LECTURE NOTES
ON
DIGITAL SIGNAL PROCESSING
6TH SEMESTER
E & TC ENGG.

DIGITAL SIGNAL PROCESSING (DSP) DI → 9.12.19

SYLLABUS

1 INTRODUCTION :-

1.1 Discuss signals, system & signal processing.

1.1.1 Explain basic element of a digital signal processing system.

1.1.2 Compare the advantage of digital signal processing over analog signal processing.

1.2 Classify signals.

1.2.1 Multi-channel & multi-dimensional signals.

1.2.2 Continuous time verses Discrete-time signal.

1.2.3 Continuous valued verses Discrete valued signals

1.3 Discuss the concept of frequency in continuous time & discrete time signals.

1.3.1 continuous time sinusoidal signals.

1.3.2 Discrete time sinusoidal signals.

1.3.3 Harmonically related complex exponential.

1.4 Discuss Analog to digital to analog conversion & Explain the following.

1.4.1 Sampling of Analog signal.

1.4.2 The Sampling theorem.

1.4.3 Quantization of continuous amplitude signals.

1.4.4 coding of quantized sample.

1.4.5 Digital to analog conversion

1.4.6 Analysis of digital systems signals Vs. discrete time signals system.

2 DISCRETE TIME SIGNALS & SYSTEMS :-

2.1 State and Explain discrete time signal.

2.1.1 Discuss some Elementary discrete time signals.

2.1.2 classify discrete time signal.

2.1.3 Discuss simple manipulation of discrete time signals.

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2.2.1 Describe input-output of system.
2.2.2 Draw block diagram of discrete time system
2.2.3 classify discrete time system.
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2.3 Discuss discrete time time invarient system.
2.3.1 Discuss different technique for the analysis of linear system.
2.3.2 Discuss the resolution of a discrete time signal into impulse
2.3.3 Discuss the response of LTI system to arbitray I/Ps. Using convolution theorem
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2.4.1 Explain recursive & non-recursive discrete time system.
2.4.2 Determine the impulse response of linear time invarient recursive system.
3. THE Z-TRANSFORM & ITS APPLICATION TO THE ANALYSIS OF LTI SYSTEM
3.1 Discuss Z-transform & its applications to LTI system
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3.1.2 State & Explain inverse Z-transform.
3.2 Discuss various properties of Z-transform.
3.3 Discuss rational Z-transform
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3.4.1 Determine inverse Z- transform by partial fractions Expansion.

4 DISCUSS FOURIER TRANSFORM : ITS APPLICATION PROPERTIES.
4.1 Discuss discrete fourier transform.
4.2 Determine frequency domain sampling & reconstruction of discrete time signals.
4.3 State & Explain discrete time fourier transformations (DTFT).
4.4 State & Explain discrete fourier transformations (DFT).
4.5 compute DFT as a linear transformations.
4.6 Relate DFT to other transform.
4.7 Discuss the property of the DFT...
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5 FAST FOURIER TRANSFORM ALGORITHM & DIGITAL FILTERS:-
5.1 Compute DFT & FFT algorithm
5.2 Explain direct compulation of DFT.
5.3 Discuss the radix -2 algorithm. (small problems)
5.4 Introductions to digital filters (FIR FILTERS)
5.5 Introductions to DSP architecture, familarsation of different types of procesor.

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INTRODUCTION

SIGNAL:-

→ A signal is a physical quantity which varies with respect to some independent variable such as time

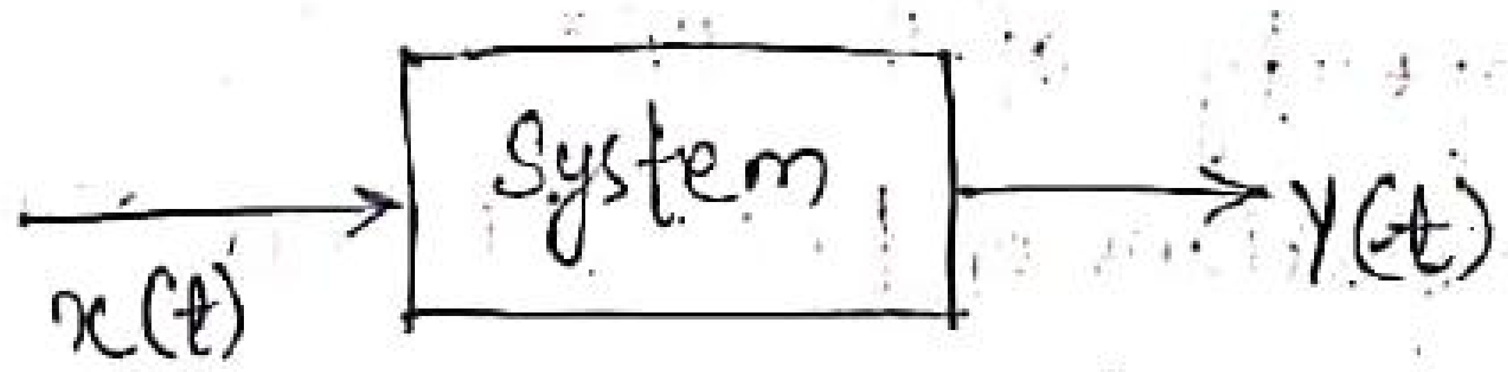
→ Each signal must contain some information

→ Example- Speech signal, video signal.

→ Each signal is a function of some independent variable such as time space etc

SYSTEM :-

A system is an Entity which process some input signal and provides some output signal.



$$y(t) = f[x(t)]$$
$$= T[x(t)]$$

→ A system produces o/p in response to an i/p signal.

→ A system can be of analog type or digital

→ Example:- Analog to digital converter, amplifier

Signal Processing :-

→ Signal processing is any operation that changes the characteristics of a signal. (Amp; freq. phase, etc).

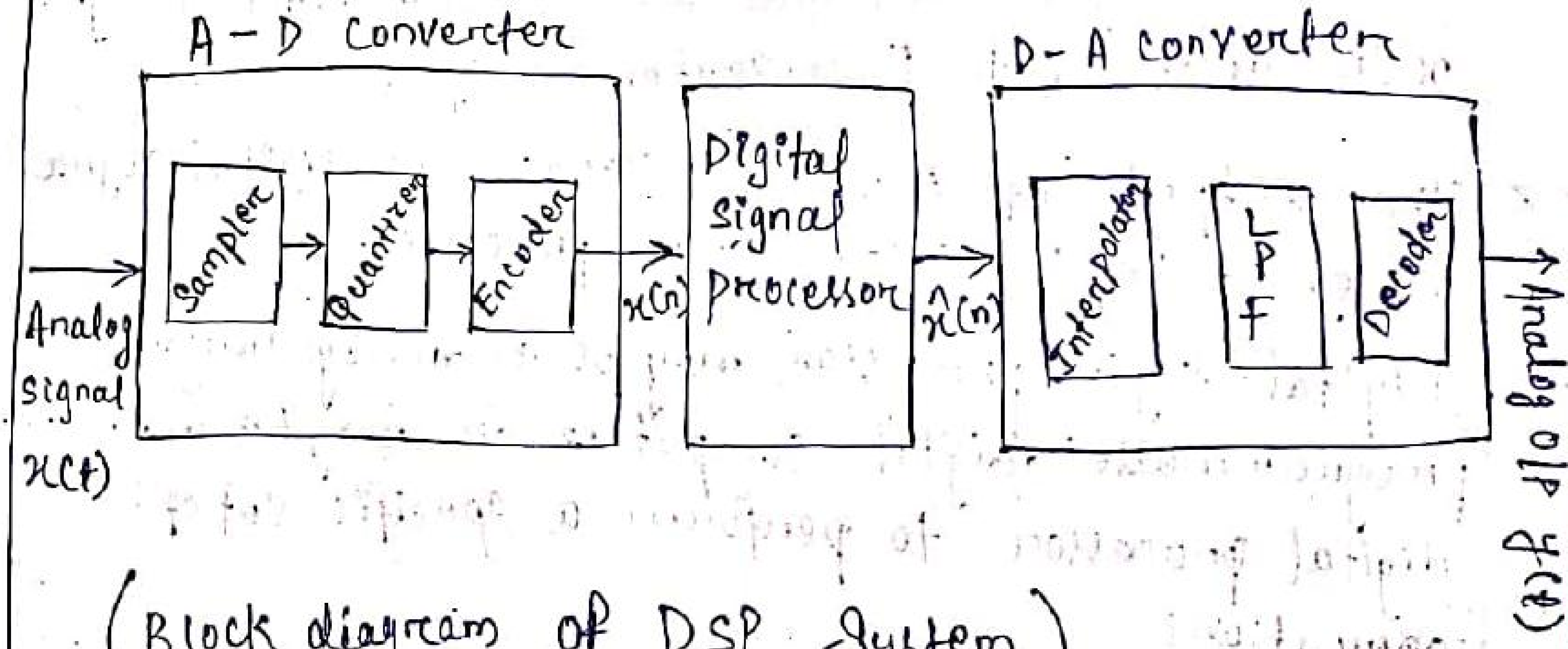
→ Signal processing can be a method of Extracting the informations from the signal;

→ Signal processing can be of two types:-
(i) Analog signal processing
(ii) Digital signal processing.

→ Example of Analog signal processing
* public address system.

→ Example of digital signal processing.
* Fingerprint recognization
* RADAR
* Speech recognization

1.1.1 Basic Elements Of A Digital Signal Processing System :-



(Block diagram of DSP System)

→ The main Elements of DSP System are :-

(i) Analog to digital converter (sampler, Quantizer, Encoder)

(ii) Digital signal processor

(iii) Digital to Analog converter (interpolator, low-pass filter, decoder):

→ First the continuous time analog signal is provided to the A to D converter.

→ The sampler converts the continuous time continuous amplitude signal into discrete time continuous amplitude signal.

→ The output of sampler is fed to quantizer which converts the discrete time continuous amplitude signal into discrete time amplitude signal.

→ The Encoder provides a particular value to the quantized signal so, that we get the digital signal $x(n)$ at the A to D converter output.

→ The output of A to D converter is digital signal which is fed to digital signal processor.

→ Digital Signal processor may be a large programmable digital computer or may be a hardware digital processor to perform a specific set of operations

→ The output of digital signal processor is digital in nature so, we have to again convert it to analog signal by the help of D to A Converter.

→ The most common type of D to A converter is zero order hold.

→ In D to A converter a proportional output is produced to that of the binary values.

→ The interpolator converts the analog samples into analog signal.

→ In zero order hold technique a given sample is hold for the total sample interval untill the next sample is arrived.

→ The Low pass filter attenuates the undesireable high frequency components.

Advantage of DSP over Analog Signal Processing :-

→ Digital signal processing have the following advantage over analog signal processing:

(i) Flexibility :-

Digital signal processing (DSP) operations by changing the program in the digital signal processor which is difficult in case of analog signal processor.

(ii) Greater Accuracy :-

Digital signal processing (DSP) have greater accuracy and control over analog signal processing.

(iii) Data Storage is Easy:-
Digital signal can store Easily on CD, Drive and any storing devices and it can reproduce Easily which is not possible in case of analog signal processing.

(iv) Easily transported:-
DSP can be transported Easily whereas analog signal is not.

(v) Implementations of sofesticated algorithm.

(vi) Digital circuits are less sensitive to Environmental changes and tolerance of component value.

(vii) DSP is independent of temperature, ageing of components.

(viii) DSP is cheaper due to mass productions cost is reproduce.

(ix) DSP can process very low frequency signal.

(x) DSP allow time sharing.

DISADVANTAGE OF DSP :-

→ System Complexity
→ Limited bandwidth

APPLICATIONS OF DSP :-

→ Speech recognization
→ Image processing
→ RADAR signal processing
→ Spectrum Analysis.
→ SONER signal processing.

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CLASSIFICATIONS OF SIGNAL :-

Broadly signals are classified into types :-
(i) Continuous time signal - $x(t)$
(ii) Discrete time signal - $x(n)$

Signals can be further classified in the following types.
(a) Deterministic & Non-deterministic
(b) periodic & Aperiodic signal.
(c) Even signal & odd signal
(d) Energy signal & power signal
(e) multi-channel signal & multi-dimensional signal.
(f) continuous value signal & discrete value signal.

MULTI-CHANNEL SIGNAL :-

→ If the signal is generated by a single source then it is called as single channel signal.
→ signals which are generated from more than one source known as multi-channel signal.

→ This signals are represented by a vector

Example :-

$$S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{bmatrix}$$

S(t) is a 3-channel signal.

Ex:-

→ The ground accelerations due to Earthquake

→ In ECG it may be Either 3-lead so, that the signals may be 3-channel or 12-channel

MULTI - DIMENSIONAL SIGNAL :-

A signal is said to be multi-dimensional if it is a functions of ~~one~~ more than one variable.

Example :-

Picture is an 2-dimensional signal.

Colour T.V signal is a 3-dimensional signal.

CONTINUOUS TIME SIGNAL :-

→ The signal that varies continuously with time is known as continuous time signal.

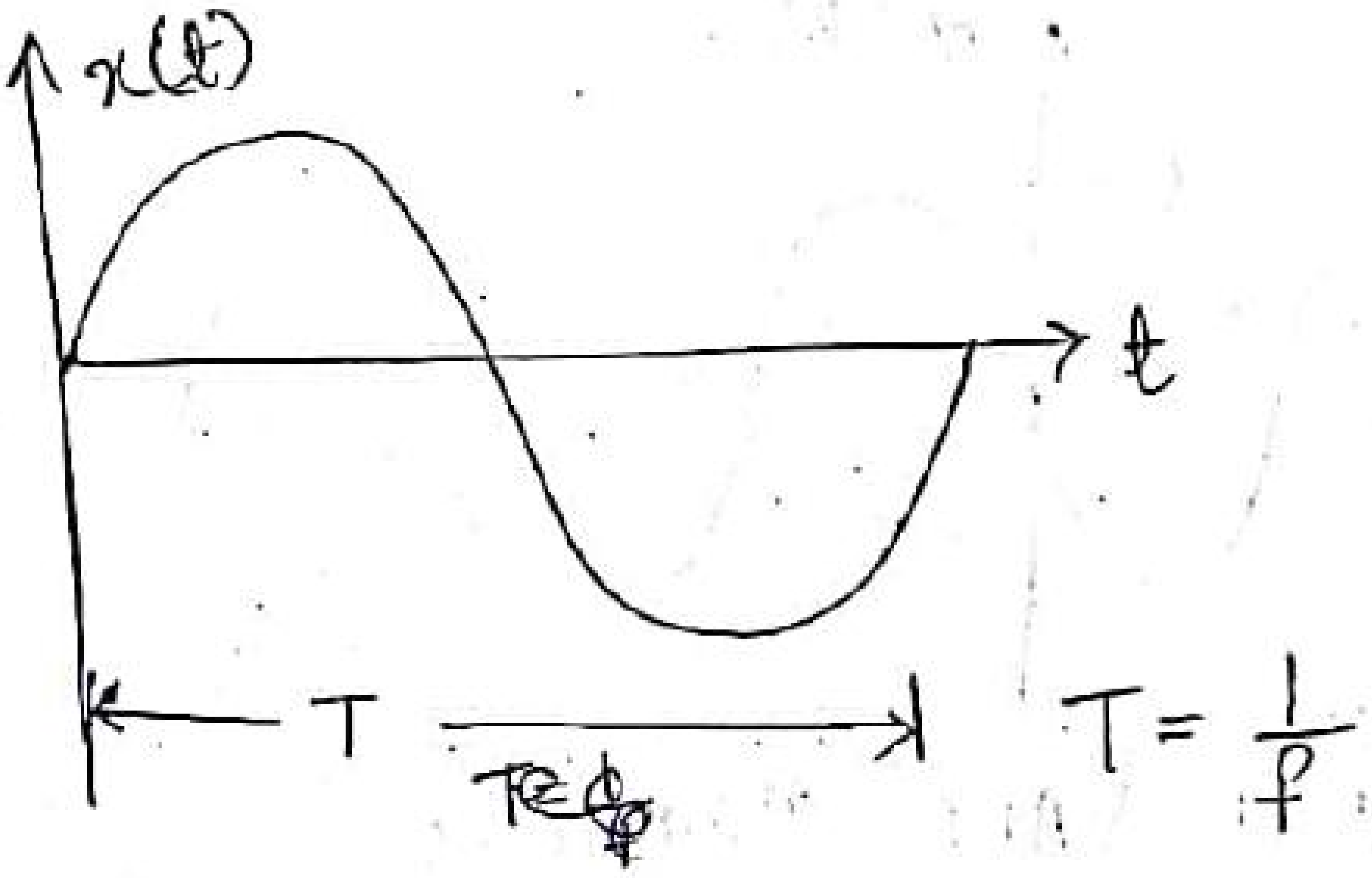
→ It is defined for each value of time

→ It takes the value in continuous interval from $-\infty$ to ∞

Example :-

$x_1(t) = \sin \pi t$

→ A continuous time signal is represented by $x(t)$ where t is the independent variable



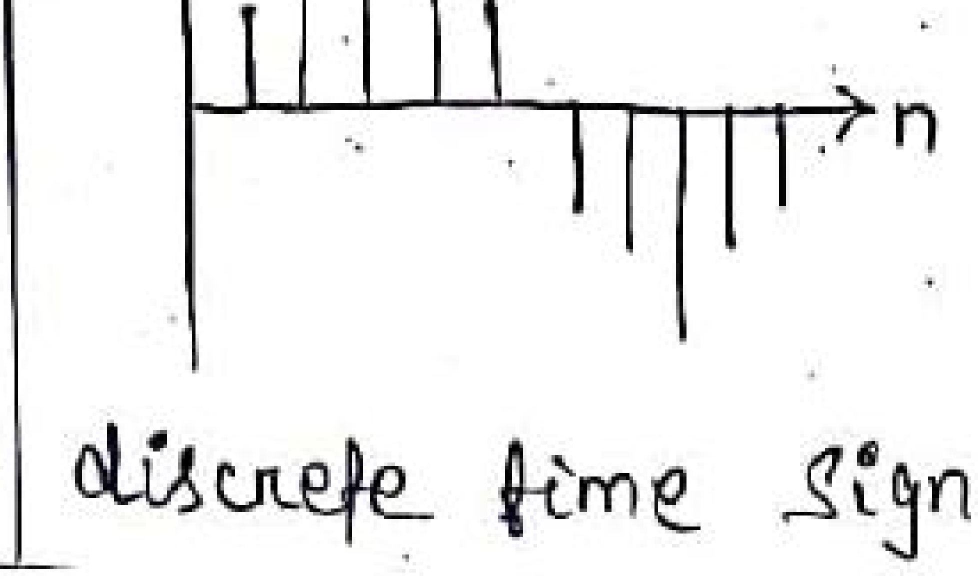
DISCRETE TIME SIGNAL :-

→ A discrete time signal is defined only at certain time instant.

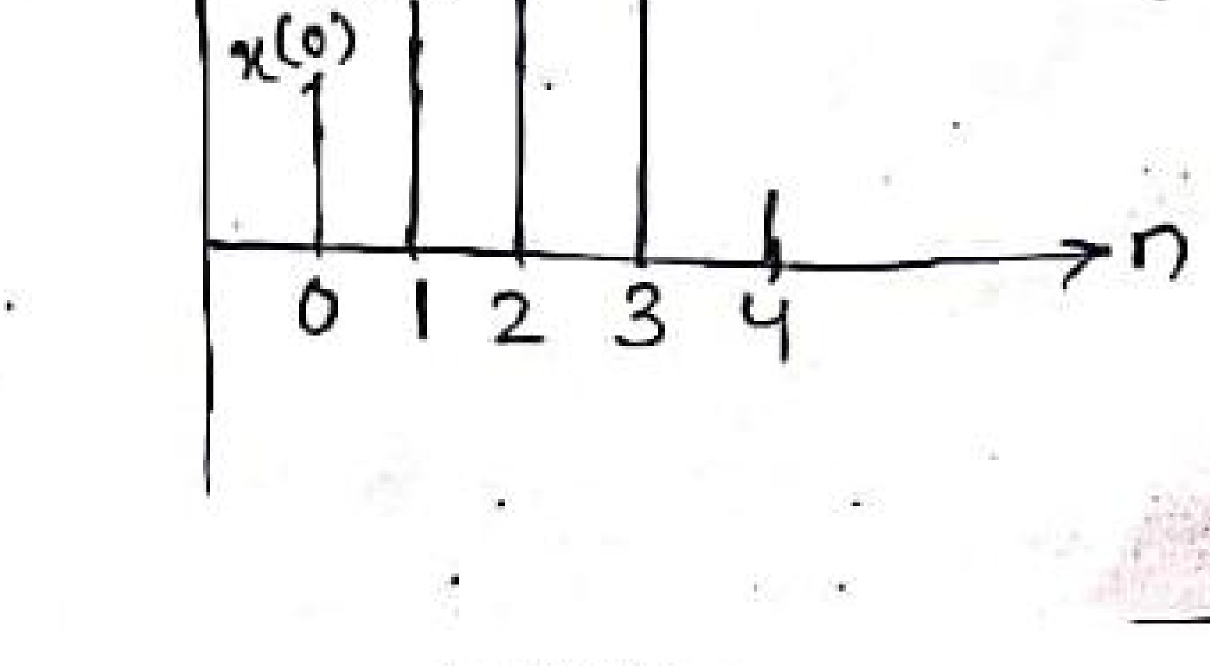
→ This time instant may be or may not be equi-distant but usually taken at equally spaced interval

→ In discrete time signal the amplitude between 2 time instant is not defined.

→ A discrete time signal is denoted by $x(n)$ where 'n' is independent variable.

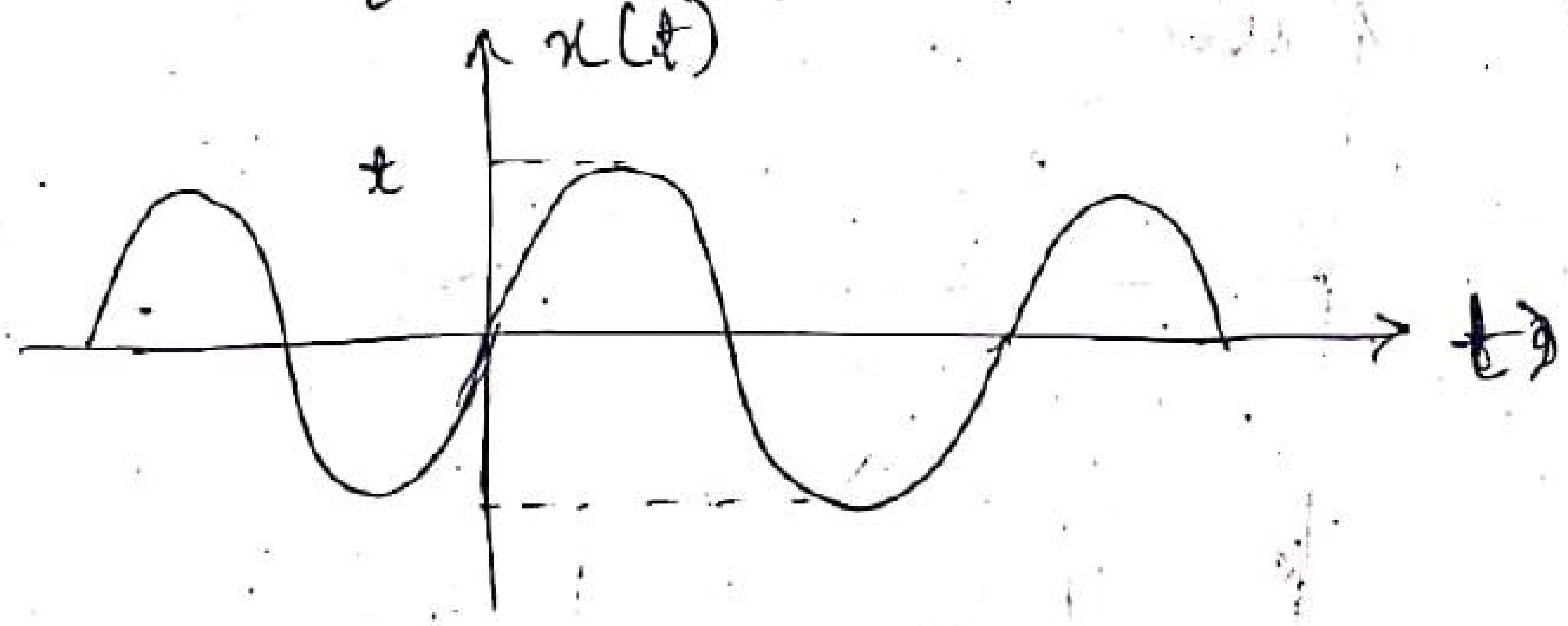


discrete time signal



CONTINUOUS VALUE SIGNAL :-

→ A signal is said to be continuous valued signal if it takes all possible value on a finite or infinite range.



DISCRETE VALE SIGNAL :-

→ Discrete value signal take some values from a given set

→ Normally, this values are equi-distance and can be expressed as a integer multiple of the distance between the two sucessive value.

CONCEPT OF FREQUENCY IN CONTINUOUS TIME SINUSOIDAL SIGNAL :-

Mathematically, a continuous time sinusoidal signal can be represented by

$$\underbrace{x_a(t)}_{\text{Analog Signal}} = \underbrace{A}_{\text{Amplitude}} \sin(\underbrace{\Omega t}_{\text{frequency in rad/sec}} + \overbrace{\theta}^{\text{phase}}) ; -\infty \leq t \leq \infty$$

where,

x_a = Analog signal

A = Amplitude

Ωt = frequency in rad/sec

θ = Phase

$$\boxed{\Omega = 2\pi F}$$

$$\boxed{F = \frac{1}{T}} \quad T = \text{Time period}$$

DISCRETE TIME SINUSOIDAL SIGNAL :-

Mathematically, it is represented by

$$x(n) = A \sin(\omega t + \theta) ; -\infty \leq n \leq \infty$$

where,

$x(n)$ = discrete time signal

A = Amplitude

ω = frequency in rad/sec

θ = phase in radian

$\omega = 2\pi f$

PROPERTY OF ANALOG SIGNAL :- (continuous time signal)

Property-1 :-

If $x_a(t) = x_a(t+T)$ then $x_a(t)$ is periodic with time period (T).

Property-2 :-

→ continuous time sinusoidal signal with different frequencies are themselves different.

Property-3 :-

→ Increasing the frequency results in an increase in the rate of oscillation of the signal,

Property of DISCRETE TIME SIGNAL :-

Property-1 :-

→ Discrete sinusoides are periodic only if its frequency 'f_0' is a rational number

→ $\boxed{f_0 = \frac{K}{N}}$ $\boxed{x(n) = x(n+N)}$

HARMONICALLY RELATED COMPLEX EXPONENTIAL :-

→ CONTINUOUS Time Exponential :-

The basic signal for continuous time harmonically related exponential at

$$\boxed{S_K(t) = e^{jK\Omega_0 t}}$$

where,

K = Integer

$K = 0, \pm 1, \pm 2 \ldots\ldots$

The '$x_K(t)$' can be written as a in the form of harmonically related complex exponential

$$x_a(t) = \sum_{K=-\infty}^{\infty} C_K S_K(t) = \sum_{K=\infty}^{\infty} C_K e^{jK\Omega_0 t}$$

DISCRETE TIME EXPONENTIAL :-

$$S_K(n) = e^{jK\omega_0 n}, \quad K = 0, \pm 1, \pm 2$$

$$\cancel{x(n) = \sum_{n=0}^{n-1} C_K S_K}$$

$$x(n) = \sum_{n=0}^{n-1} C_K S_K(n) = \sum_{n=0}^{n-1} C_K e^{j2\pi \frac{n}{N} K}$$

: Signal)

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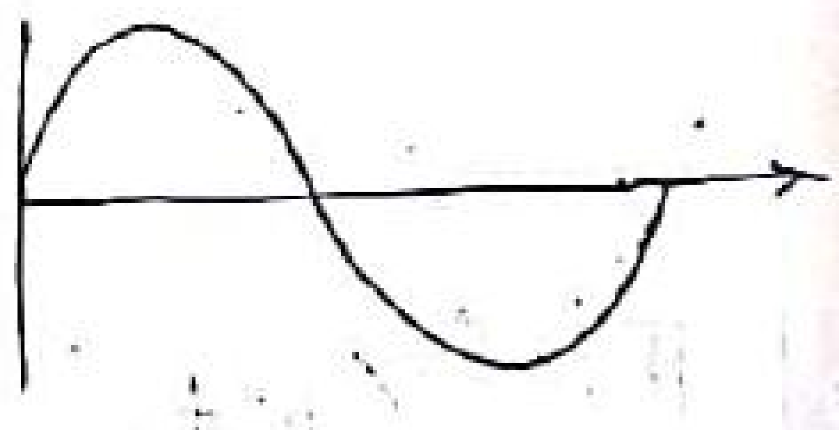
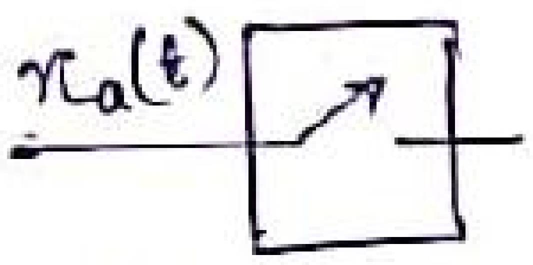
Sampling of Analog Signal :-

There are many ways to sample an analog signal but we will consider only the periodic sampling or uniform sampling mathematically we represents.

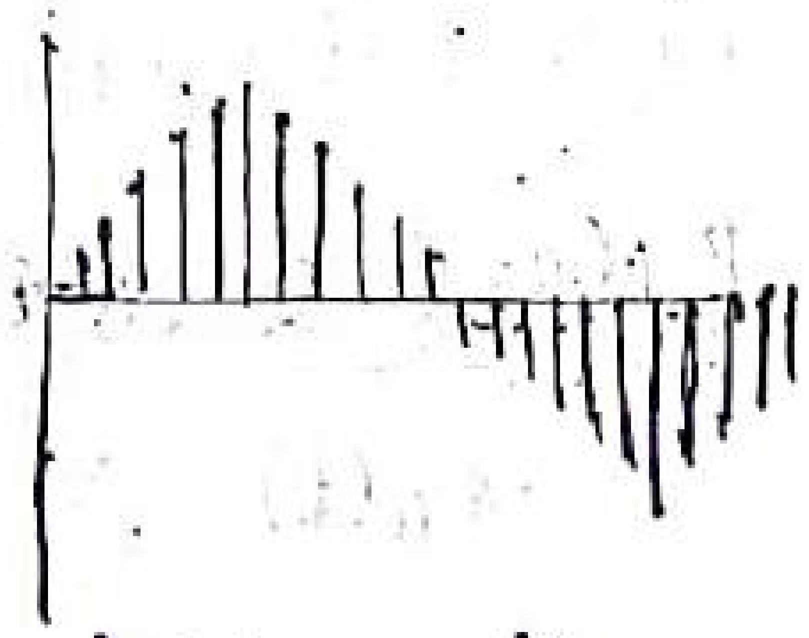
Here, $x(n)$ is the discrete time signal which is obtain by sampled of analog signal $x(t)$ at every T sec.



$$x_a(n) = x_a(nT)$$



Sampler



→ Here, T is the time interval between the successive samples known as sampling interval.

→ The relation between t and n , $t = nT$

Similarly, the relation between Ω and ω

Let us consider a analog signal $x_a(t)$

$$\boxed{x_a(t) = A\cos(2\pi F t + \theta)}$$

After sampling with a frequency $f_s = \frac{1}{T}$

$$x_a(t) = A\cos(2\pi F t + \theta)$$

$$= A\cos(2\pi F n \phi + \theta)$$

$= A \cos(2\pi \frac{F}{F_S} \cdot n + \theta)$

$= A \cos(2\pi f n + \theta$

$$\boxed{f = \frac{F}{F_S}}$$ F = continuous, f = Discrete

$$\boxed{\omega = \frac{\Omega}{F_S}}$$

For continuous time sinusoidal signal the frequency F and Ω

$$-\infty < F < \infty$$
$$-\infty < \Omega < \infty.$$

For discrete time sinusoidal signal the range of frequency variable

$$\omega = -\pi < \omega < \pi$$

$$f = -\pi/2 < F < \pi/2$$

So, we conclude that the frequency of continuous time sinusoidal when sample at a rate of $F_s = \frac{1}{T}$ must be in the range of $-\pi/2$ and $\pi/2$

Sampling theorem:-

Sampling theorem states that a band limited continuous time signal is may be in its samples and can also be recovered from its sampling frequency, $\boxed{F_s \geq 2f_m}$

Here, f_s = Sampling frequency and
f_m = maximum frequency present in the signal.

→ When sampling rate is Exactly equal to $2f_m$ then the Sampling rate is known as nyquist rate

→ Nyquist rate is also called as minimum sampling rate which is represented by

$$\boxed{F_s = 2f_m}$$

→ maximum Sampling interval is known as Nyquist interval $T_s = \frac{1}{2f_m}$

* Interpolation :- The process of reconstructing a continuous time signal $x(t)$ from its samples is known as interpolation.

Quantization of continuous Amplitude signals:-

→ A digital signal is a sequence of number in which Each number is represented by a finite number of digits.

→ The process of converting a discrete time continuous amplitude signal into a discrete time discrete amplitude signal by Expressing Each Sample value as a finite number of digits is called Quantization.

→ In this process some errors are generated which is called Quantization Error.

→ If the input of quantizer is $x(n)$ and output of quantizer is $x_q(n)$ then we can defined quantization Error as the difference of quantized value and sampled value

$$\boxed{e_q(n) = x_q(n) - x(n)}$$

Coding of quantized Sample :-

In coding process a unique binary number is assigned to each quantization level,

→ If we have n levels then we need at leasts n different binary number with a word length of 'b' no. of bits and we can create 2^b binary number,

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1. Digital signal processing Algorithm and Application — J.G. Proakis
2. Digital signal processing - Ramesh Babu

Q. consider a Analog signal $x_a(t) = 3\cos 100\pi t$

(a) Determine the minimum Sampling rate to avoid aliasing

(b) Suppose the signal is sampled at the rate of $f_s = 200\,Hz$ then what is the discrete time signal after sampling

$x_a(t) = 3 \cos 100\pi t$ $\quad (\because A \cos \omega t = A \cos 2\pi f t)$

$A = 3$

$f_m = 50Hz$

(a) Minimum Sampling rate or Nyquist rate

$$\boxed{f_s = 2f_m}$$

$$f_s = 2 \times 50 = 100Hz$$

(b) $A \cos \frac{f}{f_s} n$

$$x(n) = A \cos \frac{2\pi f}{f_s} n$$

$$= 3 \cos \frac{100\pi}{200} n$$

$$\boxed{x(n) = 3 \cos \frac{\pi}{2} n}$$

Q Consider an analog signal $x_a(t) = 3\cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$ then find out the nyquist rate for the particular signal.

$$3 \cos 50\pi t = A \cos 2\pi f_1 t$$

$$2f_1 = 50$$

$$f_1 = 25 Hz$$

$$10 \sin 300\pi \cancel{t} = A \cos 2\pi f_2 \cancel{t}$$

$$2f_2 = 300$$

$$f_2 = 150 Hz.$$

$$\cos 100\pi t = A \cos 2\pi f_3 t$$

$$2f_3 = 100$$

$$f_3 = 50\,Hz$$

$\therefore$ Here, $f_m = 150\,Hz$

$$f_s = 2f_m = 2 \times 150 = 300\,Hz$$

Digital to Analog conversion

To convert a digital signal into an analog signal a D to A converter is used.

The function of D to A converter is to interpolate the samples.

Sampling theorem specifies the optimum interpolation for a band limited signal.

A zero order volt in the simple type of D-A converter

Analysis of Digital System Signal Vs Descrete time signal system :

we know that the values of digital signal are taken from a finite set up possible value, this signal is used in digital computer.

Generally computer operate on the number which are represented by string of zero and one.

The length of this string is fixed and finite.

Sometimes this finite word length cause communication in the analysis of DSP system.

To avoid this complication we neglate the quantized nature of digital signal and system in various case and consider then descrete time signal and system.

Question

1. what is signal processing? Draw the block diagram and Explain the digital signal processing system / Elements of DSP? L10
2. Describe the advantage of digital signal processing over analog signal processing? L10
- State sampling theorem
- Draw and Explain the principle of analog to digital converter? L5
- what is quantizations?

what is multi-channel and multi dimensional signal?

Classify different signal and Explain briefly.

what are the application of signal processing

Differentiate discrete signal and digital signal?

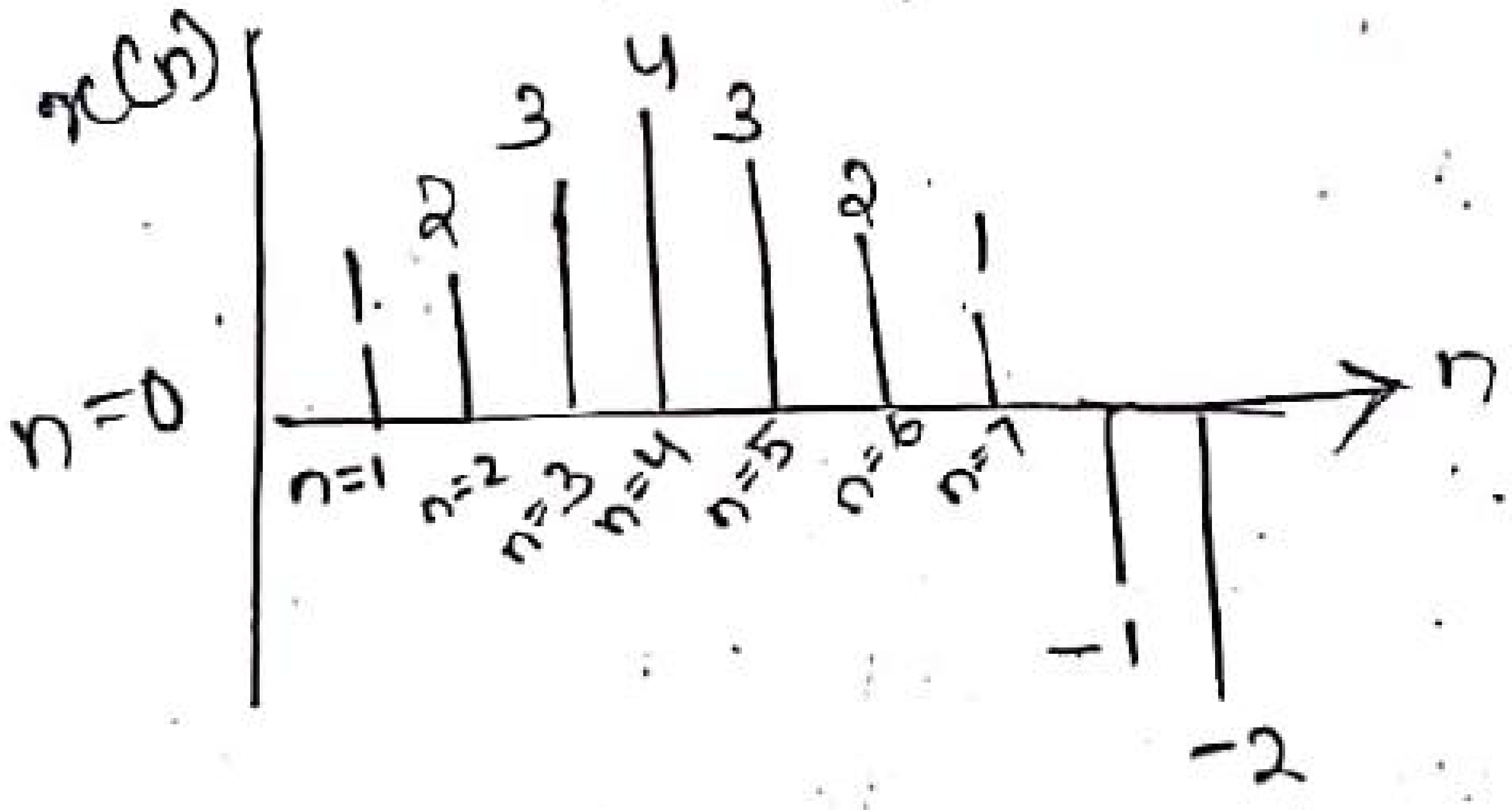
DISCRETE TIME SIGNAL

A discrete time signal s(n) is a function of an independent variable 'n'

Discrete time signal can be represented in the following 4-ways.

1. graphical representation
2. functional representation
3. Tabular representation
4. sequential representation

1. Graphical Representation :-



2. Functional Representations :-

$$x(n) \begin{cases} 1, & n=1,7 \\ 2, & n=2,6 \\ 3, & n=3,5 \\ 4, & n=4 \\ 7, & n=8 \\ 2, & n=9 \end{cases}$$

3 Tabular Representation :-

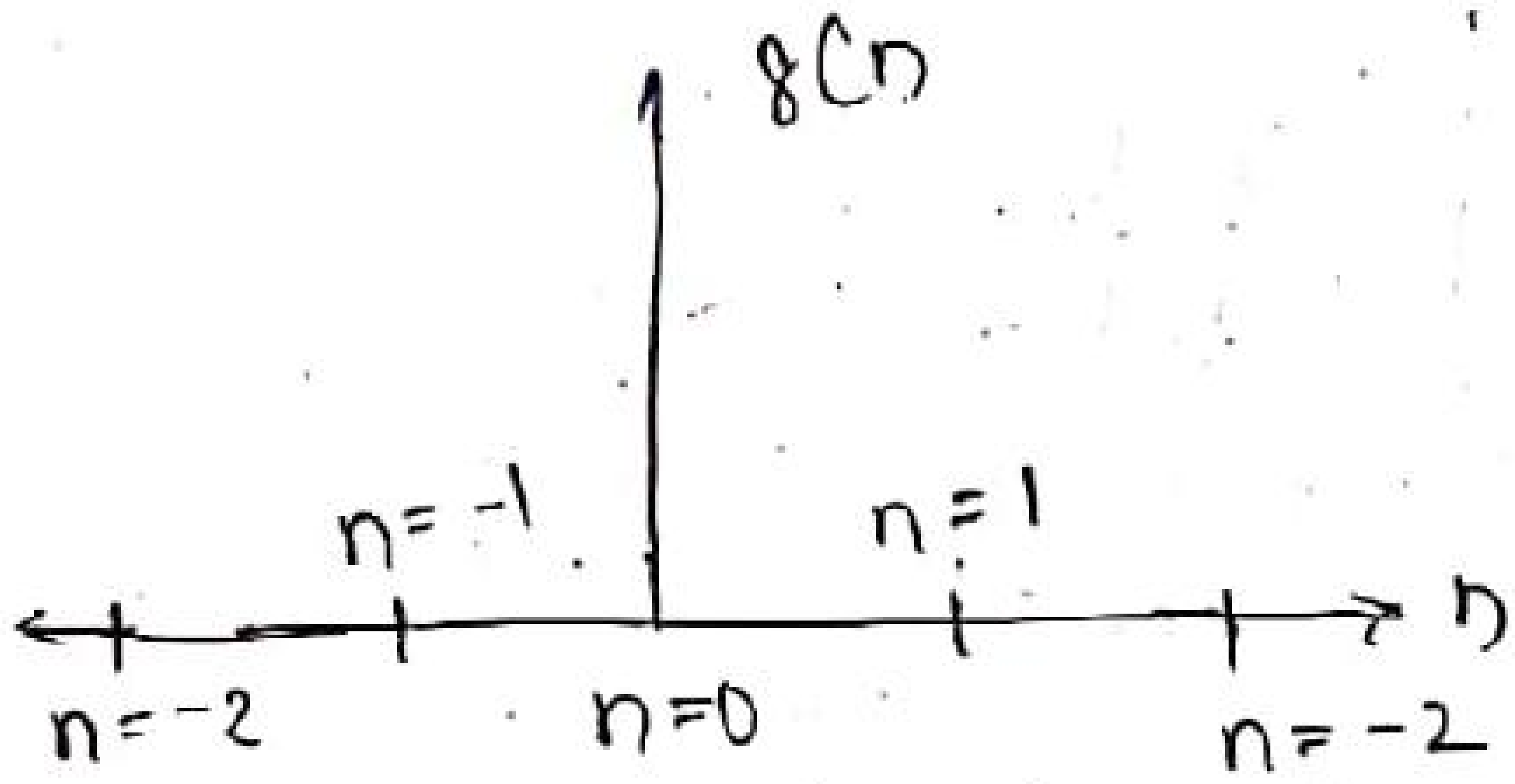
n	1	2	3	4	5	6	7	8	9
x(n)	1	2	3	4	3	2	1	-1	-2

(4) Sequential Representations :-

$$x(n) = \{ \underset{\uparrow}{0}, 1, 2, 3, 4, 3, 2, 1, -1, -2$$

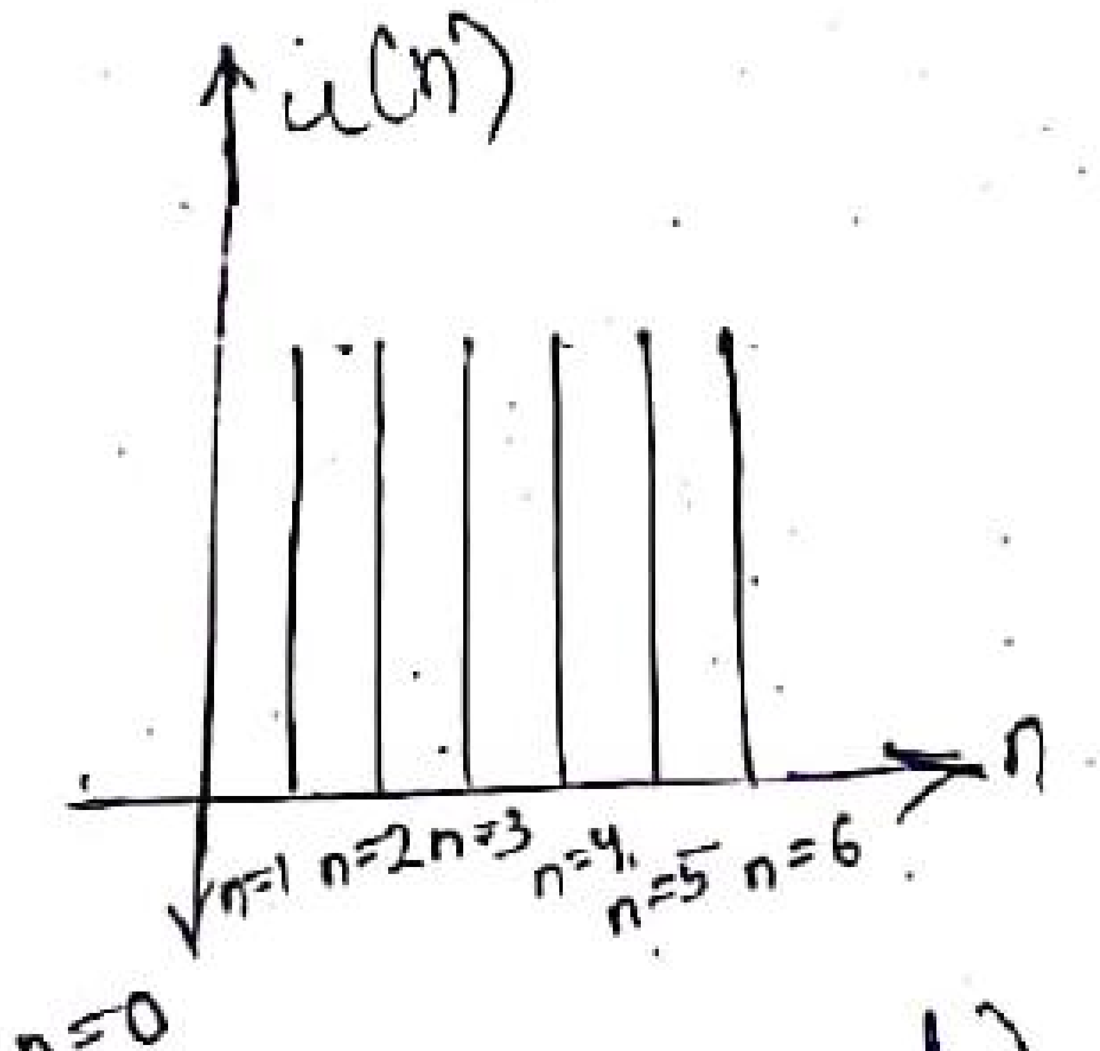
origin

Some Elementary discrete time signal :-



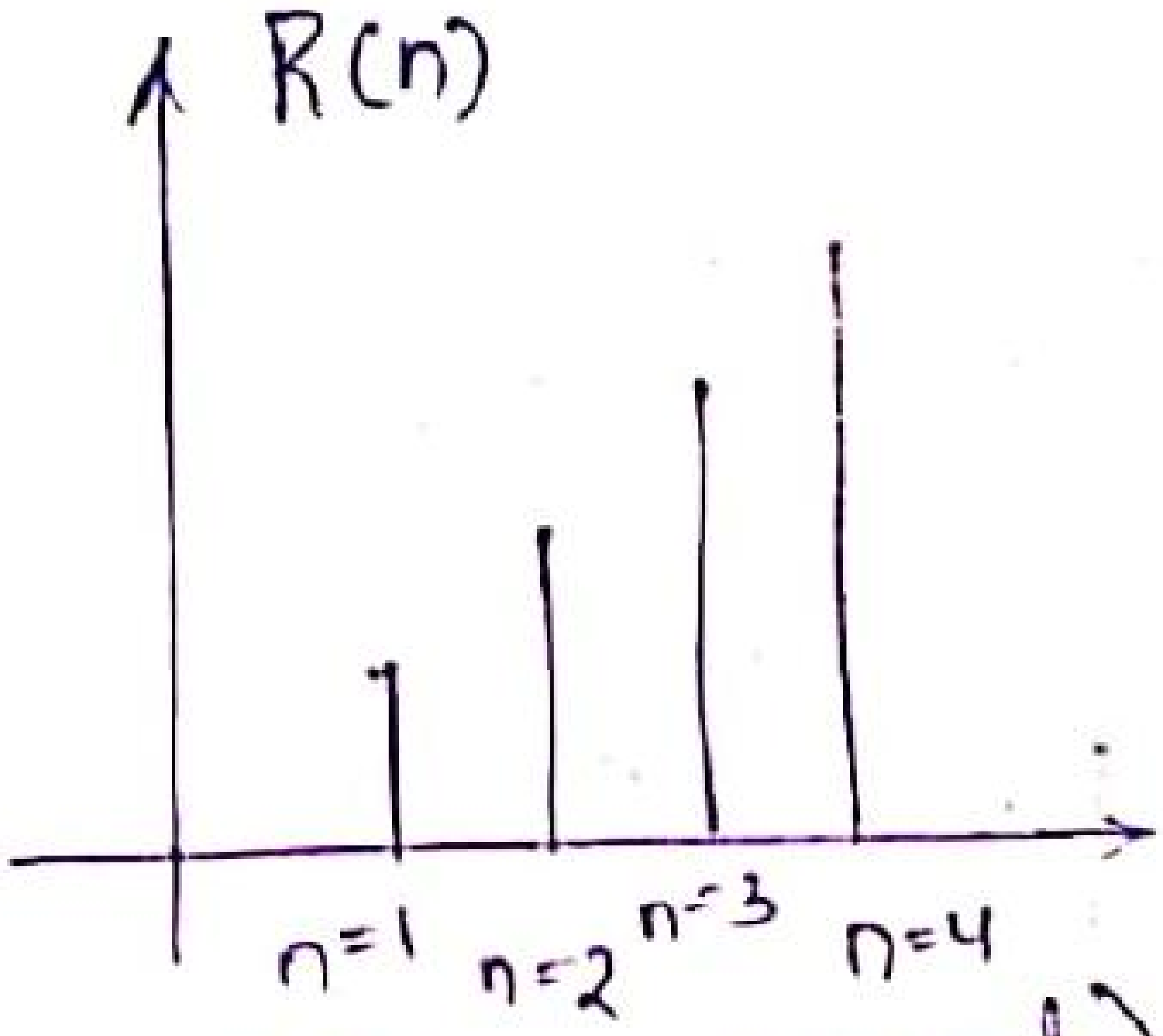
$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

(Unit impulse signal)



$$u(n) = \begin{matrix} 1, & n \geq 0 \\ 0, & n < 0 \end{matrix}$$

(Unit Step Signal)



$$R(n) = \begin{cases} n, & n \geq 0 \\ 0 & n < 0 \end{cases}$$

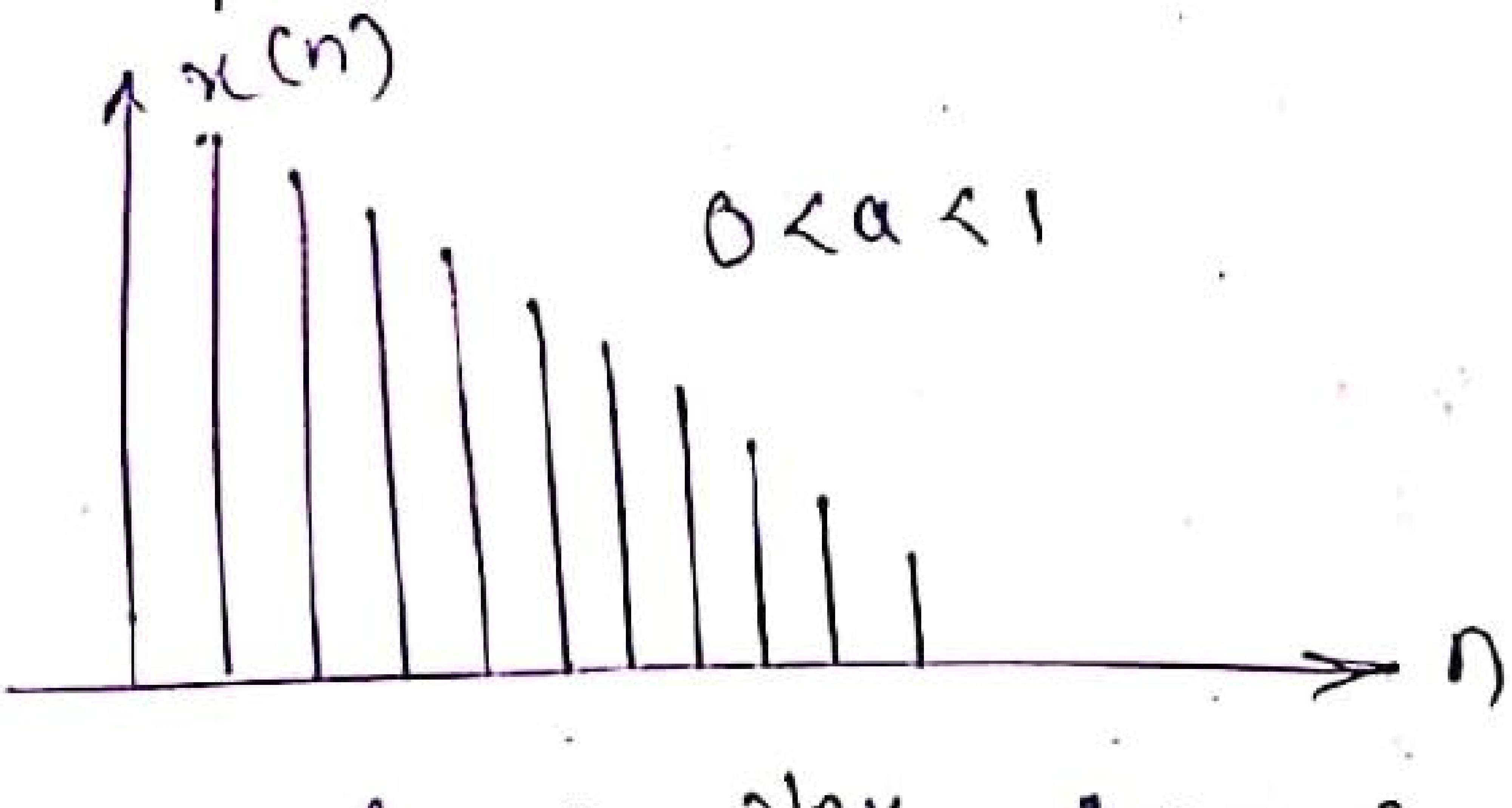
(unit Ramp signal)

Unit Exponential Signal :-

An Exponential signal is represented by $x(n) = a^n$ for all values of 'n'

If 'a' is real then $x(n)$ is real

If 'a' is a complex value then $x(n)$ is a complex value function



when 'a' is complex i.e.; $a = re^{J\theta}$

then $x(n) = r^n e^{J\theta n}$

$= r^n (\cos \theta n + J \sin \theta n)$

Classification of discrete time signal :-

Energy signal and power signal :-

→ The Energy of a discrete time signal $x(n)$ is defined as $\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$

→ A signal $x(n)$ is called as Energy signal if the Energy is finite $(0 < E < \infty)$ and power equal to zero.

→ The average power of a discrete time signal is defined as $\boxed{P = \lim_{N=\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x(n)|^2}$

where $\begin{cases} 0 < P < \infty \\ E = \infty \end{cases}$

→ The signal $x(n)$ is called as power signal if and only if the power is finite and Energy is infinite

Infinite Summetion formula :-

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2}, \quad |a| < 1$$

$$\sum_{n=0}^{\infty} n^2 a^2 = \frac{a^2 + a}{(1-a)^3}$$

Finite Summetion formula :-

① $$\sum_{n=0}^{N} a^n = \frac{1-a^{n+1}}{1-a}, \quad a \neq 1$$

② $$\sum_{n=0}^{N} 1 = N+1$$

③ $$\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1$$

④ $$\sum_{n=0} n = \frac{N(N+1)}{2}$$

⑤ $$\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

Q. Check whether the unit step signal is Energy or power signal.

Sol^n $x(n) = u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\Rightarrow \sum_{n=0}^{\infty} |u(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^2 = \sum_{n=0}^{\infty} 1 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$\Rightarrow \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{N} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot N+1$$

$$\Rightarrow \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$\Rightarrow \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

Q. Find the Energy and power of a signal

$x(n) = \left(\frac{1}{3}\right)^n u(n)$

Soln:- $\sum_{n \to -\infty}^{\infty} |x(n)|^2$

$\Rightarrow \sum_{n \to -\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n)$

$= \sum_{n \to 0}^{\infty} \left(\frac{1}{3}\right)^n$

$\Rightarrow \frac{1}{1-a}$

$= \frac{1}{1-\frac{1}{3}}$

$\Rightarrow P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n \to -N}^{N} |x(n)|^2$

PERIODIC SIGNAL AND APERIODIC SIGNAL

- A signal $x(n)$ is periodic with period N when $x(n+N) = x(n)$ for all value of n --- (1)
- The smallest value of N for which equation (1) is valid is known as fundamental period.
- If the eqⁿ (1) is not satisfied for any value of N then the signal is Aperiodic signal.

we know that $\omega_0 N = 2\pi K$

$$\omega_0 = \frac{2\pi k}{N}$$

$$2\pi f_0 = \frac{2\pi k}{N}$$

$$\boxed{f_0 = \frac{K}{N}}$$

Q. Determine whether the following signals are periodic or not and also find the fundamental period.

(1) $x(n) = \cos 0.01\pi n$

$$= \cos \omega_0 n$$

$$N = \frac{2\pi k}{\omega_0}$$

$$= \frac{2\pi k}{0.01\pi}$$

$$= \frac{2 k}{0.01}$$

$$\boxed{N = 200k}$$

The signal is periodic withea because it is multiple of 2π

If the minimum value of K is taken then fundamental period is 200.

Dt → 7.01.20

Example :-

(1). $x(n) = \sin\left(\frac{\pi}{4}\right)^{n} = \sin \omega_n$

$K, N = ?$

$\omega = \pi/4$

$2\pi f = \pi/4$

$f = \frac{1}{8} = \frac{K}{N}$

$K = 1$

$N = 8$

2. $x(n) = \cos 2\pi/3\, n + \sin \pi/2\, n$

$\cos 2\pi/3\, n$

$\omega = 2\pi/3$

$2\pi f = 2\pi/3$

$f = \frac{1}{3} = \frac{K}{N_1}$

$K = 1, N_1 = 3$

$\sin \pi/2\, n$

$\omega = \pi/2$

$2\pi f = \pi/2$

$f = \frac{1}{4} = \frac{K}{N_2}$

$K = 1$

$N_2 = 4$

$N = \text{Lcm}(N_1, N_2) \Rightarrow N = 12$

* A signal is periodic if $F = \frac{K}{N}$ is a rational number.

* All periodic signals are power signals.
* All aperiodic signals are Energy signal.
* RAMP signals are neither power nor Energy
* δn is aperiodic signal

Ex:- $x(n) = a^n u(n)$

$$E = \sum_{n\to -\infty}^{\infty} |x(n)|^2$$

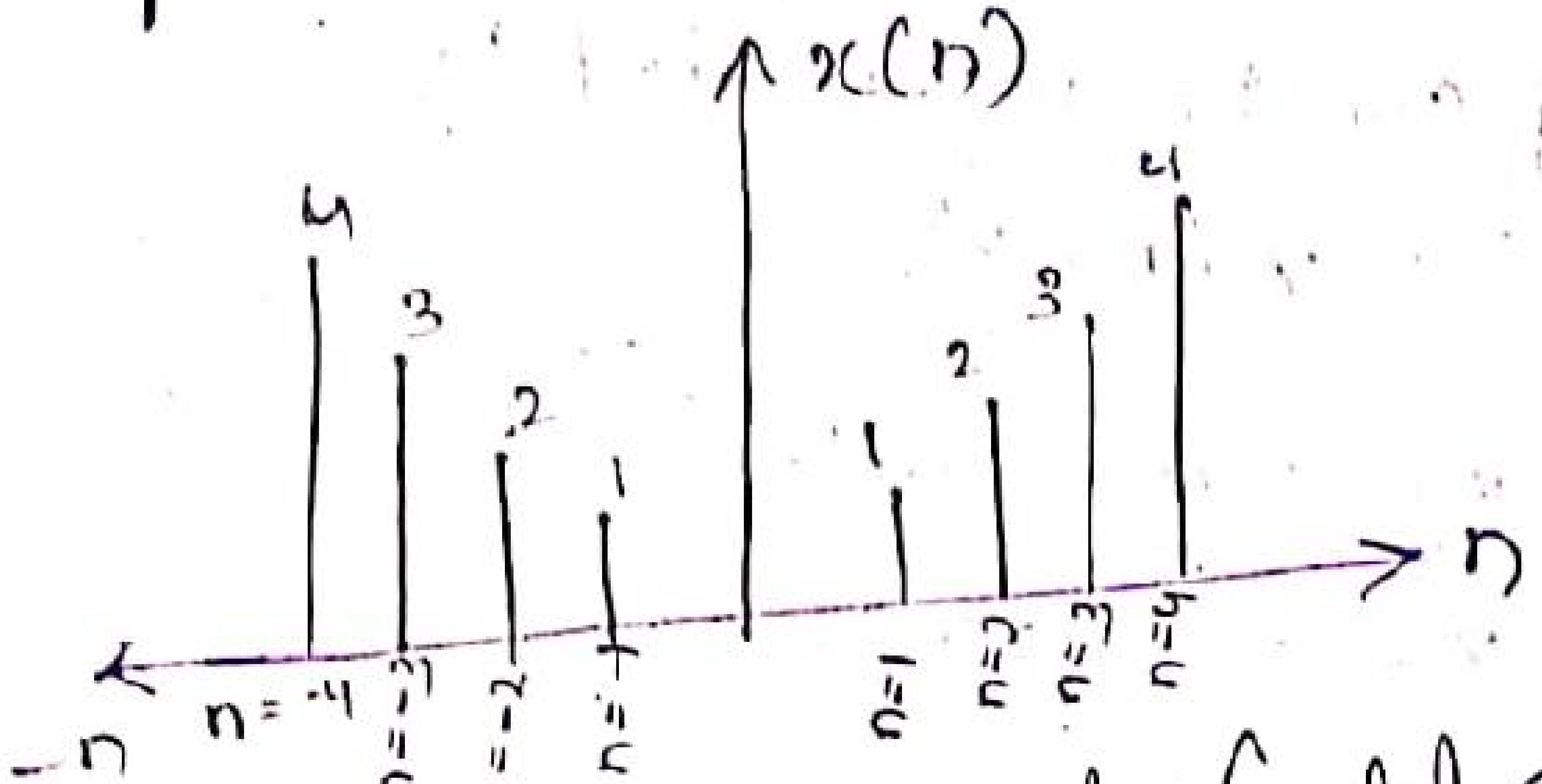
$$= \sum_{n\to -\infty}^{\infty} |a^n u(n)|^2$$

$$= \sum_{n\to 0}^{\infty} |a^n|^2$$

$$= \sum_{n\to 0}^{\infty} |a^2|^n$$

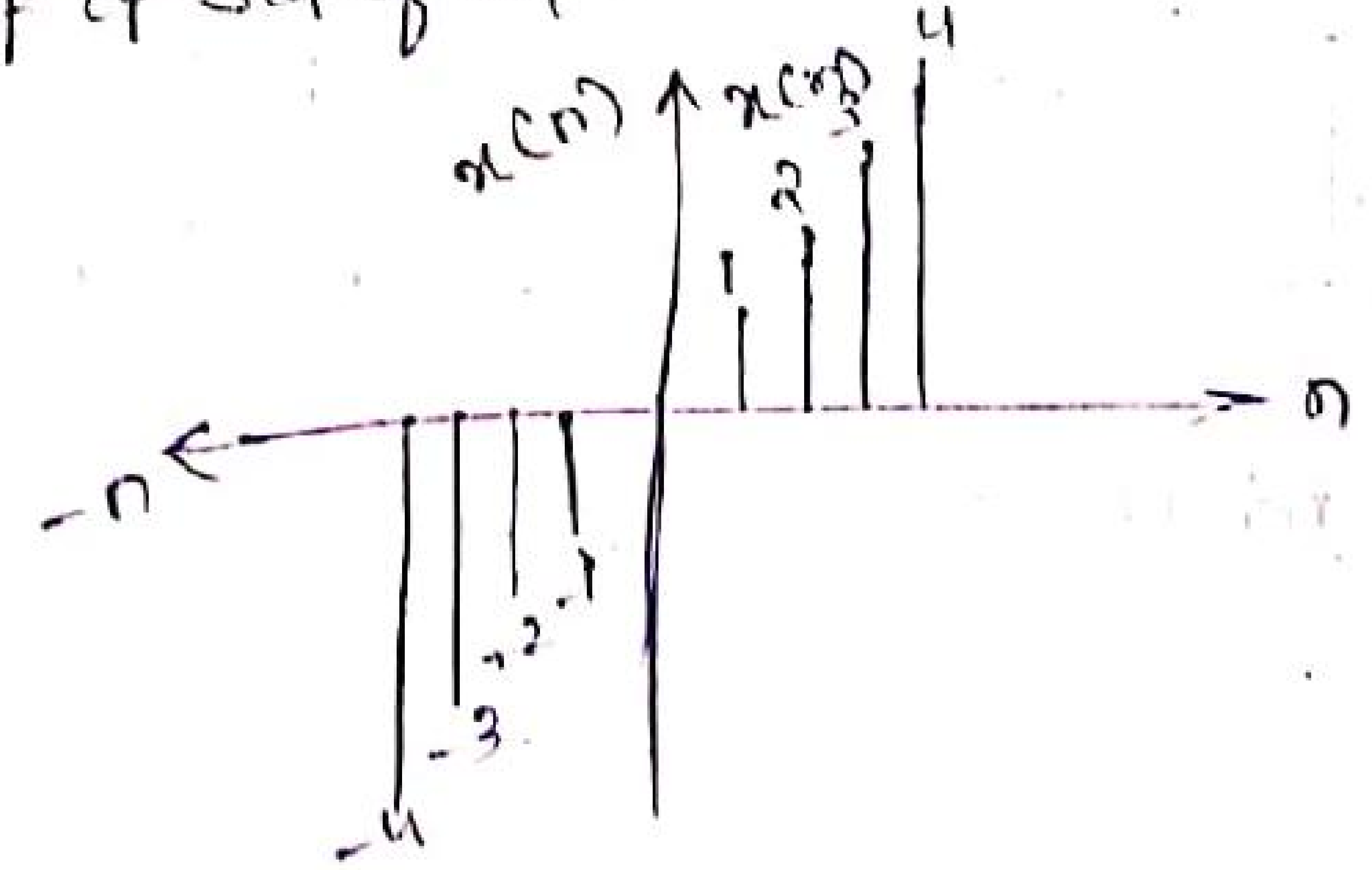
$$= \sum_{n\to 0}^{\infty} \frac{1}{1-a^2}$$

A discrete time signal is said to be symmetric (Even) if it satisfy $\boxed{x(-n) = x(n)}$ for all values of n.



Anti- symmetric signal (odd signal) :-

A discrete time signal is said to be odd signal if it satisfied the condition. for all values of n.



* Even signal is symmetric above x - axis and odd signal is symmetric about origin

1. Shifting :-

→ The Shifting operations shift the values of the sequence by an integer variable.

→ Shifting can be Either delay or advance.

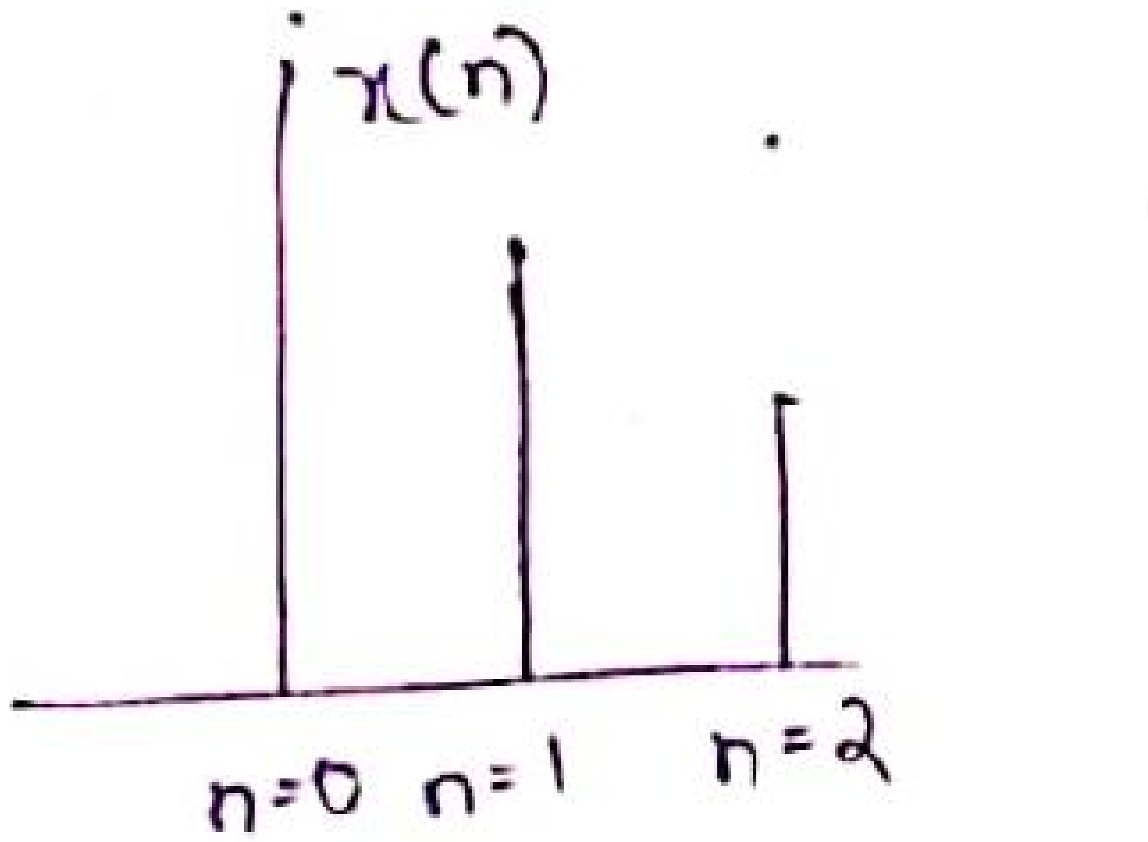
→ The shifting can be represented by

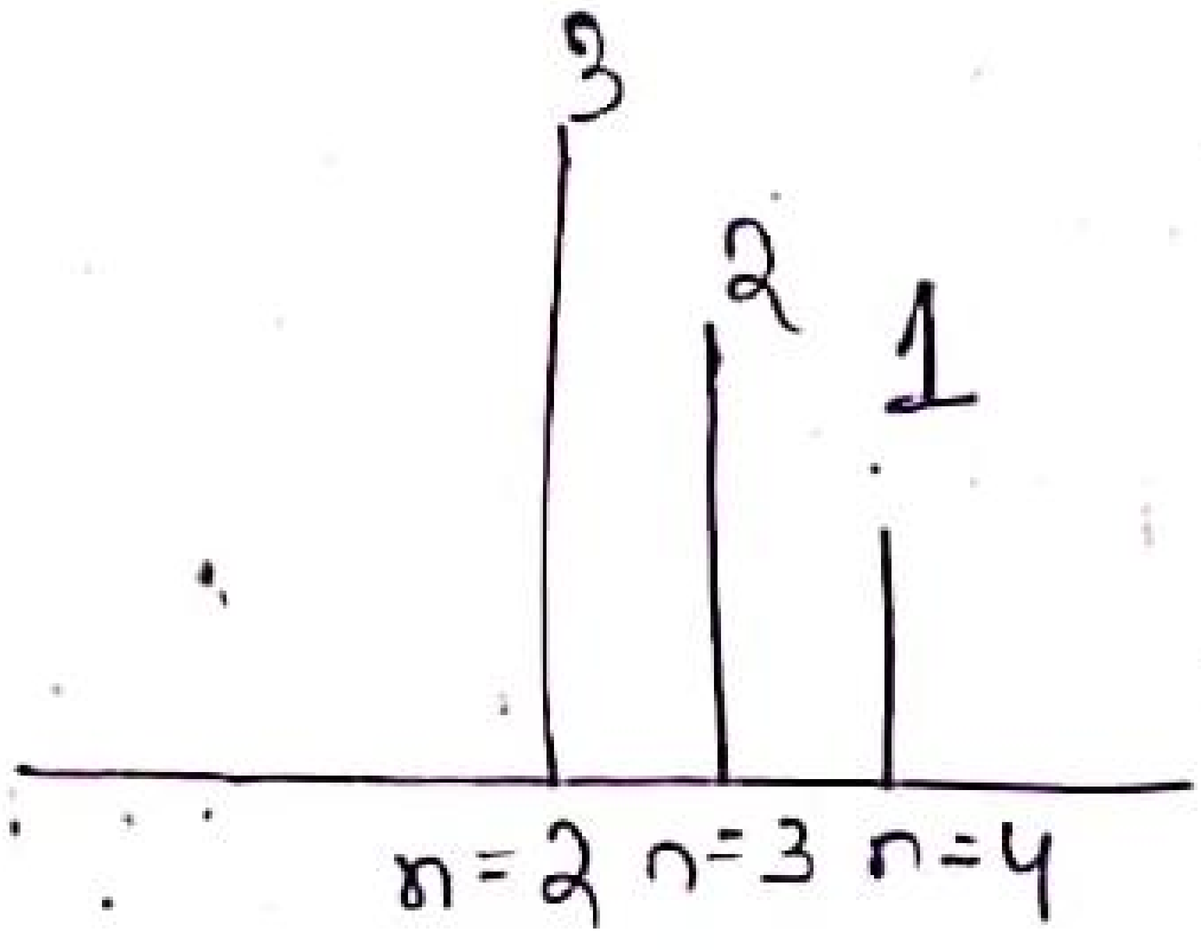
$$y(n) = x(n-k).$$

$K = +ve$ – delay

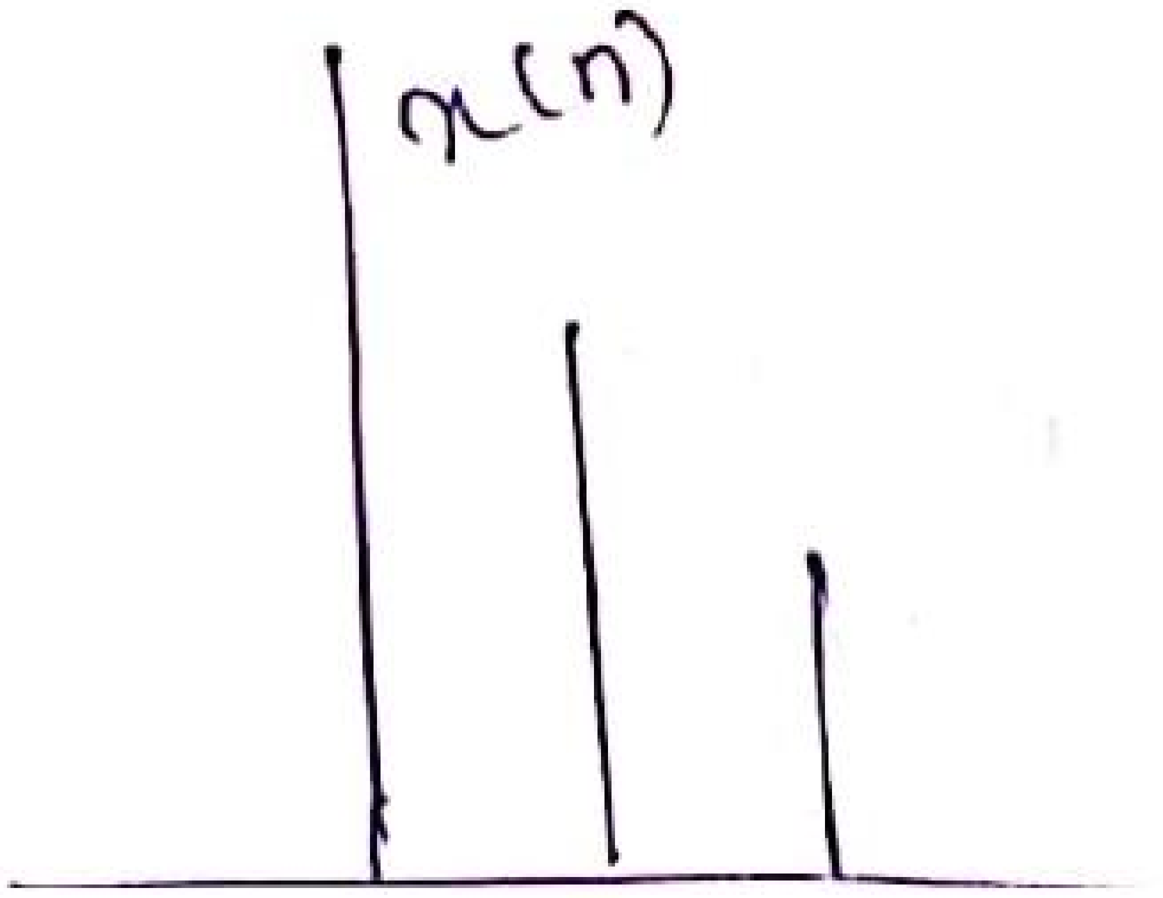
$K = -ve$ – advance

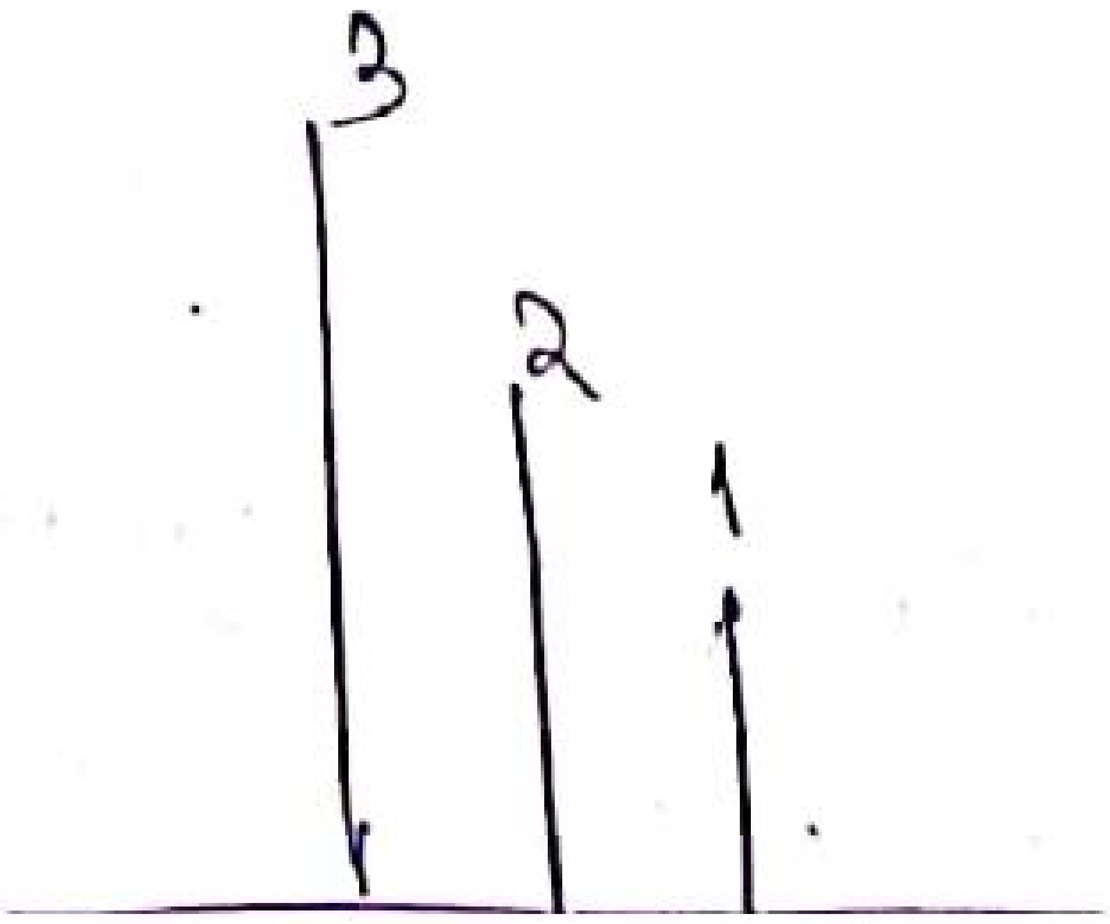
→ $y(n) = x(n-2$





$y(n) = x(n+4)$

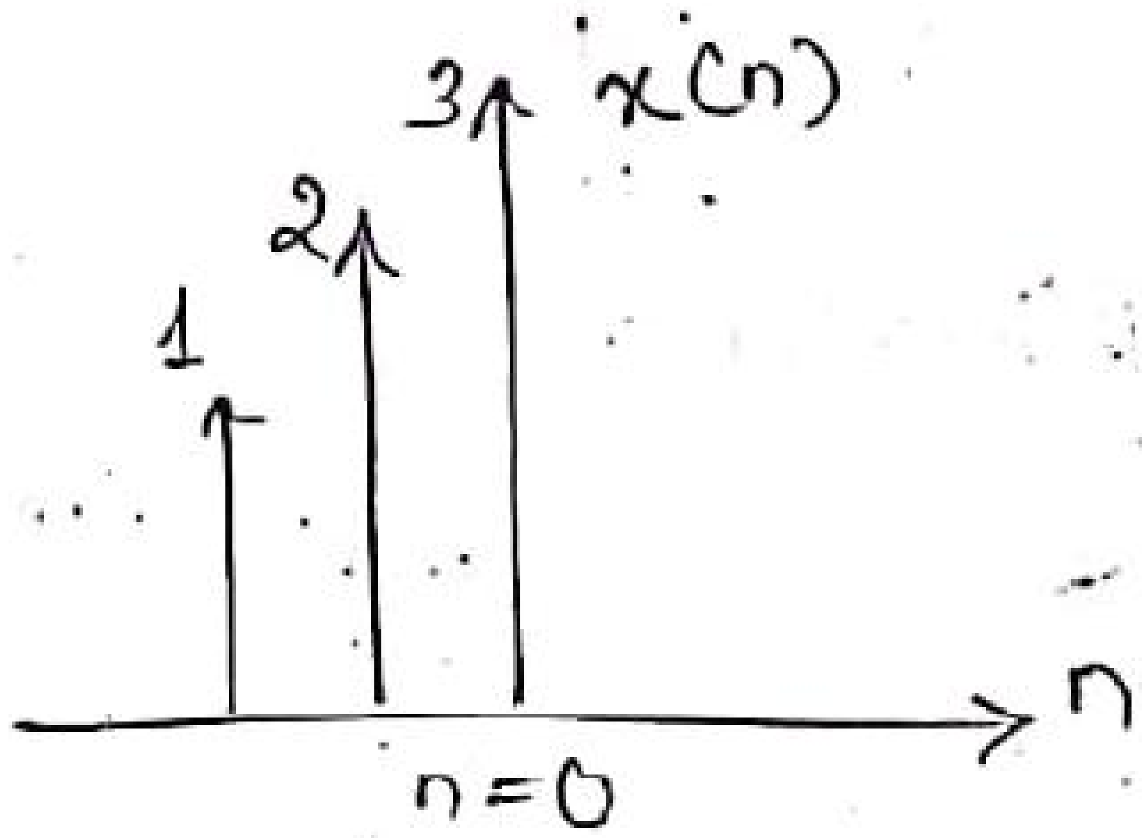




Time Reversal (Folding):-

Time Reversal of the sequence is done by holding the sequence about $n=0$.

OR



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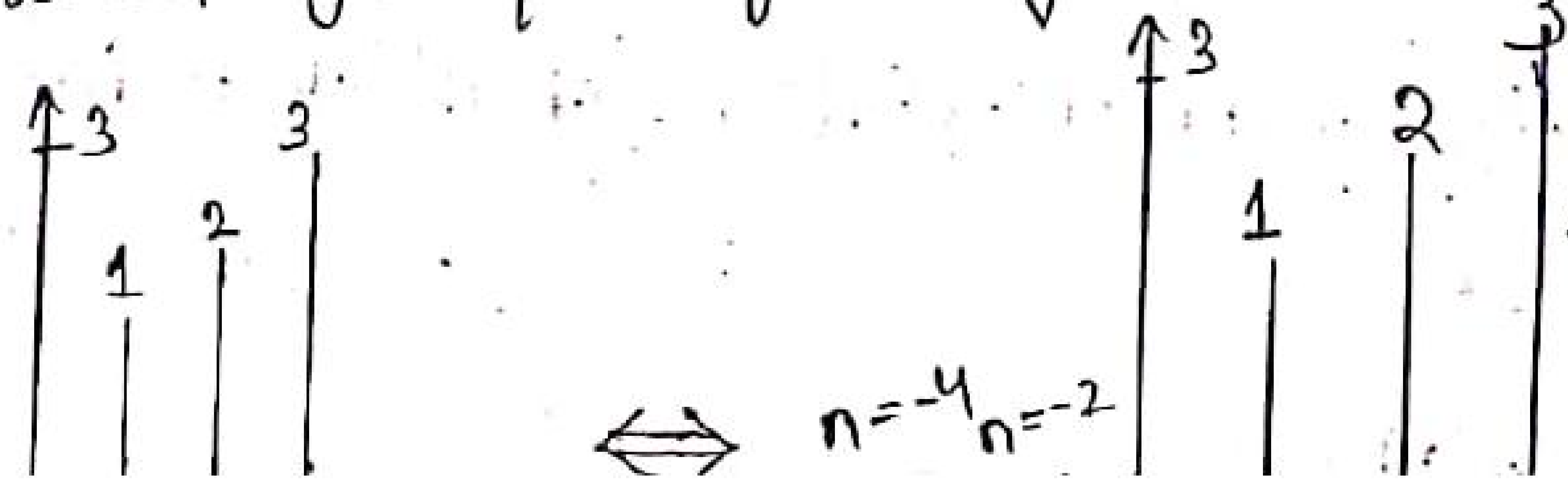
(III) Time Scaling :-

→ It is obtained by replacing n by λn

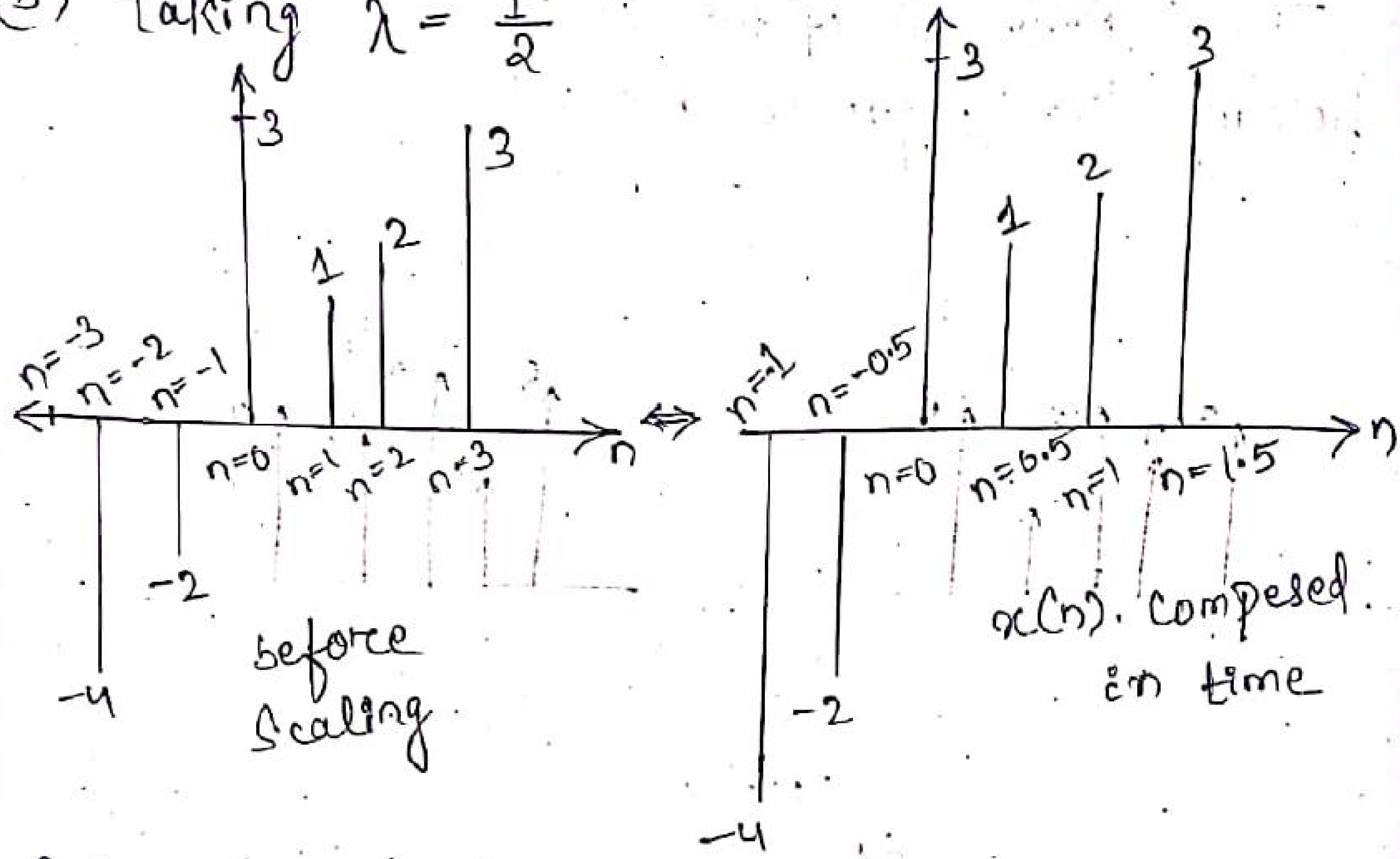
(a) before scaling



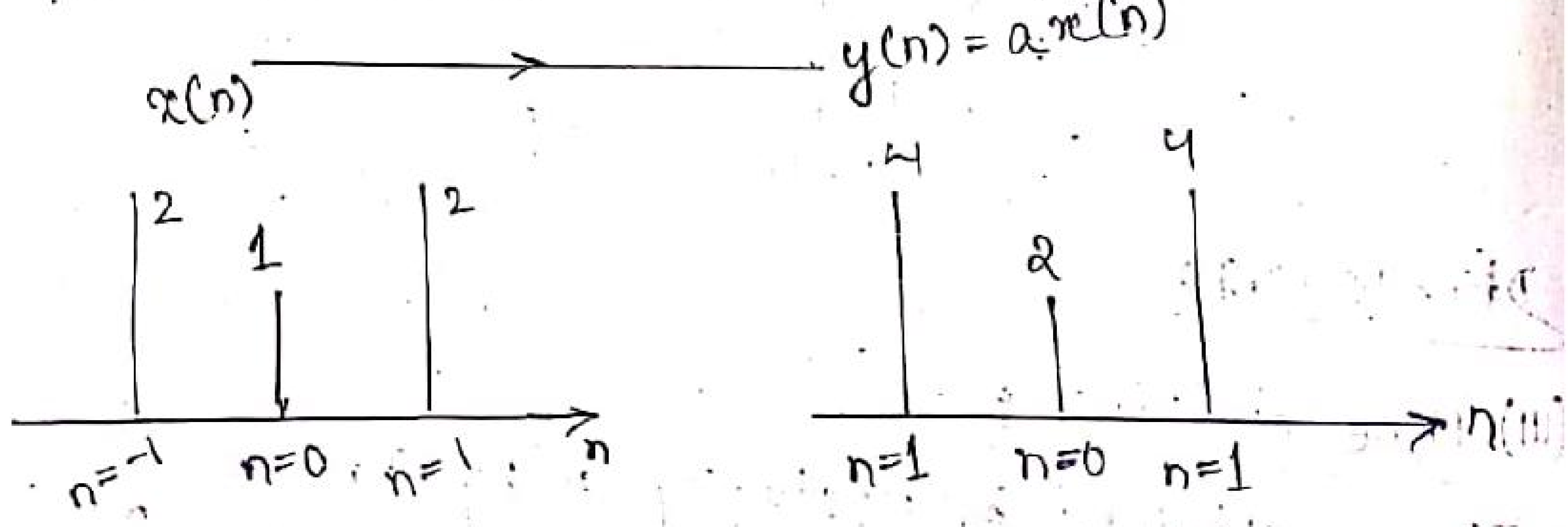
after scaling $\lambda=2$

→ Here $x(n)$ changed to $x(\lambda n)$

(b) Taking $\lambda = \frac{1}{2}$

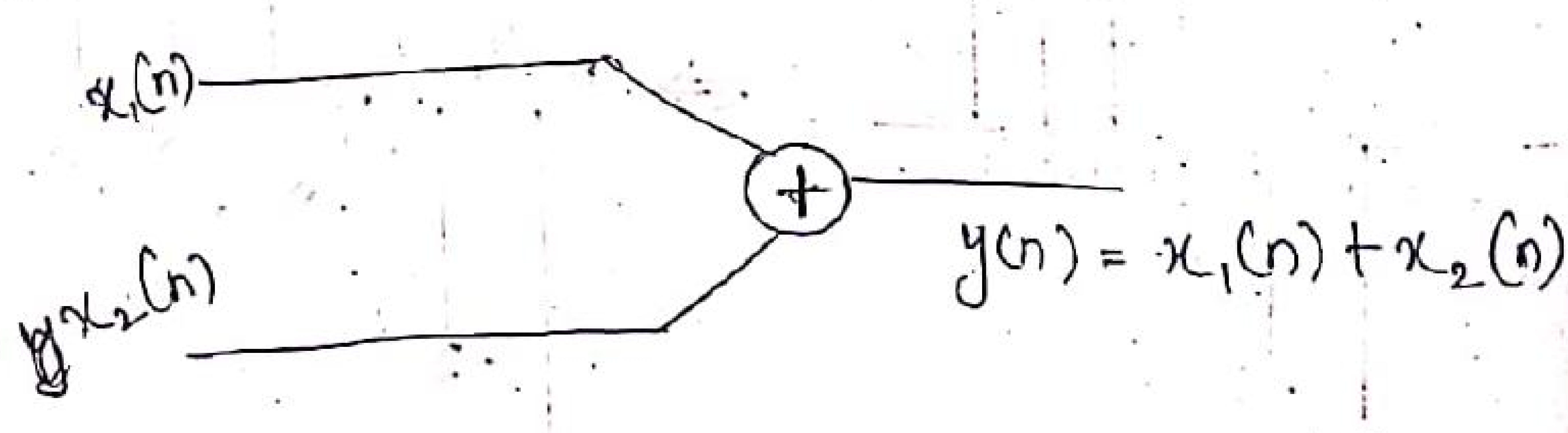


(iv) Scalar Multiplication :-



Block diagram representations of discrete time Signal :-

⟹ Adder



Example :-

(a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 1, 2\}$

$x_2(n) = \{\underset{\uparrow}{2}, 1, 3, 4\}$

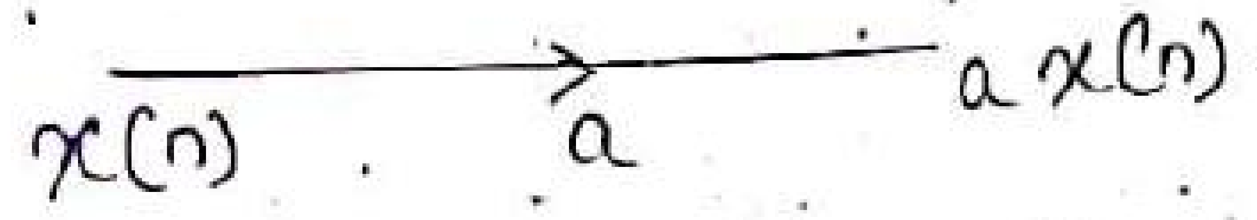
$y(n) = \{\underset{\uparrow}{3}, 3, 4, 6\}$

(b) $x_1(n) = \{\underset{\uparrow}{1}, 2, 1, 2\}$

$x_2(n) = \{2, \underset{\uparrow}{1}, 3, 4\}$

$y(n) = \{2, \underset{\uparrow}{2.5}, 5, 2\}$

⇒ **Constant Multiplier :-**

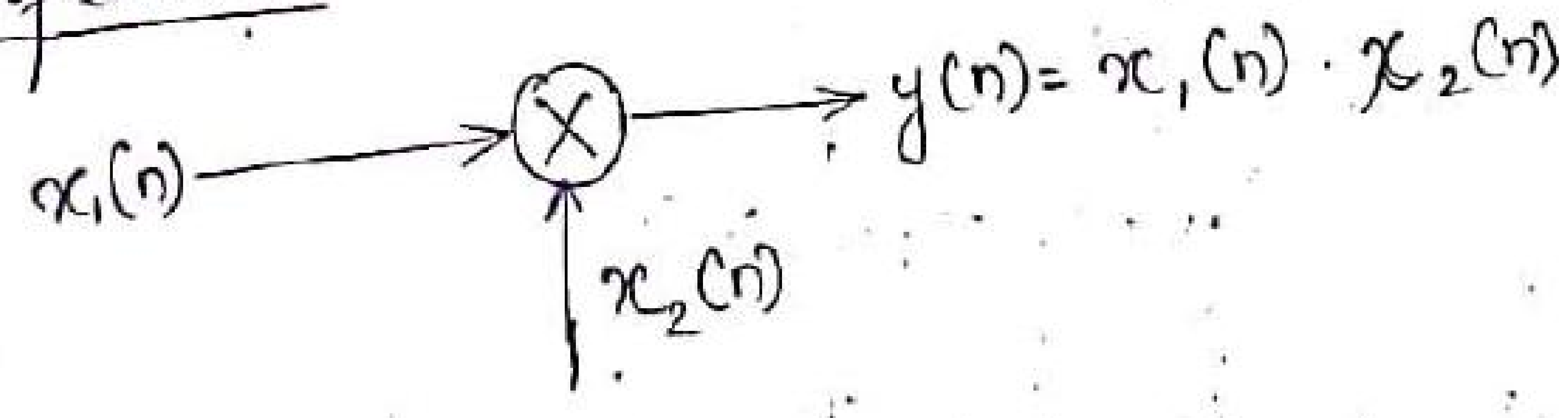


$x(n) = \{2, 4, 6\}$

$a = 2$

$a\,x(n) = \{4, 8, 12\}$

⇒ **Multiplier :-**

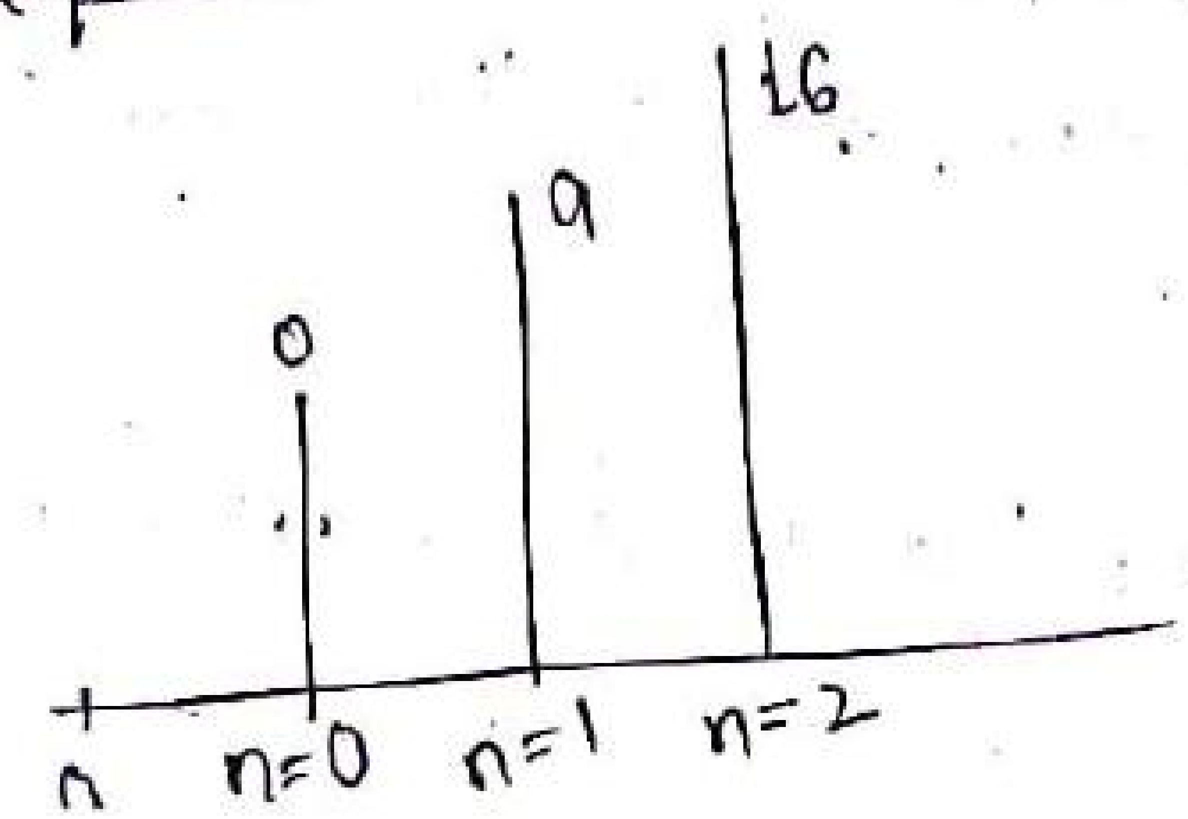


$x_1(n) = \{\underset{\uparrow}{2}, 3, 4, 5\}$

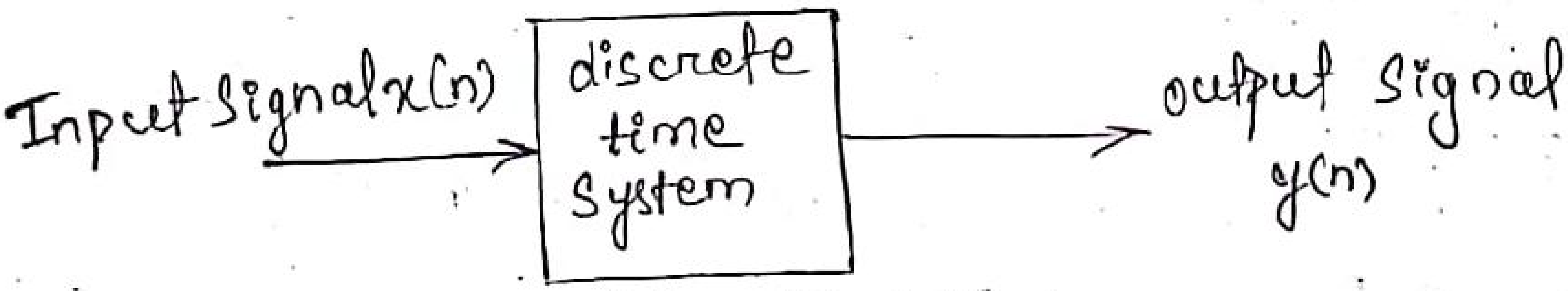
$x_2(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$y(n) = \{0, \underset{\uparrow}{4}, 9, 16, 0\}$

Graphical Representations :-



→ A discrete time system is a device that operates on a discrete time input signal $x(n)$ process the input signal with some definite functions and provides an output $y(n)$ which is also discrete in nature.



$$y(n) = T[x(n)]$$

→ Symbol T denotes the transformations.

Input output descriptions of a System :-

→ There is a mathematical relations between the input and output of the discrete time system.

→ In order to give the input the input terminal is used and in order to get the output the output terminal is used.

→ The input and output both must be discrete in nature.

→ We can Express $x(n) \xrightarrow{T} y(n)$, which means $y(n)$ is the response of the system and $x(n)$ denotes the Excitations.

Classifications of discrete time system :-

→ Discrete time system are classified depending on its various properties and characteristics as follows:-

(I) Static and dynamic system
(II) Time varient and time invarient system
(III) Causal and non-causal system
(IV) Linear and non-linear system.
(V) FIR and IIR system
(VI) Stable and unstable system

Static And Dynamic System:-

→ A discrete time system is called static or memory less system if the output at any instant and depends on the input sample at same instant, but not on the past and future values of the sample.

→ In ~~other~~ other case the system having memory is called as dynamic system

→ Example :- (I) $y(n) = ax(n) \rightarrow$ Static system

(II) $y(n) = 2x^3(n) \longrightarrow$ Static system

(III) $y(n) = 5x(n-1) + 2x(n-2) \rightarrow$ Dynamic system.

(ii) Time Varient and Time invarient System :-

→ A System is said to be time invarient if its input output characteristics does not change with time.

→ If the response to a delayed input and the delayed response are equal then, the system is said to be time invarient.

→ Let the input of the system is $x(n)$ and output of the system is $y(n)$.

→ The response to a delayed input is denoted by

$$\boxed{y(n,k) = T[x(n-k)]}$$

→ Here, the input is delayed by k samples now the output is delayed by k samples, so, $y(n-k)$.

→ If $y(n,k) = y(n-k)$ for all values of k then the system is time invarient.

→ If $y(n,k) \neq y(n-k)$ for all values of k then the system is time varient.

Q. Test wheathere the system is time varient or invarient

$$y(n) = x(n) + x(n-1)$$

Ans:- $y(n,k) = x(n-k) + x(n-k-1)$

$y(n,k) = x(n-k) + x(n-k-1)$

$\Rightarrow y(n,k) = y(n-k)$

So, the system is time invarient

Q. $y(n) = x(-n)$

$y(n, k) = x(-n-k)$

$y(n-k) = x[-n(n-k)]$

$= x(-n+k)$

$\Rightarrow y(n,k) \neq y(n-k)$

So, the system is time varient

Q. $y(n) = n\,x(n)$

$y(n,k) = n\,x(n-k)$

$y(n-k) = (n-k)\,x(n-k)$

$\Rightarrow y(n,k) \neq y(n-k)$

So, the system is time varient

Home task

Q. $y(n) = x(n)\cos\omega_0 n$

Q. $y(n) = x(-n-2)$

(iii) Causal and Non-Causal System :-

→ A system is said to be causal if the output depends on the present and past input of the sample.

→ If the output of the system depends on future input along with present and past input then the system is said to be non-causal system.

Q. $y(n) = x(n) + \frac{1}{x(n-1)}$

Ans:- Taking $n=0$

$\Rightarrow y(0) = x(0) + \frac{1}{x(0-1)} = x(0) + \frac{1}{x(-1)}$

Taking $n=1$

$\Rightarrow y(1) = x(1) + \frac{1}{x(1-1)} = x(1) + \frac{1}{x(0)}$

Taking $n=-1$

$y(-1) = x(-1) + \frac{1}{x(-1-1)} = x(-1) + \frac{1}{x(-2)}$

So, the system is causal as the output depends on past and present value.

Q. $y(n) = x(n^2)$

Ans:- Taking $n=0$

$\Rightarrow y(0) = x(0^2) = x(0)$

Taking $n=1$

$y(1) = x(1^2) = x(1)$

Taking $n=2$

$y(2) = x(2^2) = x(4)$

Taking $n=-1$

$\Rightarrow y(-1) = x(-1^2) = x(1)$

Here, the output depends on present and future value so the system is non-causal.

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Linear And Non-linear System:

A system that satisfy the principle of superposition is said to be linear position.

Super position principle said that the response of the system to a weighted some of signal is equal to the corresponding weighted some of output signal to each of individual output

Ex:- $a_1 x_1(n) \longrightarrow a_1 y_1(n)$

$$T[a_1 x_1(n) + a_2 x_2(n)] \longrightarrow T[a_1 x_1(n)] + T[a_2 x_2(n)]$$

OR

Linear System

$$\Rightarrow \boxed{T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]}$$

The System which does not follow the principle of superposition is known as ~~superpo~~ Non-linear system.

Q. Test whether the following system is linear or non-linear

$$y(n) = x^2(n)$$

$$y_1(n) = x_1^2(n)$$

$$y_2(n) = x_2^2(n)$$

$$a_1 y_1(n) = a_1 x_1^2(n)$$

$$a_2 y_2(n) = a_2 x_2^2(n)$$

$$y(n) = a_1 x_1^2(n) + a_2 x_2^2(n) \quad \text{------} \quad (1)$$

$$y(n) = [a_1 x_1(n) + a_2 x_2(n)]^2 \quad \text{------} \quad (2)$$

Equation (1) ~~and~~ $\neq$ (11) $\rightarrow$ Non-linear

Q. $y(n) = n\,x(n)$

$$y_1(n) = n\,x_1(n)$$

$$y_2(n) = n\,x_2(n)$$

$$y_1(n) = a_1 n x_1(n) + a_2 n x_2(n) \quad \text{------} \quad (1)$$

$$T[a_1 x_1(n) + a_2 x_2(n)]$$

$$\Rightarrow \quad = a_1 n x_1(n) + a_2 n x_2(n) \quad \text{------} \quad (2)$$

Equation (1) = (2) $\rightarrow$ Linear

Q. $y(n) = x(n^2)$

$$y_1(n) = x_1(n^2)$$

$$y_2(n) = x_2(n^2)$$

weighted some of output :-

$$y(n) = a_1 x_1(n^2) + a_2 x_2(n^2) \quad \text{------} \quad (1)$$

output due to weighted input

$$y(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$

$$= a_1 x_1(n^2) + a_2 x_2(n^2) \quad \text{------} \quad (2)$$

Equation (1) and (2) $\rightarrow$ linear.

Q $y(n) = a x(n) + b$

Stable System and Unstable system :-

LDI System is said to be stable if it produces bounded output for AB for bounded.

→ The necessary conditions for stability is summession. $\sum_{n=-\infty}^{\infty} h(n) < \infty$

Q Test stability of system whose impulse system $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$= \sum_{n=-\infty}^{\infty} h(n)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)$$

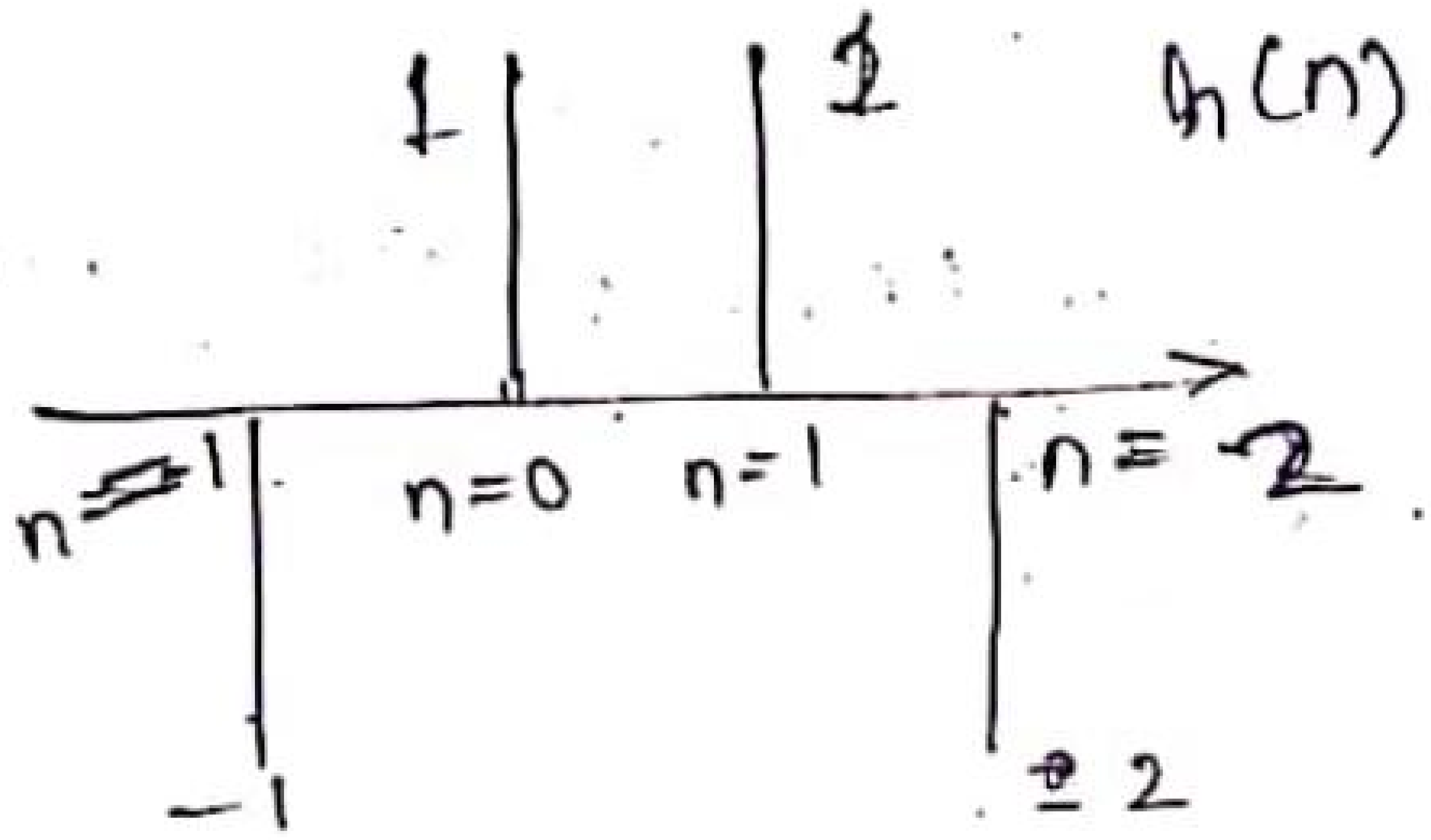
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \cdot 1$$

$$= \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{2-1}{2}} = 2$$

As $\sum_{n=-\infty}^{\infty} h(n) < \infty = 2$, So, the system is stable

Finite impulse Response and Infinite impulse Response :-

→ If the impulse response is of finite duration. the system is said to be finite impulse response system.

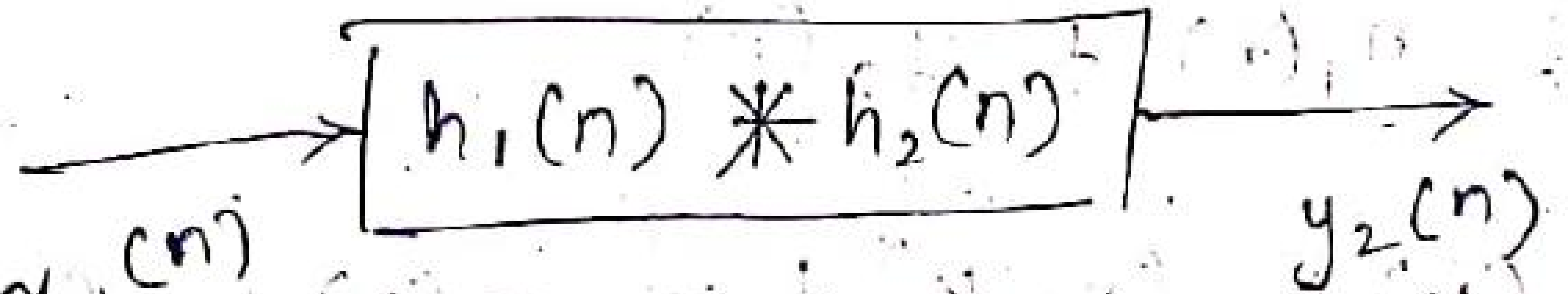


$$h(n) = \begin{cases} 1, & n = 0, 1 \\ -2, & n = 2 \\ -1, & n = -1 \end{cases}$$

An infinite impulse response system have infinite duration sequence.

Interconnections of discrete time systems:-

consider to LDI system with impulse response $h_1(n)$ and $h_2(n)$ connected in cascade



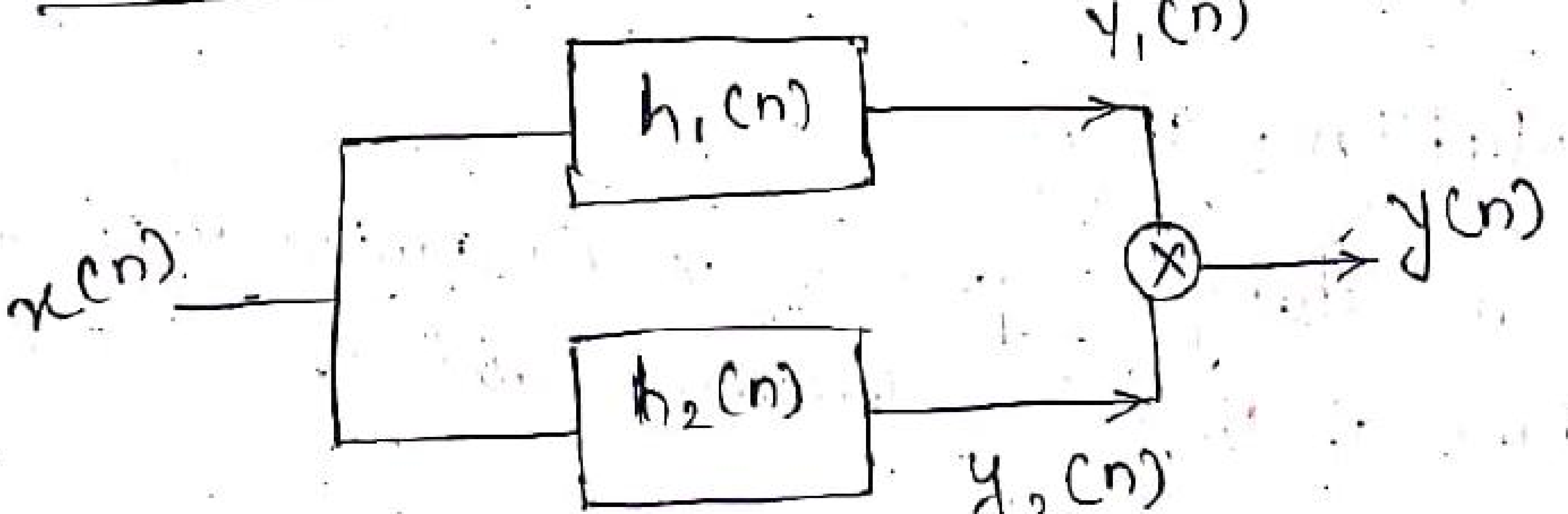
$$y_2(n) = x_1(n) * h(n)$$

$$h(n) = \sum_{n=0}^{\infty} h_1(k)\, h_2(n-k)$$

$$= \underbrace{h_1(n) * h_2(n)}_{\text{convolutions}}$$

→ Here, the impulse response of 2 LTI system connected in cascade is the convolution of individual impulse response

II. PARALLEL INTERCONNECTION :-



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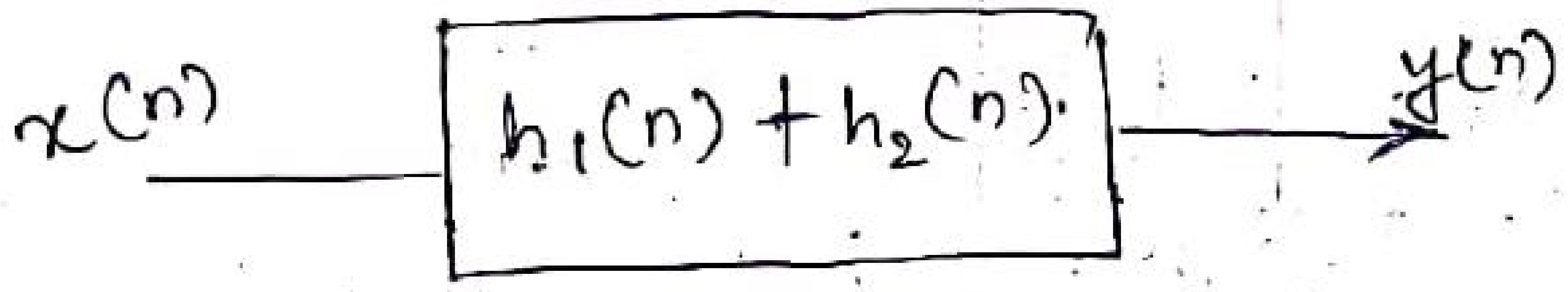


$$y_1(n) = x(n) * h_1(n)$$

$$y_2(n) = x(n) * h_2(n)$$

$$y(n) = y_1(n) + y_2(n)$$

$$y(n) = \sum_{n=-\infty}^{\infty} x(K)\, h_1(n-K) + \sum_{n=-\infty}^{\infty} x(K)\, h_2(n-K)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} x(K)\, h(n-K)$$

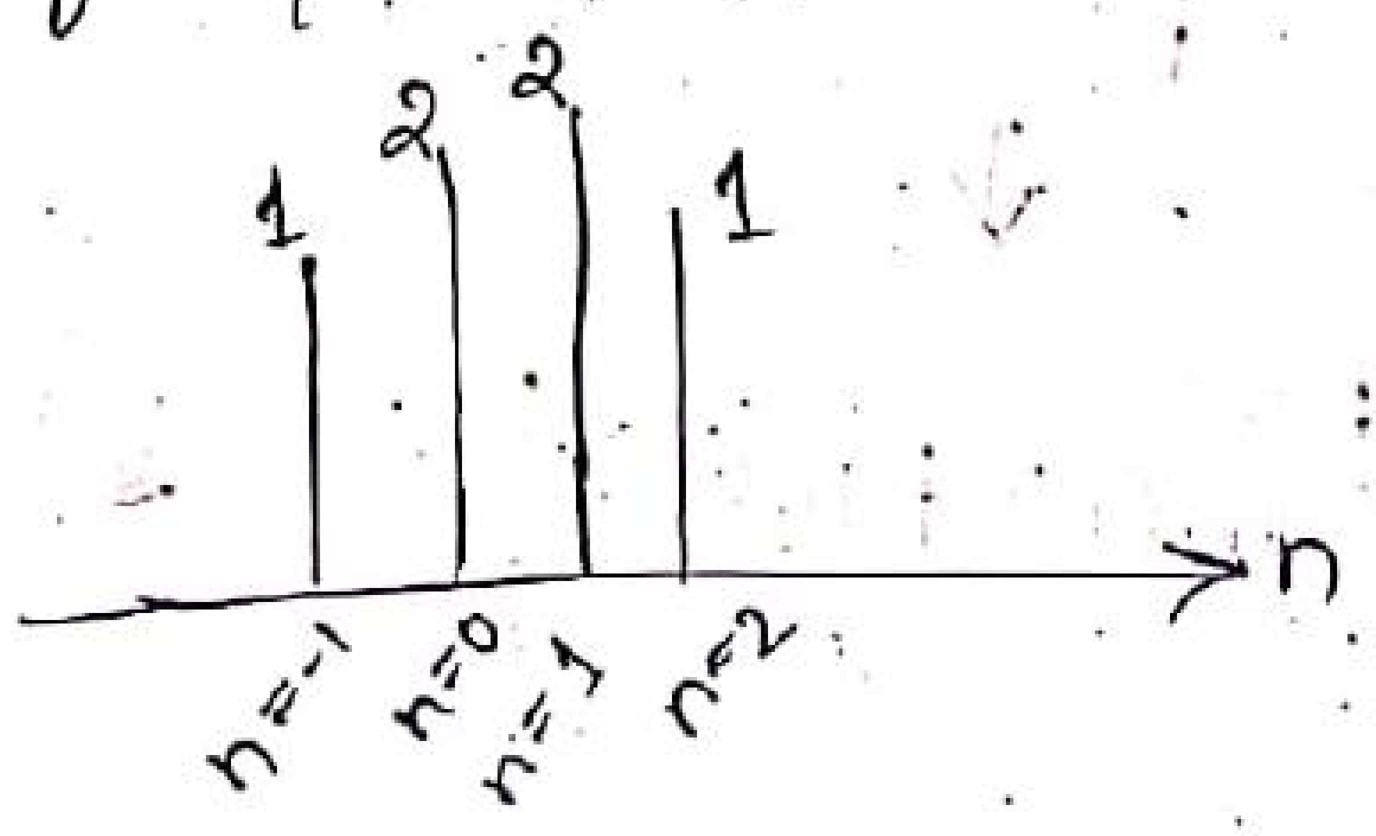
$$y(n) = x(n) * h(n)$$

→ So, when system are connected in parallel the overall Impulse response is the summetion of individual Impulse response.

RESPONSE OF LTI SYSTEM TO ARBITARY INPUTS :-

Convolution : Sum :-

Any arbitary sequence $x(n)$ can be represente in terms of Impulse sequence $\delta(n)$.



$$x(n) = \left.\begin{array}{ll} 2, & n = 0, 1 \\ = 1, & n = -1, 2 \end{array}\right\}$$

$$2\,\delta(n) + 1\,\delta(n+1) + 2\,\delta(n-1) + 1\,\delta(n-2)$$

→ A discrete time system performs an operations on the input signal based on some predefined criteria to produce a modified output.

→ If the input of a system is unit impulse then the output of the system is denoted by $h(n)$.

$$h(n) = T[\delta(n)] \text{ ----- (I)}$$

→ we can write the arbitary sequence as a weighted sum of discrete impulse,

$$y(n) = \sum_{n=-\infty}^{\infty} x(k)\,\delta(n-k) \text{ ----- (II)}$$

→ For a linear system, we can write

$$\boxed{y(n) = \sum_{n=\infty}^{\infty} x(n) + T[\delta(n-k)]}$$

→ For time invarient system.

$$\boxed{h(n, k) = h(n-k)}$$

→ The convolution sum can be represented as

$$\boxed{y(n) = x(n) * h(n)}$$

STEPS INCLUDED IN CONVOLUTION :-

The process of convolution between the sequence $x(k)$ and $h(k)$ involves the following steps :-

I. **Folding :-**
$h(k)$ is folded and we obtain $h(-k)$.

II. **Shifting :-**
Shift $h(-k)$ by n.

III. **Multiplication :-**
Multiply $x(k)$ by $h(n-k)$.

IV. **Summation :-**
Sum all the value of product sequence

Q. Determine the convolution sum of two sequence. (matrix method).

$$x(n) = \{\underset{\substack{\uparrow \\ n=0}}{3}, 2, 1, 2\}$$

$$h(n) = \{1, \underset{\substack{\uparrow \\ n=0}}{2}, 1, 2\}$$

Starting point of $x(n) = 0$

$h(n) = 0$

$y(n) = 0 + (-1) = -1$

N_1 = Number of Elements of $x(n) = 4$

N_2 = Number of Elements of $h(n) = 4$

N = Number of Elements of $y(n)$

$N = N_1 + N_2 - 1$

$\Rightarrow N = 4 + 4 - 1 = 8 - 1 = 7$

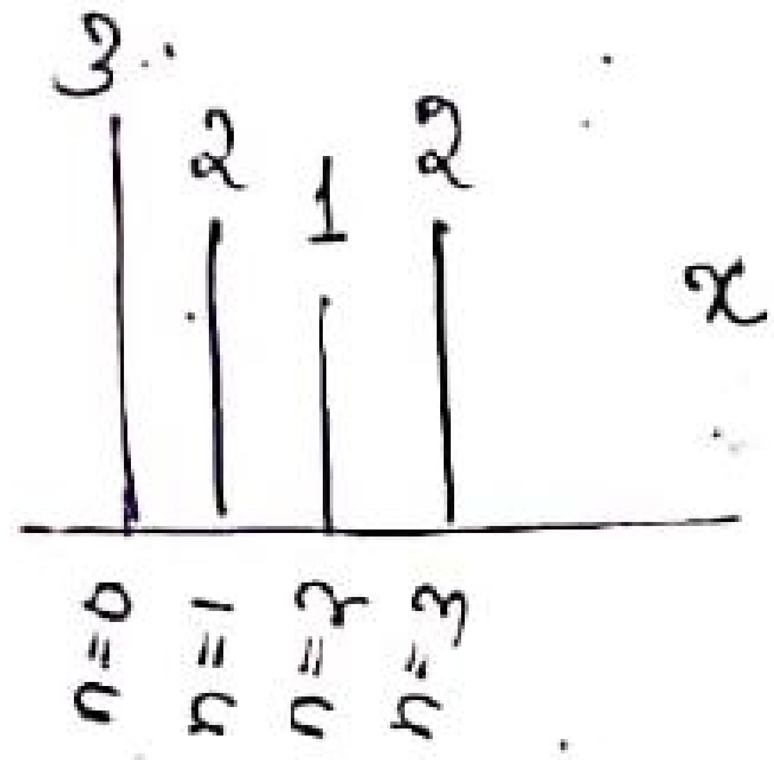
$$\begin{array}{c|cccc} & 3 & 2 & 1 & 2 \quad x(n) \\ \hline 1 & 3 & 2 & 1 & 2 \\ 2 & 6 & 4 & 2 & 4 \\ h(n) \quad 1 & 3 & 2 & 1 & 2 \\ 2 & 6 & 4 & 2 & 4 \end{array}$$

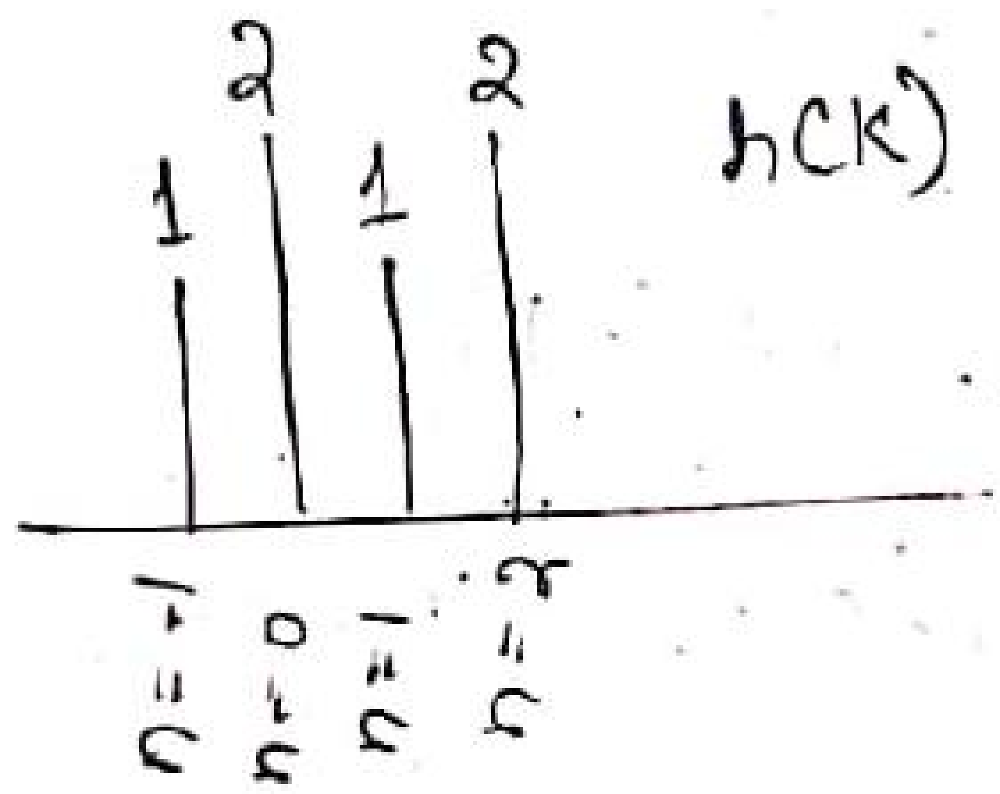
$$\{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\}$$

$\{3, 8, 8, 12, 9, 4, 4\}$
$\uparrow$

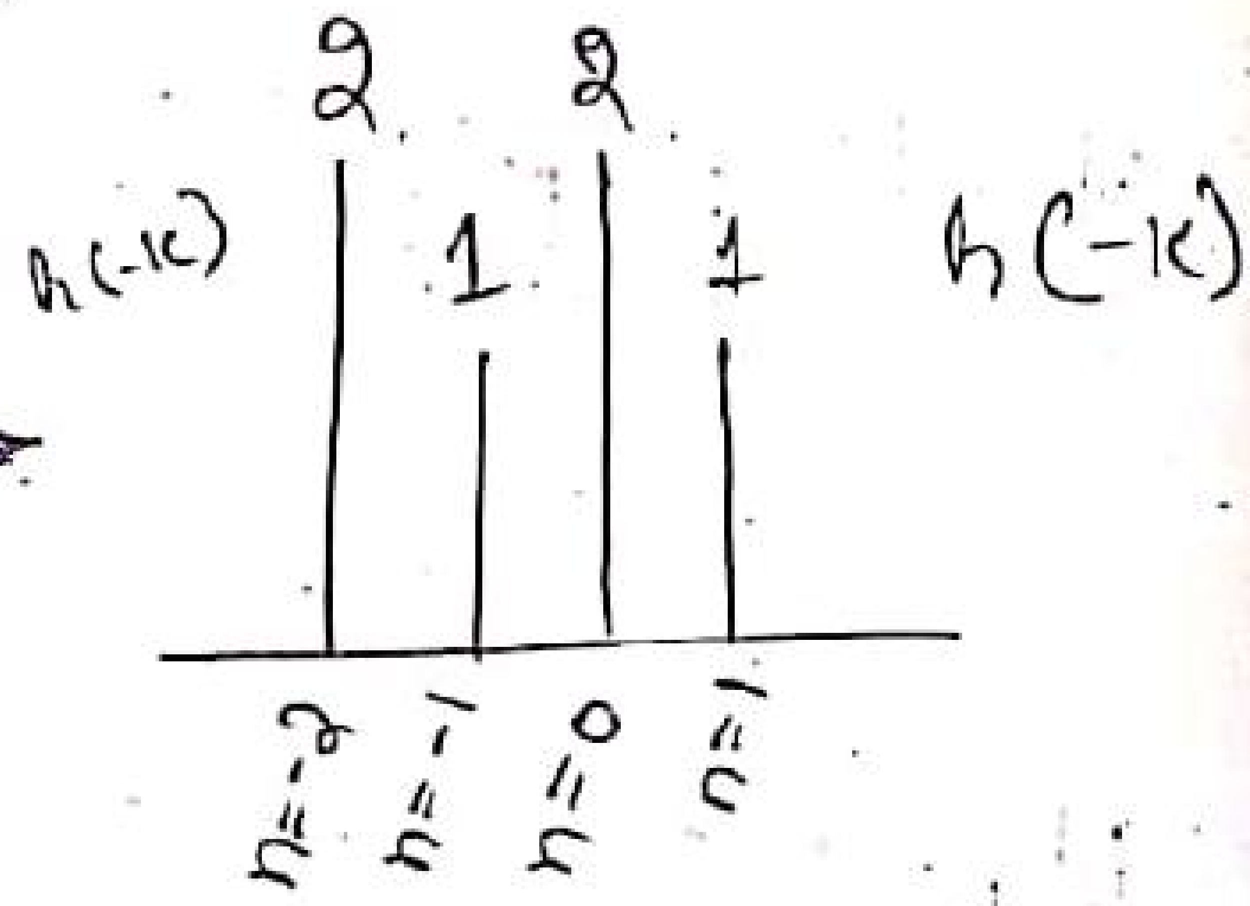
Graphical method :-

(constant)





folding $\longrightarrow$



(II) Shifting $n=0$

$h(n-K)$
$h(0-K)$
$h(-K)$

$y(n) = \sum x(K) * h(n-K)$
$y(0) = 0$

(III) Multiplications

$y(0) = x(K)\, h(-K)$
$= 2+6 = 8$

$h(n-K)$
$h(-1-K)$
$-1-K=0$
$K=1$

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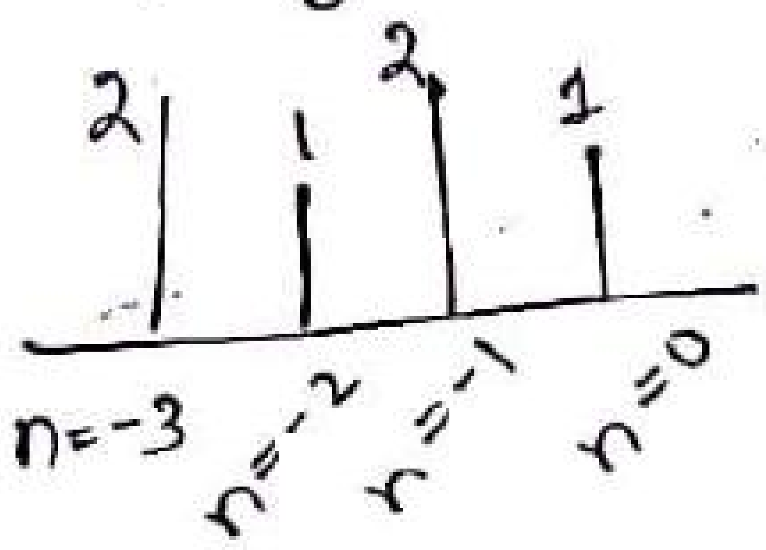
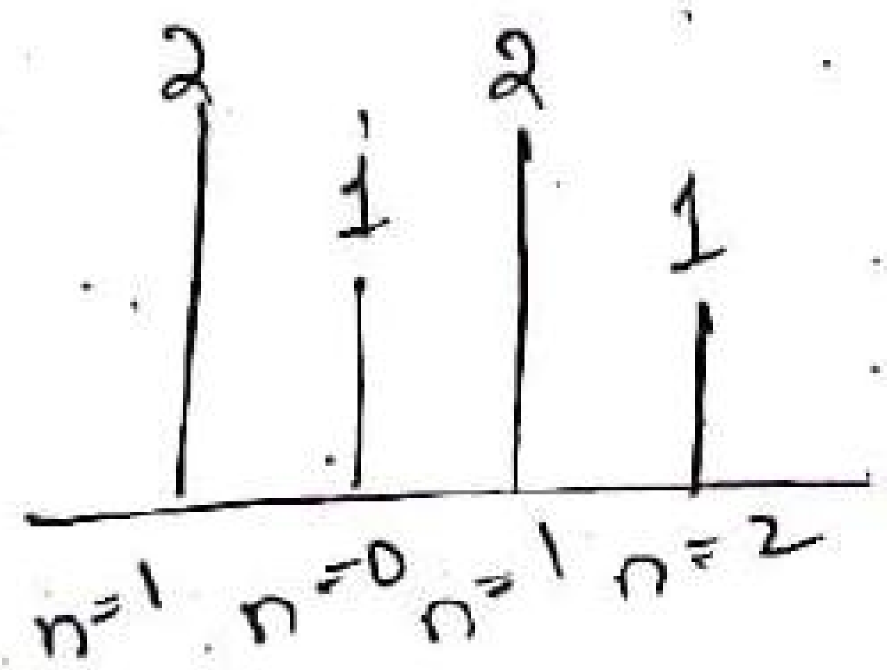
Shifting of $h(-k)$



$$y(-1) = x(k)\,h(-1-k)$$
$$= 3 \cdot 1$$
$$= 3$$



$n=1$

$h(n-k)$

$h(\cancel{n}-k)$

$k=1$

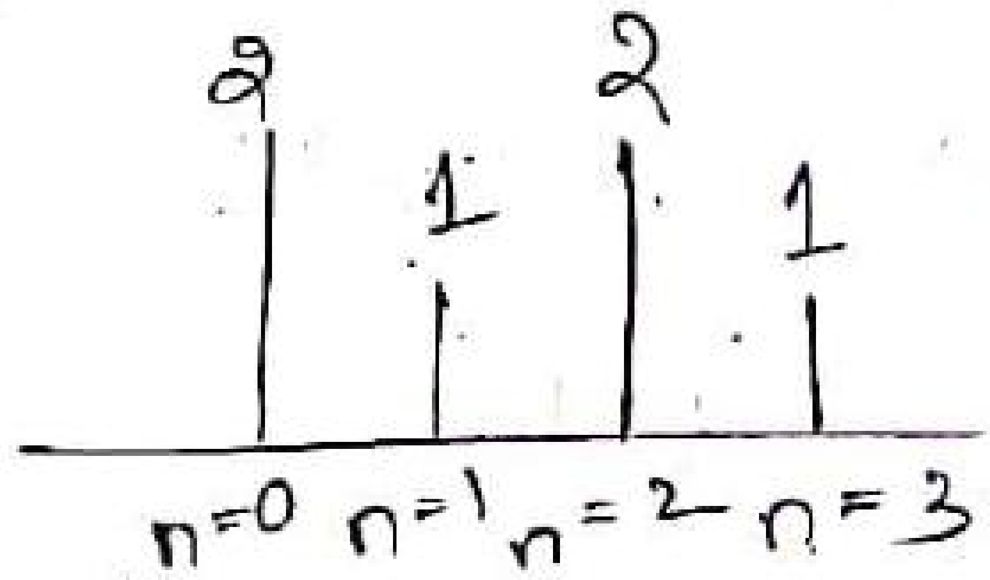
$$y(1) = x(k)\,h(-k)$$
$$= 3+4+1$$
$$= 8$$

$n=2$

$h(n-k)$

$h(2-k)$

$k=2$



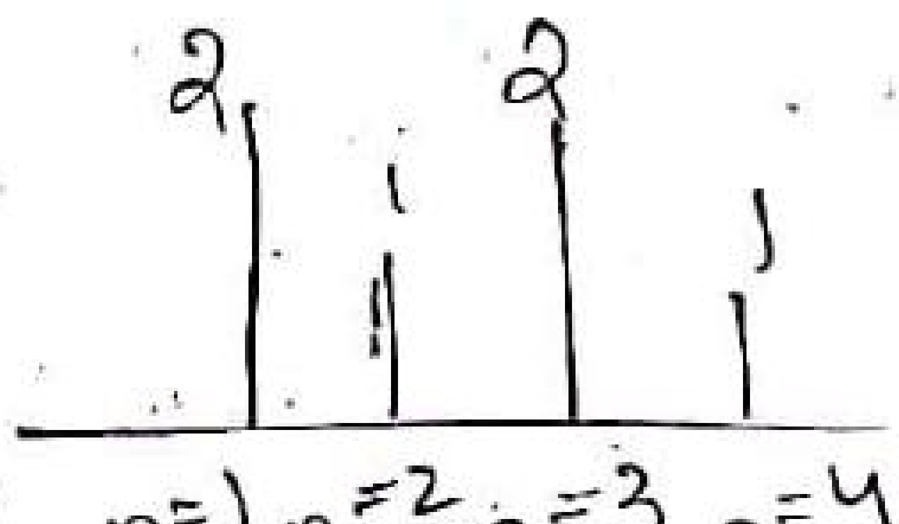
$$y(2) = x(k)\,h(2-k)$$
$$= 6+2+2+2$$
$$= 12$$

$n=3$

$h(n-k)$

$h(3-k)$

$k=3$



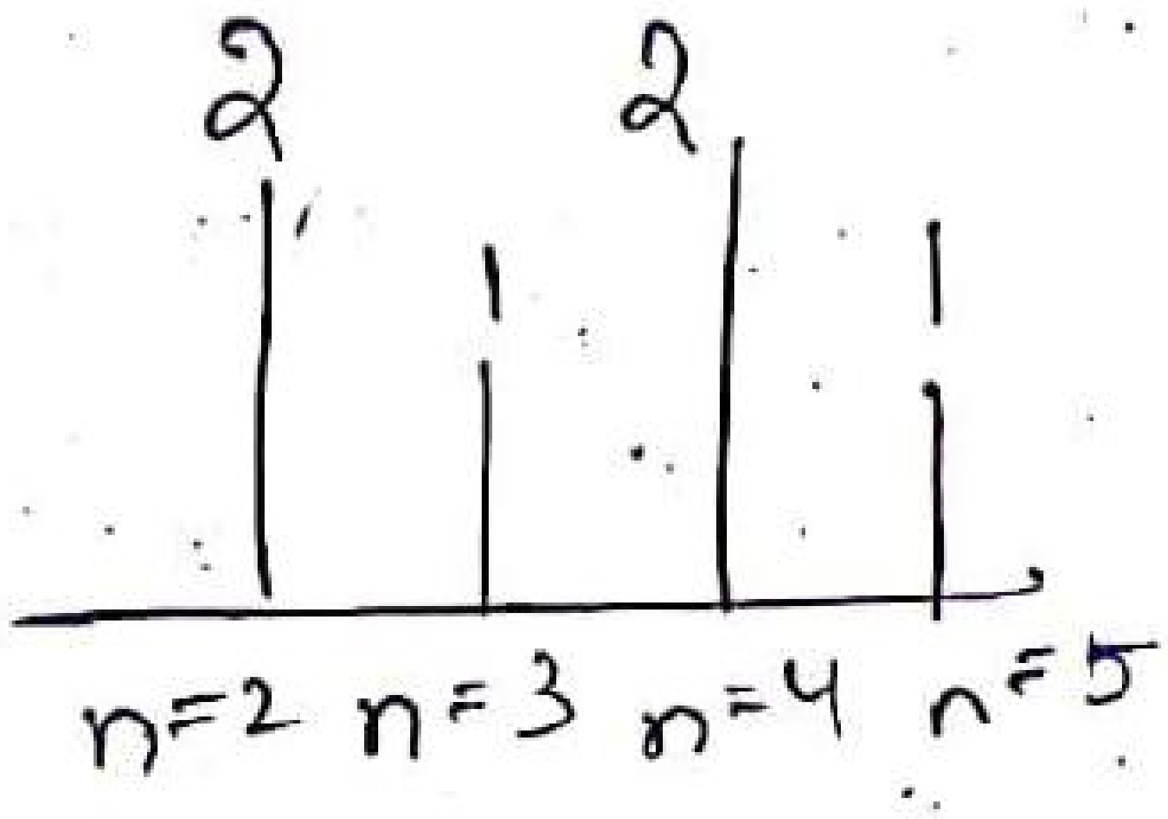
$$y(3) = x(k)\,h(n-k)$$
$$= 4+1+4$$
$$= 9$$

$\underline{n=4}$

$h(n-k)$

$h(4-k)$

$k=4$



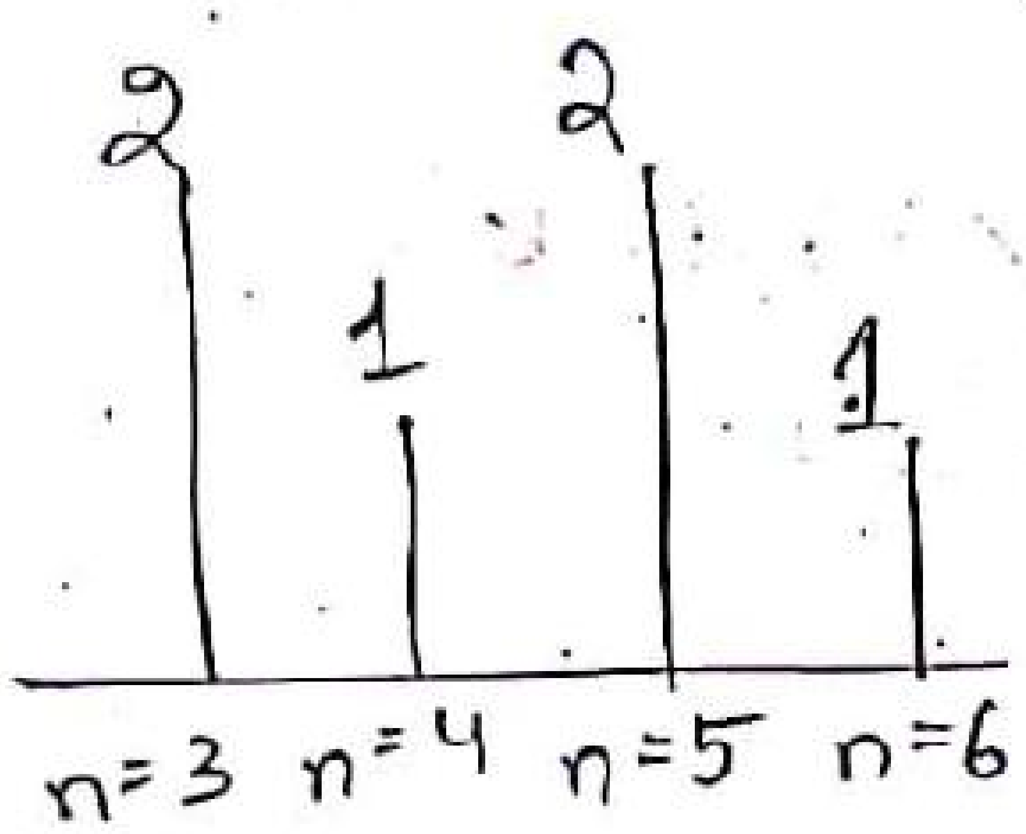
$$y(4) = x(k)\,h(n-k)$$
$$= 2+2$$
$$= 4$$

$\underline{n=5}$

$h(n-k)$

$h(5-k)$

$k=5$



$$y(5) = x(k)\,h(n-k)$$
$$=4$$

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Properties of convolution :-

(i) commutative law
(ii) Associative law
(iii) distributive law.

COMMUTATIVE LAW :-

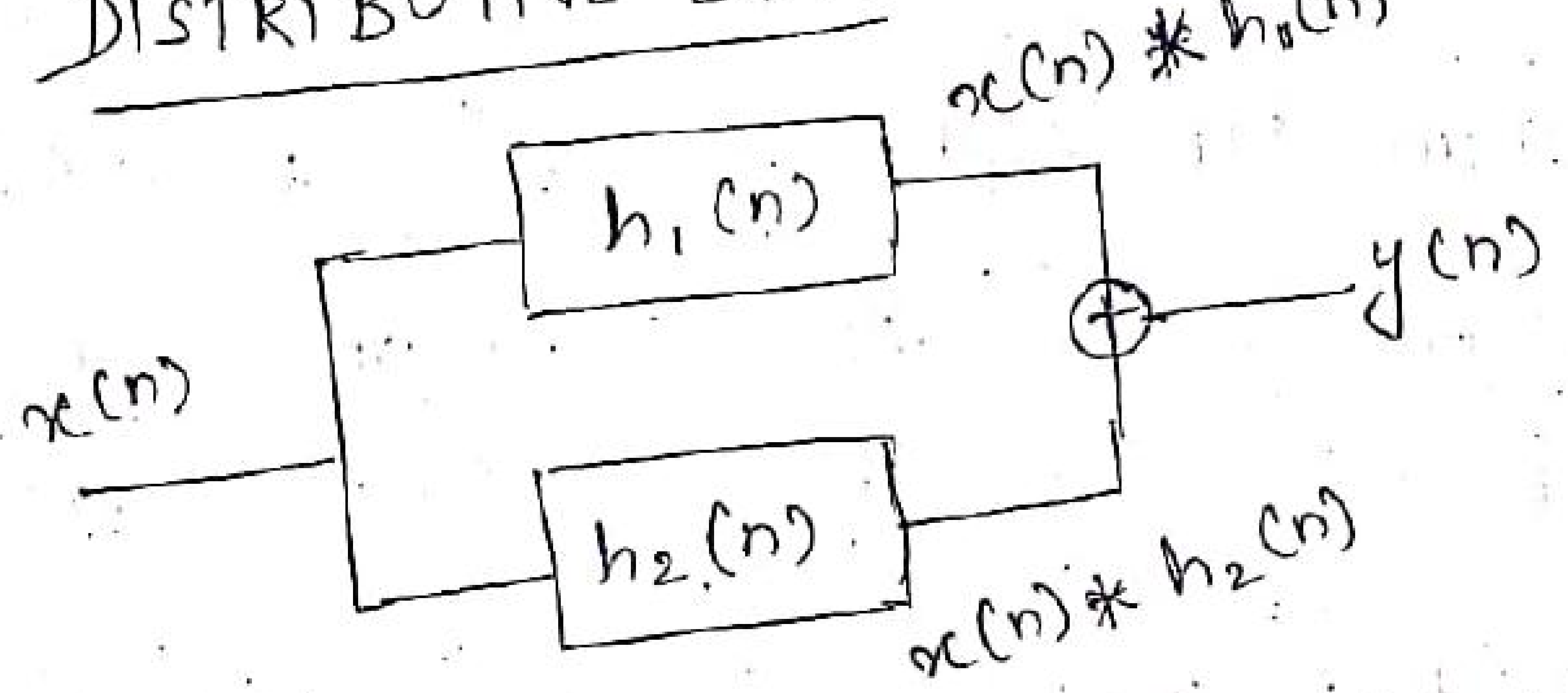
$$x(n) * h(n) = h(n) * x(n).$$

$$\sum x(k)\, h(n-k) = \sum h(k)\, x(n-k).$$

ASSOCIATIVE LAW :-

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

DISTRIBUTIVE LAW :-



$$x(n) * h_1(n) + x(n) * h_2(n) = x(n)[h_1(n) + h_2(n)]$$

Interconnections of LTI system :-

1. cascade Interconnections
II. parallel
III. series - parallel. Interconnections.

INFINITE DURATION IMPULSE RESPONSE :-

→ According to the duration of Impulse response LTI System can be classified into two types.

1. Finite impulse Response System

— If the Impulse response sequence is of finite durations.

Example :-

$$h(n) = \begin{cases} 1, & n=0 \\ 2, & n=3 \end{cases}$$

→ For causal FIR system $h(n) = 0$ for $n < 0$

The system whose Impulse response is infinite is known as infinite impulse.

RECURSIVE AND NON- RECURSIVE SYSTEM :-

Recursive System :-

A system whose present output $y(n)$ depends on any number of past output values is called as Recursive System.

Example :- $y(n) = F[x(n), y(n-1), y(n-2)]$

NON - RECURSIVE SYSTEM :-

A system whose present output $y(n)$ depends on present and past values of Input signal is known as Non- Recursive system.

SOLUTION OF LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATION :-

we can solve it by two methods :-

1. Direct method.
2. Indirect method.

DIRECT METHOD :-

In this method, the solution of difference equations consists of two parts.

1. Homogeneous part - $y_h(n)$
2. particular Integral part - $y_p(n)$

$$\boxed{y(n) = y_h(n) + y_p(n)}$$

INDIRECT METHOD :-

In this method, Z transform is used to solve the difference equations.

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HOMOGENEOUS SOLUTION OF A DIFFERENCE EQUATION :-

Homogeneous solutions is obtained by putting $x(n) = 0$ so, that,

$$\sum_{K=0}^{N} a_K \, y(n-k) = 0 \quad \text{——— (I)}$$

Assume that the solutions of this equations is in Exponential form

$$y_h(n) = \lambda^n \quad \text{——— (II)}$$

putting equation (II) in equation (I)

$$\sum_{K=0}^{N} a_K \, \lambda^{(n-K)} = 0$$

$$= a_0 \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} \ldots \ldots a_N \lambda^{n-N}$$

$$= \lambda^{n-N} \left[a_0 \lambda^N + a_1 \lambda^{N-1} + a_2 \lambda^{N-2} \ldots \ldots + a_n \right] = 0$$

The polynomial in the bracket is known as characteristic equations

Case-1 :- ROOTS ARE DISTINCT :

If roots are $\lambda_1, \lambda_2, \lambda_3 \ldots \ldots \lambda_n$. The general solutions is in the form ~~$y_h(n)$~~

$$y_h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n + \cdots C_n \lambda_N^n$$

where, $C_1, C_2, C_3 \ldots$ are weighting coefficient

Case-2 :- REPEATED ROOTS

let λ_1 is repeated for m times so, the general solutions $y_h(n) = \lambda_1 [C_1 + C_2 n$

Case-3 :- COMPLEX ROOTS

If the roots are complex conjugate

$$\lambda_1 = a + ib$$

$$\lambda_2 = a - ib$$

Then, the solutions $y_h(n) = r^n (\lambda_1^n \cos n\theta + \lambda_2^n \sin n\theta)$

here $r = \sqrt{a^2+b^2}$

$$\boxed{\theta = \tan^{-1} b/a}$$

PARTICULAR SOLUTION :-

This can be obtained by assuming a form that depends on the input $x(n)$. There are some general structure given below.

$x(n)$

$A \rightarrow K$

$AM^n \rightarrow KM^n$

$An^M \rightarrow K_0 n^m + K_1 n^{m-1} + \cdots$

$A^n n^m \rightarrow A^n [K_0 n^m + K_1 n^{m-1} + \cdots]$

$A\cos\omega_0 n + A\sin\omega_0 n \longrightarrow K_1\cos\omega_0 n + K_2\sin\omega_0 n$

Q. Determine the homogeneous solutions of the system described by the first order differential equation is $y(n) + a_1 y(n-1) = x(n)$

$y_h(n) = y(n) + a_1 y(n-1) = 0$ —— (i)

$\Rightarrow \lambda^n + a_1\lambda^{n-1} = 0$

$\Rightarrow \lambda^{n-1}(\lambda^n + a_1) = 0$

$\Rightarrow \lambda = -a_1$

Case - 3 :- COMPLEX ROOTS

If the roots are complex conjugate

$\lambda_1 = a + ib$

$\lambda_2 = a - ib$

Then, the solution $y_h(n) = r^n(\lambda_1^n \cos n\theta + \lambda_2^n \sin n\theta)$

here $r = \sqrt{a^2+b^2}$

$$\boxed{\theta = \tan^{-1} b/a}$$

$y_h(n) = C_1\lambda^n$

$\Rightarrow y_h(n) = C_1(-a_1)^n$ —— (ii)

value of C_1 in $n=0$ in equation (ii)

$$\boxed{y(0) = C_1 \times 1 = C_1}$$ —— (iii)

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put $n=0$ in equations ①,

$$y(0) + a_1 y(-1) = 0$$

$$y(0) = -a_1 y(-1)$$

Since, $y_0 = C_1$ (from eq^n (iii))

$$\boxed{C_1 = -a_1 y(-1)}$$

$$y_h(n) = C_1 (\lambda)^n$$

$$y_h(n) = C_1 (-a_1)^n$$

$$y_h(n) = (-a_1)^1 y(-1)(-a_1)^n$$

$$\boxed{y_h(n) = (-a_1)^{n+1} y(-1)}$$

Q. Determine the impulse response $h(n)$ for the system described by the difference equations

$$y(n) - 2y(n-1) = x(n) + x(n-1)$$

$$y_h(n) = y(n) - 2y(n-1) = 0 \quad \text{——— ①}$$

putting $\lambda^n = y(n)$ $\lambda^n - 2\lambda^{n-1} = 0$

$$\Rightarrow \lambda^{n-1}(\lambda - 2) = 0$$

$$\boxed{\lambda = 2}$$

$$y_h(n) = C_1 \lambda^n$$

$$\Rightarrow y_h(n) = C_1 2^n \quad \text{——— (ii)}$$

putting $n=0$ in equations (iii)

$$\boxed{y(0) = C_1}$$

Putting $n=0$ in equations ①

$$y(n) - 2y(n-1) = 0$$

$$y(0) = 2y(-1).$$

$$\boxed{C_1 = y(0) = 2y(-1)}$$

$$\Rightarrow \boxed{y_h(n) = C_1 2^n = 2y(-1)2^n}$$

Q. Consider a causal stable system whose input is $x(n)$ and output is $y(n)$ are related by the difference equations.

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n) \text{ ———— ①}$$

Then find the step response of the system:-

The total solution $y(n)$
$= y_p(n) + y_h(n)$

Step response of the system
$y(n) = y_p(n) + y_h(n)$

As the input is step-input. $y_p(n) = K$.
So, that the difference equation is written as,

$$K - \frac{1}{6}(K-1) - \frac{1}{6}(K-2)$$

$$\Rightarrow K - \frac{1}{6}(K) - \frac{1}{6}(K) = 1$$

$$\Rightarrow \frac{6K - K - K}{6} = 1$$

$\Rightarrow \frac{4K}{6} = 1$

$\Rightarrow K = \frac{6}{4} = \frac{3}{2} = 1.5$

For homogeneous solutions

$y(n) - \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = 0$

$\Rightarrow \lambda^n - \frac{1}{6}\lambda^{n-1} - \frac{1}{6}\lambda^{n-2} = 0$

$\Rightarrow \lambda^{n-2}\left[\lambda^2 - \frac{1}{6}\lambda^1 - \frac{1}{6}\right] = 0$

$\Rightarrow \lambda^2 - \frac{1}{6}\lambda - \frac{1}{6} = 0$

$\Rightarrow \frac{6\lambda^2 - \lambda - 1}{6} = 0$

$\Rightarrow 6\lambda^2 - \lambda - 1 = 0$

$\Rightarrow 6\lambda^2 - 3\lambda + 2\lambda - 1 = 0$

$\Rightarrow 3\lambda(2\lambda - 1) + 1(2\lambda - 1) = 0$

$\Rightarrow (3\lambda + 1)(2\lambda - 1) = 0$

$\Rightarrow \boxed{\lambda_2 = -\frac{1}{3}}$ or $\boxed{\lambda_1 = \frac{1}{2}}$

$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n$

$= C_1\left(\frac{1}{2}\right)^n + C_2\left(-\frac{1}{3}\right)^n$ —— (11)

According to case-I of homogeneous solution putting $n=0$ in equation (ii)

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{3}\right)^n$$

$$\Rightarrow y(0) = c_1\left(\frac{1}{2}\right)^0 + c_2\left(-\frac{1}{3}\right)^0$$

$$\Rightarrow y(0) = c_1 + c_2 \quad \text{———} (iii)$$

putting $n=0$ in equation (i),

$$y(n) - \frac{1}{6}\, y(\cancel{n}^{0}-1) - \frac{1}{6}\, y(\cancel{n}^{0}-2)$$

$$y(0) - \frac{1}{6}\, y(-1) - \frac{1}{6}\, y(-2) = 1$$

$$y(0) = 1 \quad \text{———} (iv)$$

comparing equation (iii) & (iv), we get,

$$c_1 + c_2 = 1 \quad \text{———} (v)$$

putting $n=1$ in equation (i),

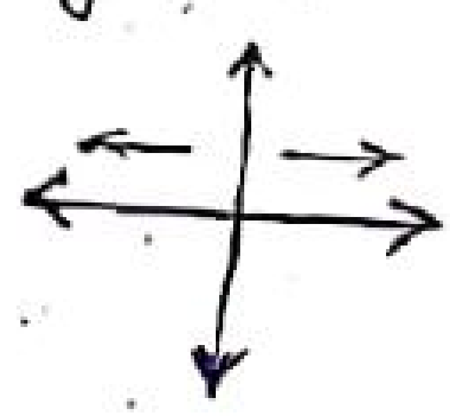
$$y(n) - \frac{1}{6}\, y(n-1) - \frac{1}{6}\, y(n-2)$$

$$\Rightarrow y(1) - \frac{1}{6}\, y(0) - \frac{1}{6}\, y(\cancel{-1}^{0}) = 1$$

$$\Rightarrow y(1) - \frac{1}{6}\times 1 = 1$$

$$\Rightarrow y(1) = \frac{1}{6}\cancel{\times} + 1 = \frac{7}{6} \quad \text{———} (vi)$$

causal system presence of only +ve value



and act the -ve values will be zero

putting $n=1$ in equation (ii),

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{3}\right)^n$$

$$\Rightarrow y(1) = c_1\left(\frac{1}{2}\right)^1 + c_2\left(-\frac{1}{3}\right)^1$$

$$\Rightarrow y(1) = c_1\left(\frac{1}{2}\right)^1 \phi - \frac{1}{3} c_2 \quad \text{———} (VII).$$

comparing equations (VI) & (VII),

$$c_1\left(\frac{1}{2}\right) - c_2\left(\frac{1}{3}\right) = 7/6$$

comparing equation (V) and (VII), we get,

$$c_1 + c_2 = 1 \quad \text{———} (V)$$

$$\frac{1}{2} c_1 - \frac{1}{3} c_2 = 7/6 \quad \text{———} (VIII)$$

Multiplying 1/2 in equation (V), we get

$$\frac{1}{2}\cancel{c_1} + \frac{1}{2} c_2 = \frac{1}{2}$$

$$\frac{1}{2}\cancel{c_1} - \frac{1}{3} c_2 = \frac{7}{6}$$

$$\frac{1}{2}\cancel{c_2} - \frac{1}{3}\cancel{c_2} = c_1 = 9/5$$

$$c_2 = -4/5$$

putting the value of c_1 and c_2 in equation (II),

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{3}\right)^n$$

$$= \frac{9}{5}\left(\frac{1}{2}\right)^n - \frac{4}{5}\left(\frac{1}{3}\right)^n$$

$$y(n) = y_p(n) + y_h(n)$$

$$= \frac{3}{2} + \left(\frac{9}{5}\right)\left(\frac{1}{2}\right)^n - \frac{4}{5}\left(-\frac{1}{3}\right)^n$$

MODEL QUESTIONS / PREVIOUS YEAR Q

① What are the Elementary discrete time signal ? L

② What are the different representations style of discrete time signal ? L

③ Define Recursive and Non-recursive discrete time system?

④ Find the convolution between the sequence

$$x(n) = \begin{cases} 1, & \text{for } n = 2, 0, 1 \\ 2, & \text{for } n = -1 \\ 0, & \text{Else where} \end{cases}$$

$$h(n) = \delta(n) - \delta(n-1) - \delta(n-2) + \delta(n+1)$$

$$x(n) = \left\{ \frac{1}{n=-2}, \frac{2}{n=-1}, \underset{\uparrow}{\frac{1}{n=0}}, \frac{1}{n=1} \right\}$$

$$h(n) = \left\{ \frac{1}{n=-1}, \underset{\uparrow}{\frac{1}{n=0}}, \frac{-1}{n=1}, \frac{-1}{n=2} \right\}$$

Q. Defin energy signal and power signal

Q. check whether the signal $x(n) = \left(\frac{1}{3}\right)^n u(n)$ is a Energy signal or power signal.

Q Define Even signal and odd signal.

Q what are the properties of convolution,

Q. Define Stable system ?

Q Describe the parallel connections of system

Q If signal $x(n) = \{1, 2, 6, 4, 3, 7, 5\}$ then find $x(2n)$, $x\left(\frac{n}{2}\right)$, $x(n+2)$, $x(3-n)$, $3x(n-1)$.

Q Define pole and zero.

Q ~~Define Recursion~~

Q Find the convolutions sum of the given sequence

$$x(n) = \{1, 2, \underset{\uparrow n=0}{3}, 4\}$$

$$h(n) = \{1, \underset{\uparrow n=0}{2}, 1, 2\}$$

Q Find the homogeneous solution and particular equation of a difference equation a

$$y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n).$$

Q classify discrete time system

Q Define causal and non-causal system?

Q Determine the system described by equations

$y(n) = x(n) + \frac{1}{x(n)}$ is linear or non-linear.

Q Find the Energy and power of unit step signal?

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \qquad P = \frac{1}{\lim_{N\to\infty} 2N+1} \sum^{N} |x(n)|^2$$

Z- Transform :-

— z-transform is used to analyse the LTI analog discrete time signal.

— z-transform converts the discrete time signal into frequency domain

→ It is used for both stable and unstable system.

→ In z-domain the convolution of 2-sequence is ~~equal into~~ equivalent to multiplications of their corresponding z-transform.

Direct z-transform :-

→ Z-transform of a discrete time signal $x(n)$ can be written as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{—— (i)}$$

where z- is a complex variable.

→ Z-transform of a signal $x(n)$ is denoted by

$$X(z) = Z[x(n)]$$

$$X(z) \overset{z}{\longleftrightarrow} x(n)$$

equation (i) refers to two sided z-transform

→ If $x(n)$ is causal then $x(n) = 0$ for $n < 0$ and the z-transform can be written as

$$\boxed{X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}} \quad \text{—— (ii)}$$

→ If $x(n)$ is causal then

→ The z-transform Exist for those values of for which series converse

Example

Q. Find the z-transform of the given sequence

$$x(n) = \{1, 2, 3, \underset{\substack{\uparrow \\ n=0}}{4}, 1, 2\}$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-3}^{2} x(n) z^{-n}$$

$$= x(-3) z^{-(-3)} + x(-2) z^{-(-2)} + x(-1) z^{-(-1)}$$
$$+ x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2}$$

$$\boxed{x(z) = 1z^3 + 2z^2 + 3z^1 + 4 + 1z^{-1} + 2z^{-2}}$$

ROC (Region of Convergence):-

→ The region of convergene of $x(z)$ is the set of all values of z for which $x(z)$ attains a finite value.

properties:-

→ ROC is a Concentric ring or a circular disk in a z-plane centered at origin.

→ ROC does not contain any pole

→ If $x(n)$ is causal sequence then the ROC is Entire z-plane, Except $z=0$.

→ If $x(n)$ is two sided sequence then ROC is Entire z-plane Except $z=0$, and ∞.

→ If $x(n)$ is non-causal sequence then ROC is Entired z-plane Except $z=\infty$.

→ If $x(n)$ is an infinite duration two sided sequence then ROC consist of a circular ring in the z-plane bounded on the Enterior or Exterior by a pole but does not contain any pole.

→ The ROC of LTI stable system contains the unit circle

INVERSE Z-TRANSFORM :-

→ The process used for transforming the z-domain signal into time domain signal is known as inverse z-transform.

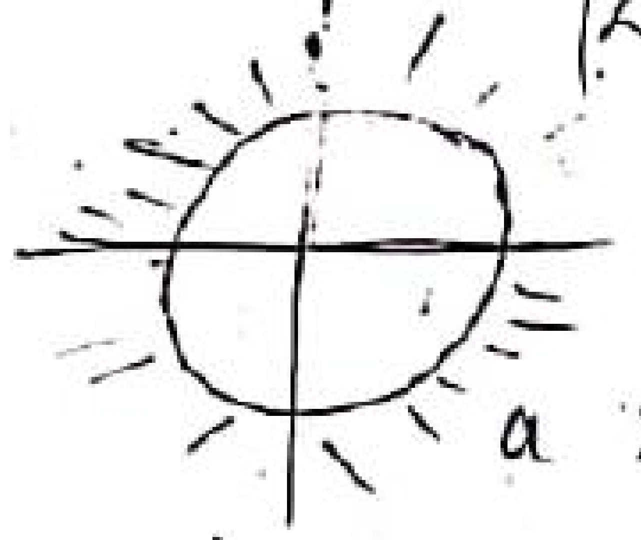
I.V.I PROBLEMS ON INFINITE DURATION SEQUENCE :-

Q Find the z-transform and ROC of $x(n) = a^n u(n)$

Soln:- $x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$

$\Rightarrow \sum_{n=0}^{\infty} a^n . 1 . z^{-n}$



$\Rightarrow \sum_{n=0}^{\infty} a^n z^{-n}$

$\Rightarrow \sum_{n=0}^{\infty} (a z^{-1})^n$

$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{1-az^{-1}}$

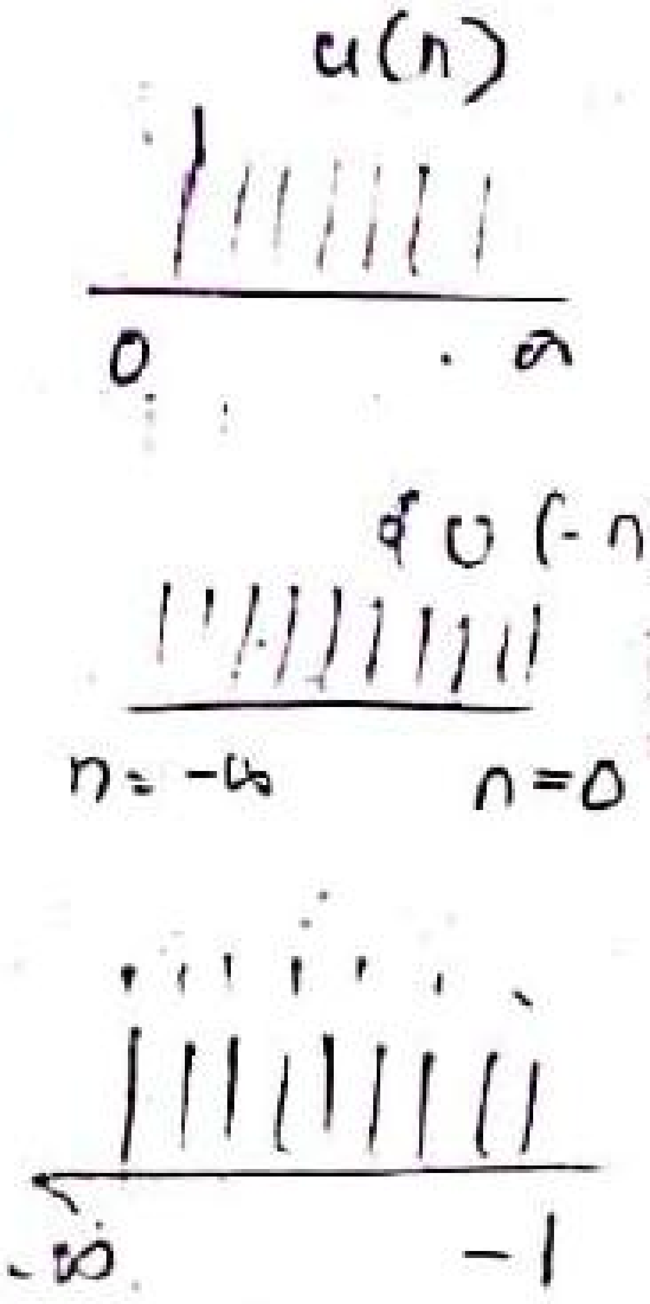
$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{1-\frac{a}{z}} \Rightarrow \frac{1}{\frac{z-a}{z}} \Rightarrow \frac{z}{z-a}$

$\therefore \boxed{x(z) = \frac{z}{z-a}}$ $\boxed{ROC : |z| > |a|}$

Q. Find the z-transform and ROC of the signal

$$x(n) = -b^n u(-n-1)$$

Sol^n:- $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$



$$= \sum_{n=-\infty}^{\infty} -b^n u(-n-1) z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -b^n \, 1 \, z^{-n}$$

$$\Rightarrow \sum_{n=-\infty}^{-1} -b^n z^{-n}$$

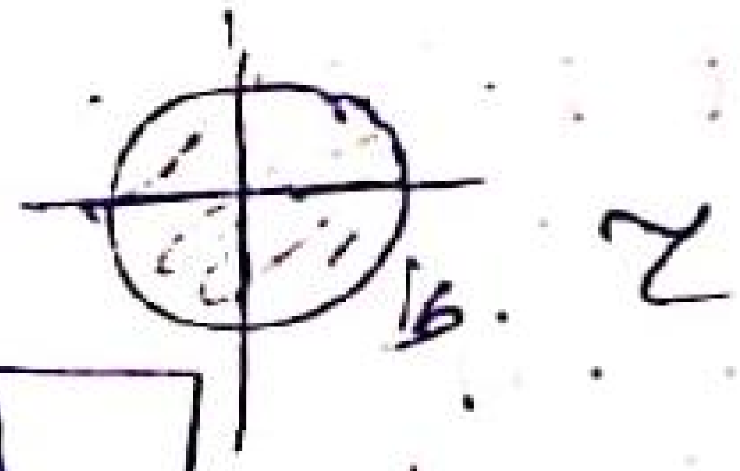
$$\Rightarrow \sum_{n=-\infty}^{-1} (-b z^{-1})^n \Rightarrow \sum_{n=1}^{\infty} (b^{-n} z^n)$$

$$\Rightarrow \sum_{n=1}^{\infty} (b^{-1} z)^n$$

$$\Rightarrow \sum_{n=0}^{\infty} (b^{-1} z)^n - 1 \Rightarrow \frac{1}{1-b^{-1}z} - 1$$

$$\Rightarrow \frac{1}{1-\frac{z}{b}} - 1 \Rightarrow \frac{1}{\frac{b-z}{b}} - 1$$

$$\Rightarrow \frac{b}{b-z} - 1 \Rightarrow \frac{b-b+z}{b-z} = \frac{z}{b-z}$$

$$x(z) = \frac{z}{b - z}$$ 

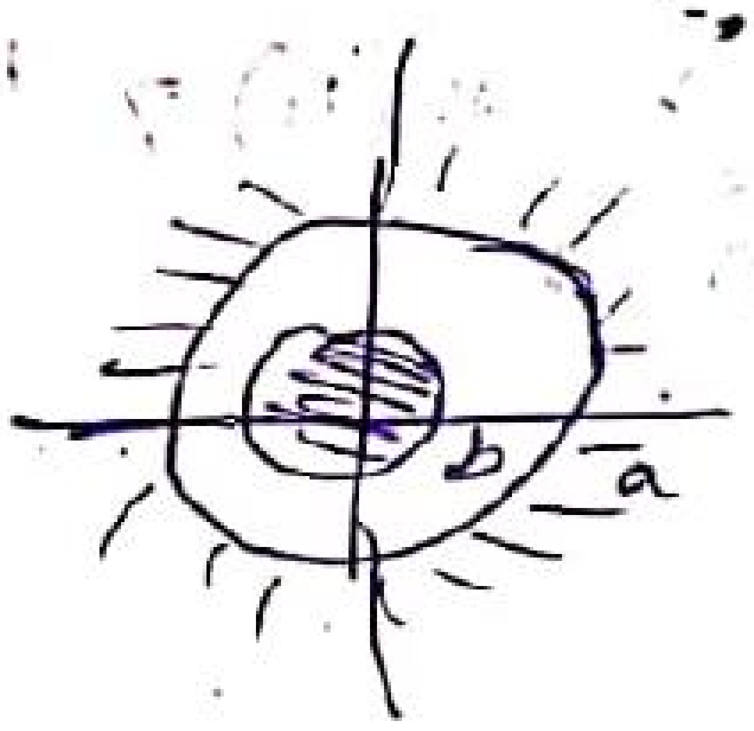
$\boxed{ROC: |z| < |b|}$

find the z-transform of ROC $x(n) =$

$$x(n) = a^n u(n) + [-b^n u(-n-1)]$$

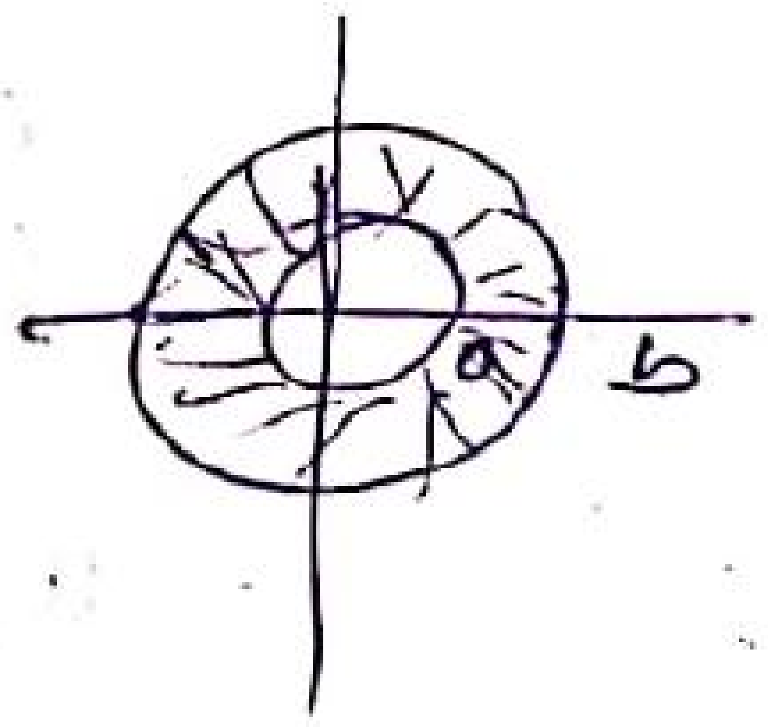
$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\cancel{x(z) = \sum_{n=-\infty}^{\infty} a^n u(n) +}$$



$$x(z) = \frac{z}{z-a} + \frac{z}{b-z}$$

$$= \frac{z}{z-a} - \frac{z}{z-b}$$



$a < |z| < b$

$a < b$

Dt → 10.2.20

1. Linearity

$$x_1(n) \xleftrightarrow{z} X_1(z)$$

$$x_2(n) \xleftrightarrow{z} X_2(z)$$

$$[a_1 x_1(n) + a_2 x_2(n)] \longleftrightarrow a_1 X_1(z) + a_2 X_2(z)$$

Proof:-

$$\sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} [a_1 x_1(n) + a_2 x_2(n)] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [a_1 x_1(n) z^{-n} + a_2 x_2(n) z^{-n}]$$

$$= \sum_{n=-\infty}^{\infty} a_1 x_1(n) z^{-n} + \sum_{n=-\infty}^{\infty} a_2 x_2(n) z^{-n}$$

$$= a_1 X_1(z) + a_2 X_2(z) \text{ proved}$$

Multiplication by an Exponential Sequence :-

$$x(n) \xleftrightarrow{z} X(z).$$

$$a^n x(n) \xleftrightarrow{z} X(a^{-1} z)$$

Proof: $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) (a^{-1} z)^{-n} = X(a^{-1} z)$$

proved

Time reversal :-

$$x(n) \xleftrightarrow{z} X(z)$$

$$x(-n) \longleftrightarrow X(z^{-1})$$

proof:- $\sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$

$= \sum_{n=-\infty}^{\infty} x(l) z^{l}$

$= \sum_{n=-\infty}^{\infty} x(l) (z^{-1})^{l}$

$= X(z^{-1})$ proved

Inverse transform :-

The process through which $X(z)$ is converted back to $x(n)$. This can be done by the following methods.

I. Long division method
II. partial fraction method
III. Residue method
IV. Convolution method

Partial fraction method:

Find the inverse z-transform of

$$x(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}, \text{ ROC} = |z| > 2$$

Proof:-

multiplying z^2 up and down

$$x(z) = \frac{z^2+3z}{z^2+3z+2}$$

$$x(z) = \frac{z(z+3)}{z^2+3z+2}$$

$$\Rightarrow \frac{x(z)}{z} = \frac{z+3}{z^2+3z+2}$$

$$\Rightarrow \frac{x(z)}{z} = \frac{z+3}{z^2+2z+z+2}$$

$$= \frac{z+3}{z(z+2)+1(z+2)}$$

$$\Rightarrow \frac{x(z)}{z} = \frac{z+3}{(z+2)(z+1)}$$

$$\frac{z+3}{(z+2)(z+1)} = \frac{A}{z+2} + \frac{B}{z+1}$$

$$A = \frac{z+3}{(z+2)(z+1)} \times (z+2) \Big|_{z=-2}$$

$$= \frac{-2+3}{-2+1} = \frac{1}{-1} = -1$$

$$B = \frac{z+3}{(z+2)(z+1)} \times (z+1) \Big|_{z=-1}$$

$$= \frac{-1+3}{-1+2} = \frac{2}{1} = 2$$

$$\frac{X(z)}{z} = \frac{-1}{z+2} + \frac{2}{z+1}$$

$$\boxed{\begin{array}{l} a^n u(n) = \frac{z}{z-a} \\ \frac{z}{z+a} = (-a)^n u(n) \end{array}}$$

$$\Rightarrow X(z) = \frac{-z}{z+2} + \frac{2}{z+1}$$

$$= \frac{2z}{z+1} - \frac{z}{z+2}$$

$$\boxed{2(-1)^n u(n) - (-2)^n u(n) = x(n)}$$

Properties of z-transform :-

Differentiation in z-Domain :-

$$x(n) \xleftrightarrow{z} X(z)$$

then, $n\,x(n) \xleftrightarrow{z} -z \frac{dX(z)}{dz}$

Proof :-

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Differentiating above on both side with respect to z.

$$\frac{d x(z)}{dz} = \frac{d}{dz}\left\{\sum_{n=-\infty}^{\infty} x(n) z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{d}{dz}\left\{z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} -n\, x(n)\, z^{-n-1}$$

$$-z \frac{d x(z)}{dz} = \sum_{n=-\infty}^{\infty} n\, x(n)\, z^{-n}$$

comparing both equations $-z \frac{d x(z)}{dz}$ is the z-transform of $n x(n)$

11. Parseval's Theorem :-

$$\sum_{n=-\infty}^{\infty} x(n)\, x^{*}(n) = \int_{-\pi}^{\pi} f(z)\, f^{R}(z)\, dz$$

Parseval's relation tells us that the Energy of a signal is equal to the Energy of its fourier transform

TIME SHIFTING :-

If $x(n) \xleftrightarrow{z} x(z)$ with ROC = R then

$$x(n-m) \longleftrightarrow z^{-m} x(z) \text{ with ROC} = R$$

proof :- $Z\left[x(n-m)\right] = \sum_{n=-\infty}^{\infty} x(n-m) z^{-n}$

Let $n-m = p$

$$= \sum_{n=-\infty}^{\infty} x(p) z^{-(p+m)}$$

$$= z^{-m} \sum_{n=-\infty}^{\infty} x(p) z^{p} = z^{-m} x(z)$$

IV. Convolution property :-

$x_1(n) \xleftrightarrow{z} y_1(z)$ with ROC = R_1

$x_2(n) \xleftrightarrow{z} x_2(z)$ with ROC = R_2

then $x_1(n) * x_2(n) \xleftrightarrow{z} x_1(z) \cdot x_2(z)$ with ROC containing $R_1 R_2$.

Proof :-

$$Z\left[x_1(n) * x_2(n)\right] = \sum_{n=-\infty}^{\infty} \left\{x_1(n) * x_2(n)\right\} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left\{ \sum_{m=-\infty}^{\infty} x_1(m) \cdot x_2(n-m) \right\} z^{-m}$$

Interchanging the order of Σ

$$z\left[x_1(n) * x_2(n)\right] = \sum_{n=-\infty}^{\infty} x_1(m) \left\{ \sum_{n=-\infty}^{\infty} x(n-m) z^{-n} \right\}$$

$$= \sum_{m=-\infty}^{\infty} x_1(m) \left\{ z^{-m} x_2(z) \right\}$$

Since the time shifting property

$$x_2(z) \left\{ \sum_{m=-\infty}^{\infty} x_1(m) z^{-m} \right\} = x_1(z) \cdot x_2(z)$$

V. Initial value theorem :-

$$x_t(z) = Z[x(n)]$$

then $x(0) = \lim_{z\to\infty} x_t(z)$

VI Final value theorem :-

$$x_1(z) = Z[x(n)]$$

then $x(0) = \lim_{z\to 1} (z^{-1}) x_t(z)$

(1) Find the inverse z-transform of

$$x(z) = \frac{z(z^2-4z+5)}{(z-3)(z-1)(z-2)}$$

ROC : $2 < |z| < 3$

$|z| > 3$

$|z| < 1$

Solutions :- By partial fraction method

$$x(z) = \frac{z(z^2-4z+5)}{(z-3)(z-1)(z-2)}$$

$$\frac{x(z)}{z} = \frac{z^2-4z+5}{(z-3)(z-1)(z-2)} = \frac{A}{z+3} + \frac{B}{z-1} + \frac{C}{z}$$

$$A = \frac{z^2-4z+5}{\cancel{(z-3)}(z-1)(z-2)} \times \cancel{(z-3)} \Big|_{z=3}$$

$$= \frac{9-12+5}{2\cdot 1} = \frac{2}{2} = 1$$

$$B = \frac{z^2-4z+5}{(z-3)\cancel{(z-1)}(z-2)} \times \cancel{(z-1)} \Big|_{z=1}$$

$$= \frac{1-4+5}{-2\cdot -1} = \frac{2}{2} = 1$$

$$C = \frac{z^2-4z+5}{(z-3)(z-1)(z-2)} \times (z-2) \Big|_{z=2}$$

$$= \frac{4-8+5}{-1\cdot 1} = \frac{1}{-1} = -1$$

$$\frac{x(z)}{z} = \frac{1}{z-3} + \frac{1}{z-1} - \frac{1}{z-2}$$

$$x(z) = \frac{z}{z-3} + \frac{z}{z-1} - \frac{1}{z-2}$$

$$z\left[a^n u(n)\right] = \frac{z}{z-a}$$

$3^n u(n)$

$1^n u(n) = 1\cdot u(n) = u(n)$

$2^n u(n)$

Case-1:- for ROC : $2 < |z| < 3$

$$x(z) = \frac{z}{z-3} + \frac{z}{z-1} - \frac{z}{z-2}$$

$$x(n) = u(n) - 2^n u(n) + 3^n u(-n)$$

Case-2:- for ROC: $|z| > 3$

$$x(n) = u(n) - 2^n u(n) + 3^n u(-n)$$

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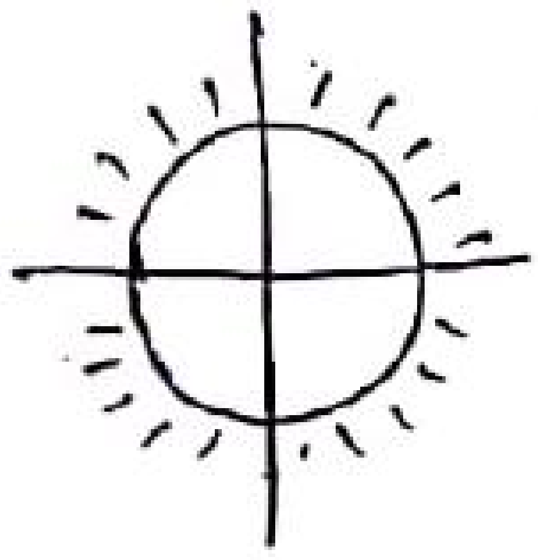
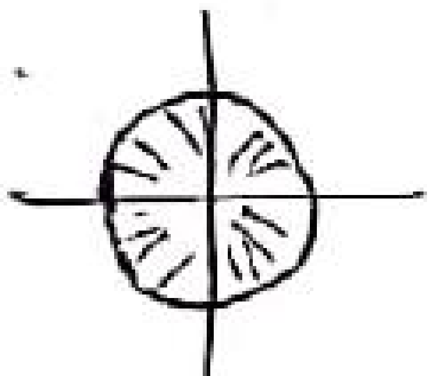
∴ causal sequence



Case-3 :- For ROC : $|z| < 1$

$$x(n) = -u(-n-1) - 2^n u(n) + 3^n u(n)$$

∴ Non - causal sequence



Q. Solve by long division method

$$x(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}} \quad \text{find } z^{-1}$$

causal sequence $= \dfrac{1}{1-2z^{-1}+z^{-2}} + \dfrac{2z^{-1}}{1-2z^{-1}+z^{-2}}$

$$1-2z^{-1}+z^{-2} \;\Big)\; 1 \;\Big(\; 1+2z^{-1}+3z^{-2}$$

$$\begin{array}{l} 1 - 2z^{-1} + z^{2} \\ - \quad + \\ \hline 2z^{-1} - 2z^{-2} \\ 2z^{-1} - 4z^{-2} + 2z^{-3} \\ - \quad\quad + \quad\quad - \\ \hline 3z^{-2} - 2z^{-3} \\ 3z^{2} - 2z^{-3} \\ \hline \end{array}$$

$$\{ \underset{\uparrow}{1}, 2, 3, 4 \}$$

$$1-2z^{-1}-1z^{-2} \,\big|\, 2z^{-1} \,\big|\, 2z^{-1}+4z^{-2}+6z^{-3}$$

$$-2z^{-1}-4z^{-2}+2z^{-3}$$
$$+ \quad -$$

$$4z^{-2}-2z^{-3}$$
$$\pm 4z^{-2} \mp 8z^{-3} \pm 4z^{-4}$$

$$6z^{-3}-4z^{-4}$$

$\{2,4,6,8\}$

or

$\{3,6,9,12...\}$

Non- causal sequence

$$z^{-2}-2z^{-1}+1 \,\big|\, 1 \,\big|\, z^{2}+2z^{3}$$

$$1-2z^{-1}+z^{2}$$
$$- \quad + \quad -$$

$$2z^{-1}-z^{2}$$
$$\underline{-}2z^{-1} \underset{+}{-} 4z^{2} \underset{-}{+} 2z^{3}$$

$$3z^{2}-2z^{3}$$

Pole :-

The value of 'z' for which x(z) becomes infinite is known as pole.

zero :-

The value of 'z' for which x(z) becomes zero is known as zero.

example :-

$$x(z) = \frac{(z-2)}{(z+3)(z+2)}$$

pole $= z+3=0$

$z=-3$

$z-2=0$

$z=2$

zero $= z-2=0$

$\Rightarrow z=2$

Sequence	Transform	ROC
$\delta(n)$	1	all value of z
$u(n)$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
$-u(-n-1)$	$\frac{z}{z-1}$	$\lvert z\rvert < 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$ or $\frac{z}{z-a}$	$\lvert z\rvert > a$
$-a^n u(n)$	$\frac{1}{1-az^{-1}}$ or $\frac{z}{z-a}$	$\lvert z\rvert < a$

Probable Semester Questions :-

(1) Define z- transform

(2) Define ROC and write down its Property

(3) write all the properties of z-transform with proof

(4) Find z-transform and ROC of $x(n) = a^n u(n) - b^n u(n)$

(5) what are the methods for inverse z- transform

(6) Find the z-transform and ROC of the signal

$x(n) = \{2, 4, 5, 7; 0, 1\}$

Starting point = -2

7. Find the inverse z-transform of

$$x(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}} \quad \begin{array}{l} \text{ROC } |z|>1 \\ |z|<0.5 \\ 0.5<|z|<1 \end{array}$$

8. Find the z-transform and Roc of
$x(n) = (n+0.5)\left(\frac{1}{3}\right)^n u(n)$

9. Find the z-transform and Roc of $x(n) = a^n u(n)$

10. Find the inverse z-transform of $x(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$

11. Find the z-transform and ROC of $x(A) = 2^n u(n) + 3^n u(-n-1)$

12. Determine the inverse z-transform of the sequence using Long division method $x(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$. If $z(n)$ is causal

24.02.20

4. FOURIER TRANSFORM

DISCRETE FOURIER TRANSFORM

> DFT is a powerful compatation tool which allows us to Evaluate the fourier transform on a digital computer or Specially designed hardware.

Fourier transform
$x(t) \xleftrightarrow{F.T} x(\omega)$
time domain — frequency domain

> DFT is used to convert $x(n)$ into $x(\omega)$

FREQUENCY DOMAIN SAMPLING AND RECONSTRUCTION OF DISCRETE TIME SIGNAL:-

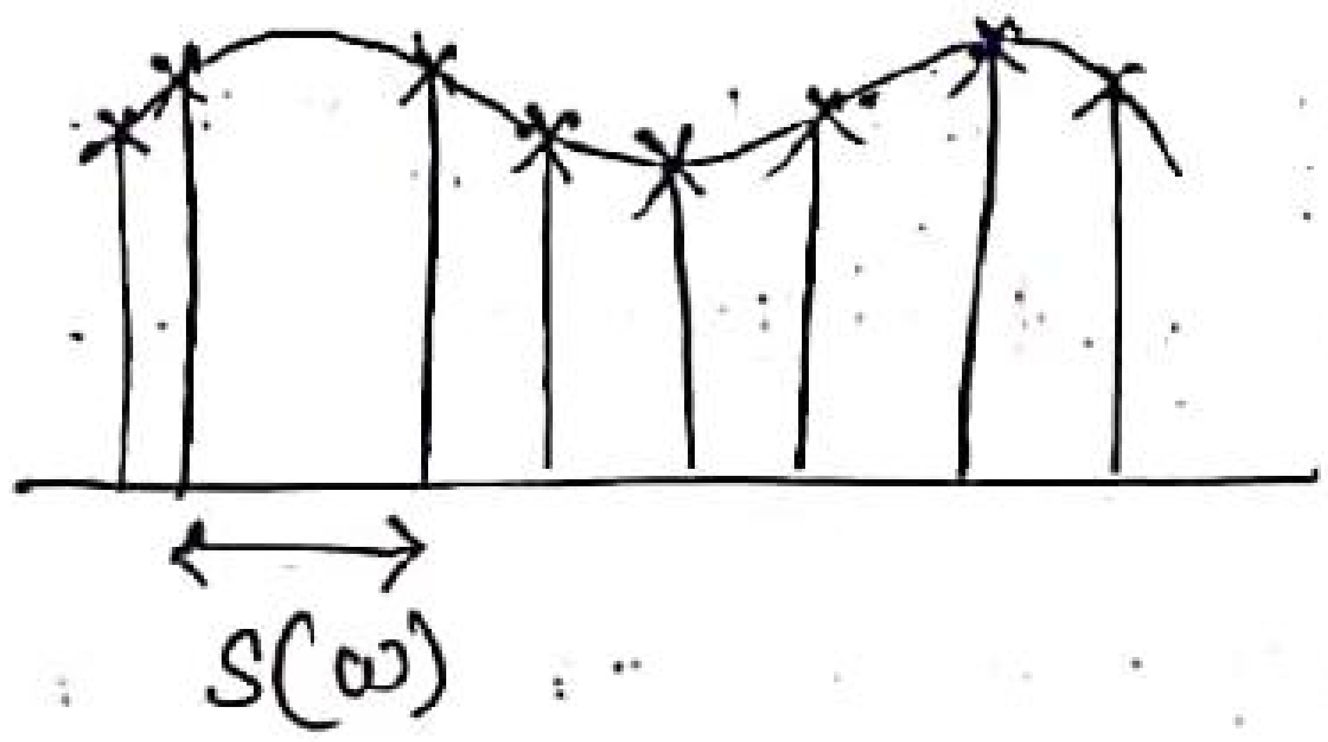
→ let us consider a periodic discrete time sequence $x(n)$ with fourier transform $x(\omega)$

$$\boxed{x(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}}$$

→ Suppose we sample periodically with a Spacing $S(\omega)$ between two sucessive samples.

$$\boxed{x(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}}$$

→ Suppose we sample periodically with a spacing $\delta(\omega)$ between two sucessive samples.



→ Since $x(\omega)$ is periodic with a period 2π so, that we require the samples only in the fundamental range.

→ we take 'N' number of samples in the interval $0-2\pi$ with a spacing,

$$\boxed{\delta(\omega) = \frac{2\pi}{N}}$$

→ If we evaluate $\boxed{\omega = \frac{2\pi k}{N}}$ then,

the equation becomes,

$$\boxed{x(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi k}{N} n}}$$

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It is used to convert the frequency domain signal back to time domain signal.

$$\boxed{x(n) = \sum_{k=0}^{N-1} x(\omega)\, e^{j\frac{2\pi nk}{N}}}$$

$$\Rightarrow \boxed{x(n) = \frac{1}{N}\sum_{K=0}^{N-1} x(K)\, e^{j\frac{2\pi nK}{N}} \qquad n = 0, 1, \ldots\ldots N-1}$$

Q. Find the DFT of a sequence $x(n) = \{\underset{\uparrow}{1}, 1, 0, 0\}$
$\quad n=0 \quad n=1 \quad n=2 \quad n=3$

Sol^n: $$\boxed{x(\omega) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}}$$

$$x(\omega) = \{\underline{2}, \underline{1-j}, \underline{0}, \underline{1+j}\}$$

$N = 4$

$$x(\omega) = \sum_{n=0}^{4-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$\Rightarrow x(\omega) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$\Rightarrow x(\omega) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{4}}$$

$$\Rightarrow x(\omega) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi kn}{2}}$$

For $k=0$:-

$$x(\omega) = \sum_{n=0}^{3} x(n) \cdot e^{0}$$

$$\Rightarrow x(\omega) = \sum_{n=0}^{3} x(n)$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1+1+0+0 = 2$$

For $k=1$:-

$$x(\omega) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi 1 n}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{\frac{-j\pi}{2}} + x(2)\, e^{-\frac{j\pi \cancel{2}}{\cancel{2}}} + x(3)\, e^{-\frac{j\pi 3}{2}}$$

$$(e^{-j\theta} = \cos\theta - j\sin\theta) = 1 + 1(\cos \pi/2 - j\sin \pi/2)$$

$$= 1 + \cos\pi/2 - j\sin\pi/2$$

$$= 1 + 0 - 1j = 1 - 1j$$

For $k=2$:-

$$x(\omega) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi 2n}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi} + 0 + 0$$

$$= 1 + 1(\cos\pi - j\sin\pi)$$

$$= 1 + 1(-1 \qquad) = 0$$

For $K = 3$:-

$$X(\omega) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi 3 n}{2}}$$

$$\cancel{= x(0)\, e^{0} + x(1)\, e^{-j\pi} + 0 + 0}$$

$$\cancel{= 1 + 1(\cos\pi - j\sin\pi)}$$

$$\cancel{= 1 + 1(}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\frac{3\pi}{2}} \cancel{+0+0}$$

$$= 1 + 1\left(\cos 3\pi/2 - j \sin 3\pi/2\right)$$

$$= 1 + 1(0 + j) = 1 + j$$

Q. Find the DFT of a sequence

$$x(n) = \begin{Bmatrix} 1, \text{ for } 0 \le n \le 2 \\ 0, \text{ otherwise} \end{Bmatrix} \text{ for } N = 4$$

$$x(n) = \{1, 1, 1, 0\}$$

$$x(n) = \left\{ \frac{1}{n=0}, \frac{1}{n=1}, \frac{1}{n=2}, \frac{0}{n=3} \right\}$$

$$X(\omega) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n k}{N}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n k}{\cancel{4}\,2}}$$

$$X(\omega) = \left\{ \overline{K=0}, \overline{K=1}, \overline{K=2}, \overline{K=3} \right\}$$

for $K = 0$

$$X(\omega) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi K n}{2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi \times 0 n}{2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{0} = \sum_{n=0}^{3} x(n)$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1+1+1+0 = 3$$

For $k=1$:-

$$x(\omega) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi \times 1n}{2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n}{2}}$$

$$= x(0)e^{0} + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j\pi} +$$

$$= 1 + 1 (\cos\pi/2 - j\sin\pi) = 1 + 1(0 - j \times 0) = 2$$

For $k=2$:-

$$x(\omega) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi \times 2n}{2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0)e^{0} + x(1)\, e^{-j\pi} + x(2)\, e^{-j2\pi} + 0$$

$$= 1 + 1(\cos\pi - \sin 2\pi) = 1 + 1(-1-$$

For $k=3$:-

$$x(\omega) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi \times 3n}{2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi 3n}{2}}$$

$$= x(0)e^{0} + x(1)\, e^{\frac{-j\pi 3}{2}} + x(2)\, e^{-j\pi 3} + 0$$

$$= 1 + 1(\cos 3\pi/2 - \sin 3\pi) = 1 + 1(0-$$

Q Find the 4-point DFT of the sequence

$x(n) = (-1)^n$

$$x(n) = \{ \underset{n=0}{1}, \underset{n=1}{-1}, \underset{n=2}{1}, \underset{n=3}{-1} \}$$

$$x(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi n \times k}{4_2}}$$

$$x(\omega) = \{ \underset{k=0}{\_\_}, \underset{k=1}{\_\_}, \underset{k=2}{\_\_}, \underset{k=3}{\_\_} \}$$

For $k=0$:-

$$x(\omega) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi kn}{2}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{\pi \times 0 n}{2}}$$

$$= \sum_{n=0}^{3} x(n) e^{0} = \sum_{n=0}^{3} x(n)$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 - 1 + 1 - 1 = 0$$

For $k=1$:-

$$x(\omega) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi \times 1 n}{2}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{\pi n}{2}}$$

$$= x(0) e^{0} + x(1) e^{-\frac{j\pi}{2}} + x(2) e^{-j\pi} + x(3) e^{-\frac{j\pi 3}{2}}$$

$$= 1 + (-1) \cos \pi/2 + 1$$

Q. Find the DFT of the given sequence.

$$x(\omega) = \{ \underset{K=0}{1}, \underset{K=1}{1-2j}, \underset{K=2}{-1}, \underset{K=3}{1+2j} \}$$

Sol^n:- $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{\frac{j2\pi nk}{N}}$

$$= \frac{1}{4} \sum_{k=0}^{3} x(k) e^{\frac{j\cancel{2}\pi nk}{\cancel{4}\,2}}$$

$$= \frac{1}{4} \left[\sum_{k=0}^{3} x(k) e^{\frac{j\pi nk}{2}} \right]$$

$$x(n) = \{ \underset{n=0}{\underline{\quad}}, \underset{n=1}{\underline{\quad}}, \underset{n=2}{\underline{\quad}}, \underset{n=3}{\underline{\quad}} \}$$

For $n=0$

$$= \frac{1}{4}\left[\sum_{k=0}^{3} x(k)\, e^{0}\right]$$

$$= \frac{1}{4}\left[x(0)+x(1)+x(2)+x(3)\right]$$

$$= \frac{1}{4}\left[1+1-2j+(-1)+1+2j\right]$$

$$= \frac{1}{4}\left[1+1-2j-1+1+2j\right]$$

$$= \frac{1}{\cancel{4}_{2}}\cdot \cancel{2}^{1} = \frac{1}{2}$$

DFT AS A LINEAR TRANSFORMATION :-

DFT :

$$x(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nk}{N}}$$

$$K = 0, 1, \ldots \ldots N-1$$

IDFT :-

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} x(K)\, e^{\frac{j2\pi nk}{N}}$$

$$n = 0, 1, 2 \ldots \ldots N-1$$

Suppose $\boxed{\omega_N = e^{-\frac{j2\pi}{N}}}$

Here, ω_N is the N^{th} root of unity then,

$$\boxed{x(K) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{nk}}$$

$$\boxed{x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(K) \cdot \omega_N^{-nk}}$$

→ The computation of each point of DFT consist of 'N' number of complex multiplication and N-1 numbers of complex addition.

→ Hence, for N-point DFT the total number of complex multiplication is 'N^2' and total number of complex addition is $N(N-1)$.

→ We can express the DFT and IDFT as a linear transformations of the sequence $x(n)$ and $x(K)$ respectively.

→ Let us consider an N-point vector 'x'

$$x(N) = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}$$

→ The N×N matrix for W_N is

$$W_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \vdots \end{bmatrix}$$

→ The N-point DFT can be expressed in the matrix form.

$$\boxed{X_N = x_N W_N}$$

where, W_N is the matrix of linear transformation

— IDFT can be expressed as,

$$\boxed{x(N) = \frac{1}{N} W_N^* \times X_N}$$

where, W_N^* is the complex conjugate of W_N.

Q. Find the 4-point DFT of the sequence

$$x(N) = \{0, 1, 2, 3\} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

Sol^n:- $$W_N = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\star\ \omega_4^1 = e^{-j\frac{2\pi}{4_2}}$$

$$= \omega_4^1 = e^{-j\pi/2}$$

$$= \omega_4^1 = \cos\pi/2 - j\sin\pi/2$$

$$= \omega_4^1 = 0 - j = -j$$

$$\star\ \omega_4^2 = e^{-j\frac{2\pi\times 2}{4_2}}$$

$$= \omega_4^2 = e^{-j\pi}$$

$$\Rightarrow \omega_4^2 = \cos\pi - j\sin\pi$$

$$\Rightarrow \omega_4^2 = -1$$

$$\star\ \omega_4^3 = e^{-j\frac{2\pi\times 3}{4_2}}$$

$$\Rightarrow \omega_4^3 = e^{-j3\pi/2}$$

$$\Rightarrow \omega_4^3 = \cos 3\pi/2 - j\sin 3\pi/2$$

$$\Rightarrow \omega_4^3 = 0 + 1j = j$$

$$\star\ \omega_4^4 = e^{-j\frac{2\pi\times 4}{4}}$$

$$\Rightarrow \omega_4^4 = e^{-j2\pi}$$

$$\Rightarrow \omega_4^4 = \cos 2\pi - j\sin 2\pi$$

$$\Rightarrow \omega_4^4 = 1 - 0 = 1$$

$$\Rightarrow \omega_4^6 = e^{-j\frac{2\pi\times \cancel{6}\,3}{4_2}}$$

$$\begin{bmatrix} 1 & \omega_N^1 & \omega_N^2 \cdots \omega_N^{N-1} \\ 1 & \omega_N^2 & \omega_N^4 \cdots \omega_N^{2(N-1)} \\ 1 & \omega_N^{N-1} & \omega_N^{2(N-1)} & \omega_N^{3(N-1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega_4^1 & \omega_4^2 \cdots & \omega_4^{4-1} \\ 1 & \omega_4^2 & \omega_4^4 \cdots & \omega_4^{2(4-1)} \\ 1 & \omega_4^{4-1} & \omega_4^{2(4-1)} & \omega_4^{3(4-1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega_4^1 & \omega_4^2 & \omega_4^3 \\ 1 & \omega_4^2 & \omega_4^4 & \omega_4^6 \\ 1 & \omega_4^3 & \omega_4^6 & \omega_4^9 \end{bmatrix}$$

$$\Rightarrow \omega_4^6 = e^{-j3\pi}$$

$$\Rightarrow \omega_4^6 = \cos 3\pi - j \sin 3\pi$$

$$x_N = x_n - \omega_N$$

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}_{4\times 1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}_{4\times 4}$$

$$\begin{bmatrix} 0\times 1 + 1\times 1 + 2\times 1 + 3\times 1 \\ 0\times 1 + 1\times(-j) + 2\times(-1) + 3\times j \\ 0\times 1 + 1\times(-1) + 2\times 1 + 3\times(-1) \\ 0\times 1 + 1\times j + 2\times(-1) + 3\times(-j) \end{bmatrix}_{4\times 1}$$

$$= \begin{bmatrix} 0 + 1 + 2 + 3 \\ 0 + (-j) - 2 + 3j \\ 0 - 1 + 2 - 3 \\ 0 + j - 2 - 3j \end{bmatrix}_{4\times 1}$$

$$= \begin{bmatrix} 6 \\ 2j - 2 \\ -2 \\ -2j - 2 \end{bmatrix}_{4\times 1}$$

Relationship to the fourier series coefficient of a periodic sequence :-

A periodic sequence $x_p(n)$ with fundamental period 'N' can be represents in a fourier series form as,

$$\boxed{x_p(n) = \sum_{k=0}^{N-1} C_k e^{\frac{j2\pi nk}{N}} \qquad -\infty < n < \infty} \quad \text{—— (I)}$$

Where the fourier series coefficient is represented by;

$$\boxed{C_K = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-\frac{j2\pi nk}{N}} \qquad K = 0, 1 \ldots N-1} \quad \text{—— (II)}$$

The relation between equation (I) and (II) is $\boxed{x(K) = C_K \cdot N}$

Comparing equation (I) and (II) and defining a sequence $x(n)$ which is identical to $x_p(n)$

The discrete fourier domain represented in Expresses

$$\boxed{x(K) = \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi nk}{N}} = NC_K}$$

$$x_p(n) = x(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)\, e^{j\frac{2\pi nk}{N}}$$

<u>Relationship of DFT with Z-transform :-</u>

→ If the sequence $x(n)$ is of infinite length then the Z-transform is $\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$ but if the sequence $x(n)$ is of finite length 'N' then the Z-transform $\boxed{X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}}$ ——— (1)

→ As per IDFT $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{-j\frac{2\pi nk}{N}}$ ——— (2)

putting the value of $x(n)$ from eqn (11) to eqn (1)

$$X(z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} x(k)\, e^{\frac{j2\pi nk}{N}}\right] z^{-n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} x(k)\sum_{n=0}^{N-1} e^{\frac{j2\pi nk}{N}} z^{-n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} x(k)\sum_{n=0}^{N-1}\left(e^{j\frac{2\pi k}{N}} z^{-1}\right)^{n}$$

$$X(z) = \frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{x(k)}{1-e^{j\frac{2\pi k}{N}} z^{-1}}$$

$$X(\omega) = \frac{1-e^{-j\omega N}}{N}\sum_{k=0}^{N-1}\frac{x(k)}{1-e^{-j\left(\omega-\frac{2\pi k}{N}\right)}}$$

$$\left[\because z = e^{j\omega}\right]$$

I. Periodicity.

If $x(k)$ is DFT of a finite duration sequence $x(n)$ then,

$$x(n) \xleftrightarrow{DFT} x(k)$$

$$x(n+N) = x(n), \text{ for all values of } n$$

$$x(k+N) = x(k), \text{ for all value of } k$$

II. linearity:

If two finite duration sequence $x_1(n)$ and $x_2(n)$ are linearly combined as,

$$x_3(n) = a x_1(n) + b x_2(n)$$

$$x_3(k) = a x_1(k) + b x_2(k)$$

III. Circularship of a sequence :-

→ The period Extension of the sequence $x(n)$ can be written as

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - ln)$$

→ The periodic signal $x_p(n)$ can be visualised by wrapping the finite duration sequence $x(n)$ around a circle in anticlockwise direction

→ The shifted version $x_p(n)$ can be written as $x_p(n-k)$ where k represents the number of times o $x_p(n)$ is shifted.

$$\boxed{x_p(n-k) = \sum_{l=-\infty}^{\infty} x(n - ln - k)}$$

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IV. Time Reversal of the sequence :-

→ The time reversal of a sequence $x(n)$ is attended by wrapping the sequence $x(n)$ ~~anticlockwise~~ around the circle in clockwise direction

→ It is denoted by $x((-n))_N = x(N-n)$ —— ①

$0 \leq n \leq N-1$

→ If the DFT of $x(n)$ is $X(K)$ then, the DFT of equation ① is $X((-K))_N = X(N-K)$.

CIRCULAR FREQUENCY SHIFT :-

If $x(n) \xleftrightarrow{DFT} X(K)$ then

$$x(n)e^{\frac{j2\pi ln}{N}} \longleftrightarrow X((K-l))_N$$

Complex conjugate property :-

If $x(n) \xleftrightarrow{DFT} X(K)$ then

$$x^*(n) \longleftrightarrow X^*((-K))_N = X^*(N-K)$$

CIRCULAR CONVOLUTION PROPERTY :-

let $x_1(n)$ and $x_2(n)$ are two finite duration sequence and their DFT can be written as $X_1(K)$ & $X_2(K)$. $x_1(n)$ & $x_2(K)$ then the circular convolution of $x_1(n)$ and $x_2(n)$ can be represented $DFT[x_1(n) \, Ⓝ \, x_2(n)] = X_1(K) \times X_2(K)$

Parsevals Theorem :-

If $DFT[x(n)] = X(K)$

$DFT[y(n)] = Y(K)$

then $$\sum_{n=0}^{N-1} x(n)\, y^*(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, Y^*(K)$$

CIRCULAR CONVOLUTION :-

Matrix method :-

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$h(n) = 1, 2, 1, 2$

$x(n) \; (N) \; h(n)$

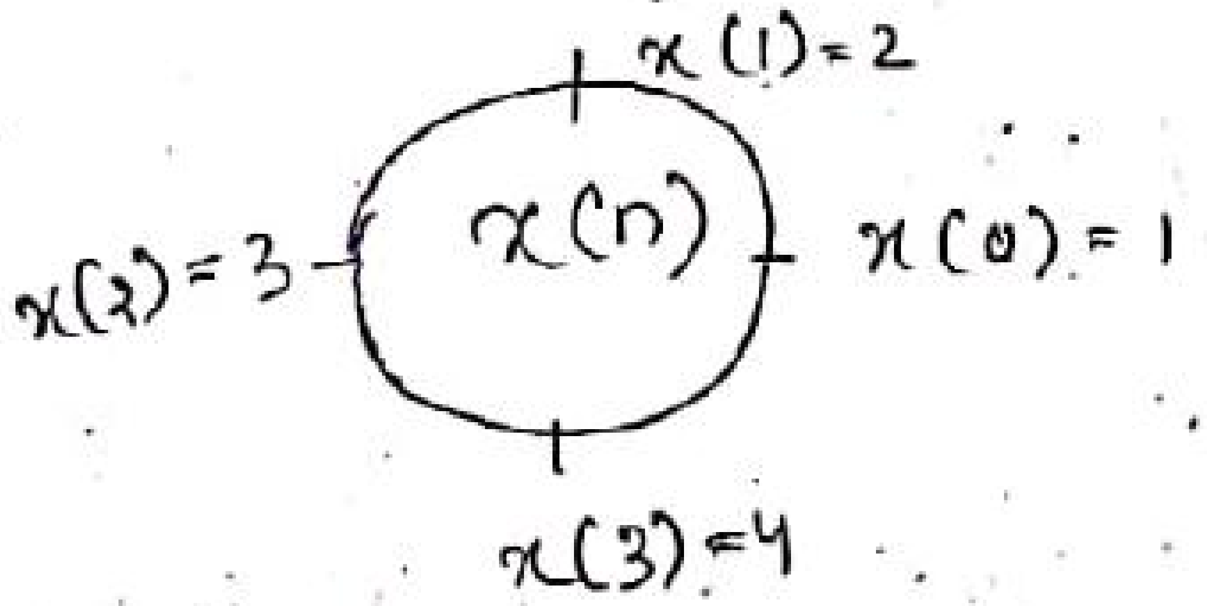
$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

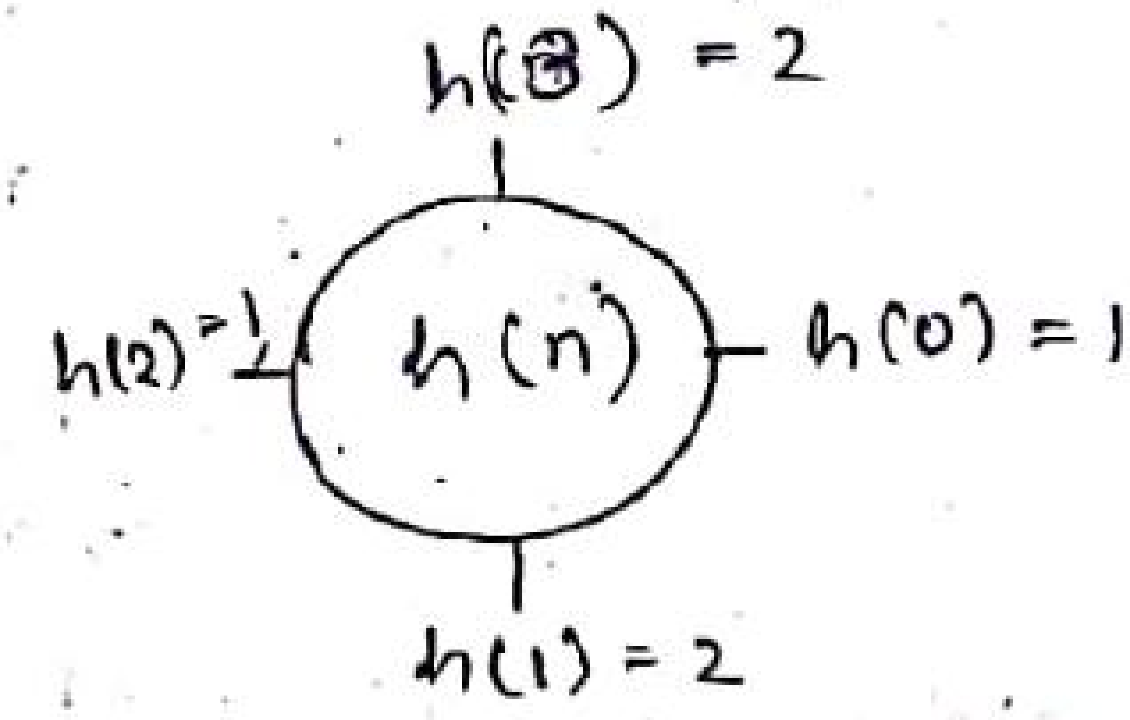
$$= \begin{bmatrix} 1+8+3+4 \\ 2+2+4+6 \\ 3+4+1+8 \\ 4+6+2+2 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 \\ 14 \\ 16 \\ 14 \end{bmatrix}$$

CONCENTRIC CIRCLE METHOD :-

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$h(n) = \{1, 2, 1, 2\}$





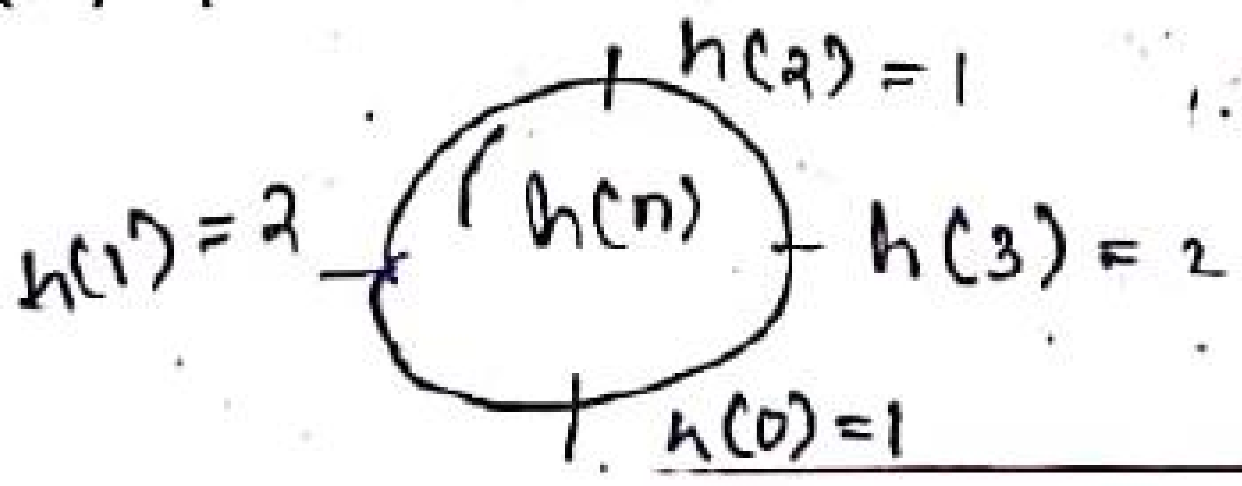
multiply the corresponding Element

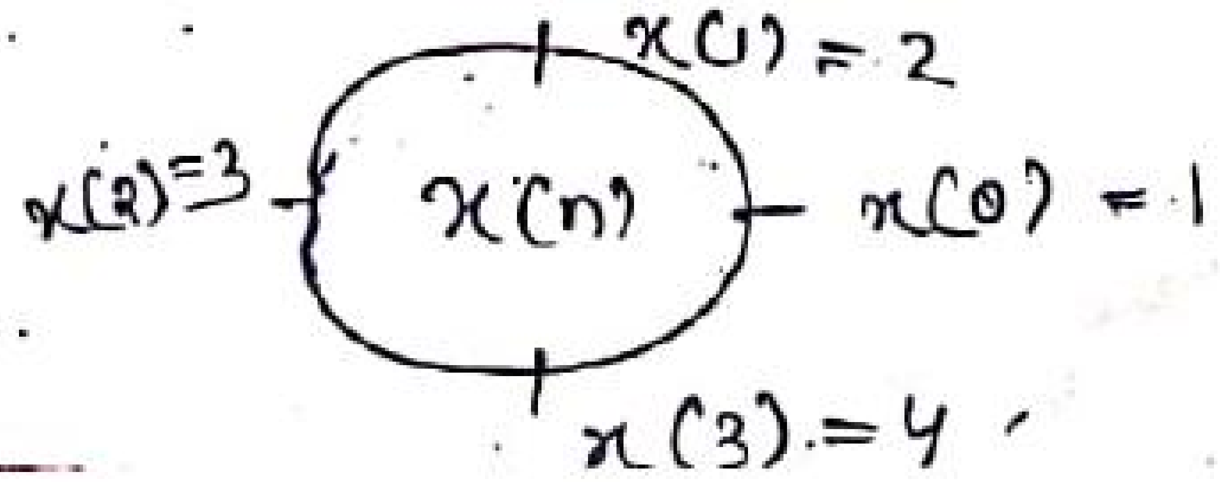
$x(1) \cdot h(3) + x(2) \cdot h(2) + x(0) \cdot h(0) + x(3) \cdot h(1)$

$= 2 \cdot 2 + 3 \cdot 1 + 1 \cdot 1 + 4 \cdot 2$

$= 4 + 3 + 1 + 8 = 16$

(ii) Rotate h(n) into clockwise direction



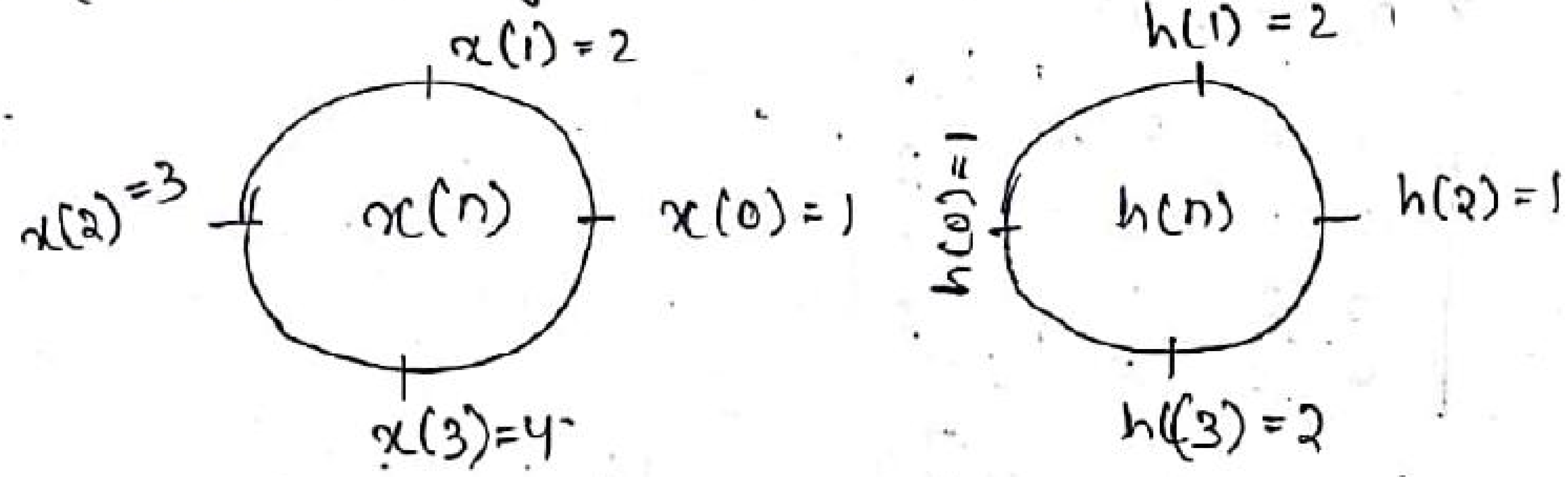


Multiply the corresponding Element $x(n) * h(n)$

$x(1)\cdot h(2) + x(2)\cdot h(1) + x(0)\cdot h(3) + x(3)\cdot h(0)$

$= 2\cdot 1 + 3\cdot 2 + 1\cdot 2 + 4\cdot 1 = 2+6+2+4 = 16$

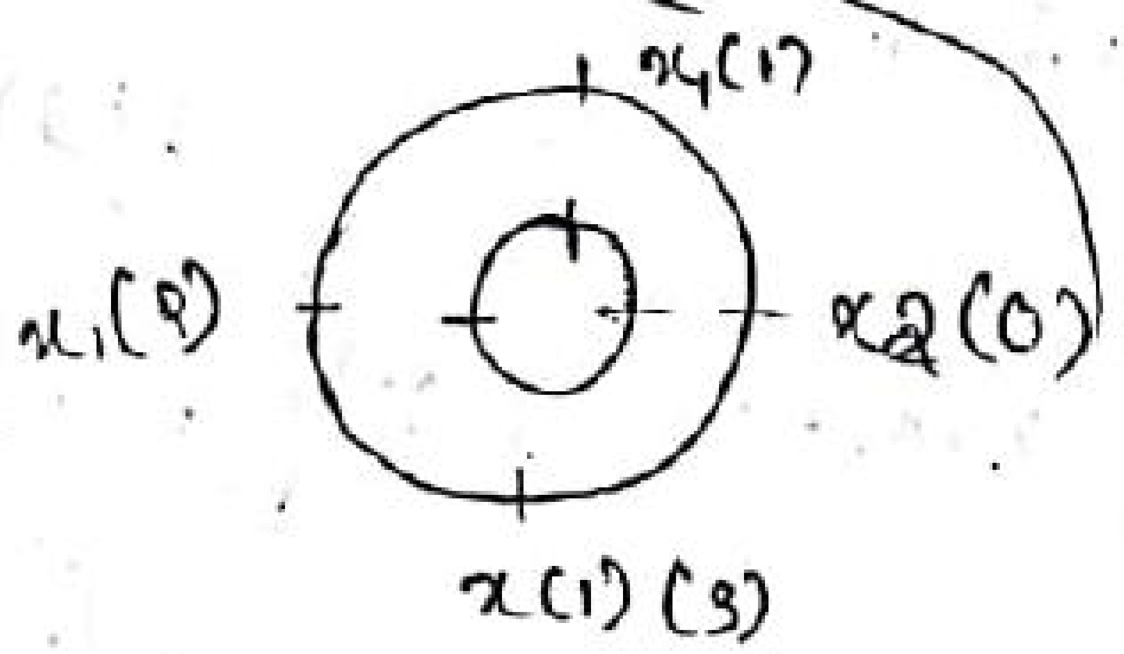
iii) Rotate $h(n)$ again into clockwise direction.



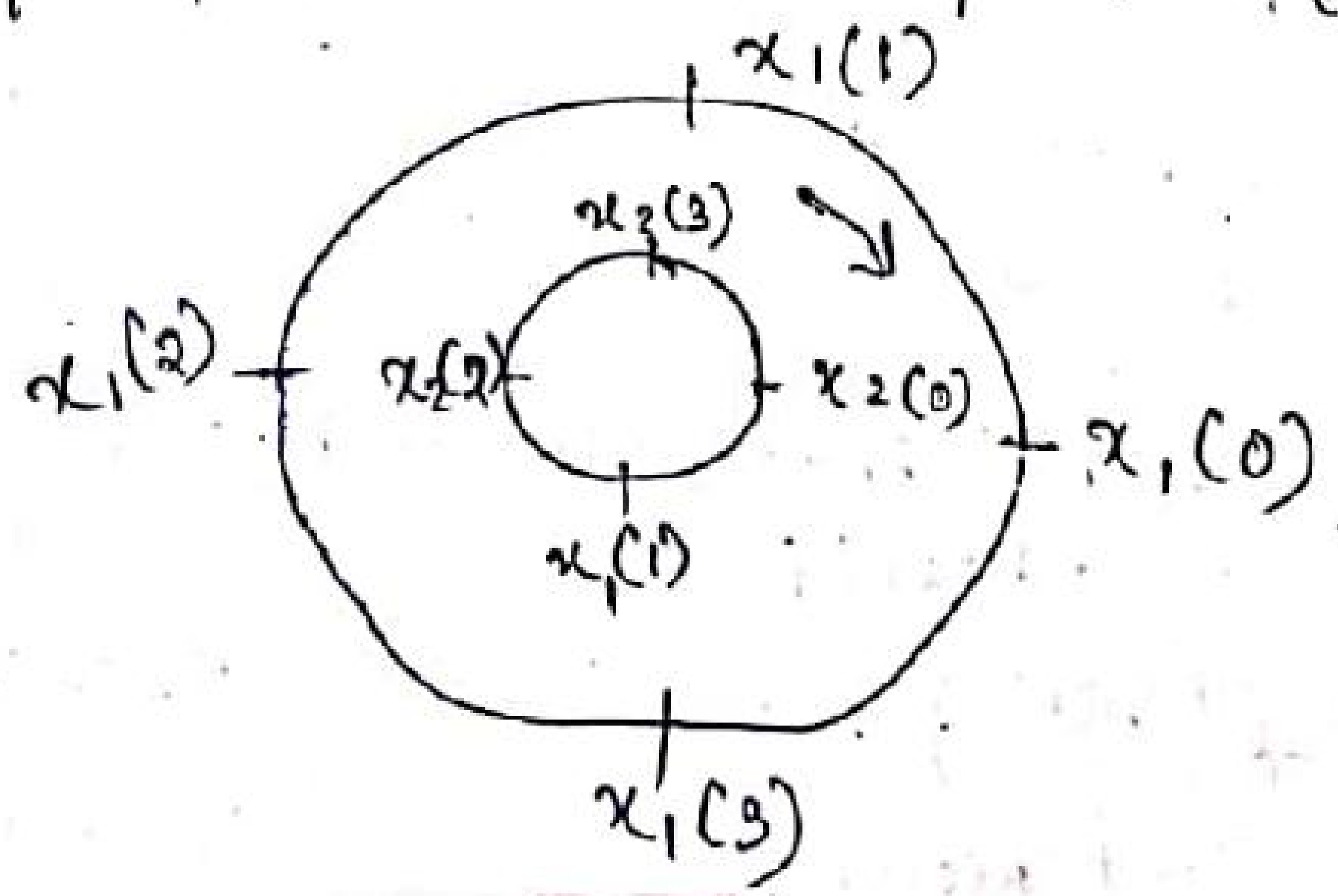
CIRCULAR CONVOLUTION BY CONCENTRIC CIRCLE METHOD:-

→ Let $x_1(n)$ and $x_2(n)$ are two given sequences.

→ Take two concentric circle plot 'N' samples of $x_1(n)$ on the circumference of the ~~number~~ outer circle maintaining equal distance between successive points in anti-clock wise directions.



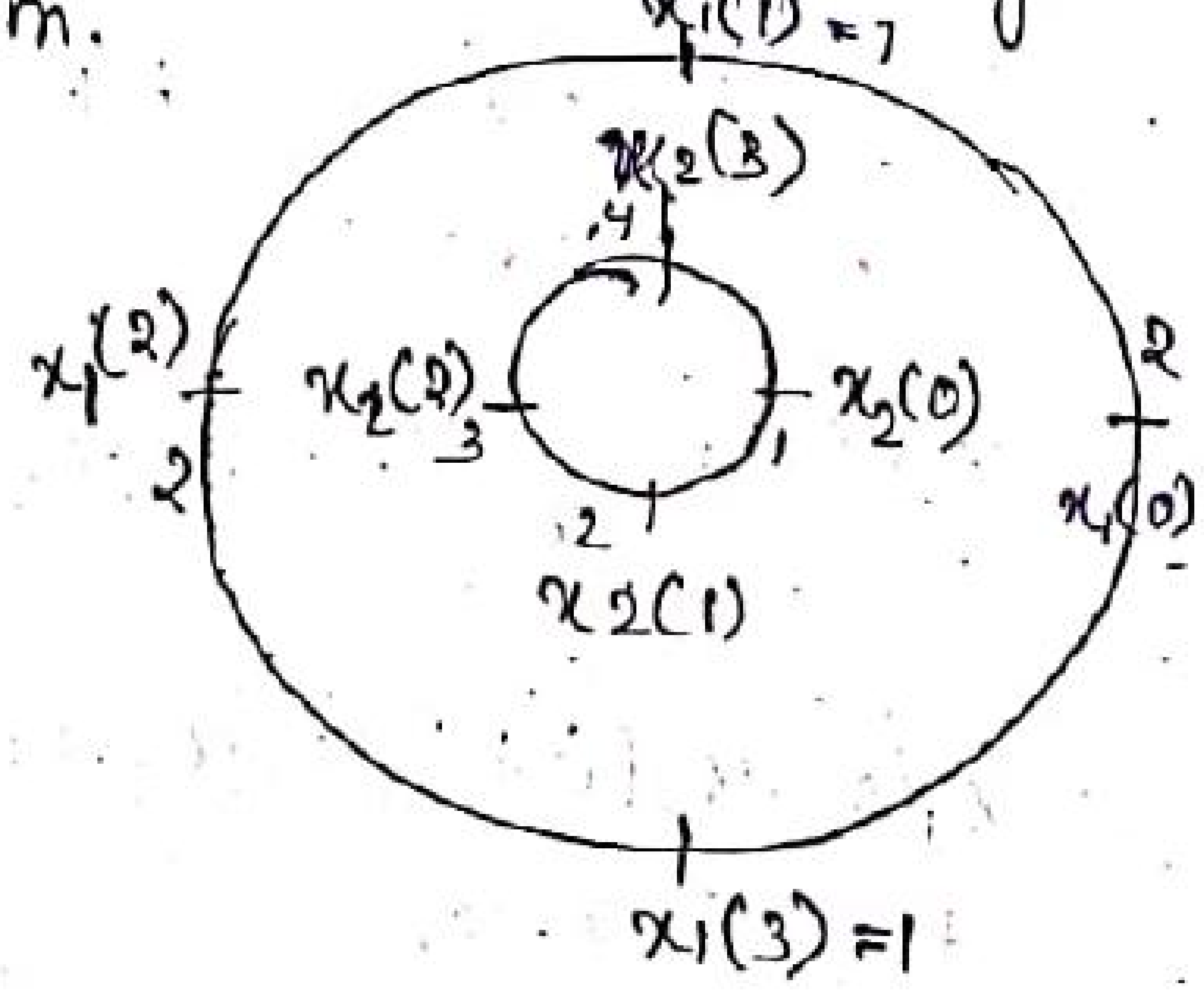
→ For plotting $x_2(n)$ plot 'N' samples of $x_2(n)$ in clock wise directions on the inner circle and the starting sample is placed at the same point as 0th sample of x_1



→ multiply the inner circle element with the outer circle element and rotate the inner circle element anti-clock wise with one sampled at a time then multiply and get the corresponding sum.

→ Eg: $x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$



Here the circular convolution $x_3(n) = x_1(n) \, (N) \, x_2(n)$

let, $m = 0$

$$x_3(0) = \sum_{n=0}^{3} x_1(n) \, x_2((-n))_4$$

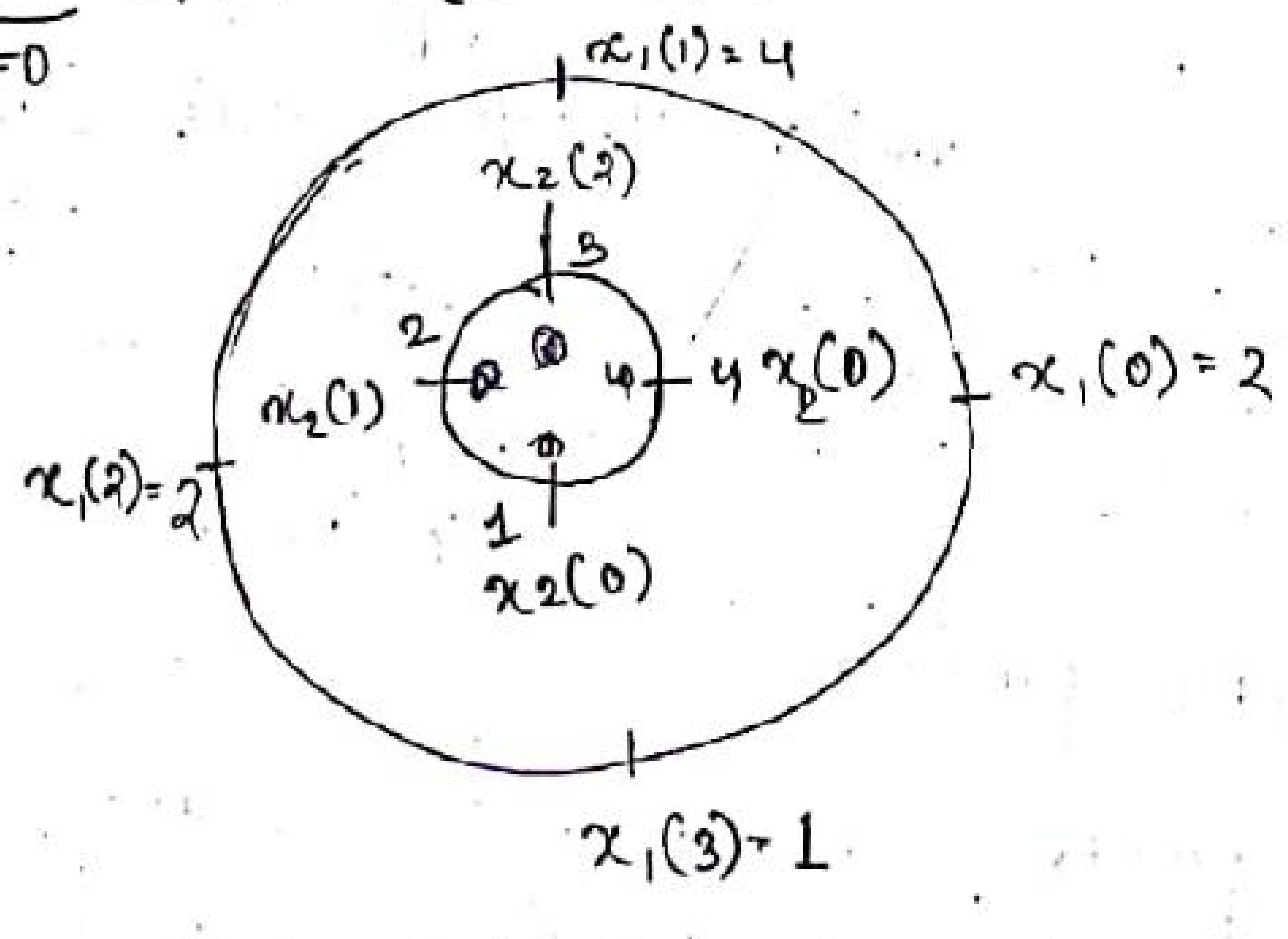
$= x_1(0).x_2(0) + x_1(3) \cdot x_2(1) + x_1(2).x_2(2) + x_1(1).x_2(3)$

$= 2\times1 + 2\times1 + 2\times3 + 1\times4$

$= 2 + 2 + 6 + 4 = 14$

for $m = 1$

$$x_3(1) = \sum_{n=0}^{3} x_1(n) \, x_2((1-n))_4$$



Here, the sequence $x_2((-n))_4$ is rotated anti-clockwise by 1 unit

$$x_3(1) = \sum_{n=0}^{3} x_1(n) \, x_2((1-n))_4$$

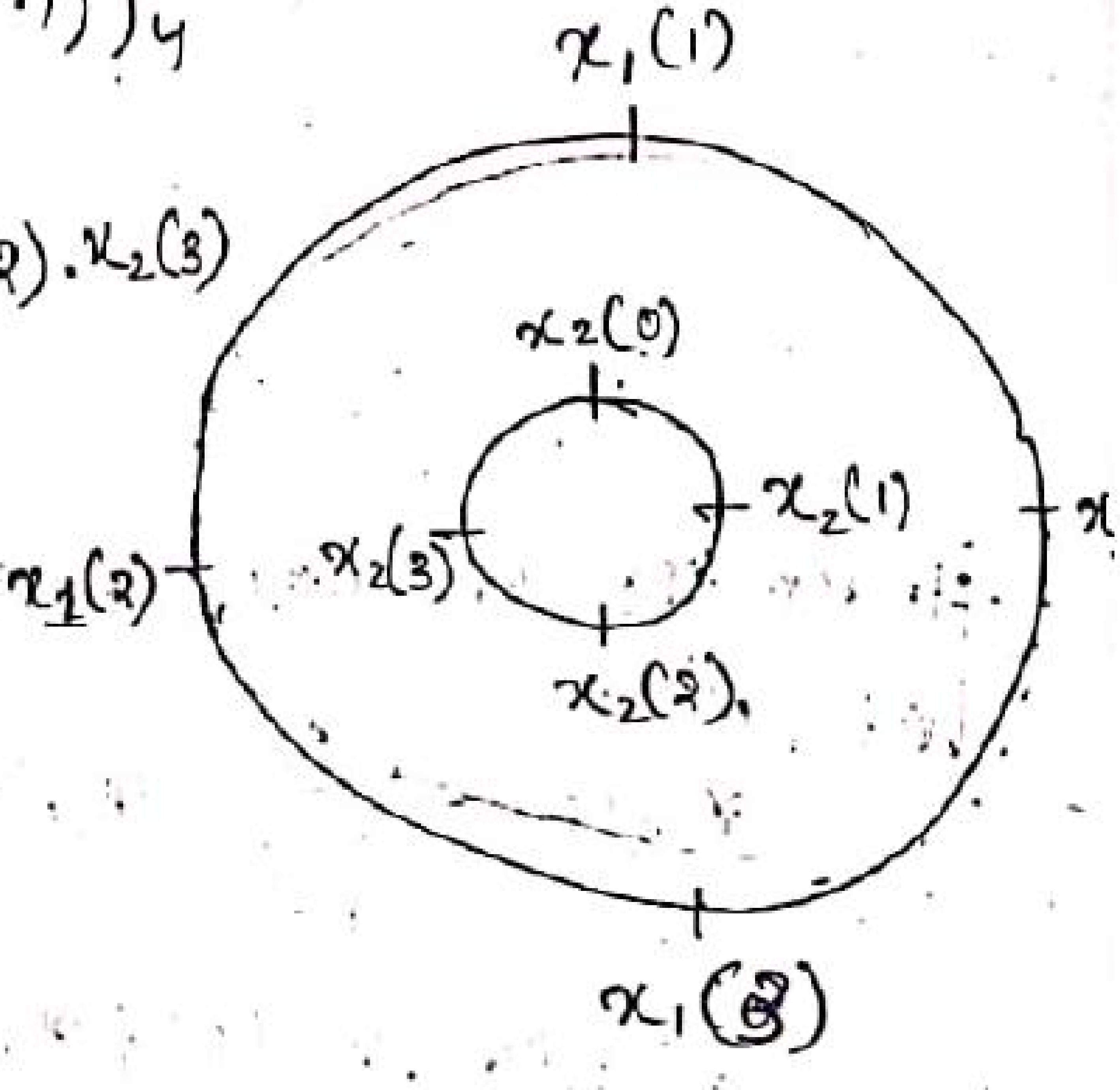
$x_1(0).x_2(3) + x_1(3).x_2(0) + x_1(2).x_2(1) . x_1(1) . x_2(2)$

$= 2.4 + 1\times1 + 2\times2 + 4\times3$

$= 8 + 1 + 4 + 12 = 26$

For m = 1

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

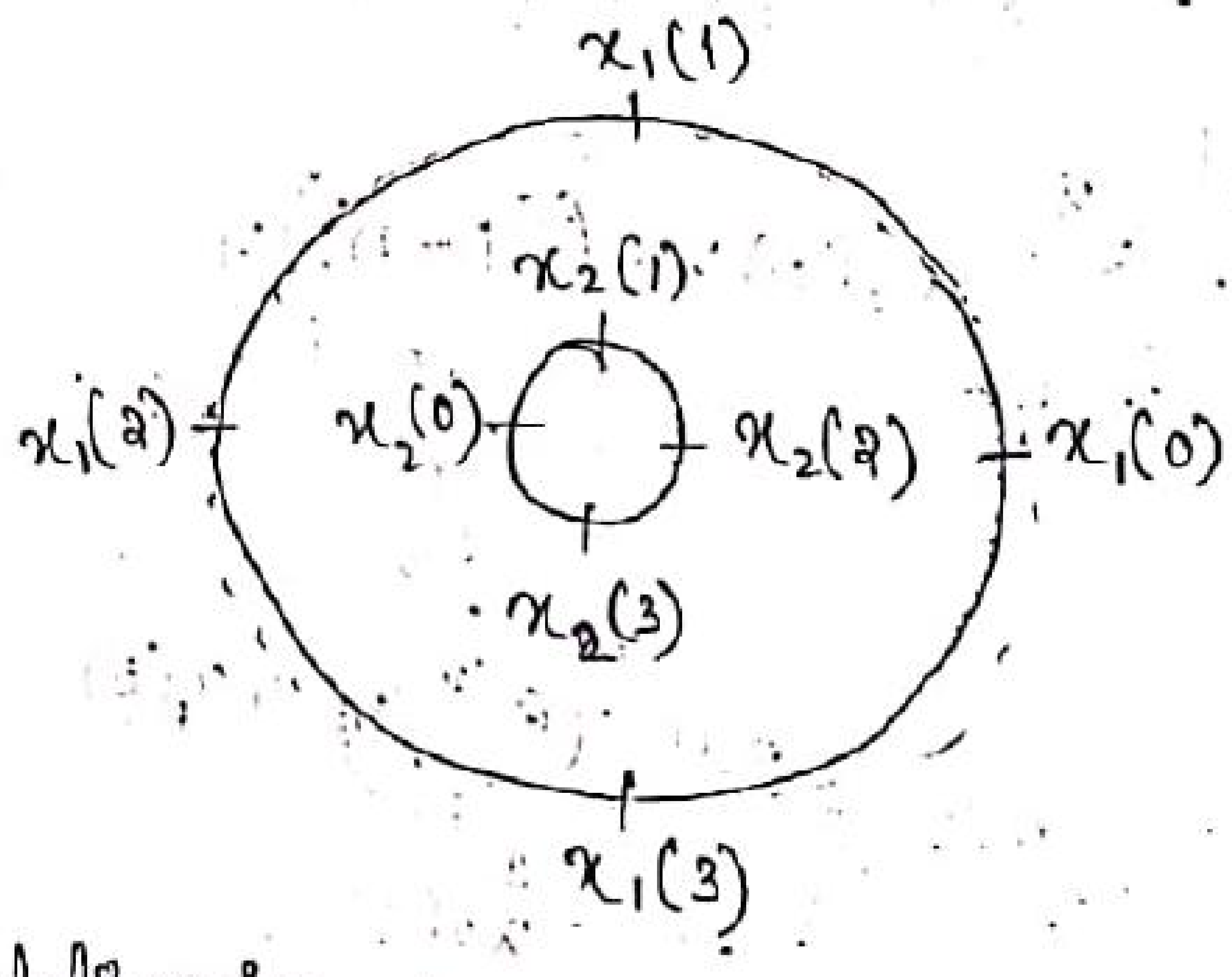


$\Rightarrow x_1(0).x_2(1) + x_1(3).x_2(2) + x_2(2).x_2(3)$
$+ x_1(1).x_2(0)$

$\Rightarrow 4 + 1 + 8 + 3 = 16$

For m = 2 :-

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$



Zero padding:-

→ The process in which zeros are added in order to make the length of the sequence equal in solv circular convolution is known as zero padding

CH→5 FAST FOURIER TRANSFORM Dt→ 11.03.2020

→ A FFT: algorithm which is used to calculate the DFT and IDFT of a sequence rapidly.

→ FFT was proposed by cooley and tucky.

→ FFT was a high efficient procedure for computing the DFT of a finite series and requires less number of computation than the direct DFT.

→ FFT is based on de-composition and drinking the transform into smaller transform and combining them to get the total transform.

→ FFT reduces the computation time and improves the performance by a factor 100 or more.

→ FFT reduces the number of complex multiplication from N^2 to $\frac{N}{2}\log_2 N$.

→ FFT algorithm are based on principle of De-composing the computation of DFT sequence of length 'N' into 'N/2' and 'N/4' sequence

→ FFT algorithm can be of two types

(I) decimation in time

(II) decimation in frequency

→ In DIT FFT the sequence for which we need the DFT is divided into smaller sequences and DFT of the sequence ~~and~~ are combined in a certain pattern to obtain the DFT of the Entire Sequence.

→ In DFT approach the frequency samples of the DFT are decomposed into smaller subsequences in a similar manner

DECIMATION IN TIME :-

→ This algorithm is also known as Radix DIT FFT algorithm which means the no. of output points 'N' can be Expressed as the power of two.

→ If $x(n)$ is an 'N' point sequence where 'N' is the power of two then decimate or break the sequence into leng of 'N/4' where the sequence consists of even value of x(and the other sequence consists of odd values of $x(n)$.

→ Here, the input sequence is shifted through beat revers

→ The no. of stages in the flow graph is given by

$$\boxed{M = \log_2 N}$$

→ Each stage consists of $\frac{N}{2}$ no. of butterflies.

→ The input and output for each butterfly are separate by 2^{m-1} where m is the step index for stage 1, m = 1 for stage - 2, ~~for~~ m = 2 for stage 3, m = 3

→ The number of complex multiplication is given by $\frac{N}{2} \log_2 N$

→ The number of complex addition is given by $N \log_2 N$

Dt → 12.03.2020

→ The twiddle factor ~~Expo~~ Exponent are a function of stage index m and given by $\boxed{K = \frac{Nt}{2^m}}$

where, $t = 0, 1 \cdots 2^{m-1}$

→ The no. of set of butterflies in each stage is given by the formula 2^{N-m}.

→ The Exponent repeat factor which is the number of times the exponent sequences associated with m and repeated is given by $\boxed{2^{N-m}}$

Radix 2-FFT :-

→ When N is the power of 2 i.e., $N = 2^K$ where $K > 1$ is an integer then the above DIT decomposition can be performed K−1 times until each DFT length is two.

→ A length 2 DFT requires no multiples the overall result is called a Radix 2 FFT.

→ A different Radix-2 FFT is derived by performing decimation in frequency.

→ The Radix-2 FFT complexity is $N \cdot \log N$ pulling together the length N DFT from the $\frac{N}{2}$ length-2 DFT's in a radix 2 FFT the only multiples needed are those used to combine two small DFT's to make a DFT as twice as long.

→ Since, there is approximately N complex multiplication need for each stage of DIT decomposition.

→ The no. of multiplication is reduced from N^2 to $\frac{N}{2} \log_2 N$

DECIMATION IN TIME:-

→ The Radix-2 decimation in time algorithm rearrange the DFT sequence into two parts

(i) A sum over the even no. of sequence

(ii) Sum over the odd no. of sequence.

→ The mathematical simplification in eqn revels that each frequency o/p of DFT can be computed as the sum of o/p of two length N/2 DFT of the even ~~radix~~ index and odd index discrete time samples $x(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}}$

$$= \sum_{n=0}^{N/2-1} x(n) e^{\frac{-j2\pi(2n)k}{N}} + \sum_{n=0}^{N/2-1} x(2n+1) e^{-j}$$

$$\Rightarrow \sum_{n=0}^{N/2-1} x(n) e^{\frac{-j2\pi(2n)k}{N}} + \sum_{n=0}^{N/2-1} x(2n+1) e^{\frac{-j2\pi(2n+1)k}{N}}$$

$$\Rightarrow DFT\frac{N}{2}\left[\left[x(0), x(2) \ldots x(N-2)\right]\right] + \omega_N^K DFT\, N/2 \left[\left[x(1), x(3) \ldots x(N-1)\right]\right]$$

→ The odd indexed DFT is multiplied by twiddle factor

$$W_N^{K} = e^{-j\frac{2\pi nK}{N}}$$

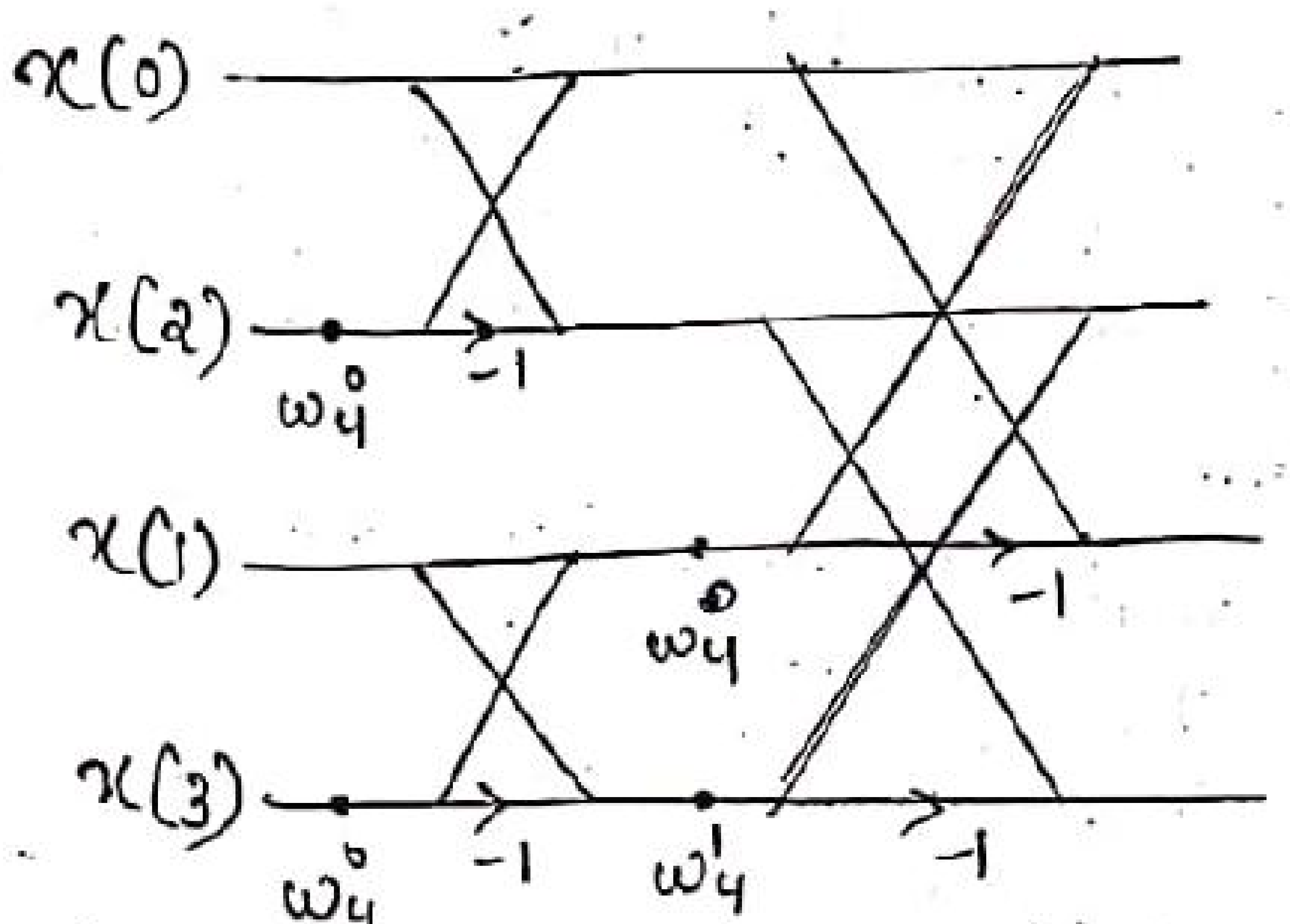
DFT - FFT algorithm :-

For 4 - point sequence

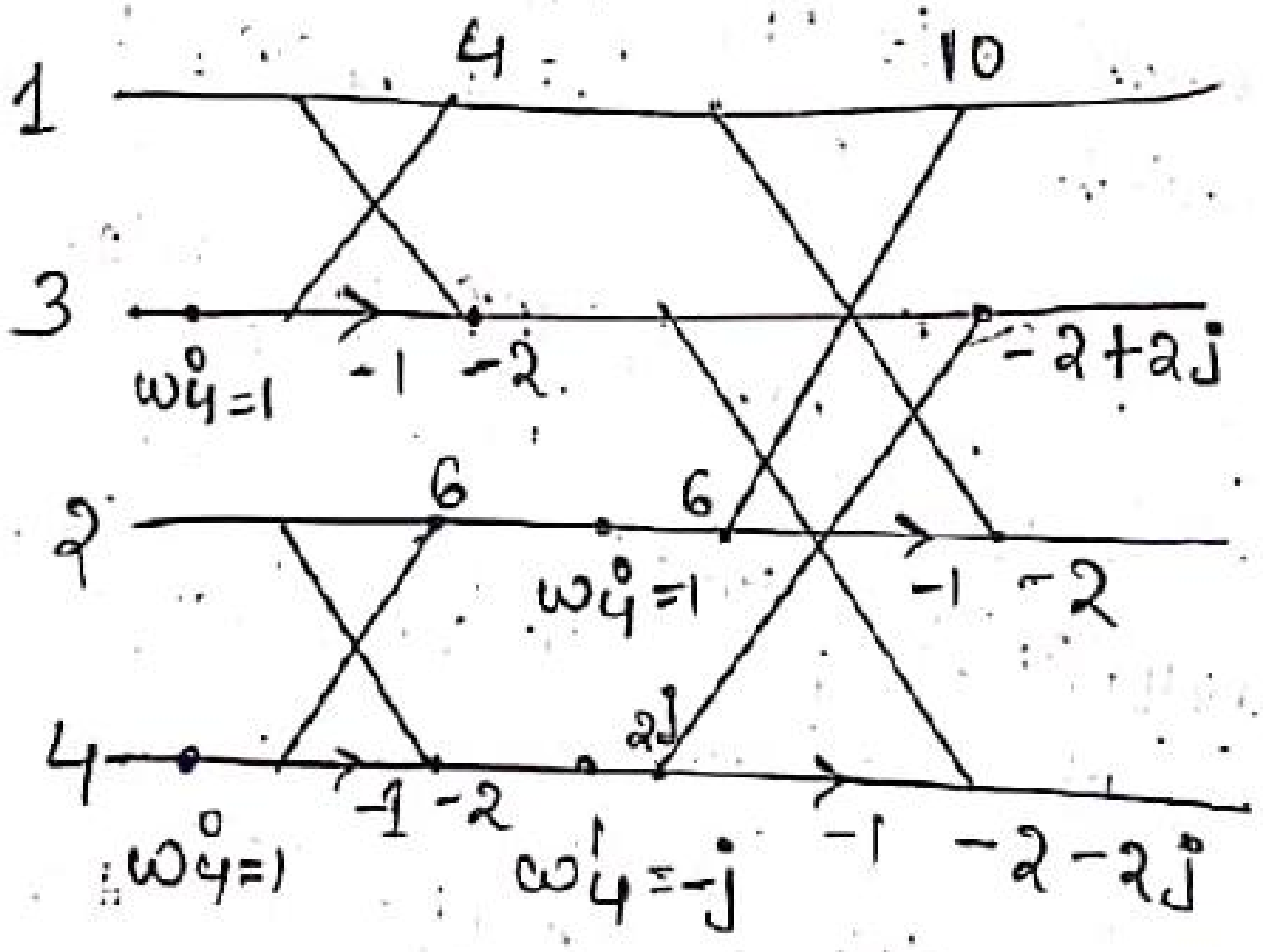
$$x(n) = \{1, 2, 3, 4\}$$
$$= \{x(0), x(1), x(2), x(3)\}$$

$$x_e(n) = \{x(0), x(2)\} = \{1, 3\}$$

$$x_o(n) = \{x(1), x(3)\} = \{2, 4\}$$



Putting the value in the above sequence / digram.

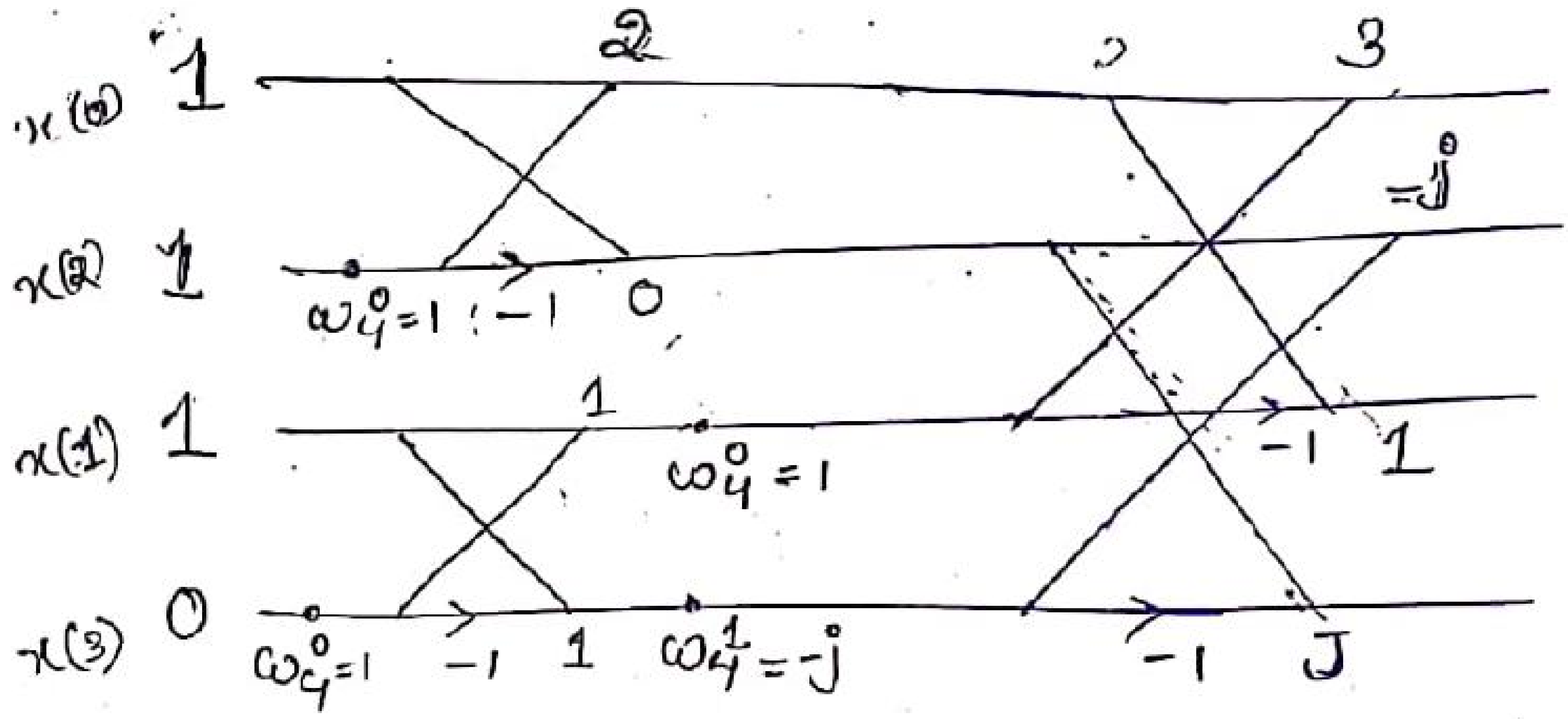


Q. $x(n) = \{1, 1, 1, 0\}$

$\{x(0), x(1), x(2), x(3)$

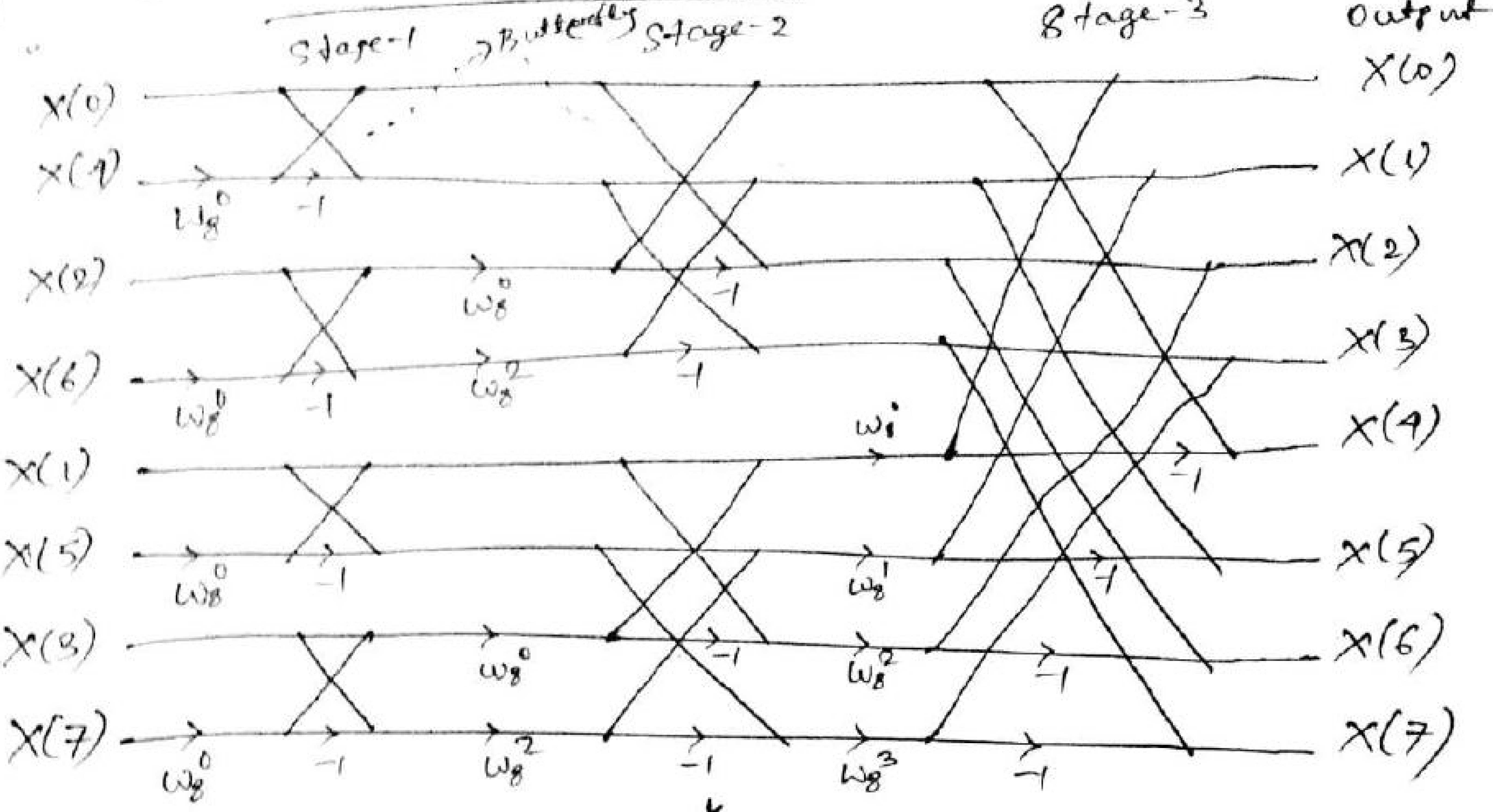
$x_e(n) = \{x(0), x(2)\} = \{1, 1\}$

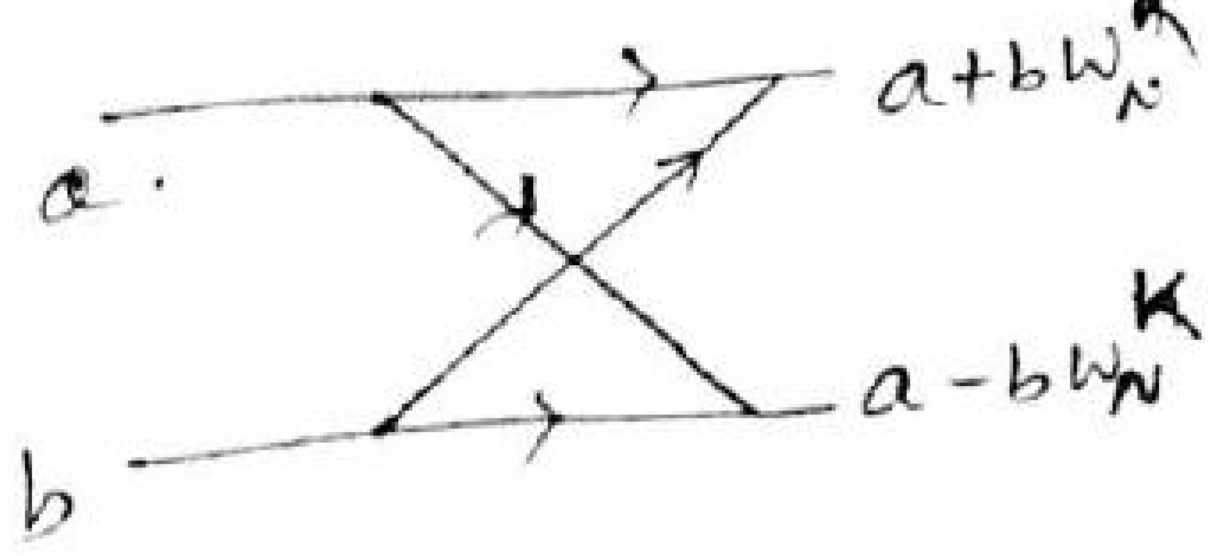
$x_o(n) = \{x(1), x(3)\} = \{1, 0\}$



8-point sequence

8 point DIT-FFT Algorithm :





Input is given in bit reversal :

Input index	bit representation	bit reversed binary	Bit reversed index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

Inputs are given in this order

Steps of Radix-2 DIT FFT algorithm: (~~N = 8~~)

1. The number of input samples $N = 2^M$, M is an integer.
2. input is suffled through bit reversal.
3. Number of stages is $M = \log_2 N$ (∵ for e.g when $N=8, M=3$)
4. Each stage has $N/2$ butterflies.
5. input/outputs for each butterfly are separated by 2^{m-1} samples, where m represents the stage index, i.e for stage 1 - $m=1$, stage 2, $m=2$.
6. No of complex multiplication is $\frac{N}{2}\log_2 N$.
7. No of complex addition is $N\log_2 N$.
8. The twiddle factor exponents are a function of the stage index 'm' and is given by $K = \frac{Nt}{2^m}$, $t = 0, 1, \cdots 2^{(m-1)}-1$
9. No of sets or sections of butterflies in each stage is given by 2^{M-m}.
10. The exponent repeat factor, which is the number of times the exponent sequence assotiated with m is repeated given by $2^{(M-m)}$.

---

for $N=8$, the above steps are:

1) $N=8$, $M=3$ 2) $x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7)$
3) $M=3$. 4) no of butterfly $= \frac{8}{2} = 4$
5) each butterfly separated by 2^{m-1}

$$\text{for stage-1} = 2^{m-1} = 2^{1-1} = 2^0 = 1$$
$$\text{stage-2} = 2^{m-1} = 2^{2-1} = 2^1 = 2.$$
$$\text{stage-3} = 2^{m-1} = 2^{3-1} = 2^2 = 4.$$

6) No of complex multiplication $= \frac{N}{2}\log_2 N = \frac{8}{2}\log_2 8 = 12$

7) No of complex addition $= N\log_2 N = 24$ (for $N=8$)

8) twiddle factor Exponent : K

$$K = \frac{Nt}{2^m}$$

for stage-1 – $m=1$, $K = \frac{8\cdot t}{2^1}$, for $t=0$, $K=0$

so twiddle factor $= W_N^K = W_8^0$

for stage-2, $m=2$, $K = \frac{8t}{2^2}$, for $t=0$, $K=0$, W_8^0.

$t=1$, $K=2$, W_8^2

for stage-3, $m=3$, $t=0,1,2,3$.

	$t=0$	$t=1$	$t=2$	$t=3$
$K=$	0			
Twiddle factor	W_8^0	W_8^1	W_8^2	W_8^3

9) Sections of Butterflies : $= 2^{M-m}$.

stage-1, $m=1 = 2^{3-1} = 2^2 = 4$.

stage-2, $m=2 = 2^{3-2} = 2^1 = 2$.

stage-3, $m=3 = 2^{3-3} = 2^0 = 1$

10) Exponent Repeat factor :

i) W_8^0 repeats $2^{M-m} = 2^{3-1} = 2^2 = 4$ times in stage-1

ii) W_8^0, W_8^2 repeats $2^{3-2} = 2^1 = 2$ times in stage-2

iii) $W_8^0, W_8^1, W_8^2, W_8^3$ repeats $2^{M-m} = 2^{3-3} = 2^0 = 1$ in stage-3

$$W_8^0 = e^{-j\left(\frac{2\pi}{8}\right)\cdot 0} = 1$$

$$W_8^1 = e^{-j\left(\frac{2\pi}{8}\right)} = \cos \pi/4 - j\sin \pi/4 = 0.707 - j0.707.$$

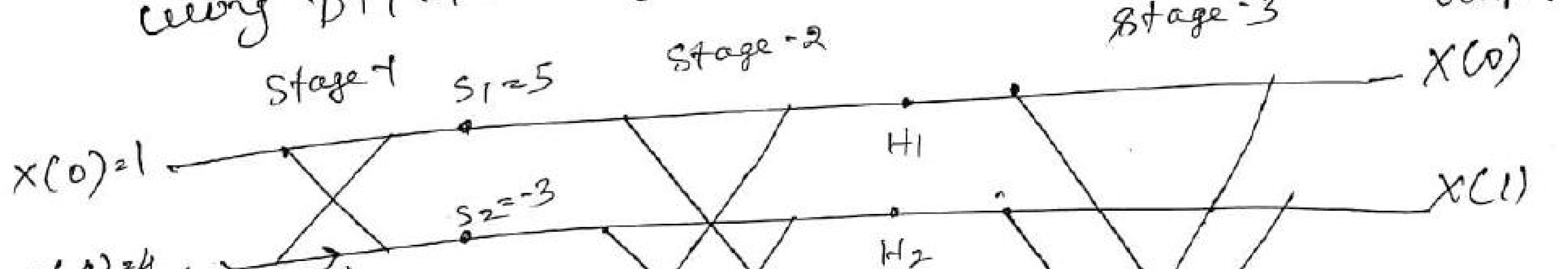
$$W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)\times 2} = e^{-j\pi/2} = \cos \pi/2 - j\sin \pi/2 = -j.$$

$$W_8^3 = e^{-j(2\pi/8)\times 3} = \cos 3\pi/4 - j\sin 3\pi/4 = -0.707 - j0.707.$$

Example:

Q. Find the DFT of sequence: $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIT-FFT algorithm.

$x(0)$ $x(1)$ $x(2)$ $x(4)$ $x(7)$

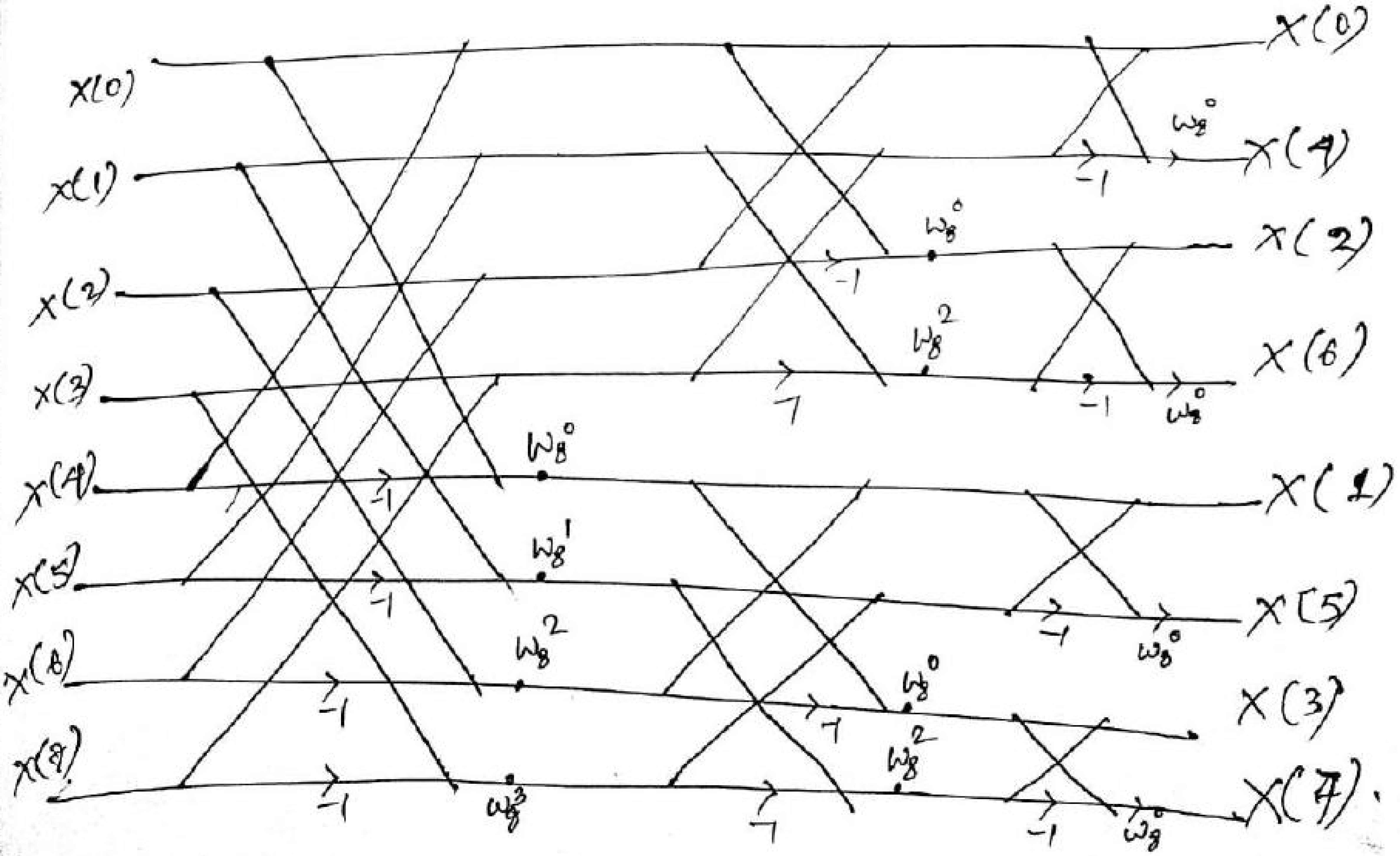


input	output of stage-1	output of stage-2	output.
$x(0)=1$	$S_1 = 1+4 = 5$	$H_1 = S_1 + W_8^0 S_3 = 10$	$X(0) = H_1 + W_8^0 H_5 = 10+10=20$
$x(4)=4$	$S_2 = 1-4 = -3$	$H_2 = S_2 + W_8^2 S_4 = -3-j.$	$X(1) = H_2 + W_8^1 \cdot H_6 = -5.828 - j2.414$

$x(2)=3$	$S_3=3+2=5$	$H_3=S_1-W_8^0S_3=0$	$X(2)=H_3+W_8^2H_7=0$
$x(6)=2$	$S_4=3-2=1$	$H_4=S_2-W_8^2S_4=-3+j$	$X(3)=H_4+W_8^3H_8$ $=-0.172-j0.414$
$x(1)=2$	$S_5=2+3=5$	$H_5=S_5+W_8^0S_7=10$	$X(4)=H_1-W_8^0H_5$ $=10-10=0$
$x(5)=3$	$S_6=2-3=-1$	$H_6=S_6+W_8^2S_8=-1-3j$	$X(5)=H_2-W_8^1H_6$ $=-0.172+j0.414$
$x(3)=4$	$S_7=4+1=5$	$H_7=S_5-W_8^0S_7=5-5$ $=0$	$X(6)=H_3-W_8^3H_7$ $=0-(-j)0=0$
$x(7)=1$	$S_8=4-1=3$	$H_8=S_6-W_8^2S_8$ $=-1+3j$	$X(7)=H_4-W_8^3H_8$ $=-5.828+j2.414.$

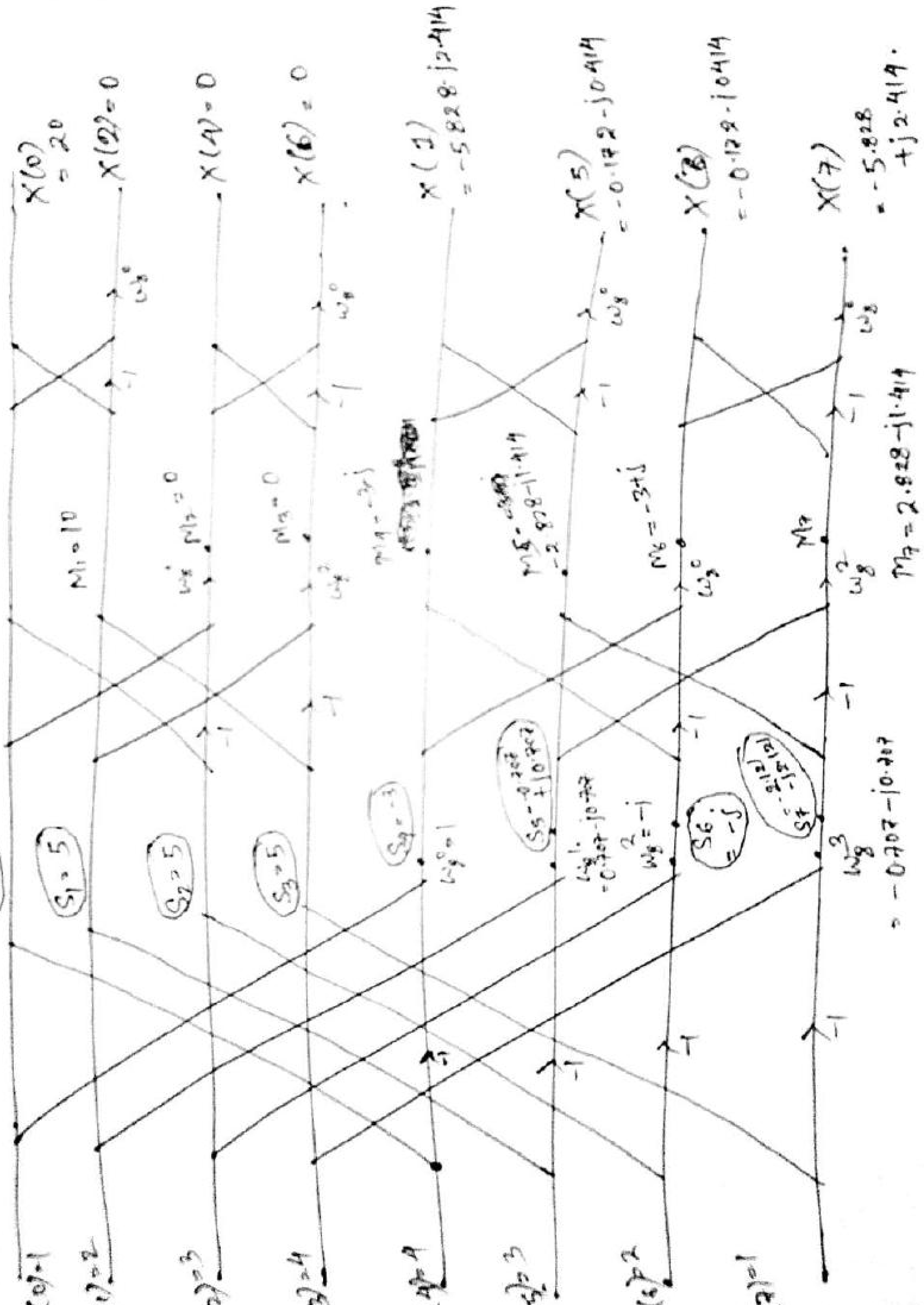
$$X(K)=\{20,\ -5.828-j2.414,\ 0,\ -0.172-j0.414,\ 0,\ -0.172+j0.414,\ 0,\ -5.828+j2.414\}$$

Flow graph of DIF-FFT algorithm : $N=8$ (DIF-FFT algorithm)



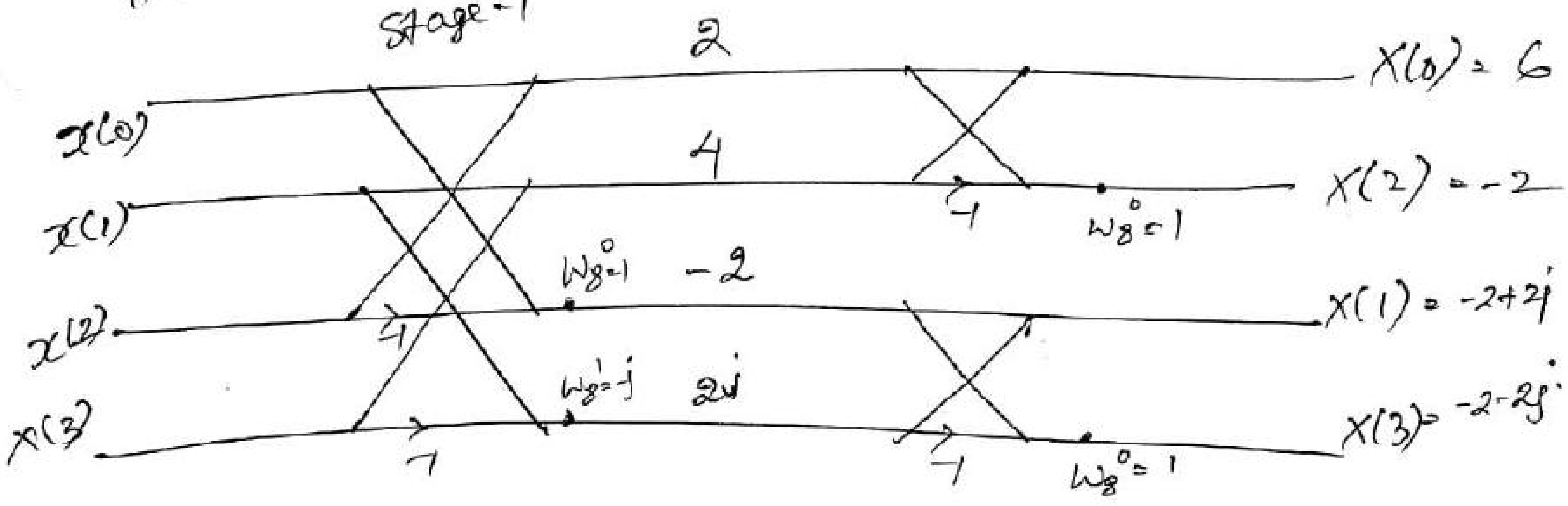
Q. Find the DFT of the sequence :

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIF-FFT algorithm



Q. Find the DFT of the sequence $x(n) = \{0, 1, 2, 3\}$ using DIF-FFT algorithm.

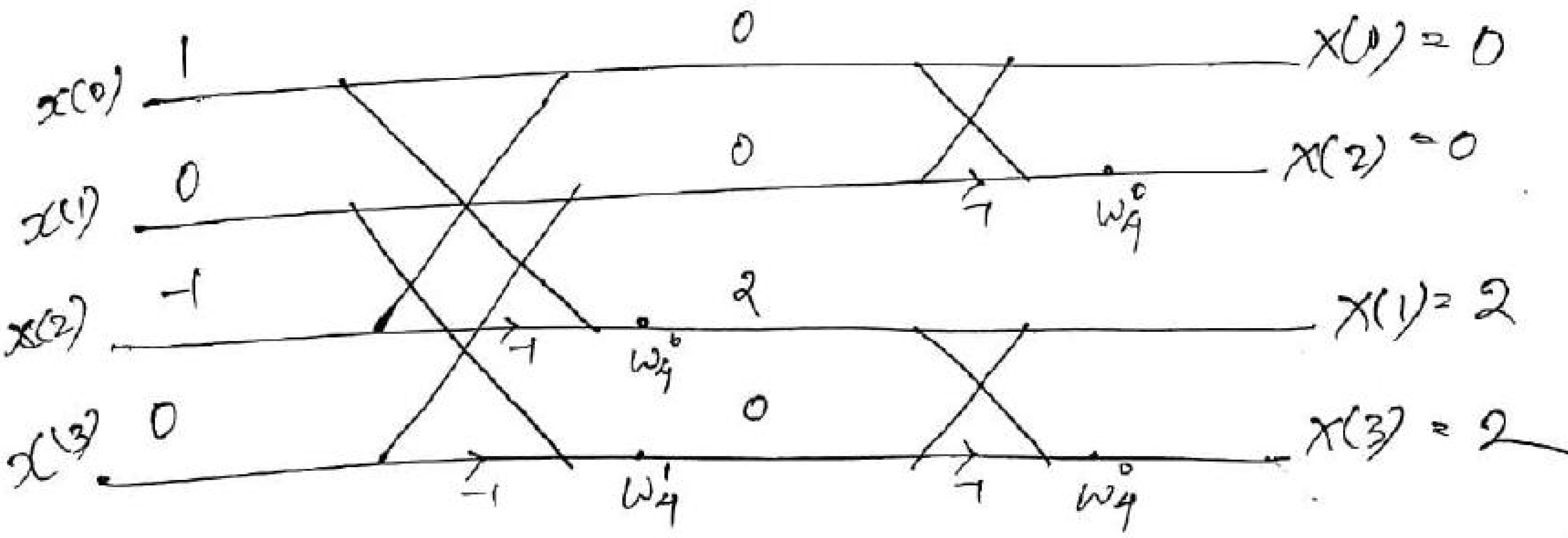
Here $N = 4$.



Q. Find DFT of the sequence $x(n) = \cos\frac{\pi n}{2}$, where $N = 4$ using DIF-FFT algorithm.

$x(n) = \cos\frac{n\pi}{2}$, and $N = 4$.

$x(n) = \{\cos 0, \cos\pi/2, \cos\pi, \cos 3\pi/2\} = \{1, 0, -1, 0\}$



5.5 Introduction to Digital filter (FIR filter)

Introduction

* Filters are two types ① analog filter & digital filter.

So we know

Fourier Transform

$$h(t) \xleftrightarrow{CTFT} H(j\omega)$$

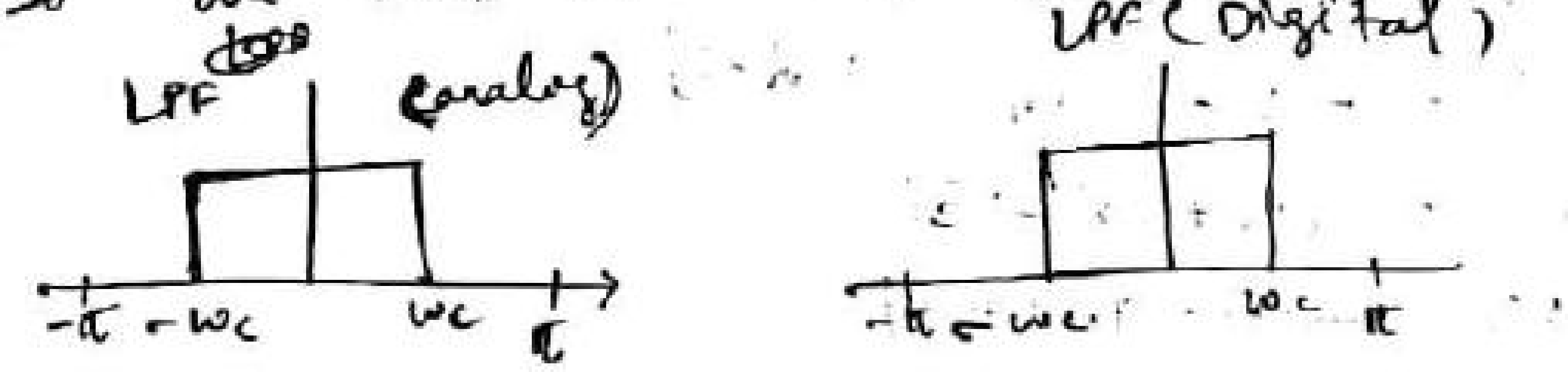
CTFT = continuous time fourier Transform

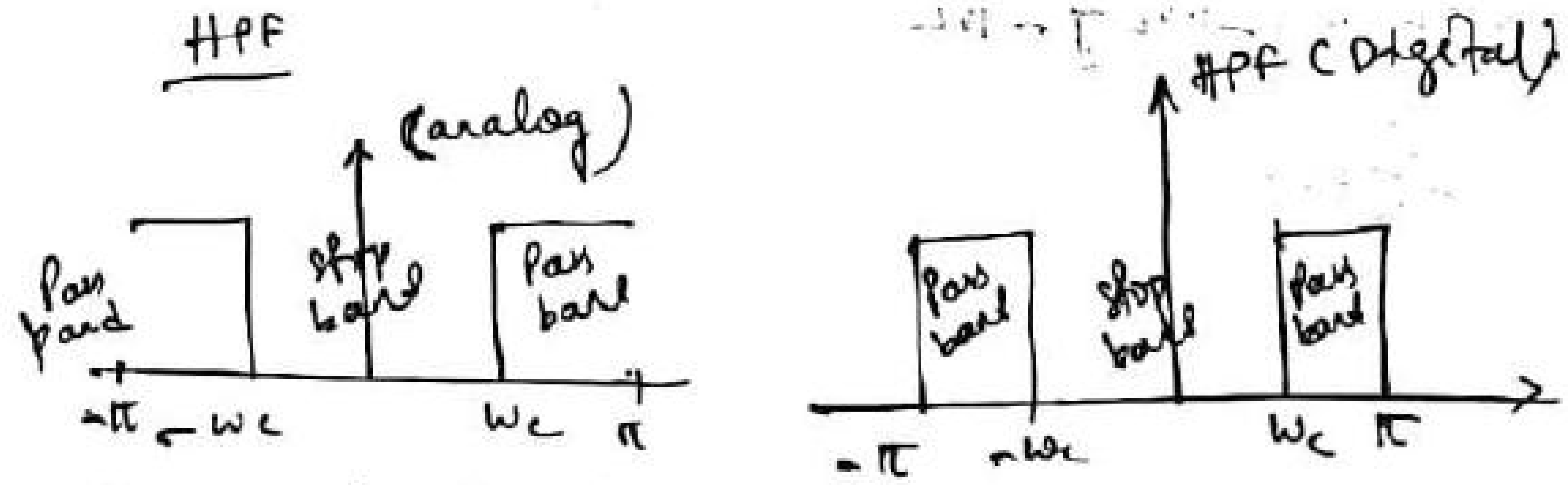
$$h(n) \xleftrightarrow{DTFT} H(e^{j\omega})$$

DTFT = Discrete Time fourier Transform

So analog filter we already know i.e. Lowpass filter, Highpass filter, band pass filter & band stop filter.

So we see





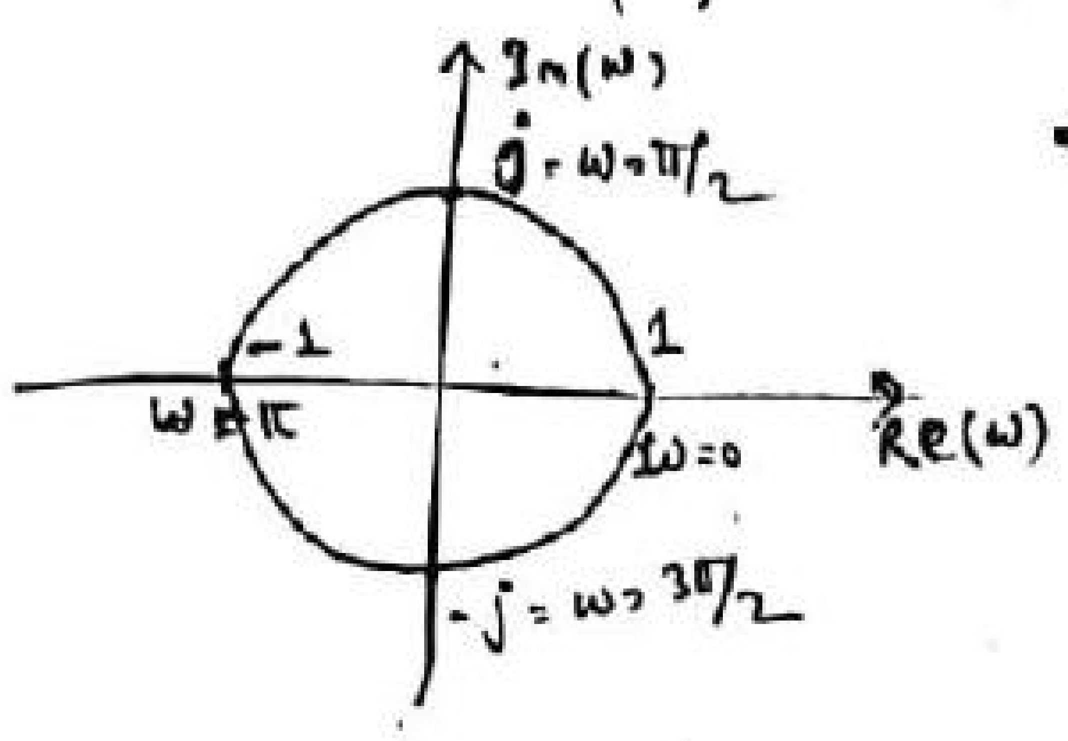
So we check at ~~low freq~~

	low freq	High freq
Analog	$w=0$	$w=\infty$
Digital	$w=0$	$w=?$

So we have know what is the angular frequency in Digital filter.

So we already study Z-Transfer.

We know $|z| = re^{j\omega}$, r = magnitude



if $|z| = r = 1$

$$\boxed{z = e^{j\omega}}$$

if we put $w = 0$.

$z = e^0$

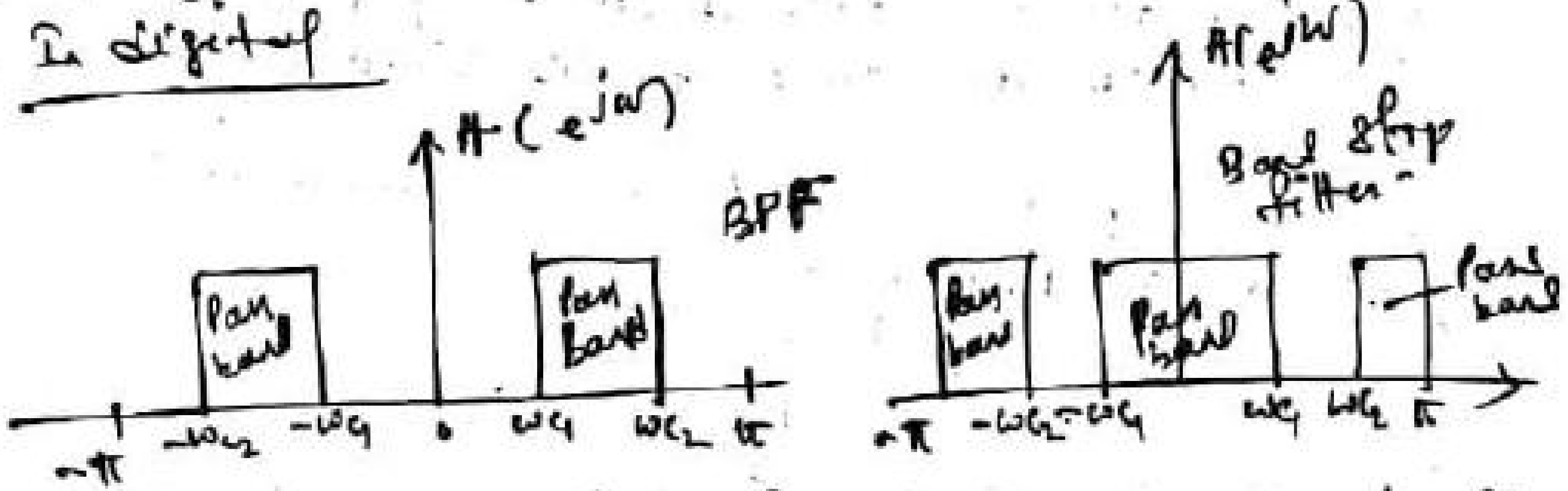
$= 1$

→ then $\omega = \pi$:
$|z| = e^{j\pi} = -1$

→ $\omega = \pi/2$
$|z| = e^{j\pi/2} = j$

we already z - Transform is discrete time function. So we study 'ω' value by putting different value

<u>In digital</u>



* Electronic circuits which perform signal processing functions, remove unwanted frequency component, to enhance wanted ones both.
→ Digital filter - performs mathematical operation on a sampled, discrete time signal to reduce or enhance certain aspects of that signal.

→ filter generally do not add frequency component to a signal.
→ Boost or attenuate selected frequency region.

<u>Classification of digital filter</u>
→ Depending on the form of the filter eqⁿ
a) linear filter Vs non-linear filter
b) Time-invariant filter Vs time-varying filter
c) Adaptive filter Vs non-adaptive filter
d) Recursive filter Vs non-recursive filter
→ Depending on the structure of implementation
a) Direct form, cascade-form, parallel form, & lattice structure.

* <u>Finite Impulse response filter</u> :-
→ FIR filter are preferred over their IIR (Infinite Impulse response filter)
* FIR ~~always~~ filter always stable.
* FIR filter with exactly linear phase can easily be designed.
* FIR filter can be realized in both recursive & non-recursive structures.
* FIR filter are free of limit cycle oscillations, when implemented on a finite word length digital system.

Analog filter	Digital filter
→ Analog filter processes Analog inputs.	→ Digital filter processes Digital input data.
→ Analog filters are constructed from Active and passive Electronic Components.	→ digital filter consists of Elements like adder, multiplier, delay unit
→ Analog filter described by differential Equations	→ It is described by difference Equation
→ frequency Response Can be modified by changing the component	→ frequency Response can be changed by changing the filter coefficients.

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system;
* Excellent design methods are available for kinds of FIR filter.

disadvantages of FIR filter are

→ The implementation of narrow transition band FIR filters are very costly, as it requires considerably more arithmetic operation and hardware components such as multiplier, adders & delay element.

→ memory requirement & execution time are very high

it operates only on previous values of the input

$$\boxed{y[n] = \sum_{k=0}^{M} b(k)\, x(n-k)}$$

5.5. Introduction to DSP architecture, familiarisation of different types of processor

Different types of processor

→ The programmable digital signal processors are general purpose microprocessor designed specifically for digital signal processing application. They contain special purpose architecture and instruction set so as to execute computation-intensive DSP algorithms more efficiently.

i) General purpose digital signal processor:-

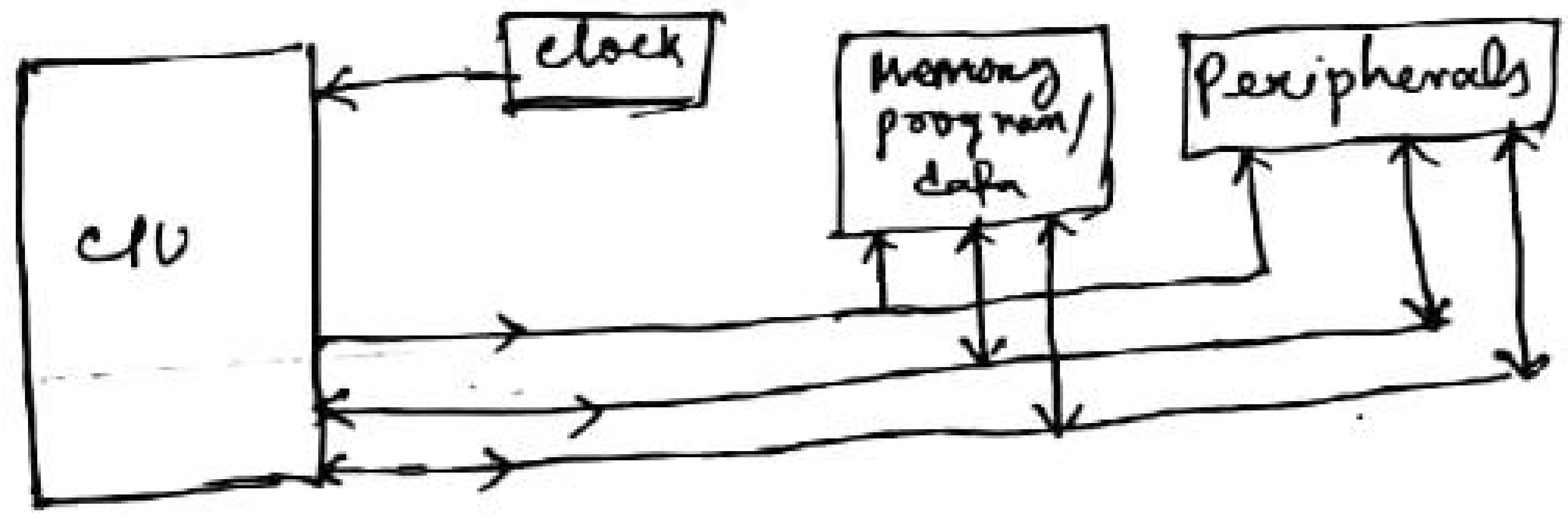
These are basically high speed microprocessors with architecture and instruction sets optimized for DSP operations. They include fixed point processors such as Texas instruments TMS320C5x, TMS320C54x & Motorola DSP563x & floating point processor such as Texas instruments TMS320C4x, TMS320C67xx & analog device ADSP21xxx.

(ii) Special purpose digital signal processors:-

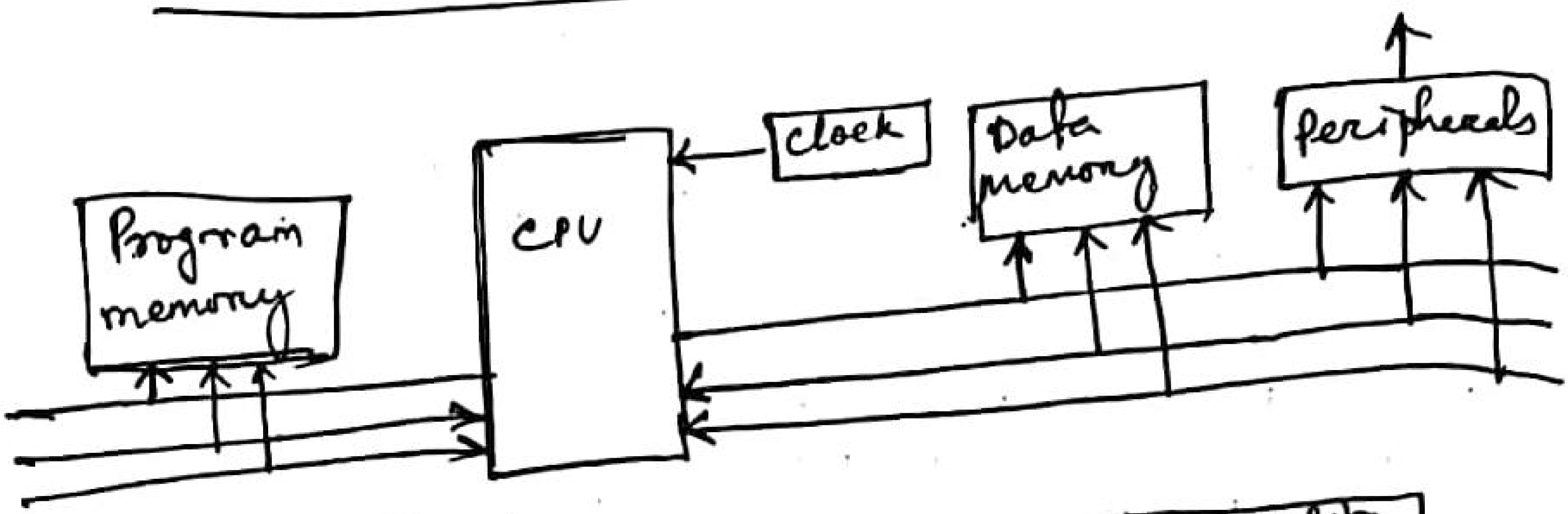
These type of processors consists of hardware i) designed for specific DSP algorithms such as FFT, (ii) hardware designed for specific applications such as PCM & filtering. Examples for special purpose DSPs are Mitel's multi channel telephony voice echo canceller (MT93001), FFT processor (PDSP 16515A, TM-44, TM-66) & programmable FIR filter (UPDSP 16256, model 3092).

Different types of Architecture:-

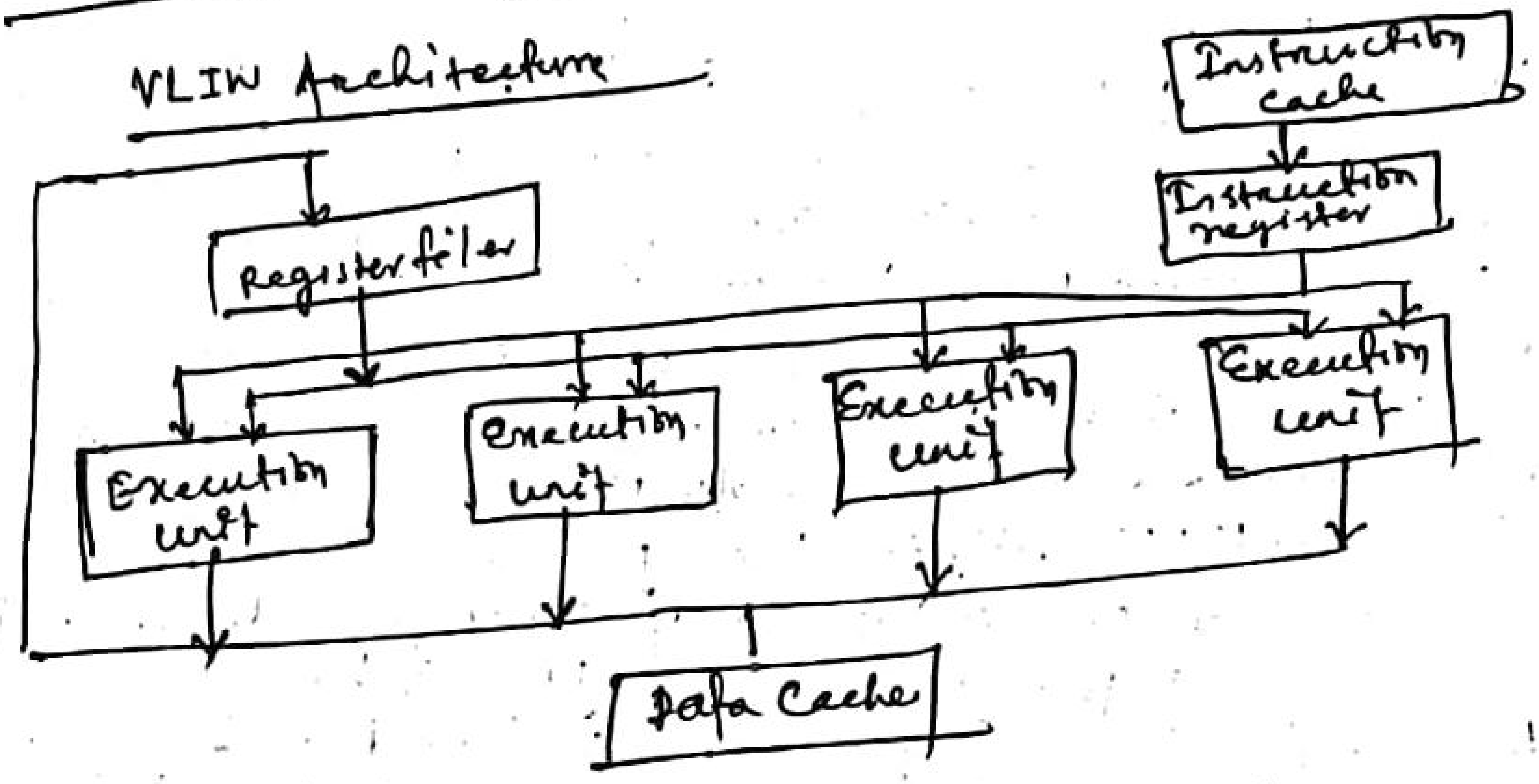
→ Von Neuman Architecture →



Harvard architecture



VLIW Architecture



VLIW (very long · Instruction word)