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MODULE-1

PRINCIPLE OF COMBINATIONAL LOGIC

Truth table representation :

1) Develop a truth table for a system with 2 bit (Binary digit) binary numbers & generate 3 outputs namely:

1^{st} output indicates, when two no. differ by two or more.
2^{nd} output indicates, when two no. are identical
3^{rd} output indicates, when first no. exceeds 2^{nd} no.

i/p				o/p		
$a\ 2^3$	$b\ 2^2$	$c\ 2^1$	$d\ 2^0$	x	y	z
0	0	0	0	0	1	0
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	0	0
0	1	1	1	1	0	0
1	0	0	0	1	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	0	0	0
1	1	0	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

2) Design the truth table representation of the system which takes 2 bit binary no. as its input generates an output to indicate when the sum of two numbers is odd.

i/p a 2^3	b 2^2	c 2^1	d 2^0	o/p x
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

3) Develop a truth table in order to represent the no. of days in a month for a non-leap year indicating the output of invalid inputs if any by zero.

Months	a^8 (i/p)	b^4	c^2	d^1	x (o/p)	y	z	P	Q
JAN	0	0	0	0	1	1	1	1	1
FEB	0	0	0	1	1	1	1	0	0
MAR	0	0	1	0	1	1	1	1	1
APR	0	0	1	1	1	1	1	1	0
MAY	0	1	0	0	1	1	1	1	1
JUN	0	1	0	1	1	1	1	1	0
JUL	0	1	1	0	1	1	1	1	1
AUG	0	1	1	1	1	1	1	1	1
SEP	1	0	0	0	1	1	1	1	0
OCT	1	0	0	1	1	1	1	1	1
NOV	1	0	1	0	1	1	1	1	0
DEC	1	0	1	1	1	1	1	1	1
Invalid	1	1	0	0	0	0	0	0	0
Invalid	1	1	0	1	0	0	0	0	0
Invalid	1	1	1	0	0	0	0	0	0
Invalid	1	1	1	1	0	0	0	0	0

3 posibilities of o/p's are 28, 30, 31

* Convert decimal to binary.

1) $(28)_{10} = 11100$

2 | 28
2 | 14 – 0
2 | 7 – 0
2 | 3 – 1
1 – 1

2) $(30)_{10} = 11110$

2 | 30 –
2 | 15 – 0
2 | 7 – 1
2 | 3 – 1
1 – 1

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3) $(31)_{10} = 11111$

2 | 31
2 | 15 – 1
2 | 7 – 1
2 | 3 – 1
1 – 1

4) Represent the truth table for 4 input system indicating when majority of its inputs are true.

a	b	c	d		x
0	0	0	0		0
0	0	0	1		0
0	0	~~0~~ 1	0		0
0	0	~~0~~ 1	1		0
0	1	0	0		0
0	1	0	1		0
0	1	1	0		0
0	1	1	1		1
1	0	0	0		0
1	0	0	1		0
1	0	1	0		0
1	0	1	1		1
1	1	0	0		0
1	1	0	1		1
1	1	1	0		1
1	1	1	1		1

5) Write a truth table for 4 i/p logic system to indicate when numbers divisible by 3 or 5 occur

i/p				o/p
a (8)	b (4)	c (2)	d (1)	x
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Defination of combinational logic:

When logic gates are connected together to produce a specified o/p for certain specified combinations to input variables with no storage involved. The resulting circuit is called the combinational logic.

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Canonical forms:

* Canonical is a word used to describe the condition of a switching eqⁿ.
* In general, the word canonical means confining to a general rule.

The rules for switching logic is that each term used in a switching eqⁿ must contain all the available input variables & literals.

Two formats generally exist for expressing switching eqⁿ in a canonical or standard form.

(1) Sum of min terms [Sum of products (SOP)]
(2) Product of max terms. [POS]

Steps to convert SOP to canonical (or) std form:-

(1) find the missing literal in each product term if any.
(2) AND each product term having the missing literal with the term formed by ORing the literal & its compliment.
(3) Expand the terms by applying the distributive law & rearrange the order.
(4) Reduce the exp. by omitting the repeated product terms of any because $A+A = A$ [a/c Boolean law]

Eg (1): Convert the given exp. into standard canonical form.

$$f(A,B,C) = AC + AB + BC$$

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→ $f(A,B,C) = AC + AB + BC$

literal B is missing

literal C is missing

literal A is missing

$= AC\cdot(B+\bar{B}) + AB\cdot(C+\bar{C}) + BC\cdot(A+\bar{A})$

$= ACB + AC\bar{B} + ABC + AB\bar{C} + ABC + \bar{A}BC$ [∵ BL]

$= ABC + \bar{A}BC + A\bar{B}C + AB\bar{C}$

<u>Steps to convert POS to canonical form:</u>

(1) find the missing literals in each sum terms if any.

(2) OR each sum term having missing literal with the term formed by ANDing the literals & its compliment.

(3) Expand the terms by applying the distributive law & rearrange the orders.

(4) Reduce the exp. by omitting the repeated sum terms if any because $A\cdot A = A$.

<u>Eg (1):</u> $f(A,B,C) = (A+B)(B+C)(A+C)$

literal C is missing

literal A is missing

literal B is missing

$= (A+B)+(C\cdot\bar{C})\cdot(B+C)+(A\cdot\bar{A})\cdot(A+C)+(B\cdot\bar{B})$

$= (A+B+C)(A+B+\bar{C})\ (A+B+C)(B+C+\bar{A})(A+C+B)$
$(A+C+\bar{B})$

$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)$

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Generation of switching eq^n from the truth table:

Variables A B C	Min terms m_i	Max terms M_i
0 0 0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0 0 1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0 1 0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0 1 1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1 0 0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1 0 1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1 1 0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1 1 1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

Min term & Max term canonical formula:

There are 2 types of boolean exp. that are directly obtained from the given truth table.

(1) Min term canonical formula
(2) Max term canonical formula

Min term canonical formula:

In this form all the literals must be present in each product term. Therefore min term canonical form is a standard SOP form with each product term consisting of all the literals in their normal or complicated form.

Now let us consider a truth table for a 3 variable boolean function.

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Eg (1):

i/p			o/p
A	B	C	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

from the truth table we see that o/p function 'f' is 1 for following i/p combination.

$0\ 0\ 0 = 1 = \bar{A}\ \bar{B}\ \bar{C}$

$0\ 1\ 1 = 1 = \bar{A}\ B\ C$

$1\ 0\ 1 = 1 = A\ \bar{B}\ C$

$1\ 1\ 0 = 1 = A\ B\ \bar{C}$

Here in the i/p combination a "0" bit is replaced by the compliment of the corresponding variable & a "1" bit is replaced by its normal variable & then we can write the 4 product terms of 3 variables as follows:

$\bar{A}\bar{B}\bar{C}$, $\bar{A}BC$, $A\bar{B}C$, $AB\bar{C}$

By using these 4 min terms we can write the min term canonical form as,

$f(A,B,C) = (\bar{A}\bar{B}\bar{C}) + (\bar{A}BC) + (A\bar{B}C) + (AB\bar{C})$

$f(A,B,C) = m_0 + m_3 + m_5 + m_6$

Then further the same min term notation can be written as:

$$f(A,B,C) = \Sigma m(m_0, m_3, m_5, m_6)$$

And this can also be written as:

$$f(A,B,C) = \Sigma m(0,3,5,6).$$

2) Write the following in boolean exp. in terms of 'm' notation [min term].

$$f(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}bc\bar{d} + a\bar{b}\bar{c}\bar{d} + ab\bar{c}\bar{d} + abcd.$$

	a b c d	f	
0 0 0 0	$\bar{a}\ \bar{b}\ \bar{c}\ \bar{d}$	1	m_0
0 0 0 1	$\bar{a}\ \bar{b}\ \bar{c}\ d$	0	m_1
0 0 1 0	$\bar{a}\ \bar{b}\ c\ \bar{d}$	0	m_2
0 0 1 1	$\bar{a}\ \bar{b}\ c\ d$	0	m_3
0 1 0 0	$\bar{a}\ b\ \bar{c}\ \bar{d}$	0	m_4
0 1 0 1	$\bar{a}\ b\ \bar{c}\ d$	0	m_5
0 1 1 0	$\bar{a}\ b\ c\ \bar{d}$	1	m_6
0 1 1 1	$\bar{a}\ b\ c\ d$	0	m_7
1 0 0 0	$a\ \bar{b}\ \bar{c}\ \bar{d}$	1	m_8
1 0 0 1	$a\ \bar{b}\ \bar{c}\ d$	0	m_9
1 0 1 0	$a\ \bar{b}\ c\ \bar{d}$	0	m_{10}
1 0 1 1	$a\ \bar{b}\ c\ d$	0	m_{11}
1 1 0 0	$a\ b\ \bar{c}\ \bar{d}$	1	m_{12}
1 1 0 1	$a\ b\ \bar{c}\ d$	0	m_{13}
1 1 1 0	$a\ b\ c\ \bar{d}$	0	m_{14}
1 1 1 1	$a\ b\ c\ d$	1	m_{15}

$\bar{a}\bar{b}\bar{c}\bar{d} = 0000 = m_0$

$\bar{a}bc\bar{d} = 0110 = m_6$

$a\bar{b}\bar{c}\bar{d} = 1000 = m_8$

$ab\bar{c}\bar{d} = 1100 = m_{12}$

$abcd = 1111 = m_{15}$

In terms of min terms notation,

$f(a,b,c,d) = m_0 + m_6 + m_8 + m_{12} + m_{15}$

Now the same exp. will be written in terms of Σm.

$f(a,b,c,d) = \Sigma m(m_0, m_6, m_8, m_{12}, m_{15})$

$= \Sigma m(0,6,8,12,15)$

3) Write the following boolean exp. in terms of min term notation.

$f = a + ab + a\bar{c}d$

→ The given exp. contains 4 variable functions. Since the given exp. is not in the standard min term canonical form we need to first convert the given eqn in terms of std. form using steps:

$f = a + ab + a\bar{c}d$

$f(a,b,c,d) = a(b+\bar{b})(c+\bar{c})(d+\bar{d}) + ab(c+\bar{c})(d+\bar{d}) + a\bar{c}d(b+\bar{b})$

$= (ab + a\bar{b})(c+\bar{c})(d+\bar{d}) + (abc + ab\bar{c})(d+\bar{d}) + (a\bar{c}bd + a\bar{c}\bar{b}d)$

$= (abc + a\bar{b}\bar{c} + ab\bar{c} + a\bar{b}c)(d+\bar{d}) + (abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d}) + (a\bar{c}db + a\bar{c}d\bar{b})$

$= (abcd + a\bar{b}\bar{c}d + ab\bar{c}d + a\bar{b}cd + abc\bar{d} + a\bar{b}\bar{c}\bar{d} + ab\bar{c}\bar{d} + a\bar{b}c\bar{d}) + (abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d}) + (ab\bar{c}d + a\bar{b}\bar{c}d)$

$= abcd + a\bar{b}\bar{c}d + ab\bar{c}d + a\bar{b}cd + abc\bar{d} + a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d}$

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Refer previous problem table.

$a\,b\,c\,d = 1\,1\,1\,1 = m_{15}$

$a\,\bar{b}\,c\,d = 1\,0\,1\,1 = m_{11}$

$a\,b\,\bar{c}\,d = 1\,1\,0\,1 = m_{13}$

$a\,\bar{b}\,\bar{c}\,d = 1\,0\,0\,1 = m_9$

$a\,b\,c\,\bar{d} = 1\,1\,1\,0 = m_{14}$

$a\,\bar{b}\,c\,\bar{d} = 1\,0\,1\,0 = m_{10}$

$a\,b\,\bar{c}\,\bar{d} = 1\,1\,0\,0 = m_{12}$

$a\,\bar{b}\,\bar{c}\,\bar{d} = 1\,0\,0\,0 = m_8$

In terms of min term notation.

$f(a,b,c,d) = m_8 + m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$

Now, the same expression can be written in terms of $\Sigma m(m_8, m_9, m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15}) = f(a, b, c, d)$.

$\therefore f(a,b,c,d) = \Sigma m(8, 9, 10, 11, 12, 13, 14, 15)$.

Maxterm Canonical form:

In this form all the literals must be present in sum term [maxterm of Boolean exp.]. Since the maxterm is of the compliment of the minterm the maxterm canonical formula is written as o/p function zero.

Consider the truth table for a 3 bit variable

i/p			o/p
A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

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from the above truth table we find that the output function 'f' is zero for the following input combinations.

000, 100, 101, 111

Here each zero bit is replaced by its normal variable & each 1 bit is replaced by its compliment variable

$$000 = A + B + C$$
$$100 = \bar{A} + B + C$$
$$101 = \bar{A} + B + \bar{C}$$
$$111 = \bar{A} + \bar{B} + \bar{C}$$

Hence we get four sum term of 3 literals each. Each which corresponds to it output function 'f' then the above 4 terms will be represented into max term.

$$f(a,b,c) = (a+b+c)(\bar{a}+b+c) - - - -$$

Now representing the same eq^n in terms of M ~~to~~ notation

$$f(a,b,c) = (M_0 \cdot M_4 \cdot M_5 \cdot M_7)$$
$$\boxed{f(a,b,c) = \pi M(0,4,5,7)}$$

1) Write the given boolean exp. in terms of M notation.

$$f(a,b,c,d) = (a+b+c+d)(a+\bar{b}+c+d)(\bar{a}+b+c+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$$

8 4 2 1

$$\rightarrow (a+b+c+d) = 0000 = M_0$$
$$(a+\bar{b}+c+d) = 0100 = M_4$$
$$(\bar{a}+b+c+d) = 1000 = M_8$$
$$(\bar{a}+\bar{b}+c+d) = 1100 = M_{12}$$
$$(\bar{a}+\bar{b}+\bar{c}+\bar{d}) = 1111 = M_{15}$$

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$f(a,b,c,d) = M_0 \cdot M_4 \cdot M_8 \cdot M_{12} \cdot M_{15}$

Now by using π notation.

$$\boxed{f(a,b,c,d) = \pi M(0,4,8,12,15)}$$

2) Convert the given boolean exp. $f = a + ab + a\bar{c}d$ in terms of M notation.

→ $f = a + ab + a\bar{c}d$

$a \rightarrow bcd$
$ab \rightarrow cd$
$a\bar{c}d \rightarrow b$

$\left.\begin{matrix} bcd \\ cd \\ b \end{matrix}\right\}$ is missing

$f(a,b,c,d) =$

$= a + (b\cdot\bar{b})(c\cdot\bar{c})(d\cdot\bar{d})$

It can be written as:

$f = (a)(a+b)(a+\bar{c}+d)$

$(a) \rightarrow bcd$
$(a+b) \rightarrow cd$
$(a+\bar{c}+d) \rightarrow b$

$f(a,b,c,d) = (a) + (b\cdot\bar{b})(c\cdot\bar{c})(d\cdot\bar{d}) \cdot (a+b) + (c\cdot\bar{c})(d\cdot\bar{d})$

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Simplification of boolean expression:

This involves reducing the boolean exp. with less no. of literals. Basically we have 2 simplification methods:

(1) Karnaugh method.
(2) Quine-Mccluskey method.

Formulation of simplification of problems:

In order to get the optimal digital circuits the following factors should be considered:

* **Network cost:**

This includes the component cost, design cost, assembly cost & manufacturing cost.

* **Network reliability:**

The digital network must be reliable to operate at a long time coz the reliable components shows less variation in their performance like variable parameters that include temp, time, environmental condition, etc. ∴ By using the reduction ~~techo~~ technique an alternative circuit is designed that takes over the control of the digital system in the event of failure.

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Power consumption:

The power consumption should be less in case of digital networks so that the source of power can be given in terms of battery or cell.

*2M-3M

Karnaugh maps:-

* It is a graphical method of simplifying the boolean functions.

* This k map method is the easiest method of determining prime implicates & implicants of a given boolean expression.

Hence the simplified boolean expression obtained by using k Map always corresponds to the minimal boolean expressions either in product or sum term [SOP or POS]

for 'n' variable boolean function the k map consists of 2^n boxes called cells. for eg: A 3 variable boolean function requires $2^3 = 8$ cells & 5 variable boolean function requires $2^5 = 32$ cells for the purpose of simplification & each cell of a K-map is assigned with product or sum term of the boolean function.

We know that for every 3 variable function 2^3 or 8 rows in the truth table & a four variable function has 16 rows in the truth table describing the function.

A 'n' variable function as 2^n or n rows in truth table.

A 'K' map for given truth table of boolean

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function is drawn such that each row of a truth table corresponds to a particular cell in the map that means a particular row of a truth table locates a particular cell in the K-map.

A bit 1 is placed in a cell corresponding to a row of the truth table for which the boolean function 'f' assumes a value 1, & '0' is placed in a cell corresponding to the row for which a function 'f' assumes '0'.

"The cells are so arranged that physically adjacent cells are also logically adjacent".

"Two terms are said to be logically adjacent if they differ only in one literal"

Eg: Let us consider 2 variable $\bar{a}b$, this term is adjacent to two other terms namely ab & $\bar{a}\bar{b}$.

Consider for a 3 variable term $a\bar{b}c$, this is adjacent to 3 other terms namely abc, $\bar{a}\bar{b}c$, $a\bar{b}\bar{c}$.

Now consider for a 4 variable terms $abcd$ this is adjacent to 4 other terms namely $\bar{a}bcd$, $a\bar{b}cd$, $ab\bar{c}d$, $abc\bar{d}$.

∴ We can say that a 'n' variable term is logically adjacent to n terms.

Hence in a K map a cell corresponds to 'n' variable term will be logically adjacent to 'n' other cells.

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One variable & Two variable k maps:-

* A one variable k map of a one variable boolean function consists of 2 cells [2^1 = 2 cells]. The cell no. is written at the top most corner in a side of the cell.

⇒ Let us consider a boolean expression f(a) & Let it be $f(a) = \bar{a}$, this function describes an Inverter, Now we write truth table of $f(a) = \bar{a}$ along with cell number.

	i/p	o/p	
Cell number	a	f(a)	
0	0	1	→ f(0)
1	1	0	→ f(1)

The above truth table can be diagramatically represented in K-map as shown in the above

0	1	→ Inputs
0	1	→ Cell number
1	0	→ Outputs

from the K-map we observed that the physically adjacent cells vary.

Hence those cells are logically adjacent. Note that input combination 0 (or) $\bar{a}$ is located in cell no. '0' & input combination 1 (or) a is located in cell no. 1.

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* A 2 variable k-map of a 2 variable boolean function consists of 4 cells [i.e., $2^2 = 4$ cells]

⇒ Now, Let us consider a following 2 variables function $f(a,b) = \bar{a}b + a\bar{b}$.

Now, we shall write the truth table along with cell number.

	i/p		o/p
cell no.	a	b	f(a,b)
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

Now K-map of above table is written as

	0	1
0	0 [0]	1 [1]
1	1 [2]	0 [3]

Three & four variable k-map:

A 3 variable k-map of a boolean function f(a,b,c) having 3 variables consists of $2^3 = 8$ cells

Consider the following 3 variable boolean expression:

$f(a,b,c) = \bar{a}\bar{b}\bar{c} + a\bar{b}c + abc$

Now, we shall write the truth table along the cell no.

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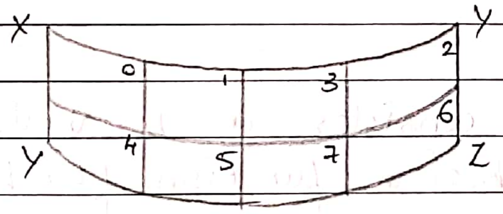
Cell no:	i/p a	b	c	o/p
0	0	0	0	1
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

Now K-map of given table is written as:

a \ bc	00	01	11	10
0	1 (0)	0 (1)	0 (3)	0 (2)
1	0 (4)	1 (5)	1 (7)	0 (6)

→ cell no. (superscript numbers); → o/p (cell values); i/p



Imagine the folding of the above K-map for a cylinder with edge x, y in contact with edge 'YZ'. This assures that cell 0 is physical adja~~stant~~cent to cell 4, cell 1, cell 2.

Cell 4 is physically ~~adjusant~~ adjacent to cell 0, cell 5, cell 6.

Cell 2 is physically adjacent to cell 0, cell 6, cell 3.

Cell 6 is physically adjacent to cell 2, cell 7, cell 4.

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The physically adjacent ~~tines~~ cells are also logically adjacent [differing by only one bit] i.e, cell 0 $(\overset{\bar{a}}{0}, \overset{\bar{b}}{0}, \overset{\bar{c}}{0})$ is logically adjacent to cell 4 $(\overset{a}{1}, \overset{\bar{b}}{0}, \overset{\bar{c}}{0})$, cell 1 $(\overset{\bar{a}}{0}, \overset{\bar{b}}{0}, \overset{c}{1})$ & cell 2 $(\overset{\bar{a}}{0}, \overset{b}{1}, \overset{\bar{c}}{0})$

cell 4 $(\overset{a}{1}, \overset{\bar{b}}{0}, \overset{\bar{c}}{0})$ is logically adjacent to cell 5 $(\overset{a}{1}, \overset{\bar{b}}{0}, \overset{c}{1})$, cell 0 $(\overset{\bar{a}}{0}, \overset{\bar{b}}{0}, \overset{\bar{c}}{0})$, cell 6 $(\overset{a}{1}, \overset{b}{1}, \overset{\bar{c}}{0})$

cell 2 $(\overset{\bar{a}}{0}, \overset{b}{1}, \overset{\bar{c}}{0})$ is logically adjacent to cell 0 $(\overset{\bar{a}}{0}, \overset{\bar{b}}{0}, \overset{\bar{c}}{0})$, cell 6 $(\overset{a}{1}, \overset{b}{1}, \overset{\bar{c}}{0})$, cell 3 $(\overset{\bar{a}}{0}, \overset{b}{1}, \overset{c}{1})$.

cell 6 $(\overset{a}{1}, \overset{b}{1}, \overset{\bar{c}}{0})$ is logically adjacent to cell 2 $(\overset{\bar{a}}{0}, \overset{b}{1}, \overset{\bar{c}}{0})$, cell 7 $(\overset{a}{1}, \overset{b}{1}, \overset{c}{1})$ & cell 4 $(\overset{a}{1}, \overset{\bar{b}}{0}, \overset{\bar{c}}{0})$.

* <u>K-map for 4 variable boolean function with the help of truth table:</u>

Now let us consider a 4 variable K-map for a 4-variable boolean function: $f(a,b,c,d)$ which has $2^4 = 16$ cells.

Now consider a 4 variable boolean function $f(a,b,c,d) = \Sigma m(0,3,5,11,12,14,15)$.

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Cell no:	i/p a	b	c	d	o/p f(a,b,c,d)
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	1 (3)	0 (2)
01	0 (4)	1 (5)	0 (7)	0 (6)
11	1 (12)	0 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	1 (11)	0 (10)

* **Karnaugh maps & canonical formula:**

Consider the following 3 variable min term canonical formula.

$$f(a,b,c) = \bar{a}\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + abc$$

$$f(a,b,c) = \Sigma m(0,0,1 + 100 + 101 + 111)$$

$$f(a,b,c) = \Sigma m(1,4,5,7).$$

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Since 'f' has 3 variables the corresponding K-map consists of $2^3 = 8$ cells.

Now enter 1's in the k-map cells corresponding to the min terms 1, 4, 5, 7.

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

fig(1)

And enter the remaining cells with 0's, Now we observe that cells no's 1, 4, 5, 7 represent the min terms of the given boolean funcⁿ i.e., these cells represents the input combination 001, 100, 101 & 111 respectively. Because these i/p combinations make the function 'f' to assume '1'.

Therefore we can say that the cell nos. 1, 4, 5 & 7 represents the min terms $\bar{a}\bar{b}c$, $a\bar{b}\bar{c}$, $a\bar{b}c$ and abc respectively.

Alternatively the above funⁿ 'f' given in min term canonical formula can be written in the max term canonical formula as

$$f(a,b,c) = \pi M(0, 2, 3, 6).$$

~~Alternati~~ We can write the k-map for this max term canonical formula. ∴ Enter zeroes in the cells ~~sense~~ corresponding to the max term represented by the decimal values 0, 2, 3 & 6. And the remaining cells should be filled with 1's.

Now we can write the k-map for the max term.

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

fig(2)

We observe from fig(1) & fig(2) that both the k-maps are one or the same.

$$\Sigma m(1,4,5,7) = \pi M(0,2,3,6).$$

Problem:

1) Write the algebraic form of the function k-map shown below.

a \ bc	00	01	11	10
0	1	0	0	1
1	1	1	0	1

→ We shall rewrite the above k-map along with cell nos.

a \ bc	00	01	11	10
0	1 (0)	0 (1)	0 (3)	1 (2)
1	1 (4)	1 (5)	0 (7)	1 (6)

from the k-map we can write the min term canonical formula in terms of

$f(a,b,c) = \Sigma m(0,2,4,5,6)$.

i.e., $f(a,b,c) = \Sigma \cancel{m}(\bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + a\bar{b}\bar{c} + a\bar{b}c + ab\bar{c})$

Alternatively from the above k-map we can write the max-term canonical formula in terms of M notation also,

$f(a,b,c) = \pi M(1,3,7)$

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$f(a,b,c) = (a+b+\bar{c})(a+\bar{b}+\bar{c})(\bar{a}+\bar{b}+\bar{c})$

Also we can write the truth table for $f(a,b,c)$ directly for the above k-map.

cell no.	a	b	c	f(a,b,c)
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

2) Write the Karnaugh - map for the following function
$f(a,b,c) = (a+c)(b+c)(\bar{b}+\bar{c})$

cell no.	a	b	c	f(a,b,c)
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

a \ bc	00	01	11	10
0	0 ^{0}	1 ^{1}	0 ^{3}	0 ^{2}
1	0 ^{4}	1 ^{5}	0 ^{7}	1 ^{6}

3) Write the k-map for $f(a,b,c) = \bar{a}\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + abc$

Cell no.	a	b	c	f(a,b,c)
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

So we observe that the given function is in min-term canonical formula. Hence no need ~~to~~ to write the truth table, we can directly write the m-notation.

$\therefore f(a,b,c) = \Sigma m(1,4,5,7)$

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

<u>NOTE</u>: If a boolean function is given as an expressions of min term canonical formula, then by writing the truth table along with cell no. also we can get the k-map.

4) Write the k-map for $f(a,b,c) = (\bar{a}+\bar{b}+c)(a+\bar{b}+\bar{c})(a+\bar{b}+c)(a+b+c)$

→ $f(a,b,c) = \pi M(6,3,2,0)$

$= \pi M(0,2,3,6)$

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

5) Write the k-map for:

(a) $f(a,b,c) = ab + \bar{b}c$

(b) $f(a,b,c,d) = \Sigma m(0,2,7,9,10,14,15)$.

(c) $f(a,b,c,d) = \bar{a}\bar{b}c\bar{d} + \bar{a}bcd + a\bar{b}\bar{c}d + ab\bar{c}d$.

(a) $f(a,b,c) = ab + \bar{b}c$

$ab(c+\bar{c}) + \bar{b}c(a+\bar{a})$

$abc + ab\bar{c} + a\bar{b}c + \bar{a}\bar{b}c$

Cell no:	a	b	c	f(a,b,c)
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	0 (4)	1 (5)	1 (7)	1 (6)

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Min terms ⇒ 1 = Σm = $\bar{0}$

Max terms ⇒ 0 = πM = $\bar{1}$

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(b) $f(a,b,c,d) = \Sigma m(0, 2, 7, 9, 10, 14, 15)$.

cell no:	a	b	c	d	f(a,b,c,d)
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

ab \ cd	00	01	11	10
00	1^{0}	0^{1}	0^{3}	1^{2}
01	0^{4}	0^{5}	1^{7}	0^{6}
11	0^{12}	0^{13}	1^{15}	1^{14}
10	0^{8}	1^{9}	0^{11}	1^{10}

(c) $f(a,b,c,d) = \bar{a}\bar{b}c\bar{d} + \bar{a}bcd + a\bar{b}\bar{c}d + ab\bar{c}d$.

$= \Sigma m(0010, \overset{0111}{\cancel{1000}}, \overset{1001}{\cancel{0110}}, \overset{1101}{\cancel{0010}})$

$= \cancel{\Sigma m(13, 8, 6, 2)}$ $\quad \Sigma m(2, 7, 9, 13)$

ab \ cd	00	01	11	10
00	0^{0}	0^{1}	0^{3}	1^{2}
01	0^{4}	0^{5}	1^{7}	0^{6}
11	0^{12}	1^{13}	0^{15}	0^{14}
10	0^{8}	1^{9}	0^{11}	0^{10}

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Product & Sum term on a k-map:

* We know that for a given n-variable boolean function we can write an 'n' variable k-map, with 2^n nos of cells.

* The given boolean function can then be simplified with less no. of literals by groping all possible no. of 1's or 0's.

* Grouping of 1's gives implecants of the given function & grouping of 0's gives impleeants of the given function. i.e., grouping of 1's gives product terms & grouping of 0's gives sum term.

* Grouping of two 1's or two 0's ~~k~~ in a k-map eliminates one variable in product or sum term.

* This means that a product term or sum term with one less variable is obtained.

* Grouping of four 1's or four 0's eliminates two variables in a term of the given boolean function, this means that a product or sum term with two less variables is obtained.

* Similarly grouping of eight 1's or eight 0's eliminates three variables of a boolean function & so on.

* In a k-map a cell filled with functional value 1 [f = 1]. is called as 1 cell & a with a

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functional value 0 (f = 0) is called as zero cell.

* Thus in general a 'n' variable boolean function which is associated with n-variable k-map consists of 2^n cells. When all the 2^n cells are grouped then all the variables of a given boolean function are eliminated & hence there are no product & or sum term exist.

* In general the dimension of the rectangular grouping of the k-map is given by $2^P \times 2^Q$ where 2^P is the no. of columns & 2^Q is the no. of rows present in the rectangular grouping.

* The dimension of the rectanglar grouping is also called as sub ~~gs~~ cubes. Therefore an 'n' variable 'k' map with rectangular grouping of 1 cells of the size $2^P \times 2^Q$ gives the product term (n - P - Q) variables, here P = 0, 1, 2, 3, ... similarly Q = 0, 1, 2, 3, ...

* following are the examples of rectangular grouping or sub cubes.

Eg1: $2^0 \times 2^0 = 1 \times 1 = 1$

This shows that when P = 0 & Q = 0, we get a sub cube containing only 1 cell & that sub cube has 1 column & 1 row in size.

∴ The size of the sub cube = 1 × 1.

Eg2: $2^0 \times 2^1 = 1 \times 2 = 2$

Here P = 0 & Q = 1, with this the sub cube contains 2 cells & the sub cube has 1 column & 2 rows & the size of sub cube = 1 × 2 = 2

Eg3: $2^1 \times 2^1 = 2 \times 2 = 4$

Here P=1 & Q=1, with this the sub cube contains 4 cells & the sub cube has 2 columns & 2 rows & size of sub cube = $2 \times 2 = 4$

Eg4: $2^1 \times 2^2 = 2 \times 4 = 8$

Here P=1 & Q=2, with this the sub cube contains 8 cells & the sub cube has 2 columns & 4 rows & size of sub cube = $2 \times 4 = 8$.

To illustrate the grouping of cells i.e., rectangular grouping or sub cubes:

ab \ cd	00	01	11	10
00	0 (0)	1 (1)	0 (3)	1 (2)
01	0 (4)	0 (5)	0 (7)	1 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	1 (10)

Let us now carry out the rectangular grouping of 1 cells as shown below.

Group G_1 $(2^0 \times 2^0 = 1 \times 1 = 1)$

ab \ cd	00	01	11	10
00	0 (0)	$\boxed{1}$ (1)	0 (3)	1 (2)
01	0 (4)	0 (5)	0 (7)	1 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	1 (10)

→ Group G_2 $(2^1 \times 2^0 = 2 \times 1 = 2)$

→ Group G_3 $(2^0 \times 2^2 = 1 \times 4 = 4)$

From the above k-map, group G_1 is the product term having four variables i.e., $G_1 = \bar{a}\bar{b}\bar{c}d$. We can verify this as follows:

The size of the subcube of group G_1 is 1 i.e., $2^0 \times 2^0 = 1 \times 1 = 1$. & it has only 1 cell with only one column & 1 row with P = 0 & Q = 0.

$\therefore$ n − P − Q

= 4 − 0 − 0

= 4.

for 4 variable boolean function, n = 4.

$\therefore$ The product term of the sub cube G_1 consists of 4 variables namely $\bar{a}, \bar{b}, \bar{c}$ & d.

Now consider the group G_2 which consists of 2 min terms namely $ab\bar{c}d$ (13) & $abcd$ (15).

These two min terms are grouped to obtain a min term with 1 less variable.

$$ab\bar{c}d + abcd$$
$$= abd(c + \bar{c}) \qquad \because \bar{c} + c = 1$$
$$= abd$$

$\therefore$ n − P − Q

= 4 − 1 − 0

= 3

With this we obtain 3 variables namely, a, b, d.

Similarly group G_3, consists of 4 min terms namely $\bar{a}\bar{b}c\bar{d}$ (2), $\bar{a}bc\bar{d}$(6), $abc\bar{d}$(13), $a\bar{b}c\bar{d}$(10).

$$= \bar{a}\bar{b}c\bar{d} + \bar{a}bc\bar{d} + abc\bar{d} + a\bar{b}c\bar{d}$$
$$= \bar{a}c\bar{d}(b + \bar{b}) + ac\bar{d}(b + \bar{b})$$
$$= \bar{a}c\bar{d} + ac\bar{d}$$
$$= c\bar{d}(a + \bar{a})$$
$$= c\bar{d}.$$

$\therefore$ n − P − Q

= 4 − 0 − ~~4~~ 2

= 2.

We obtain 2 variables namely, c & $\bar{d}$.

* Two cells grouping:

a \ bc	00	01	11	10
0	0 (0)	1 (1)	1 (3)	1 (2)
1	1 (4)	1 (5)	0 (7)	1 (6)

For a given 3 variable k-map, following figure shows all the possible grouping of 2 cells

$G_3 (2^1 \times 2^1 = 2 \times 2 = 4)$

a \ bc	00	01	11	10
0	0 (0)	1 (1)	1 (3)	1 (2)
1	1 (4)	1 (5)	0 (7)	1 (6)

$\rightarrow G_2 (2^2 \times 2^0 = 4 \times 1 = 4)$

$\rightarrow G_1 (2^2 \times 2^0 = 4 \times 1 = 4)$

From the above grouping we get the ~~foot~~ following

$G_1 = a\bar{b}\bar{c} + ab\bar{c}$

$= a\bar{c}(b+\bar{b})$

$G_1 = a\bar{c}$

$G_2 = \bar{a}bc + \bar{a}b\bar{c}$

$= \bar{a}b(c+\bar{c})$

$G_2 = \bar{a}b$

$G_3 = \bar{a}\bar{b}c + a\bar{b}c$

$= \bar{b}c(a+\bar{a})$

$G_3 = \bar{b}c$

$\therefore$ The resulting simplified boolean exp. is $f(a,b,c) = a\bar{c} + \bar{a}b + \bar{b}c$.

* Consider the following K-map consisting of

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ab \ cd	00	01	11	10
00	1 (0)	1 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	1 (11)	0 (10)

for given 4 variable k-map, following fig. shows all the possible grouping of 2 cells.

ab \ cd	00	01	11	10
00	1 (0)	1 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	1 (11)	0 (10)

G_1 $(2^2 \times 2^0 = 4 \times 1 = 4)$

G_2 $(2^1 \times 2^2 = 2 \times 4 = 8)$

G_3 $(2^1 \times 2^1 = 2 \times 2 = 4)$

G_4 $(2^2 \times 2^0 = 4 \times 1 = 4)$

from the above k-map we can get directly the product term with 1 eliminated variable in each group.

$G_1 = \overset{0010}{\bar{a}\bar{b}c\bar{d}} + \overset{0000}{\bar{a}\bar{b}\bar{c}\bar{d}}$

$= \bar{a}\bar{b}\bar{d}(c+\bar{c})$

$G_1 = \bar{a}\bar{b}\bar{d}$

$G_2 = \overset{0001}{\bar{a}\bar{b}\bar{c}d} + \overset{0101}{\bar{a}b\bar{c}d}$

$= \bar{a}\bar{c}d(b+\bar{b})$

$G_2 = \bar{a}\bar{c}d$

$G_3 = \overset{0111}{\bar{a}bcd} + \overset{1111}{abcd}$

$= bcd(a+\bar{a})$

$G_3 = bcd$

$G_4 = \overset{1001}{a\bar{b}\bar{c}d} + \overset{1011}{a\bar{b}cd}$

$= a\bar{b}d(c+\bar{c})$

$G_4 = a\bar{b}d$

$\therefore$ The resulting simplified boolean exp. is

$f(a,b,c,d) = \bar{a}\bar{b}\bar{d} + \bar{a}\bar{c}d + bcd + a\bar{b}d$

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* Grouping of 4 cells:

Consider a 4 variable k-map in which four 1 cells are grouped together to get a simplified product term with reduced variables.

ab \ cd	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	1 (11)	0 (10)

~~c̄d + cd~~

ab \ cd	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	1 (11)	0 (10)

$= ab + a\bar{b}$

$= a(b+\bar{b})$

$= a$

→ 4 cells' group 'G' [Sub cube having four 1-cell]

We observe that four cell sub cube

$\bar{c}d + cd$

$= d(c+\bar{c})$

$= d$

← → 2 columns of sub cube

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We observe that 4 cell sub cubes has 2 rows & 2 columns for the column of the subcube the simplified variable is "d" & for row simplification variable is "a". The multipling (ANDing) this two common terms we get the simplified product term with reduced no. of variables

ab \ cd	00	01	11	10
00	0 (0)	1 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	0 (7)	1 (6)
11	0 (12)	1 (13)	0 (15)	1 (14)
10	0 (8)	1 (9)	0 (11)	1 (10)

ab \ cd	00	01 (G_1)	11	10 (G_2)
00	0 (0)	1 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	0 (7)	1 (6)
11	0 (12)	1 (13)	0 (15)	1 (14)
10	0 (8)	1 (9)	0 (11)	1 (10)
		$\bar{c}d$		$c\bar{d}$

$\bar{a}\bar{b}+\bar{a}b+ab+a\bar{b}$

$=\bar{a}(\bar{b}+b)+a(b+\bar{b})$

$=\bar{a}+a$

$=1$

$G_1 = 1\cdot\bar{c}d$ $\qquad$ $G_2 = 1\cdot c\bar{d}$

$G_1 = \bar{c}d$ $\qquad$ $G_2 = c\bar{d}$

$\therefore f(a,b,c,d) = \bar{c}d + c\bar{d} = (c\oplus d)$

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Problems :-

1)

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	0	0	0	1
10	1	0	1	1

$\bar{a}bd$

G1 ← ; → G2 ; G3 ; G4

$(2^2 \times 2^0 = 4 \times 1 = 4)$

$G_1 = (2^2 \times 2^0 = 4 \times 1 = 4)$

$G_1 = \bar{a}b\bar{c}d + \bar{a}bcd$
$= \bar{a}bd(c+\bar{c})$
$= \bar{a}bd$

$G_2 = \bar{a}bc\bar{d} + abc\bar{d}$
$= bc\bar{d}(a+\bar{a})$
$= bc\bar{d}$

$G_3 = a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d}$
$= a\bar{b}\bar{d}(c+\bar{c})$
$= a\bar{b}\bar{d}$

$G_4 = \bar{a}\bar{b}cd + a\bar{b}cd$
$= \bar{b}cd(a+\bar{a})$
$= \bar{b}cd$

$f(a,b,c,d) = \bar{a}bd + bc\bar{d} + a\bar{b}\bar{d} + \bar{b}cd$

2)

ab \ cd	00	01	11	10
00	0	0	0	1
01	1	1	0	1
11	1	1	0	1
10	0	0	0	1

→ G1 ; G2

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0010 0110 1110 1010

$G_1 = \bar{a}\bar{b}c\bar{d} + \bar{a}bc\bar{d} + abc\bar{d} + a\bar{b}c\bar{d}$

$= \bar{a}c\bar{d}(b+\bar{b}) + ac\bar{d}(b+\bar{b})$

$= \bar{a}c\bar{d} + ac\bar{d}$

$= c\bar{d}(a+\bar{a})$

$G_1 = c\bar{d}$

0100 0101 1100 1101

$G_2 = \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + ab\bar{c}\bar{d} + ab\bar{c}d$

$= \bar{a}b\bar{c}(d+\bar{d}) + ab\bar{c}(d+\bar{d}) = \bar{a}b\bar{c} + ab\bar{c}$

$G_2 = b\bar{c}$

$f(a,b,c,d) = c\bar{d} + b\bar{c}$

3)

ab \ cd	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	0 (13)	0 (15)	1 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

$\rightarrow G_1$

$\rightarrow G_2$

0100 0101 0111 0110

$G_1 = \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + \bar{a}bc\bar{d}$

$= \bar{a}b\bar{c}(d+\bar{d}) + \bar{a}bc(d+\bar{d})$

$= \bar{a}b\bar{c} + \bar{a}bc$

$= \bar{a}b(c+\bar{c})$

$= \bar{a}b$

1100 1000 1110 1010

$G_2 = ab\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d} + abc\bar{d} + a\bar{b}c\bar{d}$

$= a\bar{c}\bar{d}(b+\bar{b}) + ac\bar{d}(b+\bar{b})$

$= a\bar{c}\bar{d} + ac\bar{d}$

$= a\bar{d}(c+\bar{c}) = a\bar{d}$

$f(a,b,c,d) = \bar{a}b + a\bar{d}$

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4)

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	0 (12)	0 (13)	0 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

$\rightarrow G_1$

$$G_1 = \overset{0000}{\bar{a}\bar{b}\bar{c}\bar{d}} + \overset{1000}{a\bar{b}\bar{c}\bar{d}} + \overset{0010}{\bar{a}\bar{b}c\bar{d}} + \overset{1010}{a\bar{b}c\bar{d}}$$
$$= \bar{b}\bar{c}\bar{d}(a+\bar{a}) + \bar{b}c\bar{d}(a+\bar{a})$$
$$= \bar{b}\bar{c}\bar{d} + \bar{b}c\bar{d}$$
$$= \bar{b}\bar{d}(c+\bar{c})$$
$$G_1 = \bar{b}\bar{d}$$

5)

	00	01	11	10
00	0 (0)	1 (1)	1 (3)	1 (2)
01	1 (4)	1 (5)	0 (7)	0 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	0 (9)	1 (11)	1 (10)

$\rightarrow G_3$ $\rightarrow G_2$ G_1 G''

$$G_1 = \overset{0000}{abcd} + \overset{1000}{\bar{a}bcd}$$
$$= bcd(a+\bar{a})$$
$$= bcd$$
$$G_2 = \overset{1100}{\bar{a}\bar{b}cd} + \overset{1110}{\bar{a}\bar{b}\bar{c}d}$$

$$G_1 = (a+b+c+d)(\bar{a}+b+c+d)$$
$$= (b+c+d)(a+\bar{a})$$
$$G_1 = (b+c+d)$$

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$G_2 = (\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+d)$
$\quad = (\bar{a}+\bar{b}+d)(c+\bar{c})$
$G_2 = (\bar{a}+\bar{b}+d)$

$G_3 = \cancel{a\bar{b}\bar{c}\bar{d}} + \cancel{a\bar{b}\bar{c}d}$
$\quad = \cancel{a\bar{b}\bar{c}(d+\bar{d})}$

$G_3 = (a+\bar{b}+\bar{c}+\bar{d})(a+\bar{b}+\bar{c}+d)$
$\quad = (a+\bar{b}+\bar{c})(d+\bar{d})$
$G_3 = (a+\bar{b}+\bar{c})$

$G_4 = \cancel{\bar{a}\bar{b}c\bar{d}} + \cancel{\bar{a}bc\bar{d}}$

$G_4 = (\bar{a}+\bar{b}+c+\bar{d})(\bar{a}+b+c+\bar{d})$
$\quad = (\bar{a}+c+\bar{d})(b+\bar{b})$
$G_4 = (\bar{a}+c+\bar{d})$

$f(a,b,c,d) = (b+c+d)(\bar{a}+\bar{b}+d)(a+\bar{b}+\bar{c})(\bar{a}+c+\bar{d})$

6)

	00	01	11	10	
00	0 (0)	0 (1)	0 (3)	0 (2)	→ G_1
01	1 (4)	0 (5)	0 (7)	1 (6)	→ G_2
11	1 (12)	0 (13)	0 (15)	1 (14)	
10	1 (8)	1 (9)	1 (11)	1 (10)	

$G_1 = (a+b+c+d)(a+b+c+\bar{d})(a+b+\bar{c}+\bar{d})(a+b+\bar{c}+d)$
$\quad = (a+b+c)(d+\bar{d}) + (a+b+\bar{c})(d+\bar{d})$
$\quad = (a+b)(c+\bar{c})$
$G_1 = (a+b)$

$G_2 = \cancel{\bar{a}b\bar{c}d}\ (a+\bar{b}+c+\bar{d})(a+\bar{b}+\bar{c}+\bar{d})(\bar{a}+\bar{b}+c+\bar{d})(\bar{a}+\bar{b}+\bar{c}+\bar{d})$
$\quad = (a+\bar{b}+\bar{d})(c+\bar{c})(\bar{a}+\bar{b}+\bar{d})(c+\bar{c})$
$\quad = (\bar{b}+\bar{d})(a+\bar{a})$
$G_2 = (\bar{b}+\bar{d})$

$f(a,b,c,d) = (a+b)(\bar{b}+\bar{d})$

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7)

	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	0	1	1	1
10	0	1	0	0

G_1 $\quad$ $\rightarrow G_2$

0000 $\quad$ 0100 $\quad$ 1100 $\quad$ 1000

$$G_1 = (a+b+c+d)(a+\bar{b}+c+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+b+c+d)$$
$$= (a+c+d)(b+\bar{b}) + (\bar{a}+c+d)(b+\bar{b})$$
$$= (c+d)(a+\bar{a})$$
$$G_1 = (c+d)$$

0011 $\quad$ 0010 $\quad$ 1011 $\quad$ 1010

$$G_2 = (a+b+\bar{c}+\bar{d})(a+b+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+b+\bar{c}+d)$$
$$= (a+b+\bar{c})(d+\bar{d}) + (\bar{a}+b+\bar{c})(d+\bar{d})$$
$$= (b+\bar{c})(a+\bar{a})$$
$$G_2 = (b+\bar{c})$$

$$f(a,b,c,d) = (c+d)(b+\bar{c})$$

8)

	00	01	11	10
00	0 (0)	1 (1)	1 (3)	0 (2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	1 (9)	1 (11)	0 (10)

0000 $\quad$ 1000 $\quad$ 0010 $\quad$ 1010

$$G_1 = (a+b+c+d)(\bar{a}+b+c+d)(a+b+\bar{c}+d)(\bar{a}+b+\bar{c}+d)$$
$$= (b+c+d)(a+\bar{a})(b+\bar{c}+d)(a+\bar{a})$$
$$= (b+c+d)(b+\bar{c}+d)$$
$$= (b+d)(c+\bar{c})$$
$$G_1 = (b+d)$$

$$f(a,b,c,d) = (b+d)$$

* Grouping of 8 cells:

The fig. below shows the grouping of 8 cells of a four variable k-map.

ab \ cd	00	01	11	10
00	0 (0)	1 (1)	1 (3)	0 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	1 (11)	0 (10)

→ G_1

$\bar{a}\bar{b}+\bar{a}b+ab+a\bar{b}$
$= \bar{a}(b+\bar{b})+a(b+\bar{b})$
$= a+\bar{a}$
$= 1$

$\bar{c}d+cd$
$d(c+\bar{c})$
$= d$

$f(a,b,c,d) = 1 \times d = d.$

The product term for this function,
$f(a,b,c,d) = d$

In the above eg. of grouping 8 cells, all the rows in the k-map are grouped together & hence the variable ab representing row will vanish but all the ~~coutt~~ coulmns are not grouped & only the variable 'c' is ~~available~~. eliminated.

fig. below shows the grouping of 8 cells of a four variable k-map.

ab \ cd	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

cd

ab \ cd	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$ab + a\bar{b} + \bar{a}\bar{b} + \bar{a}b$
$= a(b+\bar{b}) + \bar{a}(b+\bar{b})$
$= a + \bar{a}$
$= 1$

$\bar{c}\bar{d}\bar{b}$ $\bar{a}bc\bar{d}$

$f(a,b,c,d) = 1 \times \bar{c}\bar{d} \times c\bar{d}$

2)

cd

ab \ cd	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

cd

ab \ cd	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$\bar{a}\bar{b} + \bar{a}b + ab + a\bar{b}$
$= \bar{a}(b+\bar{b}) + a(b+\bar{b})$
$= a + \bar{a}$
$= 1$

$\bar{c}\bar{d} + \bar{c}d + cd + c\bar{d}$
$= \bar{c}(d+\bar{d}) + c(d+\bar{d})$
$= 1$

$f(a, b, c, d) = 1$.

∴ When all the cells of the k-map are grouped all the variables are eliminated. & the boolean function 'f' is $f(a, b, c, d) = 1$.

* **Minimal expressions for complete boolean functions using k-map.**

The boolean exp. consisting of only essential prime implicants ~~are~~ of essential prime implecents are called as Minimal boolean exp.

Prime implicants & Karnaugh maps:-

All possible groupings of 1's cells in the k-map gives prime implicants.

Eg: Consider the following boolean function, $f(a,b,c) = \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + abc$.
(001, 010, 011, 111)

$f(a, b, c) = m_1, m_2, m_3, m_7$
$= \Sigma m(1, 2, 3, 7)$

Now let us draw the k-map & do the grouping of 1 cells as bellow:

a \ bc	00	01	11	10
0	0_0	1_1	1_3	1_2
1	0_4	0_5	1_7	0_6

$G_1 = \bar{a}\bar{b}c + \bar{a}bc$
$= \bar{a}c(b + \bar{b})$
$G_1 = \bar{a}c$

$G_2 = \bar{a}bc + \bar{a}b\bar{c}$
$= \bar{a}b(c + \bar{c})$
$= \bar{a}b$

$G_3 = \bar{a}bc + abc$
$= bc(a + \bar{a})$
$= bc$

from the k-map we get a simplified boolean function:

$f(a,b,c) = \bar{a}c + \bar{a}b + bc$

The 3 product terms $\bar{a}c$, $\bar{a}b$ & bc are the prime implicants because these 3 terms do not subsume each other.

2) Consider the following boolean function of a four variable function, $f(a,b,c,d) = \Sigma m(1,3,5,6,7,14)$.

ab \ cd	00	01	11	10
00	0 (0)	1 (1)	1 (3)	0 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	0 (13)	0 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	0 (10)

ab \ cd	00	01	11	10
00	0 (0)	1 (1)	1 (3)	0 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	0 (13)	0 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	0 (10)

G1, G2, G3

$G_1 = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bcd$ c̄ & d is eliminated

$= \bar{a}\bar{b}d(c+\bar{c}) + \cancel{\bar{a}b\bar{c}(d+\bar{d})}\ \bar{a}bd(c+\bar{c})$

$G_1 = \cancel{a}\ \bar{a}\bar{b}d + \cancel{\bar{a}b\bar{c}}\ \bar{a}bd = \bar{a}d(b+\bar{b}) = \bar{a}d$

$G_2 = \bar{a}bcd + \bar{a}bc\bar{d}$

$= \bar{a}bc(d+\bar{d})$

$G_2 = \bar{a}bc$

$G_3 = \bar{a}bc\bar{d} + abc\bar{d}$

$= bc\bar{d}(a+\bar{a})$

$G_3 = bc\bar{d}$

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$f(a,b,c,d) = \bar{a}bc + bc\bar{d} + \bar{a}\cancel{b}d \cancel{+ abc}$.

* Essential prime implicants:

Consider once again the k-map. In this k-map we observe that cell no. 6 & 7 are already included in the sub cubes G_1 & G_3. ∴ The product term obtained by the group G_2 ~~are~~ or sub group G_2 is not an essential prime implicant.

Hence the fig. below shows the grouping of essential prime implicants.

ab \ cd	00	01	11	10
00	0^{0}	1^{1}	1^{3}	0^{2}
01	0^{4}	1^{5}	1^{7}	1^{6}
11	0^{12}	0^{13}	0^{15}	1^{14}
10	0^{8}	0^{9}	0^{11}	0^{10}

ab \ cd	00	01	11	10
00	0^{0}	1^{1}	1^{3}	0^{2}
01	0^{4}	1^{5}	$\boxed{1}^{7}$	$\boxed{1}^{6}$
11	0^{12}	0^{13}	0^{15}	$\boxed{1}^{14}$
10	0^{8}	0^{9}	0^{11}	0^{10}

Thus we get the further simplified boolean exp. with essential prime implicants only [The non-essential prime implicant term $\bar{a}bc$ is not found in this simplified boolean exp.]. Hence we can say that a boolean exp. in the SOP form with essential prime implicants has its product terms can be considered as minimal boolean exp. or Minimal sums or minimal sum of product.

Eg: Consider the boolean ~~exp.~~ function $f(a,b,c) = 1,3,4,5$

a \ bc	00	01	11	10
0	0 [0]	1 [1]	1 [3]	0 [2]
1	1 [4]	1 [5]	0 [7]	0 [6]

a \ bc	00	01	11	10
0	0 [0]	1 [1]	1 [3]	0 [2]
1	1 [4]	1 [5]	0 [7]	0 [6]

G1 G2 G3

Solution:

Observe that the one cells corresponding to cell nos. 3 & 4, These are referred to as will appear only in one sub cube. essential 1's cells.

Essential 1 cells are those 1 cells which can be part of one & only 1 cube

The prime implicant form from the sub cube containing essential 1 cells are reffered to as essential prime implicants.

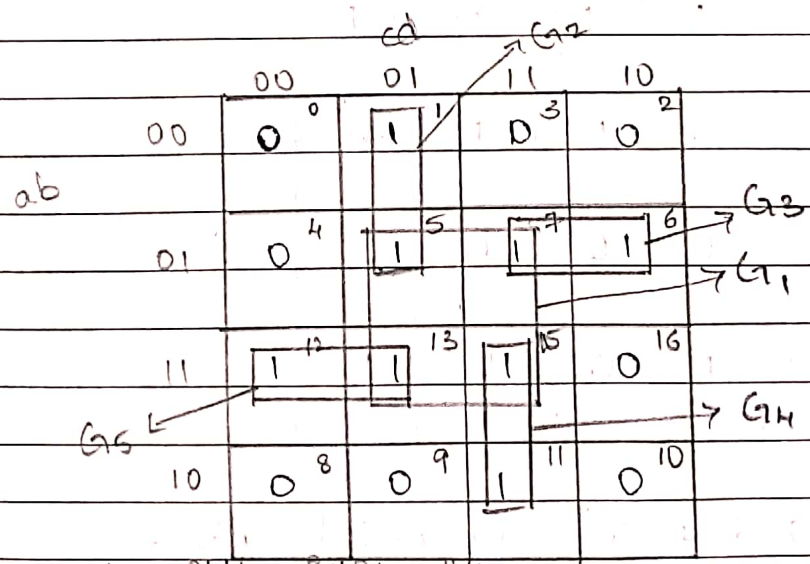
Observe in the above k-map that the 1 cells corresponding to cell nos. 1 & 5 are part of sub cubes G_1 & G_3 respectively. i.e., the cell 1 corresponding to cell no. 1 is a part of the sub cube G_1. & as well as G_2 & cell 1 corresponding to cell no. 5 is a part of sub cube G_2 as well as G_3. So these 1 cells are not essential 1 cells.

The 1-cell corresponding to 3 & 4 is a part of sub cube G_1 only. & cell no. 4 is a part of sub cube G_3 only. ∴ The essential prime implicants formed from G_1 & G_3 are the essential prime.

∴ $f(a,b,c) = \bar{a}\bar{b}c + \bar{a}bc$.

1) Consider the function $f(a,b,c,d) = \Sigma m(1,5,6,7,11,12,13,15)$.

ab \ cd	00	01	11	10
00	0^{0}	1^{1}	0^{3}	0^{2}
01	0^{4}	1^{5}	1^{7}	1^{6}
11	1^{12}	1^{13}	1^{15}	0^{16}
10	0^{8}	0^{9}	1^{11}	0^{10}



$$G_1 = \overset{0101}{\bar{a}b\bar{c}d} + \overset{0111}{\bar{a}bcd} + \overset{1101}{ab\bar{c}d} + \overset{1111}{abcd}$$

$$= \bar{a}bd(c+\bar{c}) + abd(c+\bar{c})$$

$$= bd(a+\bar{a})$$

$$G_1 = bd$$

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$G_2 = \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}d$

$= \bar{a}\bar{c}d(b+\bar{b})$

$\therefore G_2 = \bar{a}\bar{c}d$

$G_3 = \bar{a}bcd + \bar{a}bc\bar{d}$

$= \bar{a}bc(d+\bar{d})$

$G_3 = \bar{a}bc$

$G_4 = abcd + a\bar{b}cd$

$= acd(b+\bar{b})$

$G_4 = acd$

$\therefore G_5 = ab\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d}$

$= a\bar{c}\bar{d}(b+\bar{b})$

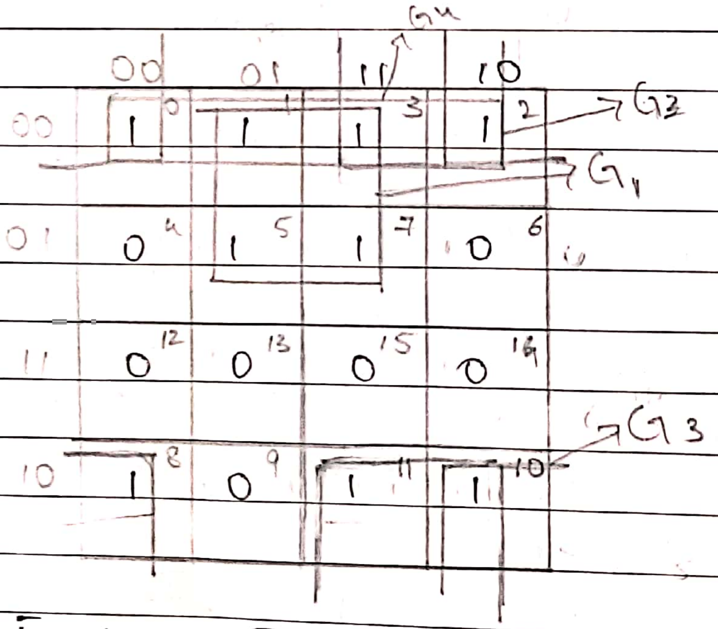
$G_5 = a\bar{c}\bar{d}$

$f(a,b,c,d) = bd + \bar{a}\bar{c}d + \bar{a}bc + acd + a\bar{c}d$

2)

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	1$	1	0
11	0	0	0	0
10	1	0	1	1

$\bar{a}d + \bar{b}c + \bar{b}\bar{d}$



$G_1 = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bcd$

$= \bar{a}\bar{b}d(c+\bar{c}) + \bar{a}bd(c+\bar{c})$

$= \bar{a}d(b+\bar{b})$

$= \bar{a}d$

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Ess $G_2 = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}\bar{b}c\bar{d}$

$= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c$

$= \bar{a}\bar{b}$

Not essential

$G_3 = a\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + a\bar{b}c\bar{d}$

$= \bar{b}\bar{c}\bar{d} + \bar{b}c\bar{d}$

$= \bar{b}\bar{d}$

$G_4 = \bar{a}\bar{b}cd + \bar{a}\bar{b}c\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d}$

$= \bar{a}\bar{b}c + a\bar{b}c$

$= \bar{b}c$

$f(a,b,c,d) = \bar{a}d + \bar{b}\bar{d} + \bar{b}c \; (+ \; \bar{a}\bar{b})$

Essential prime implicates: (POS)

ab \ cd	00	01	11	10
00	1^{0}	0^{1}	0^{3}	1^{2}
01	1^{4}	1^{5}	1^{7}	0^{6}
11	1^{12}	1^{13}	0^{15}	0^{16}
10	1^{8}	0^{9}	0^{11}	1^{10}

ab \ cd	00	01	11	10
00	1^{0}	0^{1}	0^{5}	1^{2}
01	1^{4}	1^{5}	1^{7}	0^{6}
11	1^{12}	1^{13}	0^{15}	0^{14}
10	1^{8}	0^{9}	0^{11}	1^{10}

$\to G_1$

$\to G_2$

$\to G_3$

$\to G_4$

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$G_1 = \bar{a}bc\bar{d} + \bar{a}b\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d} + \bar{a}bc\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}$

$G_1 = (a+b+c+\bar{d})(a+b+\bar{c}+\bar{d})(\bar{a}+b+c+\bar{d})(\bar{a}+b+\bar{c}+\bar{d})$

$= (a+b+\bar{d})(c+\bar{c})(\bar{a}+b+\bar{d})(c+\bar{c})$

$= b+\bar{d}\,(a+\bar{a})$

$G_1 = b+\bar{d}$

$G_2 = a\bar{b}\bar{c}d\ (a+\bar{b}+\bar{c}+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$

$= (\bar{b}+\bar{c}+\bar{d})(a+\bar{a})$

$G_2 = (\bar{b}+\bar{c}+\bar{d})$

$G_3 = (\bar{a}+\bar{b}+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+d)$

$= (\bar{a}+\bar{b}+\bar{c})(d+\bar{d})$

$G_3 = (\bar{a}+\bar{b}+\bar{c})$

$G_4 = (\bar{a}+\bar{b}+\bar{c}+\bar{d})(\bar{a}+b+\bar{c}+\bar{d})$

$= (\bar{a}+b\bar{c}+\bar{d})(b+\bar{b})$

$= (\bar{a}+\bar{c}+\bar{d})$

Non essential groups

$f(a,b,c,d) = (b+\bar{d})(\bar{b}+\bar{c}+d)(\bar{a}+\bar{b}+\bar{c})(\bar{a}+\bar{c}+\bar{d})$

Minimal sums:

Definition: A minimal sum is a minimal disjunctive normal formula describing a function consisting of prime implicants.

The advantage of obtaining minimal sum is that the logic implementation of minimal sum always represent least no. of logic gates with least no. of i/p terminals.

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The general approach for determining the minimal sum for a given min term canonical for are as following steps:

Step 1: Construct a K-map for the given boolean function described by the min-term canonical form or normal SOP form.

Step 2: Identify the essential prime implicants by locating the essential one cells i.e., first group the essential 1 cells in largest possible sub cubes.

Step 3: If all the sub cubes established at this point emcamphasis (includes) all the 1 cells of the map then the minimal sum is simply the sum of essential prime implicants.

1	1	0	0
0	0	1	1

Step 4: However if there remains 1 cells that are not included in ~~so~~ sub cube then additional sub cubes [representing prime implicants (PI)] must be selected to include the remaining cells, as guiding rules, these additional sub cubes should be as large as possible, i.e., containing as many 1 cells as possible & still satisfing the constrain that there should be a power of 2 cells. & the no. of additional subcubes should be as small as possible. The minimal sum then is the sum of product terms associated with the essential prime implicants along with the prime implicants resulting from the additional sub cubes.

As an illustration let us consider the function $f(w,x,y,z) = \bar{w}\bar{x} + \bar{w}xz + yz$.

Now let us write the k-map along with the grouping as bellow:

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

In the above k-map, the essential prime implicants are not identified clearly. Because we have not grouped the 1 cells in largest possible sub cubes. The indicated sub cubes just represent the three product terms in the given boolean function.

The fig. below ~~show~~ gives the identification of the essential prime implicants clearly because we have grouped the 1 cells in largest possible sub cubes.

wx \ yz	00	01	11	10	
00	1	1	1	1	→ G_1
01	0	1	1	0	→ G_2
11	0	0	1	0	→ G_3
10	0	0	1	0	

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Now the essential cells are cell no. 0 & cell no. 2 which gives the essential prime implicants

$G_1 = \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}y z$

$= \bar{w}\bar{x}\bar{y}(z+\bar{z}) + \bar{w}\bar{x}y(z+\bar{z})$ $\quad \bar{w}\bar{x}\bar{y}(z+\bar{z}) + \bar{w}\bar{x}y(z+\bar{z})$

$= \bar{w}\bar{x}(y+\bar{y}) \longrightarrow \quad = \bar{w}\bar{x}$

$= \bar{w}\bar{x}$

Cell no. 5 is also essential 1 cell which gives the EPI $\bar{w}z$. is

$G_3 = \bar{w}\bar{x}yz + \bar{w}xyz + wxyz + w\bar{x}yz$

$= \bar{w}yz(x+\bar{x}) + wyz(\bar{x}+x)$

$= yz$

Cell no. 15 & cell no. 11 also give EPI yz from G_3

Now from the above fig. a it is seen that all the sub cubes established will cover all the 1 cells (Refer step 3). Hence the minimal sum is simply the sum of essential prime implicants.

$\therefore \boxed{f(w,x,y,z) = \bar{w}\bar{x} + \bar{w}z + yz}$

1) Obtain the minimal sum or minimal SOP for the following min term canonical formula describing the boolean function: $f(x,y,z)$.

$f(x,y,z) = \underset{000}{\bar{x}\bar{y}\bar{z}} + \underset{011}{\bar{x}yz} + \underset{100}{x\bar{y}\bar{z}} + \underset{110}{xy\bar{z}} + \underset{111}{xyz}$.

$f(x,y,z) = \Sigma m(0, 3, 4, 6, 7)$.

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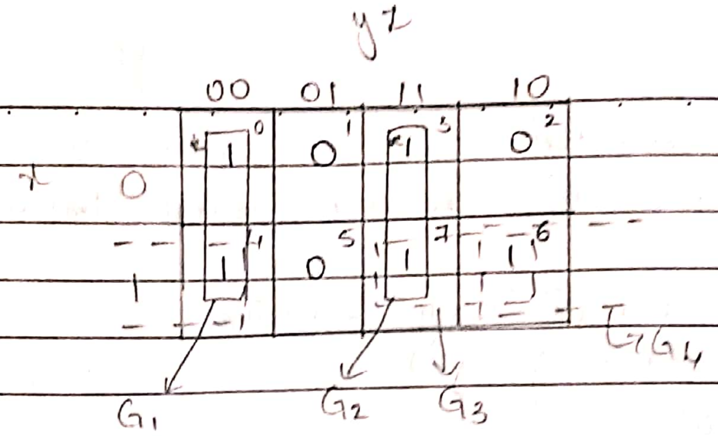

$G_3(G_2, G_4)$

$G_4(G_2, G_1)$

The two prime implicants obtained from G_1 & G_2 are essential, since these groups includes essential 1 cells denoted by (*)

The essential PI are $\overline{y}\,\overline{z}$ (G_1) & yz (G_2) respectively. After these two sub cubes are formed there is only one 1 cell that must still be grouped i.e., cell no. 6.

This cell can be placed either in sub cube G_3 representing ~~the~~ a prime implicant i.e., xy ~~are~~ or in sub cube G_4 representing a PI i.e., $x\overline{z}$, both the groups are shown in dotted lines.

Since only one of these two additional sub cubes is needed to complete the covering of all ~~one~~ 1 cells of the k-map, there are 2 minimal cells for the given function $f(x,y,z)$.

$$f(x,y,z) = \underbrace{\overline{y}\,\overline{z}}_{EPI_1} + \underbrace{yz}_{EPI_2} + \underbrace{xy}_{PI} \rightarrow (1)$$

$$f(x,y,z) = \underbrace{\overline{y}\,\overline{z}}_{EPI_1} + \underbrace{yz}_{EPI_2} + \underbrace{x\overline{z}}_{PI} \rightarrow (2)$$

NOTE: Both of the above minimal ~~set~~ sums consist of a PI in addition to EPI.
In eq^n(1) $\rightarrow xy$ & in eq^n(2) $\rightarrow x\overline{z}$.

$\bar{w}\bar{z} + w\bar{y}z + wx\bar{y} + xyz$

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Obtain the minimal sum for the following boolean function $f(w,x,y,z) = \sum m(0, 2, 4, 6, 7, 9, 12, 13, 15)$.

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	1 (4)	0 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	0 (11)	0 (10)

	ab \ cd	00	01	11	10	
	00	1 (0)	0 (1)	0 (3)	1 (2)	
	01	1 (4)	0 (5)	1 (7)	1 (6)	G_1
G_2	11	1 (12)	1 (13)	1 (15)	0 (14)	G_3
	10	0 (8)	1 (9)	0 (11)	0 (10)	G_4

$G_1 = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}xy\bar{z}$

$= \bar{w}\bar{y}\bar{z}(x+\bar{x}) + \bar{w}y\bar{z}(x+\bar{x})$

$= \bar{w}\bar{z}(y+\bar{y})$

$G_1 = \bar{w}\bar{z}$

$G_2 = wx\bar{y}\bar{z} + wx\bar{y}z$

$G_2 = wx\bar{y}$

$G_3 = \bar{w}xyz + wxyz$

$= xyz(w+\bar{w})$

$= xyz$

$G_4 = wx\bar{y}z + w\bar{x}\bar{y}z$

$= w\bar{y}z(x+\bar{x})$

$G_4 = w\bar{y}z$

$f(w,x,y,z) = \bar{w}\bar{z} + w\bar{y}z + wx\bar{y} + xyz$

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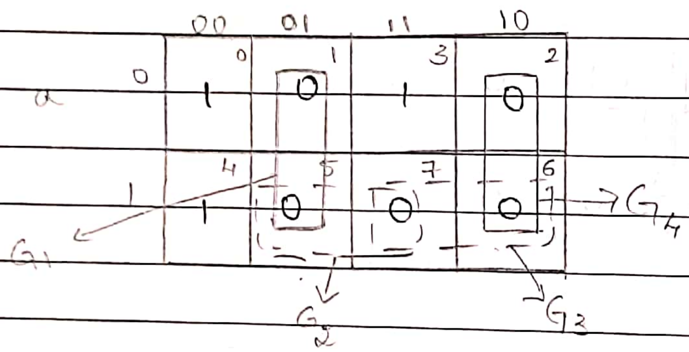
Minimal Products:

for obtaining minimal products same procedure is used to get the minimal sums can be followed but with grouping of zero cells instead of 1-cells.

1) find the minimal products of the boolean function given by $f = \pi M(1, 2, 5, 6, 7)$.

a \ bc	00	01	11	10
0	1 (0)	0 (1)	1 (3)	0 (2)
1	1 (4)	0 (5)	0 (7)	0 (6)



The zero cells ~~num~~ formed by cell no. 1 & 2 forms the essential zero cells. The uncovered zero cell in cell no: 7 can be grouped with either cell no. 5 or with cell no. 6.

∴ We can get 2

$f(a,b,c) = G_1 \cdot G_4 \cdot G_2$ &

$f(a,b,c) = G_1 \cdot G_4 \cdot G_3$.

$f(a,b,c) = (\ )(\bar{b}+c)(\bar{a}+\bar{c})$

$G_1 = (a+b+\bar{c})(\bar{a}+b+\bar{c})$

$= (b+\bar{c})(a+\bar{a})$

$G_1 = b+\bar{c}$

$G_2 = (\bar{a}+b\bar{c})(\bar{a}+\bar{b}+\bar{c})$

$= (\bar{a}+\bar{c})(b+\bar{b})$

$= (\bar{a}+\bar{c})$

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$G_3 = (\bar a+\bar b+\bar c)(\bar a+\bar b+c)$ $\quad$ $G_4 = (a+\bar b+c)(\bar a+\bar b+c)$

$= \bar a+\bar b\,(a+\bar c)$ $\quad$ $= \bar b+c\,(a+\bar a)$

$= \bar a+\bar b$ $\quad$ $= \bar b+c$

$$f(a,b,c,d) = (b+\bar c)(\bar a+\bar c)(\bar a+\bar b)(\bar b+c)$$

Minimal exp. for incomplete boolean function :-

Incomplete boolean exp. are those which evaluate to a don't care condition for sum of the input combinations.

The don't care condition is denoted by 'x'.

The PI obtained by considering the dont care cells as 1 cells in the k-map becomes the PI of incomplete boolean function, similarly the PI obtained by considering the dont care ~~as~~ cells as zero cells in the k-map becomes the PI of the incomplete boolean function.

Minimal sums of incomplete boolean function:

If an incomplete boolean function is given, then to find the minimal sum in the corresponding k-map of given B.F dont care cells are taken as 1 cells if they contribute in forming largest possible sub cubes. along with the other 1 cells.

The remaining dont care ~~sum~~ cells which donot contribute in forming posible sub cubes are simply ignored.

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Keeping this in mind the EBI EPI's are identified by first considering only the actual 1 cells as possible essential 1-cells.

* A dont care cell taken as 1 cell doesnot qualified to be an essential 1 cell.

Remember that atleast one actual 1-cell must be included in every possible sub cubes.

∴ It should be noted that there can be sub cubes with 1-cells [actual 1 cells] only

There can be sub cubes with actual 1-cells along with dont care cells. [Being considered as 1 cells]

But there can be no sub cubes with dont care cells only

Eg: Consider the following incomplete B.F.

$$f(a,b,c,d) = \Sigma m(0,2,5,8,13) + dc\,(3,7,9,10,15)$$

ab \ cd	00	01	11	10
00	1^{0}	0^{1}	X^{3}	1^{2}
01	0^{4}	1^{5}	X^{7}	0^{6}
11	0^{12}	1^{13}	X^{15}	0^{14}
10	1^{8}	X^{9}	0^{11}	X^{10}

ab \ cd	00	01	11	10
00	1 [0]	0 [1]	x [3]	1 [2]
01	0 [4]	1 [5]	x [7]	0 [6]
11	0 [12]	1 [13]	x [15]	0 [14]
10	1 [8]	x [9]	0 [11]	x [10]

$\rightarrow G_1$

$\rightarrow G_2$

In the above k-map while grouping of the cells the dont care cells x is considered as 1's coz we are going to get the minimal sums of the given incomplete B.F.

The dont care 'x' cells having the cell nos. 7 & 15 are considered as 1 cells & group with 1 cells having the cell nos. 5 & 13 to form a rectangular sub cube of size 4.

Another dont care 'x' cell having the cell no. 10 is considered as 1 cell & combined with the 1 cells having the cell no. 0, 2 & 8 to form a sub cube of size 4.

The dont care 'x' cells numbered 3 & 9 are considered as zero are ignored coz there are no ~~adjust~~ adjacent 1 cells available with which they can be grouped.

∴ The essential PI are obtained from the groups G_1 & G_2 ∴ The simplified SOP for the above given incomplete B.F is $f(a,b,c,d)$

Minimal products of incomplete boolean expressions:-

It is same as minimal sums of incomplete boolean exp. but here we group the zero cells instead of the 1-cells & with the dont care cells.

Eg: Consider the following max term incomplete boolean exp. $f(a,b,c,d) = \pi M(5,8,9,12) + dc(1,3,6,7,10,11,14,15)$

ab \ cd	00	01	11	10
00	1	X	X	1
01	1	0	X	X
11	0	1	X	X
10	0	0	X	X

$$G_1 = (a+b+c+\bar{d})(a+b+\bar{c}+\bar{d})(a+\bar{b}+c+\bar{d})(a+\bar{b}+\bar{c}+\bar{d})$$
$$= (a+b+\bar{d})(c+\bar{c})(a+\bar{b}+\bar{d})(c+\bar{c})$$
$$= (a+\bar{d})(b+\bar{b})$$
$$= (a+\bar{d})$$

$$G_2 = (\bar{a}+\bar{b}+c+d)(\bar{a}+b+c+d)(\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c}+d)$$
$$= (\bar{a}+c+d)(b+\bar{b})(\bar{a}+\bar{c}+d)(b+\bar{b})$$
$$= (\bar{a}+d)(c+\bar{c})$$
$$= (\bar{a}+d)$$

$$G_3 = (\bar{a}+b+c+d)(\bar{a}+b+c+\bar{d})(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+b+\bar{c}+d)$$
$$= (\bar{a}+b+c)(\bar{a}+b+\bar{c})$$
$$= (\bar{a}+b)(c\cdot\bar{c})$$
$$= (\bar{a}+b)$$

$$f(a,b,c,d) = (a+\bar{d})(\bar{a}+d)(\bar{a}+b)$$

1) find all the minimal sums & minimal products for the incomplete boolean function k-map method.

$$f(a,b,c,d) = \Sigma m(6,7,9,10,13) + dc(1,4,5,11,15)$$

(a)

ab \ cd	00	01	11	10
00	0 (0)	X (1)	0 (3)	0 (2)
01	X (4)	X (5)	1 (7)	1 (6)
11	0 (12)	1 (13)	X (15)	0 (14)
10	0 (8)	1 (9)	X (11)	1 (10)

$\rightarrow G_1$, $\rightarrow G_2$, $\rightarrow G_3$, $\rightarrow G_4$

$0 = \bar{a}$

$$G_1 = (\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c}+d)(a+b+\bar{c}+d)(a+\bar{b}+\bar{c}+d)$$
$$= (\bar{a}+\bar{c}+d)(a+\bar{c}+d)$$
$$G_1 = a(\bar{c}+d) \Rightarrow \bar{c}d$$

$$G_2 = (\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+b+\bar{c}+d)(\bar{a}+b+c+d)(\bar{a}+b+c+\bar{d})$$
$$= (\bar{a}+b+\bar{c})(\bar{a}+b+c)$$
$$= (\bar{a}+b) \Rightarrow \bar{a}b$$

$$G_3 = ab\bar{c}d + abcd + a\bar{b}\bar{c}d + a\bar{b}cd$$
$$= abd(c+\bar{c}) + a\bar{b}d(c+\bar{c})$$
$$= ad(d+\bar{d})$$
$$G_3 = ad$$

$$G_4 = a\bar{b}cd + a\bar{b}c\bar{d}$$
$$= a\bar{b}c(d+\bar{d})$$
$$G_4 = a\bar{b}c$$

In G_4 is simply a PI other than EPI coz a dont care cell 'x' doesnot qualify to be an essential 1-cell.

Group G_1 is an essential P.I coz the 1 cells having the cell nos. 6 & 7 are uncovered by more than 1 group are sub cube i.e., 7 & 6 are essential 1 cells. & hence G_1 is essential P.I Similarly G_3 is also an essential P.I. Again G_2 is simply a prime implicant. ∴ We have 2 minimal sums : one having P.I G_4 along with the EPI's G_1 & G_3, another having the P.I G_2 along with the EPI's G_1 & G_3.

$$f(a,b,c,d) = G_1 + G_3 + G_4$$

&

$$f(a,b,c,d) = G_1 + G_2 + G_3$$

$$f(a,b,c,d) = \bar{a}b + a\bar{b}c + \bar{c}d$$

$$f(a,b,c,d) = \bar{a}b + ad + a\bar{b}c$$

(b)

ab \ cd	00	01	11	10
00	0	X	0	0
01	X	X	1	1
11	0	1	X	0
10	0	1	X	1

find minimal products for the incomplete B.E by grouping zero cells.

G_4 is the P.I other than coz the dont care cell 'x' in cell no. 15 is not qualified as an essential zero cell.

Sub cube G_3 is entirely covered by sub cube G_2 & partialy by sub cube G_4 but the PI formed by G_3 cant be neglected coz G_2 is an essential P.I, whereas G_4 is simply a prime implicante

The condition to neglect G_3 is that both G_3 & G_4 should be essential P.I. Hence we get 2 minimal products

$$f(a,b,c,d) = G_1 \cdot G_2 \cdot G_3$$

$$\& \quad f(a,b,c,d) = G_1 \cdot G_2 \cdot G_4$$

$$G_1 = (a+b+c+d)(a+b+c+\bar{d})(a+b+\bar{c}+\bar{d})(a+b+\bar{c}+d)$$

$$= (a+b+c)(a+b+\bar{c})$$

$$G_1 = (a+b)$$

$$G_2 = (a+b+c+d)(a+\bar{b}+c+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+b+c+d)$$

$$= (a+c+d)(\bar{a}+c+d)$$

$$G_2 = (c+d)$$

$$G_3 = (\bar{a}+\bar{b}+c+d)\cancel{(\bar{a}+\bar{b}+\bar{c}+\bar{d})}(\bar{a}+\bar{b}+\bar{c}+d)$$

$$= (\bar{a}+\bar{b}+d)$$

$$G_4 = (\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$$

$$= (\bar{a}+\bar{b}+\bar{d})$$

$$f(a,b,c,d) = \underbrace{(a+b)}_{G_1}\underbrace{(c+d)}_{G_2}\underbrace{(\bar{a}+\bar{b}+d)}_{G_3}$$

or

$$f(a,b,c,d) = (a+b)(c+d)\underbrace{(\bar{a}+\bar{b}+\bar{d})}_{G_4}$$

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Find the minimal sums for a 4 variable function $f(a,b,c,d) = \Sigma m(5,6,7,8,11,12,13) + dc(0,14,15)$.

ab \ cd	00	01	11	10
00	X (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	X (15)	X (14)
10	1 (8)	0 (9)	1 (11)	0 (10)

All the groups G_1, G_2, G_3, G_4 are essential P.I. Hence the minimal sum obtained is

$$f(a,b,c,d) = G_1 + G_2 + G_3 + G_4$$

$$G_1 = \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}d + abcd$$
$$= \bar{a}bd(c+\bar{c}) + abd(c+\bar{c})$$
$$G_1 = bd$$

$$G_2 = \bar{a}bcd + \bar{a}bc\bar{d} + abcd + abc\bar{d}$$
$$= \bar{a}bc(d+\bar{d}) + abc(d+\bar{d})$$
$$= bc(a+\bar{a})$$
$$G_2 = bc$$

$$G_3 = abcd + a\bar{b}cd$$
$$= acd$$

$$G_4 = ab\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d}$$
$$= a\bar{c}\bar{d}(b+\bar{b})$$
$$= a\bar{c}\bar{d}$$

$$f(a,b,c,d) = bd + bc + acd + a\bar{c}\bar{d}.$$

Same question as previous but minimal products.

ab \ cd	00	01	11	10
00	X	0	0	0
01	0	1	1	1
11	1	1	X	X
10	1	0	1	0

G2, G1, G3, G5, G4

$$G_1 = (a+b+c+d)(a+b+c+\bar{d})(a+b+\bar{c}+\bar{d})(a+b+\bar{c}+d)$$
$$= (a+b+c)(a+b+\bar{c})$$
$$G_1 = (a+b)$$

$$G_2 = (a+b+\bar{c}+d)(\bar{a}+b+\bar{c}+d)$$
$$G_2 = (b+\bar{c}+d)$$

$$G_3 = (\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c}+d)$$
$$G_3 = (\bar{a}+\bar{c}+d)$$

$$G_4 = (a+b+c+\bar{d})(\bar{a}+b+c+\bar{d})$$
$$= (b+c+\bar{d})$$

$$G_5 = (a+b+c+d)(a+\bar{b}+c+d)$$
$$= (a+c+d)$$

$$f(a,b,c,d) = (a+b)(b+\bar{c}+d)(\bar{a}+\bar{c}+d)(b+c+\bar{d})(a+c+d)$$

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Quine Mc-cluskey:

It is the method of generating PI & implicants.

* The k-map method can be effectively used to get simplified boolean exp. only upto 6 variables but Quine - Mc-cluskey can be applied ~~ofor~~ for simplification of a B.E having any no. of variables.

* The Quine Mc-cluskey method is an algorithmic method for simplification of a B.E of n variables.

* Since the procedure is algorithmic, it can be programmed for a digital computer to simplify a given boolean exp. having n no. of variables.

* The Quine mc-cluskey method consist of two phases. In the first phase - The set of all ~~PI~~ prime implicants or prime implicates of a given B.E is systematically obtained.

In the second phase - A set of all irredendant exp. for the function is determined.

* from this set of irreduntant expressions, the minimal exp. are selected.

* Quine Mc-cluskey method for obtaining prime implicants.

NOTE: The logical adjacent theorem states that if any two product terms, in a min term canonical exp. will differs in only 1 variable

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where this variable appears complimented in one min term & remains uncomplimented in annother min term, then these two products terms can be replaced by a single term without that variable.

for eg: xy & $\bar{x}y$ can be replaced by 'y', the single term i.e., in other words we can write $xy + \bar{x}y = y$. Here x represents a single variable & y represents a product of variables.

for eg: consider the product terms or min terms:

$a\bar{b}c\bar{d}$ & $\bar{a}\bar{b}c\bar{d}$

Here $x = a$ & $y = \bar{b}c\bar{d}$, Hence $a\bar{b}c\bar{d}$ & $\bar{a}\bar{b}c\bar{d}$ can be replaced by a single product term: $\bar{b}c\bar{d}$ [without variable a].

* The logical ajacency theorem i.e., $xy + \bar{x}y = y$ can be repeatedly applied to each pair of terms of a boolean function to obtain all the prime implicants / implicates of that boolean function. This is the main principle behind Quine Mc-cluskey method of simplification of a boolean expressions of n no. of variables.

* Broadly speaking, the Quine Mc-cluskey process begins with listing of all the min terms in the first column, then every pair of ~~a~~ min terms in this column are examined to see whether $xy + \bar{x}y = y$ relationship can be applied or not. If yes a '✓' mark is placed against these pairs of min terms. & the extracted term 'y' is placed in the second column of the table. Then every pair of the product

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terms in the second column is checked to see whether $xy + \bar{x}y = y$ relationship is applicable or not. If yes, again a '✓' mark is placed against those pairs in the second column & the extracted terms 'y' are placed in the third column of the table. This process is repeated until there are no pairs left in the last column, which ~~st~~ satisfies the relationship $xy + \bar{x}y = y$.

Those terms against which there are no ~~more~~ '✓' marks are the prime implicants of the given boolean function.

As an illustration let us consider an eg & find its prime implicants using Quine Mc-clusky method.

$$f(a,b,c,d) = \Sigma m\,(0,2,3,5,8,10,11).$$

Quine - Mc clusky tabulation is shown as below

Cell no.	Column 1 Min term	Column 2	Column 3	
0	$\bar{a}\bar{b}\bar{c}\bar{d}$ ✓	$(0,2)\ \bar{a}\bar{b}\bar{d}$ ✓	$(0,2)(8,10)\ \bar{b}\bar{d}$	
2	$\bar{a}\bar{b}c\bar{d}$ ✓	$(0,8)\ \bar{b}\bar{c}\bar{d}$ ✓	$(0,8)(2,10)\ \bar{b}\bar{d}$	
3	$\bar{a}\bar{b}cd$ ✓	$(2,3)\ \bar{a}\bar{b}c$ ✓	$(2,3)(10,11)\ \bar{b}c$	
5	$\bar{a}b\bar{c}d$	$(2,10)\ \bar{b}c\bar{d}$ ✓	$(2,10)(3,11)\ \bar{b}c$	
8	$a\bar{b}\bar{c}\bar{d}$ ✓	$(3,11)\ \bar{b}cd$ ✓		
10	$a\bar{b}c\bar{d}$ ✓	$(8,10)\ a\bar{b}\bar{d}$ ✓		
11	$a\bar{b}cd$ ✓	$(10,11)\ a\bar{b}c$ ✓		

After solving, we conclude with 3 columns & we observe that in the third column the repeatative implicants are considered as a single implicant.

Hence totally we get 3 prime implicants i.e., $\bar{b}\bar{d}$, $\bar{b}c$ from 3^{rd} column & $\bar{a}b\bar{c}d$ from 1^{st} column.

Hence the simplified form of the above given min term of ~~the~~ in term boolean exp. is expressed as

$$f(a,b,c,d) = \bar{a}b\bar{c}d + \bar{b}\bar{d} + \bar{b}c.$$

ab \ bc	00	01	11	10	
00	1	0	1	1	→ G1
01	0	1	0	0	→ G3
11	0	0	0	0	
10	1	0	1	1	→ G2

$$G_1 = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d}$$
$$= \bar{a}\bar{b}\bar{d} + a\bar{b}\bar{d}$$
$$G_1 = \bar{b}\bar{d}$$

$$G_2 = a\bar{b}cd + a\bar{b}c\bar{d} + \bar{a}\bar{b}cd + \bar{a}\bar{b}c\bar{d}$$
$$= a\bar{b}c + \bar{a}\bar{b}c$$
$$= \bar{b}c$$

$$G_3 = \bar{a}b\bar{c}d$$

$$f(a,b,c,d) = \bar{b}\bar{d} + \bar{b}c + \bar{a}b\bar{c}d.$$

$$f(w,x,y,z) = \Sigma m(0,2,4,6,7,9,12,13,15)$$

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Cell no.	Column 1 min term	Column 2	Column 3
0	$\bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$ ✓	$(0,2)\ \bar{w}\bar{x}\bar{z}$ ✓	$(0,2)(4,6)\ \bar{w}\bar{z}$
2	$\bar{w}\,\bar{x}\,y\,\bar{z}$ ✓	$(0,4)\ \bar{w}\bar{y}\bar{z}$ ✓	$(0,4)(2,6)\ \bar{w}\bar{z}$
4	$\bar{w}\,x\,\bar{y}\,\bar{z}$ ✓	~~(0,9) $\bar{w}\bar{x}\bar{y}$~~	
6	$\bar{w}\,x\,y\,\bar{z}$ ✓	$(2,6)\ \bar{w}y\bar{z}$ ✓	
7	$\bar{w}\,x\,y\,z$ ✓	$(4,6)\ \bar{w}x\bar{z}$ ✓	
9	$w\,\bar{x}\,\bar{y}\,z$ ✓	$(4,12)\ x\bar{y}\bar{z}$	
12	$w\,x\,\bar{y}\,\bar{z}$ ✓	$(7,6)\ \bar{w}xy$	
13	$w\,x\,\bar{y}\,z$ ✓	~~(7,15) xyz~~ (7,15) xyz	
15	$w\,x\,y\,z$ ✓	$(9,13)\ w\bar{y}z$	
		$(12,13)\ wx\bar{y}$	
		$(13,15)\ wxz$	

$f(w,x,y,z) = \bar{w}\bar{z} + x\bar{y}\bar{z} + \bar{w}xy + \cancel{xyz} + xyz + w\bar{y}z + wx\bar{y} + wxz$

NOTE: The Quin-Mc clusky process can be written in algorithmic form for this we can represent a product term in terms of 0's, 1's. & dash [—].

for eg: $\bar{a}\bar{b}d = 00-1$

Here variable a & b are in complimented form, hence they are represented by 0's. The variable 'd' is in uncomplimented form, hence it is represented by 1. The absence of variable 'c' in the product term is represented by dash [-].

for eg: $f(a,b,c) = b\bar{c} = \_10 \Rightarrow$ index = 1 (no. of 1's)

$f(a,b,c,d,e) = bcde = \_1111 \Rightarrow$ index = 4

$f(a,b,c,d,e) = bc\bar{e} = \_11\_0 \Rightarrow$ index = 2

Similarly, $f(a,b,c,d,e) = \bar{a}\bar{b}\bar{c}e = 0\,1\,0\,\_1 \Rightarrow$ index = 2

$= \bar{a}\bar{b}\bar{c}e = 0\,0\,0\,\_1 \Rightarrow$ index = 1.

0 _ 0 _ 1

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In this eg. we can see the condition satisfying logical agecency.

Quine - Mc clusky algorithm for obtaining ~~algorithm~~ P.I:

The QM procedure for obtaining all P.I can be algorithmically expressed as follows:

Step 1: Represent each min term in its 1/0 notation.

Step 2: Write down the min terms in increasing order of their index in a column.

Step 3: Draw a line after each set of min terms with same index value.

Step 4: Seat the index value i=0

Step 5: Pick up each pair of terms with index i & i+1 to see if they differ in exactly in one position if yes, write the new single term in terms of 1's, 0's & dash (–) notation in a new column. & place a tick mark in the pervious column against those two terms having index i & i+1 which are replaced by the new single term. if no, proceed with other pairs until all the pairs with index i & i+1 have been compared Now, draw a line under the last term ~~new~~ in the new list i.e., in the new column.

Step 6: Set i = i+1 & repeat step 5, the impliment of i is continued until all the terms are compared.

Step 7: Repeat steps 4, 5 & 6 on the new list to form another column of listing.

Step 8: Terminate the process when no new list is formed.

Step 9: The P.I of the boolean function are all those terms without a tick mark against them in any of the columns of listing.

- As an eg for finding P.I of a given boolean function using QM algorithm consider the following boolean exp.

$$f(a,b,c,d) = \sum m(0,2,3,4,8,10,12,13,14).$$

Step 1:

Decimal no.	Min term	0/1 notation	Index
$\bar{a}/\bar{b}/\bar{c}/\bar{d}$	0 0 0 0		
0	$\bar{a}\bar{b}\bar{c}\bar{d}$	0 0 0 0	0
2	$\bar{a}\bar{b}c\bar{d}$	0 0 1 0	1
3	$\bar{a}\bar{b}cd$	0 0 1 1	2
4	$\bar{a}b\bar{c}\bar{d}$	0 1 0 0	1
8	$a\bar{b}\bar{c}\bar{d}$	1 0 0 0	1
10	$a\bar{b}c\bar{d}$	1 0 1 0	2
12	$ab\bar{c}\bar{d}$	1 1 0 0	2
13	$ab\bar{c}d$	1 1 0 1	3
14	$abc\bar{d}$	1 1 1 0	3

Step 2: List the min terms in increasing order of their min terms.

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Decimal no.	0/1 notation	index
0	0 0 0 0 ✓	0
2	0 0 1 0 ✓	1
4	0 1 0 0 ✓	
8	1 0 0 0 ✓	
3	0 0 1 1 ✓	2
10	1 0 1 0 ✓	
12	1 1 0 0 ✓	
13	1 1 0 1 ✓	3
14	1 1 1 0 ✓	

Step 3: Draw a line after each set of min

Step 4: Set index i = 0

Step 5: Now pick up each pair of terms with index i = 0 & index i = i+1. & see if they differ by only 1 bit position.

If yess, place a tick mark against these terms & place the new single term in a new list.

	Coulumn 2
(0,2)	0 0 _ 0
(0,4)	0 _ 0 0
(0,8)	_ 0 0 0

Step 6: Set i = ~~0~~ 1, pick up each pair of terms with index 1 & compare with i = i+1 & compare with i = 2. & repeat. step 5.

	Column
(2,3)	0 0 1 –
(2,10)	– 0 1 0
(4,12)	– 1 0 0
(8,10)	1 0 – 0
(8,12)	1 – 0 0

Step 7: Set i = 2, pick up each pair of terms with an index 3

(10,14)	1 – 1 0
(12,13)	1 1 0 –
(12,14)	1 1 – 0

Decimal no:	0/1 notation	
(0,2)	0 0 – 0 ✓	index 0
(0,4)	0 – 0 0 ✓	
(0,8)	– 0 0 0 ✓	
(2,3)	0 0 1 –	index 1
(2,10)	– 0 1 0 ✓	
(4,12)	– 1 0 0 ✓	
(8,10)	1 0 – 0 ✓	
(8,12)	1 – 0 0 ✓	
(10,14)	1 – 1 0 ✓	index 2
(12,13)	1 1 0 –	
(12,14)	1 1 – 0 ✓	

Step 7: Repeat steps 4, 5 & 6 on the new list column 2 & set i = 0 & pick up each term with index 0 & 1 in the new list & repeat step 5.

	a b c d
(0,2)(8,10)	– 0 – 0
(0,8)(2,10)	– 0 – 0
(0,4)(8,12)	– – 0 0
(0,8)(4,12)	– – 0 0
(8,10)(12,14)	1 – – 0
(8,12)(10,14)	1 – – 0

Note that combining (0,2)(8,10) or (0,8)(2,10) results in the same single term i.e., –0–0. & also combining (0,4)(8,12) or (0,8)(4,12) results in the same single term i.e., – –00 & also we observe that 001– i.e., (2,3) having index 1 does not pair with any other term having index 0. that does not go with index 1. Hence no tick mark.

Note that combining (8,10)(12,14) or (8,12)(10,14) results in the same single term & (12,13) is untick & [110–] having index 2 does not pair with any other term

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QM method for obtaining prime implicates:

The simple approach of applying QM method to find the prime implicates for a given boolean function 'f' in max term canonical form is to apply the algorithm on the complement of the function 'f'. is '$\bar{f}$' will be in min term canonical form.

$\therefore \bar{f}$ gives the prime implicants.

The prime implicates of 'f' are then found by inverting each prime implicant of '$\bar{f}$' using De-morgans theorem.

NOTE: In the case of incomplete boolean function, the don't care cells are taken as zero cells.

Eg: find all the prime implicates of the function $f(a,b,c,d) = \pi M(0,2,3,4,5,12,13) + dc(8,10)$.

→ To determine prime implicates we have
$f = \pi M(0,2,3,4,5,8,10,12,13)$.

first we shall find the compliment function '$\bar{f}$' of the given function 'f' then we apply QM method to '$\bar{f}$'.

$\bar{f} = \pi M(0,2,3,4,5,8,10,12,13)$.

Step 1: Represent each min terms in 0/1 notation in the column.

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Decimal no.	Min term	0/1 notation	Index
0	$\bar{a}\bar{b}\bar{c}\bar{d}$	0000	0
2	$\bar{a}\bar{b}c\bar{d}$	0010	1
3	$\bar{a}\bar{b}cd$	0011	2
4	$\bar{a}b\bar{c}\bar{d}$	0100	1
5	$\bar{a}b\bar{c}d$	0101	2
8	$a\bar{b}\bar{c}\bar{d}$	1000	1
10	$a\bar{b}c\bar{d}$	1010	2
12	$ab\bar{c}\bar{d}$	1100	2
13	$ab\bar{c}d$	1101	3

Step 2: List the min terms in increasing order of their index

Decimal no.	Min term	0/1 notation	Index	
0	$\bar{a}\bar{b}\bar{c}\bar{d}$	0000	0	} 0
2	$\bar{a}\bar{b}c\bar{d}$	0010	1	} 1
4	$\bar{a}b\bar{c}\bar{d}$	0100	1	
8	$a\bar{b}\bar{c}\bar{d}$	1000	1	
3	$\bar{a}\bar{b}cd$	0011	2	} 2
5	$\bar{a}b\bar{c}d$	0101	2	
10	$a\bar{b}c\bar{d}$	1010	2	
12	$ab\bar{c}\bar{d}$	1100	2	
13	$ab\bar{c}d$	1101	3	} 3

Step 4: Set index $i = 0$

Step 5: Now pick up each pair of terms with index $i = 0$ & index $i = i+1$ & see if they differ by only 1 bit position.

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	% notation
(0,2) ✓	0 0 – 0
(0,4) ✓	0 – 0 0
(0,8) ✓	– 0 0 0
(2,3)	0 0 1 –
(2,10) ✓	– 0 1 0
(4,5) ✓	0 1 0 –
(4,12) ✓	– 1 0 0
(8,10) ✓	1 0 – 0
(8,12) ✓	1 – 0 0
(5,13) ✓	– 1 0 1
(12,13) ✓	1 1 0 –

Step 7: Repeat steps 4,5 & 6 on the new list & set i = 0 & pick up each term with index 0 & 1 in the new list & repeat step 5.

	a b c d
(0,2)(8,10) ✓	– 0 – 0
(0,4)(8,12)	– – 0 0
(0,8)(4,12)	– – 0 0
(0,8) (2,10)	– 0 – 0
(4,5)(12,13)	– 1 0 –
(4,12)(5,13)	– 1 0 –

Now we can rewrite the column 3 such that in that column the repeative terms are considered as single terms.

(0,2)(8,10)	– 0 – 0	
(0,4)(8,12)	– – 0 0	
(4,5)(12,13)	– 1 0 –	

a b c d

∴ The prime implicants of $\bar{f}$ are in terms of in any column without a '✓' mark against them. In that sense the prime implicants of $\bar{f}$ are $\bar{a}\bar{b}c$ from column 2, $\bar{b}\bar{d}$, $\bar{c}\bar{d}$ & $b\bar{c}$ from the rewritten column no. 3.

$\overline{\bar{a}\bar{b}c}$, $\overline{\bar{b}\bar{d}}$, $\overline{\bar{c}\bar{d}}$, $\overline{b\bar{c}}$

$(a+b+\bar{c})$, $(b+d)$, $(c+d)$, $(\bar{b}+c)$

Now verification using k-map technique

$$f(a,b,c,d) = \pi M(0,2,3,4,5,12,13) + dc(8,10)$$

ab \ cd	00	01	11	10
00	0	1	0	0
01	0	0	1	1
11	0	0	1	1
10	X	1	1	X

$G_1 = (a+b+c+d) + (a+b+\bar{c}+d) + (\bar{a}+b+c+d) + (\bar{a}+b+\bar{c}+d)$

$= (a+b+d) + (\bar{a}+b+d)$

$G_1 = b+d$

$G_2 = (a+b+\bar{c}+\bar{d})(a+b+\bar{c}+d)$

$= (a+b+\bar{c})$

$G_3 = (a+\bar{b}+c+d)(a+\bar{b}+c+\bar{d})(\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+c+\bar{d})$

$= (a+\bar{b}+c)(\bar{a}+\bar{b}+c)$

$G_3 = (\bar{b}+c)$

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$$G_4 = (a+b+c+d)(a+\bar{b}+c+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+b+c+d)$$
$$= (a+c+d)(b\cdot\bar{b})(\bar{a}+c+d)(b\cdot\bar{b})$$
$$= (c+d)(a\cdot\bar{a})$$
$$G_4 = (c+d)$$

$$f(a,b,c,d) = (b+d)(a+b+\bar{c})(\bar{b}+c)(c+d)$$

<u>Prime implicant tables for obtaining irredundent exp.</u>

for cost minimization based on some cost criteria all the prime implicants / prime implicates generated by QM method or k-map method are not always required to form irreduntant minimal exp.

We need to develop a procedure by which we could choose those prime implicants / prime implicates which constitute minimal exp.

A tabulation which gives the relationship b/w the prime implicants / prime implicates of the complete B.E for the min term / max term of the function is called as prime implicates / prime implicants table.

* Consider the B.E $f(a,b,c) = \Sigma m\ (2,3,4,5,7)$ by using k-map or QM method, the prime implicants where found to be $\bar{a}b$, $a\bar{b}$, ac & bc.

→ We see that the min terms of the above function are:

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Min terms

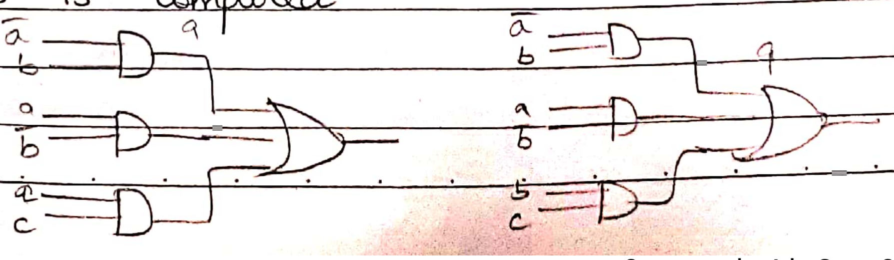
PI's	m_2 $\bar{a}b\bar{c}$	m_3 $\bar{a}bc$	m_4 $a\bar{b}\bar{c}$	m_5 $a\bar{b}c$	m_7 abc
$\bar{a}b$	X	X			
$a\bar{b}$			X	X	
ac				X	X
bc		X			X

Consider a min term 'm_2' $\bar{a}b\bar{c}$ & prime implicants $\bar{a}b$ we observe that this min term m_2 subsumes the considered prime implicant $\bar{a}b$, hence put a X mark in the location in the table corresponding to the first row first column. Similar checkings are made with other remaining ~~terms~~ ~~&~~ min terms & prime implicants. from the table we observe that both min terms $a\bar{b}\bar{c}$ & $a\bar{b}c$ subsumes the PI $a\bar{b}$. Also we observe that the min term $a\bar{b}c$ subsumes the prime implicants $a\bar{b}$ & ac.

Similarly abc subsumes ac & bc in case of incomplete boolean function, min terms correspondings to don't care condition are not placed along the horizontal axis & only min terms for which the function equals to 1 are placed along the horizontal axis in the PI table. from the above PI table we observe that together with $\bar{a}b$ & $a\bar{b}$ by either choosing ac or bc all the min terms will be covered then we get

$$f(a,b,c) = \bar{a}b + a\bar{b} + ac$$
$$f(a,b,c) = \bar{a}b + a\bar{b} + bc$$

then the minimal sum SOP in terms of no. of gates is compared



• 5 Variable K-map :

ab \ cde	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

X

Y

A

BC \ DE	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

BC \ DE	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

• Simplify the following functions using 5 variable K-map.

1) $f(a,b,c,d,e) = \sum m(5,7,13,15,21,23,29,31)$

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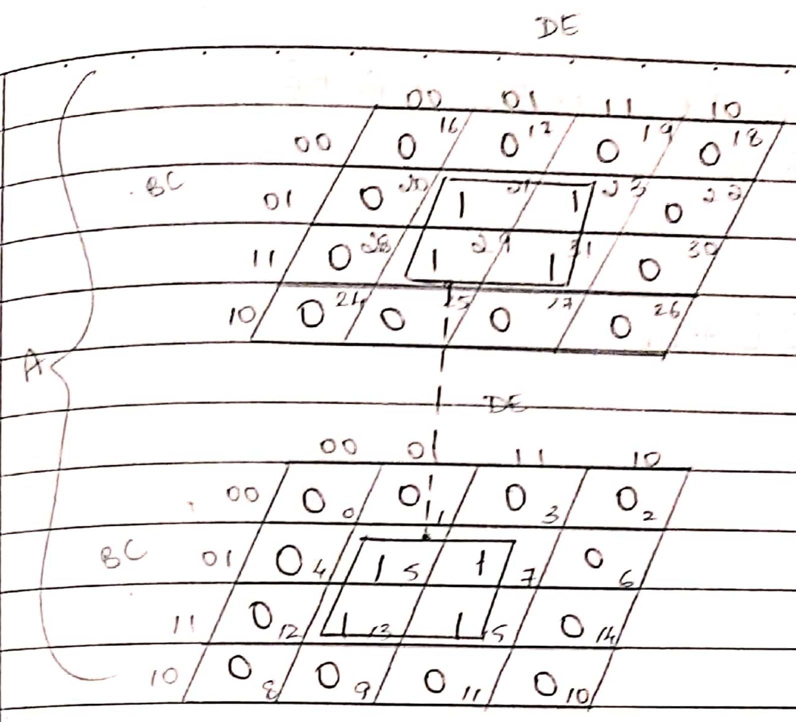
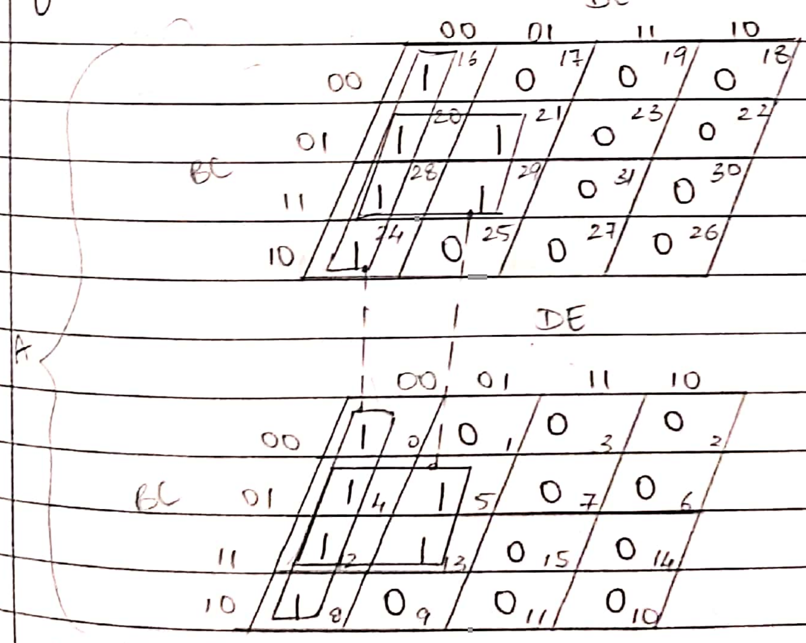


$= \bar{B}C\bar{D}E + \bar{B}CDE + BC\bar{D}E + BCDE$

$= \bar{B}CE(D+\bar{D}) + BCE(D+\bar{D})$

$= CE$

2) $f(a,b,c,d,e) = \Sigma m(0,4,5,8,12,13,16,20,21,24,28,29)$.



$G_1 = \bar{B}C\bar{D}\bar{E} + \bar{B}C\bar{D}E + BC\bar{D}\bar{E} + BC\bar{D}E$

~~$BC\bar{D}\bar{E} + BC\bar{D}E$~~

$= \bar{B}C\bar{D} + BC\bar{D}$

$= C\bar{D}$

$G_2 = \bar{B}\bar{C}\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E} + BC\bar{D}\bar{E} + B\bar{C}\bar{D}\bar{E}$

$= \bar{B}\bar{D}\bar{E} + B\bar{D}\bar{E}$

$= \bar{D}\bar{E}$

$f(a,b,c,d) = \bar{D}\bar{E} + C\bar{D}$