation.
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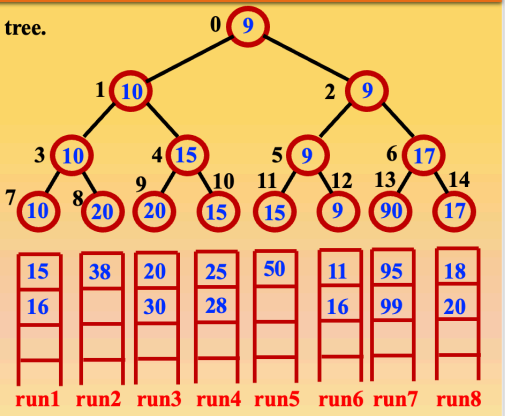

Output: 6 8 9 9

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What is the procedure to construct min winner tree?

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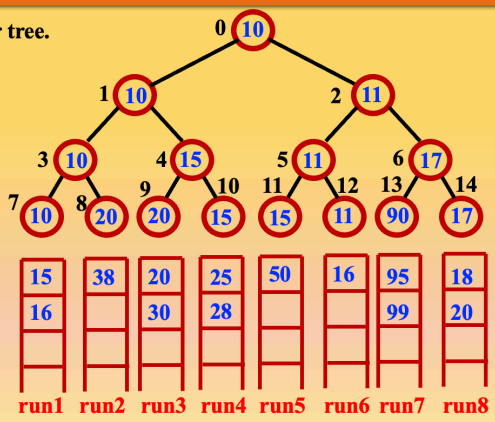


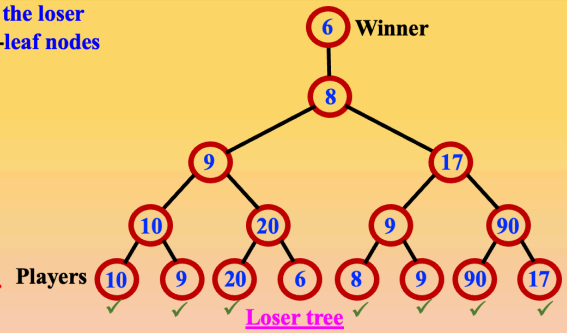
Output: 6 8 9 9 10 and so on.

tournament tree (Winner tree) sort

What is loser tree?

Definition: A selection tree where each internal node represents the loser is called loser tree. A loser tree is a complete binary tree with n-leaf nodes and n - 1 internal nodes where

- Each internal node records the loser of the match.
- To determine the loser of the match, we assume that each player is associated with a value.
- In a loser tree, each internal node represents the loser. i.e., we select a player who loses the match.
- The final player who has not lost any match is the winner of the tournament and it is written right above the root node.

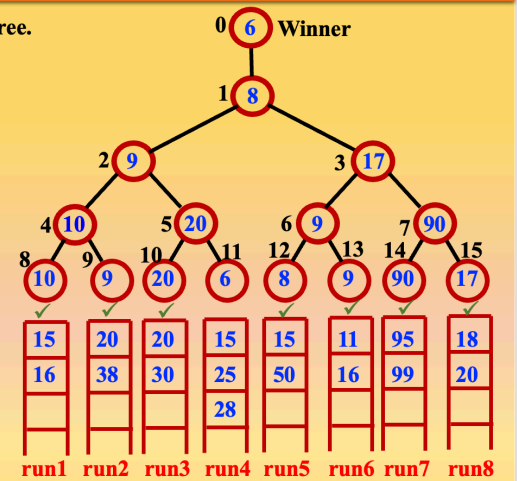


Number of leaves = 8 = n

Number of internal nodes = 7 = n - 1

What is the procedure to construct a loser tree?

- A set of records arranged in ascending order is input to get a loser tree. These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Among the players, select a player who lost the match and elevate to parent position.
- Each non-leaf node in the tree represent the loser in the tournament and a person who has not lost is the winner of the tournament.
- The winner is indicated by parent of root node and contains the smallest item.
- The loser tree may be represented using sequential representation (array representation) for binary trees
- The number by the side of the node indicates the position of that node in array representation.
- The parent of root contains smallest item. Remove the root node and output.
- After removing the smallest node, obtain the next node from corresponding run.



Output: 6

What is the procedure to construct a loser tree?

- A set of records arranged in ascending order is input to get a loser tree.
  These sets of records arranged in ascending are called runs.
- Initially, take the first item from each run and treat them as leaves of a tree.
- Among the players, select a player who lost the match and elevate to parent position.
- Each non-leaf node in the tree represent the loser in the tournament and a person who has not lost is the winner of the tournament.
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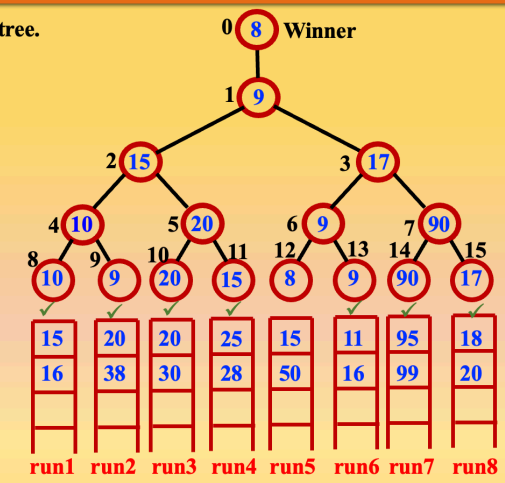

Output: 6 8

What is the procedure to construct a loser tree?

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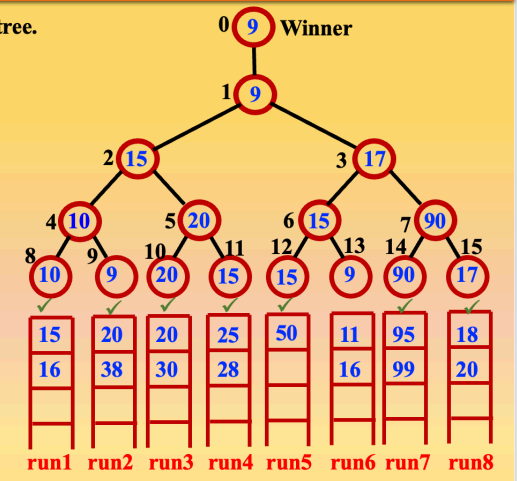

Output: 6 8 9

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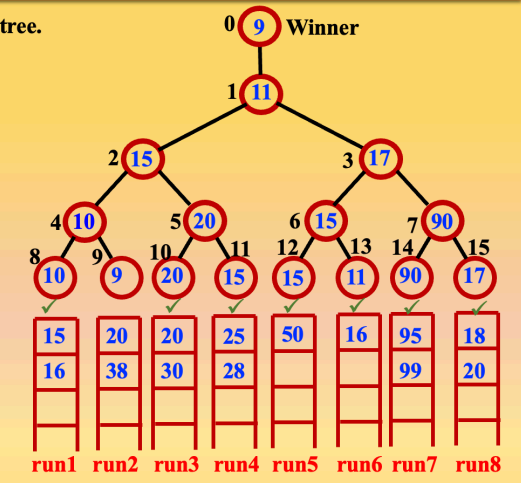

Output: 6 8 9 9

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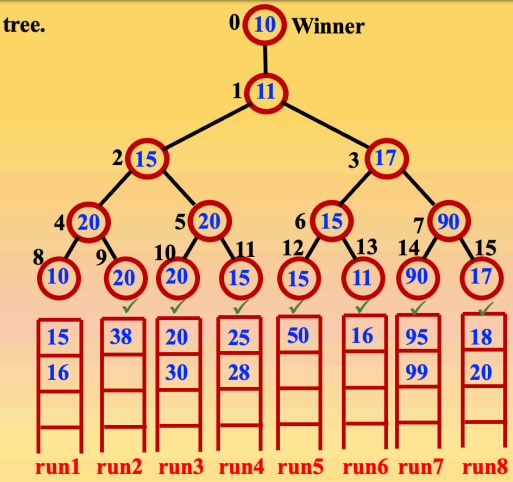

Output: 6 8 9 9 10

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What is the procedure to construct a loser tree?

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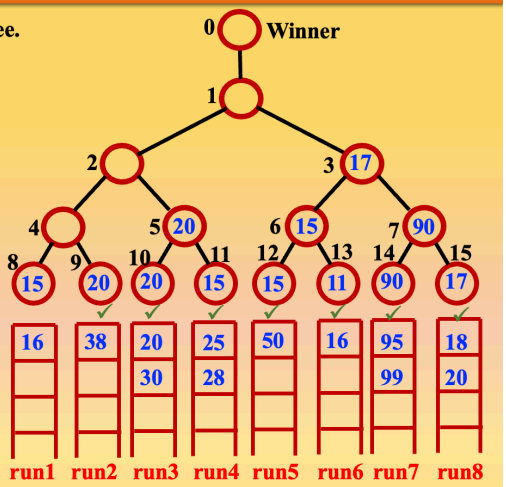


Output: 6 8 9 9 10 and so on.

tournament tree (Loser tree) sort

Chapter 11: Graphs

What are we studying in this chapter?

- Definitions
- Terminologies
- Matrix and Adjacency List Representation of Graphs
- Elementary Graph operations
- Traversal methods:
  - Breadth First Search
  - Depth First Search

11.1 Introduction

In this chapter, let us concentrate another important and non-linear data structure called graph. In this chapter, we discuss basic terminologies and definitions, how to represent graphs and how graphs can be traversed.

11.2 Graph Theory terminology

First, let us see "What is a vertex?"

Definition: A vertex is a synonym for a node. A vertex is normally represented by a circle. For example, consider the following figure:

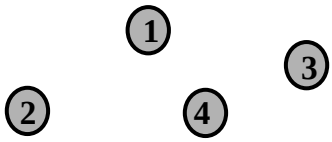


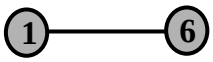
Fig Vertices

In the above figure, there are four nodes identified by 1, 2, 3, 4. They are also called vertices and normally denoted by a set V = {1, 2, 3, 4}.

Now, let us see "What is an edge?"

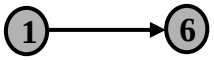
Definition: If *u* and *v* are vertices, then an *arc* or a *line* joining two vertices *u* and *v* is called an *edge*.

Example 1: Consider the figure:



Observe the following points from above figure:

- *There is no direction* for the edge between vertex 1 and vertex 6 and hence it is undirected edge.
- The undirected edge is denoted by an ordered pair (1, 6) where 1 and 6 are called *end points of the edge* (1, 6). In general, if e = (u, v), then the nodes u and v are called *end points of directed edge.*
- In this graph, edge (1, 6) is same as edge (6, 1) since there is no direction associated with that edge. So, (u, v) and (v, u) represent same edge.

Example 2: consider the figure: 

Observe the following points from above figure:

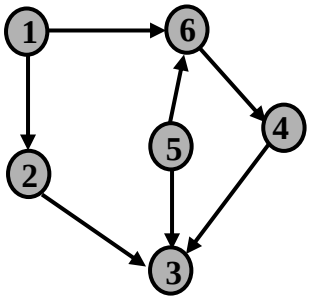
- *There is a direction* for the edge originating at vertex 1 (called tail of the edge) and heading towards vertex 6 (called head of the edge) and hence it is called directed edge.
- The directed edge is denoted by the directed pair <1, 6> where 1 is called *tail of the edge* and 6 is the *head of the edge*. So, the directed pair <1, 6> is not same as directed pair <6, 1>.
- In general, if a directed edge is represented by directed pair <u, v>, *u* is called the *tail of the edge* and *v* is the *head of the edge*. So, the directed pair <u, v> is different from the directed pair <v, u>. So, <u, v> and <v, u> represent two different edges.

Now, let us see "What is a graph?"

Definition: Formally, a graph **G** is defined as a pair of two sets V and E denoted by

G = (V, E)

where V is set of vertices and E is set of edges. For example, consider the graph shown below:



Here, graph G = (V, E) where

- V = {1, 2, 3, 4, 5, 6} is set of vertices
- E = { <1, 6>, <1, 2>, <2, 3>, <4, 3>, <5, 3>, <5, 6>, <6, 4> } is set of directed edges

Note:

- |V| = |{1, 2, 3, 4, 5, 6}| = 6 represent the number of vertices in the graph.

- |E| = |{<1, 6>, <1, 2>, <2, 3>, <4, 3>, <5, 3>, <5, 6>, <6, 4> }| = 7 represent the number of edges in the graph.

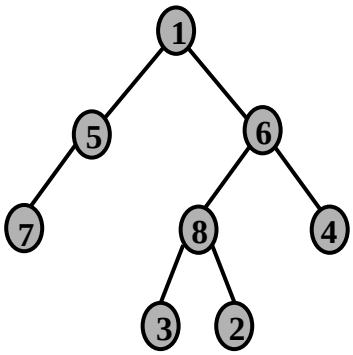
Now, let us see "What is a directed graph? What is an undirected graph?"

Definition: A graph G = (V, E) in which every edge is directed is called a directed graph. The directed graph is also called digraph. A graph G = (V, E) in which every edge is undirected is called an undirected graph. Consider the following graphs:

Here, graph G = (V, E) where

- V = {0, 1, 2} is set of vertices
- E = {<0, 1>, <1, 0>, <1, 2>} is set of edges

Note: Since all edges are directed it is a directed graph. In directed graph we use angular brackets < and > to represent an edge



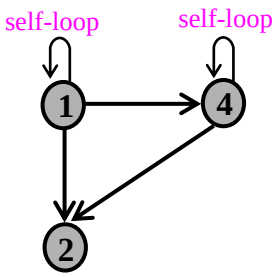
Here, graph G = (V, E) where

- V = {1, 2, 3, 4, 5, 6, 7, 8} is set of vertices
- E = {(1, 5), (1, 6), (5, 7), (6, 8), (6, 4), (8, 3), (8, 2)} is set of edges

Note: Since all edges are undirected, it is an undirected graph. In undirected graph we use parentheses (and) to represent an edge (u, v).

Now let us see "What is a self-loop (or self-edge)?

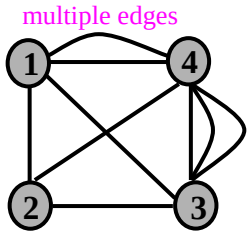
Definition: A loop is an edge which starts and ends on the same vertex. A loop is represented by an ordered pair (i, i). This indicates that the edge originates and ends in the same vertex. A loop is also called self-edge or self-loop. In the given graph shown below, there are two self-loops namely, <1, 1> and <4, 4>.



Now, let us see "What is a multigraph?"

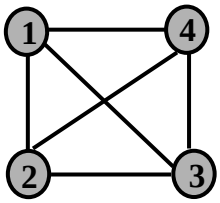
11.4 Graphs

Definition: A graph with multiple occurrence of the same edge between any two vertices is called multigraph. Here, there are two edges between the nodes 1 and 4 and there are three edges between the nodes 4 and 3.

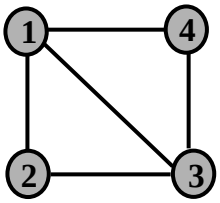


Now, let us see "What is a complete graph?"

Definition: A graph G = (V, E) is said to be a complete graph, if there exists an edge between every pair of vertices. The graph (a) below is complete. Observe that in a complete graph of *n* vertices, there will be n(n-1)/2 edges. Substituting *n* = 4, we get 6 edges. Even if one edge is removed as shown in graph (b) below, it is not complete graph.



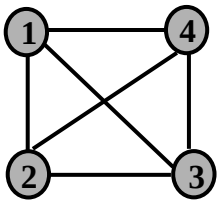
Complete graph



Not a complete graph

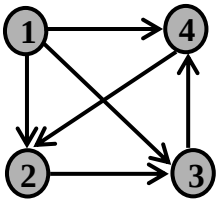
Now, let us see "What is a path?"

Definition: Let G = (V, E) be a graph. A *path* from vertex *u* to vertex *v* in an undirected graph is a sequence of adjacent vertices (u, v_0, v_1, v_2,….v_k, v) such that: (u, v_0), (v_0, v_1),….(v_k, v) are the edges in G. Consider the following graph:



In the graph, the path from vertex 1 to 4 is denoted by: 1, 2, 3, 4 which can also be written as (1, 2), (2, 3), (3, 4).

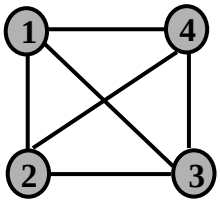
Definition: Let G = (V, E) be a graph. A *path* from vertex *u* to vertex *v* in a directed graph is a sequence of adjacent vertices <u, v_0, v_1, v_2,….v_k, v> such that <u, v_0>, <v_0, v_1>,….<v_k, v> are the edges in G. Consider the following graph:



In the graph, the path from vertex 1 to 3 is denoted by 1, 4, 2, 3 which can also be written as <1, 4>, <4, 2>, <2, 3>

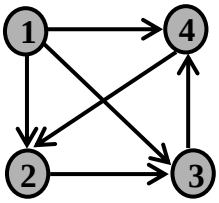
Now, let us see "What is simple path?"

Definition: A *simple path* is a path in which all vertices except possibly the first and last are distinct. Consider the undirected and directed graph shown below:



Ex 1: In the graph, the path 1, 2, 3, 4 is simple path since each node in the sequence is distinct.

Ex2: In the graph, the path 1, 2, 3, 2 is not a simple path since the nodes in sequence are not distinct. The node 2 appears twice in the path



Ex 1: In the graph, the path 1, 4, 2, 3 is simple path since each node in the sequence is distinct.

Ex 2: The sequence 1, 4, 3 is not a path since there is no edge <4, 3> in the graph.

Now, let us see "What is length of the path?"

Definition: The *length* of the path is the number of edges in the path.

Ex 1: In the above undirected graph, the path (1, 2, 3, 4) has length 3 since there are three edges (1, 2), (2, 3), (3, 4). The path 1, 2, 3 has length 2 since there are two edges (1, 2), (2, 3).

Ex 2: In the above directed graph, the path <1, 2, 3, 4> has length 3 since there are three edges <1, 2>, <2, 3>, <3, 4>. The path <1, 4, 2> has length 2 since there are two edges <1, 4>, <4, 2>.

Now, let us "Define the terms cycle (circuit)?"

Definition: A cycle is a path in which the first and last vertices are same.

For example, the path <4, 2, 3, 4> shown in above directed graph is a cycle, since the first node and last node are same. It can also be represented as <4, 2>, <2, 3>, <3, 4> <4, 2>.

Note: A graph with at least one cycle is called a cyclic graph and a graph with no cycles is called ***acyclic*** graph. A tree is an acyclic graph and hence it has no cycle.

Now, let us see "What is a connected graph?"

11.6 Graphs

Definition: In an undirected graph G, two vertices *u* and *v* are said to be connected if there exists a path from *u* to *v*. Since G is undirected, there exists a path from *v* to *u* also. A graph G (directed or undirected) is said to be connected if and only if there exists a path between every pair of vertices.

For example, the graphs shown in figure below are connected graphs.

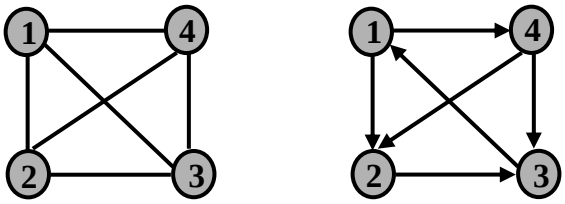
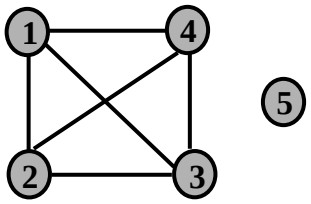


Figure Connected graphs

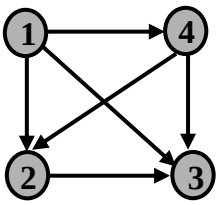
Now, let us see "What is a disconnected graph?"

Definition: Let G = (V, E) be a graph. If there exists at least one vertex in a graph that cannot be reached from other vertices in the graph, then such a graph is called disconnected graph. For example, the graph shown below is a disconnected graph.



Not connected

Since vertex 1 is not reachable from 3, the graph is not connected



11.3 Representation of graph

Now, let us see "What are the different methods of representing a graph?" The graphs can be represented in two different methods:

Representation of graph → Adjacency matrix
Representation of graph → Adjacency linked list

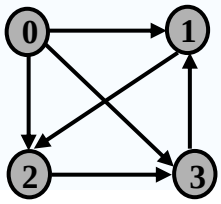
Let us see "What is an adjacency matrix? explain with example"

Definition: Let G = (V, E) be a graph where V is set of vertices and E is set of edges. Let N be the number of vertices in graph G. The adjacency matrix A of a graph G is formally defined as shown below:

$$A[i][j] = \begin{cases} 1 & \text{if there is an edge from vertex } i \text{ to vertex } j. \\ 0 & \text{if there is no edge from vertex } i \text{ to vertex } j. \end{cases}$$

- It is clear from the definition that an adjacency matrix of a graph with *n* vertices is a Boolean square matrix with *n* rows and *n* columns with entries 1's and 0's (bit-matrix)
- In an undirected graph, if there exists an edge (i, j) then a[i][j] and a[j][i] is made 1 since (i, j) is same as (j, i)
- In a directed graph, if there exists an edge <i, j> then a[i][j] is made 1 and a[j][i] will be 0.
- If there is no edge from vertex *i* to vertex *j*, then a[i][j] will be 0.

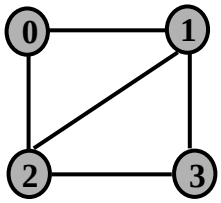
Note: The above definition is true both for directed and undirected graph. For example, following figures shows the directed and undirected graphs along with equivalent adjacency matrices:



(a) Directed graph

	0	1	2	3
0	0	1	1	1
1	0	0	1	0
2	0	0	0	1
3	0	1	0	0

Adjacency matrix



(b) Undirected graph

	0	1	2	3
0	0	1	1	0
1	1	0	1	1
2	1	1	0	1
3	0	1	1	0

Adjacency matrix

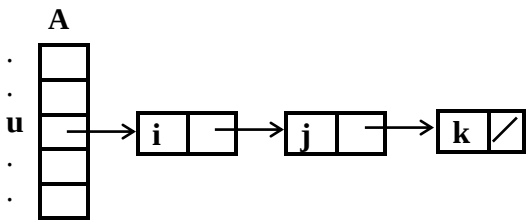
Fig. Graphs and equivalent adjacency matrices

Now, let us see "What is an adjacency list? explain with example"

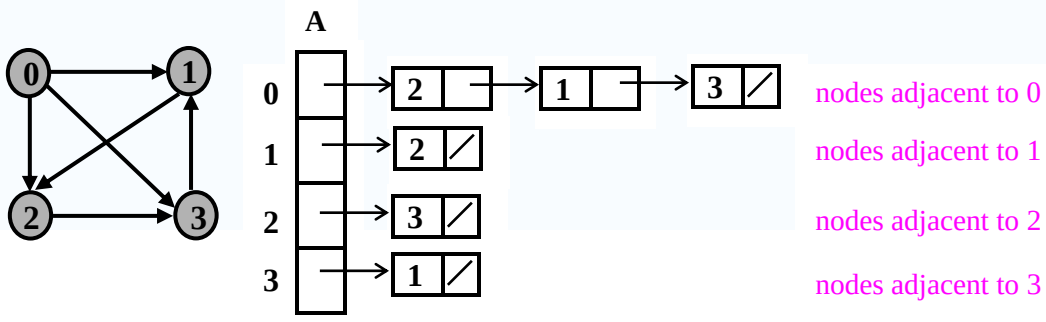
11.8 Graphs

Definition: Let G = (V, E) be a graph. An *adjacency linked list* is an array of *n* linked lists where *n* is the number of vertices in graph G. Each location of the array represents a vertex of the graph. For each vertex $u \in V$, a linked list consisting of all the vertices adjacent to *u* is created and stored in A[u]. The resulting array A is an adjacency list.

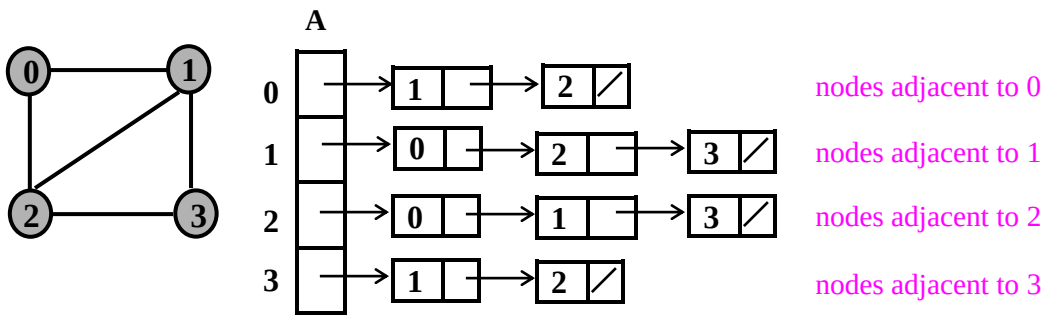
Note: It is clear from the above definition that if *i*, *j* and *k* are the vertices adjacent to the vertex *u*, then *i*, *j* and *k* are stored in a linked list and starting address of linked list is stored in A[u] as shown below:



For example, figures below shows the directed and undirected graphs along with equivalent adjacency linked list:



(a) Directed graph **Adjacency linked list**



(b) Undirected graph **Adjacency linked list**

Fig.: Graphs and equivalent adjacency linked lists

Now, let us see "Which graph representation is best?" The graph representation to be used depends on the following factors:

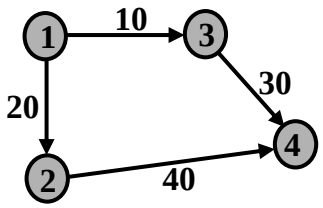
- Nature of the problem
- Algorithm used for solving
- Type of the input.
- Number of vertices and edges:
  - If a graph is *sparse*, less number of edges are present. In such case, the adjacency list has to be used because this representation uses lesser space when compared to adjacency matrix representation, even though extra memory is consumed by the pointers of the linked list.
  - If a graph is *dense*, the adjacency matrix has to be used when compared with adjacency list since the linked list representation takes more memory.

Note: So, based on the nature of the problem and based on whether the graph is *sparse* or *dense*, one of the two representations can be used.

Now, let us see "What is a weighted graph?"

Definition: A graph in which a number is assigned to each edge in a graph is called weighted graph. These numbers are called costs or weights. The weights may represent the cost involved or length or capacity depending on the problem.

For example, in the following graph shown in figure the values 10, 20, 30 and 40 are the weights associated with four edges <1,3>, <1,2>, <3,4> and <2,4>



Let us see "How the weighted graph can be represented?" The weighted graph can be represented using adjacency matrix as well as adjacency linked list. The adjacency matrix consisting of costs (weights) is called cost adjacency matrix. The adjacency linked list consisting of costs (weights) is called cost adjacency linked list. Now, let us see "What is cost adjacency matrix?"

Definition: Let G = (V, E) be the graph where V is set of vertices and E is set of edges with *n* number of vertices. The cost adjacency matrix A of a graph G is formally defined as shown below:

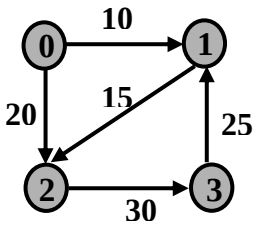
$$A[i][j] = \begin{cases} w & \text{if there is a weight associated with edge from vertex } i \text{ to vertex } j. \\ \infty & \text{if there is no edge from vertex } i \text{ to vertex } j. \end{cases}$$

11.10 Graphs

It is clear from the above definition that

- The element in i^{th} row and j^{th} column is weight *w* provided there exist an edge from i^{th} vertex to j^{th} vertex with cost *w*
- ∞ if there is no edge from vertex *i* to vertex *j*.
- The cost from vertex *i* to vertex *i* is ∞ (assuming there is no loop).

For example, the weighted graph and its *cost adjacency matrix* is shown below:



(a) Weighted graph

	0	1	2	3
0	∞	10	20	∞
1	∞	∞	15	∞
2	∞	∞	∞	30
3	∞	25	∞	∞

Note: Diagonal values can be replaced by 0's

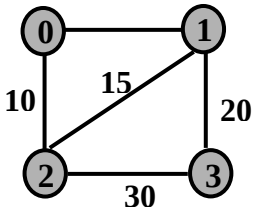
(b) Adjacency matrix

Figure A weighted digraph and the cost adjacency matrix

For the undirected graph, the elements of the cost adjacency matrix are obtained using the following definition:

$$A[i][j] = \begin{cases} w & \text{if there is a weight associated with edge (i, j) or (j, i)} \\ \infty & \text{if there is no edge from vertex } i \text{ to vertex } j. \end{cases}$$

The undirected graph and its equivalent adjacency matrix is shown below:



(a)

	0	1	2	3
0	∞	25	10	∞
1	25	∞	15	20
2	10	15	∞	30
3	∞	20	30	∞

Diagonal values can be replaced by 0's

(b)

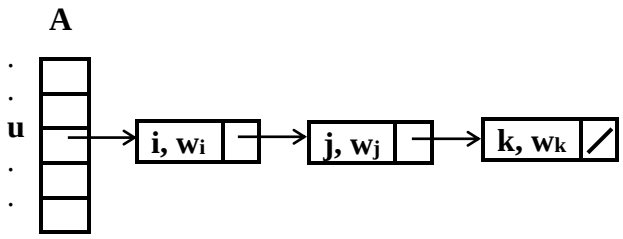
Figure: Weighted undirected graph and the adjacency matrix

Note: The cost adjacency matrix for the undirected graph is symmetric (i.e., a[i, j] is same as a[j, i]) whereas the cost adjacency matrix for a directed graph may not be symmetric.

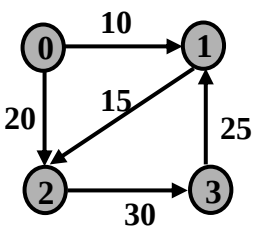
Note: For some of the problems, it is more convenient to store 0's in the main diagonal of cost adjacency matrix instead of ∞.

Now, let us see "What is cost adjacency linked list?"

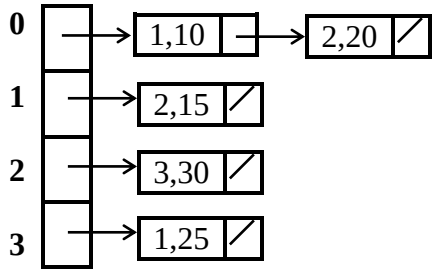
Definition: Let G = (V, E) be a graph where V is set of vertices and E is set of edges with *n* number of vertices. A *cost adjacency linked list* is an array of *n* linked lists. For each vertex u ∈ V, A[u] contains the address of a linked list. All the vertices which are adjacent from vertex *u* are stored in the form of a linked list (in an arbitrary manner) and the starting address of first node is stored in A[u]. If *i, j* and *k* are the vertices adjacent to the vertex u, then *i, j* and *k* are stored in a linked list along with the weights in A[u] as shown below:



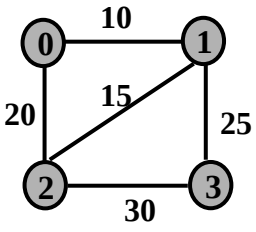
For example, the figure below shows the weighted diagraph and undirected graph along with equivalent adjacency list.



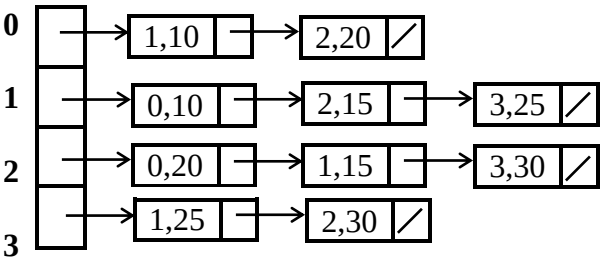
(a) weighted digraph



(b) adjacency list



(c) weighted undirected graph



(d) adjacency list

Figure weighted graph and equivalent adjacency list

Now, the function to read an adjacency matrix can be written as shown below:

Example 11.1: Function to read adjacency matrix

```
void read_adjacency_matrix(int a[10][10], int n)
{
        int     i, j;

        for (i = 0; i < n; i++)
        {
                for (j = 0; j < n; j++)
                {
                        scanf("%d", &a[i][j]);
                }
        }
}
```

The function to read adjacency list can be written as shown below:

Example 11.2: Function to read adjacency list

```
void read_adjacency_list (NODE a[], int n)
{
        int     i, j, m, item;

        for (i = 0; i < n; i++)
        {
                printf("Enter the number of nodes adjacent to %d:", i);
                scanf("%d", &m);

                if (m ==0) continue;

                printf("Enter nodes adjacent to %d : ", i);

                for (j = 0; j < m; j++)
                {
                        scanf("%d", &item);

                        a[i] = insert_rear(item, a[i]);
                }
        }
}
```

11.4 Graph traversals

Now, we concentrate on a very important topic namely graph traversal techniques and see "What is graph traversal? Explain different graph traversal techniques"

Definition: The process of visiting each node of a graph systematically in some order is called graph traversal. The two important graph traversal techniques are:

- **B**readth **F**irst **S**earch (**BFS**)
- **D**epth **F**irst **S**earch (**DFS**)

11.4.1 Breadth First Search (BFS)

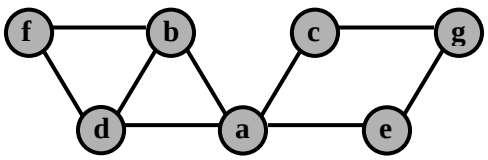
Now, let us see "What is breadth first search (BFS)?"

Definition: The breadth first search is a method of traversing the graph from an arbitrary vertex say *u*. First, visit the node *u*. Then we visit all neighbors of *u*. Then we visit the neighbors of neighbors of *u* and so on. That is, we visit all the neighboring nodes first before moving to next level neighbors. The search will terminate when all the vertices have been visited.

BFS traversal can be implemented using a queue. As we visit a node, it is inserted into queue. Now, delete a node from a queue and see the adjacent nodes which have not been visited. The unvisited nodes are inserted into queue and marked as visited. Deleting and inserting operations as discussed are continued until queue is empty.

Now, let us take an example and see how BFS traversal can be used to see what are all the nodes which are reachable from a given source vertex.

Example 11.3: Traverse the following graph by breadth-first search and print all the vertices reachable from start vertex *a*. Resolve ties by the vertex alphabetical order.



Solution: It is given that source vertex is *a*. Perform the following activities:

Initialization: Insert source *vertex a into queue* and *add a to S* as shown below:

	(i) u = del(Q)	(ii) v = adj. to u	(iii) Nodes visited S	(iii) queue
Initialization	-	-	a	a

Step 1: i): Delete an element *a* from queue
ii): Find the nodes adjacent to *a* but not in S: i.e., *b, c, d* and *e*
iii): Add *b, c, d* and *e* to S, insert into queue as shown in the table:

	(i) u = del(Q)	(ii) v = adj. to u	(iii) Nodes visited S	(iii) queue
	-	-	a	a
Step 1	a	b, c, d, e	a, b, c, d, e	b, c, d, e

Step 2: i): Delete *b* from queue
ii): Find nodes adjacent to *b* but not in S: i.e., f
iii): Add *f* to S and insert *f* into queue as shown in table:

	(i) u = del(Q)	(ii) v = adj. to u	(iii) Nodes visited S	(iii) queue
	-	-	a	a
Step 1	a	b, c, d, e	a, b, c, d, e	b, c, d, e
Step 2	b	f	a, b, c, d, e, f	c, d, e, f

Stage 3: i) : Delete *c* from queue
ii): Find nodes adjacent to *c* but not in S: i.e., g
iii): Add *g* to S, insert *g* into queue as shown in table

	(i) u = del(Q)	(ii) v = adj. to u	(iii) Nodes visited S	(iii) queue
	-	-	a	a
Step 1	a	b, c, d, e	a, b, c, d, e	b, c, d, e
Step 2	b	f	a, b, c, d, e, f	c, d, e, f
Step 3	c	g	a, b, c, d, e, f, g	d, e, f

The remaining steps are shown in the following table:

(i) (ii) ←——————(iii)——————→

	u = del(Q)	v = adj. to u	Nodes visited S	queue
Initialization	-	-	a	a
Step 1	a	b, c, d, e	a, b, c, d, e	b, c, d, e
Step 2	**b**	a, d, **f**	a, b, c, d, e, f	c, d, e, f
Step 3	c	a, **g**	a, b, c, d, e, f, g	d, e, f, g
Step 4	d	a, b, f	a, b, c, d, e, f, g	e, f, g
Step 5	e	a,g	a, b, c, d, e, f, g	f, g
Step 6	f	b, d	a, b, c, d, e, f, g	g
Step 7	g	c. e	a, b, c, d, e, f, g	empty

Thus, the nodes that are reachable from source *a*: **a, b, c, d, e, f, g**

11.4.1.1 Breadth First Search (BFS) using adjacency matrix

The above activities are shown below in the form of an algorithm along with pseudocode in C when graph is represented as an adjacency matrix.

```
no node is visited to start with                // int s [10] = {0};

insert source u to q                            // f = 0, r = -1, q[++r] = u
print u                                         // print u
mark u as visited i.e., add u to S              // s[u] = 1
while queue is not empty                        // while f <= r

    Delete a vertex u from q                    //      u = q[f++]
    For every v adjacent to u                   //      for each v, if a[u][v] == 1
    If v is not visited                         //          if s[v] == 0
        print v                                 //              print v
        mark v as visited                       //              s[v] = 1
        Insert v to queue                       //              q[++r] = v
    end if                                      //          endif
                                                //      endif
end while                                       // end while
```

The above algorithm can be written using C function as shown below:

Example 11.4: C function to show the nodes visited using BFS traversal

```
void bfs(int a[10][10], int n, int u)
{
        int     f, r, q[10], v;
        int     s[10] = {0};   /* initialize all elements in s to 0 i.e, no node is visited */

        printf("The nodes visited from %d : ", u);
        f = 0, r = -1;              // queue is empty
        q[++r] = u;                 // Insert u into queue

        s[u] = 1;                   // insert u to s
        printf("%d  ", u);          // print the node visited

        while ( f <= r )
        {
                u = q[f++];                             // delete an element from q
                for (v = 0; v < n; v++)
                {
                        if (a[u][v] == 1)               // If v is adjacent to u
                        {
                                if (s[v] == 0)  // If v is not in S i.e., v has not been visited
                                {
                                        printf("%d  ", v);   // print the node visited
                                        s[v] = 1;            // add v to s, mark it as visited
                                        q[++r] = v;          // Insert v into queue
                                }
                        }
                }
        }
        printf("\n");
}
```

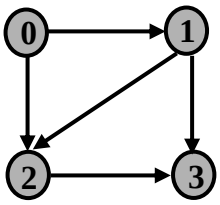
Now, the C program that prints all the nodes that are reachable from a given source vertex is shown below:

Example 11.5: Algorithm to traverse the graph using BFS

```
#include <stdio.h>
/* Insert: Example  11.1: Function to read an adjacency  matrix*/
/* Insert: Example  11.4: Function to traverse the graph in BFS */
```

```
void main()
{
        int     n, a[10][10], source, i, j;

        printf("Enter the number of nodes : ");
        scanf("%d", &n);

        printf("Enter the adjacency matrix:\n");
        for (i = 0; i < n; i++)
        {
                for (j = 0; j < n; j++) scanf("%d", &a[i][j]);
        }

        for (source = 0; source < n; source++)
                bfs(a, n, source);
}
```

Now, let us see how to obtain the nodes reachable from each node of the following graph using the above program:



Given graph

	0	1	2	3
0	0	1	1	0
1	0	0	1	1
2	0	0	0	1
3	0	0	0	0

Adjacency matrix

Output

```
Enter the number of nodes: 4
Enter the adjacency matrix:
0 1 1 0
0 0 1 1
0 0 0 1
0 0 0 0
The nodes visited from 0: 0  1  2  3
The nodes visited from 1: 1  2  3
The nodes visited from 2: 2  3
The nodes visited from 3: 3
```

11.4.1.2 Breadth First Search (BFS) using adjacency list

We know that BFS traversal uses queue data structure which require insert rear function and delete front function. We can use insert rear function given in example 8.6. But, the delete front function shown in example 8.5 is modified after deleting the printf() function.

Example 11.6: C function to delete an item from the front end of singly linked list

```
NODE delete_front(NODE first)
{
        NODE temp;

        if ( first == NULL )   return NULL;

        temp = first;                   /* Retain address of the node to be deleted */
        temp = temp->link;              /* Obtain address of the second node */
        free(first);                    /* delete the front node */
        return temp;                    /* return address of the first node */
}
```

The algorithm for BFS along with pseudocode when a graph is represented as an adjacency list can be written as shown below:

```
no node is visited to start with        // int s [10] = {0}
insert source u to q                    // q = NULL, q = insert_rear(u, q);
mark u as visited i.e., add u to S      // s[u] = 1, printf("%d  ", u);
while queue is not empty                // while q != NULL
                                                u = q->info;
        Delete a vertex u from q        //      q = delete_front(q),
                                                list = a[u]; // list of vertices adj. to u
        Find vertices v adjacent to u   //      while (list != NULL)
                                                        v = list->info;
        If v is not visited             //              if (s[v] == 0)
                print v                 //                      print v
                mark v as visited       //                      s[v] = 1
                Insert v to queue       //                      q = insert_rear(v, q);
        end if                          //              endif
                                                        list = list->link
                                        //      end while
end while                               // end while
```

Now, the complete C function to traverse the graph using BFS when a graph is represented as adjacency list can be written as shown below:

Example 11.7: C function to show the nodes visited using BFS traversal

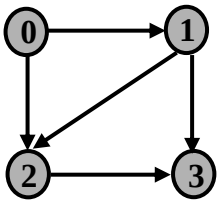
```
void bfs(NODE a[], int n, int u)
{
        NODE        q, list;
        int         v;

        int     s[10] = {0};   /* initialize all elements in s to 0 i.e, no node is visited */

        printf("The nodes visited from %d : ", u);

        q = NULL;                       // queue is empty
        q = insert_rear(u, q);          // Insert u into queue

        s[u] = 1;                       // insert u to s
        printf("%d  ", u);              // print the node visited
        while ( q != NULL )                             // as long as queue is not empty
        {
                u = q->info;                            // delete a node from queue
                q = delete_front(q);

                list = a[u];                            // obtain nodes adjacent to u
                while (list != NULL)                    // as long as adjacent nodes exist
                {
                        v = list->info;                 // v is the node adjacent to u
                        if (s[v] == 0)  // If v is not in S i.e., v has not been visited
                        {
                                printf("%d  ", v);      // print the node visited

                                s[v] = 1;                // add v to s, mark it as visited
                                q = insert_rear(v, q);  // Insert v into queue
                        }
                        list = list->link;
                }
        }
        printf("\n");
}
```

Now, the complete C program to see the nodes reachable from each of the nodes in the graph can be written as shown below:

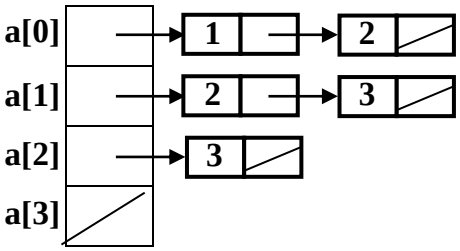
Example 11.8: Program to print nodes reachable from a vertex (bfs using adjacency list)

```
#include <stdio.h>
#include <stdlib.h>
struct node
{
        int             info;
        struct node     *link;
};

typedef struct node *NODE;

/* Insert: Example  8.2: Function to get a node */
/* Insert: Example  8.6: Function to insert an element into queue */
/* Insert: Example  11.2: Function to read adjacency list */
/* Insert: Example  11.6: Function to delete an element from front end of queue */
/* Insert: Example  11.7: Function to traverse the graph in BFS (adjacency list) */
void main()
{
        int             n, i, source;
        NODE            a[10];

        printf("Enter the number of nodes : ");
        scanf("%d", &n);

        for (i = 0; i < n; i++) a[i] = NULL;            // Graph is empty to start with

        read_adjacency_list(a, n);

        for (source = 0; source < n; source++)
                bfs(a, n, source);
}
```

Now, let us see how to obtain the nodes reachable from each node of the following graph using the above program:



Given graph



Adjacency linked list

Input

```
Enter the number of nodes: 4
Enter the number of nodes adjacent 0: 2
Enter nodes adjacent to 0: 1  2

Enter the number of nodes adjacent 1: 2
Enter nodes adjacent to 1: 2  3

Enter the number of nodes adjacent 2: 1
Enter nodes adjacent to 2: 3

Enter the number of nodes adjacent 3: 0
Enter nodes adjacent to 3:
```

Output

```
The nodes visited from 0: 0  1  2  3
The nodes visited from 1: 1  2  3
The nodes visited from 2: 2  3
The nodes visited from 3: 3
```

11.4.2 Depth First Search (DFS)

The depth first search is a method of traversing the graph by visiting each node of the graph in a systematic order. As the name implies *depth-first-search* means "to search deeper in the graph". Now, let us see "What is depth first search (DFS)?"

Definition: In DFS, a vertex *u* is picked as source vertex and is visited. The vertex *u* at this point is said to be unexplored. The exploration of the vertex *u* is postponed and a vertex *v* adjacent to *u* is picked and is visited. Now, the search begins at the vertex

v. There may be still some nodes which are adjacent to *u* but not visited. When the vertex *v* is completely examined, then only *u* is examined. The search will terminate when all the vertices have been examined.

Note: The search continues deeper and deeper in the graph until no vertex is adjacent or all the vertices are visited. Hence, the name DFS. Here, the exploration of a node is postponed as soon as a new unexplored node is reached and the examination of the new node begins immediately.

Design methodology The iterative procedure to traverse the graph in DFS is shown below:

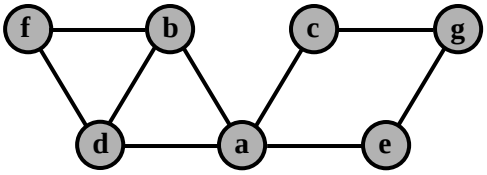
Step 1: Select node *u* as the start vertex (select in alphabetical order), push *u* onto stack and mark it as visited. We add *u* to S for marking

Step 2: While stack is not empty

```
        For vertex u on top of the stack, find the next immediate adjacent vertex.
        if v is adjacent
            if a vertex v not visited then
                push it on to stack and number it in the order it is pushed.
                mark it as visited by adding v to S
            else
                ignore the vertex
            end if
        else
            remove the vertex from the stack
            number it in the order it is popped.
        end if
    end while
```

Step 3: Repeat step 1 and step 2 until all the vertices in the graph are considered

Example 11.9: Traverse the following graph using DFS and display the nodes reachable from a given source vertex



Solution: Since vertex *a* is the least in alphabetical order, it is selected as the start vertex. Follow the same procedure as we did in BFS. But, there are two changes:

- Instead of using a queue, we use stack
- In BFS, all the nodes adjacent and which are not visited are considered. In DFS, only one adjacent which is not visited earlier is considered. Rest of the procedure remains same.

Now, the graph can be traversed using DFS as shown in following table

	Stack	**v = adj(s[top])**	**Nodes visited S**	**pop(stack)**
Initial step	a	-	a	
Stage 1	a	b	a, b	-
Stage 2	a, b	d	a, b, d	-
Stage 3	a, b, d	f	a, b, d, f	-
Stage 4	**a, b, d, f**	-	a, b, d, f	f
Stage 5	a, b, d	-	a, b, d, f	d
Stage 6	a, b	-	a, b, d, f	b
Stage 7	a	c	a, b, d, f	-
Stage 8	a, c	g	a, b, d, f, g	-
Stage 9	a, c, g	e	a, b, d, f, g, e	-
Stage 10	a, **c, g, e**	-	a, b, d, f, g, e	e
Stage 11	a, c, g	-	a, b, d, f, g, e	g
Stage 12	a, c	-	a, b, d, f, g, e	c
Stage 13	a_1	-	a, b, d, f, g, e	$a_{1,7}$

11.4.2.1 Depth First Search (DFS) using adjacency matrix

It is clear from the above example that the *stack* is the most suitable data structure to implement DFS. Whenever a vertex is visited for the first time, that vertex is pushed on to the stack and the vertex is deleted from the stack when a dead end is reached and the search resumes from the vertex that is deleted most recently. If there are no vertices adjacent to the most recently deleted vertex, the next node is deleted from the stack and the process is repeated till all the vertices are reached or till the stack is empty.

The recursive function can be written as shown below: (Assuming adjacency matrix *a*, number of vertices *n* and array *s* as global variables)

Example 11.10: Program to print nodes reachable from a vertex (dfs - adjacency matrix)

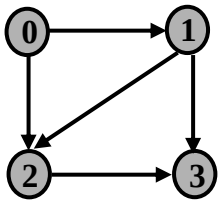
```
void dfs(int u)
{
        int v;

        s[u] = 1;

        printf("%d ", u);

        for (v = 0; v < n; v++)
        {
                if (a[u][v] == 1 && s[v] == 0) dfs(v);
        }
}
```

The complete program that prints the nodes reachable from each of the vertex given in the graph can be written as shown below:

Example 11.11: Program to print nodes reachable from a vertex (dfs - adjacency matrix)

```
#include <stdio.h>

int     a[10][10], s[10], n;            // Global variables

/* Insert: Example 11.1: Function to read an adjacency  matrix*/
/* Insert: Example 11.10: Function to traverse the graph in DFS */

void main()
{
        int     i, source;

        printf("Enter the number of nodes in the graph : ");
        scanf("%d", &n);
        printf("Enter the adjacency matrix:\n");
        read_adjacency_matrix(a, n);

        for (source = 0; source < n; source++)
        {
                for (i = 0; i < n; i++) s[i] = 0;

                printf("\nThe nodes reachable from %d: ", source);
                dfs(source);
        }
}
```

Now, let us see how to obtain the nodes reachable from each node of the following graph using the above program:



Given graph

	0	1	2	3
0	0	1	1	0
1	0	0	1	1
2	0	0	0	1
3	0	0	0	0

Adjacency matrix

Output

```
Enter the number of nodes: 4
Enter the adjacency matrix:
0 1 1 0
0 0 1 1
0 0 0 1
0 0 0 0
The nodes visited from 0: 0  1  2  3
The nodes visited from 1: 1  2  3
The nodes visited from 2: 2  3
The nodes visited from 3: 3
```

11.4.2.2 Depth First Search (DFS) using adjacency linked list

The procedure remains same. But, instead of using adjacency matrix, we use adjacency list. The recursive function can be written as shown below: (Assuming adjacency list *a*, number of vertices *n* and array *s* as global variables.)

Example 11.12: Program to print nodes reachable from a vertex (dfs - adjacency list)

```
void dfs(int u)
{
        int v;
        NODE temp;

        s[u] = 1;

        printf("%d ", u);

        for (temp = a[u];   temp != NULL;   temp = temp->link)
                if (s[temp->info] == 0) dfs(temp->info);
}
```

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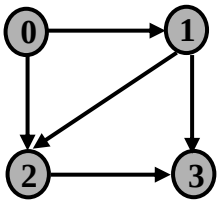
The complete program that prints the nodes reachable from each of the vertex given in the graph using DFS represented using adjacency list can be written as shown below:

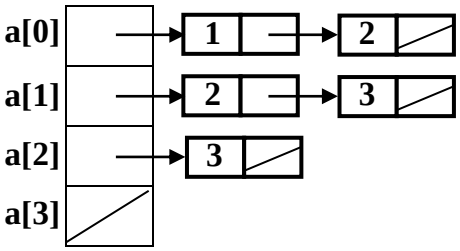
Example 11.13: Program to print nodes reachable from a vertex (dfs - adjacency matrix)

```
#include <stdio.h>
#include <stdlib.h>

struct node
{
        int             info;
        struct node    *link;
};

typedef struct node *NODE;

NODE        a[10];
int         s[10], n;               // Global variables

/* Insert: Example 8.2: Function to get a node */

/* Insert: Example 8.6: Function to insert an element into queue */

/* Insert: Example 11.2: Function to read adjacency list */

/* Insert: Example 11.12: Function to traverse the graph in DFS */
void main()
{
        int     i, source;

        printf("Enter the number of nodes in the graph : ");
        scanf("%d", &n);

        printf("Enter the adjacency list:\n");
        read_adjacency_list(a, n);

        for (source = 0; source < n; source++)
        {
                for (i = 0; i < n; i++) s[i] = 0;

                printf("\nThe nodes reachable from %d: ", source);
                dfs(source);
        }
}
```

Now, let us see how to obtain the nodes reachable from each node of the following graph using the above program:



Given graph



Adjacency linked list

Input

```
Enter the number of nodes: 4
Enter the number of nodes adjacent 0: 2
Enter nodes adjacent to 0: 1  2

Enter the number of nodes adjacent 1: 2
Enter nodes adjacent to 1: 2  3

Enter the number of nodes adjacent 2: 1
Enter nodes adjacent to 2: 3

Enter the number of nodes adjacent 3: 0
Enter nodes adjacent to 3:
```

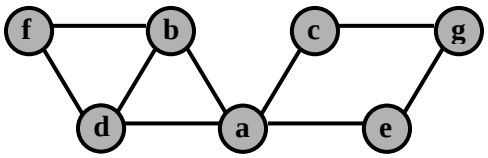
Output

```
The nodes visited from 0: 0  1  2  3
The nodes visited from 1: 1  2  3
The nodes visited from 2: 2  3
The nodes visited from 3: 3
```

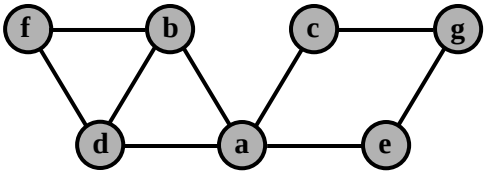
Exercises

1) Define the terms: a) vertex b) edge c) graph d) directed graph
   e) undirected graph
2) Define the terms: a) self-loop (or self-edge) b) multigraph c) complete graph
3) Define the terms: a) path b) simple path c) length of the path
4) Define the terms: a) cycle (circuit) b) Connected graph c) disconnected graph
5) What are the different methods of representing a graph?
6) What is an adjacency matrix? explain with example

7) What is an adjacency list? Explain with example
8) What is a weighted graph?
9) How the weighted graph can be represented?
10) What is cost adjacency matrix? What is cost adjacency linked list?
11) What is graph traversal? Explain different graph traversal techniques
12) What is breadth first search (BFS)?"
13) Traverse the following graph by breadth-first search and print all the vertices reachable from start vertex *a*. Resolve ties by the vertex alphabetical order.



14) Write a C function to show the nodes visited using BFS traversal (adjacency matrix)
15) Write a C function to show the nodes visited using BFS traversal (adjacency list)
16) What is depth first search (DFS)?"
17) Traverse the following graph using DFS and display the nodes reachable from a given source vertex



18) Write a program to print nodes reachable from a vertex (dfs - adjacency matrix)
19) Write a program to print nodes reachable from a vertex (dfs - adjacency matrix)