

ELECTROMAGNETIC THEORY

It deals with physics which deals with electric and magnetic phenomenon.

- Electromagnetic is the study of interactions between electric charges at rest and ~~electre~~ in motion.
- It is the study of some particular field
- A field is a function which will specify a particular physical quantity everywhere in the given region.
- A physical quantity is a property of a particular body or substance or a phenomena that can be quantified by measurement.
eg - velocity, mass, weight, acceleration, electric potential, gravitational field.

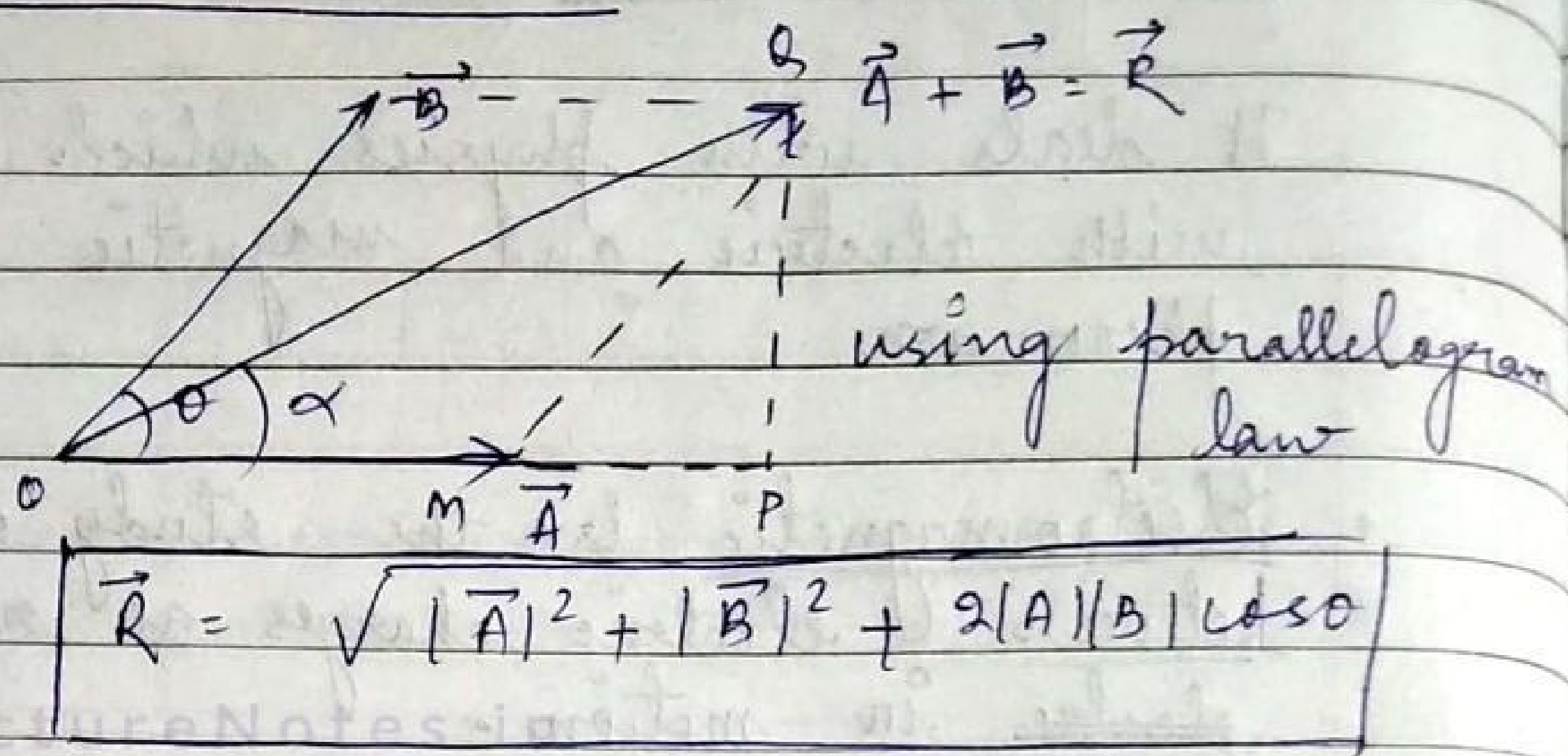
eg -

Scalar field - temp distribution in a room, sound distribution in a theatre.

Vector field - gravitational force acting on a region or body, velocity of rain drops.

$$\vec{A} = |\vec{A}| \cdot \hat{a} \quad \hat{a} = \frac{\vec{A}}{|\vec{A}|} = 1 \quad (\text{unit vector})$$

Vector Addition



$$\tan\alpha = \frac{PQ}{OP}$$

$$\Delta MPQ =$$

$$\sin\theta = \frac{PQ}{B}$$

for ΔMPQ sub ΔMPQ

$$\Delta OPQ$$

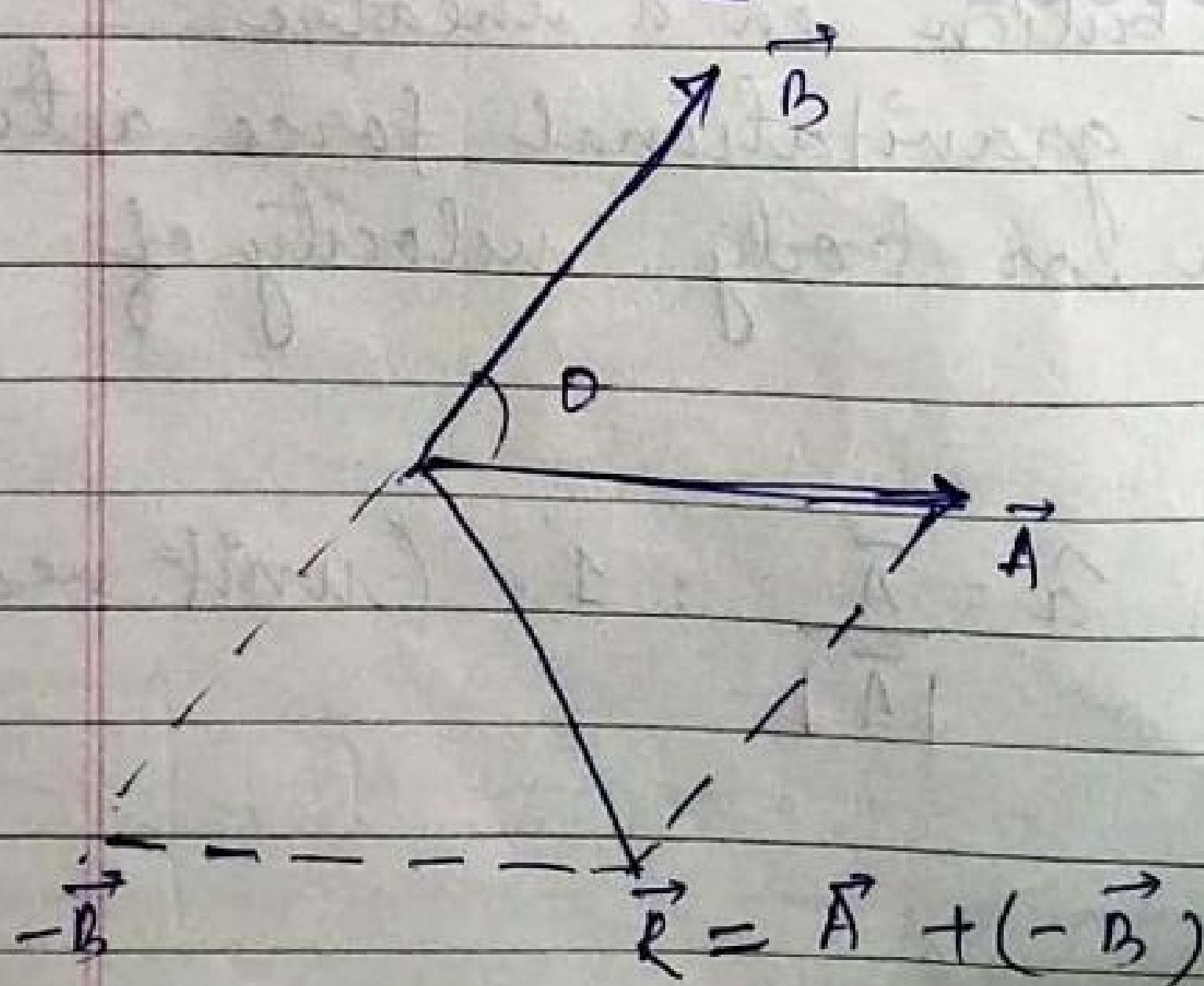
$$= \frac{B \sin\theta}{OM + MP}$$

$$\cos\theta = \frac{MP}{MB} = \frac{MP}{B}$$

$$\tan\alpha = \frac{B \sin\theta}{A + B \cos\theta}$$

$$\alpha = \tan^{-1} \left(\frac{B \sin\theta}{A + B \cos\theta} \right)$$

Subtraction



$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$R = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$\alpha = \tan^{-1} \frac{B \sin \theta}{A - B \cos \theta}$$

Product :

- 1) Cross product
- 2) Dot product

Dot product

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Properties

- i) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- ii) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
- iii) $\vec{A} \cdot \vec{A} = A^2$

Cross product

$$\vec{A} \times \vec{B} = AB \sin \theta (\hat{a}_n)$$

$\hat{a}_n$ = unit vector
gives direction

properties

- (i) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (anti commutative)
- (ii) It is associative as well as distributive
- (iii) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

$$(iii) (\vec{A} + \vec{B}) \times (\vec{C} + \vec{D}) = (\vec{A} \times \vec{C}) + (\vec{A} \times \vec{D}) + (\vec{B} \times \vec{C}) + (\vec{B} \times \vec{D})$$

(Associative).

Scalar Tripple product :

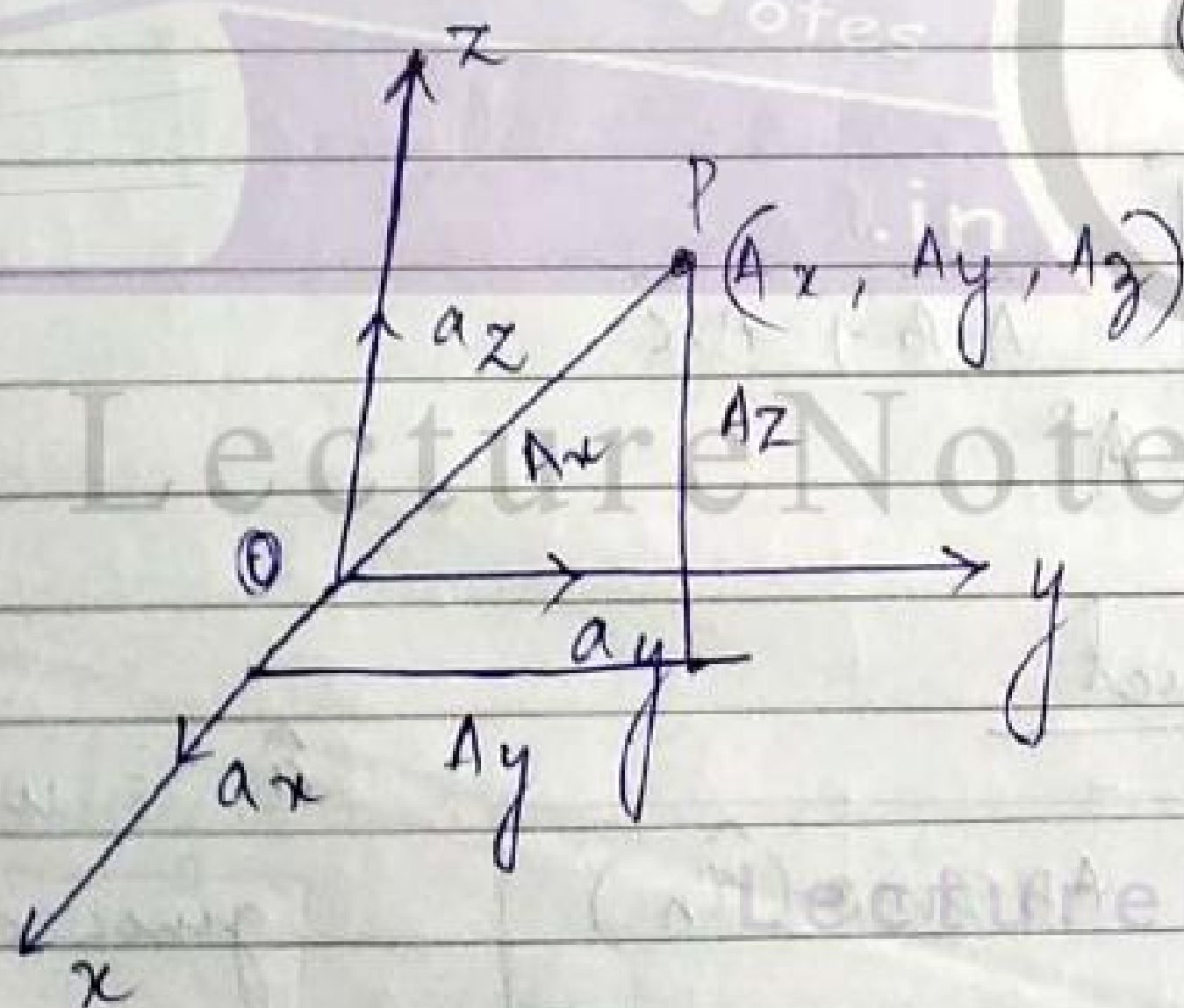
$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

Vector Tripple product :

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$$

Rectangular coordinate system (x, y, z)

(Cartesian coordinate system).



$$\vec{A} (A_x, A_y, A_z)$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{a} = \frac{|\vec{A}|}{A}$$

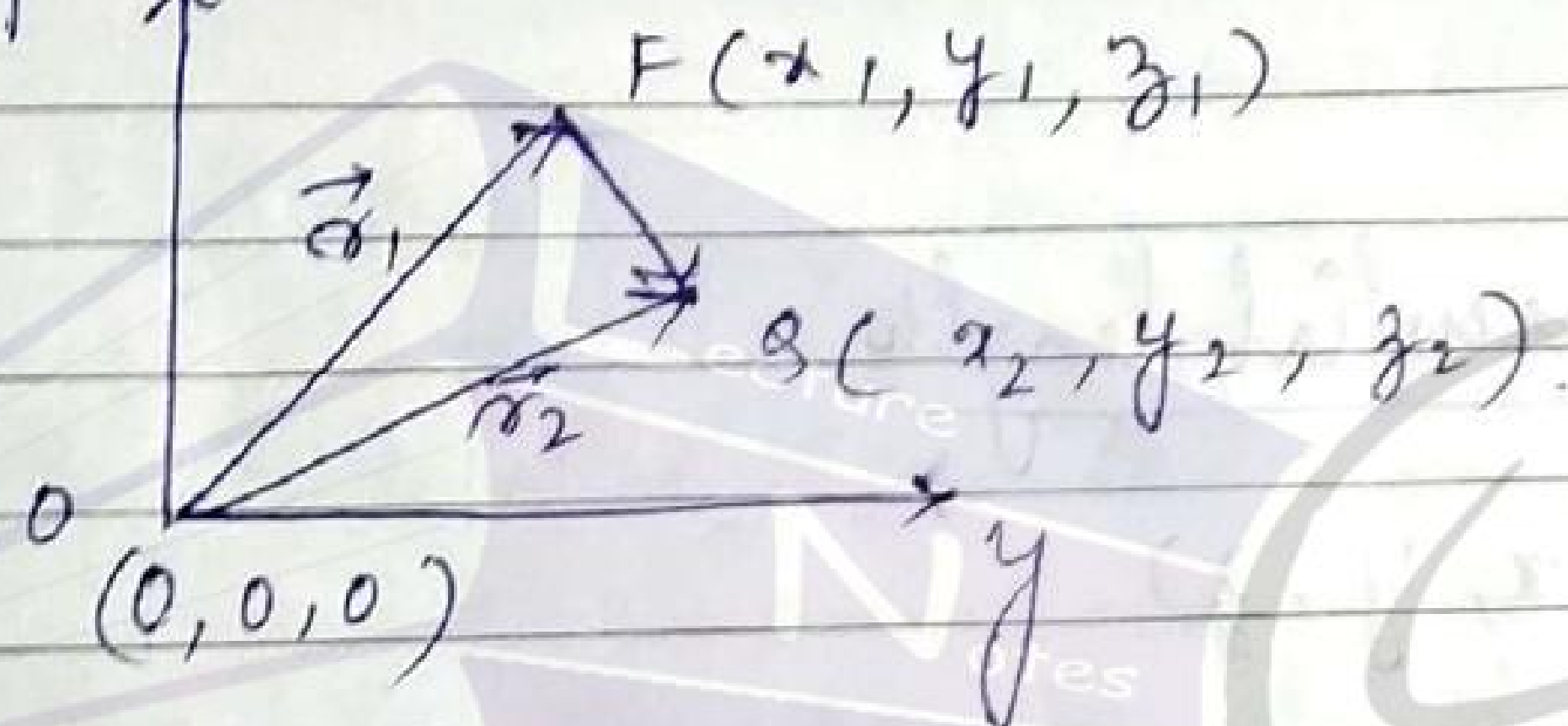
$$\begin{aligned} -\infty &\leq x < \infty \\ -\infty &\leq y < \infty \\ -\infty &\leq z < \infty \end{aligned}$$

Position vector

$$\vec{OP} = (x-0)\hat{a}_x + (y-0)\hat{a}_y + (z-0)\hat{a}_z$$

$$\vec{OP} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

Displacement vector



$$\vec{OP} + \vec{PB} = \vec{OB}$$

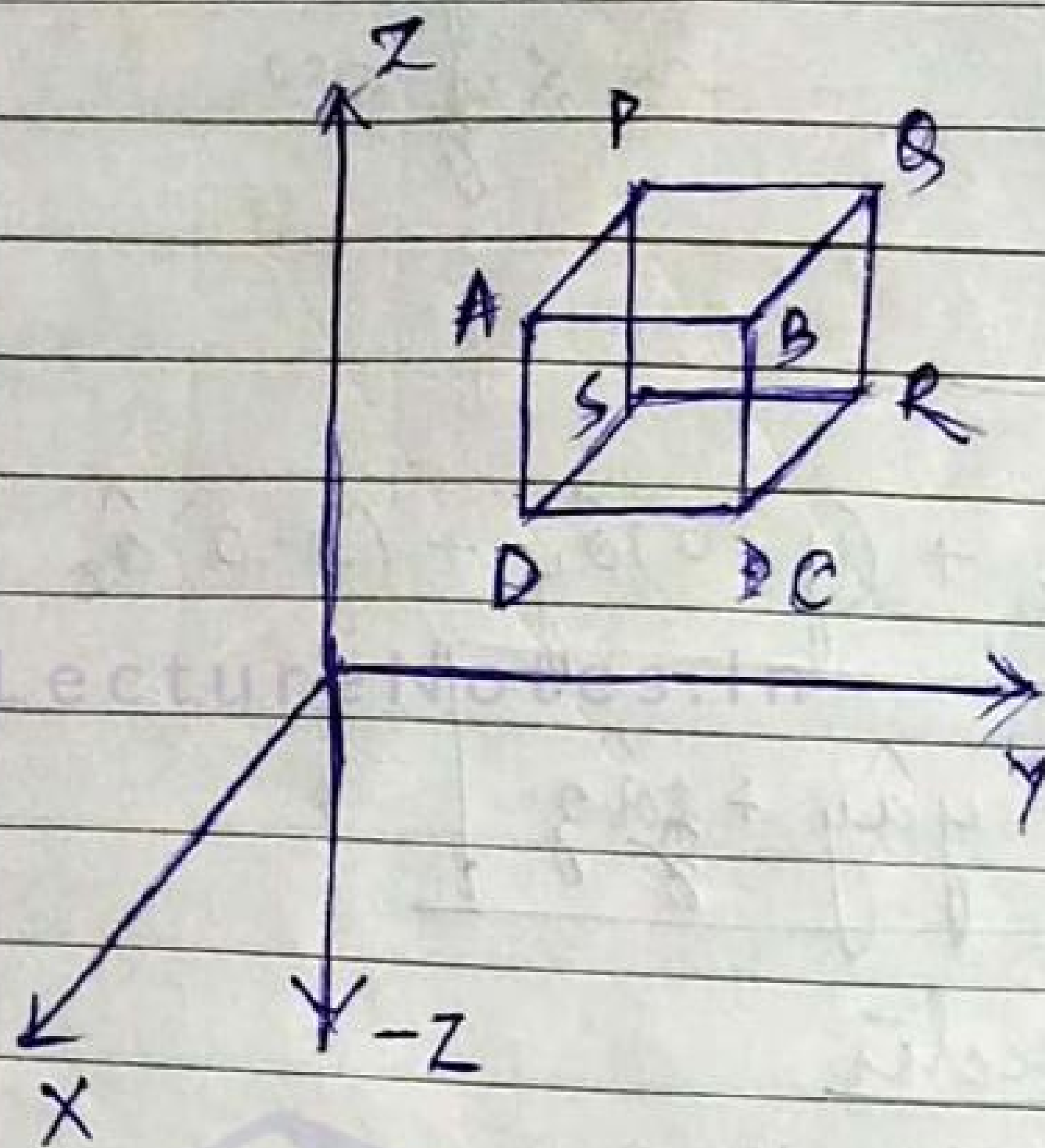
$$\vec{PB} = \vec{OB} - \vec{OP}$$

$$\vec{PB} = \vec{r}_2 - \vec{r}_1$$

$$= (x_2\hat{a}_x + y_2\hat{a}_y + z_2\hat{a}_z) - (x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z)$$

$$\vec{PB} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

Vector Calculus



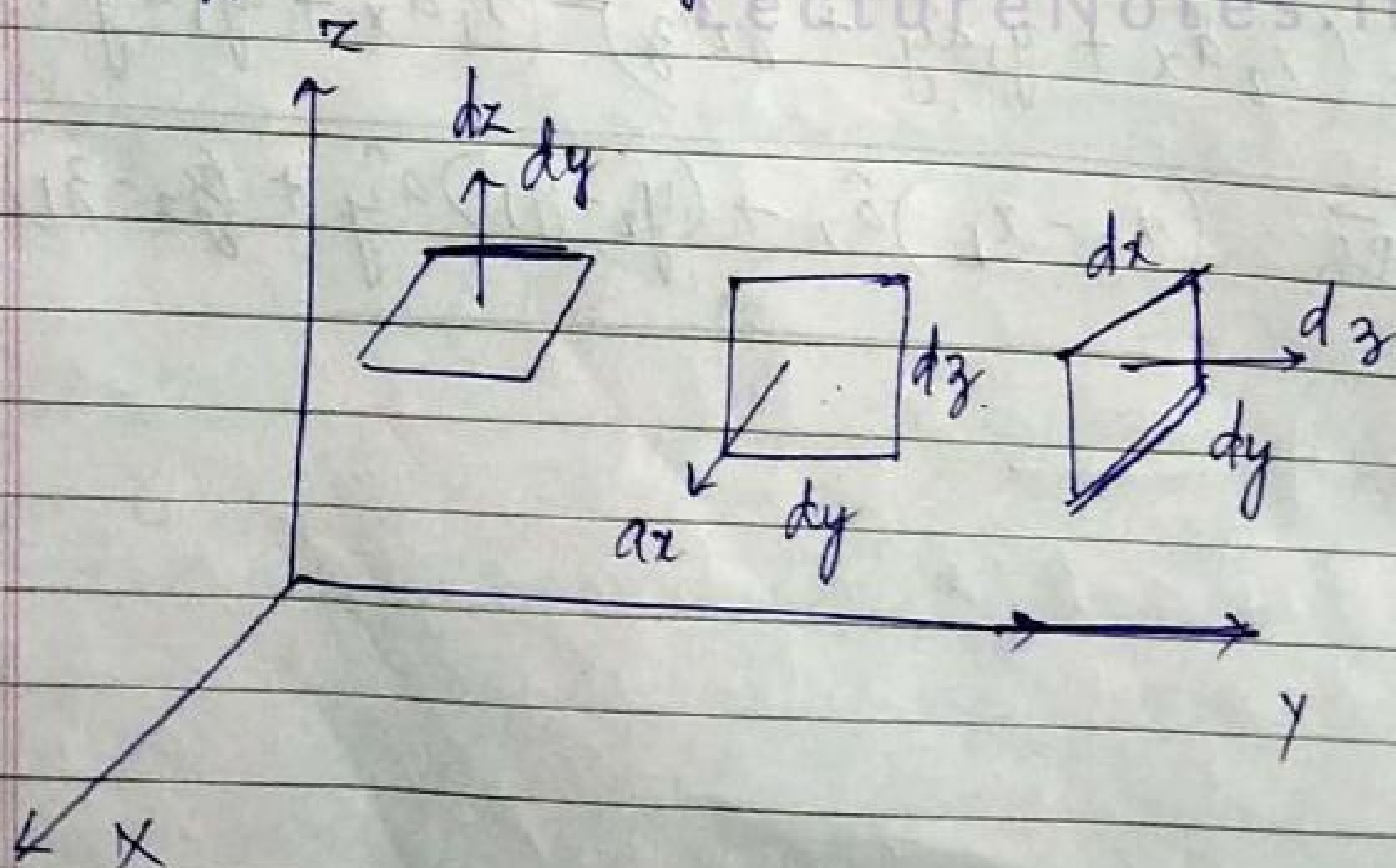
Differential length

$$r(x, y, z)$$

$$B(x+dx, y+dy, z+dz)$$

$$dl = dx \mathbf{a}_x + dy \mathbf{a}_y + dz \mathbf{a}_z$$

② Differential surface



$$\boxed{ds = dy dz \mathbf{a}_x} \quad \left. \begin{array}{l} \text{Vector} \\ \text{from } d\mathbf{l} \end{array} \right\}$$

$$= dx dz \mathbf{a}_y = dx dy \mathbf{a}_z$$

② Differential volume

$$\boxed{dv = dx dy dz} \rightarrow \text{scalar}$$

Cylindrical coordinate

$$A(\rho, \phi, z)$$

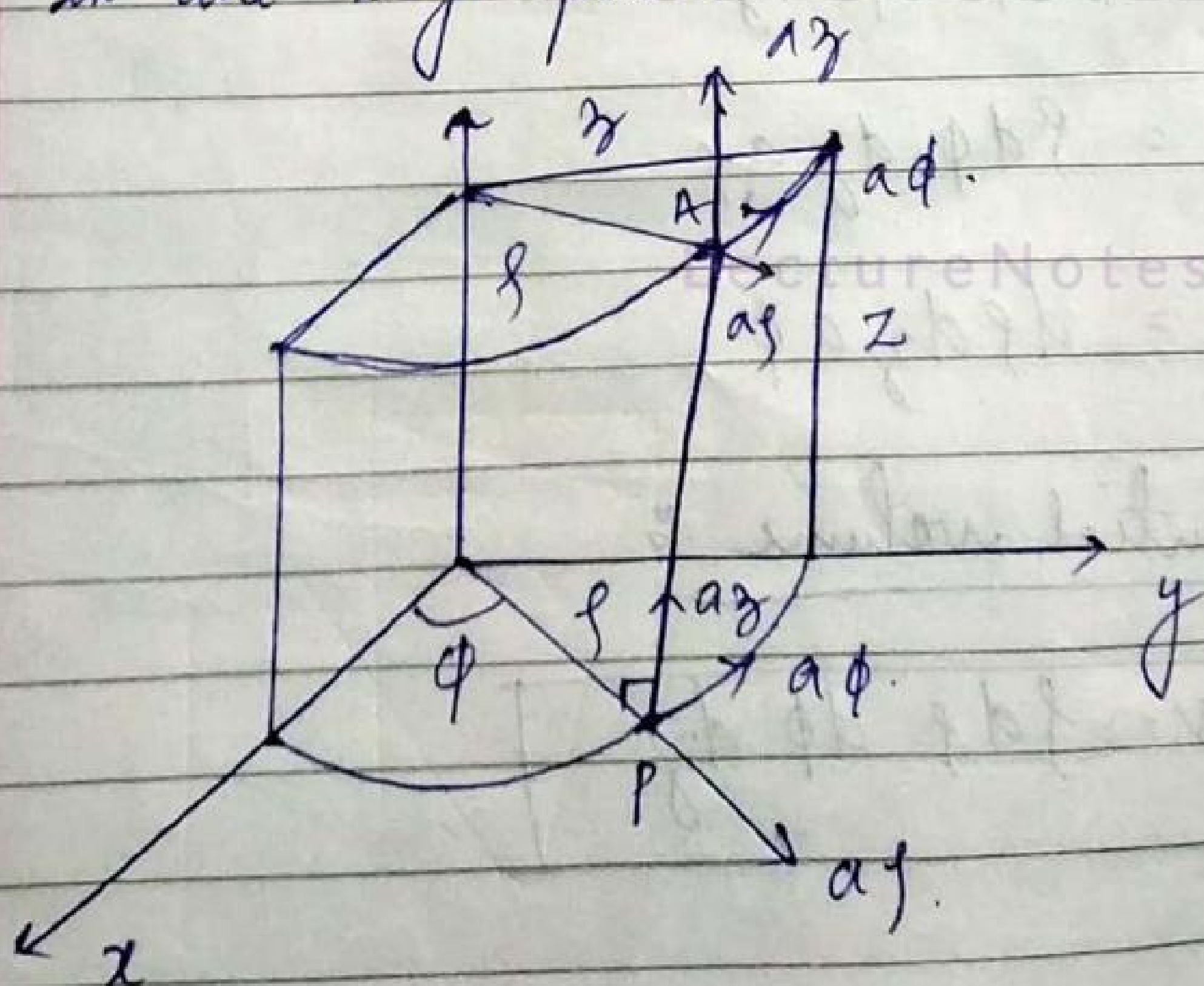
$$\vec{A} = A_\rho \mathbf{a}_\rho + A_\phi \mathbf{a}_\phi + A_z \mathbf{a}_z$$

let ρ be the radius of the cylinder

$z \rightarrow$ height of the cylinder

$\phi \rightarrow$ Azimuthal angle

It is the angle measured from x -axis in the x - y plane

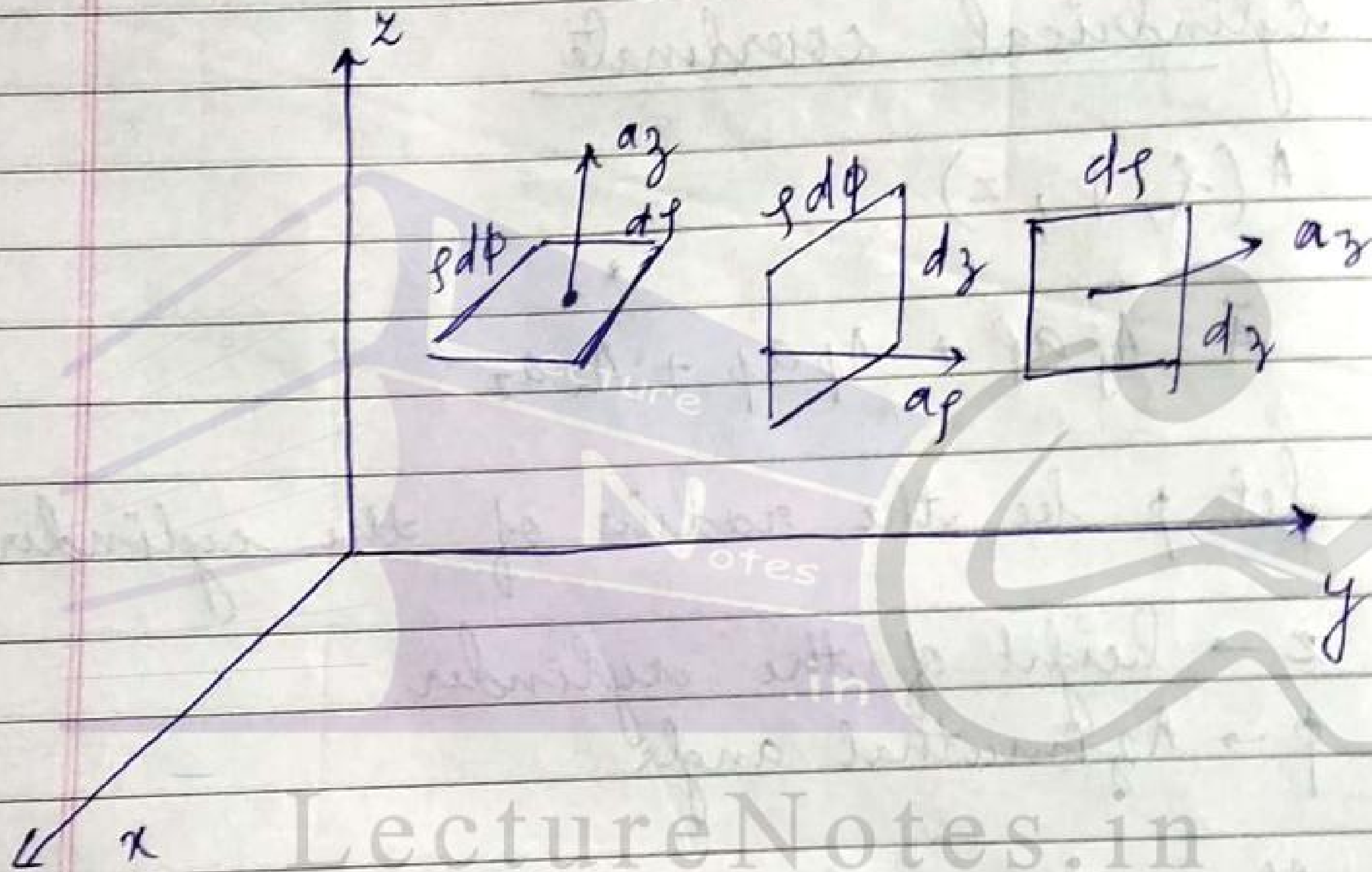


$$|\vec{A}| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$

① Differential length :

$$dl = d\rho a_\rho + \rho d\phi a_\phi + dz a_z$$

② Differential surface :



$$ds = \rho d\rho d\phi dz$$

$$= \rho d\phi dz a_\rho$$

$$= d\rho dz a_\phi$$

③ Differential volume :

$$dv = \rho d\rho d\phi dz$$

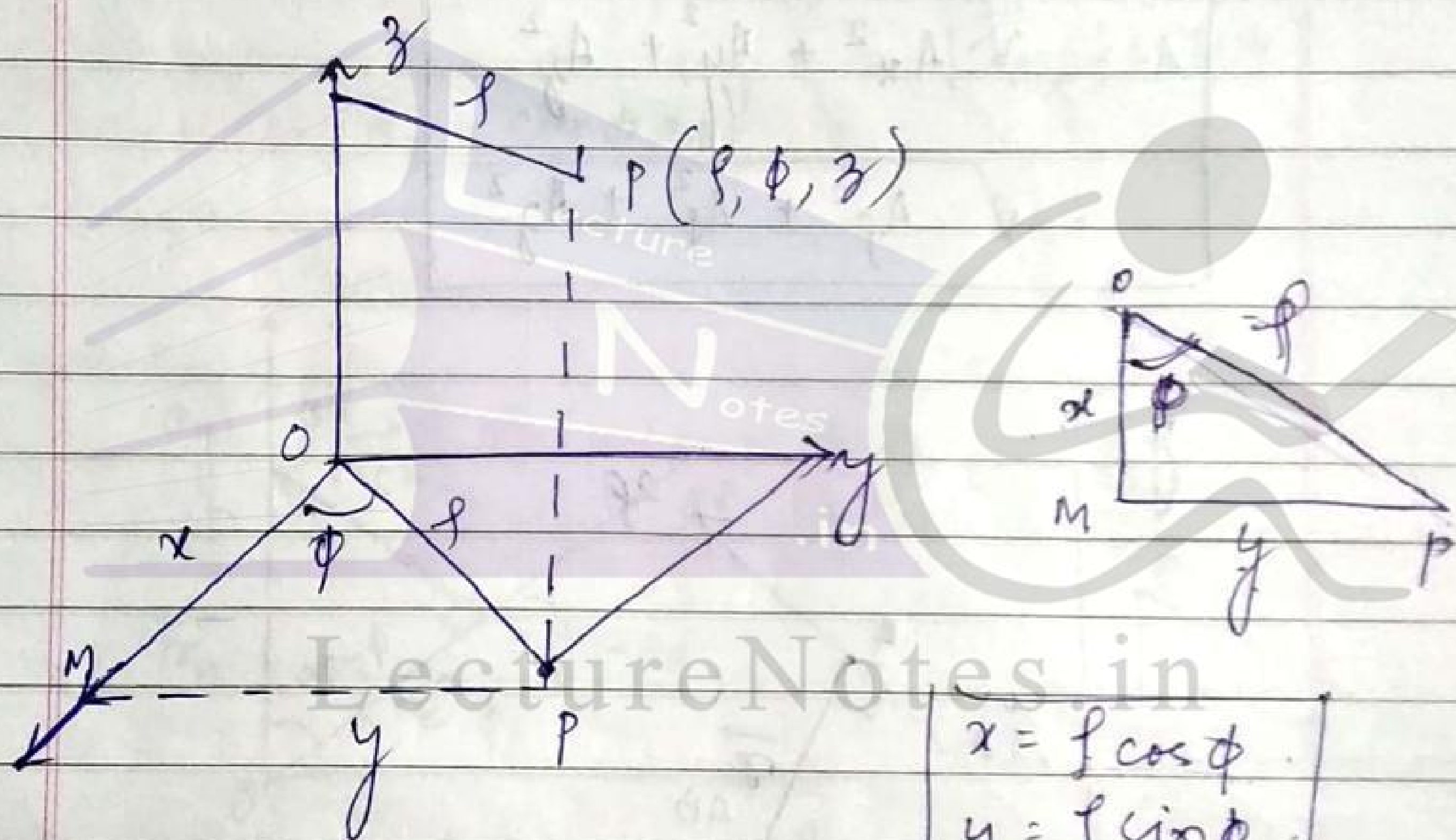
PAGE NO. _____
DATE: ____/____/____

Conversion from cartesian to cylindrical coordinate system.

$$A(x, y, z) \rightarrow A(\rho, \phi, z)$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\vec{B} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$



$$\rho = \sqrt{x^2 + y^2}$$

$$\tan \phi = \frac{y}{x}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$

$$P(x, y, z) \rightarrow (\rho, \phi, z)$$

$$|OA| = |OB| \cos \phi$$

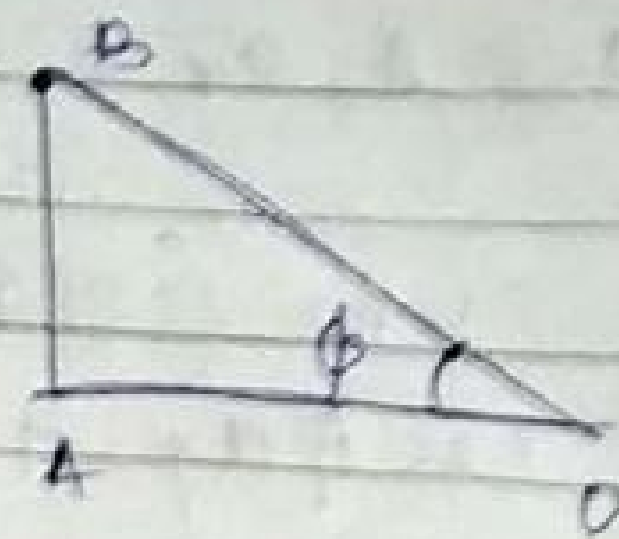
$$|\vec{r}_A| = \cos \phi$$

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{AB}$$

$$a_y = \cos \phi a_x + \sin \phi (-a_\phi)$$

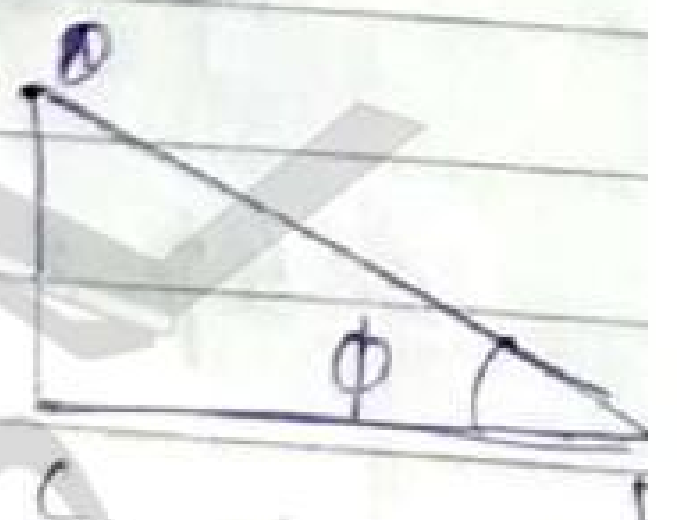
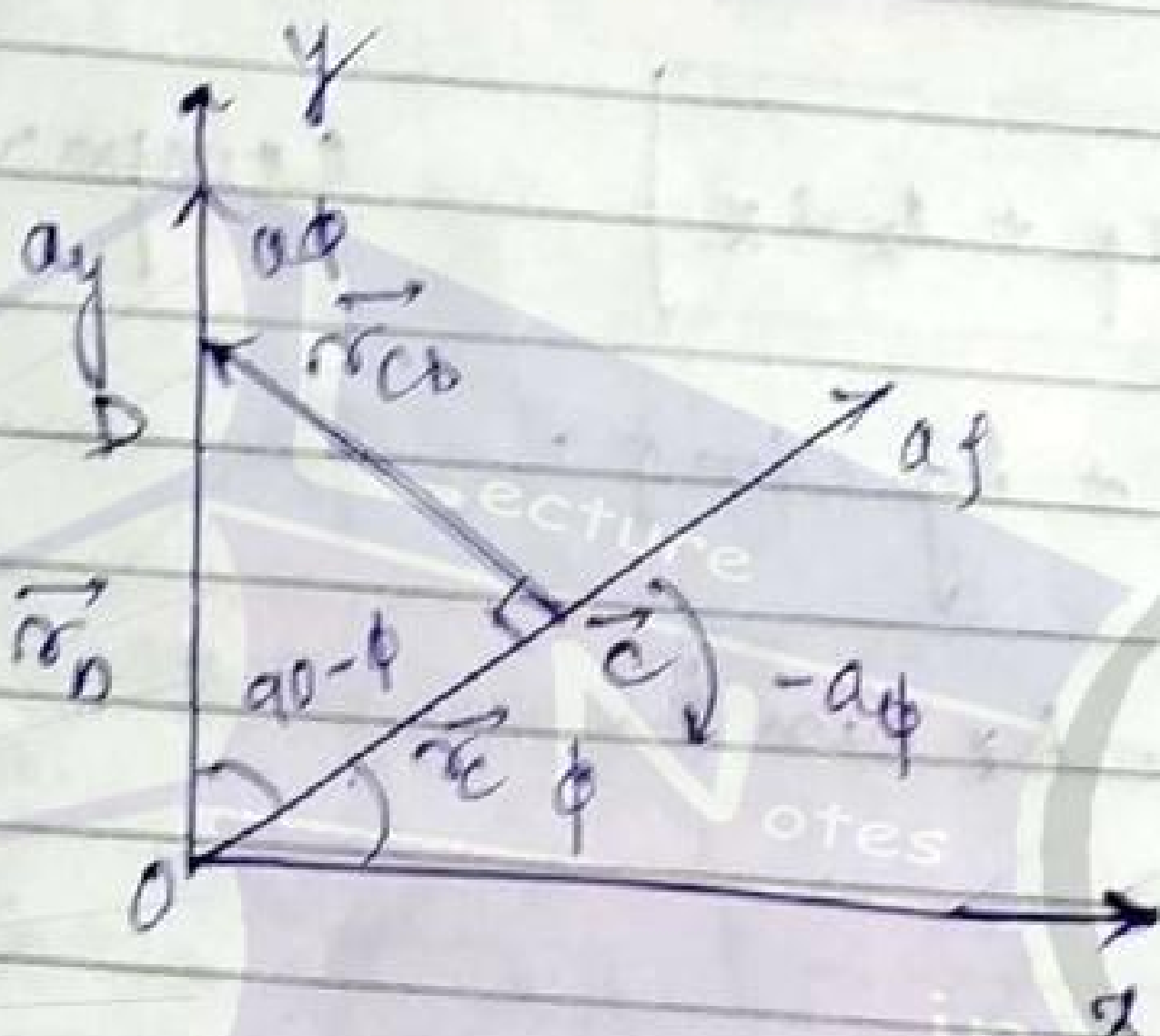
$$a_y = \cos \phi a_x - \sin \phi a_\phi$$



$$\sin \phi = \frac{AB}{OB}$$

$$|\vec{r}_{AB}| = |\vec{r}_{AB}| \sin \phi$$

$$|\vec{r}_{AB}| = \sin \phi$$



$$\vec{OD} = \vec{OC} + \vec{OB}$$

$$\vec{r}_D = \vec{OB} = |\vec{OD}| a_y$$

$$\vec{r}_D = a_y$$

$$\vec{r}_D = \vec{r}_C + \vec{r}_{CD}$$

$$a_y = |\vec{r}_C| a_x + |\vec{r}_{CD}| a_\phi$$

$$a_y = \sin \phi a_x + \cos \phi a_\phi$$

$$\cos \phi = \frac{|CD|}{|OD|}$$

$$CD = \cos \phi$$

$$\sin \phi = \frac{OC}{OD}$$

$$OC = \sin \phi$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$= A_x (\cos \phi \hat{a}_r - \sin \phi \hat{a}_\phi) + A_y (\sin \phi \hat{a}_r + \cos \phi \hat{a}_\phi) + A_z \hat{a}_z$$

$$= A_x \cos \phi \hat{a}_r - A_x \sin \phi \hat{a}_\phi + A_y \sin \phi \hat{a}_r + A_y \cos \phi \hat{a}_\phi + A_z \hat{a}_z$$

$$\vec{A} = (A_x \cos \phi + A_y \sin \phi) \hat{a}_r + (A_y \cos \phi - A_x \sin \phi) \hat{a}_\phi + A_z \hat{a}_z$$

$$\vec{A} = A_r \hat{a}_r + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

comparing both eqⁿ.

$$A_r = A_x \cos \phi + A_y \sin \phi$$

$$A_\phi = A_y \cos \phi - A_x \sin \phi$$

$$A_x = A_z$$

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\vec{A}(x, y, z) \Rightarrow \vec{A}(r, \phi, z)$$

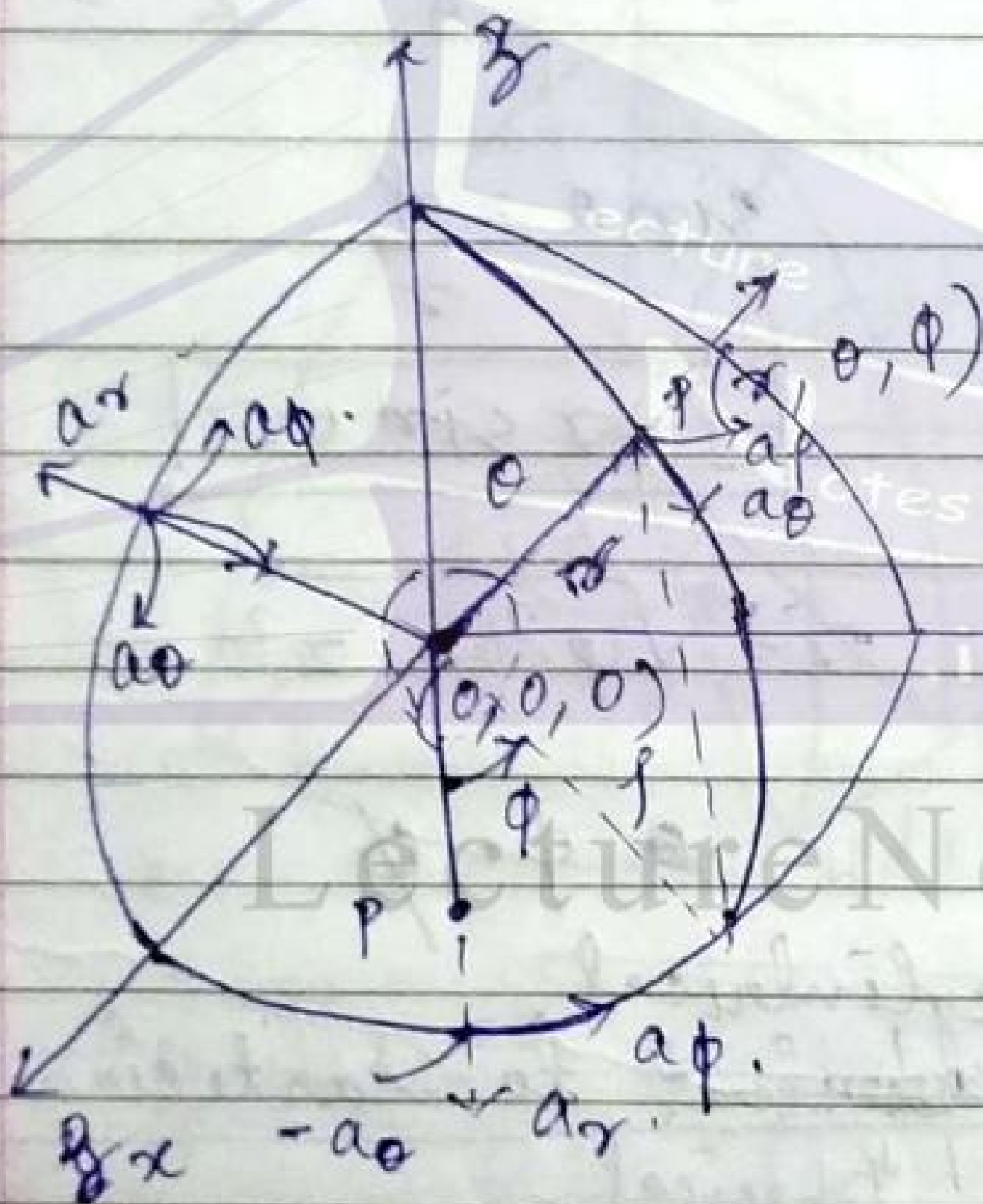
$$a_x = \cos \phi a_r - \sin \phi a_\phi \quad (\text{if given}).$$

$$| a_y \cdot a_f = \sin \phi. | \Rightarrow (\sin \phi a_f + \cos \phi a_\phi) \cdot a_f = \sin \phi.$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\phi \\ A_\theta \\ A_z \end{bmatrix}$$

$$T(x, y, z) \rightarrow$$

Spherical coordinate : (r, θ, ϕ)



$$\left\{ \begin{array}{l} 0 \leq \theta \leq \infty \\ 0 \leq \theta \leq \pi \\ 0 \leq \phi \leq 2\pi \end{array} \right.$$

- $\mathbf{r}$ is the position vector from the origin to the point P .

$$P = (r, \theta, \phi)$$

[θ = co-altitude angle]

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi.$$

- It is the angle measured from the z-axis to the position vector (R) is called co-altitude angle (θ).

~~Range~~

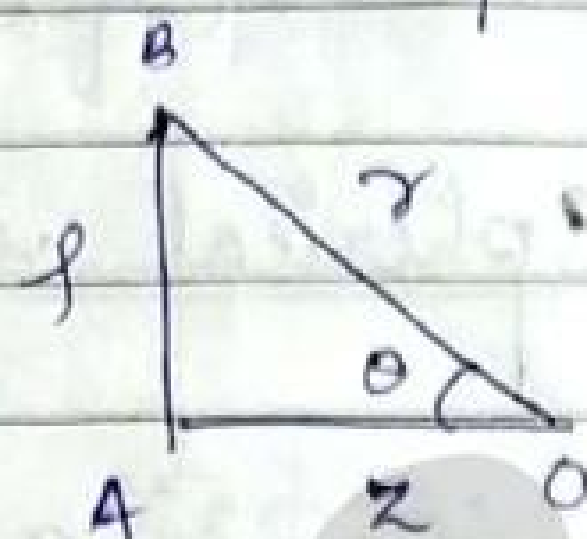
- ϕ is the azimuthal angle measured from the x-axis

22.01.19 Conversion from cylindrical to spherical

$$(\rho, \phi, z) \rightarrow (r, \theta, \phi \dots)$$

$\triangle OAB$

$$\cos \theta = \frac{z}{r}$$



$$z = r \cos \theta$$

$$\sin \theta = \frac{\rho}{r}$$

$$\tan \theta = \frac{\rho}{z}$$

$$\rho = r \sin \theta$$

$$\theta = \tan^{-1} \left(\frac{\rho}{z} \right)$$

$$r = \sqrt{\rho^2 + z^2}$$

$$\cancel{r = \tan^{-1} \left(\frac{\rho}{z} \right)}$$

$$\phi = \phi$$

Conversion from ~~spherical~~ cylindrical to cartesian

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

cartesian to spherical

$$(x, y, z) \rightarrow (r, \theta, \phi)$$

$$x = r \cos \phi$$

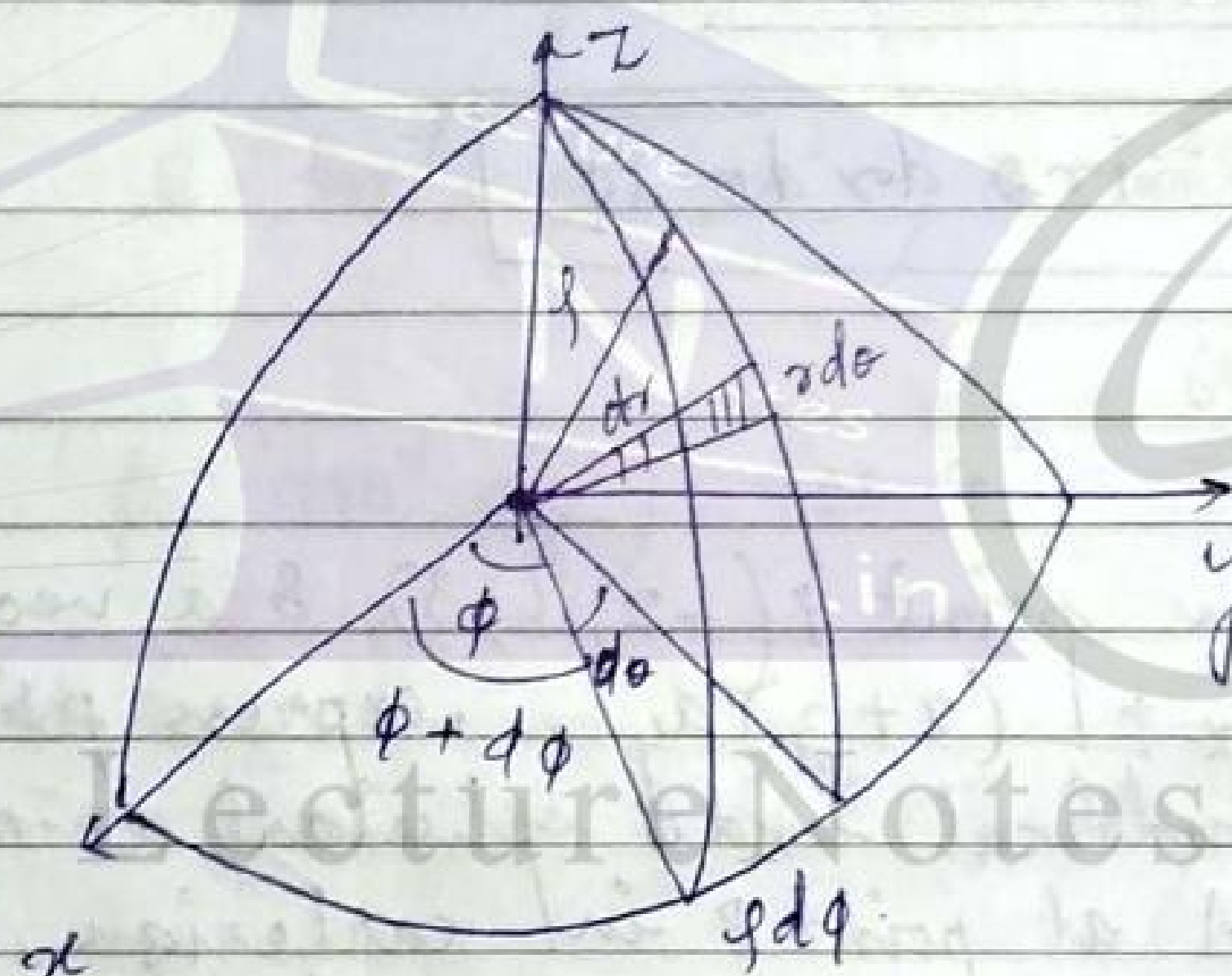
$$\boxed{x = r \sin \theta \cos \phi}$$

$$y = r \sin \phi \quad \boxed{y = r \sin \theta \sin \phi}$$

$$\boxed{z = r \cos \theta}$$

$$(r, \theta, \phi) \rightarrow (x, y, z)$$

vector calculus for spherical co-ordinate:

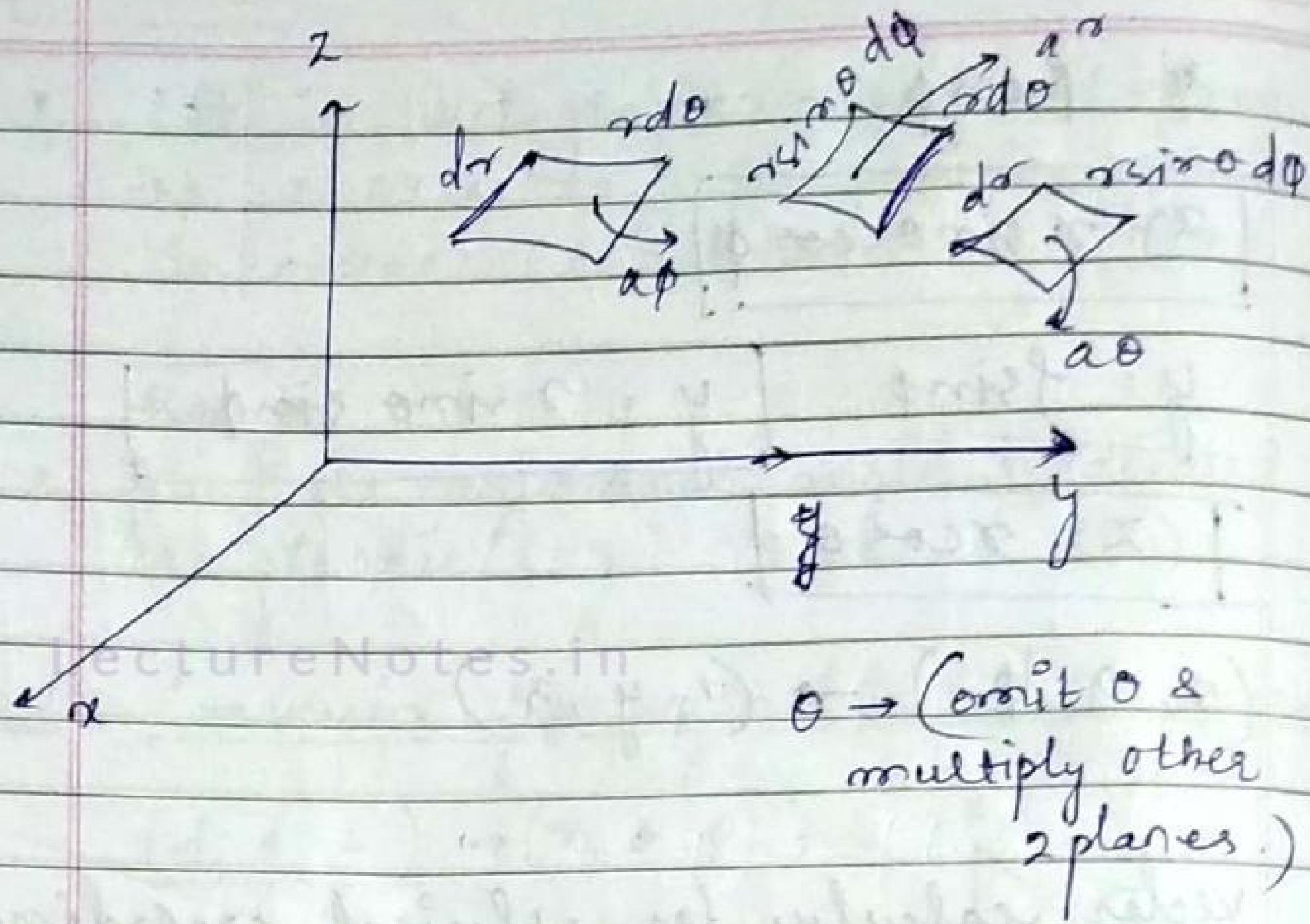


① Differential length

$$\boxed{dl = dr \mathbf{a}_r + r d\theta \mathbf{a}_\theta + r \sin \theta d\phi \mathbf{a}_\phi}$$

② Differential surface

$$\begin{aligned} ds_1 &= r dr d\theta d\phi \\ &= r^2 \sin \theta d\theta d\phi \mathbf{a}_r \\ &= r \sin \theta dr d\phi \mathbf{a}_\theta \end{aligned}$$



③ Differential volume :

$$dv = r^2 \sin \theta \, dr \, d\theta \, d\phi$$

23.01.19

Q) A given point $P(-2, 6, 3)$ & a vector $\vec{A} = y\vec{a}_x + (x+z)\vec{a}_y$, express P & $\vec{A}$ in cylindrical and spherical co-ordinate. Evaluate A at point P in cartesian, cylindrical & spherical coordinate.

47 It is in cartesian system

$$1) P(x, y, z) \rightarrow P(r, \theta, \phi) \\ P(r, \theta, \phi)$$

for P

$$\text{given: } x = -2, y = 6, z = 3$$

$$P(x, y, z) \rightarrow P(r, \theta, \phi)$$

$$\boxed{\rho = \sqrt{x^2 + y^2}} = \sqrt{(-2)^2 + (6)^2} = \sqrt{4 + 36} = \sqrt{40} = 6.32$$

$$\boxed{\phi = \tan^{-1} \frac{y}{x}}$$

$$= \tan^{-1} \left(\frac{6}{-2} \right) = 108.4^\circ$$

$$\boxed{z = 3} \quad z = 3$$

$$P(x, y, z) \rightarrow P(r, \theta, \phi)$$

$$r = \sqrt{x^2 + y^2 + z^2} = 7$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right) = \tan^{-1} \left(\frac{\sqrt{(-2)^2 + (6)^2}}{3} \right) = 64.5^\circ$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right) = 108.4^\circ$$

for $\vec{A}$

$$\vec{A} = y a_x + \theta(x + z) a_y$$

$$\vec{A} \big|_{P(-2, 6, 3)} = 6 a_x + (-2 + 3) a_y$$

$$\boxed{\vec{A} = 6 a_x + a_y} \rightarrow \text{cartesian}$$

for cylindrical:

$$\vec{A} = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

$$P(6.32, 108.4, 3)$$

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = A_x \cos\phi + A_y \sin\phi$$

$$A_r = y \cos\phi + (x+z) \sin\phi$$

$$= r \sin\phi \cos\phi + (r \cos\phi + z) \sin\phi$$

$$= 6.32 \sin(108.4^\circ) \cos(108.4^\circ) +$$

$$\{ 6.32 \times \cos(108.4^\circ) + 3 \} \sin(108.4^\circ)$$

$$= -0.9487$$

$$A_\phi = -\sin\phi A_x + \cos\phi A_y$$

$$= -y \sin\phi + (x+z) \cos\phi$$

$$= -r \sin\phi \sin\phi + (r \cos\phi + z) \cos\phi$$

$$= -r \sin^2\phi + r \cos^2\phi + z \cos\phi$$

$$= -6.32 \sin^2(108.4^\circ) + 6.32 \cos^2(108.4^\circ) +$$

$$3(\cos(108.4^\circ))$$

$$= -6$$

$$\therefore \boxed{\vec{A} = -0.94 a_r - 6.0 a_\phi}$$

cylindrical

for spherical :

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

(r, θ, ϕ)

$$(7, 0.45, 108.4)$$

Imp

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_\theta \\ A_\phi \end{bmatrix}$$

ay

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -0.94 \\ -6 \\ 0 \end{bmatrix}$$

$$\begin{aligned} A_r &= \sin\theta \times (-0.94) + 0 \\ &= \sin(64.5) \times (-0.94) \\ &= -0.85 \end{aligned}$$

$$\begin{aligned} A_\theta &= \cos\theta (-0.94) \\ &= \cos(64.5) (-0.94) \\ &= -0.4 \end{aligned}$$

$$A_\phi = -6$$

$$\therefore \vec{A} = -0.85 \vec{a}_x - 0.4 \vec{a}_y - 6 \vec{a}_\phi$$

$$Q) \vec{A} = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} \vec{a}_x - \frac{yz}{\sqrt{x^2+y^2+z^2}} \vec{a}_y$$

Transform the $\vec{A}$ into spherical & cylindrical coordinate.

As spherical coordinate

$$\vec{A} = A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = A_x \sin\theta \cos\phi + A_y \sin\theta \sin\phi + A_z \cos\theta$$

$$A_x = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}, \quad r = \sqrt{x^2+y^2+z^2}$$

$$= \frac{r \sin\theta}{r} = \sin\theta \quad x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$A_y = 0, \quad A_z = \frac{-yz}{\sqrt{x^2+y^2+z^2}} \quad \sqrt{x^2+y^2} = \sqrt{r^2 \sin^2\theta \cos^2\phi + r^2 \sin^2\theta \sin^2\phi}$$

$$A_z = -\frac{r \sin\theta \sin\phi r \cos\phi}{r} = -\sqrt{r^2 \sin^2\theta} [\cos^2\phi + \sin^2\phi]$$

$$= -r \sin\theta$$

$$A_z = -r \sin \theta \sin \phi \cos \theta$$

$$\therefore A_x = \sin \theta \sin \phi \cos \phi - r \sin \theta \cos \theta \sin \phi \cos \theta$$

$$A_x = \sin^2 \theta \cos \phi - r \sin \theta \cos^2 \theta \sin \phi$$

$$A_\theta = A_x \cos \theta \cos \phi + A_y(0) - \sin \theta A_z$$

$$= \sin \theta \cos \theta \cos \phi + r \sin^2 \theta \cos \theta \sin \phi$$

$$A_\phi = -A_x \sin \phi - 0 - 0$$

$$= -\sin \theta \cdot \sin \phi$$

$$\vec{A} = (\sin^2 \theta \cos \phi - r \sin \theta \cos^2 \theta \sin \phi) \hat{a}_x$$

$$+ (\sin \theta \cos \theta \cos \phi + r \sin^2 \theta \cos \theta \sin \phi) \hat{a}_\theta$$

$$- \sin \theta \sin \phi \hat{a}_\phi$$

cylindrical :

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = \cos \phi A_x + A_y \sin \phi + 0$$

$$= \cos \phi A_x$$

$$= \cos \phi \sin \theta$$

$$A_\phi = -\sin \phi A_x$$

$$= -\sin \theta \sin \phi$$

$$A_z = A \cos \theta$$

$$= -r \sin \theta \sin \phi \cos \theta$$

$$\therefore \vec{A} = A_\rho \hat{a}_\rho + A_\theta \hat{a}_\theta + A_z \hat{a}_z$$

$$= \cos \theta$$

$$a) \vec{A} = \frac{f}{\sqrt{f^2 + z^2}} \left[\cos \phi \hat{a}_f - \sin \phi \hat{a}_\phi - z \sin \phi \hat{a}_z \right]$$

into spherical.

$$\vec{A} = \frac{f \cos \phi}{\sqrt{f^2 + z^2}} \hat{a}_f - \frac{f \sin \phi}{\sqrt{f^2 + z^2}} \hat{a}_\phi - \frac{z \sin \phi}{\sqrt{f^2 + z^2}} \hat{a}_z$$

$$\left\{ \begin{array}{l} f = r \sin \theta \\ z = r \cos \theta \\ r = \sqrt{f^2 + z^2} \end{array} \right.$$

$$A_f = \frac{f \cos \phi}{\sqrt{f^2 + z^2}} = \frac{r \sin \theta \cos \phi}{r} = \sin \theta \cos \phi$$

$$A_\phi = -\frac{f \sin \phi}{\sqrt{f^2 + z^2}} = -\frac{r \sin \theta \sin \phi}{r} = -\sin \theta \sin \phi$$

$$A_z = \frac{-rz \sin \phi}{\sqrt{r^2 + z^2}} = \frac{r \sin \theta \cos \theta \sin \phi}{r}$$

$$= \sin \theta \cos \theta \sin \phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$A_r = \sin \theta A_r + \cos \theta A_z$$

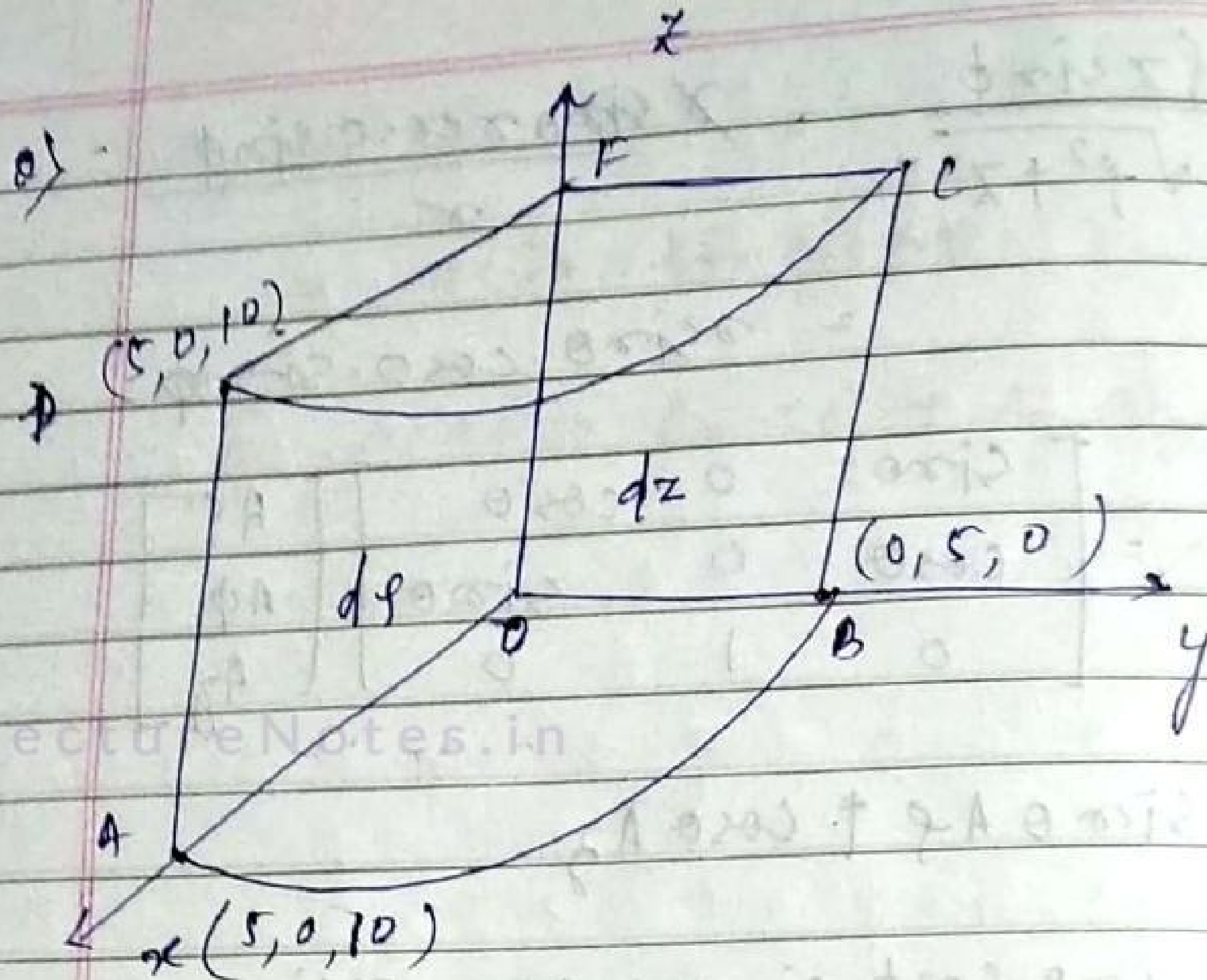
$$\begin{aligned} &= \sin \theta \cos \phi \sin \theta + \cos \theta \cdot r \sin \theta \cos \theta \sin \phi \\ &= \sin^2 \theta \cos \phi + \cos^2 \theta \cdot \sin \theta \sin \phi \end{aligned}$$

$$\begin{aligned} A_\theta &= \cos \theta A_r - A_z \sin \theta \\ &= \sin \theta \cos \phi \cos \theta - r \sin \theta \cos \theta \sin \phi \sin \theta \end{aligned}$$

$$\begin{aligned} A_\phi &= A_\phi \\ &= \sin \phi \sin \theta \end{aligned}$$

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

$$\begin{aligned} &= (\sin^2 \theta \cos \phi) \hat{a}_r + (\sin \theta \cos \phi \cos \theta - r \sin \theta \cos \theta \sin \phi) \hat{a}_\theta \\ &\quad + (\sin \phi \sin \theta) \hat{a}_\phi \end{aligned}$$



consider the object shown in the figure & calculate.

- (i) length of BC
- (ii) length of CD.
- (iii) surface area ABCD.
- (iv) S.A of ABO.
- (v) S.A of ADFD
- (vi) Volume.

AY $A(5, 0, 0) \rightarrow A(5, 0, 0)$

cartesian to cylindrical.

$$\left\{ \begin{array}{l} \rho = \sqrt{x^2 + y^2} = \sqrt{5^2} = 5 \\ \phi = \tan^{-1} \frac{0}{5} = 0 \\ z = z = 0 \end{array} \right\} (\rho, \phi, z)$$

$$B(0, 5, 0) \rightarrow B(5, \pi/2, 10)$$

$$C(0, 5, 10) \rightarrow C(5, \pi/2, 10)$$

$$D(5, 0, 10) \rightarrow D(5, 0, 10)$$

(i) length of BC (along z axis).

$$d\vec{l} = dx \vec{a}_x + y d\phi \vec{a}_\phi + dz \vec{a}_z$$

$$d\vec{l} = dz \vec{a}_z$$

$$BC = \int_0^{10} d\vec{l} = [z]_0^{10} = 10$$

(ii) length of CD (along y-axis).

$$d\vec{l} = dx \vec{a}_x + y d\phi \vec{a}_\phi + dz \vec{a}_z$$

$$d\vec{l} = y d\phi \vec{a}_\phi$$

$$CD = \int_{\phi=0}^{\pi/2} y d\phi = 5 \int_0^{\pi/2} d\phi = 9.05$$

(iii) surface area of ABCD.

$$\int dS = \int y d\phi dz = 5 \int_0^{\pi/2} d\phi \int_0^{10} dz$$

(iv) S.A of ABO.

$$ds = \int_0^r f dr \int_0^{\pi/2} d\phi.$$

v) S.A of AOED.

$$ds = \int f d\phi dz.$$

$$= r \int d\phi \times \int dz$$

$$\boxed{\text{S.A. of AOED} = 5 \times 10 = 50.}$$

vi) $dv = \int f d\phi d\phi dz$ (volume).

$$= \int_0^r f dr \int_0^{\pi/2} d\phi \int_0^{10} dz$$

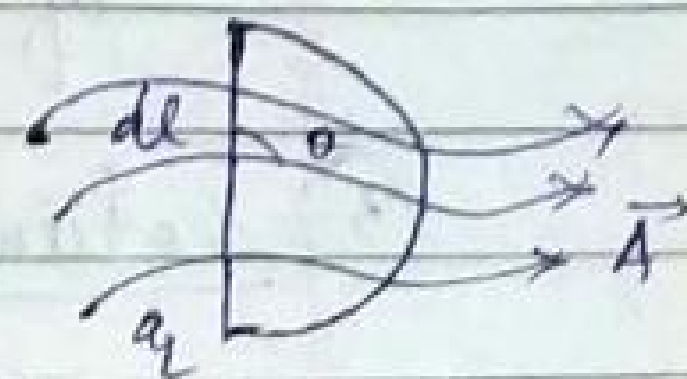
$$= \frac{f^2}{2} \Big|_0^5 \times \theta \Big|_0^{\pi/2} \times z \Big|_0^{10}.$$

$$\boxed{v = 125 \frac{\pi}{2} = 62.5\pi}$$

CHAPTER - 2

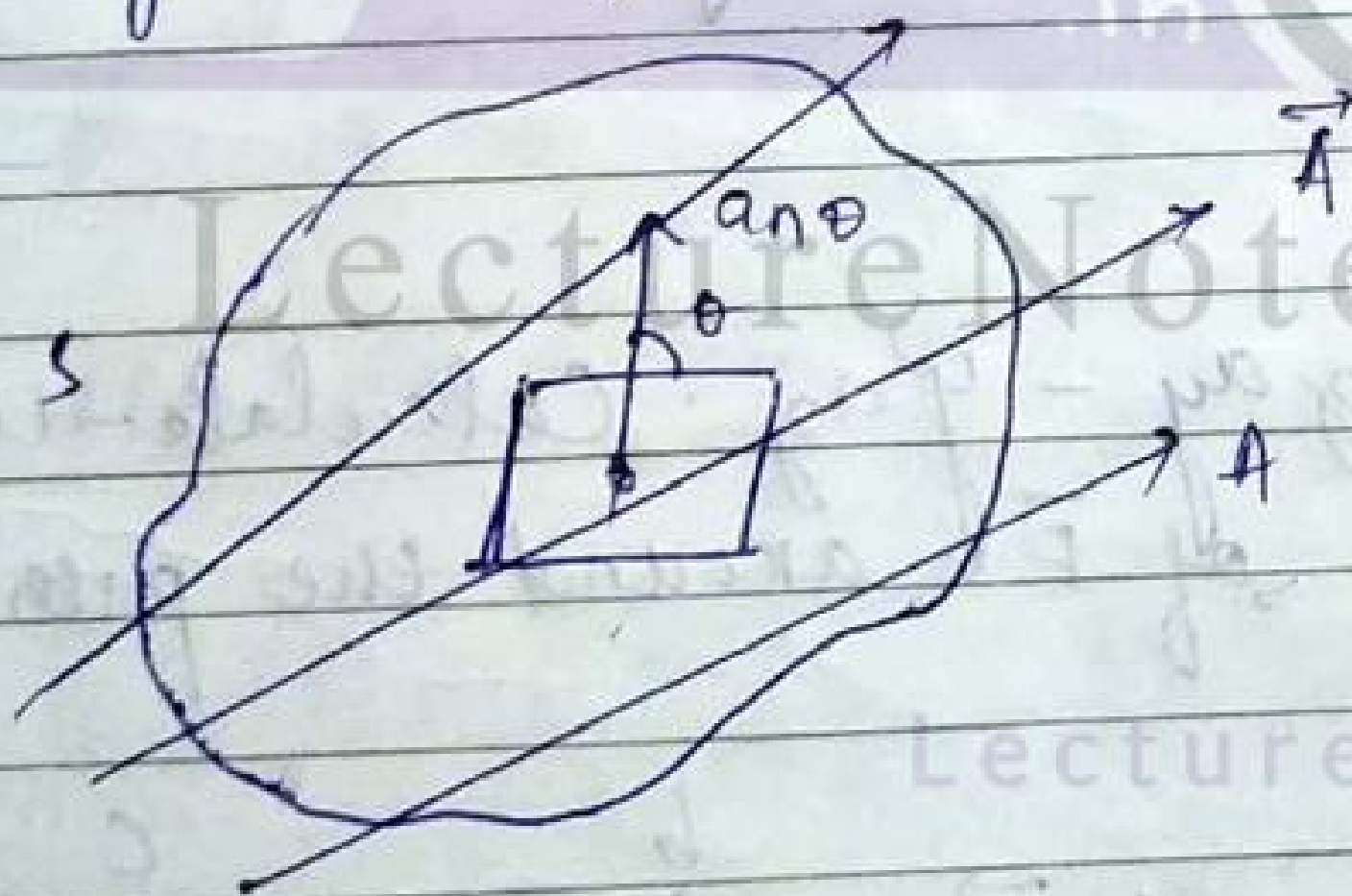
- 1) Line Integral : It is defined as the integral of the tangential component of $\vec{A}$ around the curve L .

$$\int_L \vec{A} \cdot d\vec{l} = \int_a^b |\vec{A}| \cos \theta \cdot dl$$



- * If the path is a closed curve, then $\oint_L \vec{A} \cdot d\vec{l}$ is called as circulation of $\vec{A}$ around the path L .
- * If $\oint_L \vec{A} \cdot d\vec{l} = 0 \rightarrow$ it is called as conservative field / lamellar field.

2) Surface Integral :



$$\int_S \vec{A} \cdot d\vec{s} = \int_S |\vec{A}| \cos \theta \cdot ds = \int_S \vec{A} \cdot \vec{a}_n ds$$

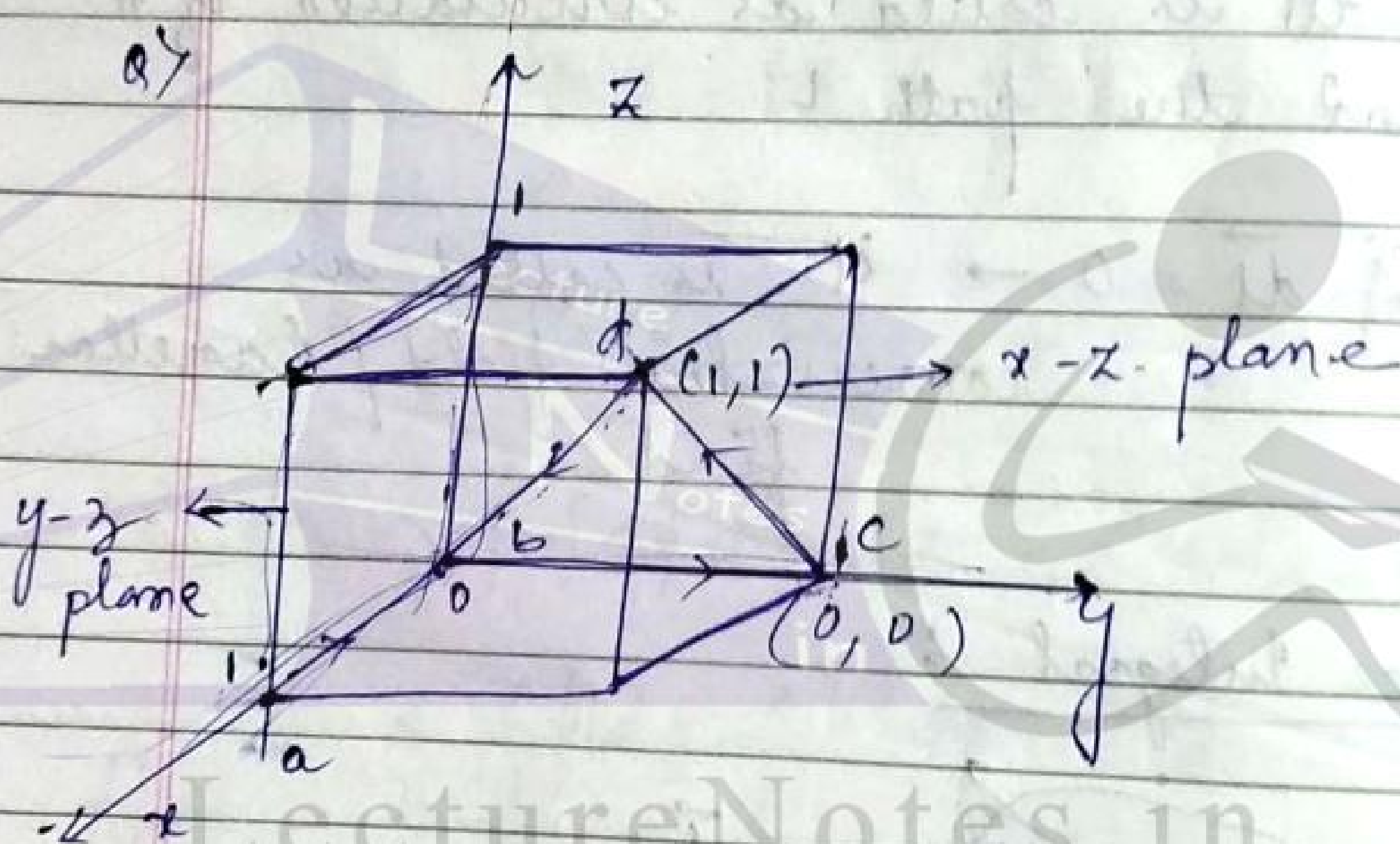
$$\psi = \int_S \vec{A} \cdot d\vec{s} = \text{flux of } \vec{A} \text{ through } S.$$

$$\boxed{\Psi_{\text{net outward}} = \oint_S \vec{A} \cdot d\vec{s}}$$

for a close surface, the surface integral will refer as the net outward flux of $\vec{A}$ from S .

③ Volume Integral.

$$\boxed{\int_V \rho_v dv}$$



$\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - y^2 \vec{a}_z$. Calculate the circulation of $\vec{F}$ around the path shown.

$$\oint_L \vec{F} \cdot d\vec{l} = \int_{\text{abcd}} \vec{F} \cdot d\vec{l} = \int_a^b \vec{F} \cdot d\vec{l} + \int_b^c \vec{F} \cdot d\vec{l} + \int_c^d \vec{F} \cdot d\vec{l} + \int_d^a \vec{F} \cdot d\vec{l}$$

segment ab.

$$y=0, z=0 \text{ (x-axis)}$$

$$\therefore \vec{F} = x^2 \vec{a}_x \quad \vec{dl} = dx \vec{a}_x$$

$$\vec{F} \cdot d\vec{l} = x^2 \cdot dx$$

$$\therefore \int_a^b \vec{F} \cdot d\vec{l} = \int_1^0 x^2 \cdot dx = \left[\frac{x^3}{3} \right]_1^0 = -\frac{1}{3}$$

segment bc : $(x=0, z=0)$ y-axis

$$\vec{F} = -y^2 \vec{a}_z \quad d\vec{l} = dy \vec{a}_y$$

$$\vec{F} \cdot d\vec{l} = -y^2 \vec{a}_z \cdot dy \vec{a}_y = 0$$

$$\int_a^b \vec{F} \cdot d\vec{l} = \int_0^1 0 = 0$$

segment cd :

$$y=1, x=z$$

$$\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - \vec{a}_z$$

$$d\vec{l} = dx \vec{a}_x + dz \vec{a}_z$$

$$\vec{F} \cdot d\vec{l} = (x^2 \vec{a}_x - xz \vec{a}_y - \vec{a}_z) \cdot (dx \vec{a}_x + dz \vec{a}_z)$$

$$= x^2 dx - dz$$

$$\vec{F} \cdot d\vec{l} = x^2 dx - dz = (x^2 - 1) dx$$

$$\int \vec{F} \cdot d\vec{l} = \int_0^1 (x^2 - 1) \cdot dx$$

$$= \left[\frac{x^3}{3} - x \right]_0^1 = -\frac{2}{3}$$

segment dl is $(1, 1) (0, 0)$

$$x=1, \quad z=y \Rightarrow dy=dz$$

$$\vec{F} = x\vec{a}_x - z\vec{a}_y - y^2\vec{a}_z \quad d\vec{l} = dy\vec{a}_y + dz\vec{a}_z$$

$$\vec{F} \cdot d\vec{l} = -z dy - y^2 dz$$

$$= -y dy - y^2 dy$$

$$= dy(-y - y^2)$$

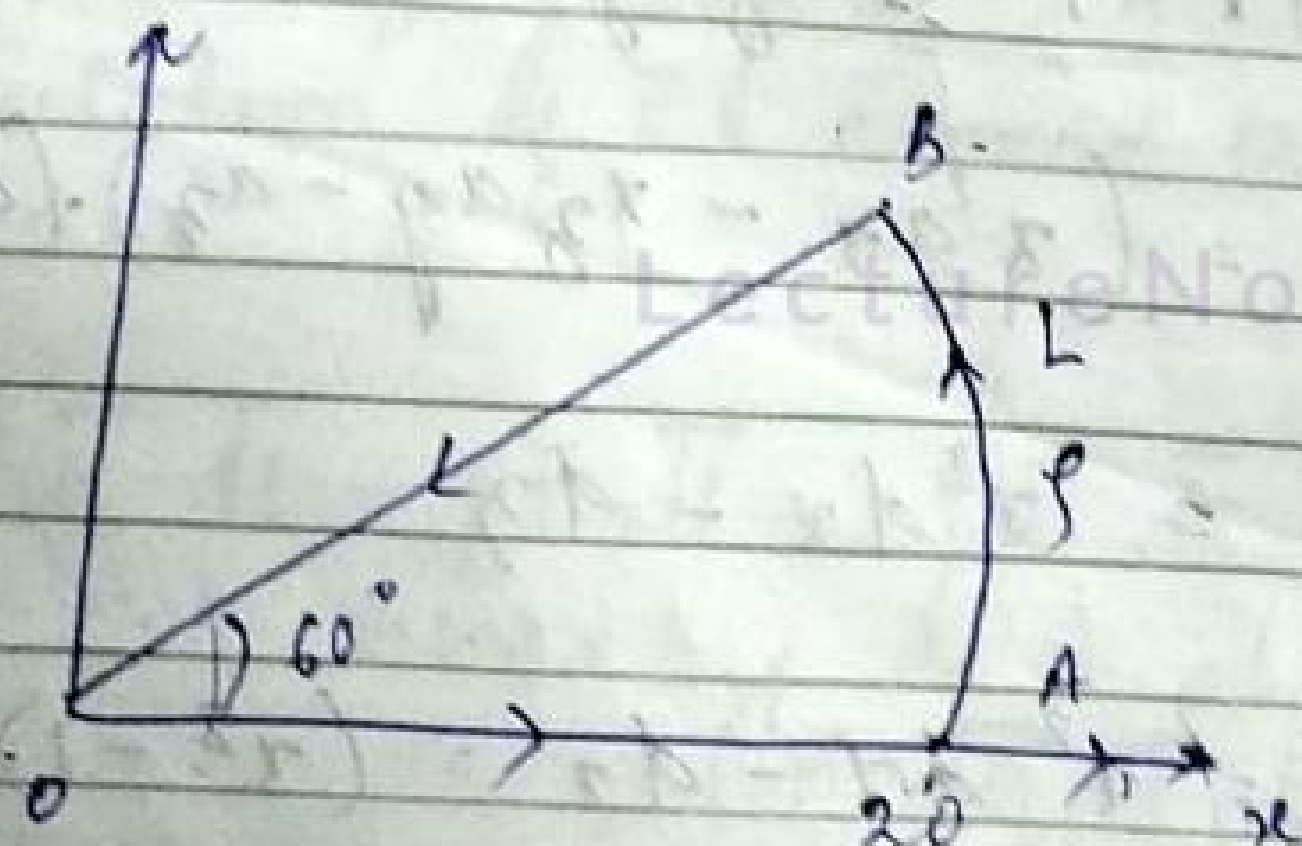
$$= \int_0^1 \vec{F} \cdot d\vec{l} = \int_0^1 (-y - y^2) \cdot dy = \left[-\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1$$

$$= -\frac{5}{6}$$

$$\therefore \oint \vec{F} \cdot d\vec{l} = -\frac{1}{3} + 0 - \frac{2}{3} + \frac{5}{6} = -\frac{1}{6}$$

$$\oint_{abcd} \vec{F} \cdot d\vec{l} = -\frac{1}{6}$$

30.01.19



Calculate the circulation of $\vec{A} = f \cos \phi \vec{a}_\phi + f \sin \phi \vec{a}_z$
around L ; $0 \leq f \leq 2$
 $0 \leq \phi \leq 60$ $z=0$

$$A) \oint_{OABO} \vec{A} \cdot d\vec{l} = \left[\int_0^4 + \int_4^6 + \int_6^0 \right] \vec{A} \cdot d\vec{l}$$

segment OA : $\phi = 0, z = 0$

$$\vec{A} = f \cos 0 \, a_\phi = f a_\phi$$

$$d\vec{l} = dr a_r + r d\phi a_\phi + dz a_z$$

$$= dr a_r$$

$$\vec{A} \cdot d\vec{l} = f a_\phi \cdot dr a_r$$

$$= f dr$$

$$\therefore \int_0^2 \vec{A} \cdot d\vec{l} = \int_0^2 f \cdot dr = \frac{f^2}{2} = 2$$

segment AB :

$$f = 2, z = 0$$

$$\vec{A} = 2 \cos \phi \, a_\phi \quad dl = r d\phi a_\phi$$

$$\int_4^6 \vec{A} \cdot d\vec{l} = 0$$

segment BO :

$$\phi = 60^\circ, z = 0$$

$$\vec{A} = f \cos 60^\circ \quad a_\phi = \frac{f}{2} a_\phi$$

$$d\vec{l} = dr a_r \quad \int \vec{A} \cdot d\vec{l} = \int_0^2 \frac{f}{2} dr = -1$$

$$\oint \vec{A} \cdot d\vec{l} = 2 + 0 - 1 = 1$$

OABO

* Del operator (∇) : It is given by:

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

The unit (m^{-1}) or scalar

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{1}{r} \frac{\partial}{\partial r} a_r + \frac{\partial}{\partial z} a_z$$

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{1}{r} \frac{d}{dr} a_r + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} a_\theta$$

Gradient of a scalar

Let v be a scalar quantity in cartesian system.

then,

$$\Delta v = \frac{\partial v}{\partial x} a_x + \frac{\partial v}{\partial y} a_y + \frac{\partial v}{\partial z} a_z$$

Properties of ∇ operator :

- 1) Gradient of a scalar gives rise to a vector field.
- 2) The magnitude of ∇v is equal to the maximum rate of change of v per unit distance.
- 3) The direction of ∇v is along the maximum rate of change of v .

NOTE

Directional Derivative

1) It is the projection of ∇v in the direction of unit vector.

$$\nabla v \cdot \hat{a}_n = \text{Directional derivative}$$

2) If $A = \nabla v$. Then v is said to be the scalar potential of A .

$$\nabla v = \frac{dv}{dr} a_r + \frac{1}{r} \frac{dv}{d\phi} a_\phi + \frac{\partial v}{\partial z} a_z$$

$$\nabla v = \frac{\partial v}{\partial r} a_r + \frac{1}{r} \frac{\partial v}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial v}{\partial \phi} a_\phi$$

Imp

Mathematical property:

$$(1) \nabla(v+u) = \nabla v + \nabla u$$

$$(2) \nabla(vu) = v \nabla u + u \nabla v$$

$$(3) \nabla \left(\frac{v}{u} \right) = \frac{u \nabla v - v \nabla u}{u^2}$$

$$(4) \nabla v^n = n v^{n-1} \nabla v$$

* Divergence of a vector :

cartesian

$$\nabla \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$$

cylindrical

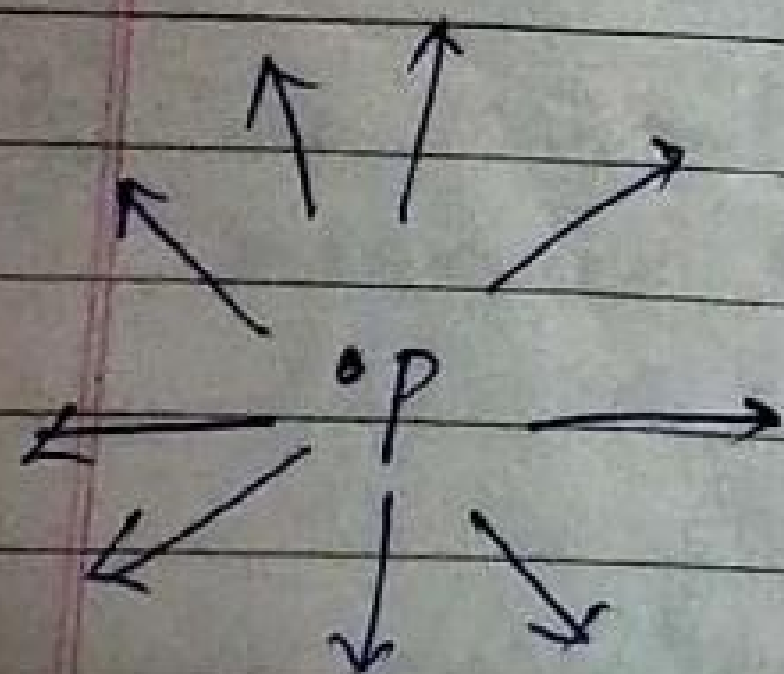
$$\nabla \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

spherical :

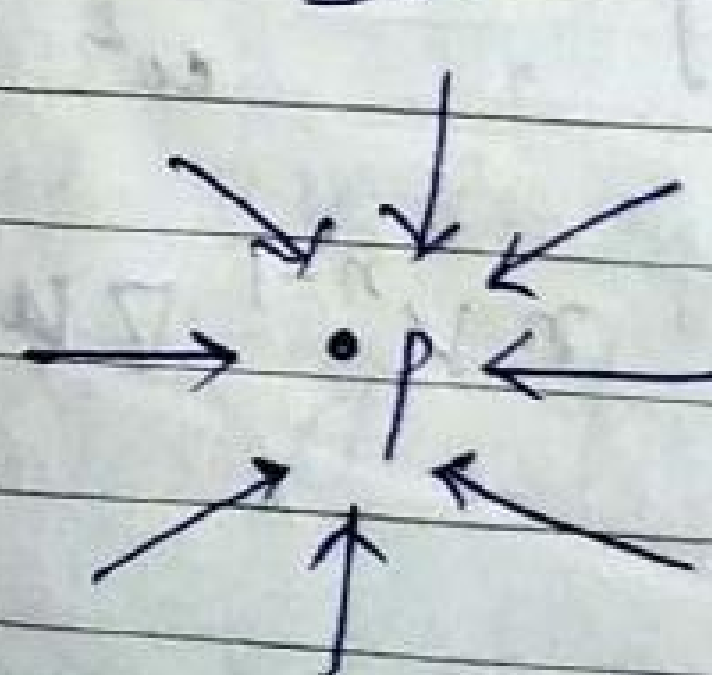
$$\nabla \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

* Divergence of a vector field at a point is the net outward flux per unit volume when the volume shrinks to a point (tends to zero).

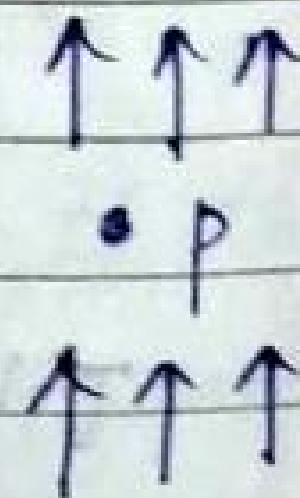
$$\nabla \vec{A} = \lim_{\Delta V \rightarrow 0} \frac{\oint \vec{A} \cdot d\vec{l}}{\Delta V}$$



positive divergence



-ve divergence / convergence



eq-
angular
zero divergence

NOTE :

- * If the divergence of a given vector field is zero i.e. $\nabla \cdot \vec{A} = 0$. Then the field is said to be solenoidal field / incompressible.

Properties of divergence :

- 1) The divergence of a vector field gives rise to another scalar field.
- 2) The divergence of $\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B}$
- 3) $\nabla \cdot (v \vec{A}) = v (\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla v$
- 4) The divergence of electric flux density is equal to volume charge density i.e. $\nabla \cdot \vec{D} = \rho_v$
- 5) The divergence of magnetic flux density is equal to zero i.e. $\nabla \cdot \vec{B} = 0$

date V.V. Imp

01.02.19

Divergence theorem (Gauss Theorem) :

Statement : This states that the total outward flux of a vector field $\vec{A}$ through the ~~flux of a vector field~~ closed surface 'S' is same as the volume integral of divergence of the vector field $\vec{A}$ over the entire volume.

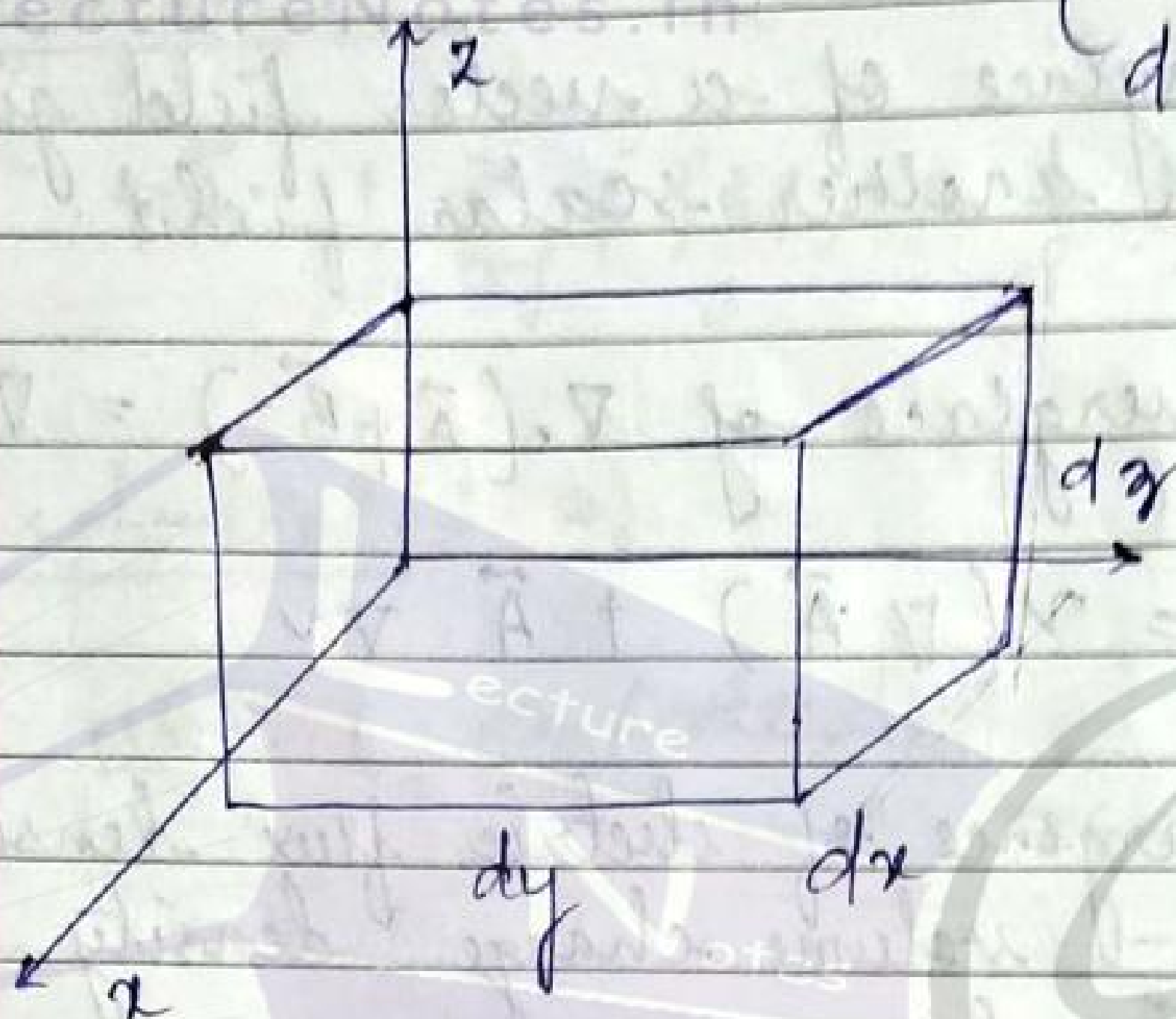
$$\oint_C \vec{A} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{A}) \cdot dV$$

net outward
flux

div of $\vec{A}$

Proof

$$\left\{ \begin{array}{l} ds_x = dy \cdot dz \\ ds_y = dx \cdot dz \\ ds_z = dx \cdot dy \end{array} \right\}$$



R.H.S

$$\int_V (\nabla \cdot \vec{A}) \cdot dV$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\int_V \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dx dy dz$$

$$= \int_V \frac{\partial A_x}{\partial x} dx dy dz + \frac{\partial A_y}{\partial y} dx dy dz +$$

$$\frac{\partial A_z}{\partial z} dx dy dz$$

$$= \iiint \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dx dy dz$$

$$= \iint A_x dy dz + \iint A_y dx dz + \iint A_z dx dy$$

$$= \iint A_x ds_x + \iint A_y ds_y + \iint A_z ds_z$$

$$= \iint (A_x ds_x + A_y ds_y + A_z ds_z)$$

$$= \iint (A_x ds_x + A_y ds_y + A_z ds_z)$$

$$= \iint [A_x a_x + A_y a_y + A_z a_z] \cdot (ds_x a_x + ds_y a_y + ds_z a_z)$$

$$= \iint \vec{A} \cdot d\vec{s}$$

$$= \oint \vec{A} \cdot d\vec{s}$$

$$= L.H.S$$

* Advantages of divergence theorem :

1) A surface integral can be converted into volume integral.

2) It can be applied to any closed surface.

3) It is applicable to any vector field whose partial derivative exists.

Disadvantages:-

1. It is not applicable to open surface
2. It is not applicable to a vector field whose partial derivative doesn't exist

Q.4

Curl of a vector

Curl of a vector $\vec{A}$ is an axial vector or normal vector whose magnitude is the maximum circulation of $\vec{A}$ per unit area when the area tends to zero and its direction is in the normal direction of the area; when the area is oriented to make the maximum circulation.

$$\text{curl } \vec{A} = \left(\lim_{\Delta S \rightarrow 0} \frac{\oint \vec{A} \cdot d\vec{l}}{\Delta S} \right) \cdot \hat{a}_n$$

(in max.)

(In cartesian)

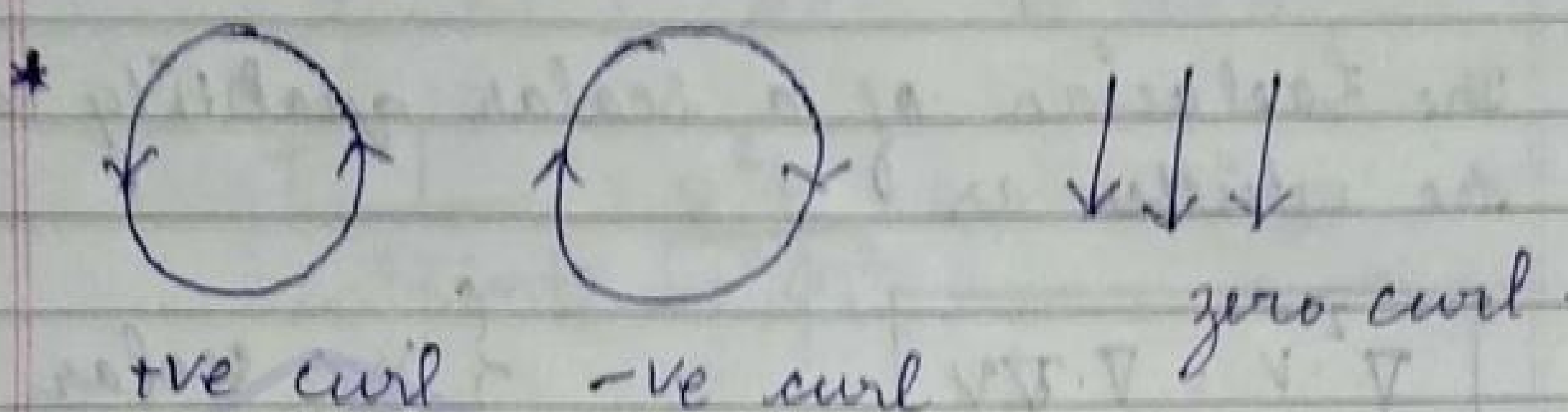
$$(\nabla \times \vec{A}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

In cylindrical

$$(\vec{\nabla} \times \vec{A}) = \frac{1}{r} \begin{vmatrix} a_\phi & a_\rho & a_z \\ \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \rho} & \frac{\partial}{\partial z} \\ A_\phi & A_\rho & A_z \end{vmatrix}$$

In spherical :

$$(\nabla \times \vec{A}) = \frac{1}{r^2 \sin \theta} \begin{vmatrix} a_r & r a_\theta & r \sin \theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$



v.v. Imp

$$\frac{\nabla \cdot \vec{A}}{\nabla \times \vec{A}}$$

Properties of curl

- 1.) The curl of a vector field is another vector field.
- 2.) The curl of addition of 2 vector fields is equals to $\nabla \times (A + B) = \nabla \times A + \nabla \times B$
- 3.) $\nabla \times (A \times B) = A(\nabla \cdot B) - B(\nabla \cdot A) + (B \cdot \nabla)A - (A \cdot \nabla)B$
- 4.) The divergence of curl of a vector field is equals to zero. $\nabla \cdot (\nabla \times \vec{A}) = 0$.
- 5.) curl of the gradient of a scalar field $(\nabla \times (\nabla V)) = 0$.

NOTE :

If $\boxed{\nabla \times \vec{A} = 0}$ then the field is said to be irrotational

* Laplacian operator :

The Laplacian of a scalar quantity can be written as

$$\boxed{\nabla^2 \cdot V = \nabla \cdot \nabla V}$$

$\left\{ \begin{array}{l} V \rightarrow \text{scalar} \\ A \rightarrow \text{vector} \end{array} \right.$

Laplacian of a vector is :

$$\boxed{\nabla^2 A = \nabla(\nabla \cdot \vec{A}) - (\nabla \times \nabla \times \vec{A})}$$

* Laplacian of a scalar in cartesian system

scalar

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

In vector

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

For cylindrical :

only for scalar :

$$\nabla^2 v = \frac{1}{r} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \phi^2} + \frac{\partial^2 v}{\partial z^2}$$

spherical
scalar :

$$\nabla^2 v = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v}{\partial \theta} \right)$$

05.02.19 STOKE'S THEOREM (Imp)

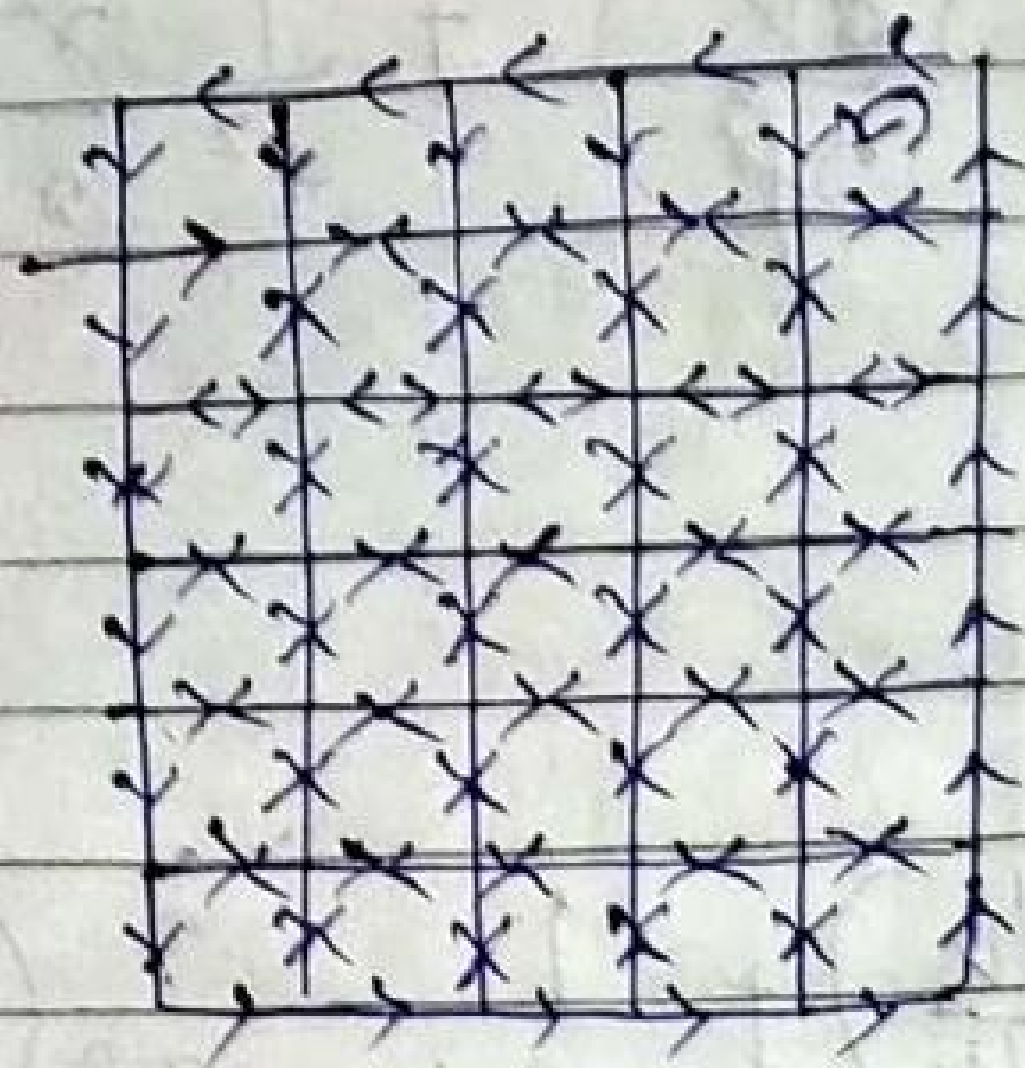
It states that the line integral of any vector field $\vec{A}$ around a closed path 'L' is equal to the integral of curl of $\vec{A}$ over the opened surface enclosed by the closed path 'L'.

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

To prove the Stokes theorem :

$$\int_{ds_1} \vec{A} \cdot d\vec{l}_1 + \int_{ds_2} \vec{A} \cdot d\vec{l}_2 + \dots \int_{ds_n} \vec{A} \cdot d\vec{l}_n = \oint \vec{A} \cdot d\vec{l} \quad \text{--- (1)}$$

Let us consider an open surface S which is divided into small surface $ds_1, ds_2, ds_3, \dots, ds_n$.



- Consider anticlockwise orientation of all the surface.
 - Perform integral of $\vec{A} \cdot d\vec{l}$ vector over the contour of $ds_1, ds_2, ds_3, \dots, ds_n$.
 - When the answers of these integrals are added then they get cancelled throughout except the boundary.
- we know that,

$$\nabla \times \vec{A} = \lim_{ds \rightarrow 0} \frac{\oint \vec{A} \cdot d\vec{l}}{ds}$$

$$\boxed{\oint \vec{A} \cdot d\vec{l} = (\nabla \times \vec{A}) \cdot ds} \quad \text{--- (2)}$$

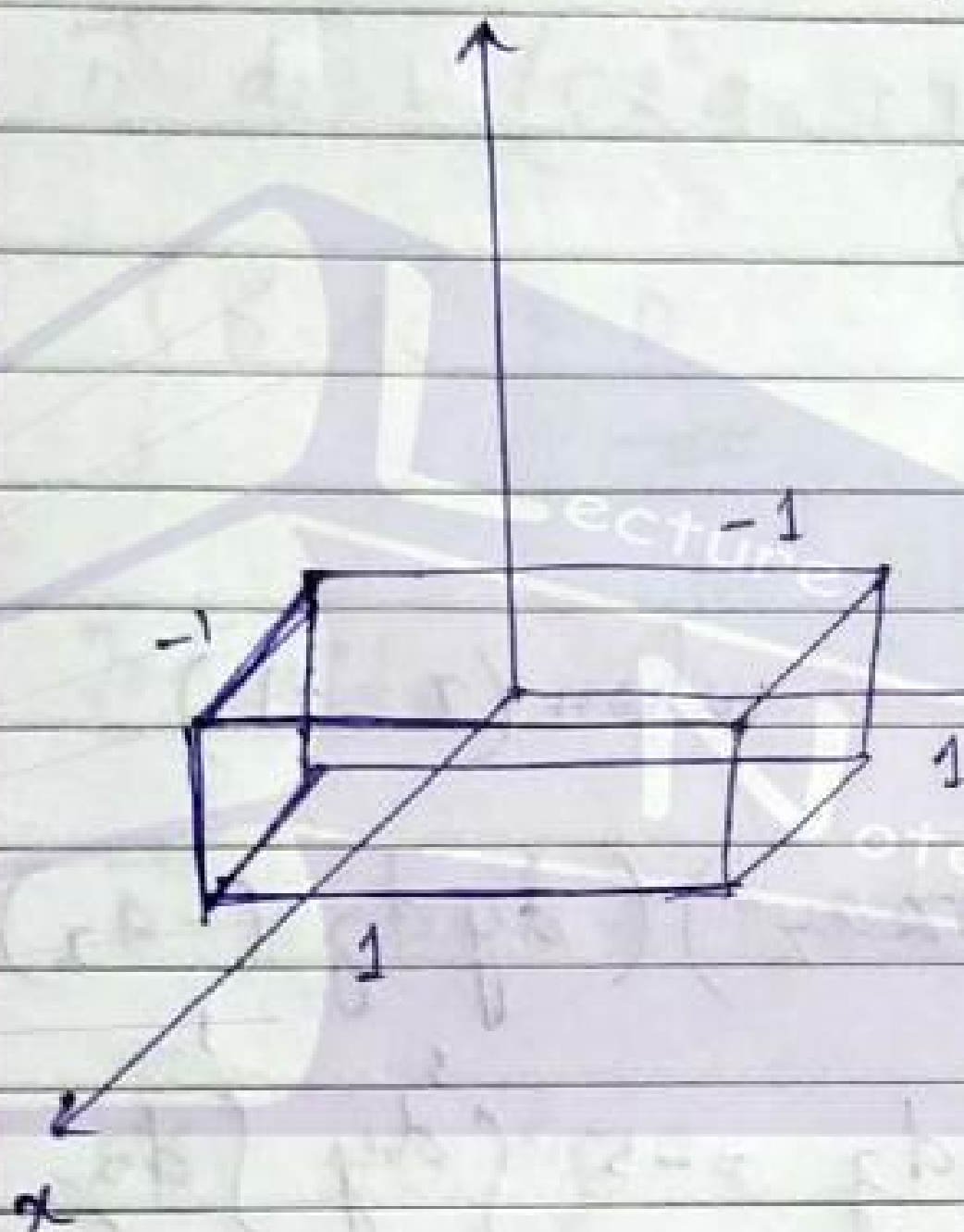
Applying eqⁿ (2) in eqⁿ (1).

eqⁿ (1) will become

$$(\nabla \times \vec{A}) ds_1 + (\nabla \times \vec{A}) ds_2 + \dots + (\nabla \times \vec{A}) ds_n = \oint \vec{A} \cdot d\vec{l}$$

$$\boxed{\int_S (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_L \vec{A} \cdot d\vec{l}} \quad \text{proved}$$

Q ① Given, $\vec{D} = 5x^2 \hat{a}_x + 10z \hat{a}_x \text{ C/m}^2$. Find the net outward flux across the surface of a cube of 2m side, on an edge centre at the origin and the edges of the cube parallel to the axis. Verify using divergence theorem.



$$\Psi = \int_S \vec{D} \cdot d\vec{s}$$

$$\Psi = \int_{\text{front}} \vec{D} \cdot d\vec{s} + \int_{\text{back}} \vec{D} \cdot d\vec{s} + \int_{\text{top}} \vec{D} \cdot d\vec{s} +$$

$$\int_{\text{bottom}} \vec{D} \cdot d\vec{s} + \int_{\text{right}} \vec{D} \cdot d\vec{s} + \int_{\text{left}} \vec{D} \cdot d\vec{s}$$

Front side ($x=1$)

$$d\vec{s} = dy dz \hat{a}_x \quad \vec{D} = 5 \hat{a}_x + 10z \hat{a}_x$$

$$\vec{D} \cdot d\vec{s} = (5a_x + 10za_z) \cdot (dydz \cdot a_x)$$

$$\int_S \vec{D} \cdot d\vec{s} = \int_S 5 dydz = 5 \int_{-1}^1 dy \int_{-1}^1 dz$$

$$= 5 [y]_{-1}^1 [z]_{-1}^1$$

$$= 5(-2)(-2)$$

$$= \underline{\underline{20C}}$$

Back side ($x = -1$)

$$d\vec{s} = dydz(-a_x)$$

$$\vec{D} = 5a_x + 10za_z \quad \text{when } (x = -1)$$

$$\vec{D} \cdot d\vec{s} = (5a_x + 10za_z) \cdot (dydz(-a_x))$$

$$\int_S \vec{D} \cdot d\vec{s} = -\int_S 5 dydz = -5 \int_{-1}^1 dy \int_{-1}^1 dz$$

$$= -5(-2)(-2)$$

$$= \underline{\underline{-20C}}$$

Right side ($y = 1$)

$$\vec{D} = 5x^2a_x + 10za_z$$

$$d\vec{s} = dx dz (a_y)$$

$$\vec{D} \cdot d\vec{s} = (5x^2a_x + 10za_z) \cdot (dx dz a_y)$$

$$\int \vec{D} \cdot d\vec{s} = 0$$

Left side ($y = -1$)

$$\vec{D} = 5x^2 a_x + 10z a_z$$

$$d\vec{s} = dx dz (-a_y)$$

$$\vec{D} \cdot d\vec{s} = (5x^2 a_x + 10z a_z) (dx dz (-a_y))$$

$$\int \vec{D} \cdot d\vec{s} = 0$$

Top side ($z = 1$)

$$\vec{D} = 5x^2 a_x + 10a_z$$

$$d\vec{s} = dx dy a_z$$

$$\vec{D} \cdot d\vec{s} = (5x^2 a_x + 10a_z) (dx dy a_z)$$

$$\vec{D} \cdot d\vec{s} = 0 + \int 10 dx dy$$

$$= 10 \int_{-1}^1 dx \int_{-1}^1 dy = 10(-2)(-2) = \underline{\underline{40C}}$$

Bottom side ($z = -1$)

$$\vec{D} = 5x^2 a_x - 10a_z$$

$$d\vec{s} = dx dy (-a_z)$$

$$\vec{D} \cdot d\vec{s} = (5x^2 - 10xz) (dx dy (-dz))$$

$$\int \vec{D} \cdot d\vec{s} = \int_{-1}^1 \int_{-1}^1 10 dx dy = 10 \int_{-1}^1 dx \int_{-1}^1 dy$$

$$= 10 (-2) (-2)$$

$$= 40C$$

LectureNotes.in

$$\psi = 20C + 20C + 0 + 0 + 40C + 40C$$

$$\boxed{\psi = 80C}$$

06.02.19

According to divergence theorem

$$\oint \vec{D} \cdot d\vec{c} = \int_V (\nabla \cdot \vec{D}) \cdot dV$$

E.H.S

$$dV = dx dy dz$$

$$(\nabla \cdot \vec{D}) = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial}{\partial x} (5x^2) + \frac{\partial}{\partial y} (0) + \frac{\partial}{\partial z} (10z)$$

$$= 10x + 10$$

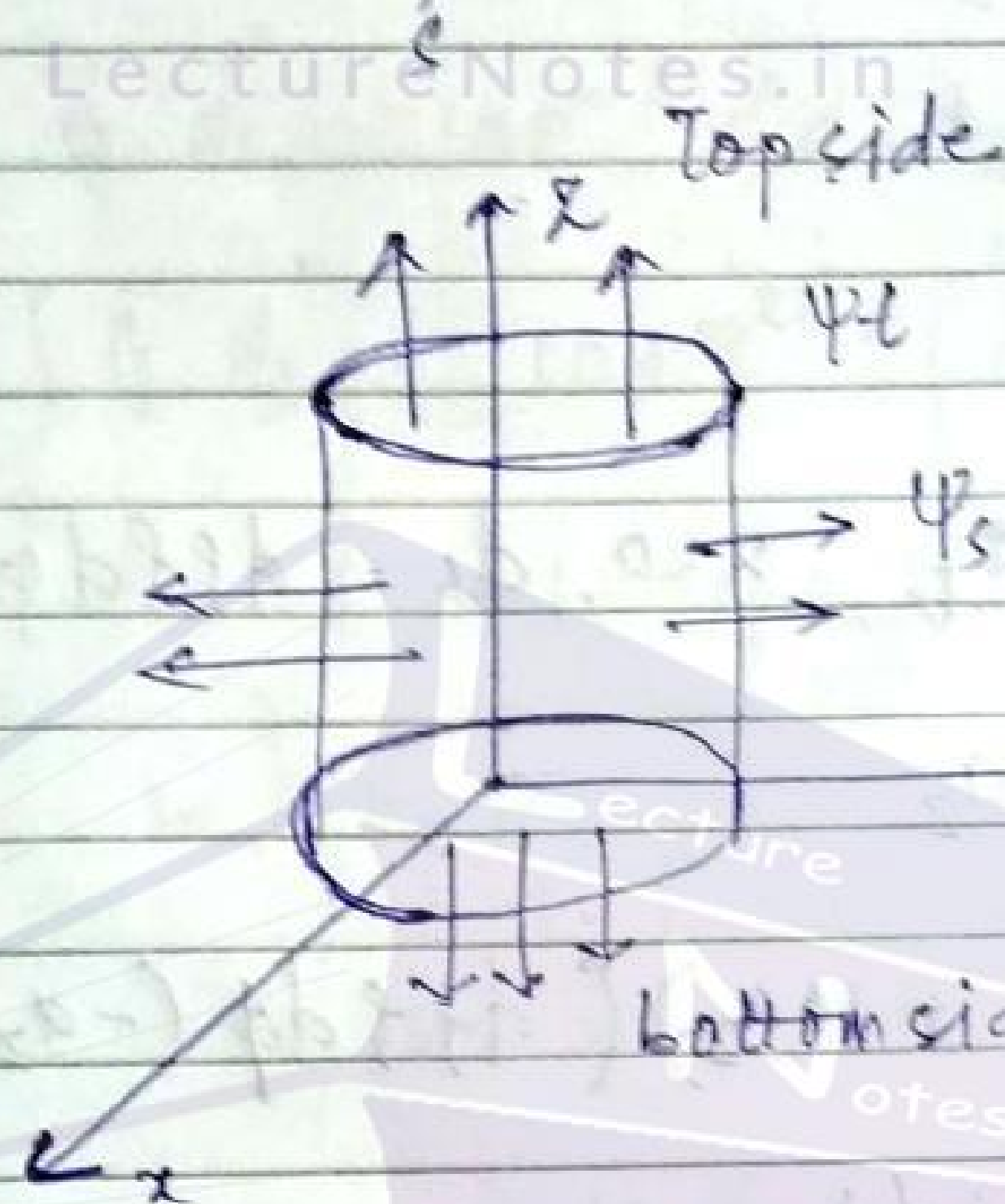
$$\int_V (\nabla \cdot \vec{D}) \cdot dV = \int_V (10x + 10) dx dy dz$$

$$= \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (10x + 10) dx dy dz = 80C$$

given $\vec{G} = 10e^{-z}(\rho \vec{a}_\rho + \vec{a}_z)$.

Determine the flux of $\vec{G}$ out of the entire surface of the cylinder $\rho = 1$, $(0 \leq z \leq 1)$ confirm the result using divergence theorem.

$$\Psi = \int \vec{G} \cdot d\vec{s}$$



Top side

$$\Psi_t, z=1$$

$$d\vec{s} = d\rho \, d\phi \, dz$$

$$\Psi_t = \int \vec{G} \cdot d\vec{s} =$$

$$\vec{G} = 10e^{-z}(\rho \vec{a}_\rho + \vec{a}_z)$$

$$\vec{G} \cdot d\vec{s} = (10e^{-z}(\rho \vec{a}_\rho + \vec{a}_z)) \cdot (d\rho \, d\phi \, dz)$$

$$\vec{G} \cdot d\vec{s} = 10\rho e^{-z} d\rho \, d\phi$$

$$\int_0^1 \vec{G} \cdot d\vec{s} = 10e^{-2} \int_0^1 \rho \, d\rho \int_0^{2\pi} d\phi$$

$$= 10e^{-2} \left[\frac{\rho^2}{2} \right]_0^1 \left[\phi \right]_0^{2\pi}$$

$$= 10e^{-2} \times \frac{1}{2} \times 2\pi$$

$$= \underline{\underline{10\pi e^{-2}}}$$

Bottom side ϕ_b , $z=0$, $d\vec{s} = d\rho \, d\phi (-a_z)$

$$\vec{G} = 10\rho a_\rho + 10a_z$$

$$\vec{G} \cdot d\vec{s} = (10\rho a_\rho + 10a_z) \cdot (d\rho \, d\phi (-a_z))$$

$$\vec{G} \cdot d\vec{s} = -10\rho \, d\rho \, d\phi$$

$$\int_0^1 \vec{G} \cdot d\vec{s} = -10 \int_0^1 \rho \, d\rho \int_0^{2\pi} d\phi$$

$$= -10 \left[\frac{\rho^2}{2} \right]_0^1 \left[\phi \right]_0^{2\pi}$$

$$= -10 \times \frac{1}{2} \times 2\pi$$

$$= \underline{\underline{-10\pi}}$$

Curvature side

$$\psi, f=1, \vec{ds} = f d\phi dz \vec{a}_\phi$$

$$\vec{G} = 10e^{-2z} \vec{a}_\phi + 10e^{-2z} \vec{a}_z$$

$$\vec{G} \cdot \vec{ds} = (10e^{-2z} \vec{a}_\phi + 10e^{-2z} \vec{a}_z) (f d\phi dz \vec{a}_\phi)$$

$$\vec{G} \cdot \vec{ds} = 10e^{-2z} f d\phi dz$$

$$\int \vec{G} \cdot \vec{ds} = 10e^{-2z} f \int_0^{2\pi} d\phi \int_0^1 e^{-2z} dz$$

$$= 10f \left[\frac{e^{-2z}}{-2} \right]_0^1 \left[\phi \right]_0^{2\pi}$$

$$= 10f \left[\frac{e^{-2}}{-2} - \frac{e^0}{-2} \right] 2\pi$$

$$= 10f \frac{e^{-2} - 1}{-2} \times 2\pi$$

$$= 10\pi - 10\pi(e^{-2})$$

$$= \underline{\underline{10\pi(1 - e^{-2})}}$$

$$\psi = \oint \vec{G} \cdot \vec{ds}$$

$$= 10\pi e^{-2} - 10\pi + 10\pi - 10\pi e^{-2} = 0$$

divergence theorem.

$$\oint \vec{G} \cdot \vec{ds} = \int_V (\nabla \cdot \vec{G}) dV$$

$$\nabla G = \frac{1}{f} \frac{\partial}{\partial f} (f \phi) + \frac{1}{f} \frac{\partial}{\partial \phi} (G \phi) + \frac{2}{r z} (h z)$$

$$= \frac{1}{f} \frac{\partial}{\partial f} (f 10 e^{-2z}) + \frac{1}{f} \frac{\partial}{\partial \phi} (0) +$$

$$\frac{2}{r z} (10 e^{-2z})$$

$$= \frac{1}{f} 10 e^{-2z} [2f] + 0 + 10 e^{-2z} \cdot \frac{2}{r z} \cdot x - 2$$

$$= 0$$

Q. $w = x^2 y^2 + xyz$. Compute (∇w) & the directional derivative in the direction of $3a_x + 4a_y + 12a_z$ at $(2, -1, 0)$

$$w = x^2 y^2 + xyz$$

$$\nabla w = \frac{\partial w}{\partial x} a_x + \frac{\partial w}{\partial y} a_y + \frac{\partial w}{\partial z} a_z$$

$$= \frac{\partial}{\partial x} [x^2 y^2 + xyz] a_x + \frac{\partial}{\partial y} [x^2 y^2 + xyz] a_y + \frac{\partial}{\partial z} [x^2 y^2 + xyz] a_z$$

$$\nabla w = (2xy^2 + yz) a_x + (2x^2 y + xz) a_y + xy a_z$$

directional derivative of $w = \nabla w \cdot \hat{a}$

$$\boxed{\nabla w = 4a_x - 8a_y - 2a_z} \quad [\text{putting } (2, -1, 0)]$$

$$a = \frac{3a_x + 4a_y + 12a_z}{\sqrt{(3)^2 + (4)^2 + (12)^2}}$$

$$= \frac{3a_x + 4a_y + 12a_z}{\sqrt{169}}$$

LectureNotes.in

$$= \frac{3}{13} a_x + \frac{4}{13} a_y + \frac{12}{13} a_z$$

$$= \nabla w \cdot \hat{a}$$

$$= \left[(2xy^2 + yz)a_x + (2x^2y + xz)a_y + xy a_z \right]$$

$$\left[\frac{3}{13} a_x + \frac{4}{13} a_y + \frac{12}{13} a_z \right]$$

$$= (2xy^2 + yz) \frac{3}{13} + (2x^2y + xz) \frac{4}{13} + \frac{12}{13} xy$$

$$= \frac{4 \times 3}{13} - \frac{8 \times 4}{13} - \frac{2 \times 12}{13}$$

$$= \frac{12}{13} - \frac{32}{13} - \frac{24}{13}$$

$$= \frac{12 - 32 - 24}{13}$$

$$= \frac{12 - 56}{13} = -\frac{44}{13}$$

[Handwritten signature and scribbles]

Q. Prove that divergence of curl of any vector field is zero.

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

L.H.S
 $\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{a}_x \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) - \hat{a}_y \left(\frac{\partial}{\partial x} A_z - \frac{\partial}{\partial z} A_x \right) + \hat{a}_z \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right)$$

$$\nabla \cdot (\nabla \times \vec{A}) = \frac{\partial}{\partial x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) - \frac{\partial}{\partial y} \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_x}{\partial y \partial z}$$

$$= 0 \quad \text{R.H.S proved}$$

$$\text{L.H.S} = \text{R.H.S}$$

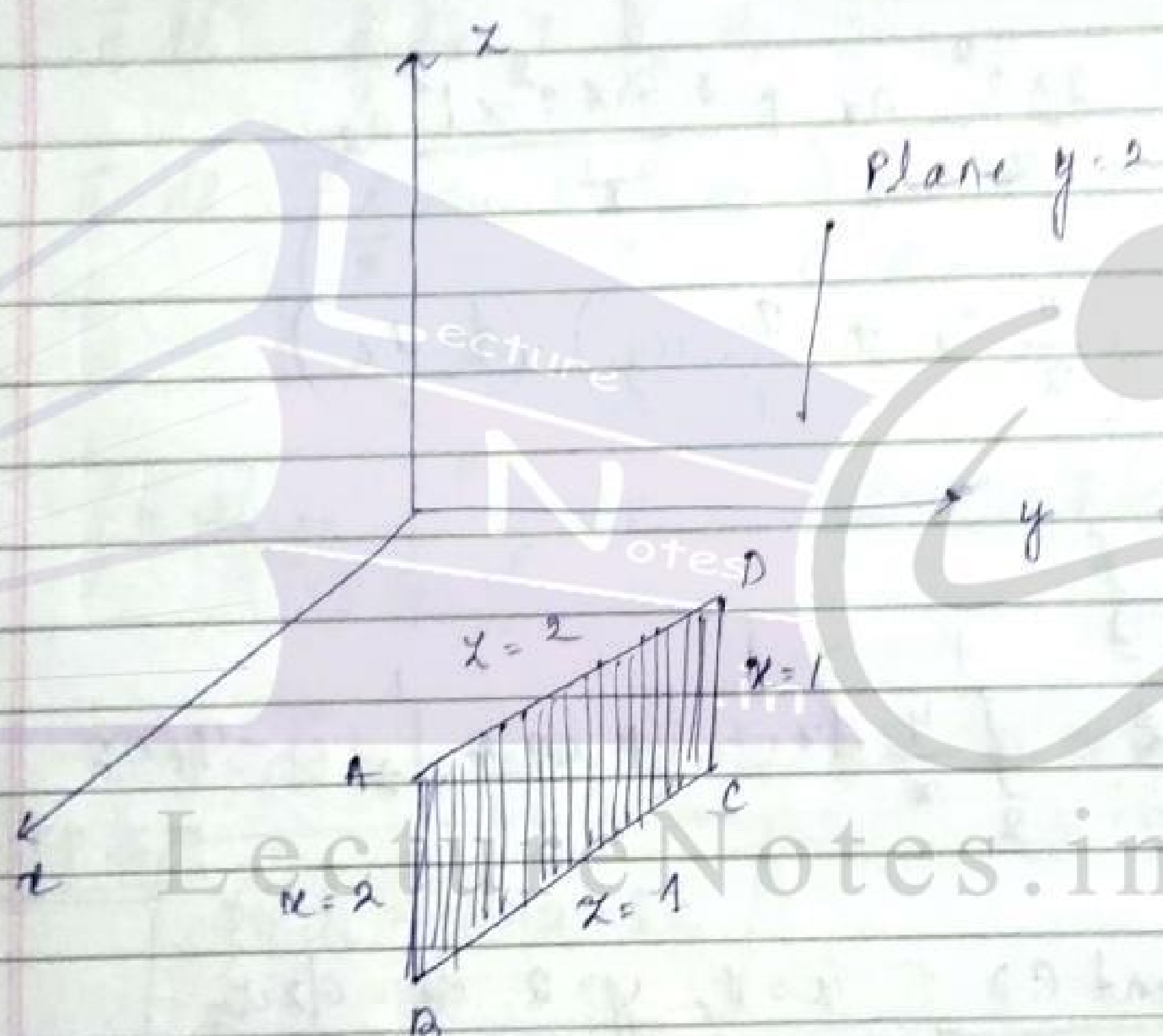
08.02.19

Q Evaluate both sides of Stokes theorem of
 $\vec{H} = y^2 z \vec{a}_x + 0.5 y^2 z^2 \vec{a}_x = 1/m$

crossing the square surface in the plane
 $y=2$ bounded by $x=z=1$ & $x=z=2$

Ans According to Stokes theorem

$$\oint_L \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{s}$$



L.H.S

$$\oint_L \vec{H} \cdot d\vec{l} = \int_A^B + \int_B^C + \int_C^D + \int_D^A$$

segment AB. $y=2, x=2$

$$d\vec{l} = dz \vec{a}_z$$

$$\vec{H} = \cancel{2^2} z \vec{a}_x + \frac{0.5 \times \cancel{2^2} \times z^2}{\cancel{2^2}} \vec{a}_x$$

$$\vec{H} = 2x a_x + \frac{z^2}{2} a_z$$

$$\vec{H} \cdot d\vec{l} = \left(2x a_x + \frac{z^2}{2} a_z \right) dz a_z$$

$$\vec{H} \cdot d\vec{l} = \frac{z^2}{2} dz$$

$$\int_1^3 \vec{H} \cdot d\vec{l} = \int_1^3 \frac{z^2}{2} dz = \left[\frac{z^3}{6} \right]_1^3 = -1.1667$$

Segment BC $z=1, y=2, d\vec{l} = dx a_x$

$$\vec{H} = \frac{1 \times 2^2}{x} a_x + \frac{0.5 \times 2^2 \times 1^2}{x^2} a_z$$

$$\vec{H} = \left(\frac{4}{x} a_x + \frac{2}{x^2} a_z \right) \cdot (dx a_x)$$

$$\int_2^1 \vec{H} \cdot d\vec{l} = \int_2^1 \frac{4}{x} dx = \left[4 \ln x \right]_2^1 = -4 \ln 2$$

$$= -2.77$$

Segment CD $x=1, y=2, d\vec{l} = dz a_z$

$$\vec{H} = \frac{2^2 \times z}{1} a_x + \frac{0.5 \times 2^2 \times z^2}{1^2} a_z$$

$$= 4z a_x + 2z^2 a_z$$

$$\vec{H} \cdot d\vec{l} = 2z^2 dz \quad \left[\frac{2z^3}{3} \right]_1^3 = 4.667$$

$$\int_C^D \vec{H} \cdot d\vec{l} = \int_1^3 2z^2 dz = 4.667$$

segment DA.

$$z = 2, y = 2 \quad dl = dx a_x$$

$$\vec{H} = \frac{2^2 \times 2}{x} a_x + \frac{0.5 (2)^2 (2)^2}{x^2} a_x$$

$$\vec{H} = \frac{8}{x} a_x + \frac{0.5 \times 16}{x^2} a_x$$

$$\vec{H} = \frac{8}{x} a_x + \frac{8}{x^2} a_x$$

$$\vec{H} \cdot d\vec{l} = \frac{8}{x} a_x + \frac{8}{x^2} a_x$$

$$\vec{H} \cdot d\vec{l} = \left(\frac{8}{x} a_x + \frac{8}{x^2} a_x \right) (dx a_x)$$
$$= \frac{8}{x} dx$$

$$\int \vec{H} \cdot d\vec{l} = \int_1^2 \frac{8}{x} dx = 8 \ln 2 - 8 \ln 1$$
$$= 5.54$$

$$\oint \vec{H} \cdot d\vec{s} = -1.1667 - 2.77 + 4.667 + 5.54$$

$$\oint \vec{H} \cdot d\vec{s} = 6.275$$

R.H.S

$$\int_S (\nabla \times \vec{H}) \cdot d\vec{s}$$

$$d\vec{s} = dx dz a_y$$

$$\nabla \times \vec{H} = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{y^2 z}{x} & 0 & \frac{0.5 y^2 z^2}{x^2} \end{vmatrix}$$

$$\nabla \times \vec{H} = a_y \left[\frac{\partial}{\partial y} \left[\frac{0.5 y^2 z^2}{x^2} \right] - 0 \right] -$$

$$a_y \left[\frac{\partial}{\partial x} \left(\frac{0.5 y^2 z^2}{x^2} \right) - \frac{\partial}{\partial z} \left(\frac{y^2 z}{x} \right) \right]$$

$$+ a_x \left[\frac{\partial}{\partial y} \left(\frac{y^2 z}{x} \right) - 0 \right]$$

$$\nabla \times \vec{H} = \frac{0.5 z^2}{x^2} 2y a_x - \left(0.5 y^2 z^2 \left(\frac{-1}{x^3} \right) - \frac{y^2}{x} \right) a_y$$

$$- \frac{2yz}{x} a_z$$

$$\vec{\nabla} \times \vec{H} = \frac{2yz}{x^2} a_x + \left[\frac{y^2 z^2}{x^3} + \frac{y^2}{x} \right] a_y - \frac{2yz}{x} a_z$$

$$(\vec{\nabla} \times \vec{H}) \cdot d\vec{c} = \left(\frac{y^2 z^2}{x^3} + \frac{y^2}{x} \right) dx dz$$

$$\int_C (\vec{\nabla} \times \vec{H}) \cdot d\vec{c} = \int_{x=1}^2 \int_{z=1}^2 \left(\frac{y^2 z^2}{x^3} + \frac{y^2}{x} \right) dx dz$$

$$\int_1^2 \int_1^2 \left(\frac{y^2 z^2}{x^3} + \frac{y^2}{x} \right) dz dx$$

$$= y^2 \int_1^2 \left\{ \int_1^2 \left(\frac{z^2}{x^3} + \frac{1}{x} \right) dz \right\} dx$$

$$= y^2 \int_1^2 \left(\frac{z^3}{3x^3} + \frac{z}{x} \right) \Big|_1^2 dx$$

$$= y^2 \int_1^2 \left(\frac{z^3}{3x^3} + \frac{z}{x} - \frac{1}{3x^3} - \frac{1}{x} \right) dx$$

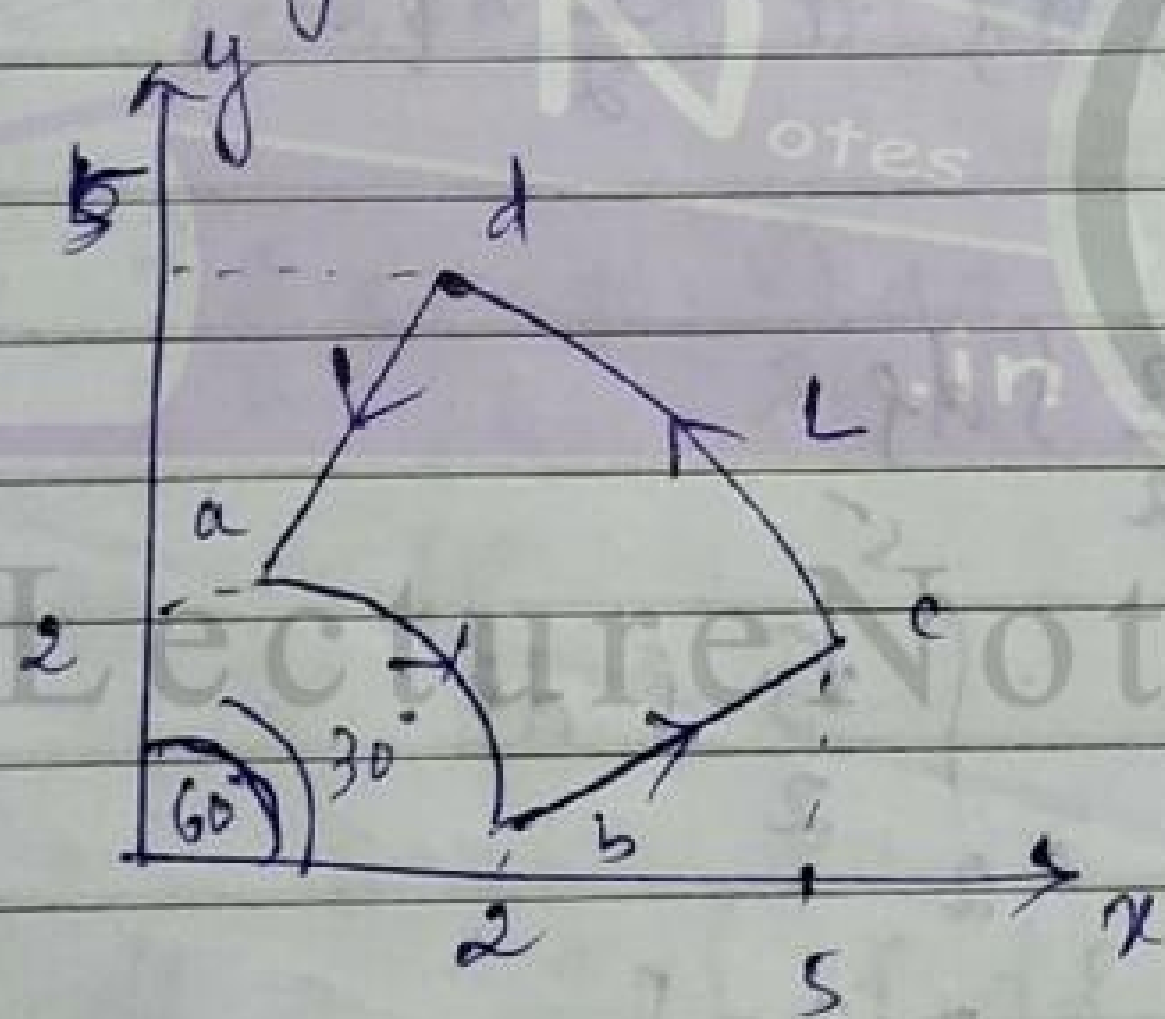
$$= y^2 \int_1^2 \left(\frac{7}{3x^3} + \frac{18}{x} \right) dx$$

$$= y^2 \left[\frac{7}{3} \left(\frac{-2}{x^2} \right) + 18(\ln x) \right]_1^2$$

$$= y^2 \left[\frac{7}{3} \times \left(\frac{-2}{2} \right) + 18(\ln 2) + \frac{7}{3} \left(\frac{2}{1} - 18 \ln 1 \right) \right]$$

$$= \underline{6.2725}$$

12.02.19 If $\vec{A} = f \cos \phi \, a_\phi + \sin \phi \, a_\phi$. Evaluate $\oint \vec{A} \cdot d\vec{l} = ?$ around the path shown L in the figure and confirm the result using Stokes theorem.



given that

$$\vec{A} = f \cos \phi \, a_\phi + \sin \phi \, a_\phi$$

$$\oint_L \vec{A} \cdot d\vec{l} = \oint_{abcd} \vec{A} \cdot d\vec{l} = \left[\int_a^b + \int_b^c + \int_c^d + \int_d^a \right] \vec{A} \cdot d\vec{l}$$

Along ab. $f: 2$, $d\vec{l} = f d\phi a_\phi$

$$\vec{A} = 2 \cos \phi a_f + \sin \phi a_\phi$$

$$\vec{dl} \cdot \vec{A} = f \sin \phi d\phi$$

$$\int_a^b \vec{A} \cdot d\vec{l} = 2 \int_{60^\circ}^{30^\circ} \sin \phi d\phi = -0.828$$

$$\Rightarrow 2(-\cos 30^\circ + \cos 60^\circ) = -0.73$$

Along bc $\phi: 30^\circ$, $d\vec{l} = d f a_f$

$$\vec{A} = f \cos 30^\circ a_f + \sin 30^\circ a_\phi$$

$$\vec{A} = \frac{\sqrt{3}}{2} f a_f + \frac{1}{2} a_\phi$$

$$\vec{A} \cdot d\vec{l} = \frac{\sqrt{3}}{2} f df$$

$$\int_b^c \vec{A} \cdot d\vec{l} = \int_2^5 \frac{\sqrt{3}}{2} f df$$

$$= \frac{\sqrt{3}}{2} \left[\frac{f^2}{2} \right]_2^5 = \frac{\sqrt{3}}{2} \left[\frac{25}{2} - \frac{4}{2} \right]$$

$$= \frac{\sqrt{3}}{2} \times \frac{21}{2} = \frac{21\sqrt{3}}{4} = \underline{9.09}$$

Along cd $f = 5$, $d\vec{l} = f d\phi a_\phi$

$$\vec{A} = 5 \cos \phi a_\phi + \sin \phi a_\phi$$

$$\vec{A} \cdot d\vec{l} = \sin \phi f d\phi$$

$$\int_c^d \vec{A} \cdot d\vec{l} = 5 \int_{30^\circ}^{60^\circ} \sin \phi \cdot d\phi$$

$$= 5 [\cos \phi]_{30^\circ}^{60^\circ}$$

$$= 5 [-\cos 60^\circ - (-\cos 30^\circ)]$$

$$= \underline{\underline{1.83}}$$

Along da ~~$f = 5$~~ $\phi = 60^\circ$, $d\vec{l} = df a_f$

$$\vec{A} = f \cos 60^\circ a_f + \sin 60^\circ a_\phi$$

$$\vec{A} = f \frac{1}{2} a_f + \frac{\sqrt{3}}{2} a_\phi$$

$$\vec{A} \cdot d\vec{l} = \frac{f}{2} df$$

$$\int_a^2 \vec{A} \cdot d\vec{l} = \frac{1}{2} \int_1^2 f \cdot df$$

$$= \frac{1}{2} \left[\frac{f^2}{2} \right]_1^2 = \frac{1}{2} \left[\frac{4}{2} - \frac{1}{2} \right]$$

$$= \frac{1}{2} \times \frac{3}{2} = \frac{3}{4} = \underline{\underline{0.75}}$$

$\oint \vec{A} \cdot d\vec{l} = -0.73 + 9.09 + 1.83 - 0.75$
along da

$$= \underline{\underline{4.94}} \quad (\text{total})$$

Stokes theorem. (acc. to.)

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

$$d\vec{s} = \hat{e}_r dr d\phi + \hat{e}_\phi r dz$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{e}_r & \hat{e}_\phi & \hat{e}_z \\ \frac{\partial}{\partial r} & \frac{1}{r} \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{r} \left[\hat{e}_r \left(\frac{\partial}{\partial \phi} A_z - \frac{\partial}{\partial z} (r A_\phi) \right) - \hat{e}_\phi \left(\frac{\partial}{\partial r} A_z + \frac{\partial}{\partial z} A_r \right) + \hat{e}_z \left(\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial}{\partial \phi} A_r \right) \right]$$

$$= \frac{1}{r} \begin{vmatrix} \hat{e}_r & \hat{e}_\phi & \hat{e}_z \\ \frac{\partial}{\partial r} & \frac{1}{r} \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ r \cos \phi & r \sin \phi & 0 \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{r} \left[\hat{e}_r \left(\frac{\partial}{\partial \phi} (0) - \frac{\partial}{\partial z} (r \sin \phi) \right) - \hat{e}_\phi \left(\frac{\partial}{\partial r} (0) + \frac{\partial}{\partial z} (r \cos \phi) \right) + \hat{e}_z \left(\frac{\partial}{\partial r} (r \sin \phi) - \frac{\partial}{\partial \phi} (r \cos \phi) \right) \right]$$

$$= \frac{1}{r} \left[0 - 0 + \hat{e}_z (r \sin \phi + r \cos \phi) \right]$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{f} \left[a_f (0-0) - f a_\phi (0-0) + a_z (\sin \phi + f \sin \phi) \right]$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{f} a_z (\sin \phi + f \sin \phi)$$

$$\vec{\nabla} \times \vec{A} = \frac{\sin \phi}{f} [1 + f] a_z$$

$$\text{R.H.S} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$$

$$= \int_S \frac{\sin \phi}{f} (1+f) \cdot df \, f \, d\phi$$

$$= \int_{30^\circ}^{60^\circ} \sin \phi \, d\phi \cdot \int_2^5 (1+f) \cdot df$$

$$= (-\cos 60^\circ + \cos 30^\circ) \left[f + \frac{f^2}{2} \right]_2^5$$

$$= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \left\{ \left(5 + \frac{25}{2} \right) - \left(2 + \frac{4}{2} \right) \right\}$$

$$= \frac{0.732 \times 17.5}{2}$$

$$\underline{\underline{6.44}}$$

Q. A circular loop lies in the $z=0$ plane has a radius of 0.1 m . Determine the flux if $\vec{B} = 0.2 \sin 10^3 t \hat{a}_z$

$$r = 0.1\text{ m}$$

$$\phi = \int \vec{B} \cdot d\vec{s} \quad B = 0.2 \sin 10^3 t \hat{a}_z$$

$$\phi = \int_S (0.2 \sin 10^3 t \hat{a}_z) \cdot (\vec{ds} \hat{a}_z)$$

$$= 0.2 \sin 10^3 t \int ds$$

$$= 0.2 \sin 10^3 t [\pi \times (0.1)^2]$$

$$= 0.2 \times 0.01 \times \pi \sin 10^3 t$$

$$= 0.006 \sin 10^3 t$$

$$= 6.28 \sin 10^3 t \text{ mwb.}$$

Q. Determine the nature of the following field. $F = \frac{150}{r^2} \hat{a}_r + 10 \hat{a}_\phi$

Case ① $\vec{\nabla} \cdot \vec{F} = 0$, solenoid.

$\vec{\nabla} \cdot \vec{F} \neq 0$, non-solenoid.

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r} \frac{\partial}{\partial r} (r F_r) + \frac{1}{r} \frac{\partial}{\partial \phi} (r F_\phi) + \frac{\partial}{\partial z} (F_z)$$

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{150}{r^2} \right) + \frac{1}{r} \frac{\partial}{\partial \phi} (10) + 0$$

$$\vec{\nabla} \cdot \vec{F} = \frac{150}{r} \left(-\frac{1}{r^2} \right) + \frac{1}{r} (0)$$

$$\boxed{\vec{\nabla} \cdot \vec{F} = -\frac{150}{r^3}}$$

$\vec{\nabla} \cdot \vec{F} \neq 0$ non-solenoid

case ② . $\vec{\nabla} \times \vec{F} = 0$, irrotational
 $\vec{\nabla} \times \vec{F} \neq 0$, rotational.

$$\vec{\nabla} \times \vec{F} = \frac{1}{f} \begin{vmatrix} a_f & f a_\phi & a_z \\ \frac{\partial}{\partial f} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_f & f F_\phi & F_z \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \frac{1}{f} \begin{vmatrix} a_f & f a_\phi & a_z \\ \frac{\partial}{\partial f} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{150}{f^2} & 10 \cdot f & 0 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \frac{1}{f} \left[a_f (0 - 0) - f a_\phi (0 - 0) + a_z (10 - 0) \right]$$

$$\boxed{\vec{\nabla} \times \vec{F} = \frac{10 a_z}{f}} \neq 0$$

$\therefore \vec{\nabla} \times \vec{F} \neq 0$ (nonzero)
 rotational.

13.02.19

ELECTROSTATIC FIELD

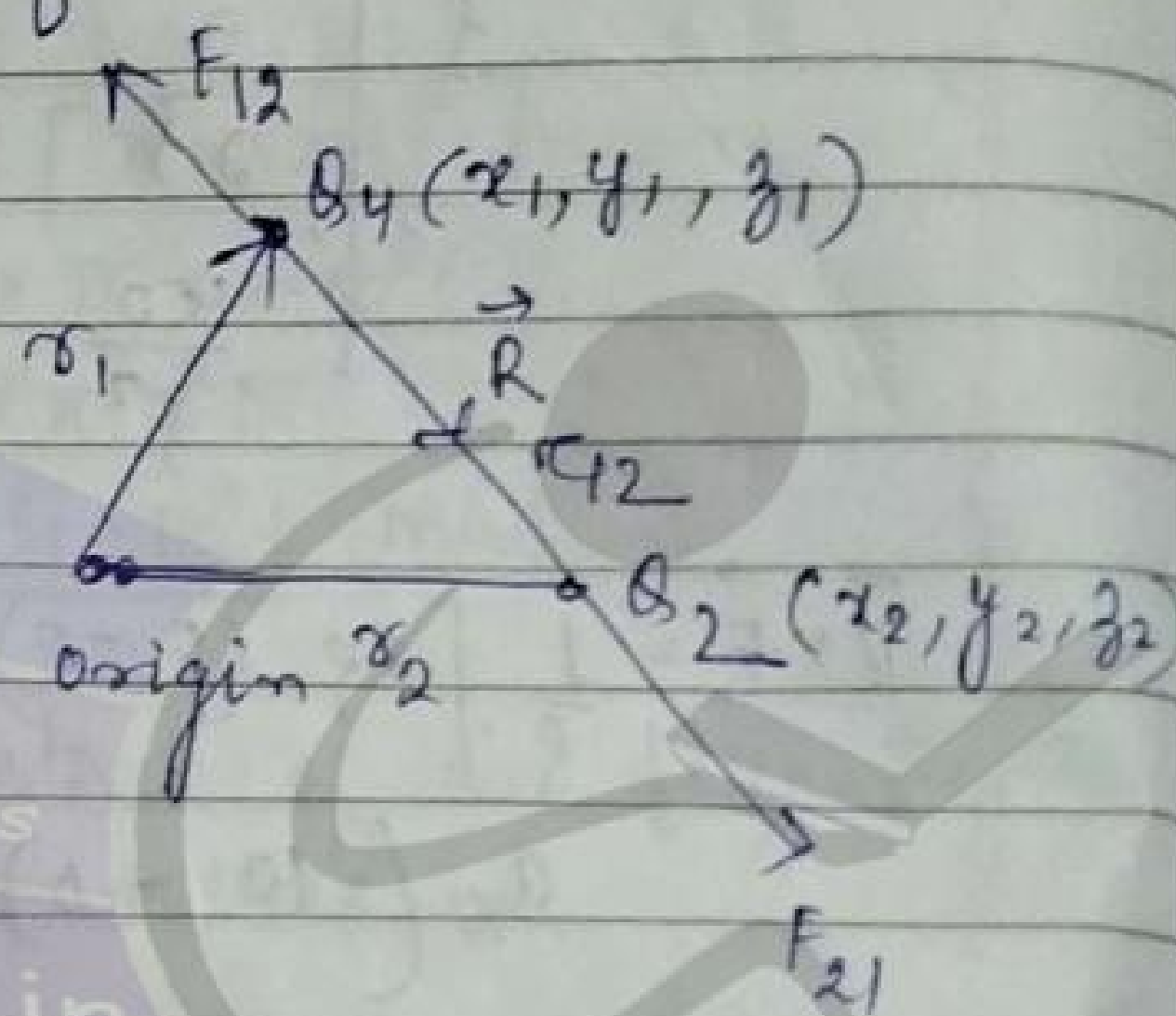
State Coulomb's law.

- Coulomb's law states that the force between 2 point charges Q_1 & Q_2 is
 - line joining between them
 - directly proportional to the product of Q_1 & Q_2 and inversely proportional to the square of the distance between them.

$$F \propto Q_1 Q_2$$

$$\propto \frac{1}{r^2}$$

$$|F| = \frac{k Q_1 Q_2}{r^2} \hat{r}$$



$$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_{12} = -\vec{r}_{21}$$

$$\vec{r}_{12} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

$$\vec{F}_{21} = \frac{k Q_1 Q_2}{|\vec{r}_{12}|^2} \hat{a}_{r12}$$

$$= \frac{k Q_1 Q_2}{|\vec{r}_{12}|^2} \times \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

$$\vec{F}_{21} = \frac{k Q_1 Q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3}$$

$$= \frac{k Q_1 Q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3}$$

$$\boxed{F_{21} = -F_{12}}$$

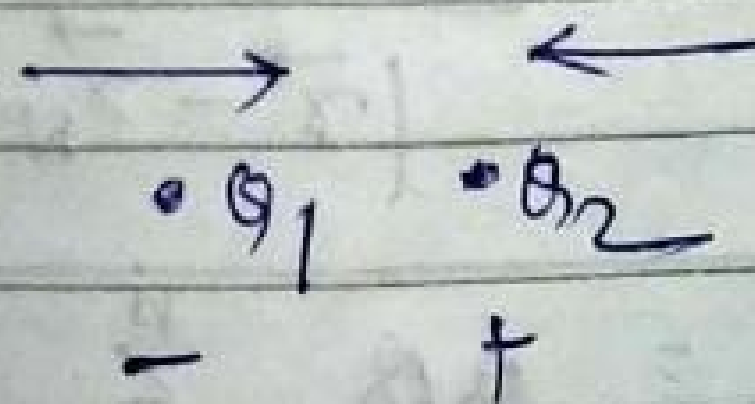
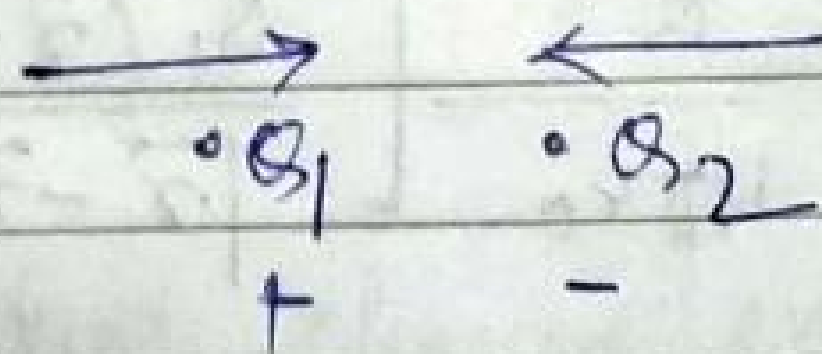
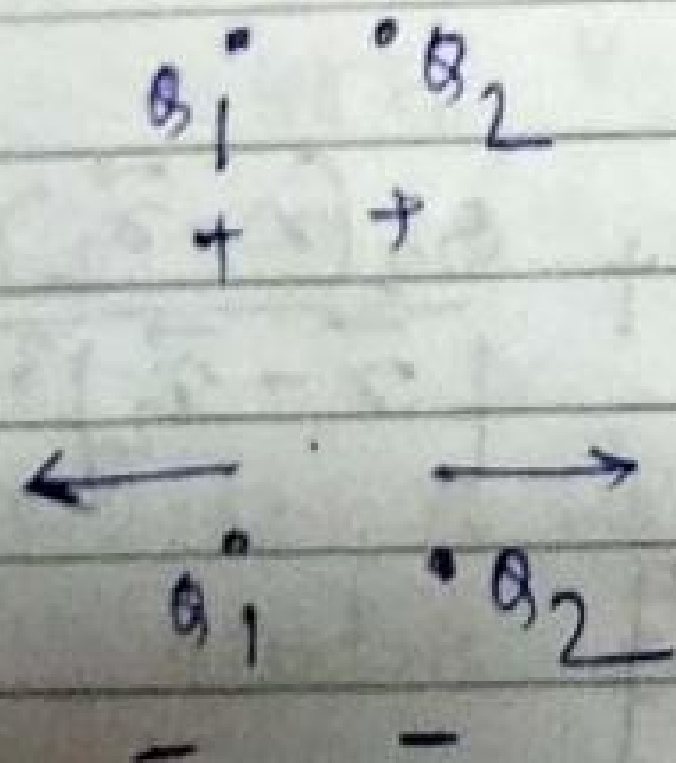
$$K = \frac{1}{4\pi\epsilon}$$

Lecture Notes.in

for free space $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N/m}^2$

$$K = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9$$

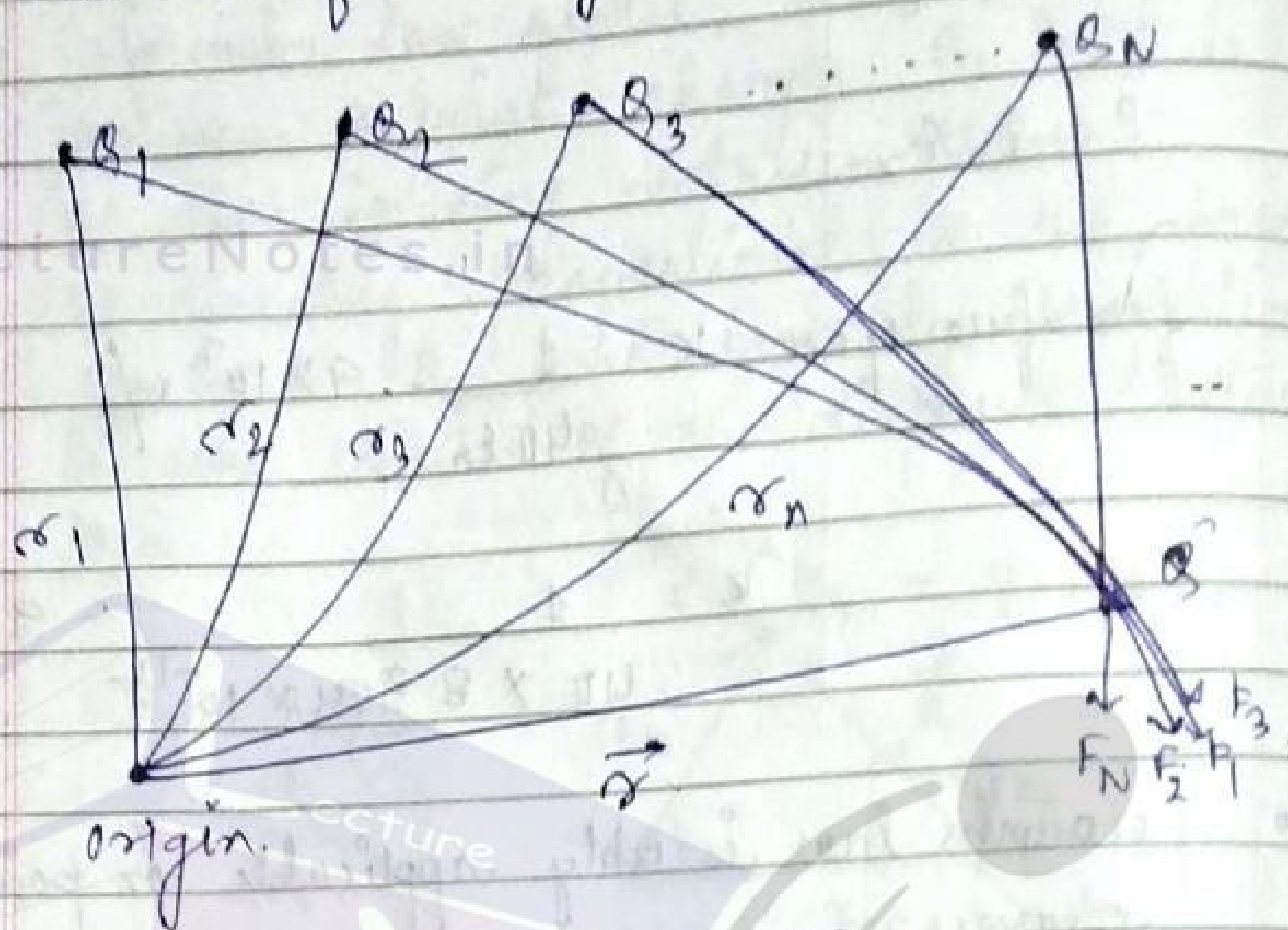
- Coulombs law is only applicable on point-charges.
- A point charge is a charge located on a body whose dimensions are much smaller than other relevant dimensions.
- If Q_1 & Q_2 are like charges which provide force of repulsion and unlike charges provides force of attraction.



Principle of Superposition

- Q_1 and Q_2 should be static charges.

Let in a given space there are n no. of charges



$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n$$

$$\vec{F} = \left[\frac{Q_1 Q_2 (\vec{r} - \vec{r}_1)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_1|^2} + \frac{Q_2 Q_3 (\vec{r} - \vec{r}_2)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_2|^2} + \dots + \frac{Q_N Q_n (\vec{r} - \vec{r}_n)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_n|^2} \right] \hat{r}$$

$$\vec{F} = \frac{Q}{4\pi\epsilon_0} \left[\frac{Q_1 (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} + \frac{Q_2 (\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3} + \dots + \frac{Q_N (\vec{r} - \vec{r}_N)}{|\vec{r} - \vec{r}_N|^3} \right]$$

$$\boxed{\vec{F} = \frac{Q}{4\pi\epsilon_0} \sum_{k=1}^n \frac{Q_k (\vec{r} - \vec{r}_k)}{|\vec{r} - \vec{r}_k|^3}}$$

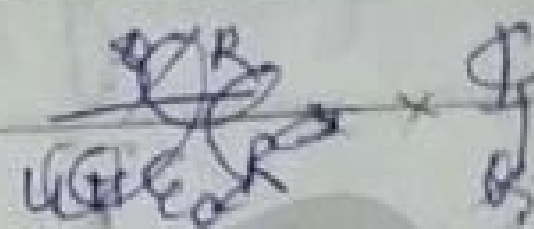
Principle of superposition

It states that if there are n charges Q_1 to Q_n located with position vectors r_1 upto r_n , the resultant force act on a charge Q located at point r is the sum of forces exerted on Q by each individual charges.

Electric field Intensity / Electric field strength

It is the electric force on per unit charge

$$\vec{E} = \frac{\vec{F}}{Q} \text{ N/C}$$



$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_R$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 (R^3)} \vec{R}$$

on applying principle of superposition on the electric field intensity

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n Q_k \frac{(\vec{r} - \vec{r}_k)}{|\vec{r} - \vec{r}_k|^3}$$

where Q is the test charge on which electric $\vec{E}$ is to be calculated.

- ① Calculate electric field $\vec{E}$ at a point $N(3, -4, 2)$ in the free space caused by
- (a) $Q_1 = 2 \text{ nC}$ at $P_1(0, 0, 0)$
 - (b) $Q_2 = 8 \text{ nC}$ at $P_2(-1, 2, 3)$
 - (c) due to both Q_1 & Q_2

Ans (a) $\vec{E} = \frac{Q}{4\pi\epsilon_0 |\vec{R}|^3} \vec{R}$

$\vec{R}$ is the distance between the test charge & ~~charge~~ the point charge.

$$\vec{R} = (3-0)a_x + (-4-0)a_y + (2-0)a_z$$

$$\boxed{\vec{R} = 3a_x - 4a_y + 2a_z} \quad \text{due to } Q_1.$$

$$|\vec{R}| = \sqrt{3^2 + 4^2 + 2^2}$$

$$= \sqrt{9 + 16 + 4} = 5.4$$

$$\vec{E} = \frac{2 \times 10^{-6} \times 9 \times 10^9 \times (3a_x - 4a_y + 2a_z)}{(5.4)^3}$$

$$\Rightarrow \vec{E} = 114.31 (3a_x - 4a_y + 2a_z)$$

$$\Rightarrow \vec{E} = 342.9a_x - 457.2a_y + 228.6a_z$$

$$\Rightarrow \boxed{\vec{E}_1 = 345a_x - 460a_y + 230a_z \text{ V/m}}$$

$$(b) \vec{E} = (3+1)a_x + (-4-2)a_y + (2-3)a_z$$

$$\boxed{\vec{E} = 4a_x - 6a_y - a_z}$$

$$|\vec{E}| = \sqrt{(4)^2 + (-6)^2 + (-1)^2}$$

$$\sqrt{16 + 36 + 1} = 7.28$$

$$\vec{E} = \frac{3 \times 10^{-6} \times 9 \times 10^9 \times (4a_x - 6a_y - a_z)}{(7.28)^3}$$

$$\vec{E} = 69.97 (4a_x - 6a_y - a_z)$$

$$\vec{E} = 279.6a_x - 419.4a_y - 69.9a_z$$

$$\boxed{\vec{E}_2 = 280a_x - 420a_y - 70a_z}$$

(c) due to both Q_1 & Q_2

$$\vec{E}_1 + \vec{E}_2 = (345 + 280)a_x - (460 + 420)a_y + (230 - 70)a_z$$

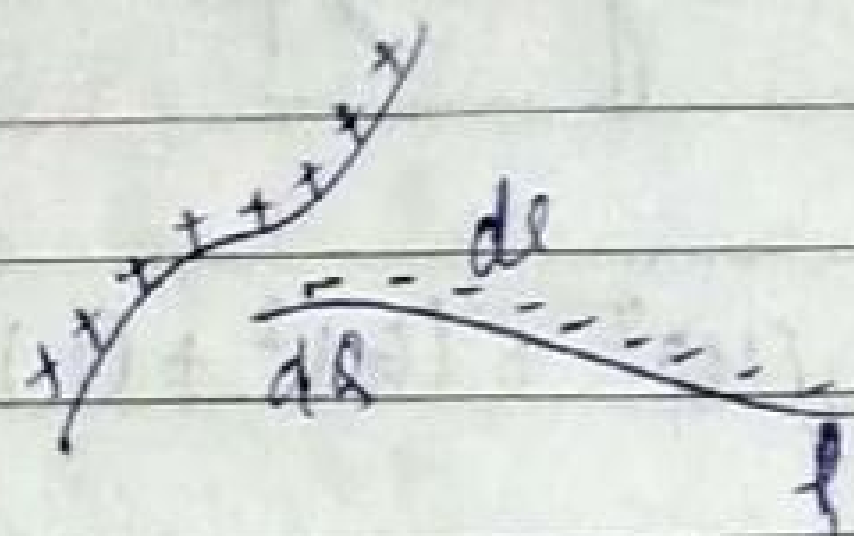
$$\boxed{\vec{E}_{\text{total}} = 625a_x - 880a_y + 160a_z}$$

4.02.19.

Continuous charge distribution

1. > Point charge

2. > Line charge



λ_L = line charge density

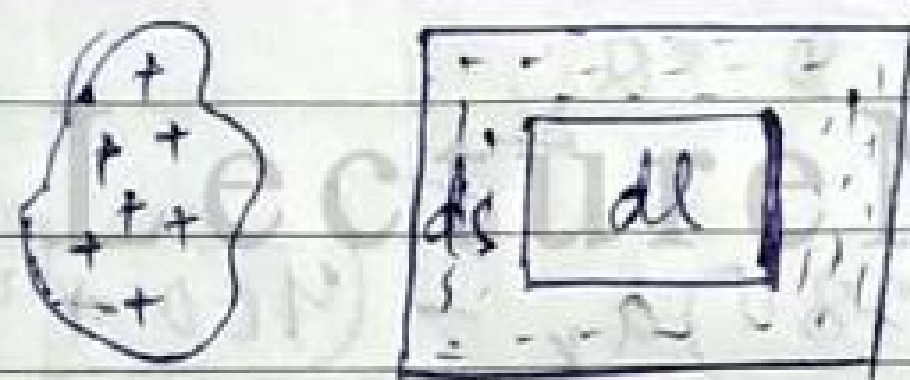
$$\lambda_L = \frac{\text{Total charge}}{\text{Unit length}} \quad \frac{C}{m}$$

$$\lambda_L = \frac{dq}{dl}$$

$$dq = \lambda_L \cdot dl$$

$$Q = \int \lambda_L \cdot dl$$

3. > Surface charge



λ_s = surface charge density

$$\lambda_s = \frac{\text{Total charge}}{\text{Unit surface Area}} \quad \frac{C}{m^2}$$

$$\lambda_s = \frac{dq}{ds}$$

$$dq = \lambda_s \cdot ds$$

$$Q = \int \lambda_s \cdot ds$$

4) Volume charge

ρ_v = volume charge density

$$\rho_v = \frac{\text{Total charge}}{\text{Unit volume}} \quad \frac{C}{m^3}$$

$$\rho_v = \frac{dq}{dv}$$

$$* \quad Q = \int_V \rho_v \cdot dv$$

- Electric field intensity at any point on the axis of an uniformly charged ring:

Consider a circular loop of wire of radius 'a', negligible thickness, centre held per to the plane of paper.

let 'Q' is the total charge uniformly distributed over the circumference.

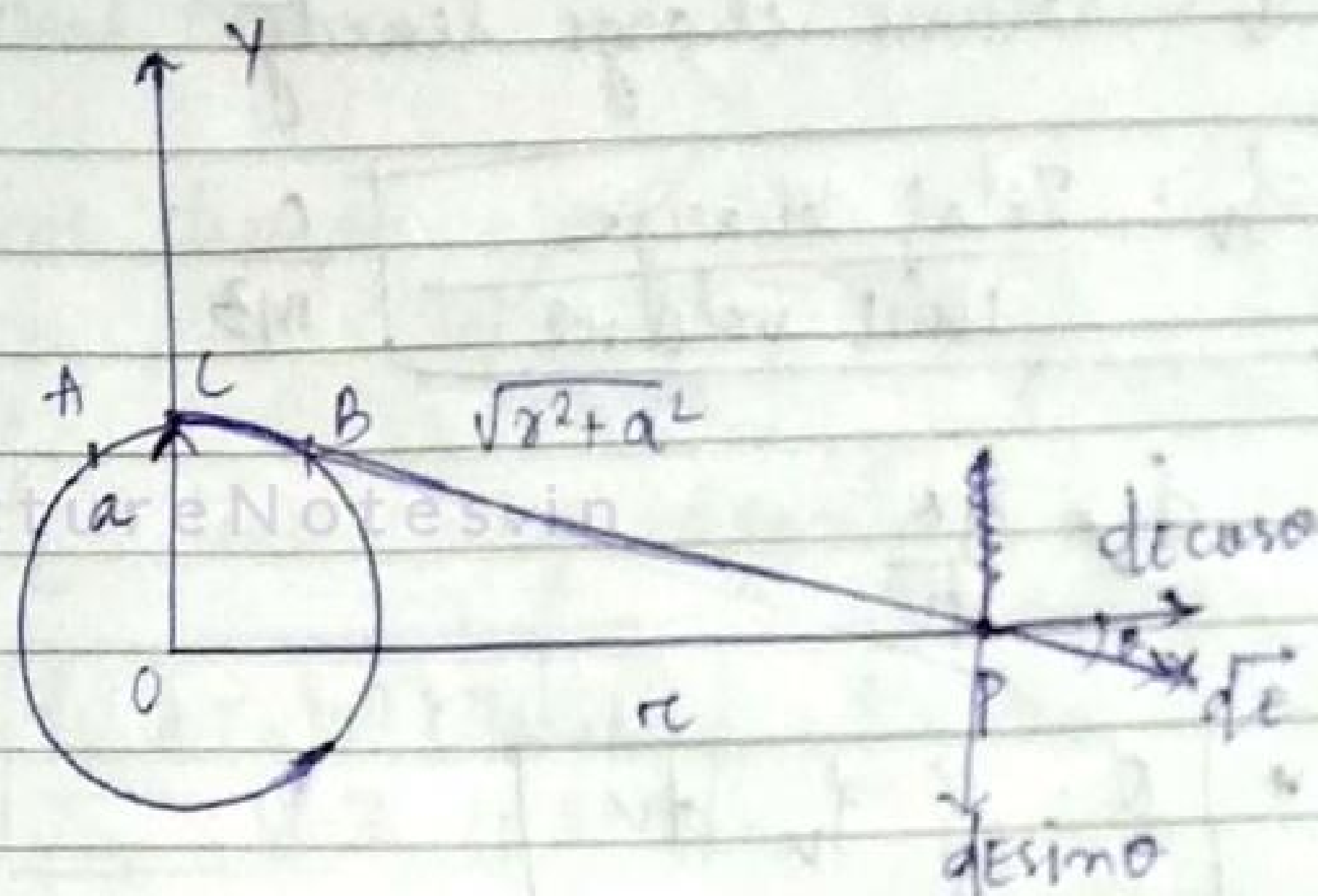
'P' is the point at which ($\vec{E}$) is need to be calculated

The distance between OP is x

$$\boxed{OP = x}$$

let ρ_L is the ^{line} charge density

$$\lambda = \frac{q}{2\pi a}$$



Consider a differential length AB ,
having center at C .

let dl is the differential length and
 dq is the charge distributed over
 dl .

$$\left[\begin{array}{l} \text{Total charge} = q \\ \text{Unit length} = \frac{q}{2\pi a} \\ \text{dl length} = \frac{q}{2\pi a} \times dl \end{array} \right]$$

let the $d\vec{E} = \vec{E}$ due to dq

$$d\vec{E} = \sum dE \sin \theta + \sum dE \cos \theta$$

$$\boxed{\sum d\vec{E} = \sum dE \cos \theta}$$

$d\vec{E}$ resolves into 2 components

$$d\vec{E} \sin\theta, d\vec{E} \cos\theta$$

For a pair of diametrically opposite element of the loop $d\vec{E} \sin\theta$ component cancels.

$$\therefore \boxed{\vec{E} = \int d\vec{E} \cos\theta}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$= \frac{k dq}{(cp)^2} \frac{cp}{cp}$$

$$r = cp$$

$$\vec{E} = \frac{k}{r^2 + a^2} \frac{r}{\sqrt{r^2 + a^2}} \int dq$$

$$\frac{k r}{(r^2 + a^2)^{3/2}} \leq \frac{a dl}{2\pi a}$$

$$\frac{k r q}{2\pi a (r^2 + a^2)^{3/2}} \leq dl$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{r q}{(r^2 + a^2)^{3/2}} \frac{2\pi a}{2\pi a}}$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{r q}{(r^2 + a^2)^{3/2}}}$$

by taking $q = 2\pi a I$

$$\vec{E} = \frac{r \times 2\pi a I}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}}$$

$$\boxed{\vec{E} = \frac{I a \cdot r}{2\epsilon_0 (r^2 + a^2)^{3/2}}}$$

NUMERICAL

- (1) Determine the total charge on the line $0 \leq x \leq 5$ if $\lambda = 12x^2 \text{ mc/m}$.

$$Q = \int_L \lambda \cdot dl \quad dl = dx$$

$$Q = \int_0^5 12x^2 \cdot dx$$

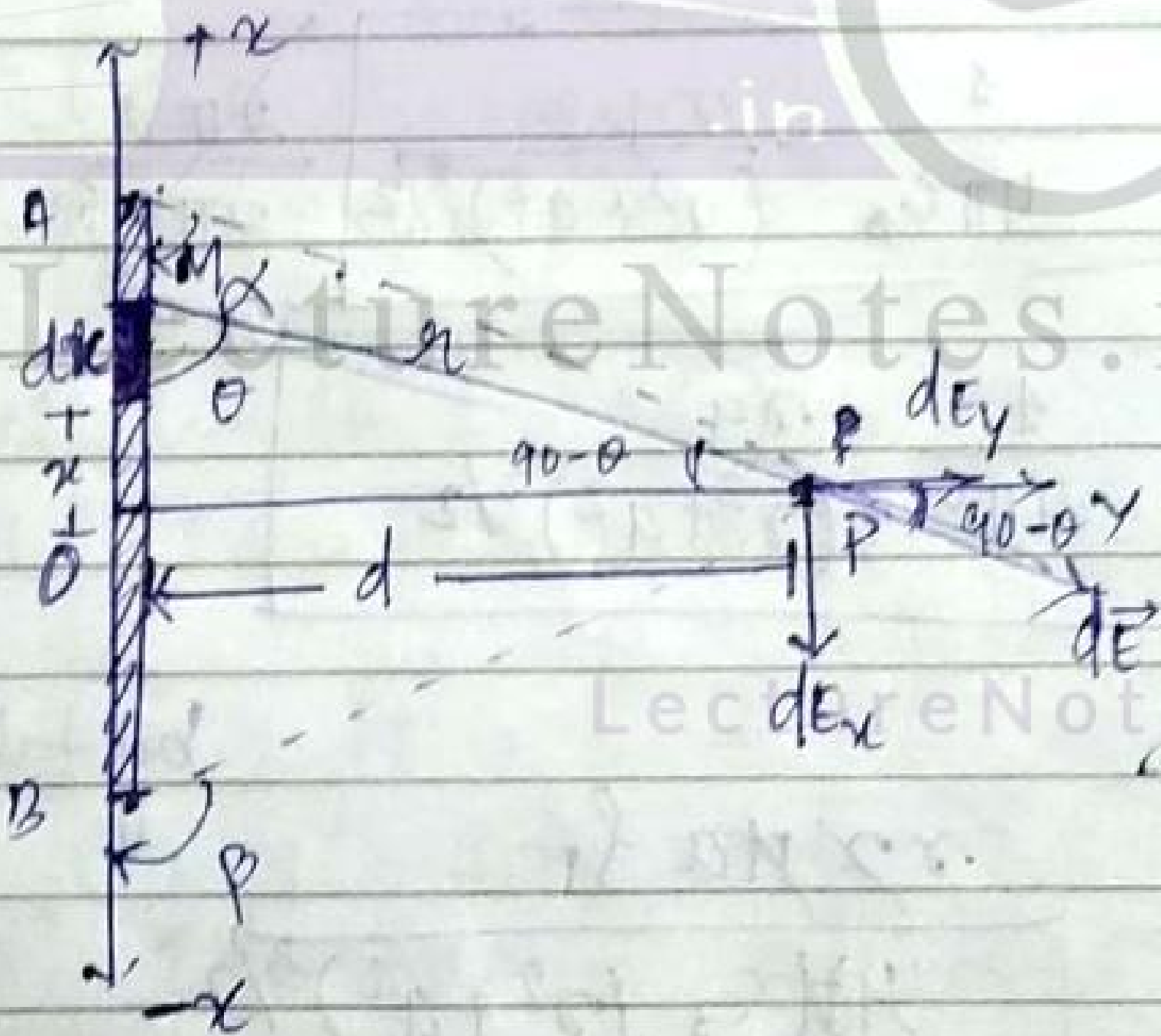
$$Q = \left[\frac{12x^3}{3} \right]_0^5$$

$$Q = 4(5)^3 \quad \boxed{Q = 500 \text{ mc}}$$

20.02.19

(derivation)

Imp. Electrical field intensity due to an
Infinitely long line charge



Consider a line charge AB in x-y plane

$$\lambda = \text{line charge density (C/m)} = \frac{dq}{dl}$$

let P is the point at which d is to be calculated

d = distance between the origin and the point P .

consider a differential element dx on the line charge.

let dq is the charge on dx

$$\boxed{d\vec{E} = d\vec{E}_x + d\vec{E}_y}$$

$d\vec{E}$ is the $\vec{E}$ intensity due to the differential element dx .

$$\vec{E} = \vec{E}_x + \vec{E}_y$$

$$\vec{E} = |E_x|\hat{a}_x + |E_y|\hat{a}_y$$

to compute dE_y due to dx due to dx , then compute for E_y & E_x .

$$dE_y = dE \cos(90^\circ - \theta)$$

$$dE_y = dE \sin \theta$$

$$dE = \frac{k dq}{r^2}$$

$$dq = \frac{dq}{dl} dl$$

$$dq = \lambda dl$$

$$\boxed{dE_y = \frac{k dq}{r^2} \sin \theta}$$

$$\because dq = \lambda dl \quad (\text{substitute in above eq})$$

$$= \lambda dx$$

$$dE_y = \frac{k \lambda dl}{r^2} \sin \theta$$

$$\boxed{dE_y = \frac{k l_L dx \sin \theta}{x^2}}$$

In AOMP

$$\Rightarrow \tan \theta = \frac{d}{x} \quad \text{a triangle with } d \text{ and } x$$

Lecture Notes in

$$x = \frac{d}{\tan \theta}$$

$$dx =$$

$$dx =$$

$$x = d \cot \theta$$

$$\boxed{dx = -d \operatorname{cosec}^2 \theta \cdot d\theta}$$

$$\Rightarrow \sin \theta = \frac{d}{x} \quad x = \frac{d}{\sin \theta} = d \operatorname{cosec} \theta$$

$$\boxed{x = d \operatorname{cosec} \theta}$$

$$dE_y = \frac{-k l_L d \operatorname{cosec}^2 \theta \cdot d\theta}{d^2 \operatorname{cosec}^2 \theta} \sin \theta$$

$$dE_y = -\frac{k l_L d\theta \sin \theta}{d}$$

$$E_y = -\int \frac{k l_L}{d} \sin \theta \cdot d\theta$$

$$E_y = -\frac{k l_L}{d} \int_p^{\alpha} \sin \theta \cdot d\theta$$

[θ varies from $p \rightarrow \alpha$ when line charge varies along $+x$.
 θ varies from $\alpha \rightarrow p$ when line charge varies along $-x$]

$$E_y = -\frac{k \lambda L}{d} [\cos \theta]_p^\alpha$$

$$\star \boxed{E_y = \frac{k \lambda L}{d} [\cos \alpha - \cos p]}$$

Similarly,

$$E_x = -\frac{k \lambda L}{d} \int_p^\alpha \cos \theta \cdot d\theta$$

$$E_x = -\frac{k \lambda L}{d} [\sin \theta]_p^\alpha$$

$$\star \boxed{E_x = -\frac{k \lambda L}{d} [\sin \alpha - \sin p]}$$

- For infinite line charge $A \rightarrow +\infty$, $B \rightarrow -\infty$
Hence, $\alpha \rightarrow 0$ and $p \rightarrow 180^\circ$ or π

$$E_x = -\frac{k \lambda L}{d} [\sin 0 - \sin \pi]$$

$$\boxed{E_x = 0}$$

$$E_y = \frac{k \lambda L}{d} [\cos 0 - \cos \pi]$$

$$E_y = \frac{k \int_L}{d} [10 + (-1)]$$

$$E_y = \frac{2k \int_L}{d}$$

$$E_y = \frac{1}{4\pi\epsilon_0} \times \frac{2 \int_L}{d} a_y$$

$$|E_y| = \frac{\int_L}{2\pi\epsilon_0 d}$$

Note:

- $\vec{E}$ at any point has no component in the direction parallel to the line along which the charge is present located at infinite.

Electric Field Intensity due to Surface Charge
(charge sheet) / (surface disc)

let us consider a disc consists of
no. of rings.

let for a particular ring of radius a at of distance x from the centre with uniform charge density is σ .

P is the point at which electric field need to be calculated.

P is the point distance from the origin

each ring is assumed to be formed by or consist of no. of point charges.

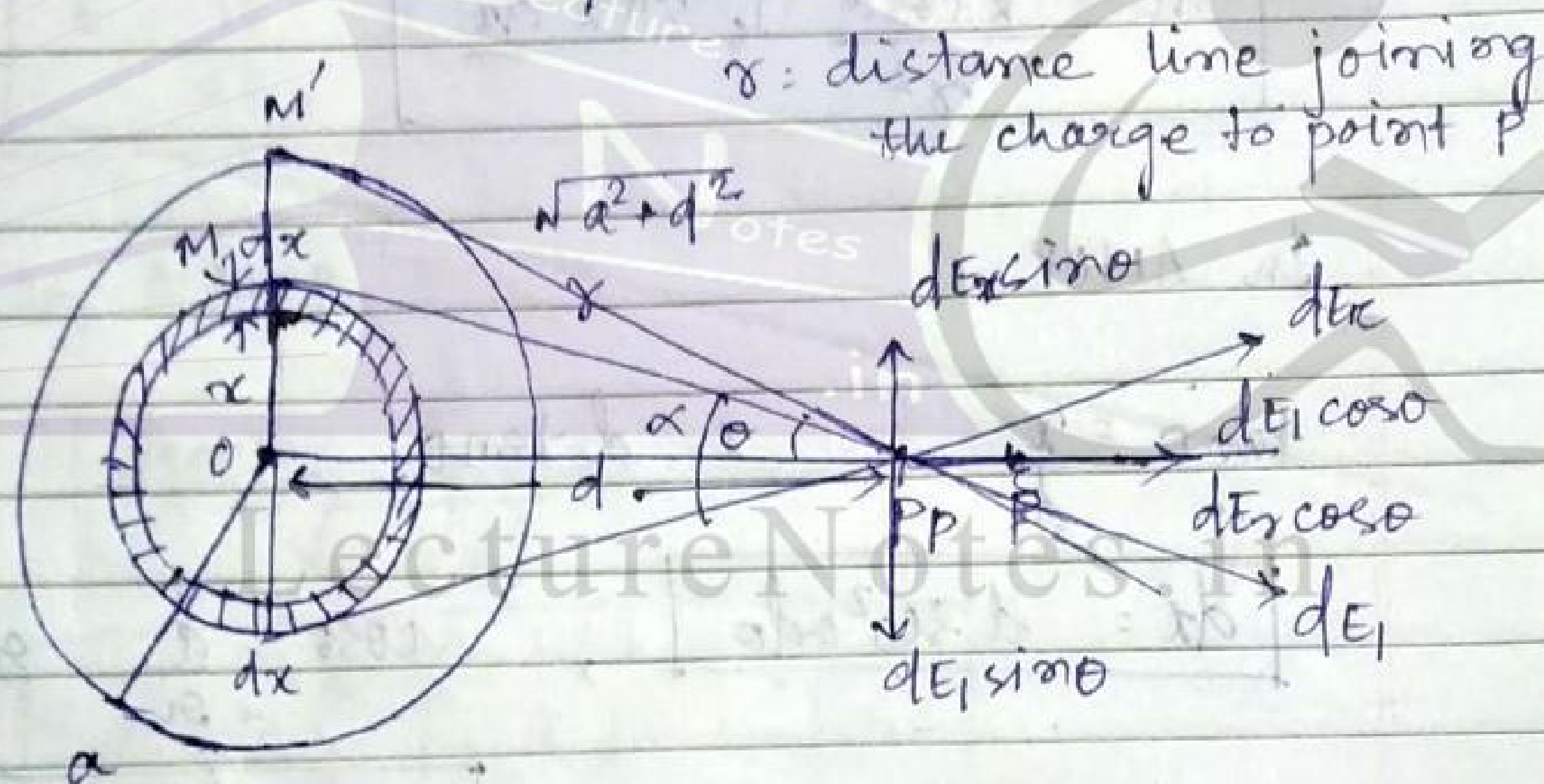
Consider the ring with radius a and the thickness dx .

Let there are n no. of point charges $dq_1, dq_2, dq_3, \dots, dq_n$.

Then, the Electric field intensity ($\vec{E}$) due to $dq_1 \rightarrow d\vec{E}_1, dq_2 \rightarrow d\vec{E}_2, \dots, dq_n \rightarrow d\vec{E}_n$

All the $\vec{E}$'s resolved into 2 components
normal & radial components.

The Normal components cancel out each other and the radial components are added.



Let $d\vec{E}$ is the resultant $\vec{E}$

$$d\vec{E} = dE_1 \cos \theta + dE_2 \cos \theta + \dots + dE_n \cos \theta$$

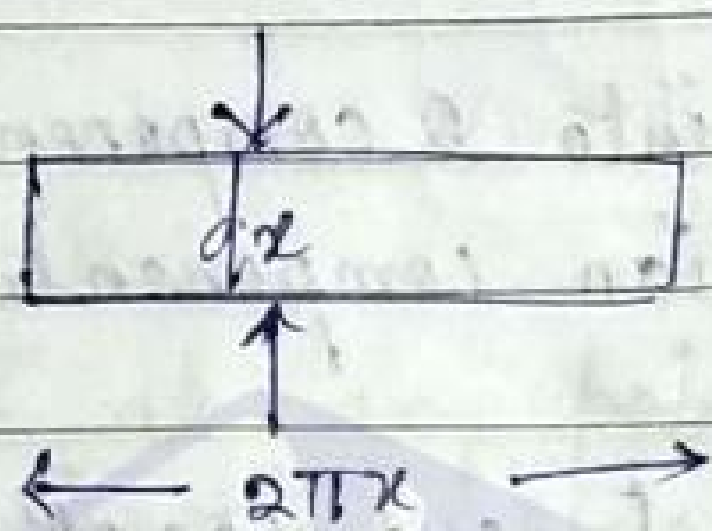
$$= \frac{k dq_1}{r^2} \cos \theta + \frac{k dq_2}{r^2} \cos \theta + \dots + \frac{k dq_n}{r^2} \cos \theta$$

22.02.19

$$d\vec{E} = \frac{k \cos \theta}{r^2} [\pm dq_1 + dq_2 + \dots dq_n]$$

$$d\vec{E} = \frac{k \cos \theta}{r^2} \cdot dq \quad \therefore \boxed{dq = \int_s ds}$$

$$\boxed{d\vec{E} = \frac{k \cos \theta}{r^2} \cdot \int_s ds}$$



$$ds = 2\pi x \cdot dx$$

$$dQ = \int_s 2\pi x \cdot dx$$

$$\boxed{d\vec{E} = \frac{k \cos \theta}{r^2} \int_s 2\pi x \cdot dx}$$

In ΔMOP

$$\tan \theta = \frac{x}{d} \quad x = d \tan \theta$$

$$\boxed{dx = d \sec^2 \theta d\theta}$$

$$\cos \theta = \frac{d}{r} \quad r = \frac{d}{\cos \theta}$$

$$\boxed{r = d \sec \theta}$$

$$d\vec{E} = \frac{k \cos \theta}{d^2 \sec^2 \theta} \int_s 2\pi d \tan \theta \times d \sec^2 \theta \cdot d\theta$$

$$d\vec{E} = k \cos \theta \int_s 2\pi \tan \theta \cdot d\theta$$

$$d\vec{E} = k \cos \theta \int_s 2\pi \frac{\sin \theta}{\cos \theta} \cdot d\theta$$

$$d\vec{E} = k \frac{\rho_s}{r^2} 2\pi r \sin\theta \, d\theta$$

$$\because k = \frac{1}{4\pi\epsilon_0}$$

$$d\vec{E} = \frac{\rho_s \sin\theta \, d\theta}{2\epsilon_0}$$

$$\vec{E} = \int_0^\alpha d\vec{E} = \frac{\rho_s}{2\epsilon_0} \int_0^\alpha \sin\theta \, d\theta$$

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} [-\cos\theta]_0^\alpha$$

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} [-\cos\alpha + \cos(0)]$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} (1 - \cos\alpha)} \quad (\text{Final value})$$

• In $\Delta M'OP$

$$\cos\alpha = \frac{d}{\sqrt{a^2 + d^2}}$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \left[1 - \frac{d}{\sqrt{a^2 + d^2}} \right] \hat{r}}$$

• For an infinite sheet of charge

$$a \rightarrow \infty$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_r}$$

Q (1) Find out $\vec{E}$ at a point $E(3, 2, 1)$ when the line charge is along x-axis with

$$\boxed{\rho_L = 20 \text{ nC/m}}$$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 d} \hat{a}_d$$

$$(\cancel{x}, \cancel{y}, \cancel{z})^*$$

$$\begin{aligned} \vec{d} &= (3, 2, 1) - (3, 0, 0) \\ &= (2-0)a_y + (1-0)a_z \end{aligned}$$

$$2a_y + a_z$$

$$|d| = \sqrt{5}$$

$$\vec{E} = \frac{20 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12} \times \sqrt{5}} (2a_y + a_z)$$

$$\vec{E} = \frac{160.477}{\sqrt{5}} (2a_y + a_z)$$

$$\vec{E} = 71.90 (2a_y + a_z)$$

$$\boxed{\vec{E} = 143.8 a_y + 71.90 a_z}$$

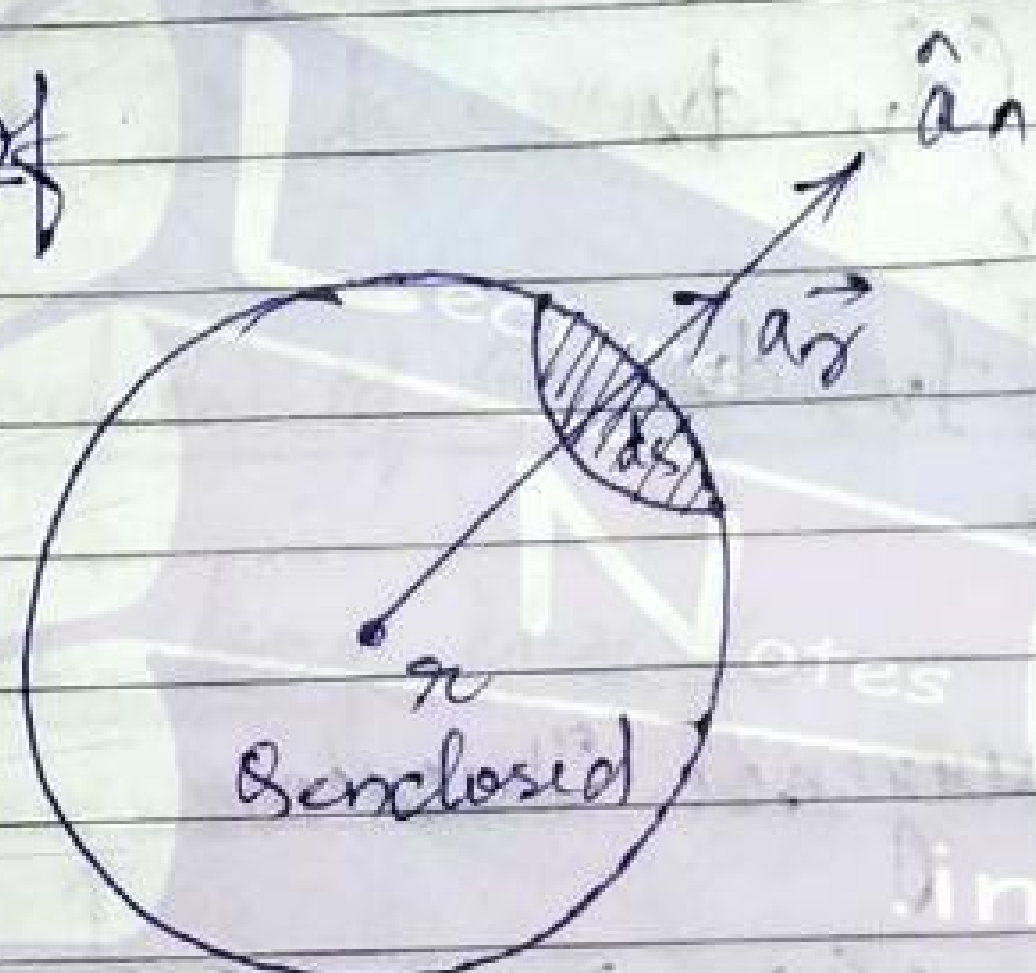
Gauss's Law

E-flux density $\boxed{\vec{D} = \epsilon_0 \vec{E}}$

- It states that the net electric or outward flux Φ passing through any closed surface is equal to the net charge enclosed by that surface.

$$\Psi = Q_{\text{enclosed}} = \oint \vec{D} \cdot d\vec{s}$$

Proof



$$\boxed{d\vec{s} = dS \hat{a}_n}$$

$$\boxed{\vec{E} = \frac{Q_{\text{enclosed}}}{4\pi\epsilon_0 r^2} \hat{a}_r}$$

$$\vec{D} = \epsilon_0 \vec{E} = \frac{Q_{\text{enclosed}}}{4\pi r^2} \hat{a}_r$$

$$\vec{D} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{4\pi r^2} \hat{a}_r dS \hat{a}_n$$

$$\vec{D} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{4\pi r^2} dS$$

$$\oint_S \vec{D} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{4\pi\epsilon^2} \int ds$$

$$\oint_S \vec{D} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{4\pi\epsilon^2} \times 4\pi\epsilon^2$$

$$\boxed{\psi = Q_{\text{enclosed}}}$$

$$\rho_v = \text{volume charge density} = \frac{Q_{\text{en}}}{dv}$$

$$Q_{\text{enclosed}} = \int_V \rho_v \cdot dv$$

$$\oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v \cdot dv$$

Applying divergence Theorem

$$\oint_S \vec{D} \cdot d\vec{s} = \int_V (\vec{\nabla} \cdot \vec{D}) dv = \int_V \rho_v dv$$

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_v}$$

* Application of $\vec{D}$ or ψ from Gauss law:

- Determination of Q_{enclosed} from $\vec{D}$ & $\vec{E}$
- Determination of E from Q
- Determination of $\vec{D}$ or ψ from Q

* Limitations & Disadvantages :

- It is applicable to gaussian surface
- It is applicable to closed surface.

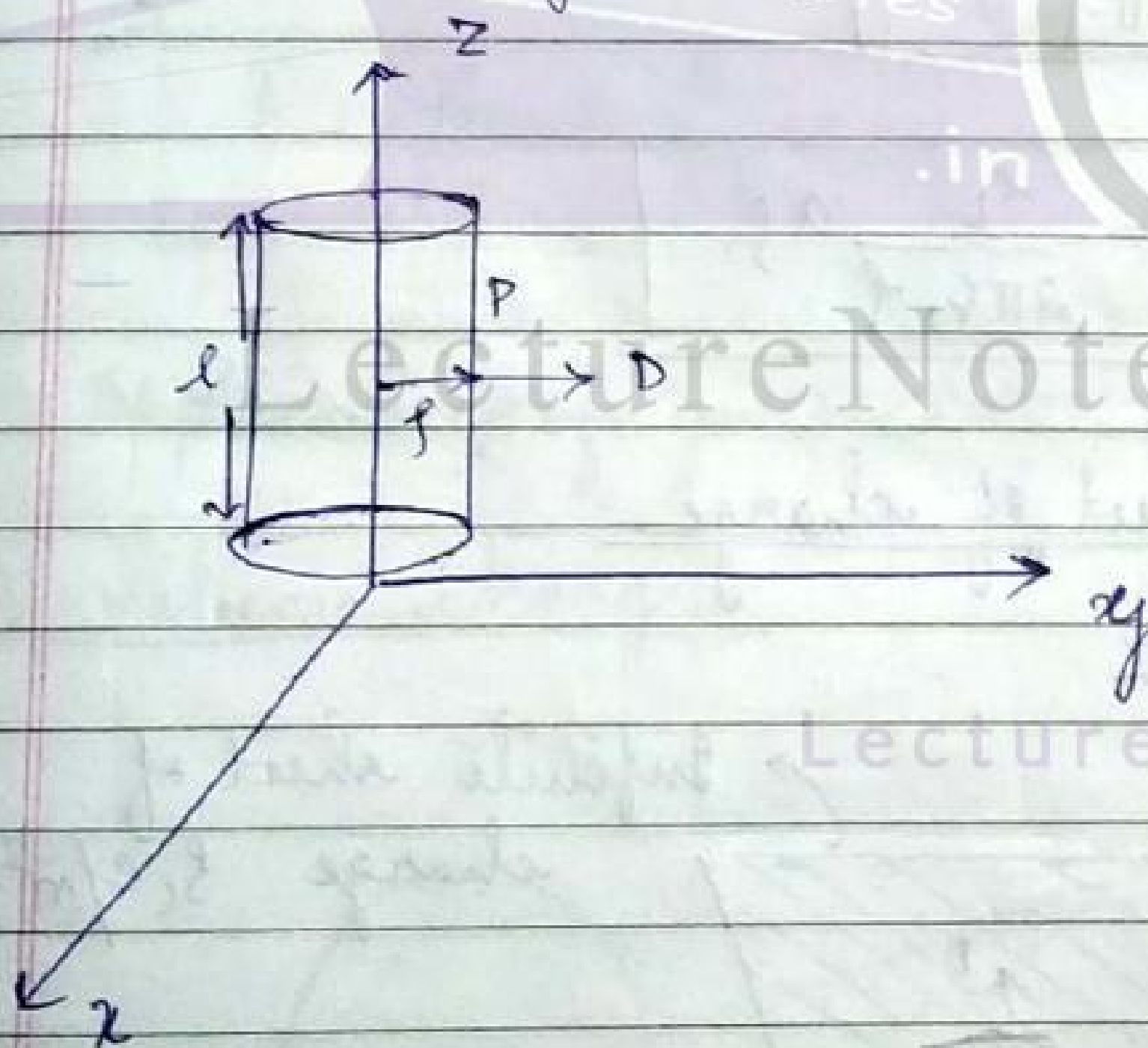
* A gaussian surface (i) is a closed surface
(ii) sufficiently large to enclose the entire charge.

(iii) At each point of the surface $\vec{D}$ is normal to the surface.

(iv) $\vec{D}$ is constant over the surface at which it is normal.

26.02.19 Application of Gauss law :

1) Infinite long charge



ϵ_l = line charge density C/m

l = length of the cylinder.

r = Radius of the cylinder.

$$\vec{D} = D_f a_f$$

$$Q = \oint_L \times l = \oint \vec{D} \cdot \vec{ds} =$$

$$ds = \int d\phi dz a_f$$

$$= \int D_f a_f \cdot \int d\phi dz a_f$$

$$= D_f \int_0^{2\pi} d\phi \int_0^l dz$$

$$l \times l = D_f \int (2\pi) (l)$$

$$D_f = \frac{l}{2\pi f}$$

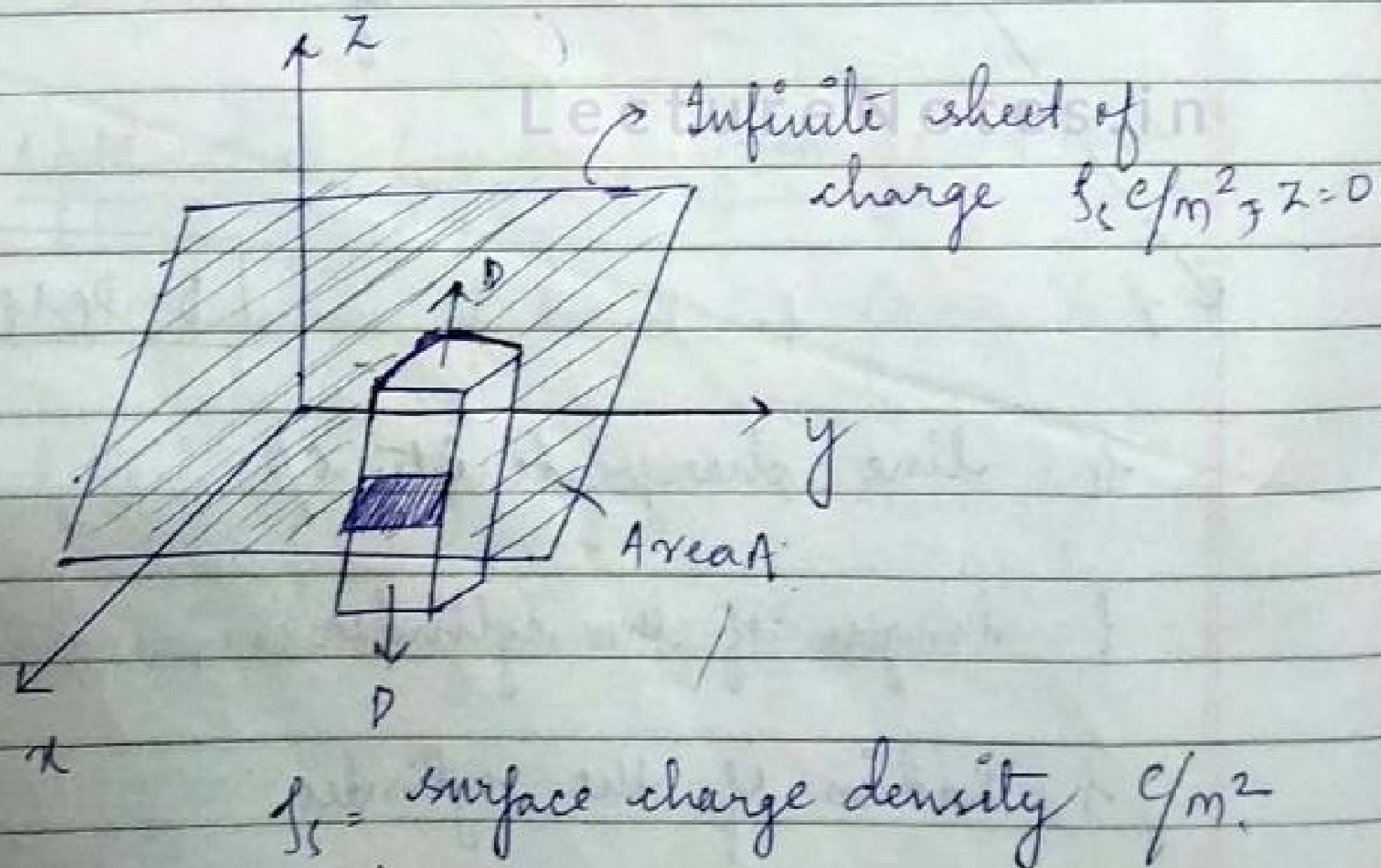
$$\vec{D} = \frac{l}{2\pi f} a_f$$

$$\vec{D} = \vec{E} \epsilon_0$$

$$E = \frac{D}{\epsilon_0}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{l}{2\pi \epsilon_0 f} a_f$$

2) Infinite sheet of charge



$$\vec{D} = D_z \hat{a}_z$$

$$\oint_S d\vec{s} = 0 \quad \left| \quad \psi = \oint_S \vec{D} \cdot d\vec{s} \right|$$

$$\oint_S A = D_z \int$$

$$\oint_S A = \int_{\text{Top}} \vec{D} \cdot d\vec{s} + \int_{\text{Bottom}} \vec{D} \cdot d\vec{s}$$

$$D_z \left[\int_{\text{top}} d\vec{s} + \int_{\text{bottom}} d\vec{s} \right]$$

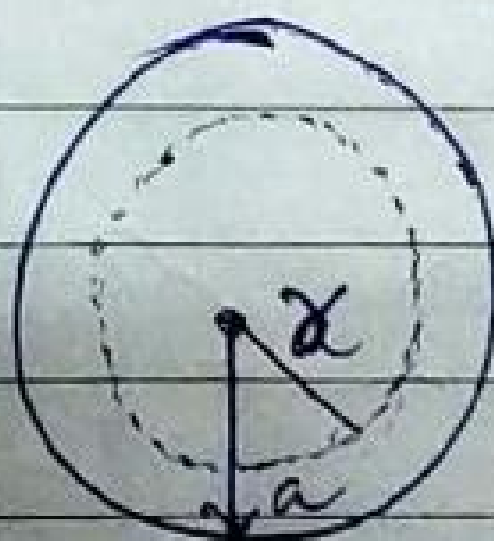
$$\oint_S A = D_z [A + A] = 2A$$

$$D_z = \frac{\rho_s}{2}$$

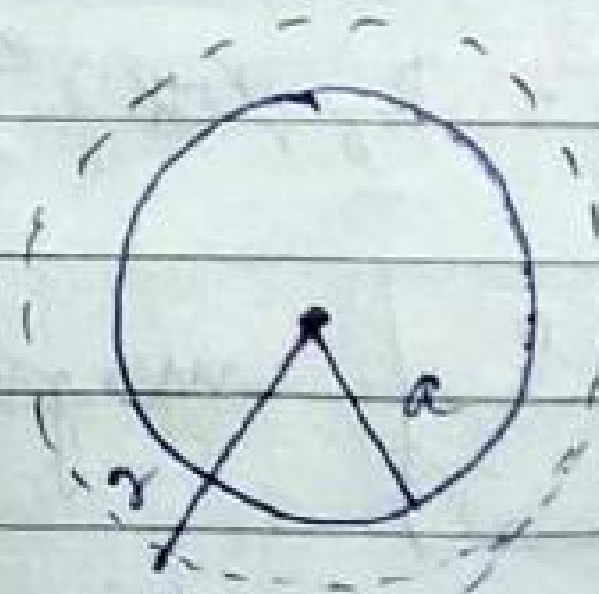
$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z$$

~~Application of Gauss law~~

3. > Volume charge



Case 1: $0 < r \leq a$



Case 2: $r > a$

case 1 : $0 \leq r \leq a$.

ρ_v : volume charge density C/m^3

$$Q_{enc} = \int_V \rho_v dv$$

$$= \rho_v \int_V dv = \rho_v \int_0^a r^2 \sin \theta dr d\theta d\phi$$

$$= \rho_v \int_0^a r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi$$

$$= \rho_v \left[\frac{r^3}{3} \right]_0^a \left[-\cos \theta \right]_0^\pi \left[\phi \right]_0^{2\pi}$$

$$= \rho_v \left[\frac{r^3}{3} - 0 \right] \left[-\cos \pi + \cos 0 \right] \left[2\pi - 0 \right]$$

$$Q_{enc} = \int_0^a \frac{4}{3} \pi r^3 \rho_v$$

$$\begin{aligned} \Phi &= \oint_S \vec{D} \cdot d\vec{s} \\ &= \oint_S D_r \phi ds \\ &= \oint_S D_r \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \end{aligned}$$

note

- r is the radius of gaussian surface
- a is the radius of the sphere

$$\int_0^a \frac{4}{3} \pi r^3 \rho_v = D_r \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi$$

$$D_r = \frac{r \rho_v}{3} \quad \text{magnitude}$$

direction is along the radius.

case 2 : $r > a$

$r = a$

$$Q_{enc} = \int_V \rho_0 dv =$$

$$= \rho_0 \int_V dv$$

$$= \rho_0 \int_V r^2 \sin \theta \cdot dr d\theta d\phi$$

$$= \rho_0 \int_0^a r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi$$

$$Q_{enc} = \rho_0 \frac{4}{3} \pi a^3$$

$$Q = \Psi = \oint_S \vec{D} \cdot d\vec{S}$$

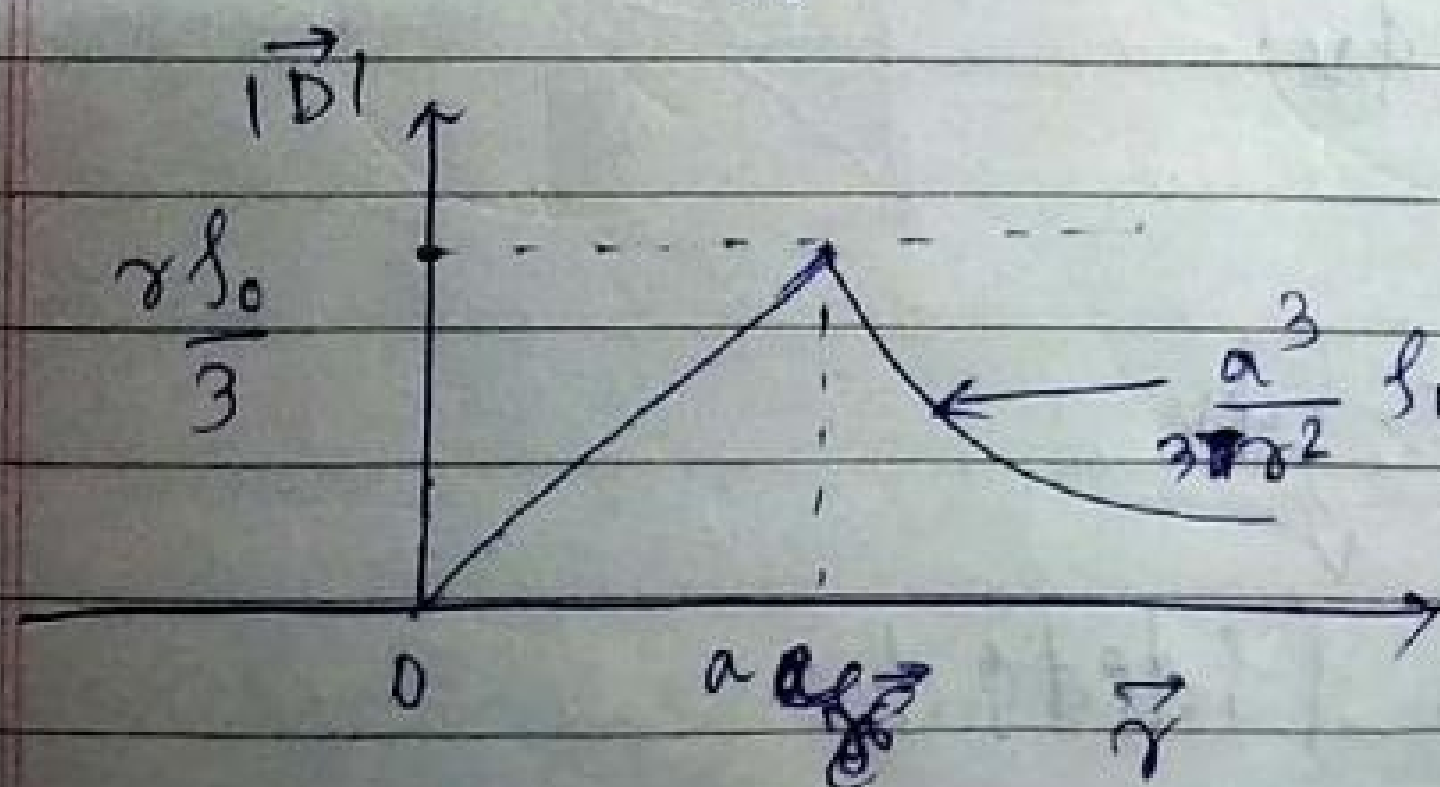
$$= D_r \oint_S dS$$

$$= D_r \int r^2 \sin \theta d\theta d\phi$$

$$\rho_0 \frac{4}{3} \pi a^3 = D_r 4\pi r^2$$

$$D_r = \frac{a^3}{3r^2} \rho_0$$

$$\vec{D} = \begin{cases} \frac{\rho_0}{3} a r & 0 \leq r \leq a \\ \frac{a^3}{3r^2} \rho_0 & r > a \end{cases}$$



imp

variation of
 $|\vec{D}|$ w.r.t. r
← (graph)

Numerical

- ① Given that $\vec{D} = z \cos^2 \phi \hat{a}_z$ C/m².
Calculate the charge density at $P(1, \pi/4, 3)$ and the total charge enclosed by the cylinder of radius 1 m which with $-2 \leq z \leq 2$.

$$\begin{aligned} \rho_v &= ? & \vec{D} &= z \cos^2 \phi \hat{a}_z & r &= 1 \text{ m} \\ P(1, \pi/4, 3) & & & -2 \leq z \leq 2 \\ P(r, \phi, z) & & & & \end{aligned}$$

$$\rho_v = \nabla \cdot \vec{D}$$

$$\rho_v = \frac{\partial}{\partial z} [z \cos^2 \phi \hat{a}_z]$$

$$\rho_v \Big|_{(1, \pi/4, 3)} = \cos^2 \phi \hat{a}_z$$

$$\rho_v \Big|_{(1, \pi/4, 3)} = 1 \cos^2 \left(\frac{\pi}{4} \right)$$

$$\boxed{\rho_v = 0.5 \text{ C/m}^3}$$

$$Q = \int_V \rho_v \cdot dV$$

$$= 0.5 \cdot \int_V dV$$

$$= 0.5 \int_V r dr d\phi dz$$

$$= 0.5 \int_0^1 f \cdot dr \int_0^{2\pi} d\phi \int_{-2}^2 dz$$

$$= 0.5 \left[\frac{r^2}{2} \right]_0^1 \left[\phi \right]_0^{2\pi} \left[z \right]_{-2}^2$$

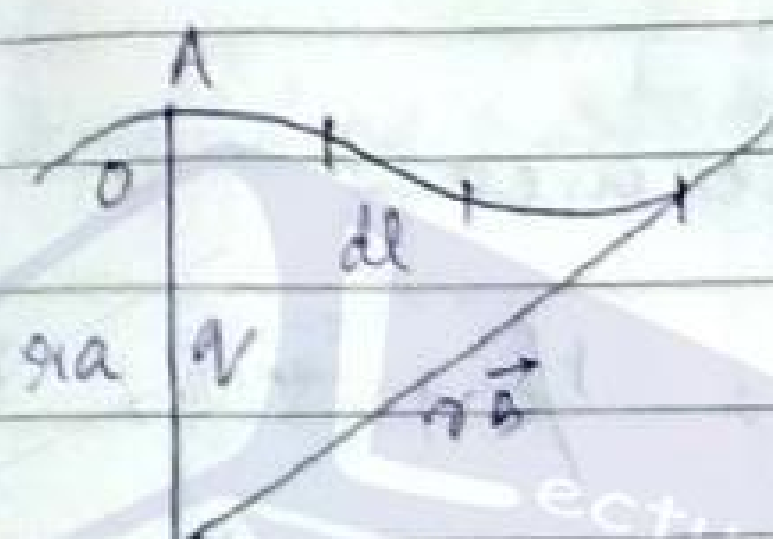
$$= 0.5 \frac{1}{2} \times 2\pi (2+0)$$

$$= 0.5 \times 4$$

$$= 2$$

$$Q_{en} = 2C$$

27.0219



$$dw = -F \cdot dl$$

$$dw = -qE \cdot dl$$

$$w = -q \int_A^B \vec{E} \cdot d\vec{l}$$

(1) A is starting point of B is the ending

(2) - sign indicates that loss of P.E

$$\frac{w}{q} = - \int_A^B \vec{E} \cdot d\vec{l}$$

$$V_{AB} = - \int_A^B \vec{E} \cdot d\vec{l}$$

Electric potential

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

$$V_{AB} = - \int_A^B \frac{q}{4\pi\epsilon_0 r^2} \cdot dr \cdot \hat{a}_r$$

$$= - \int_A^B \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0 r_B} - \frac{Q}{4\pi\epsilon_0 r_A}$$

$$V_{AB} = V_B - V_A$$

→ potential difference

If $A \rightarrow \infty$, $r_A \rightarrow \infty$, $1/r_A \rightarrow 0$, $r_B \rightarrow r$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

r' = position vector at which Q is present

r = position vector V is calculated

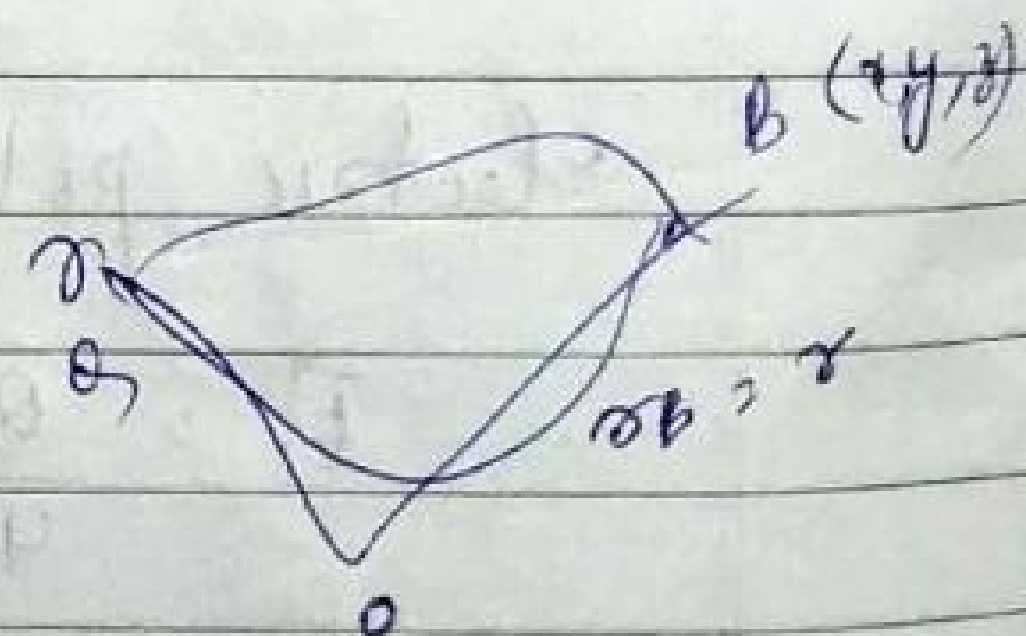
$$V(r) = \frac{Q}{4\pi\epsilon_0 |r - r'|}$$

$$V_{AB} = V_B - V_A$$

$$V_{BA} = V_A - V_B$$

$$V_{AB} = -V_{BA}$$

$$V_{AB} + V_{BA} = 0$$



$$\int_A^B \vec{E} \cdot d\vec{l} + \int_B^A \vec{E} \cdot d\vec{l} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Hence, this shows that electric potential around a closed path doesn't depend upon the path of integration. Hence it is a conservative field.

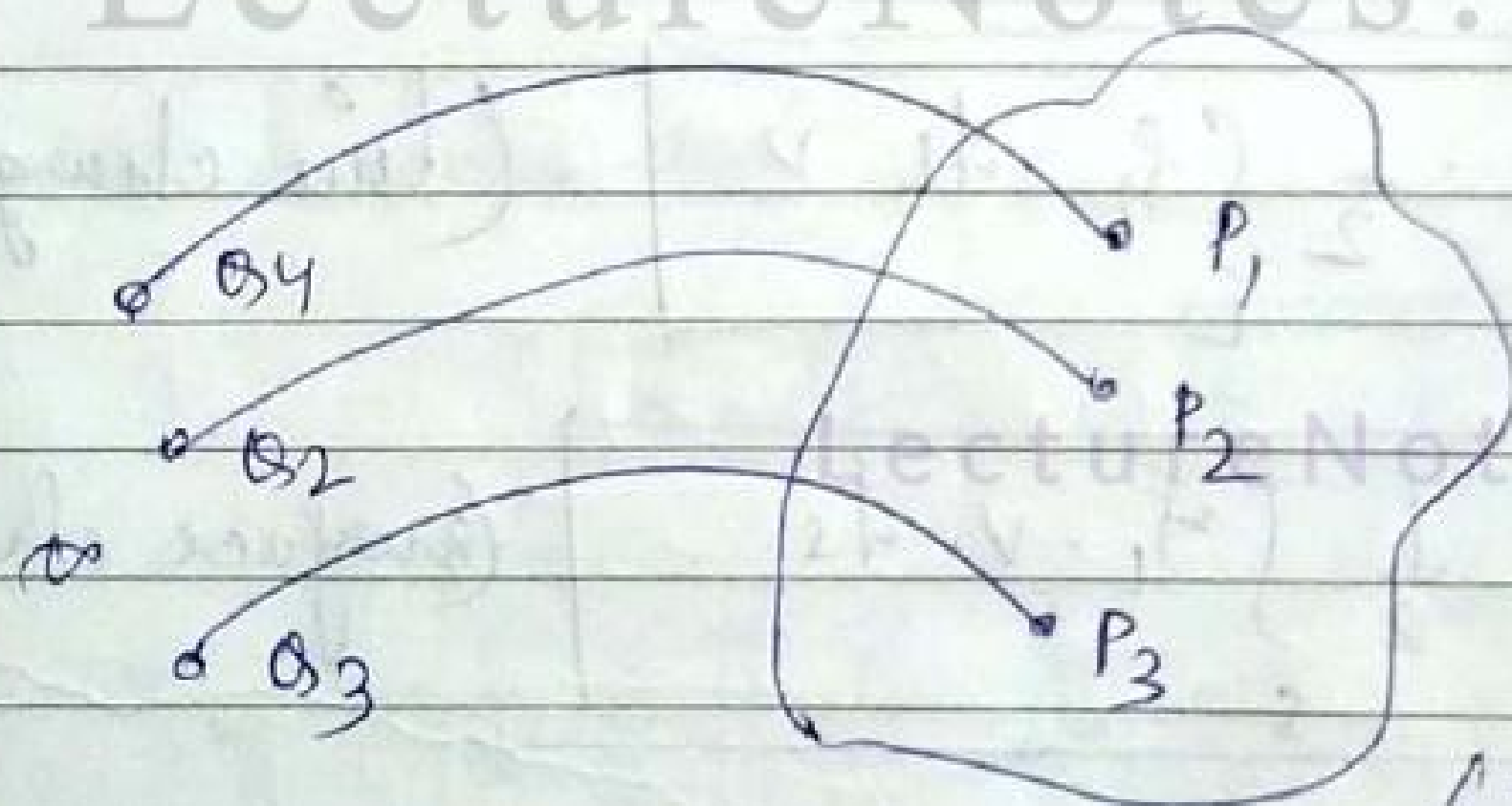
Applying Stokes theorem.

$$\oint \vec{E} \cdot d\vec{l} = \int (\nabla \times \vec{E}) \cdot d\vec{s} = 0$$

$$\nabla \times \vec{E} = 0$$

Irrrotational

Energy density



(1)

$$W_1 = 0$$

$$W_2 = Q_2 V_{21}$$

$$W_3 = Q_3 [V_{31} + V_{32}]$$

$\left\{ \begin{array}{l} V_{31} \rightarrow \\ \text{potential of} \\ 3 \text{ due to} \\ \text{charge } \end{array} \right\}$

$$\rightarrow Q_3 \rightarrow P_3, \quad Q_2 \rightarrow P_2, \quad Q_1 \rightarrow P_1$$

$$W_2 = 0$$

$$W_2 = Q_2 V_{23}$$

$$W_1 = Q_1 [V_{12} + V_{13}]$$

$$W_E = W_1 + W_2 + W_3$$

$$= 0 + Q_2 V_{21} + Q_2 [V_{21} + V_{31}] \quad \text{--- (1)}$$

$$W_E = W_1 + W_2 + W_3$$

$$= Q_1 [V_{12} + V_{13}] + Q_2 V_{23} + 0 \quad \text{--- (2)}$$

on adding (1) & (2)

$$2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$V_1 = V_{12} + V_{13}$$

$$V_2 = V_{21} + V_{23}$$

$$V_3 = V_{31} + V_{32}$$

$$\therefore W_E = \frac{1}{2} [Q_1 V_1 + Q_2 V_2 + Q_3 V_3]$$

$$W_E = \frac{1}{2} \sum_{k=1}^n Q_k V_k$$

$\therefore E$ is in joule

$$W_E = \frac{1}{2} \int_L \rho_L \cdot dL \cdot V \quad (\text{line charge})$$

$$W_E = \frac{1}{2} \int_S \rho_S \cdot V \cdot dS \quad (\text{surface charge})$$

$$W_E = \frac{1}{2} \int_V \rho_V \cdot V \cdot dV \quad (\text{volume charge})$$

Acc. to gauss law:

$$\nabla \cdot \vec{D} = \rho_V$$

$$W_E = \frac{1}{2} \int_V (\nabla \cdot \vec{D}) \cdot V \cdot dV$$

Acc. to vector identity

$$\nabla \cdot \nabla \vec{A} = \vec{A} \cdot \nabla \nabla + \nabla (\nabla \cdot \vec{A})$$

$$\nabla (\nabla \cdot \vec{A}) = \nabla \cdot \nabla \vec{A} - \vec{A} \cdot \nabla \nabla$$

$$\nabla (\nabla \cdot \vec{D}) = \nabla \cdot \nabla \vec{D} - \vec{D} \cdot \nabla \nabla$$

$$W_E = \frac{1}{2} \int (\nabla \cdot \nabla \vec{D} - \vec{D} \cdot \nabla \nabla) \cdot d\vec{v}$$

$$W_E = \frac{1}{2} \int \nabla \cdot \nabla \vec{D} \cdot d\vec{v} = \int \vec{D} \cdot \nabla \nabla \cdot d\vec{v}$$

applying divergence theorem

$$W_E = \frac{1}{2} \int \nabla \vec{D} \cdot d\vec{s} - \int (\vec{D} \cdot \nabla \nabla) \cdot d\vec{v}$$

$$V = \frac{Q}{4\pi\epsilon_0 r} \quad D = \frac{Q}{4\pi\epsilon r^2}$$

$$V \propto \frac{1}{r} \quad D \propto \frac{1}{r^2}$$

Imp In the 1st part $V \propto \frac{1}{r}$, $D \propto \frac{1}{r^2}$ & $\nabla \vec{D} \propto \frac{1}{r^3}$ while, $ds \propto r^2$, Hence for a large R^3 surface 's' the 1st term $\rightarrow 0$.

$$W_E = \frac{1}{2} \int (\vec{D} \cdot \nabla \nabla) \cdot d\vec{v}$$

we know that ;

$$V = - \int_L \vec{E} \cdot d\vec{e} \quad \because \boxed{E = -\nabla V}$$

$$W_E = \frac{1}{2} \int \vec{D} \cdot \vec{E} \cdot d\vec{v}$$

$$W_E = \frac{\epsilon_0}{2} \int E^2 \cdot d\vec{v}$$

$$\boxed{W_E = \frac{1}{2\epsilon_0} \int D^2 \cdot d\vec{v}}$$

$$\boxed{W_E = \frac{D^2}{2} = \frac{\epsilon_0 E^2}{2}} \quad \text{J/m}^3$$

201.03.19

Two point charges -4 nC & 5 nC are located at $(2, -1, 3)$ & $(10, 4, -2)$ respectively. Find the potential at $(1, 0, 1)$ assuming zero potential at infinity.

$$\begin{aligned} &-4\text{ nC} \quad (2, -1, 3) \\ &5\text{ nC} \quad (10, 4, -2) \end{aligned}$$

$$V_{\text{at}}(1, 0, 1)$$

$$V_1 = \frac{Q_1}{4\pi\epsilon_0 |r - r_1|}$$

$$V_2 = V_1 + V_2$$

$$V_1 = \frac{Q_1}{4\pi\epsilon_0 |r - r_1|}$$

$$\begin{aligned} |r - r_1| &= \sqrt{(1-2)^2 + (0-(-1))^2 + (1-3)^2} \\ &= \sqrt{6} = 2.45 \end{aligned}$$

$$V_1 = \frac{-4 \times 10^{-6} \times 9 \times 10^{-9}}{2.45}$$

$$V_1 = -1.46 \times 10^{-14}$$

$$V_2 = \frac{Q_2}{4\pi\epsilon_0 |r - r_2|}$$

$$|r - r_2| = \frac{|(1, 0, 1) - (0, 4, -2)|}{\sqrt{(1-0)^2 + (0-4)^2 + (1+2)^2}}$$

$$= \frac{\sqrt{1 + 16 + 9}}{5.09} = 5.09$$

Lecture Notes

$$V_2 = \frac{5 \times 10^{-5} \times 9 \times 10^9}{5.09}$$

$$V_2 = 8.84 \times 10^3$$

$$V = V_1 + V_2$$

$$V = -1.46 \times 10^{-14} + 8.84 \times 10^3$$

$$V = 8840 \text{ kV} \quad -5.76 \text{ kV}$$

given that $V = \frac{10}{r^2} \sin \theta \cos \phi$ (a) Find the electric flux density (D) at $(2, \pi/2, 0)$
 (b) Calculate the work done in moving a 10 Mc charge from point A $(1, 30^\circ, 120^\circ)$ to point B $(4, 90^\circ, 60^\circ)$.

$$D = \epsilon_0 E$$

$$E = -\nabla V$$

$$= - \left[\frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi \right]$$

$$= - \left[\frac{-20}{r^3} \sin \theta \cos \phi + \frac{1}{r} \frac{10}{r^2} \cos \phi \cos \theta - \frac{1}{r \sin \theta} \frac{10}{r^2} \sin \theta \sin \phi \right]$$

$$= \left[-\frac{20}{r^3} \sin \theta \cos \phi a_r + \frac{1}{r} \frac{10}{r^2} \cos \phi \cos \theta a_\theta - \frac{1}{r} \sin \theta \frac{10}{r^2} \cos \phi \sin \theta a_\phi \right]$$

$$\left(\vec{E} \right)_{(2, \pi/2, 0)} = -\frac{20}{8} a_r + 0 + 0$$

$$D = \epsilon \epsilon_0 = 8.854 \times 10^{-12} \times \frac{20}{8} a_r$$

$$D = 22.1 \times 10^{-12} \text{ C/m}^2$$

$$\boxed{\vec{D} = 22.1 \text{ pC/m}^2} a_r$$

$$W = -q \int_A^B \vec{E} \cdot d\vec{l}$$

$$= q [V_B]$$

$$= q [V_B - V_A]$$

$$= 10 \times 10^{-6} \left[\frac{10}{4^2} \sin 90^\circ \cos 60^\circ - \frac{10}{1^2} \sin 90^\circ \cos 120^\circ \right]$$

$$W = 2.812 \times 10^{-5}$$

$$\boxed{W = 28.1 \text{ nJ}}$$

A charge distribution in free space has
 $\rho_v = 2r \text{ nC/m}^3$ $0 \leq r \leq 10 \text{ m}$ and 0 otherwise.
 Determine $\vec{E}$ at $r = 2 \text{ m}$ and at $r = 12 \text{ m}$

$$\rho_v = 2r \text{ nC/m}^3 \quad 0 \leq r \leq 10 \text{ m}.$$

$\vec{E}$ at $r = 2 \text{ m}$
 $r = 12 \text{ m}$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$r = 2 \text{ m}$

$$Q_{en} = \oint_V \rho_v \cdot dV$$

$$\oint_V \rho_v \cdot dV = \epsilon_0 \oint_S \vec{E} \cdot d\vec{s}$$

$$= \epsilon_0 \cdot E \cdot 4\pi r^2$$

$$= \int_V 2r \cdot r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \int_0^2 2r^3 \, dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= \left[\frac{2r^4}{4} \right]_0^2 \left[-\cos\theta \right]_0^\pi \left[\phi \right]_0^{2\pi}$$

$$= \frac{2 \times 2^4}{4} [2] [2\pi]$$

$$= 32\pi$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\epsilon_0 E 4\pi r^2 = 32\pi$$

$$\vec{E} = \frac{32\pi}{\epsilon_0 \times 4\pi \times 2^2}$$

$$\vec{E} = 2.20 \times 10^4 \text{ N/C} \quad \boxed{E = 226 \text{ N/m}}$$

$$r = 12 \text{ m.}$$

$$\int_V \rho_v \cdot dV = \int_V \rho_v r^2 \sin\theta \cdot dr d\theta d\phi \times 10^{-9}$$

$$= \int_V 2 \times 10^{-12} r^2 \sin\theta \cdot dr d\theta d\phi + \int 0$$

$$\epsilon_0 \vec{E} 4\pi r^2 = \int_0^{10} 2 \times 10^{-12} r^3 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \times 10^{-9}$$

$$\vec{E} = 3.92 \text{ kV/m}$$

05.02.19.

→ CURRENT :

$$I = \frac{\Delta Q}{\Delta t}$$

conduction → inductor

convection current (dielectric)
↓
velocity

current density,

$$J = \frac{\Delta I}{\Delta S}$$

$$\Delta I = \int_S \vec{J} \cdot d\vec{S}$$

$$I = \int_S \vec{J} \cdot d\vec{S}$$

Ohm's law

$$V \propto I$$

$$V = RI$$

$$I = \frac{V}{R}$$

$$R = \frac{\rho L}{A}$$

$$I = \frac{VA}{\rho L}$$

$$\frac{I}{A} = \frac{V}{\rho L}$$

$$\frac{J}{A} = \frac{\sigma}{\rho} \frac{V}{L}$$

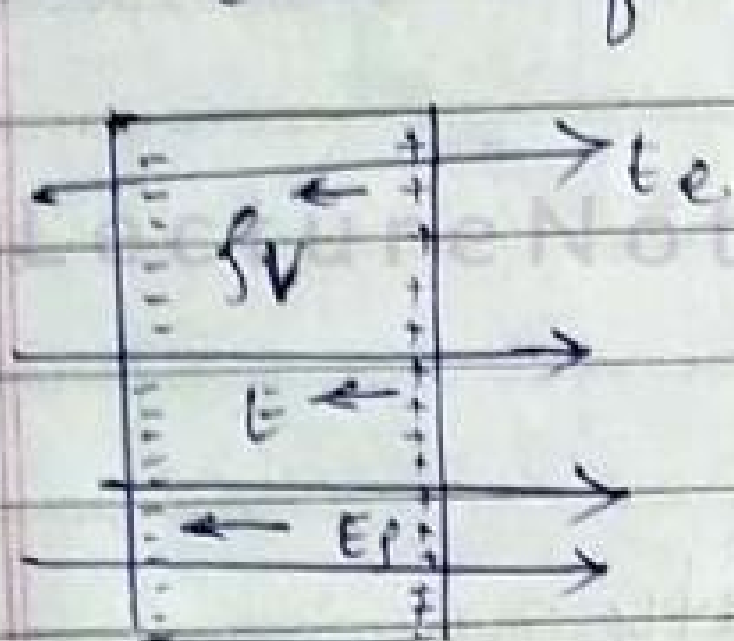
$$\boxed{J = \sigma E}$$

$$\sigma = \frac{1}{\rho} \text{ (resistivity)}$$

(conductivity) ρ

J : current density
 σ : conductivity
 E : Electric field

$$J = \frac{I}{A}$$



$$\boxed{\oint_V = 0}$$

$$\nabla \cdot \vec{B} = \oint_V$$

$$\nabla \cdot \vec{E} = 0$$

$$\boxed{E = 0}$$

$$D = 0$$

$$\epsilon_0 E = 0$$

$$\boxed{V = - \int E \cdot dl = 0}$$

$$D = \epsilon_0 E$$

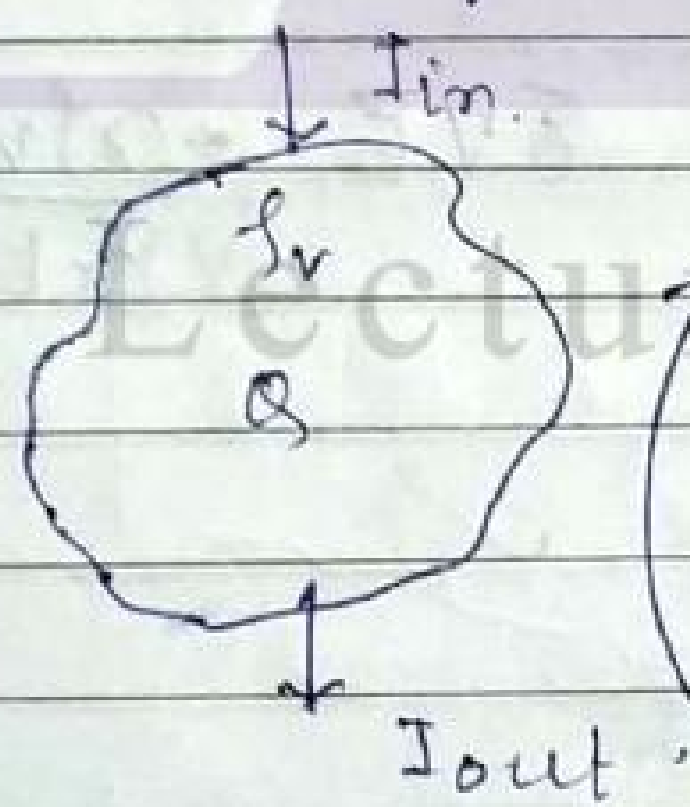
$$\textcircled{1} \quad \oint_V = 0$$

$$\textcircled{2} \quad E = 0$$

$$\textcircled{3} \quad V = 0$$

$$\begin{matrix} \text{stokes} & \text{div} \\ \oint_C \vec{S} \cdot d\vec{S} & \oint_V \vec{V} \cdot d\vec{V} \end{matrix}$$

Equation of continuity



$$\boxed{I = \oint_S \vec{J} \cdot d\vec{s} = \frac{dQ}{dt}}$$

Consider a region with uniform charge distribution ρ_v .

eq

[decrease of current]
 rate of change of charge is less

Applying divergence theorem

$$\boxed{\oint_S \vec{J} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{J}) dV}$$

$$Q = \int_V \rho_v \cdot dV$$

$$\Rightarrow \int_V (\nabla \cdot \vec{J}) dV = \frac{d}{dt} \int_V \rho_V dV$$

$$\Rightarrow \int_V (\nabla \cdot \vec{J}) dV = \int_V -\frac{\partial \rho_V}{\partial t} dV$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_V}{\partial t}}$$

eqⁿ of current continuity.

$$\vec{J} = \sigma \vec{E}$$

from gauss law; $\nabla \cdot \vec{D} = \rho_V$

$$\nabla \cdot \epsilon \cdot \vec{E} = \rho_V$$

$$\epsilon_0 \cdot \nabla \cdot \vec{E} = \rho_V \quad [\text{for homogeneous region } \epsilon \text{ is constant}]$$

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho_V}{\epsilon_0}}$$

$$\nabla \cdot \delta \vec{E} = -\frac{\partial \rho_V}{\partial t} \quad \delta \nabla \cdot \vec{E} = -\frac{\partial \rho_V}{\partial t}$$

$$\delta \frac{\rho_V}{\epsilon_0} = -\frac{\partial \rho_V}{\partial t} \quad \checkmark$$

$$\boxed{\frac{\partial \rho_V}{\partial t} + \frac{\delta \rho_V}{\epsilon_0} = 0}$$

$$\text{or } \boxed{\frac{\partial \rho_V}{\partial t} = -\frac{\delta \rho_V}{\epsilon_0}}$$

$$\boxed{\frac{\partial \rho_V}{\rho_V} = -\frac{\delta \rho_V}{\epsilon_0 \rho_V}}$$

Taking integration both sides

$$\int \frac{\sigma f_v}{f_v} = -\frac{\sigma}{\epsilon_0} \int \sigma t \quad \rightarrow \int \sigma dt$$

$$\ln(f_v) = -\frac{\sigma}{\epsilon_0} t + C$$

let $t=0$ $f_v = f_{v_0}$

$$\boxed{\ln f_{v_0} = C}$$

$$\ln f_v = -\frac{\sigma}{\epsilon} t + \ln f_{v_0}$$

$$\ln\left(\frac{f_v}{f_{v_0}}\right) = -\frac{\sigma}{\epsilon} t$$

Taking exponential both sides

$$\frac{f_v}{f_{v_0}} = e^{-\sigma/\epsilon t}$$

$$f_v = f_{v_0} e^{-\sigma/\epsilon t}$$

$$f_v = f_{v_0} e^{-t/\tau}$$

$$\boxed{f_v = f_{v_0} e^{-t/\tau}}$$

$$\boxed{\tau = \frac{\epsilon}{\sigma}}$$

Relaxation time /

Rearrangement time

$$\tau = t$$

$$f_v = f_{v_0} e^{-t/\tau} = 0.373 f_{v_0}$$

Relaxation Time - It is the time taken to reduce the charge density 37.7% of initial charge density.

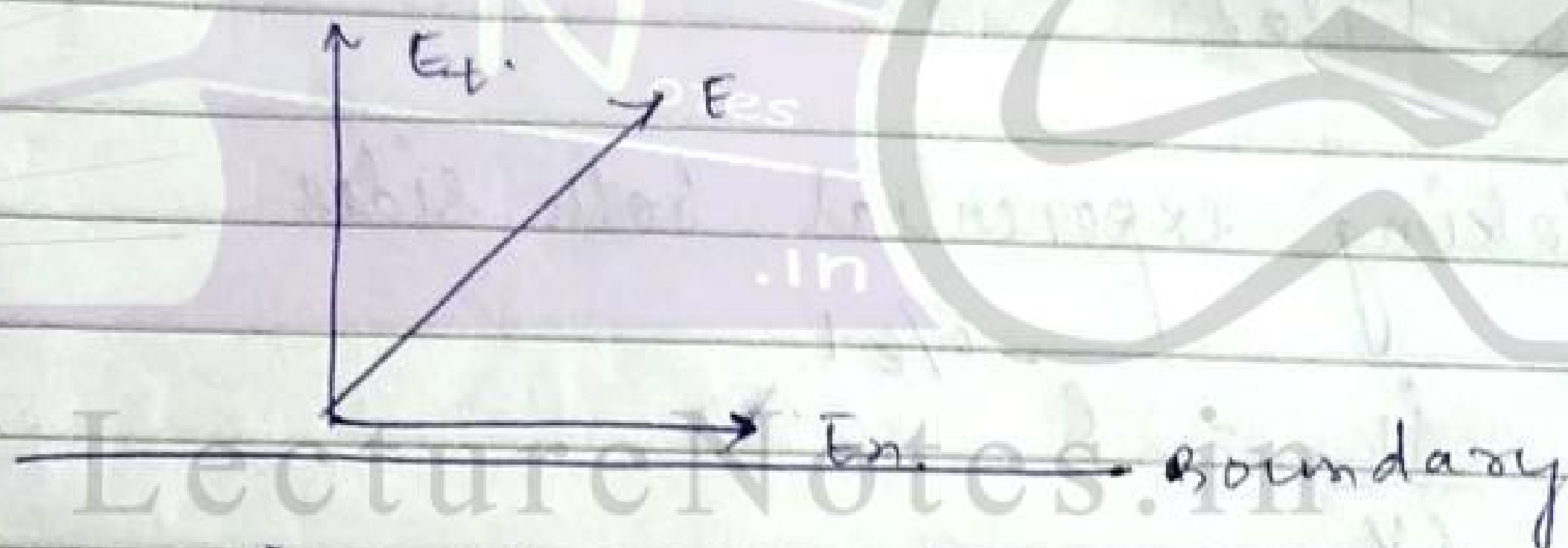
for conductor

$$\tau = \frac{\epsilon_0}{\sigma} \quad \sigma \gg \gg (\text{high}) \quad \tau \ll \ll (\text{low})$$

Boundary condition

- ① Dielectric - Dielectric condition medium.
- ② conductor - Dielectric
- ③ conductor - free space.

$$\vec{E} = \vec{E}_n + \vec{E}_t \quad \vec{E}_2 = \vec{E}_{2n} + \vec{E}_{2t}$$



$$\vec{D}_1 = \vec{D}_{1t} + \vec{D}_{1n}$$

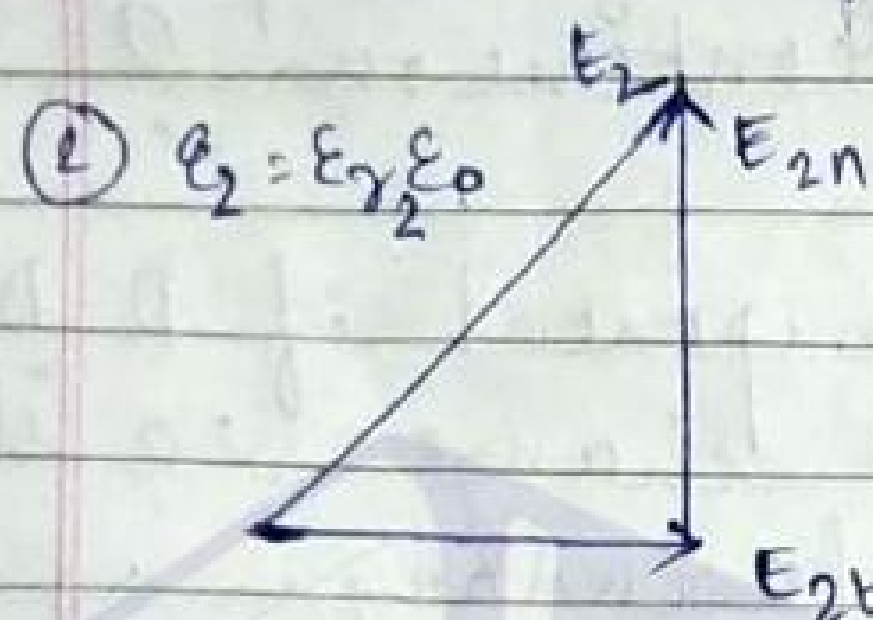
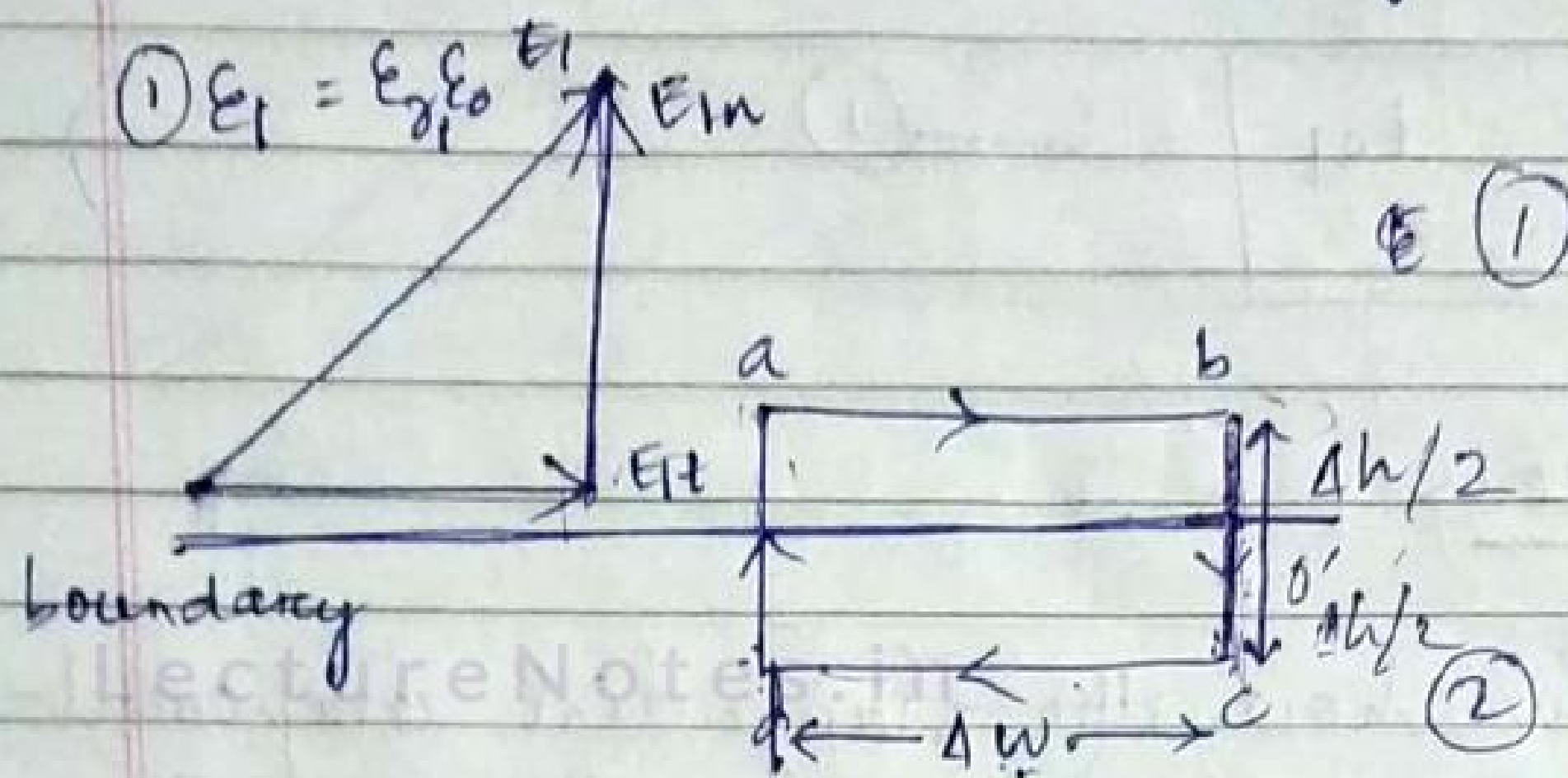
$$\vec{D}_2 = \vec{D}_{2t} + \vec{D}_{2n}$$

$$\textcircled{1} \oint_L \vec{E} \cdot d\vec{l} = 0$$

$$\textcircled{2} \nabla \cdot \vec{D} = \rho_v$$

$$\textcircled{3} \psi = \int \vec{D} \cdot d\vec{L}$$

06.03.19

Boundary condition(1) Dielectric - Dielectric Boundary

$$\vec{E}_1 = E_{1t} + E_{1n}$$

$$\vec{E}_2 = E_{2t} + E_{2n}$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

abcd

$$\Rightarrow \int_a^b \vec{E} \cdot d\vec{l} + \int_b^c \vec{E} \cdot d\vec{l} + \int_c^d \vec{E} \cdot d\vec{l} + \int_d^a \vec{E} \cdot d\vec{l} = 0$$

$$= \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b (E_{1t} + E_{1n}) \cdot d\vec{l}$$

$$[\vec{a} \cdot \vec{b} = (a||b| \cos \theta)]$$

$$= \int_a^b E_{1t} \cdot dl + 0$$

$\Delta w = \frac{\Delta h}{2}$

$$= \boxed{E_{1t} \cdot \Delta w}$$

$$\Rightarrow \cancel{E_{1t} \Delta w} - \cancel{E_{1n} \Delta h/2} - \cancel{E_{2t} \Delta w} + \cancel{E_{2n} \Delta h/2}$$

$$\Rightarrow \underbrace{E_{1t} \Delta w}_{ab} - \underbrace{E_{1n} \Delta h/2}_{bc} - \underbrace{E_{2t} \Delta w}_{cd} + \underbrace{E_{2n} \Delta h/2 + E_{1n} \Delta h/2}_{ad}$$

$$\Rightarrow E_{1t} \Delta W - E_{2t} \Delta W = 0 \Rightarrow E_{1t} \cancel{\Delta W} = E_{2t} \cancel{\Delta W}$$

$$\Rightarrow \boxed{E_{1t} = E_{2t}} \quad \text{--- (1)}$$

continuous

Q Explain how the tangential component of D-D medium is continuous?

Ans Since the tangential component of D-D medium undergoes no change so it is continuous at the boundary.

$$D = \epsilon E$$

$$\vec{D}_1 = \vec{D}_{1t} + \vec{D}_{1n}$$

$$\vec{D}_2 = \vec{D}_{2t} + \vec{D}_{2n}$$

$$\boxed{E_{1t} = \frac{D_{1t}}{\epsilon_1}}$$

$$\boxed{E_{2t} = \frac{D_{2t}}{\epsilon_2}}$$

applying above eqⁿ in eqⁿ (1)

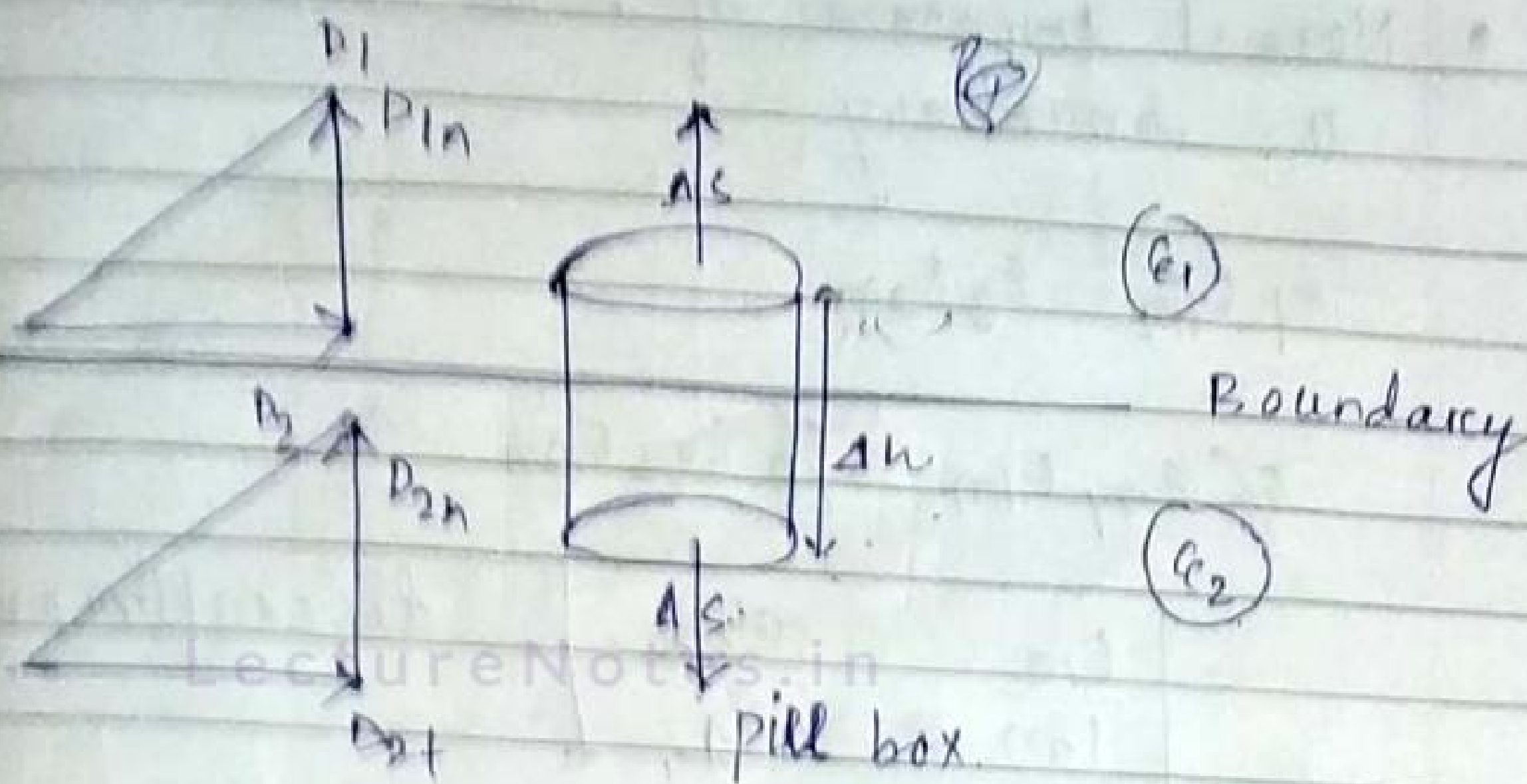
$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

$$\frac{D_{1t} \cancel{\epsilon_2}}{\cancel{\epsilon_2} \epsilon_1} = \frac{D_{2t}}{\cancel{\epsilon_2} \epsilon_2}$$

$$\boxed{\frac{D_{1t}}{D_{2t}} = \frac{\epsilon_1}{\epsilon_2}}$$

note - BPOT
• tangential component of $\vec{D}$ (electric flux density) depends

upon the permittivity of the medium hence it is discontinuous at the boundary.



Gauss law

$$\oint \vec{D} \cdot d\vec{s} = Q_{enc}$$

$$\left[\int_{top} + \int_{bottom} + \int_{surface} \right] \vec{D} \cdot d\vec{s} = Q_{enc}$$

$\Delta h \rightarrow 0$

$$D_{1n} \cdot \Delta s - D_{2n} \cdot \Delta s = Q_{enc} \quad d\vec{s} = \Delta s \cdot \vec{n}$$

$$D_{1n} - D_{2n} = \frac{Q_{enc}}{\Delta s}$$

$$D_{1n} - D_{2n} = \rho_s$$

where $\rho_s = \frac{Q_{enc}}{\Delta s}$ = surface charge density

C/m^2

Assuming charge free boundary.

$$\rho_s = 0$$

$$D_{1n} - D_{2n} = 0$$

$$\Rightarrow D_{1n} = D_{2n}$$

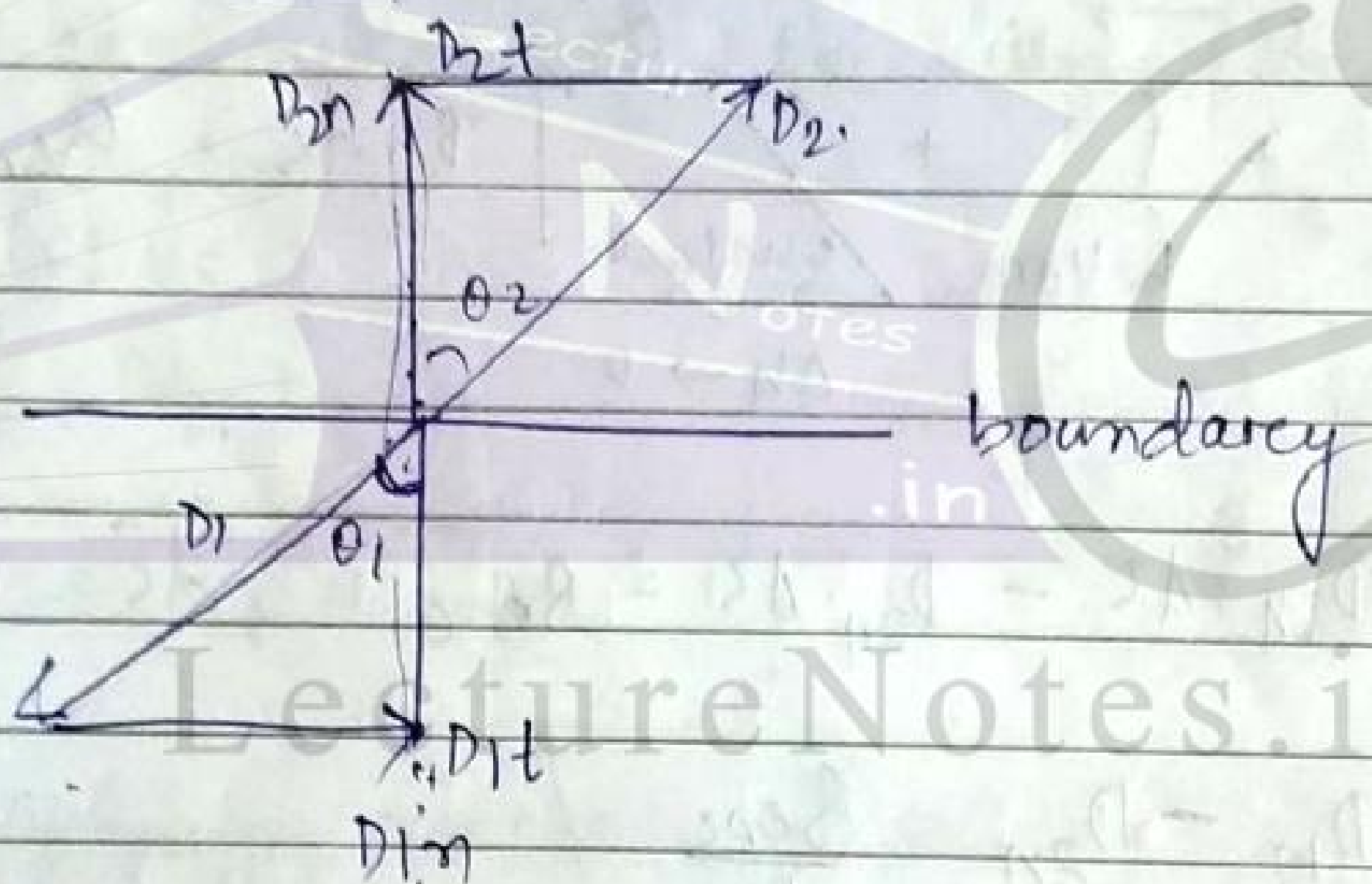
- Normal component of $\vec{D}$ is continuous at the boundary.

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\epsilon_0 \epsilon_{r1} E_{1n} = \epsilon_0 \epsilon_{r2} E_{2n}$$

$$\left[\begin{array}{cc} \frac{E_{1t}}{E_{2t}} & \frac{\epsilon_{r2}}{\epsilon_{r1}} \end{array} \right] \text{ discontinuous.}$$

$$\left[\begin{array}{cc} \frac{D_{1t}}{D_{2t}} & \frac{\epsilon_{r1}}{\epsilon_{r2}} \end{array} \right] \checkmark$$

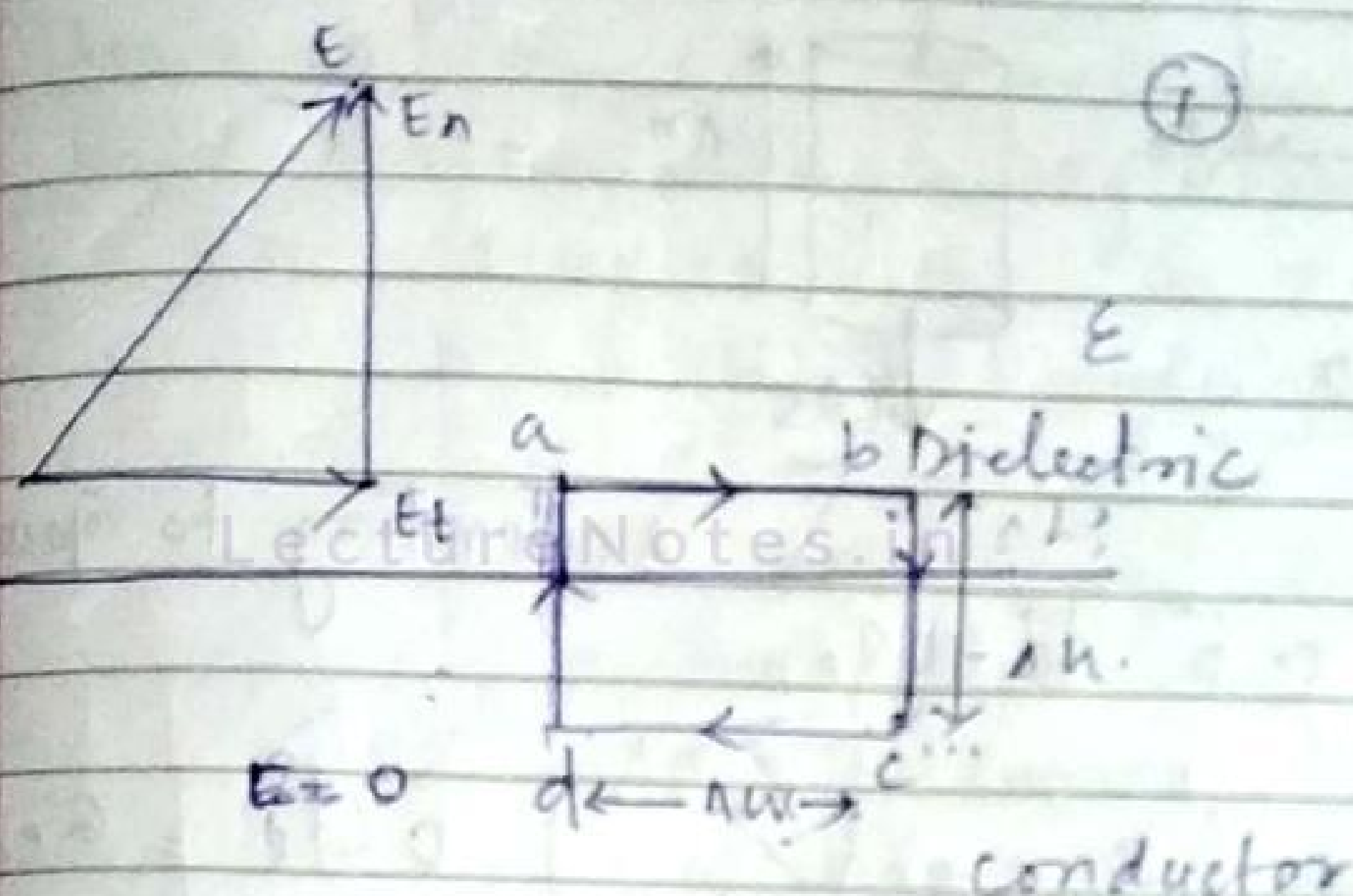


NORMAL

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{D_{1t}}{D_{1n}} \times \frac{D_{2n}}{D_{2t}} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

$$[D_{1n} = D_{2n}] \text{ when } \rho_s = 0$$

② Conductor - dielectric medium



$$\oint \vec{E} \cdot d\vec{l} = 0$$

abcd.

$$\int_a^b \vec{E} \cdot d\vec{l} + \int_b^c \vec{E} \cdot d\vec{l} + \int_c^d \vec{E} \cdot d\vec{l} + \int_d^a \vec{E} \cdot d\vec{l} = 0$$

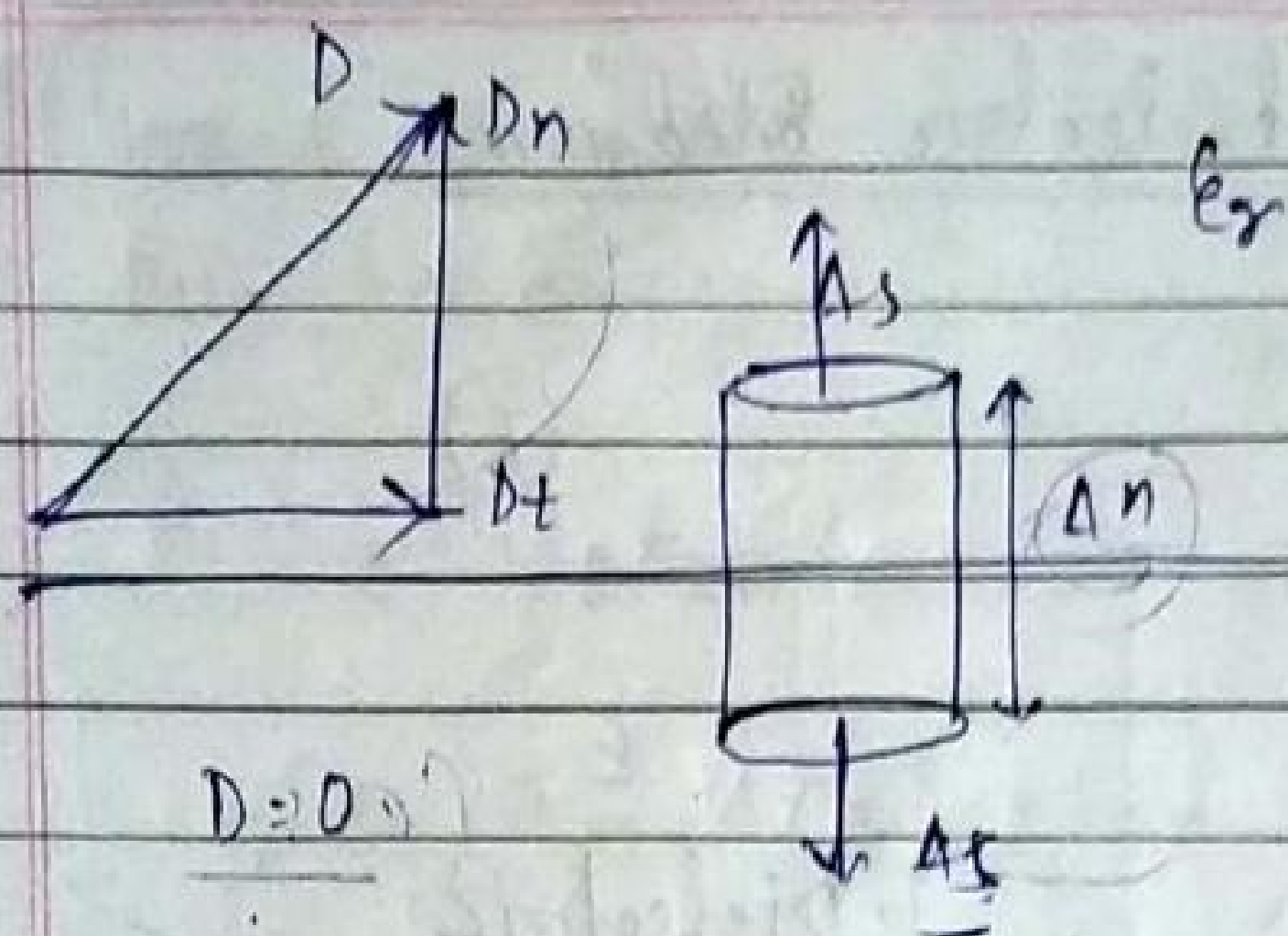
$$E_t \cdot \Delta w - E_n \frac{\Delta h}{2} + 0 + 0 + E_n \frac{\Delta h}{2} = 0$$

$$E_t \Delta w = 0$$

$$E_t = 0$$

$$D_t = 0$$

There is no tangential component of conductor - dielectric medium does not exist.



$$D = D_n$$

Lecture Notes in According to Gauss law

$$\Psi = \oint \vec{D} \cdot d\vec{s} = Q_{enc}$$

$$\left[\int_{\text{top}} + \int_{\text{Bottom}} + \int_{\text{surface}} \right] \vec{D} \cdot d\vec{s} = Q_{enc}$$

$D \rightarrow 0$ $\Delta h \rightarrow 0$

$$D_n \Delta s = Q_{enc}$$

$$D_n = \frac{Q_{enc}}{\Delta s} = \rho_s \quad \boxed{D_n = \rho_s}$$

$$\epsilon_r E_n = \rho_s$$

$$\boxed{E_n = \frac{\rho_s}{\epsilon_r}}$$

③ conductor - free space

$$\epsilon_1 E_n = \rho_s$$

$$E_n = \frac{\rho_s}{\epsilon_1}$$

$$\epsilon_1 = \epsilon_0 \epsilon_r$$

free space

$$\epsilon_r = 1$$

$$\boxed{E_n = \frac{\rho_s}{\epsilon_0}}$$

1319 Numerical

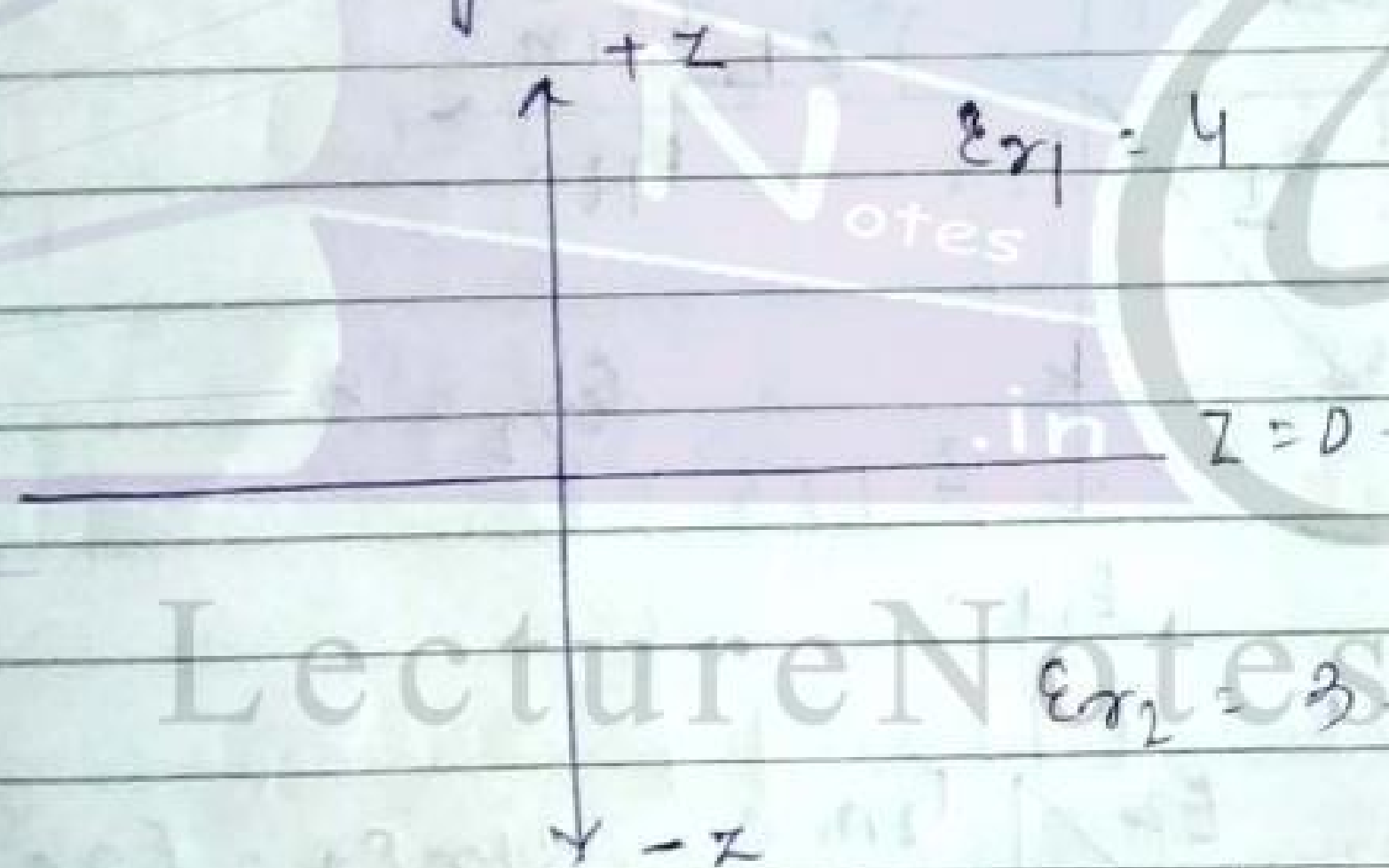
① Two extensive homogeneous isotropic dielectric meet on plane $z=0$. for $z > 0$, $\epsilon_{r1} = 4$, for $z < 0$, $\epsilon_{r2} = 3$. An uniform electric field $E_1 = 5a_x - 2a_y + 3a_z$ kV/m exists for $z > 0$. Find (a) E_2 for $z < 0$,

(b) The angles E_1 & E_2 meet with the interface.

(c) The energy densities in both dielectric

(d) The energy with a cube of side 2m. centred at $(3, 4, -5)$

$$E_1 = 5a_x - 2a_y + 3a_z \text{ kV/m.}$$



For a dielectric - dielectric boundary tangential is continuous.

$$(a) \quad E_1 = E_{1t} + E_{1n} \quad E_{1t} = E_1 - E_{1n}$$

$$E_{1n} = 3a_z \text{ kV/m} \quad E_{1t} = 5a_x - 2a_y \text{ kV/m}$$

$$E_{1t} = E_{2t} = 5a_x - 2a_y \text{ kV/m}$$

$$\frac{E_{2n}}{E_{1n}} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

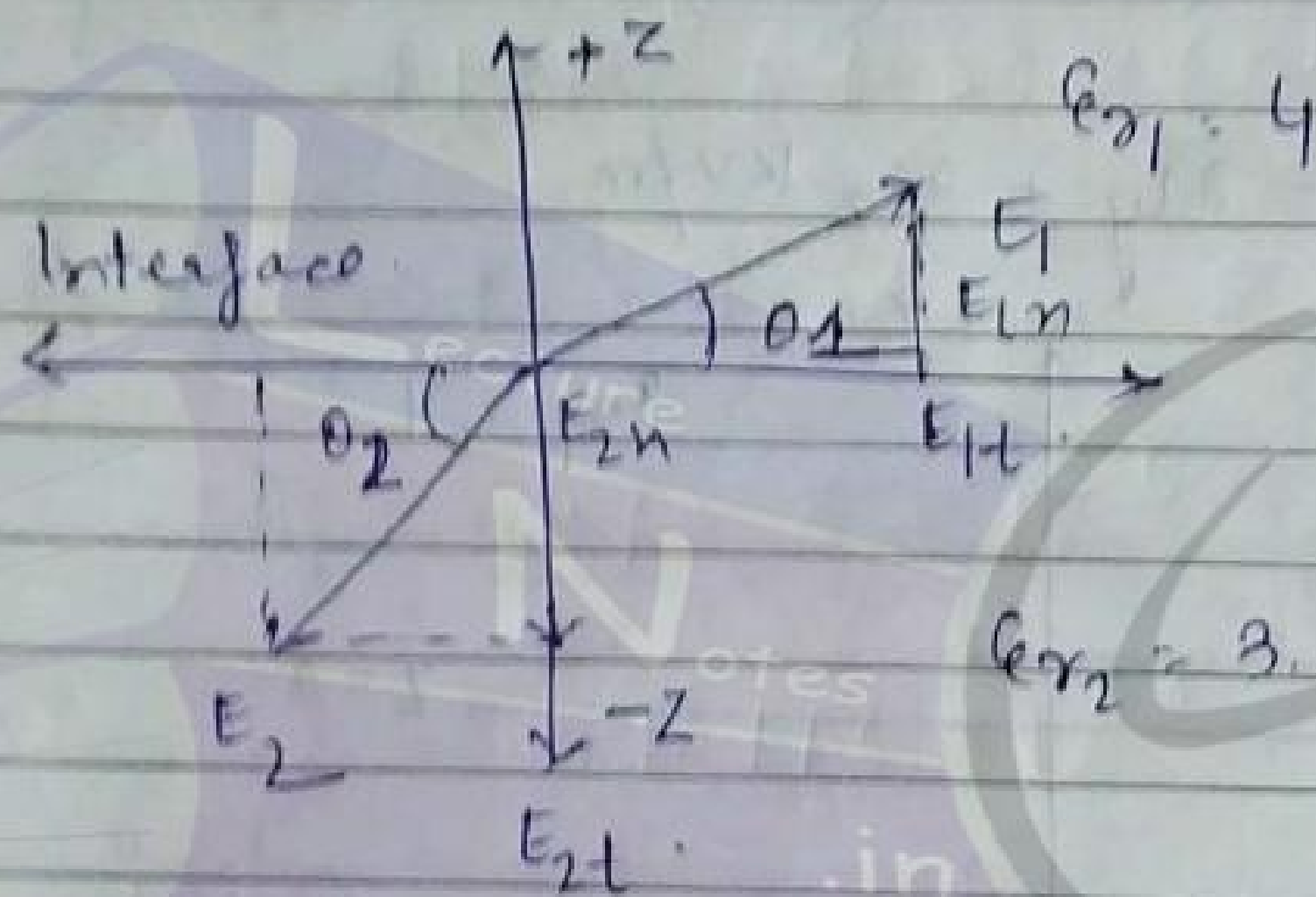
$$E_{2n} = \frac{4}{3} \times 3a_z$$

$$E_{2n} = 4a_z \text{ kV/m}$$

$$E_2 = E_{2t} + E_{2n}$$

$$E_2 = 5a_z - 2a_y + 4a_z \text{ kV/m}$$

(b)



$$\sin \theta_2 = \frac{E_{2n}}{E_2}$$

$$\cos \theta_2 = \frac{E_{2t}}{E_2}$$

$$\theta_2 = \sin^{-1} \frac{(4)}{\sqrt{(5)^2 + (-2)^2 + (4)^2}} = 36.6^\circ$$

$$\theta_2 = 36.6^\circ$$

$$\sin \theta_1 = \frac{|E_{in}|}{|E|} = \frac{3}{\sqrt{(5)^2 + (2)^2 + (3)^2}}$$

$$\theta_1 = \sin^{-1} \frac{3}{\sqrt{(5)^2 + (2)^2 + (3)^2}}$$

$$\theta_1 = 29.12^\circ$$

$$(c) \quad W_{E1} = \frac{1}{2} \epsilon_1 E_1^2 = \frac{1}{2} \times 8.854 \times 10^{-12} \times 4 \times$$

$$\epsilon_1 = \epsilon_0 \epsilon_{r1} \quad \left(\sqrt{(5)^2 + (2)^2 + (3)^2} \right)^2 \times 10^6$$

$$W_{E1} = 6.72 \times 10^{-7}$$

$$W_{E1} = 672 \text{ nJ/m}^3$$

LectureNotes.in

$$W_{E2} = \frac{1}{2} \epsilon_2 E_2^2$$

$$= \frac{1}{2} \times 8.854 \times 10^{-12} \times 3 \times (5)^2 + (2)^2 + (4)^2 \times 10^6$$

$$= 5.97 \times 10^{-7}$$

$$W_{E2} = 597 \text{ nJ/m}^3$$

$$(1) W = \int_V \rho_v \cdot dV$$

$$= \int 597 \times 10^{-6} dx dy dz$$

$$= 597 \times 10^{-6} \int_2^4 dx \int_3^5 dy \int_{-6}^{-4} dz$$

$$= 597 \times 10^{-6} [x]_2^4 \times [y]_3^5 \times [z]_{-6}^{-4}$$

$$= 597 \times 10^{-6} (4-2) \times (5-3) \times (-4+6)$$

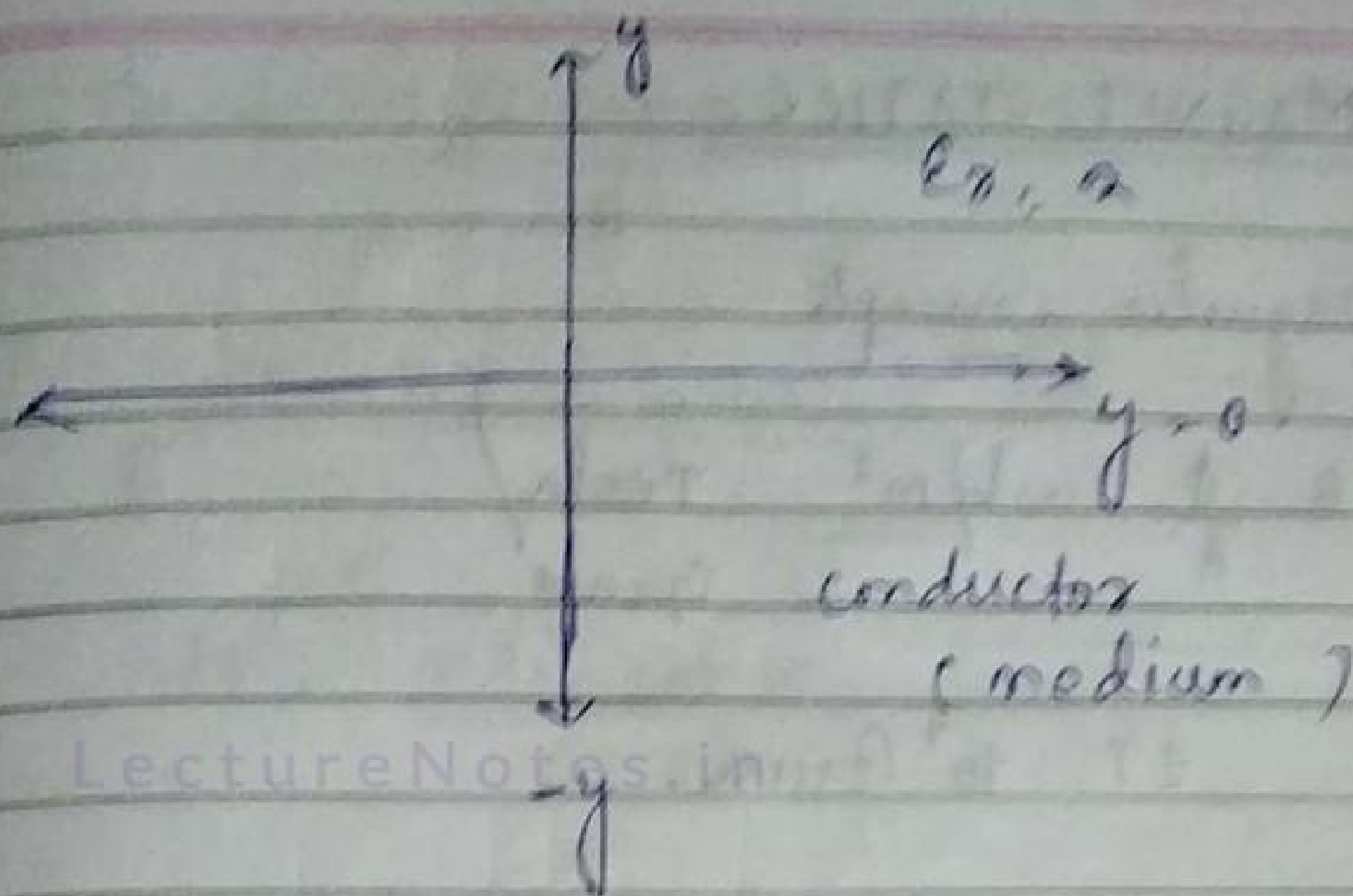
$$= 597 \times 10^{-6} \times 2 \times 2 \times (2)$$

$$= 597 \times 8 \times 10^{-6}$$

$$W = 4.776 \times 10^{-3}$$

$$\boxed{W = 4.776 \text{ mJ}}$$

- (2) If $y < 0$, consist of a perfect conductor while region $y > 0$ is a dielectric medium with $\epsilon_r = 2$. If there is a surface charge of 2 mC/m^2 on the conductor. Determine $\vec{E}$ & $\vec{D}$ at-
- (a) A(3, -2, 2)
- (b) B(-4, 1, 5).



(a) A (2, -2, 2) (taking y axis)

$$\vec{E} = 0 = \vec{D}$$

(b) B (-4, 1, 5)

$$D_m = \rho_s = 2 \text{ nC/m}^2$$

$$\boxed{\vec{D} = 2a_y \text{ nC/m}^2} \text{ in}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r}$$

$$\vec{E} = \frac{2 \times 10^{-9}}{8.854 \times 10^{-12} \times 2}$$

$$\vec{E} = 112.94 \approx 113.1 a_y \text{ V/m}$$

(3)

2.08.19.

MAGNETOSTATICS.

Magnetic concepts

$$B = \frac{\Phi}{A} \quad \text{wb/m}^2 \quad \text{Tesla/Gauss}$$

$$1 \text{ T} = 10^4 \text{ Gauss}$$

B & H (relation)

$$B = \mu H.$$

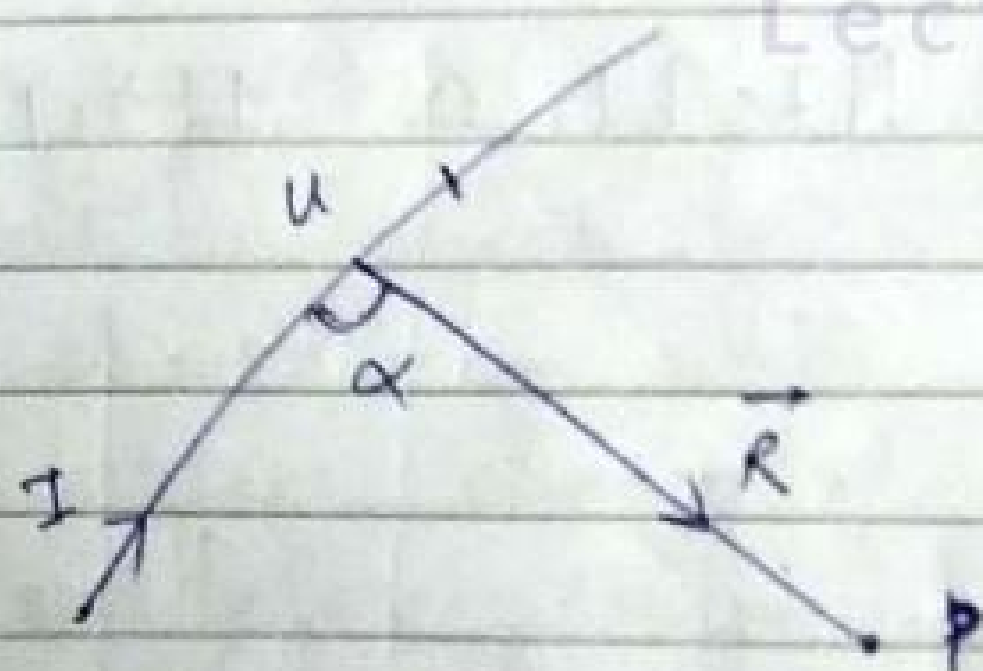
$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 4\pi \times 10^{-7}$$

μ_r = relative permeability

Biot-Savart's Law

The magnetic field H



$$dH \propto Idl$$

$$\propto \sin \alpha$$

$$\propto \frac{1}{|R|^2}$$

$$dH \propto \frac{I dl \sin \alpha}{R^2}$$

$$dH = \frac{k I dl \sin \alpha}{R^2}$$

$$k = \frac{1}{4\pi}$$

$$dH = \frac{I \vec{dl} \sin \alpha}{4\pi R^2}$$

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \alpha$$

$$dl \sin \alpha = \vec{dl} \times \vec{a_r} = |\vec{dl}| |\vec{a_r}| \sin \alpha$$

$$= dl \sin \alpha$$

$$|\vec{a_r}| = \frac{1}{|\vec{R}|}$$

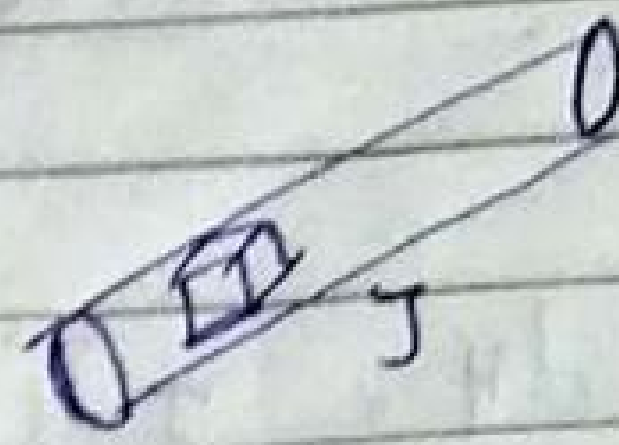
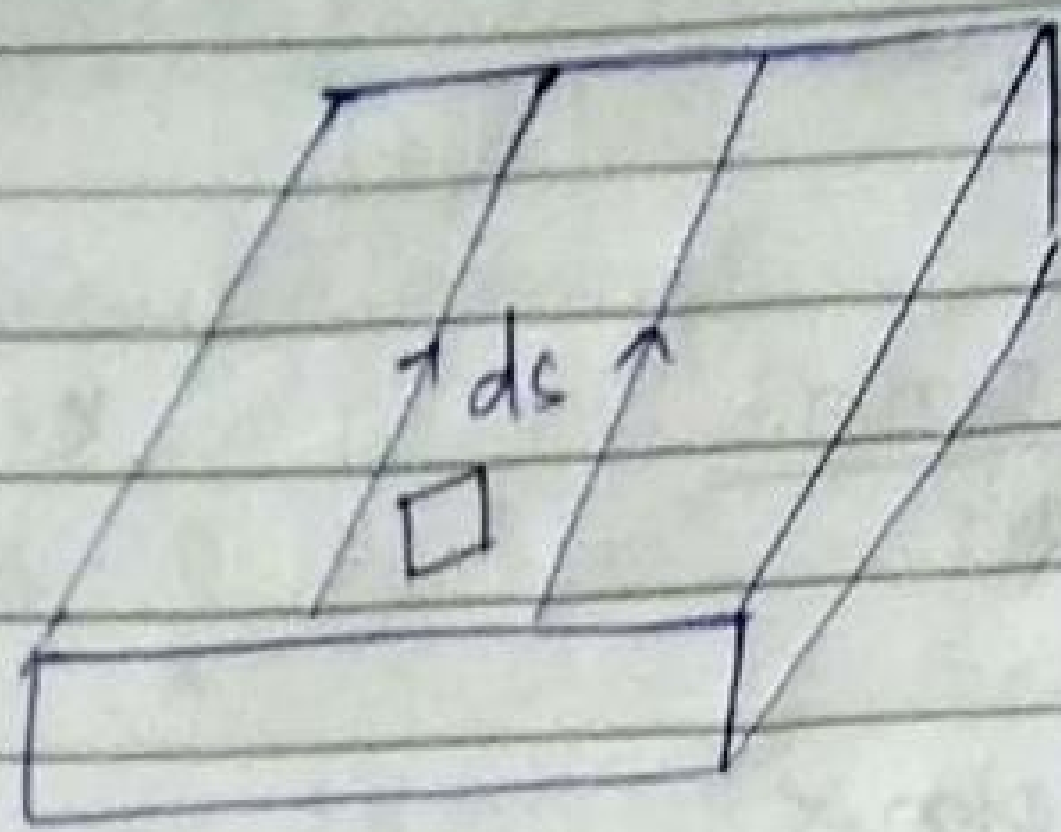
$$d\vec{H} = \frac{I \vec{dl} \times \vec{a_r}}{4\pi R^2}$$

$$\boxed{d\vec{H} = \frac{I \vec{dl} \times \vec{R}}{4\pi R^3} a_r}$$

$$\vec{H} = \int \frac{I \vec{dl} \times \vec{R}}{4\pi R^3} a_r$$

k = surface current density

y = volume current density



$$I dl = k ds = j dv$$

$$\vec{H} = \int_s \frac{k ds \times \vec{R}}{4\pi R^3}$$

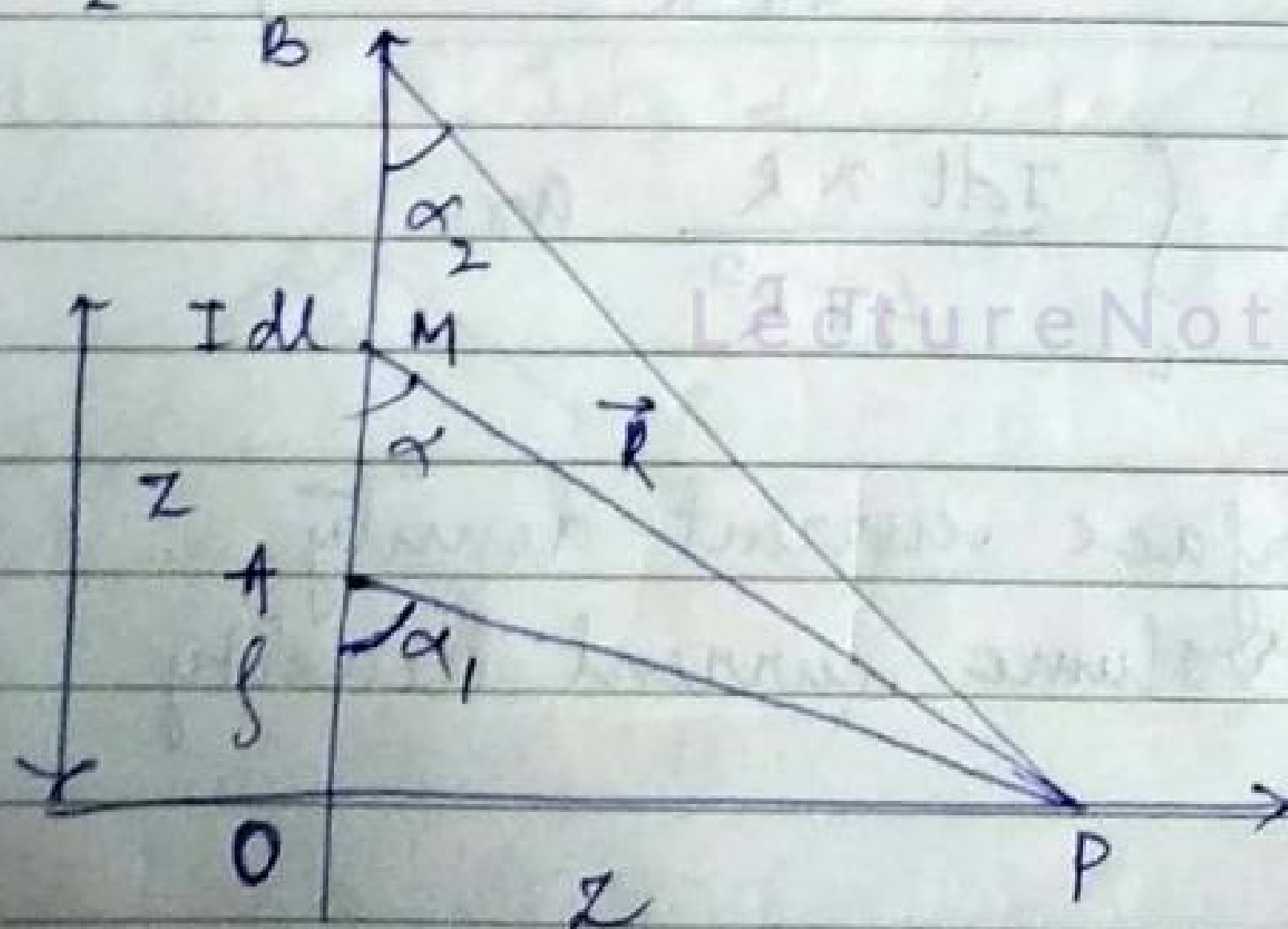
surface charge

$$\vec{H} = \int_v \frac{J dv \times \vec{R}}{4\pi R^3}$$

volume charge

Magnetic Field Intensity due to a straight filamentary conductor.

consider a current carrying element $I dl$ at a distance $(0, 0, z)$



$$\boxed{d\vec{l} = dz \hat{a}_z}$$

$$\vec{OP} = f \hat{a}_y$$

$$\vec{OM} = z \hat{a}_z$$

$$\vec{OM} + \vec{MP} = \vec{OP}$$

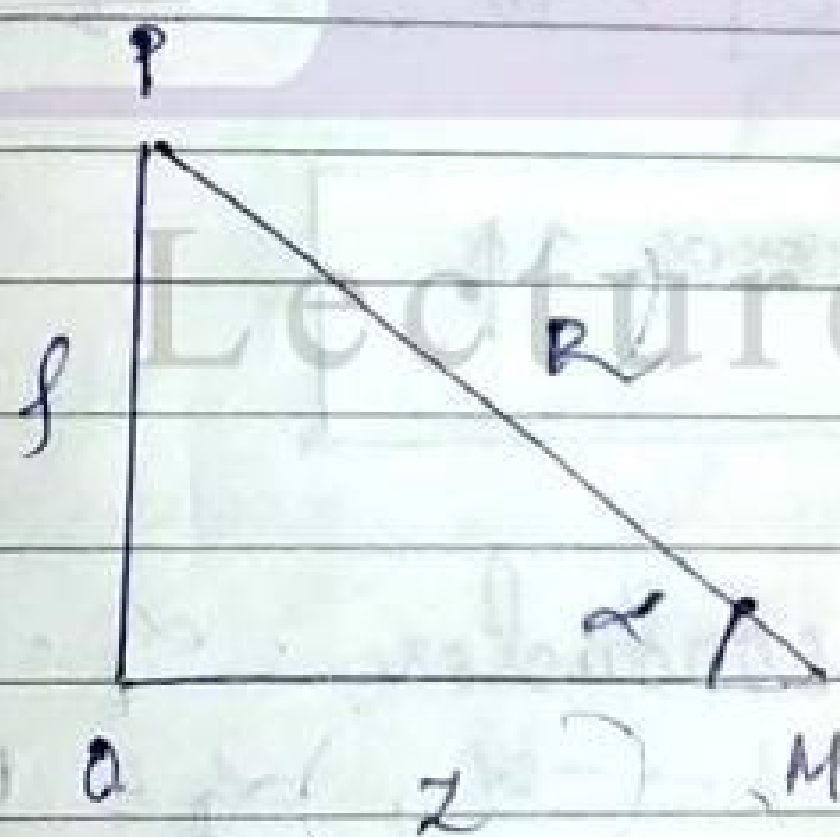
$$\vec{MP} = \vec{OP} - \vec{OM}$$

$$\boxed{\vec{R} = f \hat{a}_y - z \hat{a}_z}$$

$$\vec{H} = \int \frac{I d\vec{l} \times \vec{R}}{4\pi R^3}$$

$$\begin{aligned} d\vec{l} \times \vec{R} &= dz \hat{a}_z \times (f \hat{a}_y - z \hat{a}_z) \\ &= dz f \hat{a}_\phi \propto \theta \end{aligned}$$

$$|\vec{R}| = \sqrt{f^2 + z^2}$$



$$\cot \alpha = \frac{z}{f}$$

$$z = f \cot \alpha$$

$$dz = -f \operatorname{cosec}^2 \alpha d\alpha$$

$$\begin{aligned} |\vec{R}| &= \sqrt{f^2 + z^2} = \sqrt{f^2 + f^2 \cot^2 \alpha} = \\ &= \sqrt{f^2 (1 + \cot^2 \alpha)} \end{aligned}$$

$$|\vec{R}| = \sqrt{f^2 \operatorname{cosec}^2 \alpha} = f \operatorname{cosec} \alpha$$

$$\vec{H} = \int \frac{I dz}{4\pi R^3} a\phi$$

$$= \frac{-I}{4\pi} \int \frac{1}{f^2 \csc^2 \alpha} \csc^2 \alpha d\alpha$$

$$= \frac{-I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{1}{\csc \alpha} d\alpha$$

$$\vec{H} = \frac{-I}{4\pi f} \int_{\alpha_1}^{\alpha_2} \sin \alpha d\alpha$$

$$\boxed{\vec{H} = \frac{I}{4\pi f} [\cos \alpha_2 - \cos \alpha_1] a\phi}$$

For a semi infinite conductor ...

A → origin

$$\alpha_1 = 90^\circ \quad \alpha_2 = 0^\circ$$

$$\boxed{\vec{H} = \frac{I}{4\pi f} a\phi}$$

For an infinite conductor · $\alpha_1 = 180^\circ$

$$A \rightarrow -\infty, \quad \alpha_2 = 0^\circ$$

$$B \rightarrow \infty$$

$$\boxed{\vec{H} = \frac{I}{2\pi f} a\phi}$$

f = let distance from the current carrying conductor to the point P.

$$\vec{a}_p = \vec{a}_1 \times \vec{a}_2$$

$$\vec{a}_1 = \vec{a}_1 \times \vec{a}_1$$

$\vec{a}_1$ = unit vector along which current is flowing

$\vec{a}_2$ = unit vector along 1 m distance from the current carrying conductor to the point P.

Ampere Circuital Law

It states that the line integral of $\vec{H}$ along a closed path is the same as the net current I enclosed by the path.

~~According to Stokes theorem~~

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}}$$

According to Stokes theorem

$$\oint_L \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{s}$$

$$I_{\text{enclosed}} = \int_S \vec{J} \cdot d\vec{s}$$

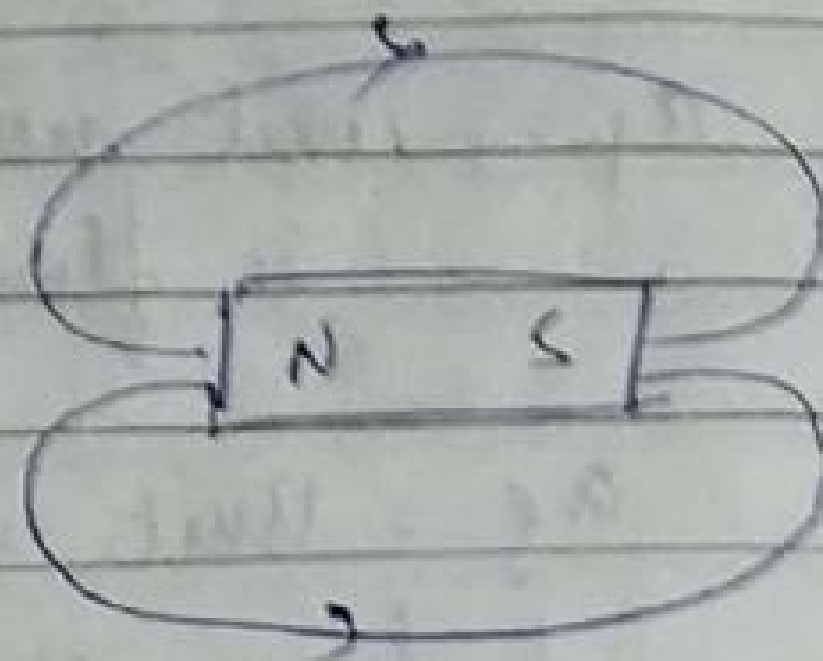
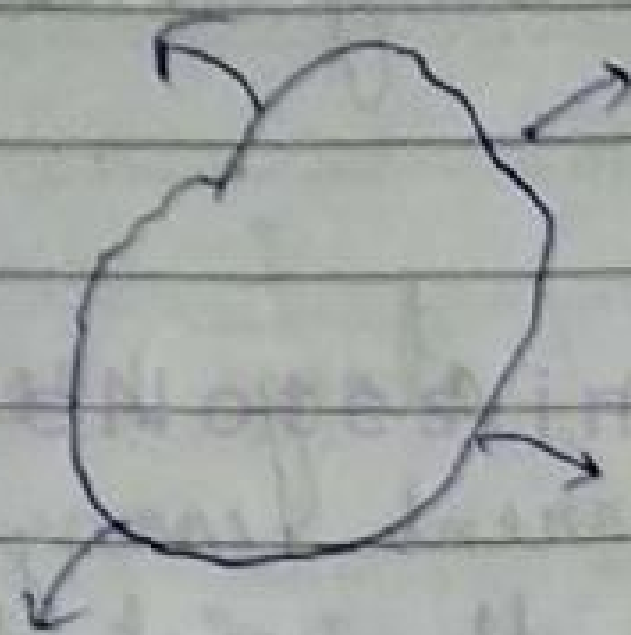
$$\int_S (\nabla \times \vec{H}) \cdot d\vec{s} = \int_S \vec{J} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

$$\boxed{\vec{\nabla} \times \vec{H} \neq 0}$$

not conservative

$$\oint_C \vec{B} \cdot d\vec{C} = 0$$



Gauss law for magnetostatics

An isolated magnetic charge does not exist, so the total ~~area~~ flux through a closed surface in a magnetic field is equal to zero.

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad \text{differential form}$$

$$\int_S (\vec{\nabla} \cdot \vec{B}) dV = 0$$

$$\oint_C \vec{B} \cdot d\vec{S} = 0 \quad \text{integral form}$$

B.03.19

Magnetic Scalar potential

$$\boxed{H = -\nabla V_m}$$

$$\boxed{J = 0}$$

V_m = Magnetic Scalar potential

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\vec{\nabla} \times (-\vec{\nabla} V_m) = 0 = \boxed{J = 0}$$

Magnetic vector potential

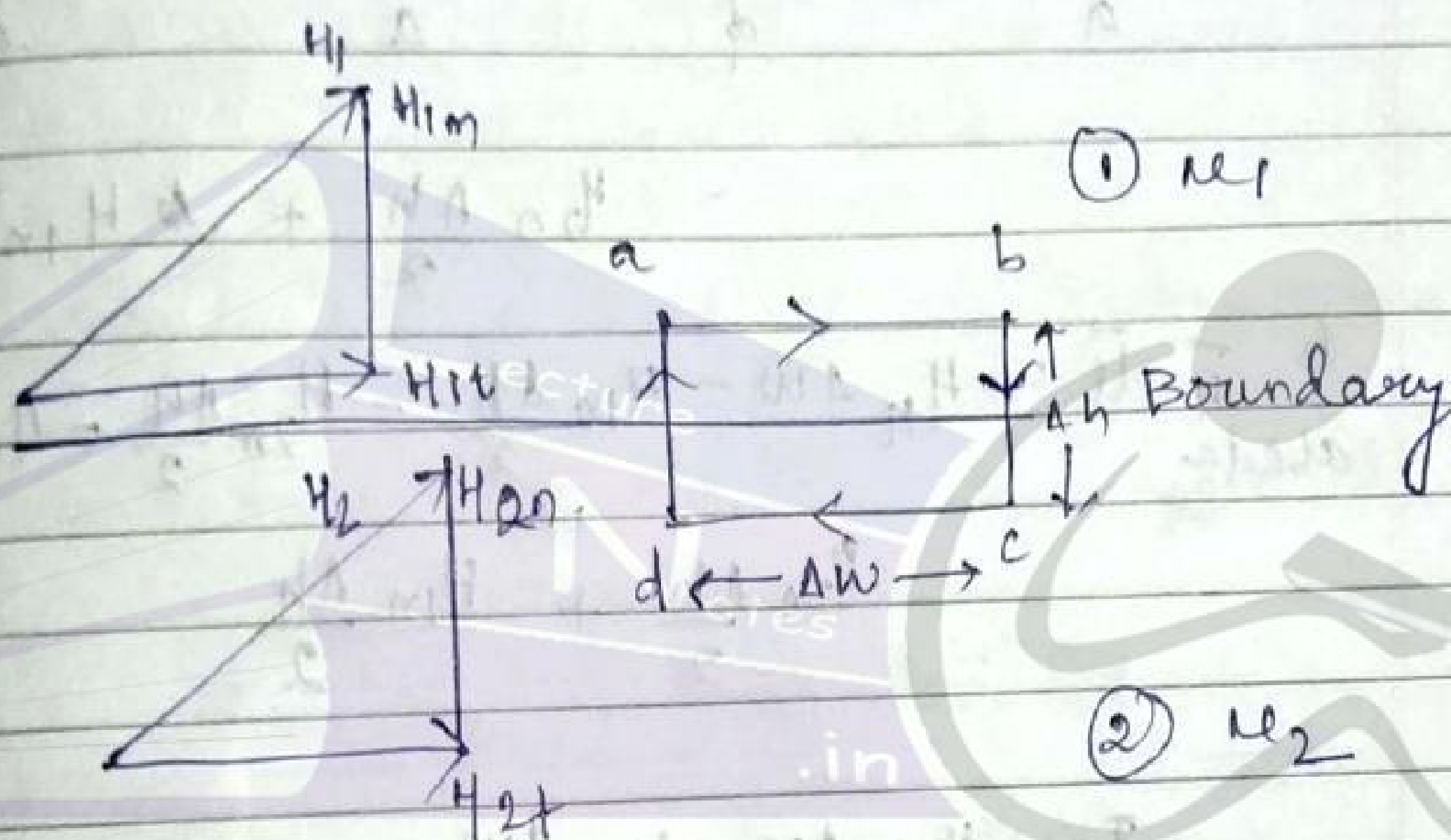
$$\nabla \times \vec{A} = \vec{B}$$

$\vec{A}$ = Magnetic vector potential (wb/m)

Boundary condition

Boundary between 2 magnetic medium

(1)



$$\vec{H}_1 = \vec{H}_{1t} + \vec{H}_{1n}$$

$$\vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n}$$

$$\oint_L \vec{H} \cdot d\vec{l} = I$$

$$\oint_{abcd} \vec{H} \cdot d\vec{l} = I$$

$$\left[\int_a^b + \int_b^c + \int_c^d + \int_d^a \right] \vec{H} \cdot d\vec{l} = I$$

$$\int_a^b \vec{H} \cdot d\vec{l} = H_{1t} \Delta w$$

$$\int_b^a \vec{H} \cdot d\vec{l} = \int_b^{b'} \vec{H} \cdot d\vec{l} + \int_{b'}^a \vec{H} \cdot d\vec{l}$$

$$= -H_{1n} \frac{\Delta h}{2} - H_{2n} \frac{\Delta h}{2}$$

$$\int_c^d \vec{H} \cdot d\vec{l} = -H_{2t} \Delta w$$

$$\int_d^c \vec{H} \cdot d\vec{l} = \int_d^{d'} \vec{H} \cdot d\vec{l} + \int_{d'}^c \vec{H} \cdot d\vec{l}$$

$$= H_{2n} \frac{\Delta h}{2} + H_{1n} \frac{\Delta h}{2}$$

$$\oint_{abca} \vec{H} \cdot d\vec{l} = H_{1t} \Delta w - H_{1n} \frac{\Delta h}{2} - H_{2n} \frac{\Delta h}{2} - H_{2t} \Delta w +$$

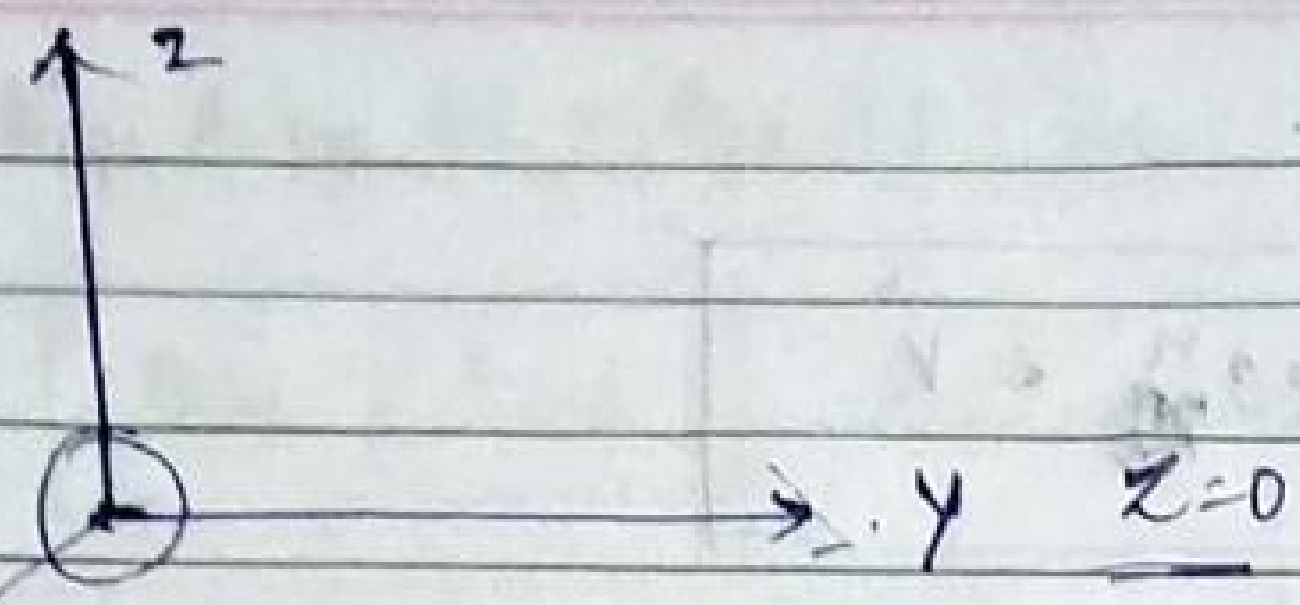
$$H_{2n} \frac{\Delta h}{2} + H_{1n} \frac{\Delta h}{2}$$

$$I = H_{1t} \Delta w - H_{2t} \Delta w$$

$$\boxed{H_{1t} - H_{2t} = \frac{I}{\Delta w}}$$

$$\frac{I}{\Delta w} = K = \text{surface current}$$

$$\boxed{H_{1t} - H_{2t} = K} \quad \text{--- (v)}$$



$$\boxed{\vec{K} = K a_x} \quad \text{--- (2)}$$

$$\vec{H}_1 = \vec{H}_{1t} + \vec{H}_{1n}$$

$$\vec{H}_1 = H_{1t} a_y + H_{1n} a_z$$

$$\vec{H}_2 = H_{2t} a_y + H_{2n} a_z$$

$$\vec{H}_1 - \vec{H}_2 = (H_{1t} - H_{2t}) a_y + (H_{1n} - H_{2n}) a_z$$

$$(\vec{H}_1 - \vec{H}_2) \times a_z = (H_{1t} - H_{2t}) a_x + 0$$

$$\cancel{(H_{1n} - H_{2n}) a_y}$$

$$\boxed{(\vec{H}_1 - \vec{H}_2) \times a_z = (H_{1t} - H_{2t}) a_x} \quad \text{--- (3)}$$

eqn (1)

multiplying a_x .

Comparing (1), (2) & (3)

$$(H_{1t} - H_{2t}) a_x = K a_x = \vec{K}$$

$$\boxed{(\vec{H}_1 - \vec{H}_2) \times a_z = \vec{K}}$$

In general.

$$(\vec{H}_1 - \vec{H}_2) \times \vec{a}_n = \vec{K}$$

$\vec{a}_n$ = normal to the boundary
for non conducting medium.

LectureNotes.in

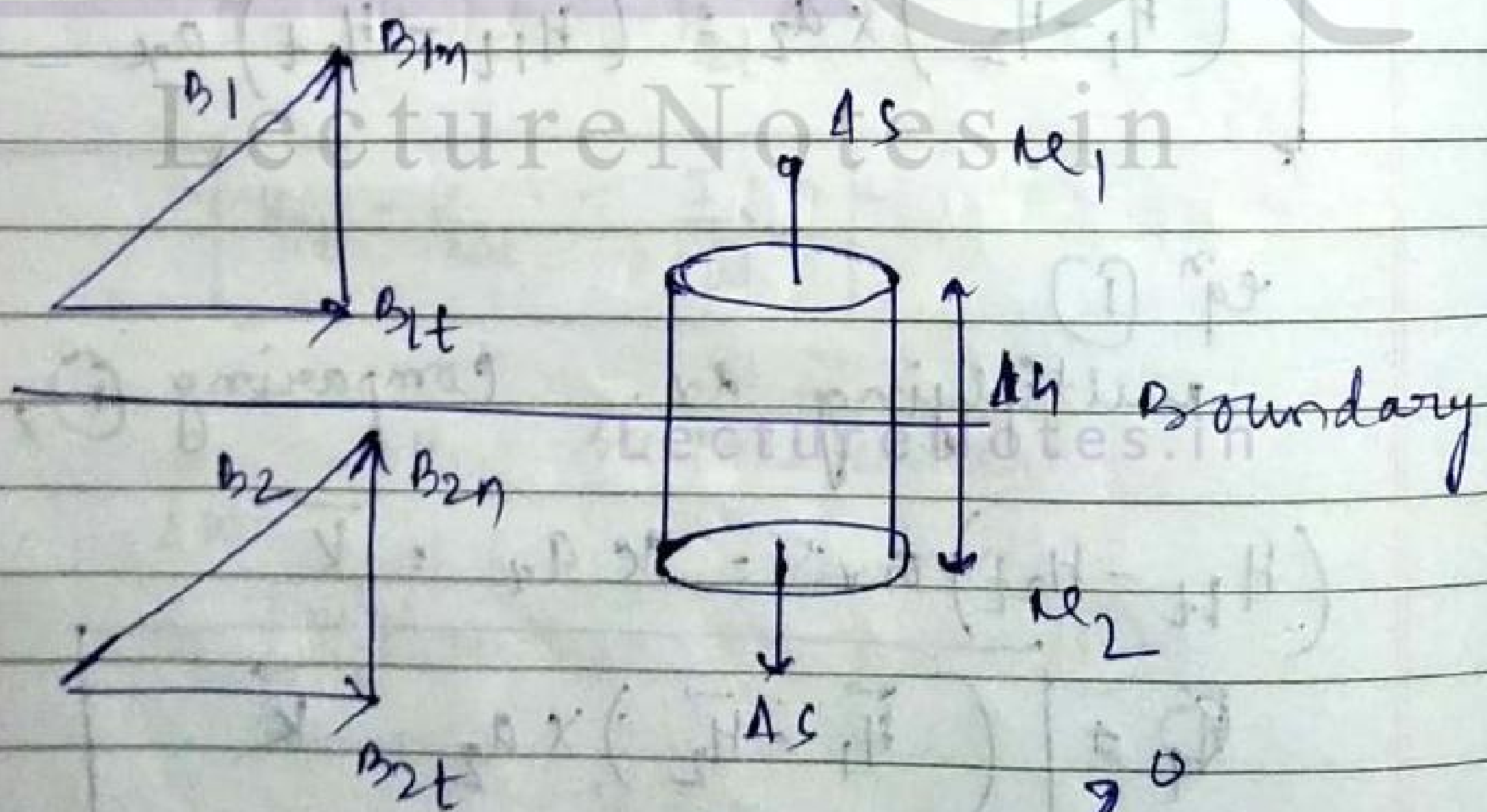
$$H_{1t} = H_{2t}$$

continuous at the boundary

$$\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

discontinuous

② $\oint_S \vec{B} \cdot d\vec{C} = 0$



$$\int_{\text{top}} \vec{B} \cdot d\vec{s} + \int_{\text{bottom}} \vec{B} \cdot d\vec{s} + \int_{\text{surface}} \vec{B} \cdot d\vec{s} = 0.$$

$\Delta h \rightarrow 0$

$$B_{1n} A_S - B_{2n} A_S = 0$$

$$\boxed{B_{1n} = B_{2n}} \quad \text{continuous}$$

$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}} \quad \text{discontinuous}$$

① Given the magnetic vector potential

$$\vec{A} = -\frac{\rho^2}{4} \hat{a}_z \text{ wb/m} \quad \text{Calculate the total}$$

magnetic flux crossing the surface

$$\phi = \phi/2, \quad 1 \leq \rho \leq 2 \text{ m} \quad \& \quad 0 \leq z \leq 5 \text{ m}$$

$$\Psi = \int_S \vec{B} \cdot d\vec{S}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & A_\phi & A_z \end{vmatrix}$$

$$\boxed{\vec{B} = \frac{\rho}{2} \hat{a}_\phi}$$

$$d\vec{S} = \rho d\phi dz \hat{a}_\rho$$

$$\Psi = \int_S \frac{\rho}{2} \rho d\phi dz$$

$$\Psi = \frac{1}{2} \int_0^2 \rho d\rho \int_0^{2\pi} d\phi \int_0^5 dz$$

$$\psi = \frac{1}{2} \left[\frac{y^2}{2} \right]^2 \left[\frac{z}{2} \right]^5$$

$$\psi = \frac{1}{2} \left(\frac{y^2}{2} - \frac{1}{2} \right) (5-0)$$

$$\psi = \frac{3}{2} \times 5$$

$$\psi = \frac{15}{4} \quad \left[\psi = 3.75 \text{ Wb} \right]$$

- ② In a current free region of electric permeability is 1 if magnetic scalar potential is given $V_m = x^2y + y^2x + z$. Calculate the value of magnitude of magnetic flux density at $H(1,1,1)$.

$$H(1,1,1) = -\nabla V_m$$

$$= -\frac{\partial}{\partial x} (x^2y + y^2x + z) \mathbf{a}_x - \frac{\partial}{\partial y} (x^2y + y^2x + z) \mathbf{a}_y - \frac{\partial}{\partial z} (x^2y + y^2x + z) \mathbf{a}_z$$

$$(1,1,1)$$

$$\mathbf{B} = \mu \mathbf{H} = 5.5 \text{ A/T}$$

5.09.19

Poisson's eqⁿ

$$\nabla \cdot \vec{D} = \rho_v \quad (1)$$

$$\vec{D} = \epsilon \vec{E} \quad (2)$$

$$\vec{E} = -\nabla V \quad (3)$$

put (2) & (3) in (1)

$$\nabla \cdot \epsilon (-\nabla V) = \rho_v$$

for homogeneous region :

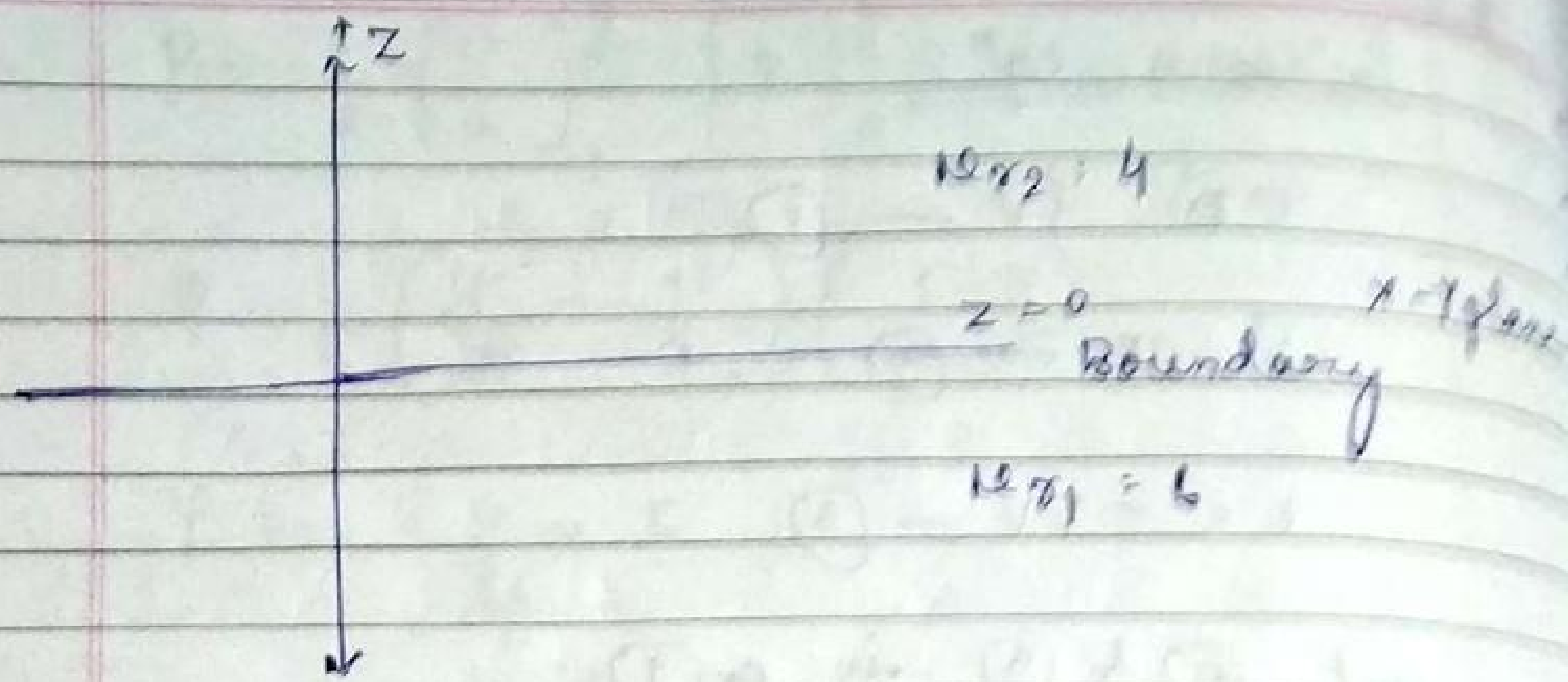
$$-\epsilon \nabla^2 V = \rho_v$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \quad \text{poisson eqⁿ}$$

For a charge free region $\rho_v = 0$.

$$\boxed{\nabla^2 V = 0} \quad \text{laplace eqⁿ}$$

- (1) The x-y plane performs at the boundary between two medium. Medium (1) ($z < 0$) is filled with material $\epsilon_1 = 6$ & medium (2) ($z > 0$) filled with material whose $\epsilon_2 = 4$. If the interface carries the current $K = 1$ A/m & $\vec{B}_2 = 5\vec{a}_x + 8\vec{a}_y$ Wb/m². Find $\vec{B}_1$ & $\vec{H}_1$.



$$\vec{B}_2 = 5a_x + 8a_z \text{ mwb/m}^2$$

$$\vec{B}_2 = \vec{B}_{2t} + \vec{B}_{2n}$$

$$\vec{B}_{2n} = 8a_z \text{ mwb/m}^2$$

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = \frac{5a_x + 8a_z}{4} \text{ mA/m}$$

$$\vec{B}_1 = B_{1x}a_x + B_{1y}a_y + B_{1z}a_z \text{ mwb/m}^2$$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{B_{1x}a_x + B_{1y}a_y + B_{1z}a_z}{6}$$

$$\vec{B}_1 = \vec{B}_{1t} + \vec{B}_{1n}$$

$$B_{1n} = B_{2n} = 8a_z \text{ mwb/m}^2$$

$$(\vec{H}_1 - \vec{H}_2) \times \hat{a}_n = \vec{K}$$

$$\vec{H}_1 \times \hat{a}_n - \vec{H}_2 \times \hat{a}_n = \vec{K}$$

$$\vec{H}_1 \times \hat{a}_n = \vec{K} + \vec{H}_2 \times \hat{a}_n$$

$$a_n = a_x$$

$$\left[\frac{B_x a_x + B_y a_y + B_z a_z}{\cancel{\mu_0} 6} \right] \times \cancel{10^{-3}} a_z = \frac{1}{\cancel{\mu_0}} a_y \times 10^{-3}$$

$$+ \left(\frac{5 a_x + 8 a_z}{4 \cancel{\mu_0}} \right) \times 10^{-3} \times a_z$$

$$\Rightarrow \frac{1}{6} [-B_x a_y + B_y a_x] \times 10^{-3} = a_y \times 10^{-3} + \left(\frac{-5}{4} \right) a_y \times \cancel{10^{-3}}$$

$$\Rightarrow \frac{1}{6} [-B_x a_y + B_y a_x] = a_y \left(1 - \frac{5}{4} \right)$$

$$\Rightarrow \frac{1}{6} \left[-\frac{B_x}{6} a_y + \frac{B_y}{6} a_x \right] = -\frac{1}{4} a_y$$

$$(x-y \text{ plane}) \quad [B_y = 0]$$

$$\cancel{\frac{B_x}{6}} a_y = \cancel{\frac{1}{4}} a_y$$

$$B_x = \frac{6}{4} = 1.5$$

$$B_x = 1.5$$

$$\boxed{\vec{B}_1 = 1.5 a_x + 8 a_z \text{ mwb/m}^2}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\vec{H}_1 = \frac{B_1}{\mu_0} = \frac{1.5 a_x + 8 a_z}{\mu_0 \cdot 6}$$

$$\boxed{\vec{H}_1 = 0.25 a_x + 1.3 a_z \text{ mA/m}}$$

In a current free region,

$$\boxed{\vec{H}_{1t} = \vec{H}_{2t}}$$

Faraday's Law of Electromagnetic Induction

LectureNotes.in

$$e \propto -\frac{d\phi}{dt}$$

$$V = \int_L \vec{E} \cdot d\vec{l}$$

$$e = -N \frac{d\phi}{dt}$$

$$-\frac{d\phi}{dt} = \oint_L \vec{E} \cdot d\vec{l}$$

$$\boxed{e = -\frac{d\phi}{dt}}$$

$$\phi = \int_S \vec{B} \cdot d\vec{s}$$

$$\oint_L \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$

$$\boxed{\oint_L \vec{E} \cdot d\vec{l} = \int_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}}$$

$$\Rightarrow \int (\nabla \times \vec{E}) \cdot d\vec{s} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

MODULE - 2

MAXWELL'S EQUATION.

(1) STATIC FIELD.

(a) Maxwells eqⁿ derived from

$$\oint_L \vec{H} \cdot d\vec{l} = I_{\text{enclosed}}$$

$$\nabla \times \vec{H} = \vec{J}$$

(b) Maxwells eqⁿ Faraday's law

$$\oint_L \vec{E} \cdot d\vec{l} = 0$$

$$\nabla \times \vec{E} = 0$$

(c) Gauss law of electrostatic

$\Psi = Q_{\text{enclosed}}$

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{\text{enc}}$$

$$\nabla \cdot \vec{D} = \rho_v$$

(d) Gauss law for magnetostatic

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\nabla \cdot \vec{B} = 0$$

19.03.19.

Maxwell's Equations for Time varying Field

(1) Derived from Ampere Circuital Law.

$$\oint_L \vec{H} \cdot d\vec{l} = I_{\text{enclosed}}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} = 0 \quad \text{--- (1)}$$

solution of continuity

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad \text{--- (2)}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \vec{J}_D}$$

modified ampere
circuital law

$\vec{J}$ = conductor current density
 $\vec{J}_D$ = displacement current density

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_D$$

$$0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_D$$

$$\nabla \cdot \vec{J}_D = -\nabla \cdot \vec{J}$$

$$\nabla \cdot \vec{J}_D = \frac{\partial \rho_v}{\partial t}$$

From Gauss law,

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \cdot \vec{J}_D = \frac{\partial}{\partial t} (\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{J}_D = \nabla \cdot \left(\frac{\partial \vec{D}}{\partial t} \right)$$

$$\boxed{\vec{J}_D = \frac{\partial \vec{D}}{\partial t}}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

modified ampere
circuitual law

② Derived from Faradays law

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

$$\int_L \vec{E} \cdot d\vec{l} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

③ Gauss Law for Electrostatics

$\psi = \phi$ enclosed

$$\oint_S \vec{D} \cdot d\vec{s} = \int_V \rho \cdot dv$$

$$\boxed{\nabla \cdot \vec{D} = \rho}$$

④ Gauss law for magnetostatics

$$\psi = \oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Statement: The net outward flux for a given closed surface is zero.
i.e magnetic monopole does not exist.

Time varying table

differential form	Integral form	Remark
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot d\vec{l} = I + I_D$	Modified ACL
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$	Faraday's law
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v \cdot dV$	Gauss law for electrostatics
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$	Gauss law for magnetostatics

Maxwell's equations for static field.

differential form	Integral form	Remark
$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = I$	ACL
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Faraday's law (Conservative)
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v \cdot dV$	Gauss law of electrostatics
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$	Gauss law of magnetostatics

Maxwells equations for free space : ($\epsilon_0 = 1, \mu_0 = 1, \delta = 0, \rho_v = 0$)

differential form	Integral form	Remark
$\nabla \times \vec{H} = \vec{J}_d$	$\oint_L \vec{H} \cdot d\vec{l} = I_d$	ACL
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot d\vec{l} = -\frac{\partial \vec{B} \cdot d\vec{s}}{\partial t}$	Faradays law (Conservative)
$\nabla \cdot \vec{D} = 0$	$\oint_S \vec{D} \cdot d\vec{s} = 0$	Gauss law of electrostatics
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$	Gauss law of magnetostatics

Maxwells eqⁿ for free space in static field

differential form	Integral form	Remark
$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = I$	ACL
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Faradays law (Conservative)
$\nabla \cdot \vec{D} = 0$	$\oint_S \vec{D} \cdot d\vec{s} = 0$	Gauss law of electrostatics
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$	Gauss law of magnetostatics

(1.) For a lossy dielectric $\sigma = 5 \text{ S/m}$ & $\epsilon_r = 1$, the electric field is $\vec{E} = 100 \sin 10^{10} t$. Find $\vec{J}_c$, $\vec{J}_d$ & Frequency at which both have equal magnitudes.

$\vec{J}_c$: conductor current density

$\vec{J}_d$: displacement current density

$$\sigma = 5 \text{ S/m} \quad \vec{E} = 100 \sin 10^{10} t$$

$$\epsilon_r = 1$$

$$\vec{J}_c = \sigma \vec{E} = 5 (100 \sin 10^{10} t)$$

$$= 500 \sin 10^{10} t$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \frac{\partial (\epsilon \vec{E})}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}_d = \epsilon_0 \epsilon_r \frac{\partial [100 \sin 10^{10} t]}{\partial t}$$

$$= 8.854 \times 10^{-12} \times 1 \times 100 \times 10^{10} \cos 10^{10} t$$

$$\boxed{\vec{J}_d = 8.854 \times \cos 10^{10} t} \text{ A/m}^2$$

$$\vec{J}_c = \sigma \vec{E}$$

$$\vec{J}_d = \epsilon \frac{\partial \vec{E}}{\partial t} = j \epsilon \omega \vec{E}$$

$$\left(\frac{\partial}{\partial t} = j\omega \right)$$

$$\frac{\vec{J}_c}{\vec{J}_d} = \frac{\sigma \vec{E}}{\epsilon \omega \vec{E}} = 1$$

$$\vec{J} = \vec{J}_c + \vec{J}_d$$

$$\vec{J} = \sigma \vec{E} + j(\epsilon \omega \vec{E})$$

$$\delta \cancel{x} = \omega \epsilon \cancel{x}$$

$$\boxed{\omega = \frac{\delta}{\epsilon}}$$

~~$$\omega = \frac{5}{100 \sin 10^{10} t}$$~~

~~$$2\pi f = \frac{5}{100 \sin 10^{10} t}$$~~

~~$$f = \frac{5}{2\pi \times 100 \sin 10^{10} t}$$~~

$$\omega = \frac{\delta}{\epsilon_0 \epsilon_r}$$

$$2\pi f = \frac{\delta}{\epsilon_0 \epsilon_r}$$

$$f = \frac{\delta}{2\pi \epsilon_0 \epsilon_r}$$

$$f = \frac{5}{2\pi \times 8.854 \times 10^{-12} \times 1}$$

$$\boxed{f = 8.99 \times 10^{10} \text{ Hz}}$$

$$89.87 \times 10^9 \text{ Hz}$$

$$\boxed{f: 89.87 \text{ GHz}}$$

② A $\vec{E} = E_m \sin x \cdot \sin t \cdot \hat{a}_z$, $\vec{H} = \frac{E_m}{\mu_0} \cos x \cos t \cdot \hat{a}_z$
 does it satisfies maxwells equation.

$$\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\mu = \mu_0 \mu_r$$

↓
= 1

L.H.S

$\nabla \times \vec{E} =$	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
	0	0	$E_m \sin x \sin t$

$$\nabla \times \vec{E} = \hat{a}_x(0) - \hat{a}_y\left(\frac{\partial}{\partial x} E_m \sin x \sin t\right) + 0$$

$$\nabla \times \vec{E} = -E_m \cos x \sin t \hat{a}_y$$

R.H.S

$$-\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$= 0 - \mu_0 \frac{\partial}{\partial t} \left[\frac{E_m}{\mu_0} \cos x \cos t \right] \hat{a}_z$$

$$= + \cancel{\mu_0} \times \frac{E_m}{\cancel{\mu_0}} \cos x \sin t \hat{a}_z$$

$$= E_m \cos x \sin t \hat{a}_z$$

$$L.H.S \neq R.H.S$$

along $\hat{a}_y$ along $\hat{a}_z$

∴ It doesnot satisfies maxwells eqⁿ.

Q) In free space the magnetic field is given by $\vec{H} = 0.4 \omega \epsilon_0 \cos(\omega t - 50x) \hat{a}_z$ A/m. Find the electric field $\vec{E}$ & the displacement current density $\vec{J}_D$?

$$\vec{H} = 0.4 \omega \epsilon_0 \cos(\omega t - 50x) \hat{a}_z \text{ A/m}$$

$$J_D = ? \quad \vec{E} = ?$$

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_D$$

$$\nabla \times \vec{H} = \cancel{\vec{E}^0} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{H} = \epsilon_0 \epsilon_r \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 0.4 \omega \epsilon_0 \cos(\omega t - 50x) \end{vmatrix}$$

$$\nabla \times \vec{H} = \hat{a}_x (0) - \hat{a}_y \left(\frac{\partial}{\partial x} 0.4 \omega \epsilon_0 \cos(\omega t - 50x) \right) + \hat{a}_z (0)$$

$$\nabla \times \vec{H} = + 0.4 \omega \epsilon_0 \times 50 \times \sin(\omega t - 50x) \hat{a}_y$$

$$\nabla \times \vec{H} = + 20 \omega \epsilon_0 \sin(\omega t - 50x) \hat{a}_y = \vec{J}_D$$

LectureNotes.in

$$\frac{\partial E}{\partial t} = 20 w \sin(\omega t - 50x) \text{ mg}$$

$$E = \int 20 w \sin(\omega t - 50x) dt$$

$$E = 20 w \int \sin(\omega t - 50x) dt$$

$$E = 20 w \left[-\frac{\cos(\omega t - 50x)}{\omega} \right]$$

LectureNotes.in

$$E = -20 \cos(\omega t - 50x)$$

LectureNotes.in

20.03.19. ELECTRO MAGNETIC WAVE (module-3)

Characteristics of Electromagnetic wave

- The Electromagnetic waves are produced by accelerated or oscillating charge.
- These waves do not require ~~in~~ any medium.
- These waves travel in free space with the speed of $(3 \times 10^8 \text{ m/s})$.
- The electric & magnetic field vectors vary sinusoidally & hence attain the maximum & minimum position at same place & at same time.

- The electric & magnetic waves are perpendicular to each other as well as the direction of propagation. Hence, they are called as Transverse electromagnetic waves.

- The cross product of $\vec{E}$ & $\vec{H}$ ($\vec{E} \times \vec{H}$) gives the direction of propagation.

- All the electromagnetic waves move with same speed in vacuum or (in air) but have different speeds in medium.

Wave Equation

$$(1) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

$$(2) \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (2)}$$

$$(3) \nabla \cdot \vec{D} = \rho_v \quad \text{--- (3)}$$

$$(4) \nabla \cdot \vec{B} = 0 \quad \text{--- (4)}$$

Taking curl on both sides of eqⁿ (1)

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

note : [The region is linear, isotropic, homogeneous, charge free].

(i) In a linear region, $\vec{D}$ varies linearly with $\vec{E}$.

② Isotropic; $\vec{D}$ & $\vec{E}$ directed along same direction.

③ Homogeneous; ϵ, μ, δ (constant) everywhere in the region

④ charge free; $\rho_v = 0$

$$\nabla \cdot (\nabla E) = \nabla^2 E = -\mu \frac{\partial}{\partial t} \left(\delta E + \epsilon \frac{\partial E}{\partial t} \right)$$

$$= -\mu \frac{\partial}{\partial t} \left\{ \delta E + \epsilon \frac{\partial (E)}{\partial t} \right\}$$

$$= -\mu \left[\delta \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \right]$$

$$\nabla^2 E = -\mu \delta \frac{\partial E}{\partial t} - \epsilon \mu \frac{\partial^2 E}{\partial t^2}$$

$$= -\frac{\partial E}{\partial t} (\mu \delta + \mu \epsilon \frac{\partial}{\partial t})$$

$$\boxed{-\nabla^2 E = -\left(\mu \delta \frac{\partial E}{\partial t} + \epsilon \mu \frac{\partial^2 E}{\partial t^2} \right)} \quad \text{--- (5)}$$

Similarly,

$$\boxed{\nabla^2 H = \mu \delta \frac{\partial H}{\partial t} + \epsilon \mu \frac{\partial^2 H}{\partial t^2}} \quad \text{--- (6)}$$

In phasor form

$$\frac{\partial}{\partial t} = j\omega$$

$$\nabla^2 E = \sigma \mu j\omega E + \mu \epsilon (j\omega)^2 E$$

$$\nabla^2 E = \left[j\omega \mu (\sigma + j\omega \epsilon) \right] E$$

$$\text{Let } \gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)} = \alpha + j\beta$$

γ = propagation constant

α = attenuation constant

β = phase constant

$$\boxed{\nabla^2 E = -\gamma^2 E} \quad \text{--- (7)}$$

$$\boxed{\nabla^2 H = -\gamma^2 H} \quad \text{--- (8)}$$

constitutes of
Electromagnetic
wave equations

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)} = \alpha + j\beta$$

$$\gamma^2 = j\omega \mu (\sigma + j\omega \epsilon) = (\alpha + j\beta)^2$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega \mu (\sigma + j\omega \epsilon)$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega \mu \sigma + j^2 \omega^2 \mu \epsilon$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega \mu \sigma - \omega^2 \mu \epsilon$$

$$\alpha^2 - \beta^2 = -\omega^2 \mu \epsilon \quad (\text{real part}) \quad \text{--- (9)}$$

$$2j\alpha\beta = j\omega \mu \sigma \quad (\text{imaginary part})$$

$$\boxed{\beta = \frac{\omega \mu \delta}{2\alpha}} \quad (10)$$

by substituting eqⁿ (10) in eqⁿ (9)

$$\alpha^2 - \beta^2 + \omega^2 \mu \epsilon = 0$$

$$\alpha^2 - \frac{\omega^2 \mu^2 \delta^2}{4\alpha^2} + \omega^2 \mu \epsilon = 0$$

$$4\alpha^2 \alpha^2 - \omega^2 \mu^2 \delta^2 + 4\alpha^2 \omega^2 \mu \epsilon = 0$$

$$(2\alpha^2)^2 - (\omega \mu \delta)^2 + 4\omega^2 \mu \epsilon \alpha^2 = 0$$

$$(2\alpha^2)^2 + (4\alpha^2) \omega^2 \mu \epsilon - (\omega \mu \delta)^2 = 0$$

$$x^2 + \underline{b}x + \underline{c} = 0$$

$$a + b + 2ab$$

$$x = 2\alpha^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\delta}{\omega \epsilon}\right)^2} - 1 \right]}} \quad \text{Np/m}$$

$$\boxed{\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\delta}{\omega \epsilon}\right)^2} + 1 \right]}} \quad \text{rad/m}$$

Attenuation constant (α) - measures the attenuation of the signal when it travels from one medium to another medium.

- The unit for attenuation constant is ~~Nepers~~ Np/m (nepers/meter)

$$1 \text{ nepers} = 8.686 \text{ dB}$$

- phase constant (β) - measures the change in phase when the wave travels from one medium to another medium.

$$\beta = \frac{\omega}{v} = \frac{2\pi f}{\cancel{\lambda}} = \frac{2\pi \text{ rad/m}}{\lambda}$$

- The unit is radian/meter. It is also given by ratio of angular frequency to velocity i.e.

Intrinsic Impedance

It is the ratio of magnitude of electric intensity to the magnetic intensity.

$$\eta = \frac{E_m}{H_m}$$

26.03.19

Intrinsic Impedance

$$\eta = \frac{E_m}{H_m}$$

η in terms of ϵ , μ

$$\left. \begin{aligned} \nabla^2 E - \mu \epsilon \frac{\partial^2 E}{\partial t^2} &= 0 \quad (1) \\ \nabla^2 H - \mu \epsilon \frac{\partial^2 H}{\partial t^2} &= 0 \quad (2) \end{aligned} \right\} \begin{array}{l} \text{Maxwell's eqns} \\ \text{eqns of waves} \end{array}$$

consider z be direction of propagation

$$\vec{E} = E_x \hat{a}_x \quad (\vec{E} \text{ along } x\text{-axis})$$

$$\vec{H} = H_y \hat{a}_y \quad (\vec{H} \text{ along } y\text{-axis})$$

$$\nabla^2 E_x \hat{a}_x = \frac{\partial^2 E_x}{\partial z^2} \hat{a}_x$$

$$\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} \right) - \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

NOTE: Since, the wave is propagating along z -axis. So, the variation of $\vec{E}$ along x & y must be zero

$$\therefore \frac{\partial^2 E_x}{\partial z^2} - \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} = 0 \quad (3)$$

$$E_x = A e^{-\gamma z} + B e^{+\gamma z}$$

{ A & B are arbitrary constant }

$$\text{If } z \rightarrow \infty, B e^{+\gamma z} \rightarrow \infty$$

$$E_x \rightarrow \infty$$

$$E_x = A e^{-\gamma z} = A e^{-(\alpha + j\beta)z} = A e^{-\alpha z} \cdot e^{-j\beta z} \quad \text{--- (1)}$$

$$\gamma = \alpha + j\beta$$

Faradays law

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} = -\mu \frac{\partial H_y}{\partial t} a_y \quad \text{--- (5)}$$

LectureNotes.in

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix}$$

$$= a_x (0) - a_y \left(0 - \frac{\partial E_x}{\partial z} \right) + a_z \left(0 - \frac{\partial E_x}{\partial y} \right)$$

$$\nabla \times E = 0 + a_y \frac{\partial E_x}{\partial z} - a_z \frac{\partial E_x}{\partial y}$$

$$\nabla \times E = \frac{\partial E_x}{\partial z} a_y - \frac{\partial E_x}{\partial y} a_z$$

LectureNotes.in

as it is along y-axis.

$$\nabla \times E = \frac{\partial E_x}{\partial z} a_y$$

LectureNotes.in

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial}{\partial z} (A e^{-\gamma z}) = -\mu \frac{\partial H_y}{\partial t} \quad \text{--- (6)}$$

$$-A \gamma e^{-\gamma z} = -\mu \frac{\partial H_y}{\partial t}$$

$$\Rightarrow H_y = \int \frac{Aze^{-\gamma z}}{ze} dt \quad \text{--- (6)}$$

$\frac{\partial}{\partial t} = j\omega$: derivative operator

$$Aze^{-\gamma z} = fze(j\omega) H_y$$

$$\boxed{H_y = \frac{Aze^{-\gamma z}}{zej\omega}} \quad \text{--- (7)}$$

$$\eta = \frac{E_z}{H_y} = \frac{Aze^{-\gamma z} \times j\omega ze}{\gamma Aze^{-\gamma z}}$$

$$\eta = \frac{j\omega ze}{\gamma}$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega ze(\delta + j\omega e)}$$

$$\eta = \frac{j\omega ze}{\sqrt{j\omega ze(\delta + j\omega e)}}$$

$$\eta = \sqrt{\frac{(j\omega ze)^2}{j\omega ze(\delta + j\omega e)}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega ze}{\delta + j\omega e}}} \quad \text{--- (8)}$$

$\omega = 2\pi f$ (angular frequency)

δ : conductivity of the medium

e : permeability

ze :

Uniform Plane Wave (UPW) :

- The electromagnetic wave in which electric & magnetic field intensities are directed in fixed directions in space & are constants in magnitude & phase to the plane ~~into~~ which perpendicular to the direction of propagation.
- The uniform plane waves only exists in free space.

① Perfect dielectric (loss less dielectric)

$$\boxed{\delta = 0, \epsilon = \epsilon_0 \epsilon_r \text{ \& \; } \mu = \mu_0 \mu_r}$$

$$(a) \quad \gamma = \sqrt{j\omega\mu(\delta + j\omega\epsilon)} = \alpha + j\beta$$

$$j\omega\sqrt{\mu\epsilon} = \alpha + j\beta$$

$$\boxed{\alpha = 0}, \quad \boxed{\beta = \omega\sqrt{\mu\epsilon}}$$

$$(b) \quad \eta = \frac{j\omega\mu}{\delta + j\omega\epsilon}, \quad \delta = 0$$

$$\frac{\sqrt{j\omega\mu}}{j\omega\epsilon} = \frac{\sqrt{\mu}}{\epsilon} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}}$$

② Free space $\epsilon_r = 1$

$$\delta = 0, \quad \epsilon = \epsilon_0 \epsilon_r = \epsilon_0 \quad \& \quad \mu = \mu_0 \mu_r = \mu_0$$

$$(a) \quad \gamma = \sqrt{j\omega\mu(\delta + j\omega\epsilon)} = \alpha + j\beta$$

$$j\omega \sqrt{\mu_0 \epsilon_0} = \alpha + j\beta$$

$$\boxed{\alpha = 0}, \quad \boxed{\beta = \omega \sqrt{\mu_0 \epsilon_0} = \frac{2\pi}{\lambda}}$$

$$(b) \quad \eta = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}}, \quad \delta = 0$$

$$\sqrt{\frac{j\omega \mu}{j\omega \epsilon_0}} = \sqrt{\frac{\mu_0 \cancel{j\omega}}{\epsilon_0 \cancel{j\omega}}} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\boxed{\eta = 120\pi = 377\Omega}$$

$$|\eta| < \theta$$

$$\eta = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

27.03.19. Propagation of VFW in good conductor

$$\eta = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

$$\eta = \sqrt{j\omega \mu \epsilon (1 + \frac{\sigma}{j\omega \epsilon})}$$

for good conductor

$$\underline{\sigma} \gg \omega \epsilon$$

$$\omega \epsilon \gg \underline{\sigma}$$

$$\frac{\omega \epsilon}{\underline{\sigma}} \ll \underline{\sigma}$$

$$\eta = \sqrt{j\omega \mu \sigma} = \sqrt{\omega \mu \sigma} \sqrt{j}$$

$$J = 1 \angle 90^\circ$$

$$\sqrt{J} = 1 \angle 45^\circ$$

$$V = \sqrt{\omega \mu \delta} \angle [1 \angle 45^\circ]$$

$$V = \sqrt{\omega \mu \delta} [\cos 45^\circ + j \sin 45^\circ]$$

$$V = \sqrt{\omega \mu \delta} \left[\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right]$$

$$V = \sqrt{\frac{\omega \mu \delta}{2}} + j \sqrt{\frac{\omega \mu \delta}{2}}$$

$$V = \sqrt{\frac{2\pi f \mu \delta}{2}} + j \sqrt{\frac{2\pi f \mu \delta}{2}}$$

$$V = \sqrt{\pi f \mu \delta} + j \sqrt{\pi f \mu \delta}$$

$$\alpha = \sqrt{\pi f \mu \delta}, \quad \beta = \sqrt{\pi f \mu \delta}$$

for a good conductor, the value of α & β are equal $[\alpha = \beta]$

$$r < \theta + p < \alpha$$

$$r < \theta \times p < \alpha$$

$$r p < \theta + \alpha$$

$$(r < \theta)^{1/2}$$

$$r^2 < \theta + \theta$$

$$\eta = \sqrt{\frac{j \omega \mu}{\delta + j \omega \epsilon}} \sqrt{\frac{j \omega \mu}{\delta [1 + j \frac{\omega \epsilon}{\delta}]}} \left[\frac{\omega \epsilon}{\delta} \ll 1 \right]$$

$$= \sqrt{\frac{j\omega\mu}{\delta}} \cdot \sqrt{\frac{\omega\mu}{\delta}} \sqrt{5} = \sqrt{\frac{\omega\mu}{\delta}} \angle 45^\circ$$

$$= \sqrt{\frac{\omega\mu}{\delta}} [\cos 45^\circ + j \sin 45^\circ]$$

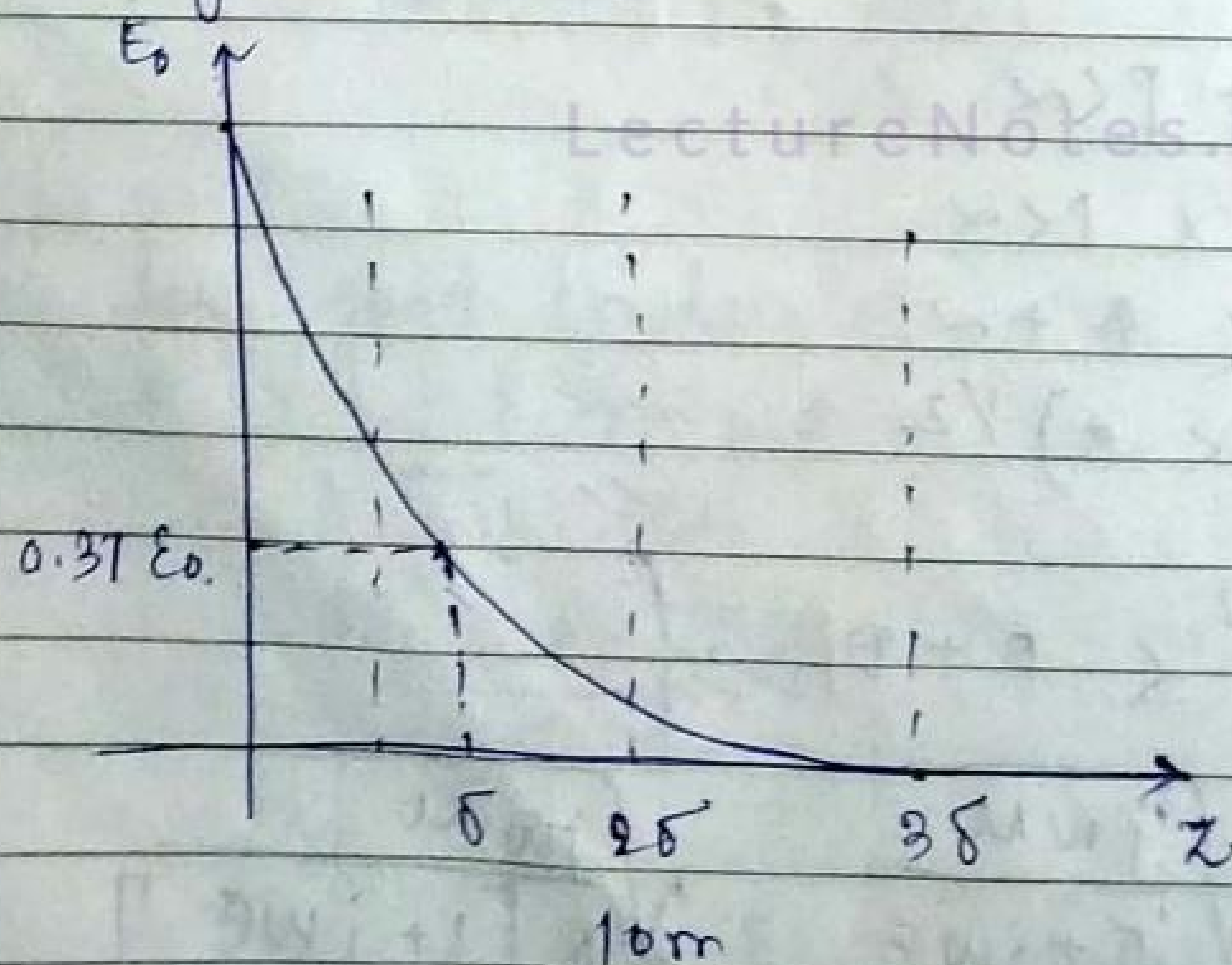
$$= \sqrt{\frac{\omega\mu}{\delta}} \left[\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right]$$

$$\therefore \eta = \sqrt{\frac{\pi f \mu}{\delta}} (1 + j)$$

Imp Hence, the electric & magnetic field vectors are at a phase difference of 45° .

Imp Skin depth : (δ)

It is defined as the depth at which the electromagnetic wave attenuates upto $1/e$ (37.7%) of its initial or original amplitude.



$$\left| \delta = \frac{1}{\alpha} \right| = \frac{1}{\sqrt{\pi f \mu \epsilon \delta}}$$

$$\boxed{\delta = \frac{1}{\alpha}}$$

$\alpha =$

$$\eta = \sqrt{\frac{\pi f \mu \epsilon \delta}{\delta}} (1 + j1)$$

$$= \sqrt{\frac{\pi f \mu \epsilon \delta}{\delta^2}} (1 + j1) \quad \frac{\sqrt{\pi f \mu \epsilon \delta}}{\delta^2}$$

$$= \frac{1}{\delta \delta} (1 + j1) \quad \cancel{\omega \delta}$$

$$= \frac{1}{\delta \delta} \sqrt{2} < 45^\circ = \frac{\sqrt{2}}{\delta \delta} < 45^\circ$$

$$Q = \frac{\omega}{\rho} = \frac{\omega}{\sqrt{\pi f \mu \epsilon \delta}} = \omega \delta$$

$$\beta = \frac{2\pi}{\lambda} \quad \checkmark$$

$$\frac{\sqrt{\pi f \mu \epsilon \delta}}{1} = \frac{2\pi}{\lambda} \Rightarrow \frac{1}{\delta} \cdot \frac{1}{\delta} = \frac{2\pi}{\lambda}$$

$$\cancel{2\pi} \cancel{2\pi} \epsilon$$

$$\boxed{\lambda = 2\pi \delta}$$

Loss Tangent :

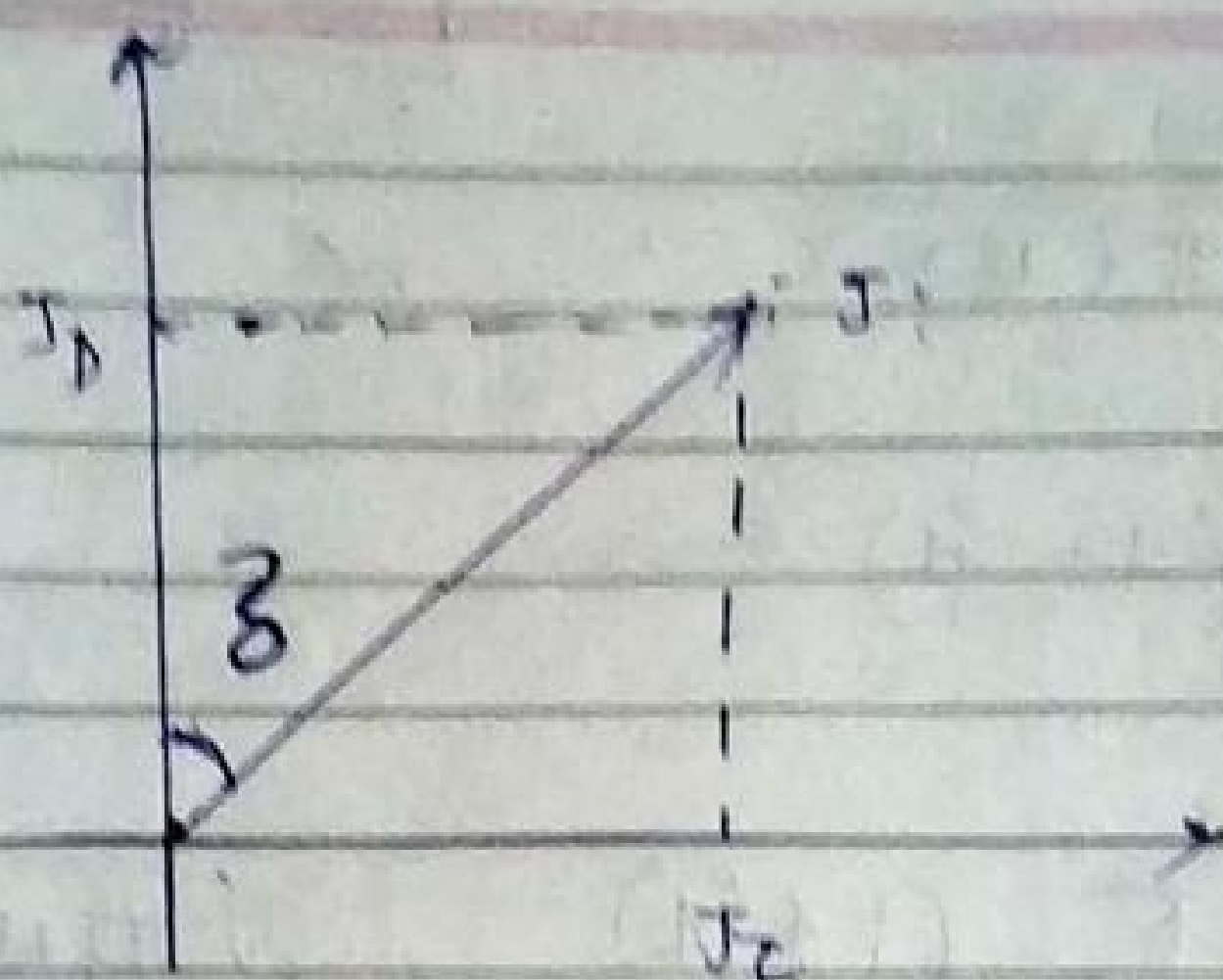
$$\frac{\partial D}{\partial t} = \epsilon \epsilon_j \omega$$

$$\vec{J} = \vec{J}_c + \vec{J}_D = \sigma E + j\omega \epsilon E$$

$$\boxed{\vec{J}_c = \sigma E}$$

$$\vec{J}_D = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t} = j\epsilon \omega E$$

$$\boxed{\vec{J}_D = j\epsilon \omega E}$$



$$\tan \delta = \frac{J_C}{J_D} = \text{loss tangent / dissipation factor}$$

$$\tan \delta = \frac{\delta E}{\epsilon \omega E} = \frac{\delta}{\epsilon \omega}$$

$$\boxed{\tan \delta = \frac{\delta}{\omega \epsilon}}$$

1. > For a good conductor $\tan \delta \gg 1$ as $\delta \gg \omega \epsilon$
2. > For a good dielectric $\tan \delta \ll 1$ as $\delta \ll \omega \epsilon$

A medium behaves as a good conductor at low frequency & a dielectric at high frequency.

Poynting Vector :

The Poynting vector is the cross product of $\vec{E}$ and $\vec{H}$ which point towards the direction of propagation.

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \quad \text{watt/m}^2$$

$$V/m \times A/m$$

$$VA/m^2$$

Page No. _____
Date: ____/____/____

The Poynting vector signifies the power density.

21.03.19 POWER DENSITY

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\vec{E}(z) = A e^{-\gamma z}$$

$$\vec{E}(t, z) = \text{Re} [A e^{-\gamma z} \cdot e^{j\omega t}]$$

$$\gamma = \alpha + j\beta$$

~~$$\vec{E}(z, t) = \text{Re} [A e^{-\alpha z} e^{-j\beta z} e^{j\omega t}]$$~~

$$\vec{E}(z, t) = \text{Re} [A e^{-\alpha z} e^{-j\beta z} e^{j\omega t}]$$

$$\vec{E}(z, t) = \text{Re} [A e^{-\alpha z} e^{j(\omega t - \beta z)}]$$

$$= \text{Re} [A e^{-\alpha z} [\cos(\omega t - \beta z) + j \sin(\omega t - \beta z)]]$$

$$[e^{j\theta} = \cos \theta + j \sin \theta]$$

$$E(z, t) = [A e^{-\alpha z} \cos(\omega t - \beta z)]$$

$$E(z, t) = E_m e^{-\alpha z} \cos(\omega t - \beta z) \quad \text{ax V/m}$$

+z axis.

$$H(z, t) = H_m e^{-\alpha z} \cos(\omega t - \beta z) \quad \text{ay A/m}$$

for free space, $\alpha = 0$

$$E(x, t) = E_m \cos(\omega t - \beta z) a_x$$

$$H(x, t) = H_m \cos(\omega t - \beta z) a_y$$

$$\eta = \frac{E_m}{H_m} \Rightarrow H_m = \frac{E_m}{\eta}$$

$$H(x, t) = \frac{E_m}{\eta} \cos(\omega t - \beta z) a_y$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= E_m \cos(\omega t - \beta z) a_x \times \frac{E_m}{\eta} \cos(\omega t - \beta z) a_y$$

$$\vec{P} = \frac{E_m^2}{\eta} \cos^2(\omega t - \beta z) a_z \quad \text{Watt/m}^2$$

Average Power Density

$$P_{avg} = \frac{1}{T} \int_0^T \vec{P} \cdot d\vec{t}$$

$$P_{avg} = \frac{1}{T} \int_0^T \frac{E_m^2}{\eta} \cos^2(\omega t - \beta z) \cdot dt$$

$$P_{avg} = \frac{E_m^2}{T\eta} \int_0^T \cos \frac{1 + \cos(2\omega t - 2\beta z)}{2} \cdot dt$$

$$P_{avg} = \frac{E_m^2}{2\eta T} \left[\int_0^T 1 \cdot dt + \int_0^T \cos(2\omega t - 2\beta z) \cdot dt \right]$$

PAGE NO
DATE

$$P_{avg} = \frac{E_m^2}{2\eta T} \left[T + \left(\frac{\sin(2\omega t - 2\beta z)}{2\omega} \right)^T \right]$$

$$P_{avg} = \frac{E_m^2}{2\eta T} \left[T + \frac{\sin(2\omega T - 2\beta z)}{2\omega} - \frac{\sin(-2\beta z)}{2\omega} \right]$$

$$P_{avg} = \frac{E_m^2}{2\eta T} \left[T + \frac{\sin 2 \times \frac{2\pi}{T} \times T - 2\beta z}{2\omega} + \frac{\sin 2\beta z}{2\omega} \right]$$

$$\frac{\sin 2\beta z}{2\omega}$$

$$\omega = 2\pi f \quad \left(\frac{1}{T} \right)$$

$$\omega = \frac{2\pi}{T}$$

$$P_{avg} = \frac{E_m^2}{2\eta T} \left[T + \frac{\sin(4\pi - 2\beta z)}{2\omega} + \frac{\sin(2\beta z)}{2\omega} \right]$$

$$P_{avg} = \frac{E_m^2}{2\eta T} \left[T - \frac{\sin 2\beta z}{2\omega} + \frac{\sin 2\beta z}{2\omega} \right]$$

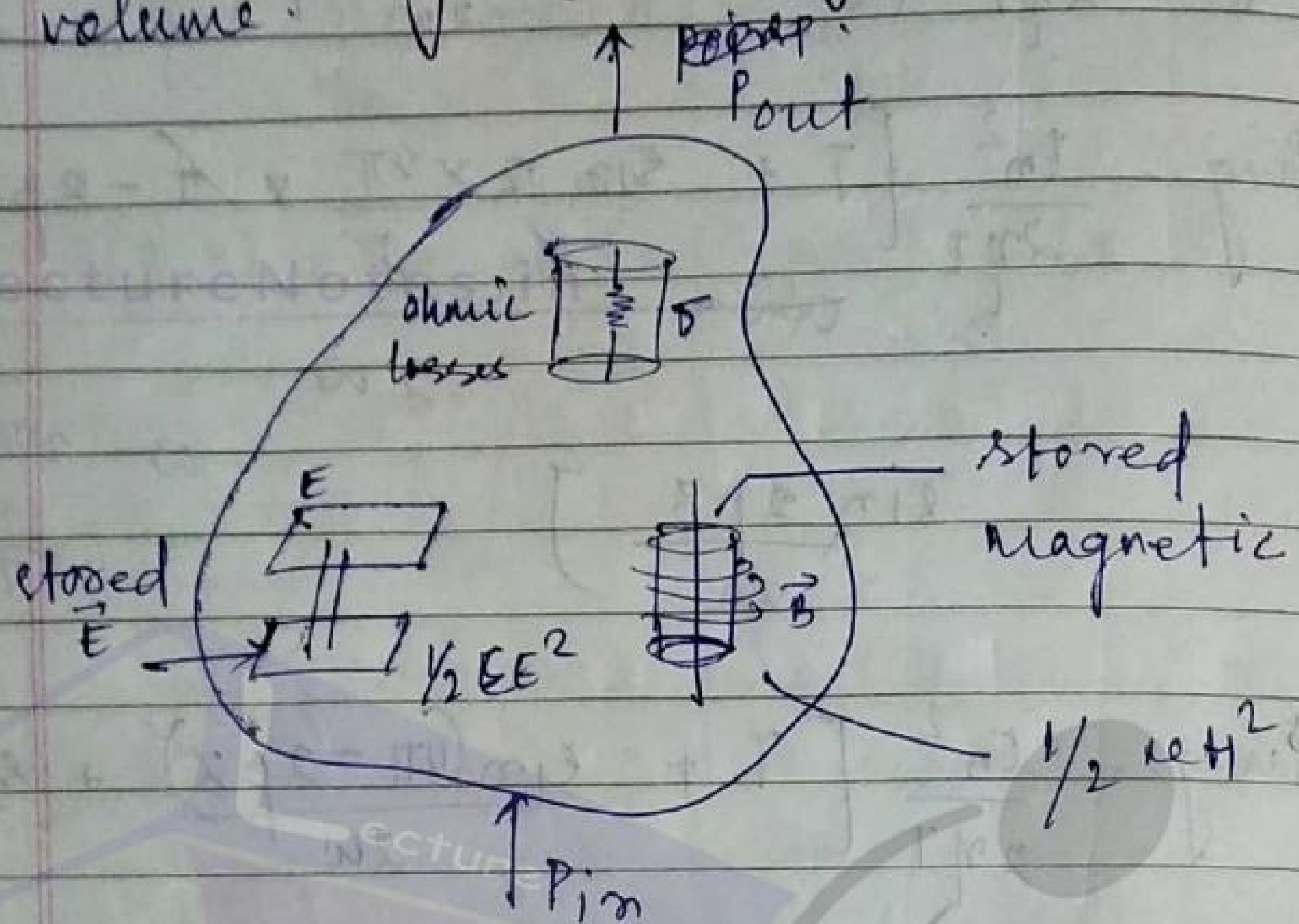
$$P_{avg} = \frac{E_m^2}{2\eta T} \times T$$

$$P_{avg} = \frac{E_m^2}{2\eta}$$

Imp Poynting Theorem :

The Poynting theorem obeys the law of energy conservation in the electro-magnetic field.

It states that the net power output of a given volume is equal to the ohmic losses - the rate of decrease of electric & magnetic energy stored in that volume.



$$\vec{P} = \vec{E} \times \vec{H}$$

$$\nabla \cdot \vec{P} = \nabla \cdot (\vec{E} \times \vec{H})$$

$$\nabla \cdot \vec{P} = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) \quad (1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad [B = \mu H] \quad (2)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad [D = \epsilon E] \quad (3)$$

$\vec{J} = \sigma \vec{E}$

Putting eqⁿ (2) & (3) in eqⁿ (1).

$$\nabla \cdot \vec{P} = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) - \vec{E} \cdot \left(\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla \cdot \vec{P} = -\mu H \frac{\partial H}{\partial t} - \delta E^2 - \epsilon E \frac{\partial E}{\partial t}$$

$$\left[\frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) = H \frac{\partial H}{\partial t} + H \frac{\partial H}{\partial t} \right]$$

$$\frac{\partial H^2}{\partial t} = 2H \frac{\partial H}{\partial t}$$

$$H \cdot \frac{\partial H}{\partial t} = \frac{1}{2} \frac{\partial H^2}{\partial t}$$

$$\left[E \cdot \frac{\partial E}{\partial t} = \frac{1}{2} \frac{\partial E^2}{\partial t} \right]$$

$$\nabla \cdot \vec{P} = -\delta E^2 - \mu \frac{1}{2} \frac{\partial H^2}{\partial t} - \epsilon \frac{1}{2} \frac{\partial E^2}{\partial t}$$

$$\boxed{\nabla \cdot \vec{P} = -\delta E^2 - \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 \right) - \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 \right)}$$

Taking the ~~surface~~ ^{volume} integration both sides

$$\int_V (\nabla \cdot \vec{P}) dV = - \int_V \delta E^2 dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 \right) dV - \int_V \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 \right) dV$$

integrating over the entire volume we get the energy distribution

applies

applying divergence theorem

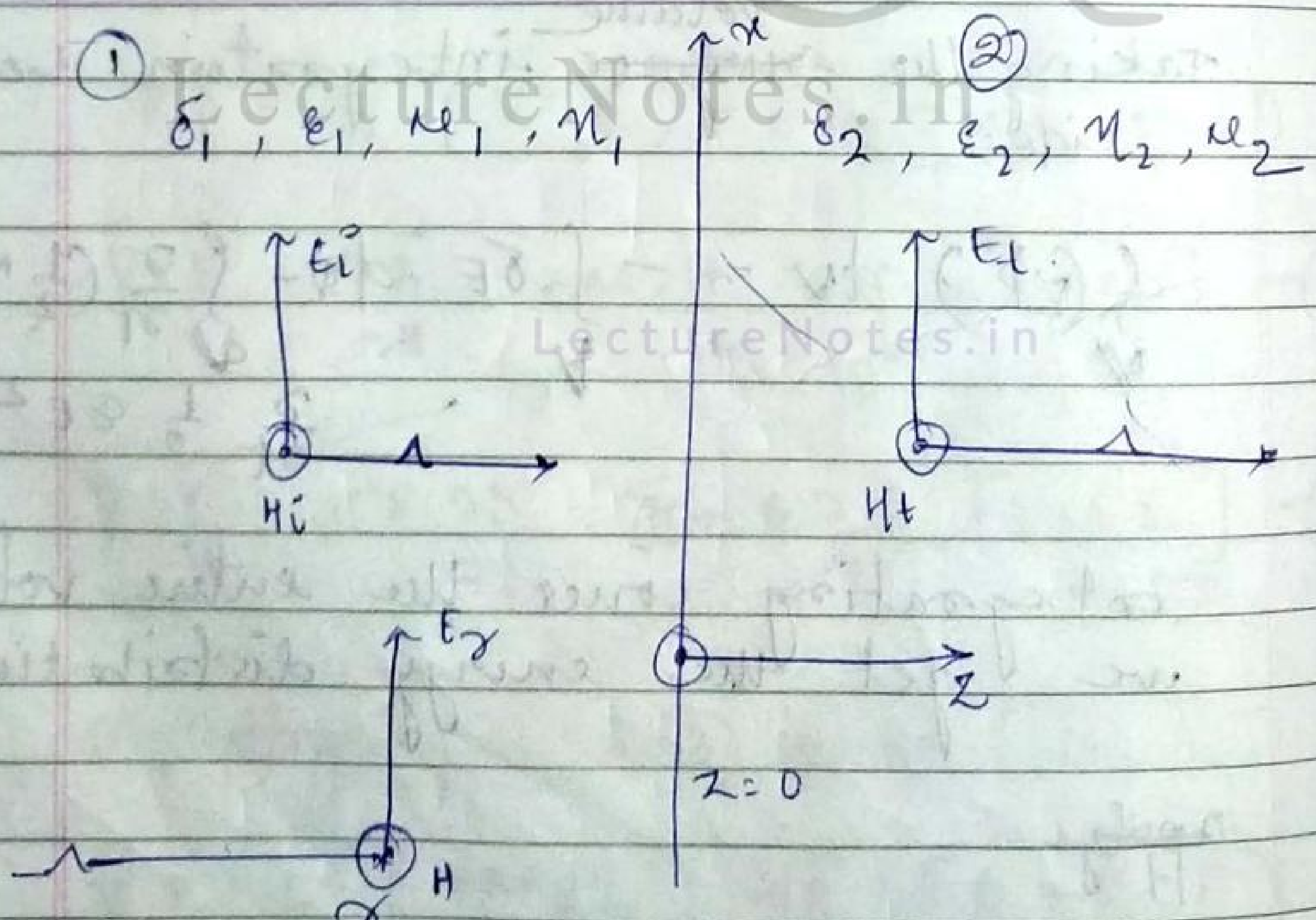
$$\oint_C \vec{p} \cdot d\vec{s} = - \int_V \sigma E^2 dv - \int_V \left\{ \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 \right) \right\} dv$$

$\oint_C \vec{p} \cdot d\vec{s}$ → output power
 $\int_V \sigma E^2 dv$ → ohmic losses
 $\int_V \left\{ \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 \right) \right\} dv$ → rate of decrease at stored E energy (Electrical)
 $\int_V \left\{ \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 \right) \right\} dv$ → rate of decrease of M-energy stored (magnetic)

generally

output power = ohmic losses + rate of decrease of stored energy

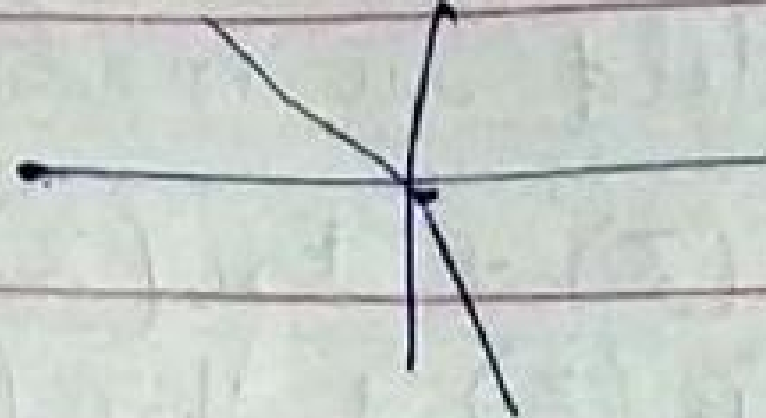
02.04.17 Reflection of UPW at normal Incidence



$$\underline{a_x = a_E + a_H}$$

$$a_z = a_x \times a_H$$

↓
-a_y



Incidence wave

$$\boxed{\vec{E}_{is}(z) = E_{i0} e^{-\gamma_1 z} \underline{a_x}} \quad \text{--- (1)}$$

$$\vec{E}_{is}(z, t) = \text{Re} [E_{i0} e^{-\gamma_1 z} e^{j\omega t}] \underline{a_x}$$

$$E_{is}(z, t) = E_{i0} e^{-\gamma_1 z} \cos(\omega t - \beta z) \underline{a_x}$$

$$\boxed{H_{is}(z) = H_{i0} e^{-\gamma_1 z} \underline{a_y} = \frac{E_{i0}}{\eta_1} e^{-\gamma_1 z} \underline{a_y}} \quad \text{--- (2)}$$

Transmitted wave

$$\boxed{\vec{E}_{ts}(z) = E_{t0} e^{-\gamma_2 z} \underline{a_x}} \quad \text{--- (3)}$$

$$\boxed{H_{ts}(z) = H_{t0} e^{-\gamma_2 z} \underline{a_y} = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z} \underline{a_y}} \quad \text{--- (4)}$$

Reflected wave

$$\boxed{\vec{E}_{rs}(z) = E_{r0} e^{\gamma_1 z} \underline{a_x}} \quad \text{--- (5)}$$

$$\boxed{H_{rs}(z) = H_{r0} e^{\gamma_1 z} (-\underline{a_y}) = -\frac{E_{r0}}{\eta_1} e^{\gamma_1 z} \underline{a_y}} \quad \text{--- (6)}$$

$\left[\eta = \frac{E_m}{H_m} \right]$

At the boundary ($z=0$),

$$E_{is} = E_{i0}, \quad E_{rs} = E_{r0}, \quad E_{ts} = E_{t0}$$

$$H_{is} = \frac{E_{i0}}{\eta_1}, \quad H_{rs} = -\frac{E_{r0}}{\eta_1}, \quad H_{ts} = \frac{E_{t0}}{\eta_2} \quad \checkmark$$

The total E. field in medium (1)

$$\vec{E}_1 = E_{is}(z) + E_{rs}(z)$$

$$H_1 = H_{is}(z) + H_{rs}(z)$$

$$H_1 = \frac{E_{is}(z)}{\eta_1} + \frac{E_{rs}(z)}{\eta_1}$$

→ The total H-field in medium (1)

The total E-field in medium (2)

$$\vec{E}_2 = E_{ts}(z)$$

The total H-field in medium (2)

$$H_2 = H_{ts}(z) = \frac{E_{ts}(z)}{\eta_2}$$

Acc. to the boundary condition, since

$\vec{E}_{1t} = \vec{E}_{2t}$, since $\vec{E}$ & $\vec{H}$ are transverse to each other

$\vec{H}_{1t} = \vec{H}_{2t}$ since $\vec{E}$ & $\vec{H}$ are transverse to the direction of propagation

$$E_{is}(0) + E_{rs}(0) = E_{ts}(0)$$

$$E_{io} + E_{ro} = E_{to} \quad \text{--- (1)}$$

$$H_{is}(0) + H_{rs}(0) = H_{ts}(0)$$

$$H_{io} + H_{ro} = H_{to}$$

or

$$\frac{E_{io}}{\eta_1} + \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$\frac{1}{\eta_1} [E_{io} + E_{ro}] = \frac{E_{to}}{\eta_2} \quad \text{--- (2)}$$

* Transmission coefficient (τ) : $\frac{E_{to}}{E_{io}}$

$$E_{io} + E_{ro} = E_{to} \quad \rightarrow \vec{E}$$

$$\frac{1}{\eta_1} E_{io} - \frac{E_{ro}}{\eta_2} = \frac{\eta_1}{\eta_2} E_{to} \quad \rightarrow \vec{H}$$

$$2E_{io} = E_{to} \left[1 + \frac{\eta_1}{\eta_2} \right]$$

$$\frac{E_{to}}{E_{io}} = \frac{2}{\left[\frac{\eta_2 + \eta_1}{\eta_2} \right]} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \frac{E_{to}}{E_{io}} =$$

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

η_2 = intrinsic Z for medium 2

η_1 = intrinsic Z for medium 1

- The ratio of magnitude of transmitted $\vec{E}$ to the magnitude of incident $\vec{E}$.

* Reflection coefficient (Γ) : $\frac{E_{ro}}{E_{io}}$

The ratio of magnitude of reflective $\vec{E}$ to the magnitude of incident $\vec{E}$:

$$\left[E_{io} + E_{ro} = E_{to} \right] \frac{\eta_1}{\eta_2}$$

$$\left[E_{io} - E_{ro} = E_{to} \left(\frac{\eta_1}{\eta_2} \right) \right]$$

$$\frac{\eta_1}{\eta_2} E_{io} + \frac{\eta_1}{\eta_2} E_{ro} = \frac{\eta_1}{\eta_2} E_{to}$$

$$E_{io} - E_{ro} = \frac{\eta_1}{\eta_2} E_{to}$$

$$\frac{\eta_1}{\eta_2} E_{io} - E_{io} + \frac{\eta_1}{\eta_2} E_{ro} + E_{ro} = 0$$

$$E_{i0} \left[\frac{\eta_1}{\eta_2} - 1 \right] + E_{r0} \left[\frac{\eta_1}{\eta_2} + 1 \right] = 0$$

$$E_{r0} \left[\frac{\eta_1}{\eta_2} + 1 \right] = -E_{i0} \left[\frac{\eta_1}{\eta_2} - 1 \right]$$

$$\frac{E_{r0}}{E_{i0}} = \frac{- \left[\frac{\eta_1}{\eta_2} - 1 \right]}{\left[\frac{\eta_1}{\eta_2} + 1 \right]}$$

$$\frac{E_{r0}}{E_{i0}} = - \frac{\left[\frac{\eta_1 - \eta_2}{\eta_2} \right]}{\left[\frac{\eta_1 + \eta_2}{\eta_2} \right]}$$

$$\star \boxed{\frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1 \quad E_{r0} = -E_{i0}$$

$$Z = \frac{E_{r0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1} = 0$$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} + \sqrt{\frac{1}{\sigma} \frac{j\omega\mu}{[1 + \frac{j\omega\epsilon}{\sigma}]}} = 0$$

Hence, the wave is totally reflected.

- The reflected wave combines with the incident wave forms a standing wave. (A standing wave stands but it doesn't travel).

$$E_2(z) = E_{i0} e^{-\gamma_2 z} + E_{r0} e^{\gamma_2 z}$$

$$E_1(z) = E_{i0} e^{-\gamma_1 z} + E_{r0} e^{\gamma_1 z}$$

$$E_1(z) = (E_{i0} e^{-\gamma_1 z} + E_{r0} e^{\gamma_1 z}) a_z$$

$$\gamma_1 = \alpha_1 + j\beta_1$$

$\alpha_1 = 0$ (if the medium is dielectric)

$$E_1(z) = (E_{i0} e^{-j\beta_1 z} + E_{r0} e^{j\beta_1 z}) a_z$$

$$E_1(z) = E_{i0} (e^{-j\beta_1 z} - e^{j\beta_1 z}) a_z$$

$$(E_1(z) = -2j E_{i0} \sin \beta_1 z) a_z$$

$$E_1(z) = (-2j E_{i0} \sin \beta_1 z) a_z$$

$$E_1(z) = E_{i0} [\cos \beta_1 z - j \sin \beta_1 z - \cos \beta_1 z - j \sin \beta_1 z]$$

$$e^{-j\theta} = \cos \theta - j \sin \theta$$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

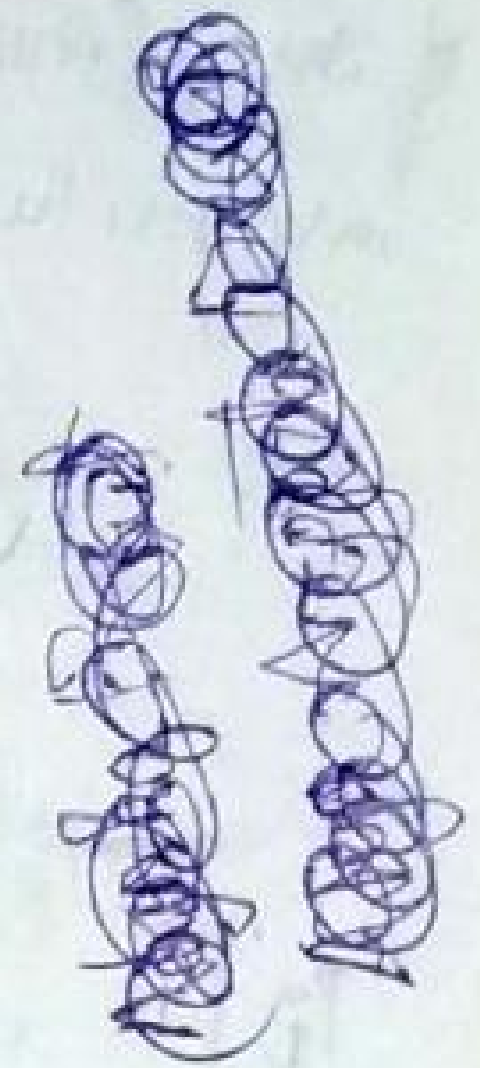
$$\vec{E}_{1s} = (-2j E_{i0} \sin \beta_1 z) a_z \quad \text{only space}$$

$$\vec{E}(z, t) = \text{Re} [\vec{E}_{1s} e^{j\omega t}]$$

$$\vec{E}(z, t) = \text{Re} [-2j E_{i0} \sin \beta_1 z (\cos \omega t + j \sin \omega t)]$$

$$\vec{E}(z, t) = 2 E_{i0} \sin \beta_1 z \sin \omega t$$

space & time



03.04.19

$$H(z, t) = (2H_0 \cos \beta z \cos \omega t) a_y$$

The standing wave ratio is the ratio of magnitude of maximum electric field to the minimum magnitude of minimum E .

$$S = \frac{|E_{\max}|}{|E_{\min}|} = \frac{|H_{\max}|}{|H_{\min}|} = \frac{1 + \Gamma}{1 - \Gamma}$$

$$P_i]_{\text{avg}} = \frac{(E_{i0})^2}{2\eta_1} \quad P_t]_{\text{avg}} = \frac{(E_{t0})^2}{2\eta_2}$$

$$P_r]_{\text{avg}} = \frac{(E_{r0})^2}{2\eta_1}$$

The ratio of transmitted to incident power

$$\frac{P_t]_{\text{avg}}}{P_i]_{\text{avg}}} = \frac{(E_{t0})^2}{2\eta_2} \times \frac{2\eta_1}{(E_{i0})^2}$$

$$= \frac{(E_{t0})^2}{(E_{i0})^2} \frac{\eta_1}{\eta_2}$$

$$= \left(\frac{E_{t0}}{E_{i0}} \right)^2 \times \frac{\eta_1}{\eta_2} = \left(\frac{2\eta_2}{\eta_1 + \eta_2} \right)^2 \times \frac{\eta_1}{\eta_2}$$

$$\boxed{\frac{P_t]_{\text{avg}}}{P_i]_{\text{avg}}} = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2}} \quad - (1)$$

The ratio of reflected to incident power

$$\frac{P_r]_{avg}}{P_i]_{avg}} = \frac{(E_{ro})^2}{2\eta_1} \times \frac{2\eta_1}{(E_{io})^2}$$

$$= \frac{(E_{ro})^2}{(E_{io})^2}$$

$$\boxed{\frac{P_r]_{avg}}{P_i]_{avg}} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right)^2} \quad \text{--- (2)}$$

by adding eqⁿ (1) & eqⁿ (2), we get-

$$\frac{P_t]_{avg}}{P_i]_{avg}} + \frac{P_r]_{avg}}{P_i]_{avg}} = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2} + \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right)^2 = 1$$

$$\frac{P_t]_{avg} + P_r]_{avg}}{P_i]_{avg}} = 1$$

$$\boxed{P_i]_{avg} = P_t]_{avg} + P_r]_{avg}}$$

Short notes on Polarisation

POLARISATION :

It is regarded as locus of the tip of the $\vec{E}$ (in a plane perpendicular to the direction of propagation) at a given point as a function of time.

Significance of Polarisation :

→ Polarisation determines the direction of $\vec{E}$ having more than one component describing the variations & direction of $\vec{E}$.

There are three types of polarisation :

→ Linear Polarisation

→ Circular Polarisation

→ Elliptical Polarisation

> Linear Polarisation : (If $\vec{E}$ traces with a straight line)

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y$$

$$E_x = a \cos(\omega t + \phi_a)$$

$$E_y = b \cos(\omega t + \phi_b)$$

Phase difference $= (\phi_b - \phi_a)$

ωt = angular position

a, b = magnitude along x & y

ϕ = phase angle

Case - 1

if $\phi_b - \phi_a = 0$

$$\boxed{\phi_b = \phi_a}$$

ratio of $\frac{E_y}{E_x} = \frac{b \cos(\omega t + \phi_a)}{a \cos(\omega t + \phi_a)}$

$$\Rightarrow \boxed{\frac{E_y}{E_x} = \frac{b}{a}}$$

$$\because \boxed{\phi_b = \phi_a}$$

case 2

if $\phi_b - \phi_a = \pi$

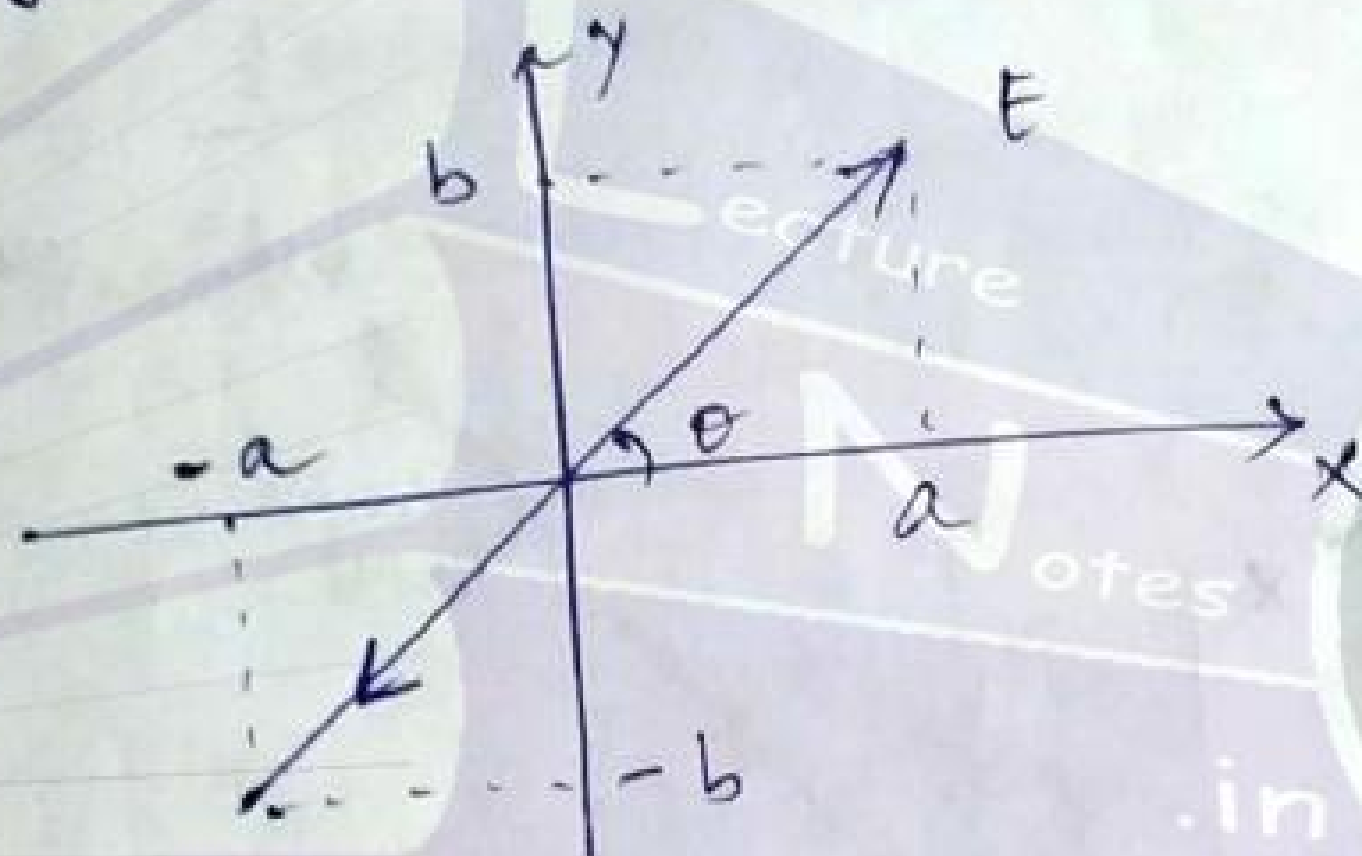
$\phi_b = \pi + \phi_a$

$\phi_b = -\phi_a$

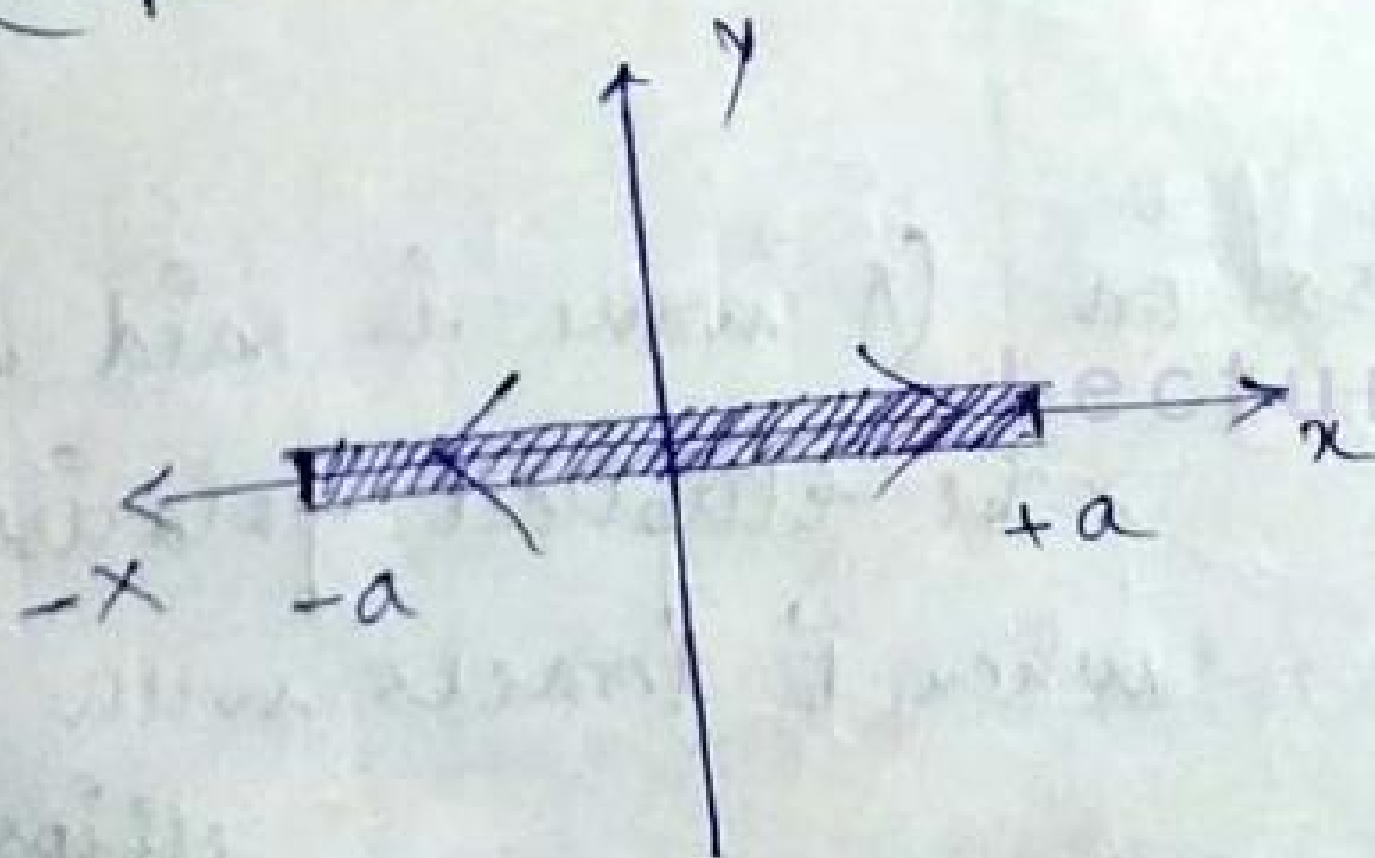
ratio of $\frac{E_y}{E_x} = \frac{b \cos(\omega t - \phi_a)}{a \cos(\omega t + \phi_a)}$

$\Rightarrow \frac{E_y}{E_x} = \pm \frac{b}{a}$

$E_y = \pm \frac{b}{a} E_x$

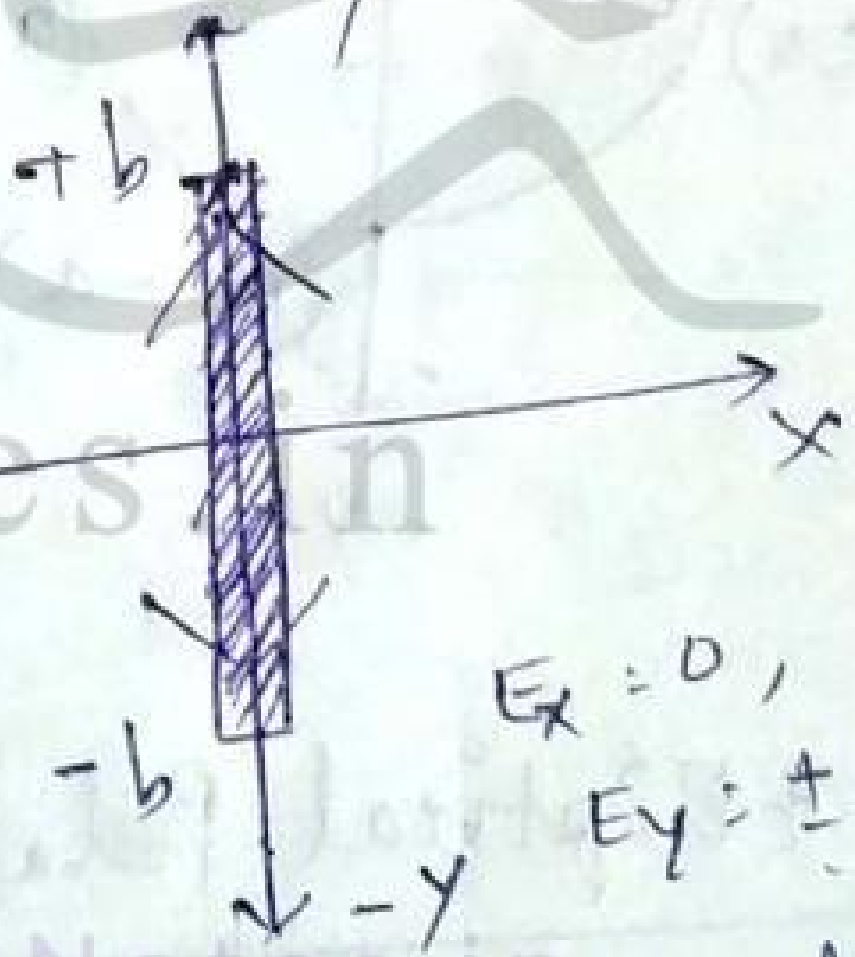


(theta polarised)



$E_y = 0 \quad E_x = \pm a$

x-polarised or horizontal polarised



$E_x = 0, \quad E_y = \pm b$

y-polarised or vertical polarised

> Circular Polarisation (If $\vec{E}$ traces with a circle)

if $\phi_b - \phi_a = 90^\circ$ $\phi_b = 90^\circ + \phi_a$

$a = b$

$E_x = a \cos(\omega t + \phi_a)$

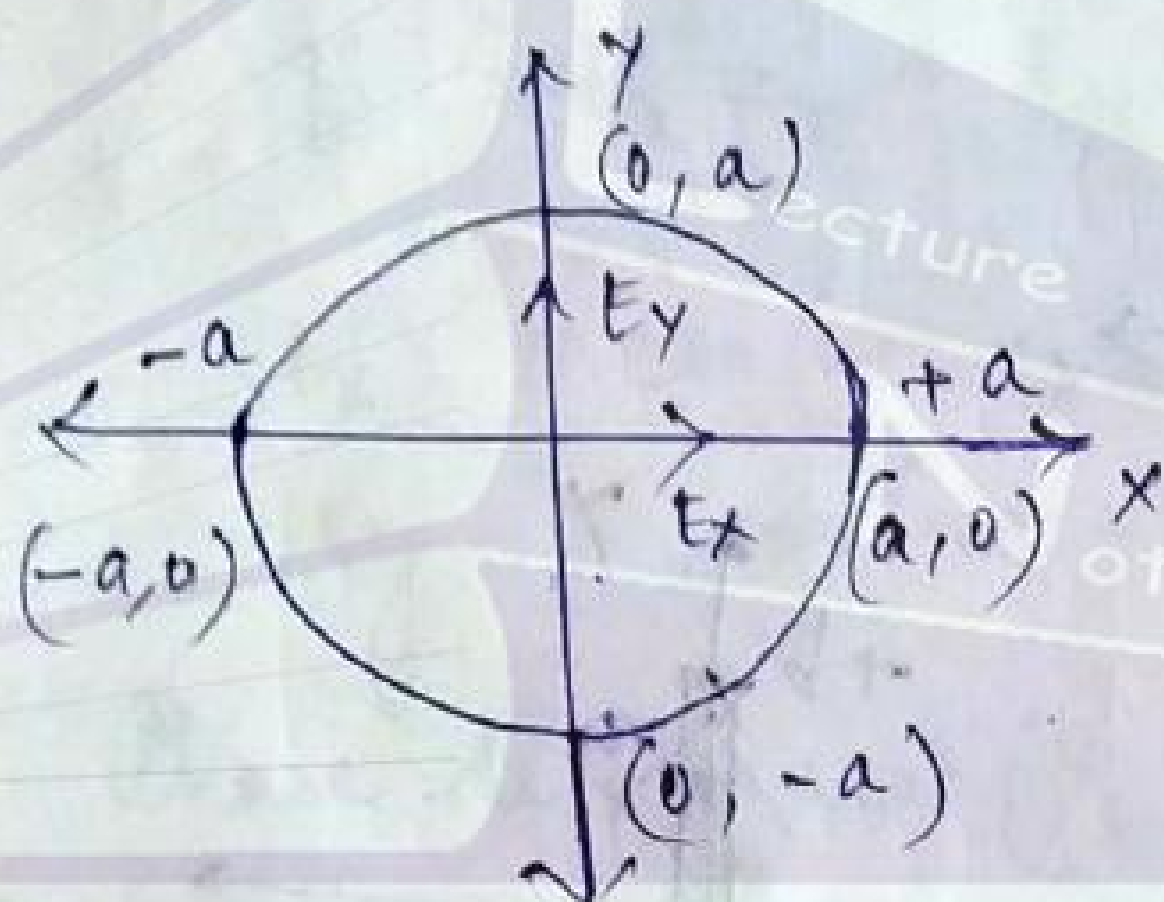
$E_y = a \cos(\omega t + 90^\circ + \phi_a)$

$E_y = -a \sin(\omega t + \phi_a)$

$\cos(90^\circ + \theta) = -\sin\theta$

$E_x^2 + E_y^2 = a^2 \cos^2(\omega t + \phi_a) + a^2 \sin^2(\omega t + \phi_a)$

$\Rightarrow E_x^2 + E_y^2 = a^2$



> Elliptical Polarisation (A wave is said to

be elliptical polarised when $\vec{E}$ traces with an ellipse)

if $\phi_b - \phi_a = 90^\circ$

$a \neq b$

$E_x = a \cos(\omega t + \phi_a)$

$E_y = b \cos(\omega t + 90^\circ + \phi_a)$

$E_y = -b \sin(\omega t + \phi_a)$

$$\frac{E_x}{a} = \cos(\omega t + \phi_a)$$

$$\frac{E_y}{b} = -\sin(\omega t + \phi_a)$$

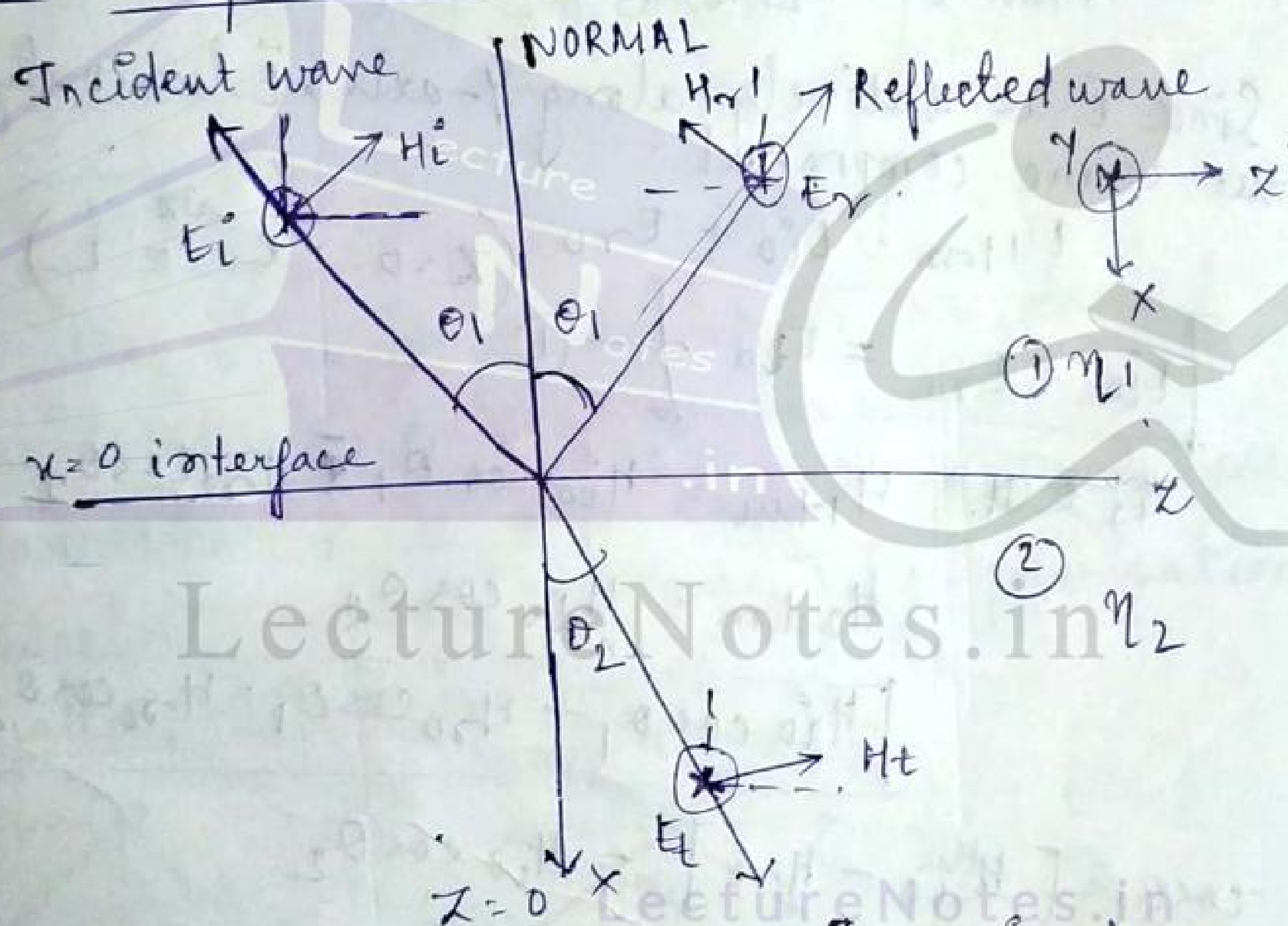
$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = \cos^2(\omega t + \phi_a) + \sin^2(\omega t + \phi_a)$$

$$\Rightarrow \boxed{\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1}$$

05.04.19 (imp)

Reflection of EM wave at oblique Incidence :

case 1 Perpendicular Polarisation



$\vec{p}_i$ = propagation vector of incident wave

$\vec{p}_t$ = propagation vector of transmitted wave

$\vec{p}_r$ = propagation vector of reflected wave

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r = E_{i0} e^{j\gamma_1 z} + E_{r0} e^{j\gamma_1 z}$$

$$E_2 \vec{E}_t = \vec{E}_t = E_{t0} e^{-j\gamma_2 z}$$

$$H_i = H_{i0} \cos \theta_1 + j H_{i0} \sin \theta_1$$

$$H_t = H_{t0} \cos \theta_2 + j H_{t0} \sin \theta_2$$

$$H_r = -H_{r0} \cos \theta_1 + j H_{r0} \sin \theta_1$$

At the boundary $z=0$

$$E_{1tan} = E_{2tan}$$

$$H_{1tan} = H_{2tan}$$

Since $\vec{E}$ is entirely along y -axis, $\vec{E}$ has only the cosine component.

$$E_{1tan} = E_{i0} + E_{r0} \quad (z=0, e^{-j\gamma z} = 1)$$

$$\boxed{E_{i0} + E_{r0} = E_{t0}} \quad (1)$$

$$H_{1tan} = H_{i0} \cos \theta_1 - H_{r0} \cos \theta_1$$

$$H_{2tan} = H_{t0} \cos \theta_2$$

$$\boxed{H_{i0} \cos \theta_1 - H_{r0} \cos \theta_1 = H_{t0} \cos \theta_2} \quad (2)$$

$$\cos \theta_1 [H_{i0} - H_{r0}] = H_{t0} \cos \theta_2$$

$$\left[\frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} \right] = \frac{E_{t0}}{\eta_2} \frac{\cos \theta_2}{\cos \theta_1}$$

$$\boxed{E_{i0} - E_{r0} = E_{t0} \frac{\eta_1}{\eta_2} \frac{\cos \theta_2}{\cos \theta_1}} \quad (3)$$

$$E_{io} - E_{ro} = E_{to} \frac{\eta_1}{\eta_2} \frac{\cos \theta_2}{\cos \theta_1} \quad (3)$$

$$E_{io} + E_{ro} = E_{to} \quad (1)$$

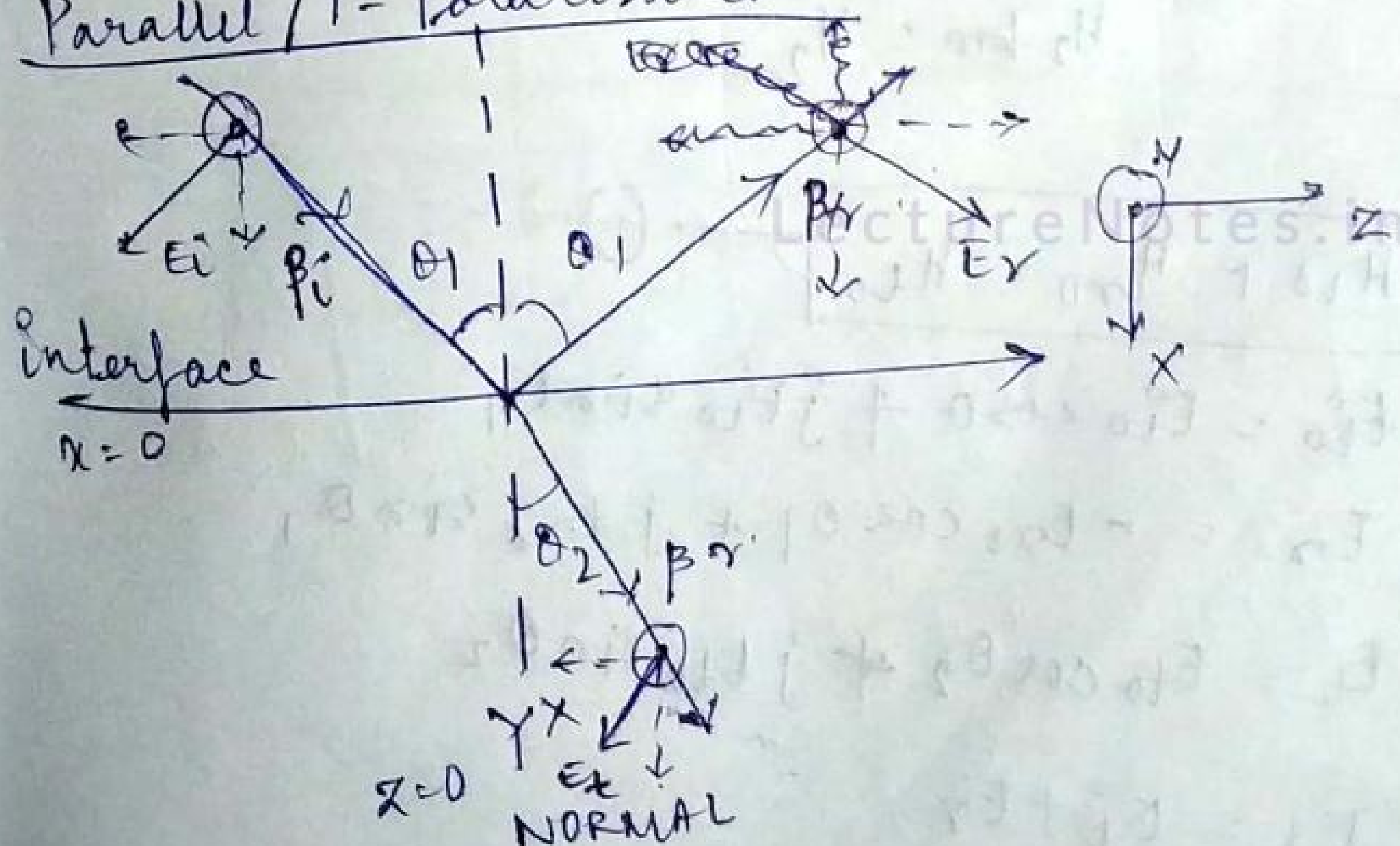
$$2E_{io} = E_{to} \left[\frac{\eta_1}{\eta_2} \frac{\cos \theta_2}{\cos \theta_1} + 1 \right]$$

$$\frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos \theta_1}{\eta_1 \cos \theta_2 + \eta_2 \cos \theta_1} = \tau_1$$

$$\Gamma = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$$

$$1 + \Gamma = \tau$$

Since the $\vec{E}$ is perpendicular to the plane of incidence is called as perpendicular polarisation; is also called as Transverse electric wave (TE wave) [s-polarisation]
Parallel / P-Polarisation:



In parallel polarisation, $\vec{E}$ is parallel to the plane of incidence hence, $\vec{B}$ is $\perp$ to the plane of incidence, & it is called a transverse magnetic wave.

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r$$

$$\vec{H}_2 = \vec{H}_t$$

$$H_1 = H_{i0} e^{-j\beta_1 z} + H_{r0} e^{+j\beta_1 z}$$

$$H_2 = H_{t0} e^{-j\beta_2 z}$$

At the boundary $z=0$.

$$H_1 = H_{i0} + H_{r0}$$

$$H_2 = H_{t0}$$

$$H_{i0} \tan \theta_1 = H_{r0} \tan \theta_1$$

Since $\vec{H}$ is entirely along y -axis,

$$H_{i0} \tan \theta_1 = H_{r0} \tan \theta_1$$

$$H_{t0} \tan \theta_2 = H_{t0} \tan \theta_2$$

$$\boxed{H_{i0} + H_{r0} = H_{t0}} \quad (1)$$

$$E_{i0} = E_{i0} \cos \theta_1 + j E_{i0} \sin \theta_1$$

$$E_{r0} = -E_{r0} \cos \theta_1 + j E_{r0} \sin \theta_1$$

$$E_{t0} = E_{t0} \cos \theta_2 + j E_{t0} \sin \theta_2$$

$$E_1 = E_i + E_r$$

$$E_2 = E_t$$

At the boundary

$$E_{1\text{tan}} = E_{2\text{tan}}$$

$$E_{i0} \cos \theta_1 = E_{r0} \cos \theta_1 = E_{t0} \cos \theta_2$$

$$\boxed{E_{i0} - E_{r0} = E_{t0} \frac{\cos \theta_2}{\cos \theta_1}} \quad \text{--- (2)}$$

$$\frac{E_{i0}}{\eta_1} + \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$$

$$\boxed{E_{i0} + E_{r0} = E_{t0} \frac{\eta_1}{\eta_2}} \quad \text{--- (3)}$$

$$E_{i0} + E_{r0} = E_{t0} \frac{\eta_1}{\eta_2}$$

$$E_{i0} - E_{r0} = E_{t0} \frac{\cos \theta_2}{\cos \theta_1}$$

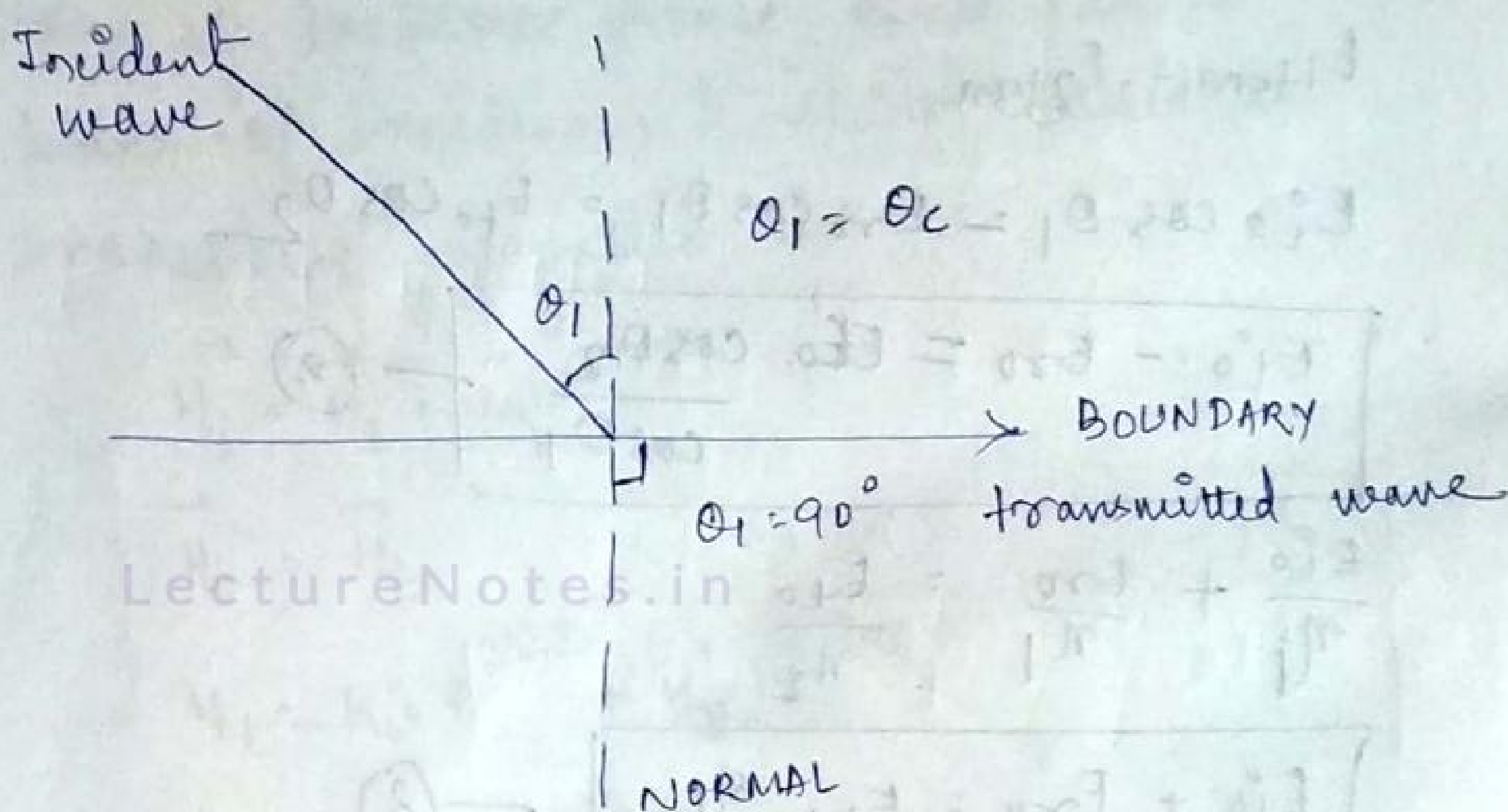
$$2E_{i0} = E_{t0} \left[\frac{\eta_1}{\eta_2} + \frac{\cos \theta_2}{\cos \theta_1} \right]$$

$$\boxed{\frac{E_{t0}}{E_{i0}} = \frac{2\eta_2 \cos \theta_1}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}} \quad \text{--- } Z_M$$

$$\boxed{\Gamma = 1 - Z}$$

09.04.19

Critical Angle



It is the angle of incidence at which the angle of transmission become 90° .

Law of reflection

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{VP_1}{VP_2} = \frac{\frac{1}{\sqrt{\mu_1 \epsilon_1}}}{\frac{1}{\sqrt{\mu_2 \epsilon_2}}} = \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$$

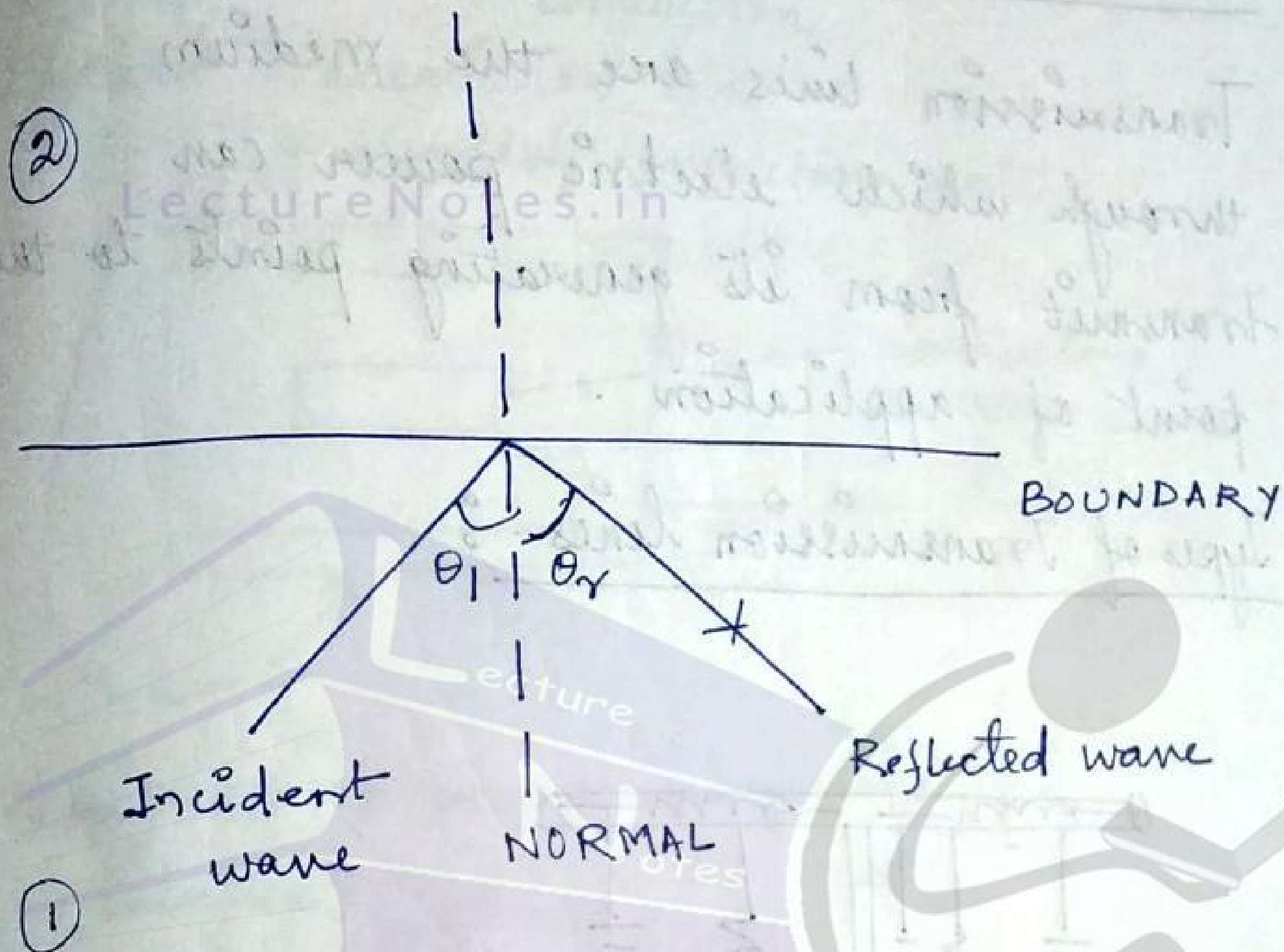
$$\sin \theta_1 = \sin \theta_c \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$$

$$\theta_1 = \theta_c, \quad \theta_2 = 90^\circ$$

$$\theta_1 = \theta_c = \sin^{-1} \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$$

Total Internal Reflection (TIR)

It is the phenomena in which the total incident wave is totally reflected back to the medium.



In total internal reflection, condition

$$\theta_i > \theta_c$$

θ_i = angle of incidence
 θ_c = angle critical angle

As to

$$r = \frac{E_{ro}}{E_{io}} = 1$$

Brewster Angle. °

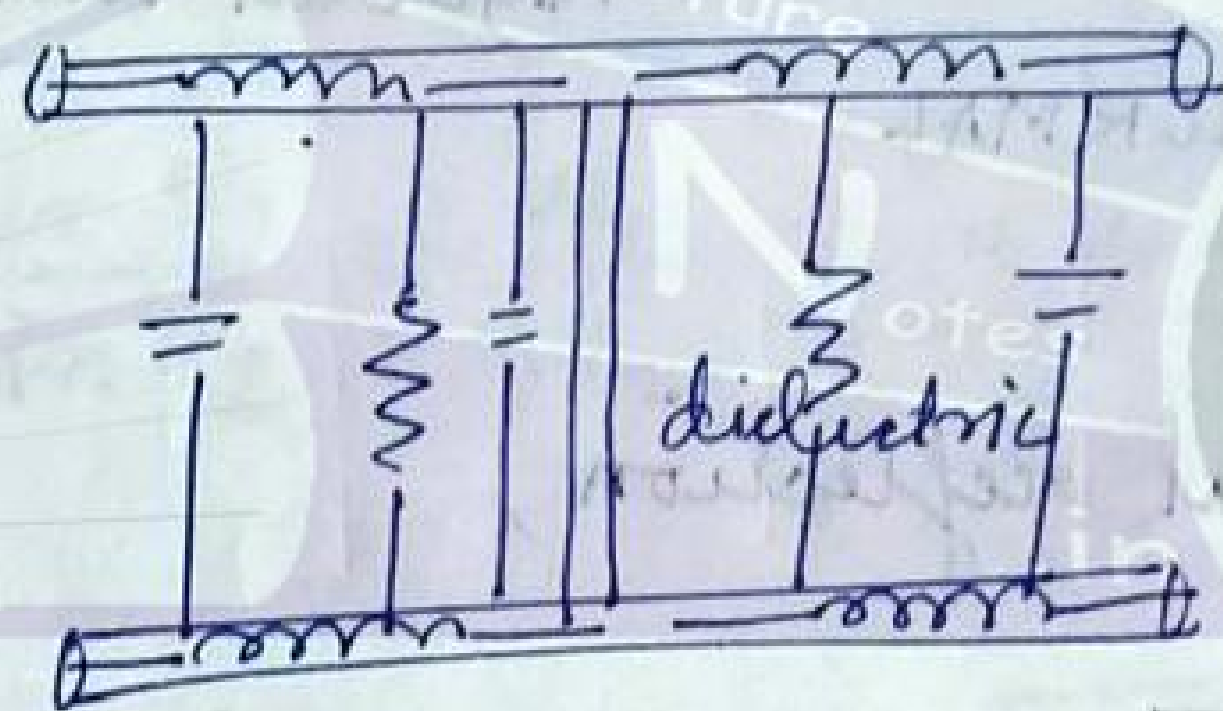
It is the angle of incidence at which the reflection coefficient becomes zero.

$$\theta_i = \theta_b = \tan^{-1} \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$$

TRANSMISSION LINE.

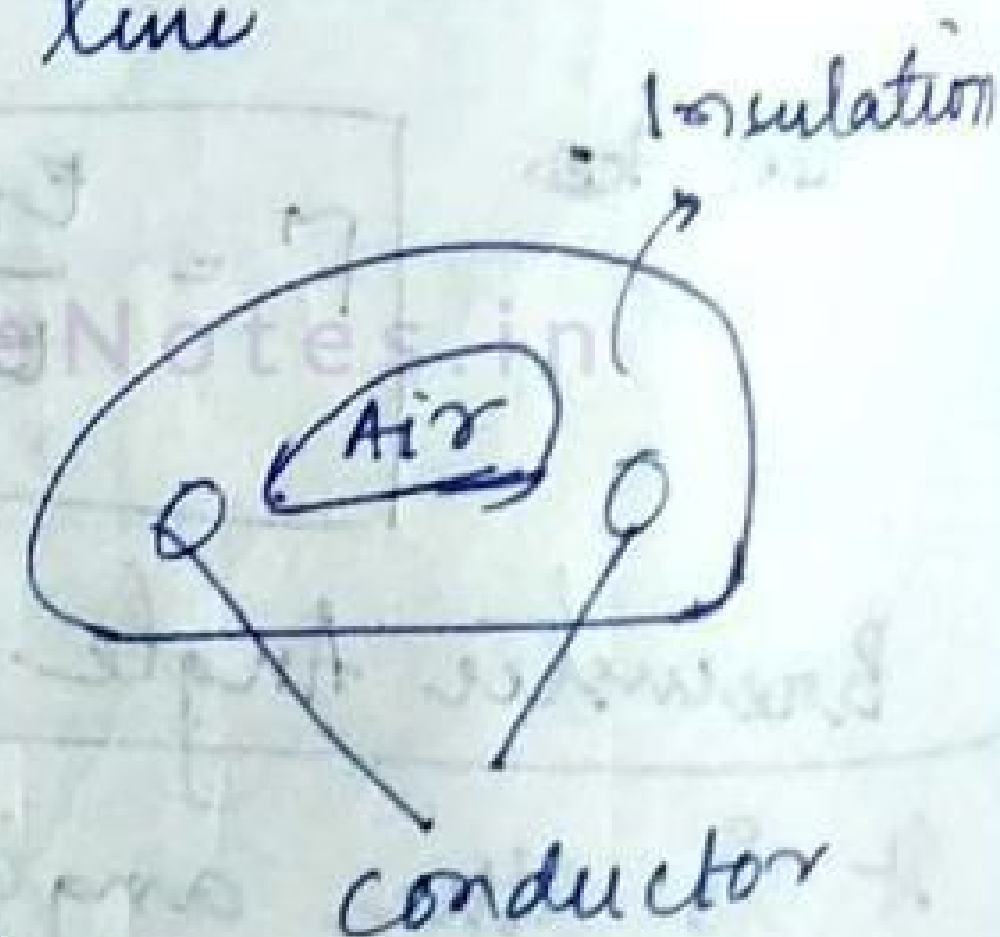
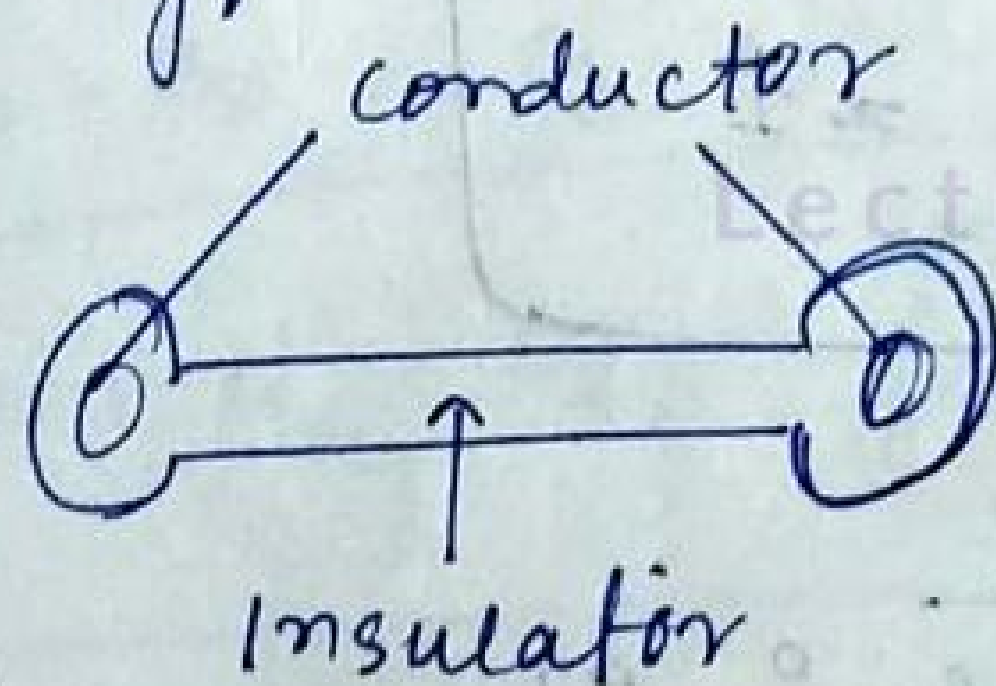
Transmission lines are the medium through which electric power can transmit from its generating points to the point of application.

Types of Transmission lines :



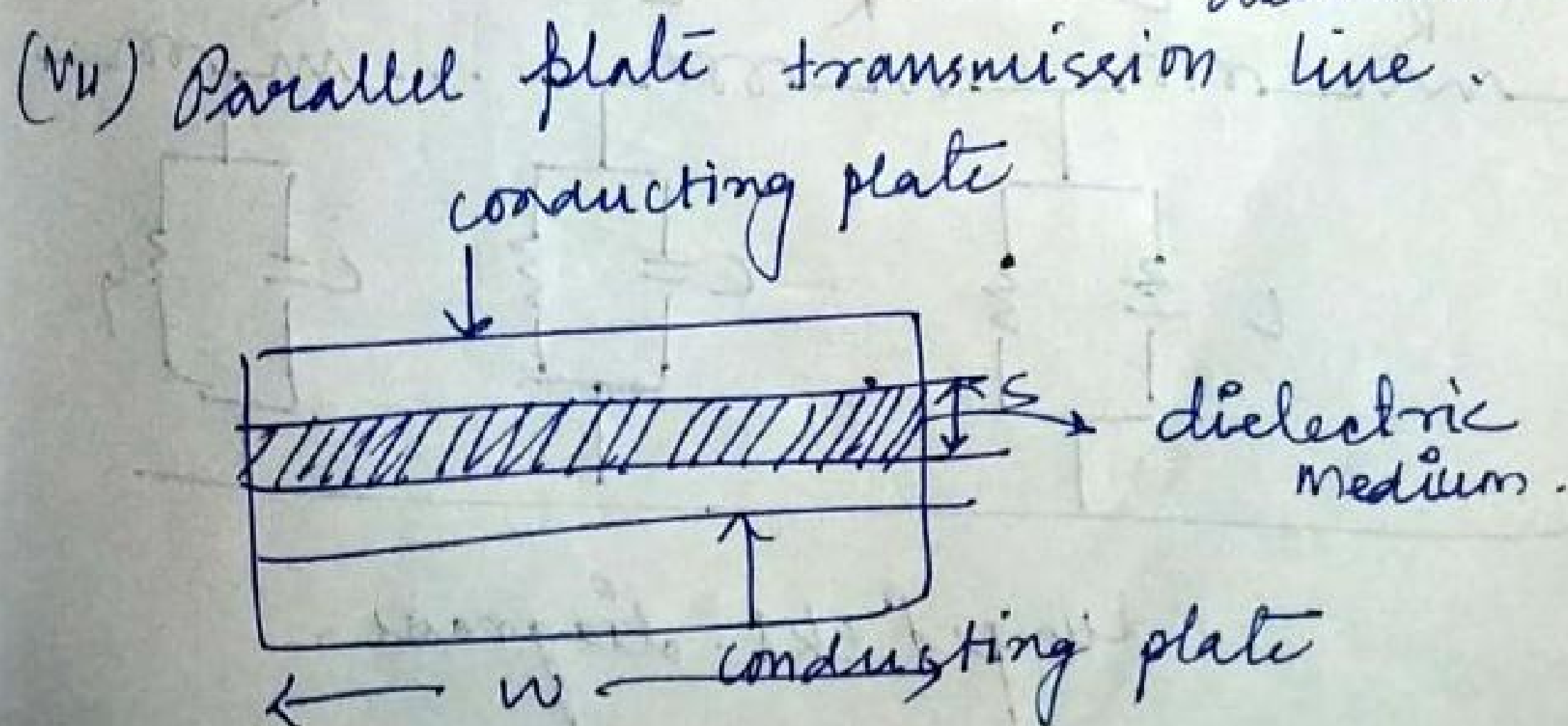
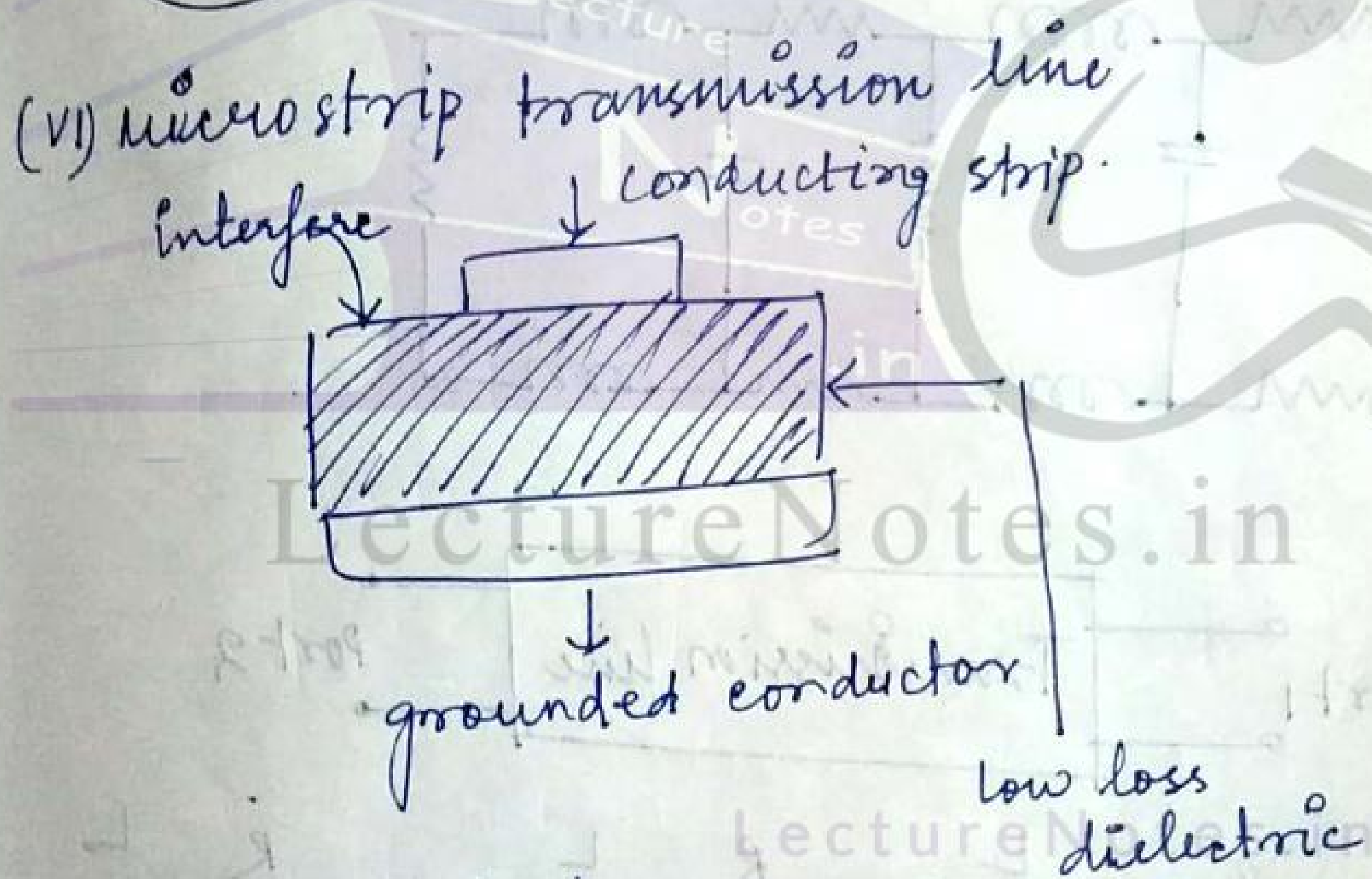
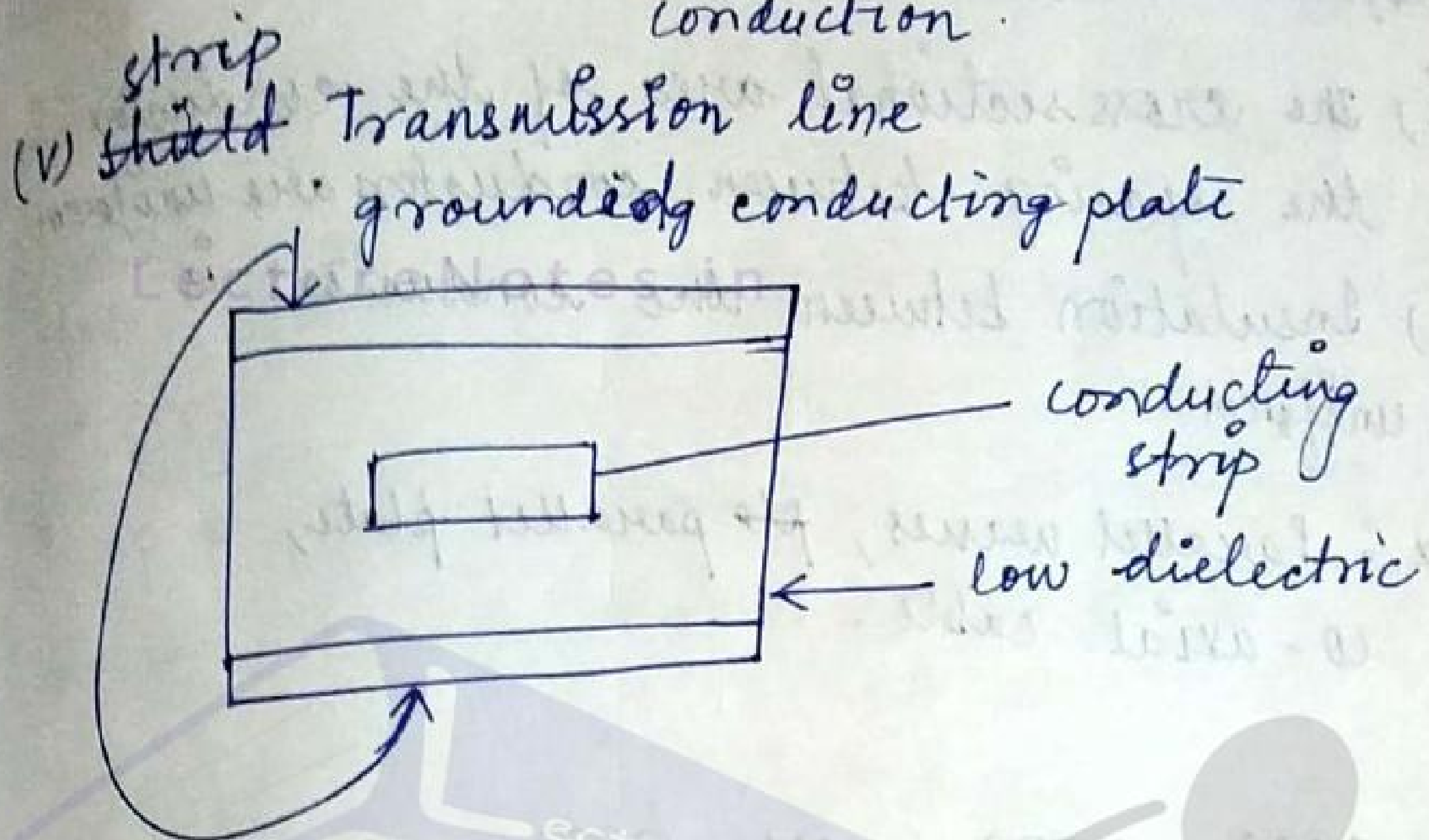
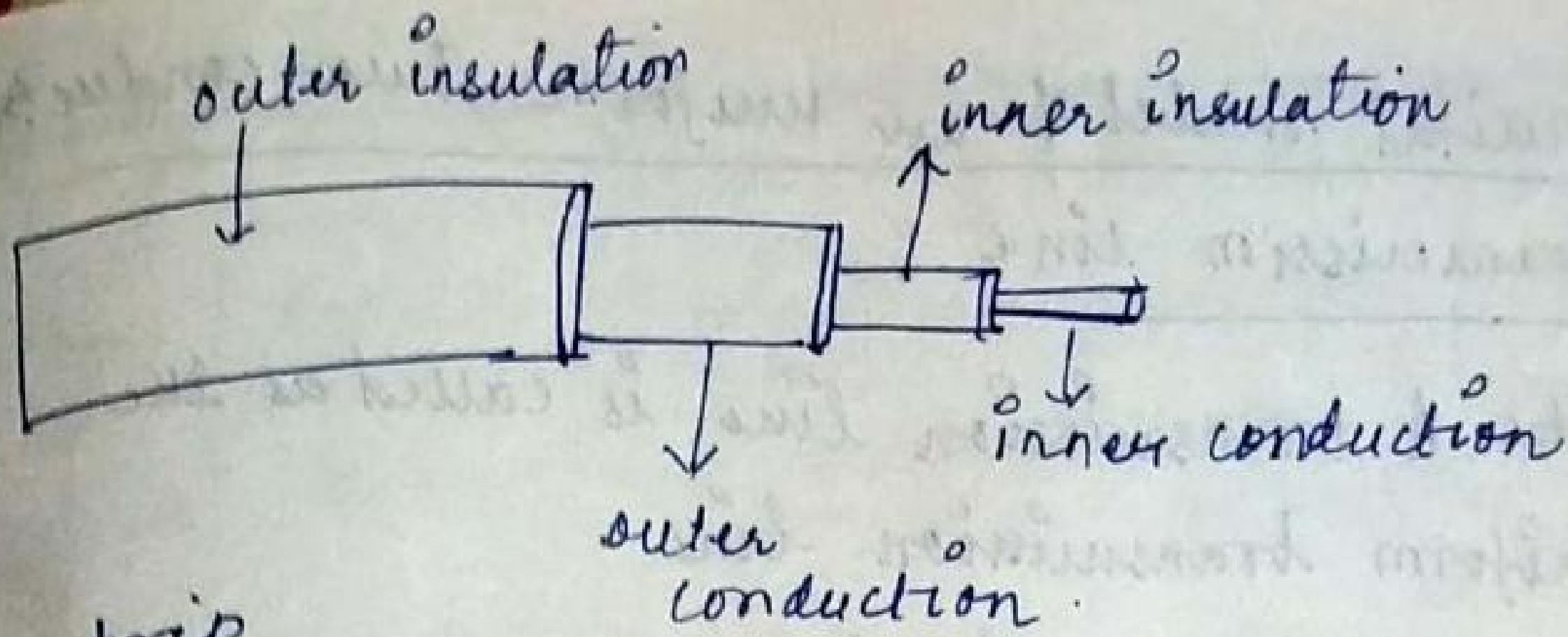
i) Parallel or Overhead Transmission Lines.

ii) Flat Type Transmission line



(iii) Tubular Type Transmission line

(iv) Coaxial Transmission line.

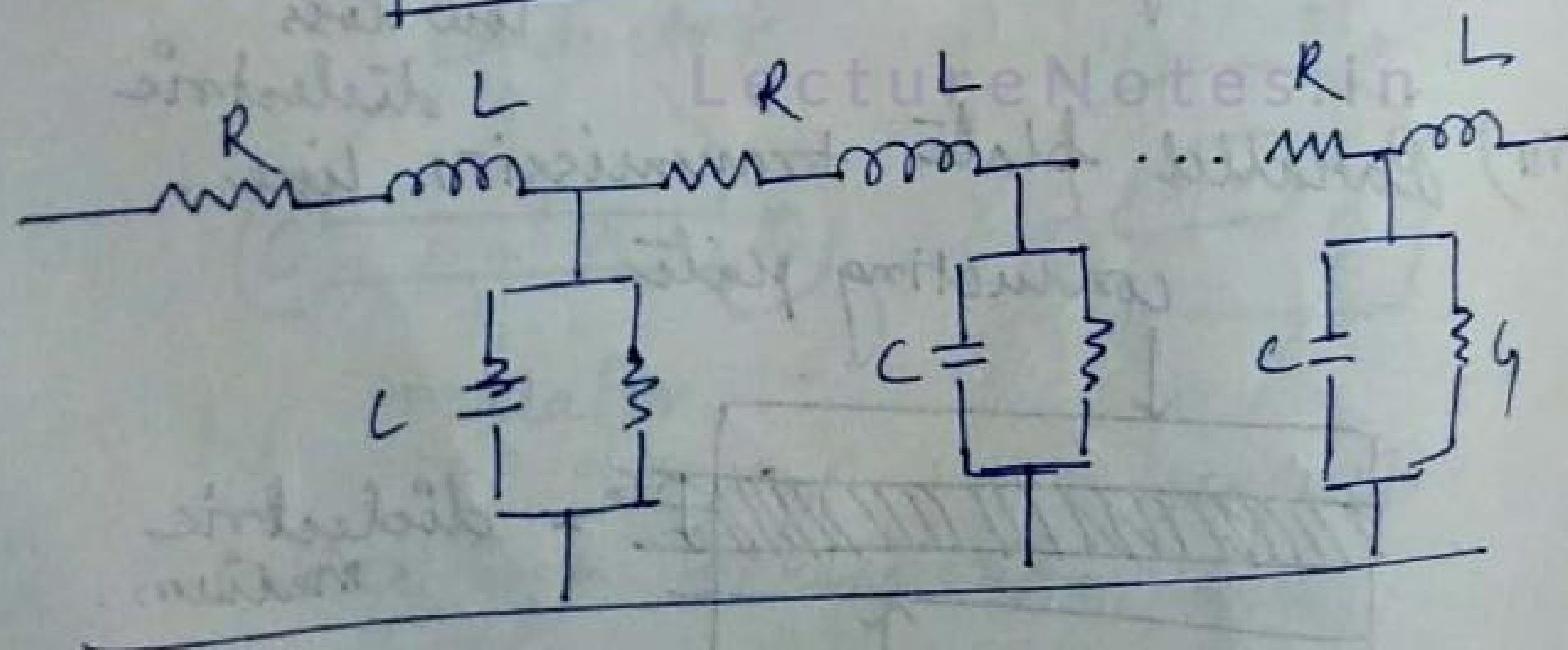
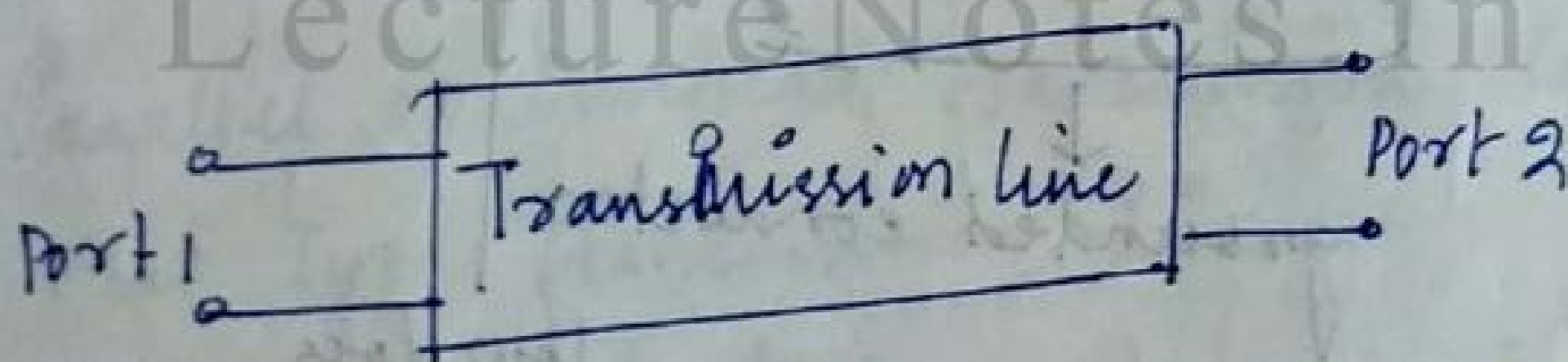
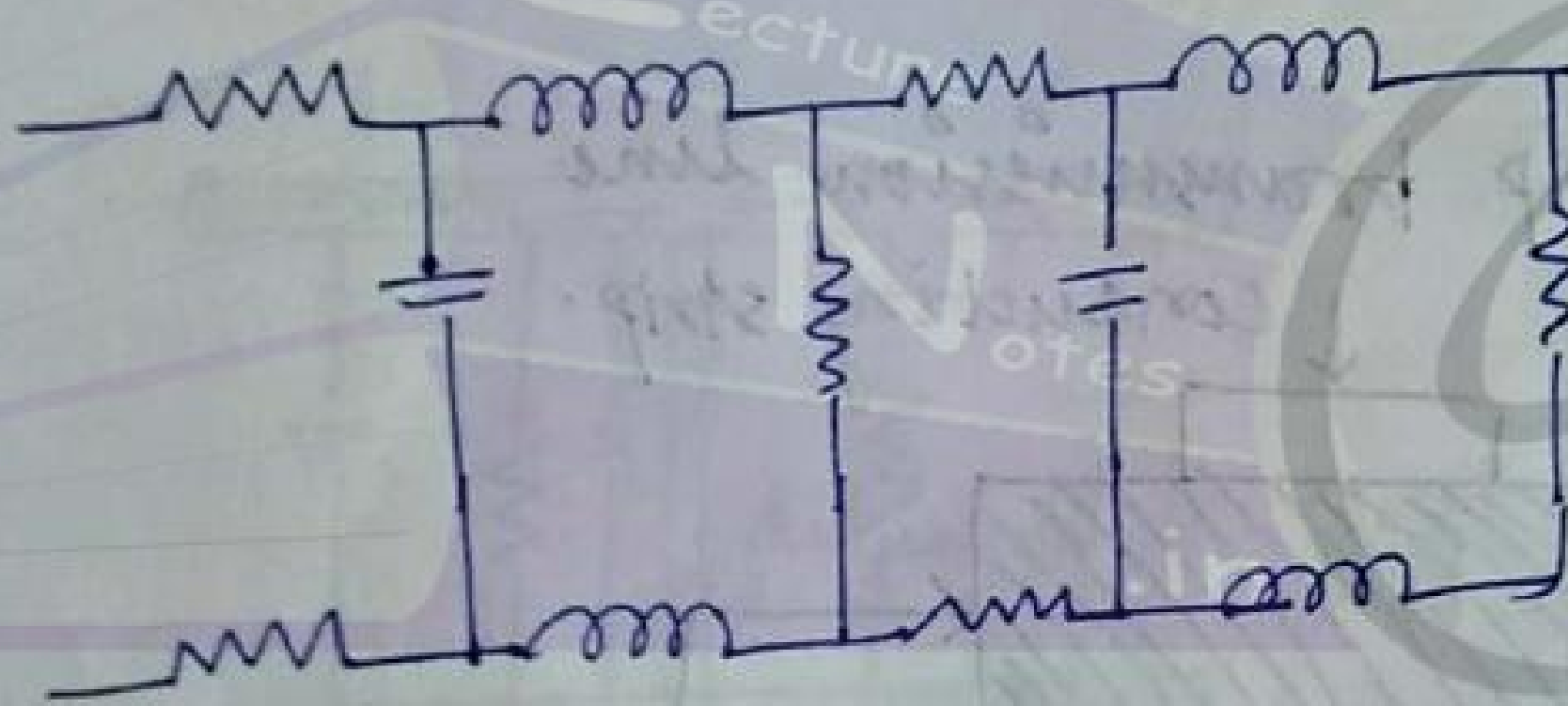


Circuit model of an uniform Two-conductor Transmission line :

A transmission line is called as the uniform transmission line.

- (i) The cross sectional area of the conductor & the spacing between conductors are uniform.
- (ii) Insulation between the conductors is uniform.

ex : Parallel wires, parallel plate, co-axial cable.



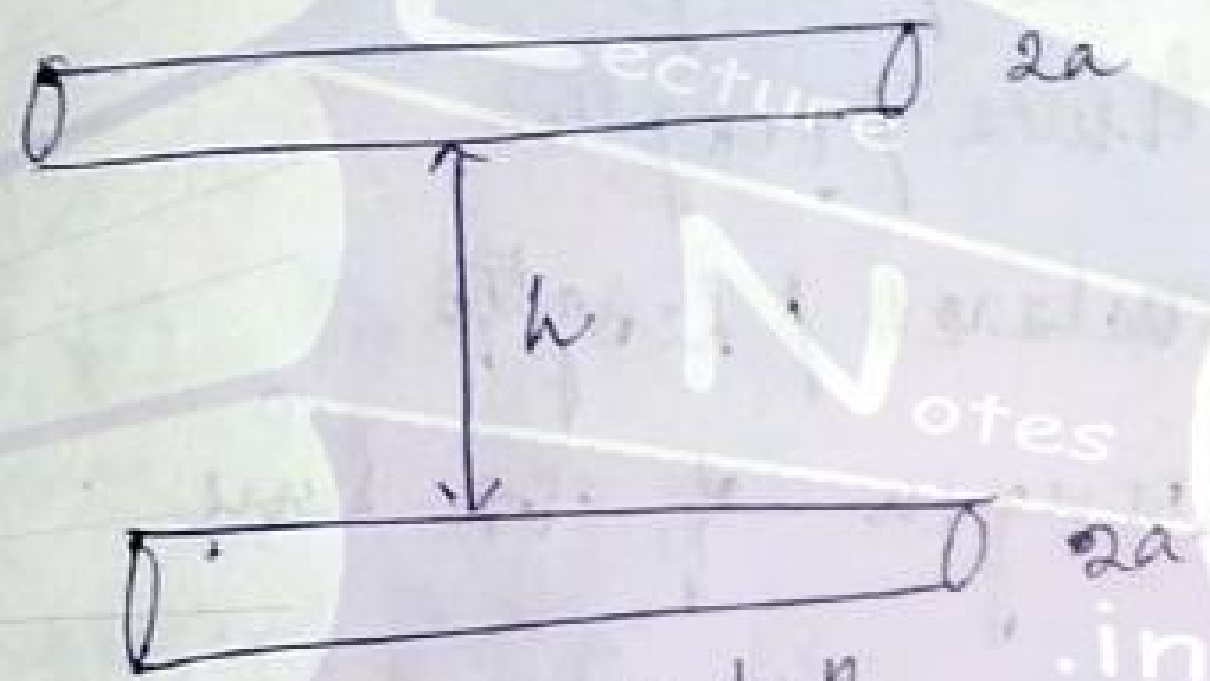
Lumped ckt. diagram.

~~Line types~~ ~~Resistance~~ ~~Inductance~~ ~~Capacitance~~ ~~Conductance~~

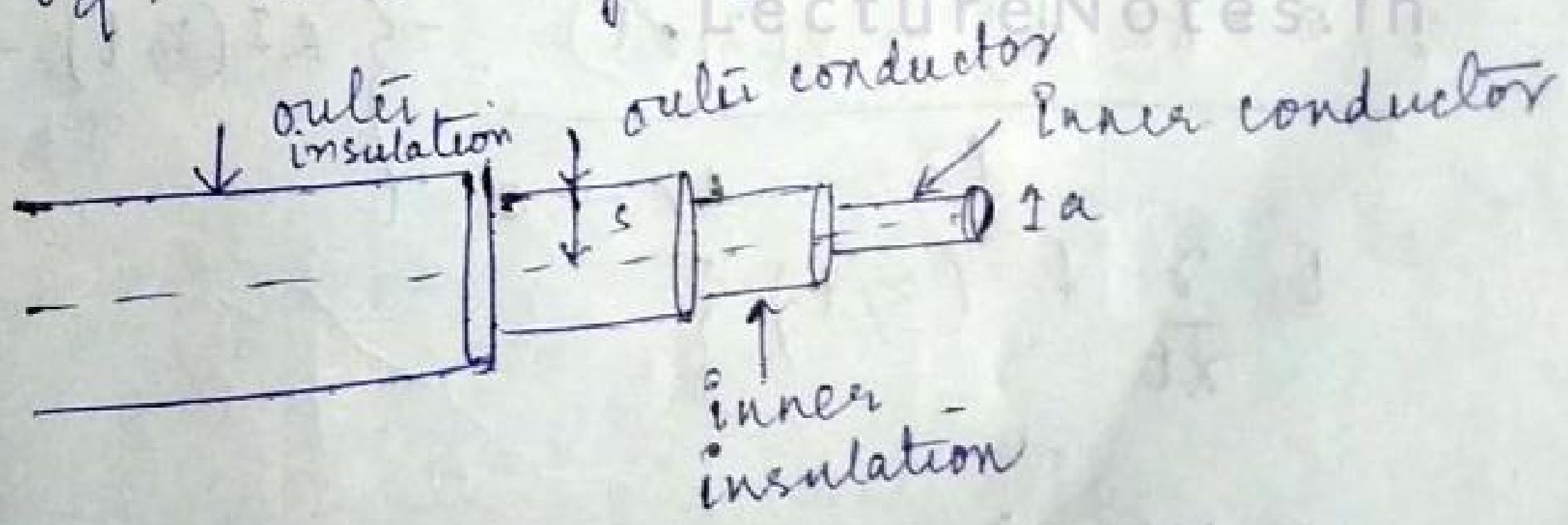
Line types	Resistance	Inductance	Capacitance	Conductance
Parallel wire	$\frac{1}{\pi a \delta \sigma_c}$	$\frac{\mu}{\pi} \ln \frac{h + \sqrt{h^2 + 4a^2}}{2a}$	$\frac{\pi \epsilon}{\ln \left[\frac{h + \sqrt{h^2 + 4a^2}}{2a} \right]}$	$\frac{\pi \sigma_d}{\ln(h/a)}$
coaxial	$\frac{1}{2\pi \delta \sigma_c} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{\mu}{2\pi} \ln \left(\frac{b}{a} \right)$	$\frac{2\pi \epsilon}{\ln(b/a)}$	$\frac{2\pi \sigma_d}{\ln(b/a)}$
parallel plate	$\frac{2}{w \delta \sigma_c}$	$\frac{\mu}{w}$	$\frac{\epsilon w}{s}$	$\frac{w \sigma_d}{s}$

10.04.19

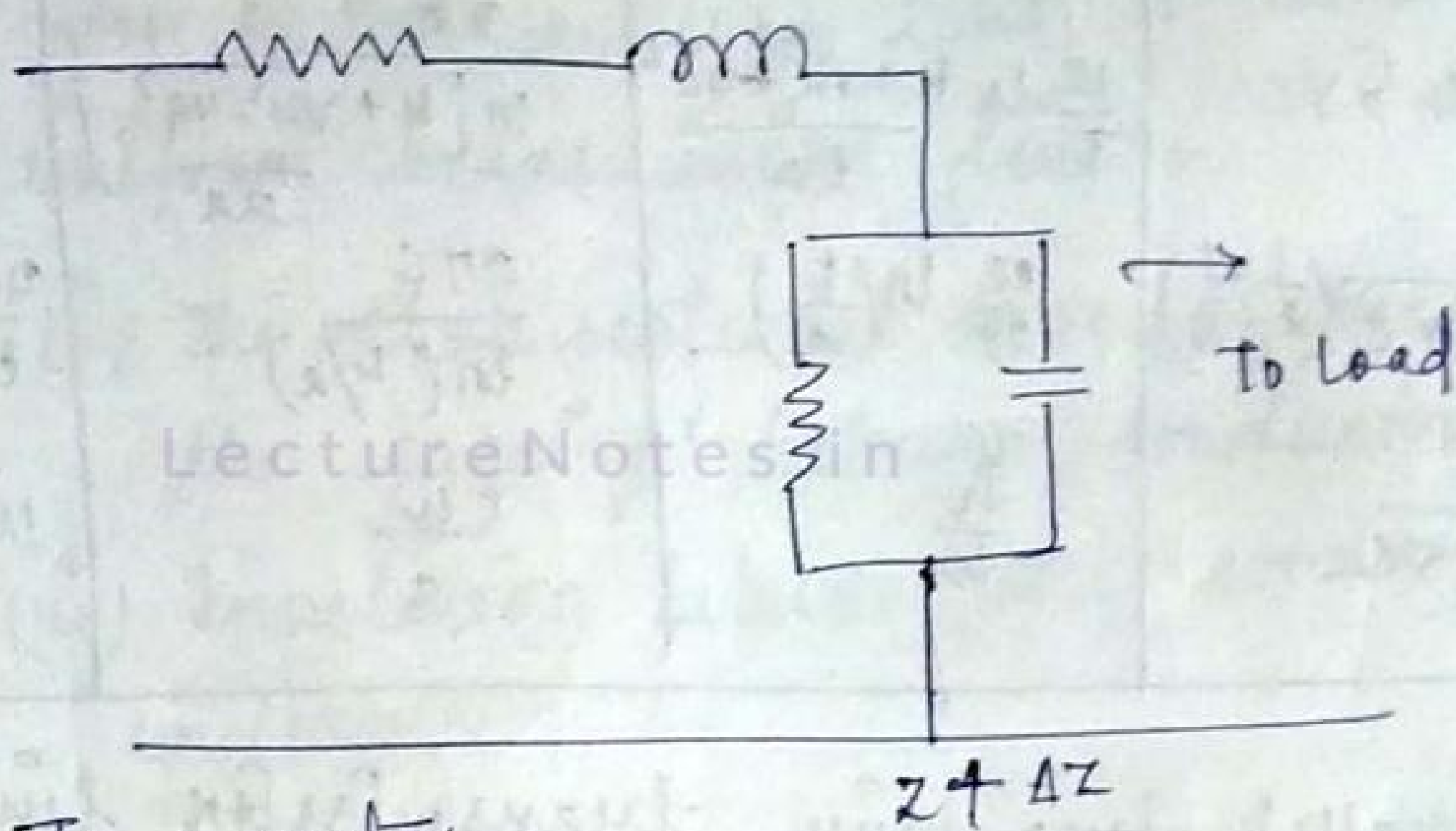
In a parallel wire transmission line
 $2a$ is the area of the wire.



- h = separation of two wires
- δ = skin depth = $\frac{1}{\sqrt{\pi f \mu \sigma_c}}$
- σ_c = conductivity of the conductor
- σ_d = conductivity of the dielectric



Transmission line equation



To generator

R = Resistance / length

L = Inductance / length

C = capacitance / length

G = conductance / length

Δz = Is the length of Tr. line.

Applying KVL to the outer loop

$$V(z, t) - R \Delta z I(z, t) - L \Delta z \frac{\partial I(z, t)}{\partial t} +$$

$$V(z + \Delta z, t) = 0$$

$$\frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = - \left\{ R I(z, t) + L \frac{\partial I(z, t)}{\partial t} \right\}$$

$\Delta z \rightarrow 0$

$$\frac{\partial V(z, t)}{\partial z} = - R I(z, t) - L \frac{\partial I(z, t)}{\partial t}$$

Applying KCL at Node :

$$I(z, t) = AI + I(z + \Delta z, t)$$

$$I(z, t) = G \Delta z V(z + \Delta z, t) + C \Delta z \frac{\partial V}{\partial t}(z + \Delta z, t) + I(z + \Delta z, t)$$

$$\frac{I(z + \Delta z, t) - I(z, t)}{\Delta z} = G V(z + \Delta z, t) + \left(\frac{\partial V}{\partial t} V(z + \Delta z, t) \right)$$

$$-\frac{\partial I}{\partial z}(z, t) = G V(z + \Delta z, t) + C \frac{\partial V}{\partial t}(z + \Delta z, t) \quad (2)$$

$$I(z, t) = \text{Re} [I_s(z) e^{j\omega t}]$$

$$V(z, t) = \text{Re} [V_s(z) e^{j\omega t}]$$

↑
harmonic form

$$\left. \begin{matrix} I_s(z) \\ V_s(z) \end{matrix} \right\} \text{ - phasor form}$$

$$-\frac{\partial I_s(z)}{\partial z} = G V_s(z) + C j\omega V_s(z) \quad (3)$$

$$-\frac{d}{dz} V_s(z) = R I_s(z) + -j\omega L I_s(z) \quad (4)$$

$$\rightarrow -\frac{d^2}{dz^2} V_s(z) = R \frac{d}{dz} I_s(z) + L j\omega \frac{d}{dz} I_s(z)$$

$$-\frac{d^2}{dz^2} V_s = (R + j\omega L) \frac{d I_s}{dz}$$

$$-\frac{d^2}{dz^2} V_s = (R + j\omega L)(-G + j\omega C) V_s(z)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Const of transmission line

$$\frac{d^2}{dz^2} V_s(z) = -\gamma^2 V_s(z)$$

$$\boxed{\frac{d^2}{dz^2} V_s(z) = -\gamma^2 V_s(z)}$$

Transmission potential eqⁿ (5)

$$\boxed{\frac{d^2}{dz^2} I_s(z) = -\gamma^2 I_s(z) = 0}$$

Transmission current eqⁿ

~~These~~ These eqⁿ are also called as Telegraph Equations or Transmission line equations.

Solⁿ of eqⁿ (5) :

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (7)$$

Solⁿ to eqⁿ (6)

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (8)$$

$+$ $\rightarrow$ +ve x-axis

$-$ $\rightarrow$ -ve x-axis

$$\boxed{V_s(z, t) = \text{Re} [V_s(z) e^{j\omega t}]}$$

$$\boxed{V_s(z, t) = V_0^+ e^{-\gamma z} \cos(\omega t - \beta z) + V_0^- e^{\gamma z} \cos(\omega t + \beta z)}$$

characteristic impedance (Z_0)

It is the ratio of positively travelling voltage ~~wave~~ ~~through~~ wave to positively travelling current wave.

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

Z_0 is given by $Z_0 = \frac{R + j\omega L}{\gamma} = \frac{V}{G + j\omega C}$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

where R_0 = characteristic resistance
 X_0 = characteristic reactance
 $Y_0 = \frac{1}{Z_0}$ = characteristic admittance

Case 1: lossless transmission line (ideal transmission line)

A transmission line is said to be lossless if the conductors of the transmission line are perfect ($\sigma_c = \infty$), and the dielectric medium separating them is lossless ($\sigma_d = 0$).

Hence $R = G = 0$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{j\omega L + j\omega C}$$

$$\gamma = j\omega\sqrt{LC} = \alpha + j\beta$$

$$\alpha = 0$$

$$\beta = \omega \sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \& \quad R_0 + jX_0$$

$$R_0 = \sqrt{\frac{L}{C}} \quad X_0 = 0$$

12.04.19 BPUT

Case 2: Distortionless Tx line.

$$\left[\frac{R}{L} = \frac{G}{C} \right]$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{R(1 + j\omega \frac{L}{R}) G(1 + j\omega \frac{C}{G})}$$

$$\gamma = \sqrt{RG} \left(1 + j\omega \frac{L}{R} \right) = \sqrt{RG} \left(1 + j\omega \frac{C}{G} \right)$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \sqrt{RG}$$

$$\beta = \sqrt{RG} \left(\frac{\omega L}{R} \right)$$

$$= \sqrt{RG} \left(\frac{\omega^2 L^2}{R^2} \right) = \omega \sqrt{\frac{L(RC)L}{R^2}}$$

$$\beta = \omega \sqrt{LC}$$

NOTE : In a distortionless line, α is frequency independent but β is dependent on frequency

$$\text{characteristic impedance } (Z_0) = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{L(1 + j\omega L/R)}{G(1 + j\omega C/G)}}$$

$$Z_0 = \sqrt{\frac{R}{G}} = R_0 + jX_0$$

$$R_0 = \sqrt{L/G} = \sqrt{L/C}$$

NOTE -

* A lossless line is also a distortionless line but a distortionless line is not necessarily lossless

Q7) An air line has a characteristic impedance (Z_0) of 70Ω & $\beta = 3 \text{ rad/m}$ at 100 MHz . Find out the inductance /m & capacitance /m of a line.

Ans) Air line $\rightarrow$ lossless line

$$R = G = 0$$

$$Z_0 = \sqrt{L/C}$$

$$\beta = \omega \sqrt{LC} =$$

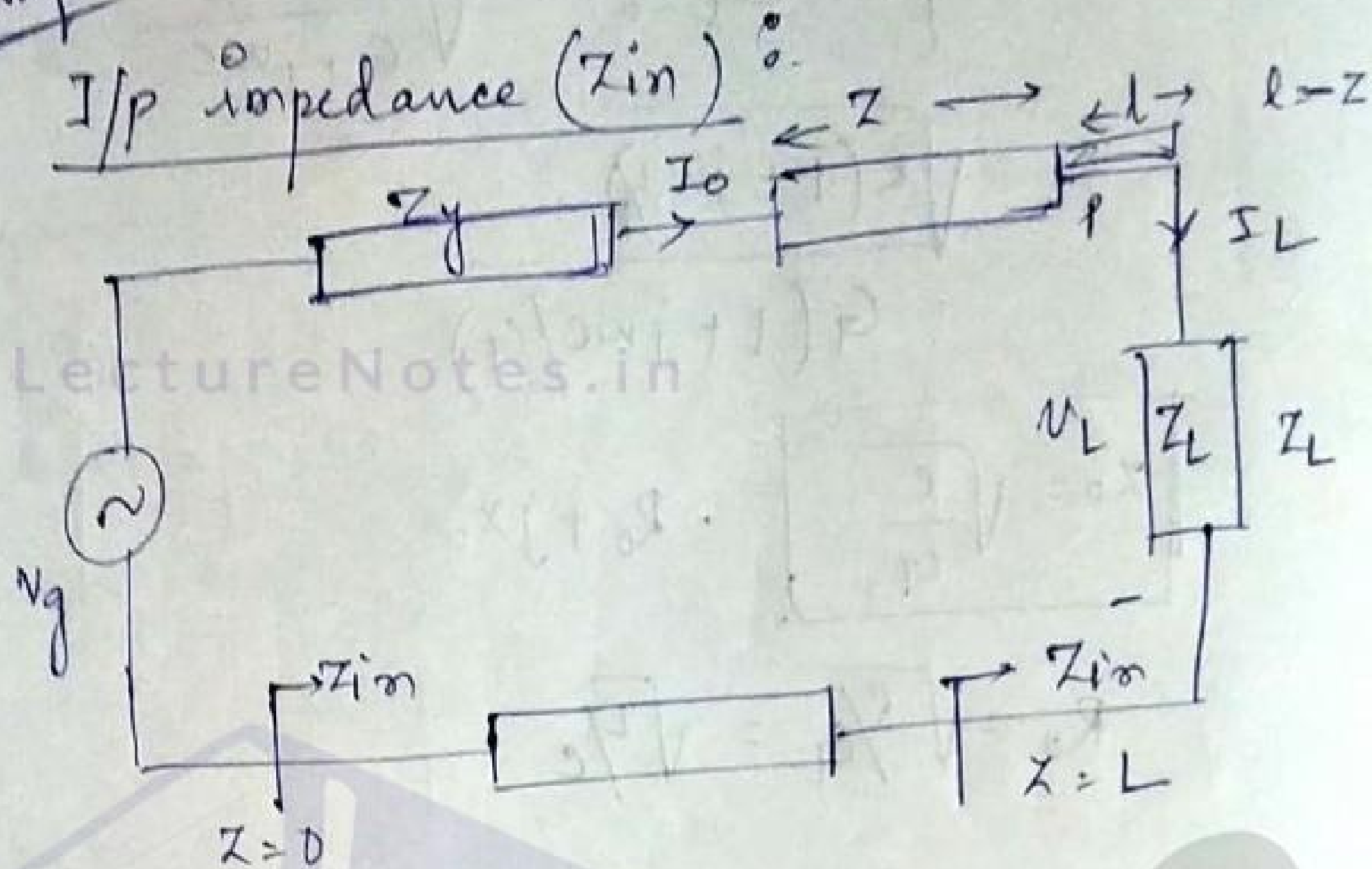
$$\left(\frac{\beta}{\omega}\right)^2 = LC = Z_0^2 \times C \times C$$

$$C = \sqrt{\left(\frac{\beta}{\omega}\right)^2 / Z_0^2} = \frac{\beta}{\omega Z_0} = \frac{3}{2\pi \times 100 \times 10^6 \times 70}$$

$$= 68.2 \text{ pF/m}$$

$$\begin{aligned}
 &= (90)^2 \times 68.2 \\
 &= 834.180 \Omega
 \end{aligned}$$

Imp.



Z_{in} = I/p Impedance

V_g = generation voltage

Z_g = generator impedance

where l is length of tr. line

Z_L is the load impedance

Z_0 is the characteristic impedance

γ is the propagation constant

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad \text{--- (1)}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} \Rightarrow I_0^+ = \frac{V_0^+}{Z_0}$$

$$Z_0 = \frac{-V_0^-}{I_0^-} \Rightarrow I_0^- = -\frac{V_0^-}{Z_0} = 0$$

$$I_s(z) = \left[\frac{V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}}{Z_0} \right] \quad (2)$$

* At the source / generator $z=0$

$$V_s(z) \big|_{z=0} = V_0$$

$$I_s(z) \big|_{z=0} = I_0$$

substitute in eqn (1) & (2)

$$(1) \quad V_0 = V_0^+ + V_0^- \quad (4)$$

$$(2) \quad I_0 = \frac{V_0^+ - V_0^-}{Z_0}$$

$$\boxed{V_0^+ = \frac{I_0 Z_0 + V_0}{2}} \quad (5)$$

similarly from (4) & (3)

$$\boxed{V_0^- = \frac{V_0 - I_0 Z_0}{2}} \quad (6)$$

At the load, $z=l$.

$$V_s(z) \big|_{z=l} = V_L; \quad I_s(z) \big|_{z=l} = I_L$$

$$(1) \quad V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \quad (7)$$

$$(2) \quad I_L = \frac{V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l}}{Z_0}$$

$$\therefore V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l} = I_L Z_0 \quad (8)$$

from (7) & (8).

$$V_0^+ = \frac{(V_L + I_L Z_0)}{2} e^{\gamma l}$$

$$V_0^- = \frac{(V_L - I_L Z_0)}{2} e^{\gamma l}$$

} (9)

} (10)

$$Z_{\text{input}} = \frac{V_L(z)}{I_L(z)}$$

At the generator ($z=0$)

$$Z_{\text{in}} = \frac{V_0^+ e^{\gamma L} + V_0^- e^{-\gamma L}}{\left(\frac{V_0^+ e^{-\gamma L} - V_0^- e^{\gamma L}}{Z_0} \right)} \Bigg|_{z=0}$$

LectureNotes.in

putting the value of V_0^+ & V_0^- from (5) & (6)

$$\boxed{Z_{\text{in}} = \frac{V_0}{I_0}}$$

Input impedance at any point :

$$Z_{\text{in}} = Z_0 \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right]$$

putting V_0^+ & V_0^- from eqn (9) & (10)

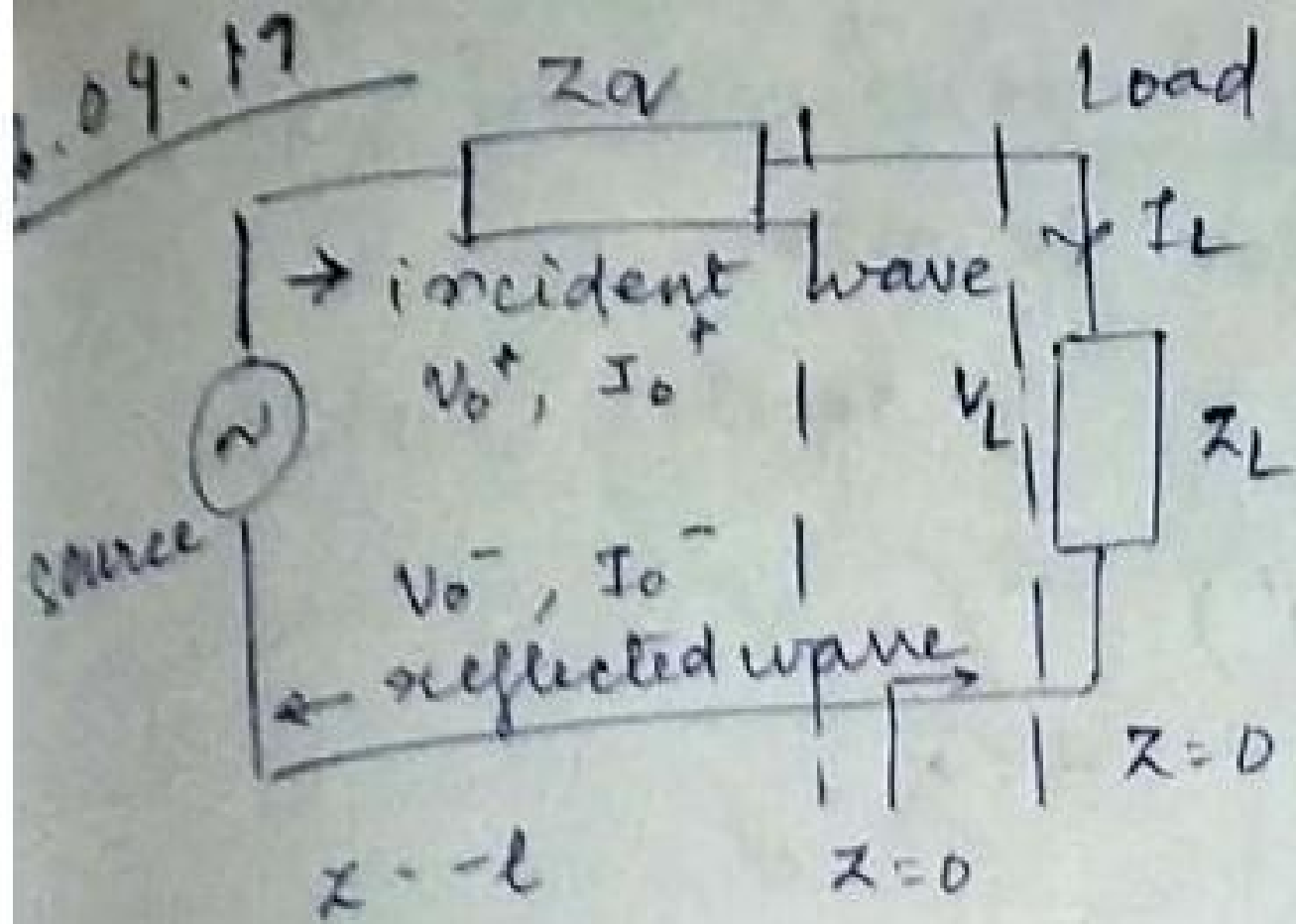
$$\boxed{Z_{\text{in}} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L} \right]}$$

Z_{in} at load with length l

$$Z_L = V_L / I_L$$

$$e^{\gamma L} + e^{-\gamma L} = 2 \cosh \gamma L$$

$$e^{\gamma L} - e^{-\gamma L} = 2 \sinh \gamma L$$



Discontinuity : $Z_L \neq Z_0$

If the impedance of the load is not equal to characteristic impedance of the transmission then the transmission line is said to be discontinuous. Hence, some part of the incident wave is reflected back & some part of wave will be absorbed by the load.

voltage reflection coefficient : It is the ratio of reflection voltage to the incident voltage.

$$\Gamma_L = \frac{V_0^- e^{-\gamma L}}{V_0^+ e^{-\gamma L}}$$

At the load, $V_0^- = (V_L - I_L Z_0) e^{-\gamma L}$

LectureNotes.in

$$V_0^+ = \frac{(V_L + I_L Z_0) e^{\gamma L}}{2}$$

from eqⁿ (1)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

where $V_L = I_L Z_L$

$$\Gamma(z) = \frac{V_0^- e^{\gamma L}}{V_0^+ e^{-\gamma L}} = \frac{V_0^-}{V_0^+} e^{2\gamma L}$$

$$\Gamma(z) = \frac{V_0^-}{V_0^+} e^{2\gamma L}$$

$$\text{At } l = l - l'$$

$$\Gamma'(z) = \frac{V_0^-}{V_0^+} e^{2\gamma(l-l')}$$

$$\Gamma'(z) = \frac{V_0^-}{V_0^+} e^{2\gamma L - 2\gamma L'}$$

$$\boxed{\Gamma(z) = \Gamma_L e^{-2\gamma L}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

At point l' = location on the Tr. line

Γ_L = Reflection coefficient at load

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Current Reflection coefficient

It is ratio of reflected ^{current} voltage to the incident voltage current

$$\frac{I_0^- e^{\gamma L}}{I_0^+ e^{-\gamma L}} = \frac{-V_0 / Z_0 e^{-\gamma L}}{V_0^+ / e^{-\gamma L}} = \frac{V_0^-}{V_0^+} e^{2\gamma L}$$

$$\boxed{[\Gamma_L]_i = [-\Gamma_L]_v}$$

$$(Z_L, Z_0) \rightarrow \Gamma$$

Voltage Transmission Coefficient

It is the ratio of voltage at the load to the incident voltage

$$\tau = \frac{V_L e^{j\beta L}}{V_0^+ e^{-j\beta L}}$$

(τ = amplitude of load voltage)

$$\tau = \frac{V_L}{\frac{(V_L + I_L Z_0) e^{j\beta L}}{2}}$$

where $V_0^+ = \frac{(V_L + I_L Z_0) e^{j\beta L}}{2}$ from eqⁿ (9)

At the load, $l = 0 = Z$

$$\tau = \frac{2V_L}{(V_L + I_L Z_0)} = \frac{2Z_L}{Z_L + Z_0}$$

$$\tau = \frac{2Z_L}{Z_L + Z_0} = 1 + \Gamma$$

$$\tau = 0.6 \quad \Gamma = 1 - \tau$$

Current Transmission Coefficient

$$\tau = \frac{I_L e^{j\beta L}}{I_0^- e^{-j\beta L}} = \frac{2Z_0}{Z_L + Z_0}$$

$$\tau = \frac{I_L}{I_0^-}, I_0^- = \frac{V_0^+}{2Z_0} = \frac{(V_L + I_L Z_0) e^{j\beta L}}{2Z_0}$$

At the load $Z = l = 0$

$$\tau = \frac{2I_L Z_0}{V_L + I_L Z_0} = \frac{2I_L Z_0}{I_L Z_L + I_L Z_0} = \frac{2Z_0}{Z_L + Z_0}$$

$$\tau = \frac{2Z_0}{Z_L + Z_0}$$

V. I_{mp}

$$V.R.C (P) = \frac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}}$$

if two tr. line are connected back to back.

Standing wave ratio :

$$SWR = \frac{V_{max.}}{V_{min}} = \frac{I_{max.}}{I_{min}}$$

It is the ratio of maximum voltage to the minimum voltage or maximum current to the minimum current.

$$V_{max} = V_0^+ + V_0^-$$

$$V_{min} = V_0^+ - V_0^-$$

$$SWR = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} = \frac{V_0^+}{V_0^+} \left[\frac{1 + V_0^-/V_0^+}{1 - V_0^-/V_0^+} \right]$$

$$VRC = \frac{V_0^- e^{2\gamma L}}{V_0^+ e^{-\gamma L}} = \frac{V_0^-}{V_0^+} e^{2\gamma L}$$

$$|VRC| = \frac{V_0^-}{V_0^+}$$

$$SWR = \frac{1 + VRC}{1 - VRC}$$

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}}$$

$$Z_0 = \frac{V_{max}}{I_{max}} \Rightarrow V_{max} = I_{max} Z_0$$

$$|Z_{in}|_{max} = \frac{I_{max} Z_0}{I_{min}}$$

$$|Z_{in}|_{max} = (SWR) Z_0$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0 I_{min}}{I_{max}} = \frac{Z_0}{SWR}$$

$$Z_0 = \frac{V_{min}}{I_{min}}$$

$$P_{av}^+ = \frac{V_0^+ I_0^+}{2} = \frac{(V_0^+)^2}{2Z_0}$$

$$I_0^+ = \frac{V_0^+}{Z_0}$$

$$P_{av}^- = \frac{V_0^- I_0^-}{2} = \frac{(V_0^-)^2}{2Z_0}$$

$$I_0^- = \frac{-V_0^-}{Z_0}$$

$$\frac{P_{av}^-}{P_{av}^+} = \frac{(V_0^-)^2}{2Z_0} \times \frac{2Z_0}{(V_0^+)^2} = \left(\frac{V_0^-}{V_0^+} \right)^2 = (VRC)^2$$

$$\frac{P_{av}^-}{P_{av}^+} = (VRC)^2$$

Transmitted Power

$$P_L)_{avg} = P_{av}^+ - P_{av}^-$$

$$\frac{P_L)_{avg}}{P_{av}^+} = 1 - \frac{P_{av}^-}{P_{av}^+}$$

$$\boxed{\frac{P_L)_{avg}}{P_{av}^+} = 1 - (VRC)^2}$$

17.04.19

Transmission Line :-

$$\frac{d^2 V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0$$

$$\frac{d^2 I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L} \right]$$

$$\boxed{VRC = \frac{Z_0 - Z_L}{Z_0 + Z_L}}$$

$$\boxed{CRC = -VRC}$$

Equations for numerical.

$$VTC = 1 + VRC$$

$$CTC = 1 - VRC$$

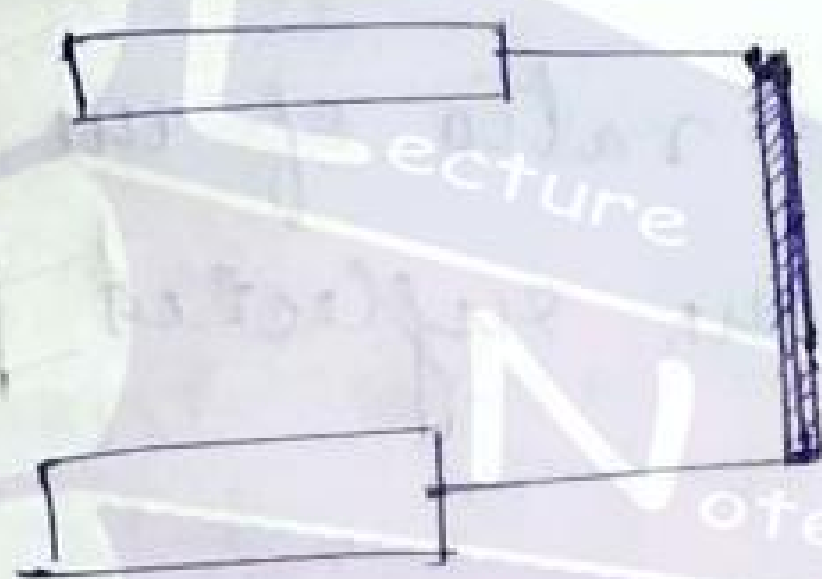
$$CTC = 1 + CRC$$

$$SWR = \frac{1 + VRC}{1 - VRC}$$

$$\frac{P_{av}^-}{P_{av}^+} = (VRC)^2$$

Case-1 :

Shorted Line ($Z_L = 0$)



$$Z_{in} = Z_0 \left[\frac{0 + Z_0 \tan h \gamma L}{Z_0 + 0} \right]$$

$$Z_{in} = Z_0 \left[\frac{Z_0 \tan h \gamma L}{Z_0} \right]$$

$$Z_{in} = Z_0 \tan h \gamma L$$

$$VRC = \frac{Z_0 - Z_L}{Z_0 + Z_L} = 1 = \frac{V_0^-}{V_0^+} \Rightarrow V_0^- = -V_0^+$$

The voltage at ~~the~~ ~~end~~ ~~of~~ ~~the~~ ~~line~~ ~~is~~ ~~totally~~ ~~reflected~~ ~~with~~ ~~change~~ ~~of~~ ~~sign~~ ~~and~~ ~~the~~ ~~wave~~ ~~is~~ ~~inverted~~

$$VRC = -VRC = -1 = \frac{I_0^-}{I_0^+} \Rightarrow \boxed{I_0^- = I_0^+}$$

• The current wave gets reflected without change in sign

• The standing wave ratio is 0 (zero)

$$\boxed{SWR = \frac{1 + VRC}{1 - VRC} = 0}$$

• The ratio of incident power = reflected power

When, the ratio of the average incident power to the reflected power is equal to 1.

$$\frac{P_{av}^-}{P_{av}^+} = (VRC)^2 = 1 \Rightarrow \boxed{P_{av}^- = P_{av}^+}$$

* Transmitted power is 0 (zero)

Case 2 :

open circuited line $[Z_L = \infty] \quad \frac{1}{\infty} = 0$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L} \right]$$

$$= Z_0 \left[\frac{\cancel{Z_L} (1 + Z_0/\cancel{Z_L} \tanh \gamma L)}{\cancel{Z_L} (Z_0/\cancel{Z_L} + \tanh \gamma L)} \right]$$

$$= Z_0 \left(\frac{1}{\tanh \gamma L} \right)$$

$$Z_{in} = Z_0 \coth \gamma L$$

$$VRC = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L \left[1 - \frac{Z_0}{Z_L} \right]}{Z_L \left[1 + \frac{Z_0}{Z_L} \right]} = 1$$

$$\Rightarrow \frac{V_o^-}{V_o^+} = 1$$

$$V_o^- = V_o^+$$

- The voltage wave gets reflected without change in sign

$$CRC = -VRC = -1 \Rightarrow I_o^- = -I_o^+$$

- The current wave gets reflected with change in sign

$$SWR = \frac{1 + VRC}{1 - VRC} = \infty$$

$$\frac{P_{av}^-}{P_{av}^+} = (VRC)^2 = 1 \Rightarrow \left. \begin{array}{l} \text{Transmitted} \\ \text{power} = 0 \end{array} \right\}$$

$$\boxed{\text{Incident power} = \text{Reflected power}}$$

Relatⁿ between open circuit (Z_{oc}) impedance & short circuit impedance (Z_{sc}):

$$Z_{oc} = Z_0 \coth \gamma L$$

$$Z_{sc} = Z_0 \tanh \gamma L$$

$$\boxed{Z_{oc} \times Z_{sc} = Z_0^2}$$

$$Z_{oc} \times Z_{sc} = Z_0 \frac{\cosh \gamma L}{\sinh \gamma L} \times Z_0 \frac{\sinh \gamma L}{\cosh \gamma L}$$

$$\Rightarrow \boxed{Z_{oc} \times Z_{sc} = Z_0^2}$$

LectureNotes.in

Case-3.

matched line

$$\boxed{Z_0 = Z_L}$$

Load impedance = characteristic impedance

In a perfectly matched transmission line, the characteristic impedance is equal to the load impedance

$$Z_{in} = Z_0 \left[\frac{Z_0 + Z_L \tanh \gamma L}{Z_L + Z_0 \tanh \gamma L} \right]$$

$$\boxed{Z_{in} = Z_0}$$

$$NRC = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \Rightarrow \frac{V_0^-}{V_0^+} = 0$$

$$\Rightarrow \boxed{V_0^- = 0}$$

- The path of the incident wave is reflected. The voltage is ~~adj~~ absorbed by the load.

$$CRC = -VRC = 0 \Rightarrow \boxed{I_0^- = 0}$$

also

$$\boxed{SWR = \frac{1 + VRC}{1 - VRC} = 1}$$

$$\frac{P_{av}^-}{P_{av}^+} = (VRC)^2 = 0 \quad \boxed{P_{av}^- = 0}$$

$$\boxed{P_{av}^+ = P_{av}} \text{ transmitted}$$

therefore $\frac{P_t}{P_{av}^+} = 1 - (VRC)^2 = 1$

$$\boxed{P_t = P_{av}^+}$$

Hence, the incident power is fully ~~abs~~ absorbed by the load.

Hence, max. power is transmitted when a transmission line is matched to the load.

Numerical :

- A transmission line of characteristic impedance 75Ω is terminated at a load of $(180 + j2000) \Omega$. Calculate VSWR and voltage reflection coefficient (VRC).

$$Z_0 = 75 \Omega$$

$$Z_L = (180 + j2000) \Omega$$

$$VRC = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{75 - 180 + j2000}{75 + 180 + j2000}$$

$$= \frac{180 + j2000 - 75}{75 + 180 + j2000 + 75}$$

$$= \frac{105 + j2000}{255 + j2000}$$

$$VRC = 1 \angle 4.26^\circ$$

$$VSWR = \frac{1 + VRC}{1 - VRC} = \frac{1 + 1 \angle 4.26}{1 - 1 \angle 4.26}$$

- ② The magnitude of o.c and s.c input impedances of a transmission line are 100Ω and 36Ω . Calculate the characteristic impedance (Z_0)?

$$Z_{oc} \times Z_{sc} = (Z_0)^2$$

$$100 \times 36 = 3600$$

$$\sqrt{3600} = Z_0$$

$$Z_0 = 60$$

Determine the input impedance of transmission line at a distance of $\lambda/8$ from a load of 400Ω

$$Z_0 = 100$$

$$Z_L = 400\Omega$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L} \right]$$

$$\beta L = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}$$

$$\gamma = \alpha + j\beta \quad \alpha = 0$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} \right]$$

$$Z_{in} = 100 \left[\frac{400 + j100 \tan \pi/4}{100 + j400 \tan \pi/4} \right]$$

$$Z_{in} = 100 \left[\frac{400 + j100}{100 + j400} \right]$$

$$Z_{in} = 100 \times \frac{412.31 \angle 14}{412.31 \angle 76}$$

$$Z_{in} = 100 \angle 80 - 14$$

$$Z_{in} = 100 \times 66$$

$$Z_{in} = 100 \times \angle 76 - 14$$

$$Z_{in} = 100 \times 62$$

$$Z_{in} = 6200$$

$$\tanh \gamma L = \frac{\sinh \alpha L \cos \beta L + j \cosh \alpha L \sin \beta L}{\cosh \alpha L \cos \beta L + j \sinh \alpha L \sin \beta L}$$

$$\tanh \gamma L = \frac{\tanh \alpha L + j \tan \beta L}{1 + j \tanh \alpha L \tan \beta L}$$

lossless $\alpha = 0$

$$\tan h \alpha L = j \tan \beta L$$

Q2) A line with $Z_0 = 100 \Omega$ is terminated with a load of $100 - j150 \Omega$. If the incident power is 200 mW. Determine the power transmitted through the load.

$$Z_0 = 100 \Omega$$

$$Z_L = (100 - j150) \Omega$$

$$P_{avg}^+ = 200 \text{ mW}$$

$$= 200 \times 10^{-6} \text{ W}$$

$$\frac{(P_{avg})}{P_{avg}^+} = 1 - (VRC)^2$$

$$VRC = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{100 - j150 - 100}{100 - j150 + 100}$$

$$= \frac{-j150}{200 - j150}$$

$$= \frac{150 \angle -90}{250 \angle -36.8}$$

$$= 0.6 \angle +36.8 \neq 90$$

$$= 0.6 \angle -53.2$$

$$= 1 - (0.6 \angle -53.2)^2$$

$$= 1 - 0.36$$

$$\frac{P_L]_{avg}}{P_{avg}^+} = 1 - (0.6)^2 = 1 - 0.36 = 0.64.$$

$$P_L]_{avg} = 0.64 P_{avg}^+$$

$$P_L]_{avg} = 0.64 \times 200 \times 10^{-6}$$

$$P_L]_{avg} = 128 \text{ mW}$$

③ A ~~distortionless~~ distortionless line has $Z_0 = 60 \Omega$, $\alpha = 20 \text{ mNp/m}$, velocity is 0.6 times of c [$v = 0.6c$]. Find the transmission line parameters & wavelength at 100 MHz.

$$Z_0 = 60 \Omega$$

$$\alpha = 20 \text{ mNp/m}$$

$$v = 0.6c$$

$$f = 100 \text{ MHz}$$

$$\begin{cases} R = ? \\ L = ? \\ C = ? \\ G = ? \end{cases} \quad \lambda = ?$$

distortionless line

$$\frac{R}{L} = \frac{G}{C}$$

$$RC = GL$$

$$\alpha = \sqrt{RG}$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\beta = \omega \sqrt{LC}$$

$$LC = v^2 \quad RG = \alpha^2$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\Rightarrow \frac{L}{C} = Z_0^2$$

$$\Rightarrow L = Z_0^2 C$$

$$\alpha = R/Z_0$$

Req

$$R = \alpha \times Z_0 = 20 \times 10^{-3} \times 60$$

$$R = 1.2 \, \Omega/m$$

$$RG = \alpha^2$$

$$G = \frac{\alpha^2}{R} = \frac{(20 \times 10^{-3})^2}{1.2} = 3.33 \, \text{mS/m}$$

$$\frac{Z_0}{n} = \frac{\sqrt{L/C}}{1/\sqrt{LC}} = \sqrt{L/C} \times LC = L$$

$$L = Z_0/v = \frac{60}{0.6 \times 3 \times 10^8} = 333 \, \text{nH/m}$$

$$L = 333 \, \text{nH/m}$$

$$\frac{R}{L} = \frac{G}{C}$$

$$C = \frac{GL}{R}$$

$$G = 3.33 \, \text{mS/m}$$

$$C = \frac{3.33 \times 10^{-6} \times 333 \times 10^{-9}}{1.2}$$

$$C = 92.4 \, \text{pF/m}$$

$$C = 92.4 \, \text{pF/m}$$

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{0.6 \times 3 \times 10^8}{100 \times 10^5}$$

$$\lambda = 1.8$$

- Q4) A 100 m long lossless transmission line at an inductance of $27.72 \mu\text{H}$ & capacitance of 18 nF .
 Determine (i) velocity of propagation
 (ii) phase constant at $f = 1000 \text{ KHz}$
 (iii) characteristic impedance of Transmission line.

lossless line $R=G=0$, $L = 27.72 \mu\text{H} = 0.2772 \mu\text{H/m}$

$C = 18 \text{ nF} = 0.18 \text{ nF/m}$

$$v = \frac{\omega}{\beta}$$

$$\beta = \omega \sqrt{LC}$$

$$v = \frac{\omega}{\omega \sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2772 \times 0.18 \times 10^{-6} \times 10^{-9}}}$$

$$v = 141.56 \times 10^6 \text{ m/sec}$$

$$\beta = \omega \sqrt{LC} = 2 \times \pi \times 1000 \times 10^3 \sqrt{0.2772 \times 0.18 \times 10^{-15}}$$

$$\beta = 4.43 \times 10^{-3} \text{ rad/m}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{27.72 \times 10^{-6}}{0.18 \times 10^{-9}}}$$

$$Z_0 = 392$$

$$Z_0 = 0.392 \times 10^{-6}$$