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|------------------------------------|-------------|
| 1. Static Electro-Magnetic Fields. | → 40% IES |
| 2. EM Waves | (9/10 ques) |
| 3. VI Waves - Transmission lines | 80% |
| 4. Guided Waves | (Gate) |
| 5. Radiated Waves - Antennas | 10% |

Text Books :-

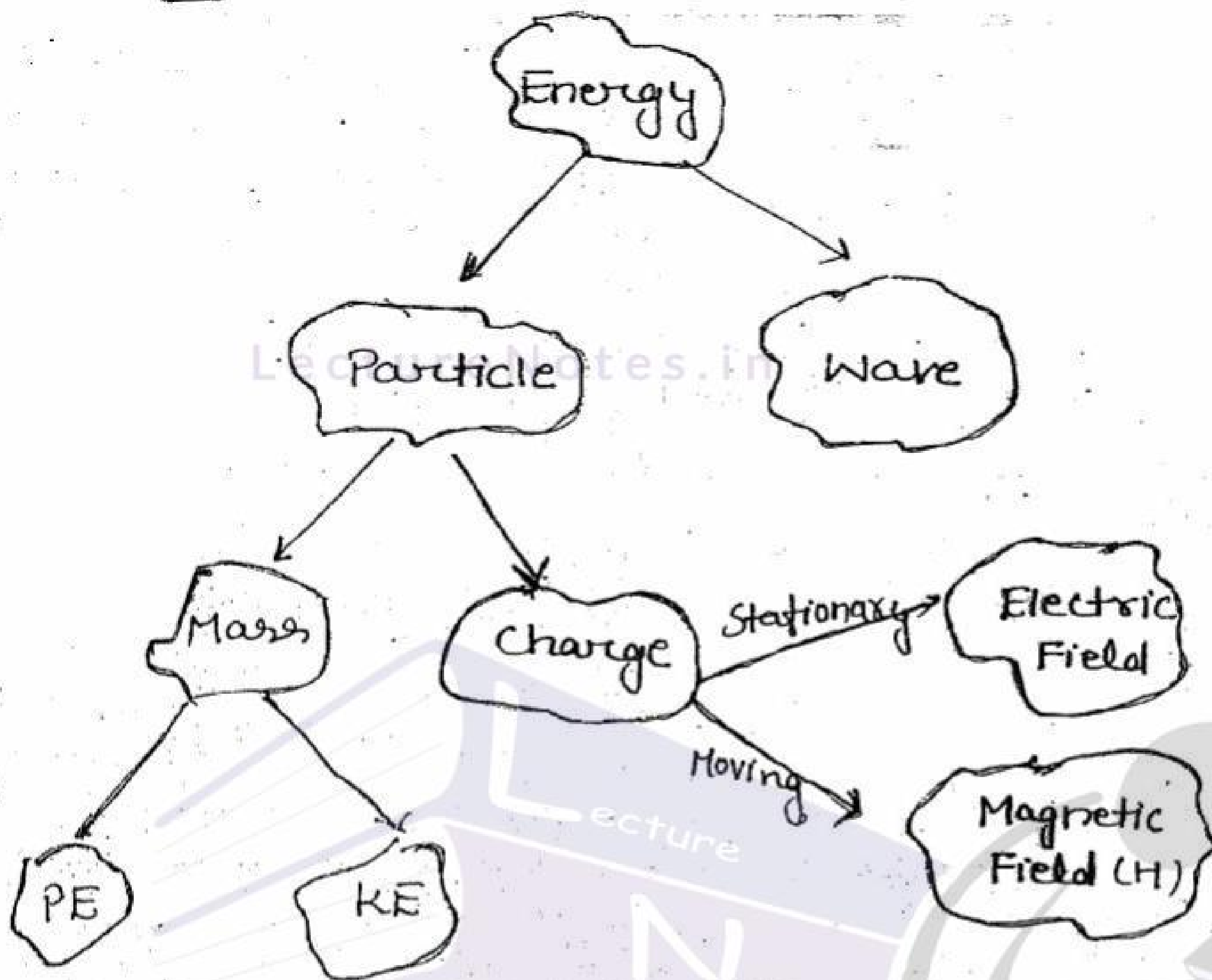
1. William Hayt (For theory of first chapter)
2. For vector calculus → N.S. Sadiku
3. For problems → Schaum Series
4. Jordan → For EM Waves & Guided Waves
5. John B. Ryder → For VI Waves
6. K. R. Prasad & Balanis → Antenna
7. Nathan Ida → For whole EMI

→ VSR S 22 @ GMAIL . COM

→ VSR Swresh

Lecture - 1

Static Electromagnetic Fields



Electric Field :-

It is a format of energy that is all around a charge & influences other charges nearby.

Magnetic Field (H) :-

It is a format of energy that is all around a moving charge & influences other moving charges nearby.

NOTE :-

The word influence in electric fields is a linear accelerating attractive or repulsive force on a charge resulting in straight line path. This is Coulomb's law and hence electric

moving charge particles

$$\vec{F} = Q\vec{E} = m\vec{a} \rightarrow \text{Linear path}$$

energised

The word influence in magnetic field is only on moving charges such that forces perpendicular to velocity and the field

$\vec{F} \perp \text{displacement}$

So that workdone = 0

This is Lorentz law and particle acquire circular path

$$\vec{F} = Q(\vec{v} \times \vec{B}) = \frac{mv^2}{r} \rightarrow \text{Circular path}$$

deflection



Summary :-

1) Charge is stationary then $\rightarrow$ Electric Field (Static)



$\rightarrow$ Time invariant

$\rightarrow$ Space Varying

eg:- (i) accumulated charge

(ii) B.C voltage

Electric field is energy in the above discussion

2. Charge is moving / flowing, without acceleration, or with constant velocity, or linearly with time

$$Q = kt$$

$$\frac{dQ}{dt} = k = I \quad (\text{DC current})$$

and DC current cause magnetic field which is static in nature

→ Magnetic field is the energy in above discussion.

3. Charge is moving with acceleration which creates Electric field $E(t)$ as well as magnetic field $H(t)$ which are power.

Basic Terms & Definitions :-

There is a measure of the field strength at any point in the field.

- (i) $\vec{E} \rightarrow$ Electric field Intensity (V/m or N/C)

↓
per unit length

- (ii) $\vec{H} \rightarrow$ Magnetic field intensity (Amp/m)

- (iii) $\vec{B} \rightarrow$ Magnetic flux density $\left[\begin{array}{l} \text{Weber/m}^2 \\ \text{or} \\ \text{Tesla} \end{array} \right]$

- (iv) $\vec{S} \rightarrow$ Electric flux density (Coulombs/m²)

↓
per unit area

- (v) $\epsilon \rightarrow$ Permittivity of the medium

The ability to permit / allow / hold electric field in that medium

(II) Current Carrying Wire :-

→ Scalar

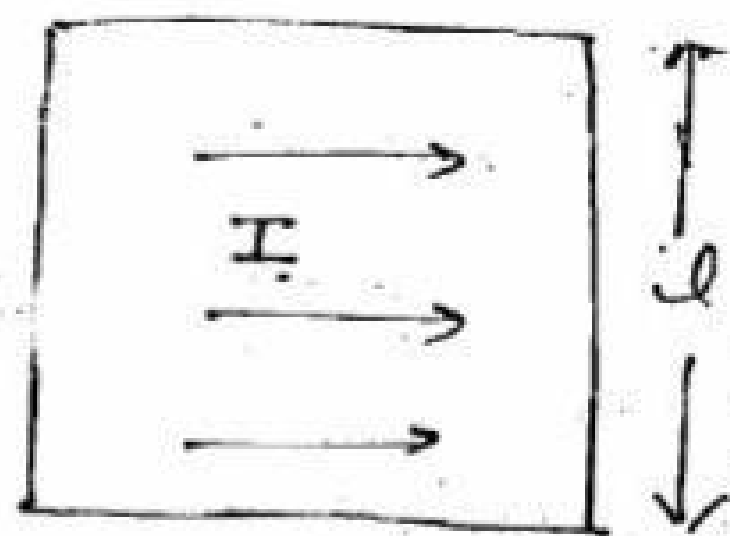
→ Amp



(III) Surface Current :-

→ $\vec{k} = \frac{dI}{d\ell} = \text{Amp/m}$

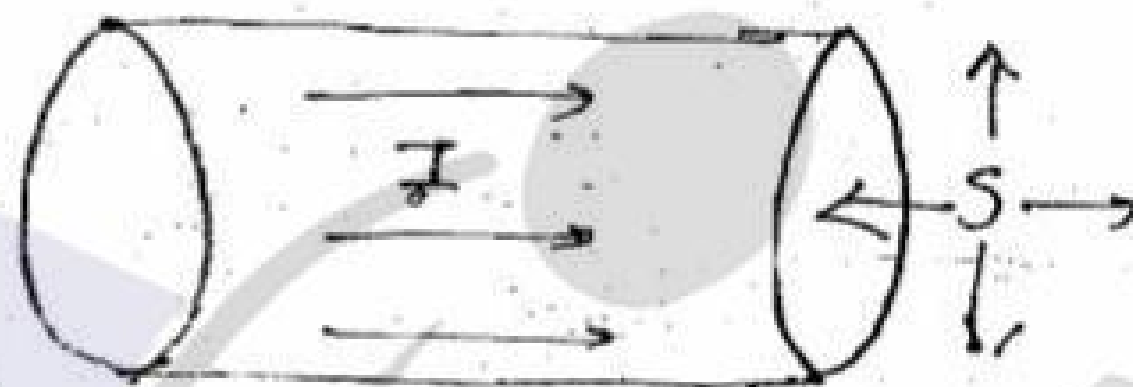
→ vector



(IV) Solid Conductor (J) :-

→ vector

→ $J = \frac{dI}{ds} = \text{Amp/m}^2$



Vector Calculus :-

It is a study of directional integrations & directional derivatives.

Directional Integration :-

It is the study of the total effects or cumulative effects of a phenomenon in a specific direction in a specific region.

Directional derivative :-

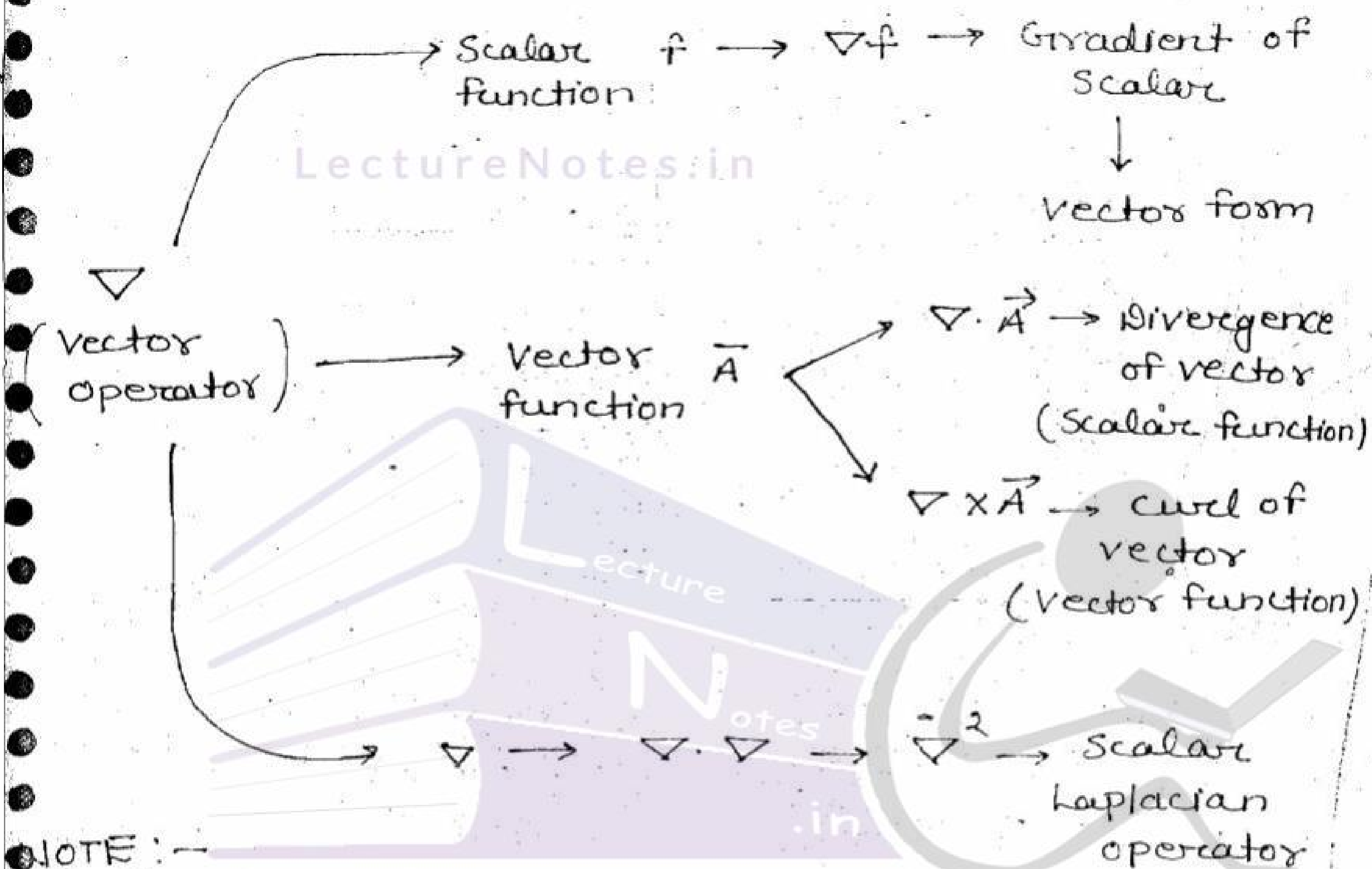
It is the study of the instantaneous or rate of change analysis of a phenomenon in a specific direction in a specific region.

eg:- Del-operator

$$\nabla \rightarrow \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

→ It is used to study the rate of change of various space varying quantities in 3d-space

• It is called as vector spatial derivative operators



NOTE: -

Vector Identities: -

$$\nabla \times \nabla f = 0$$

$$\nabla \times (\nabla f) = 0$$

$$\text{Curl (Grad. of scalar)} = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\text{Div. (Curl of vector)} = 0$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Divergence & Outflow:-

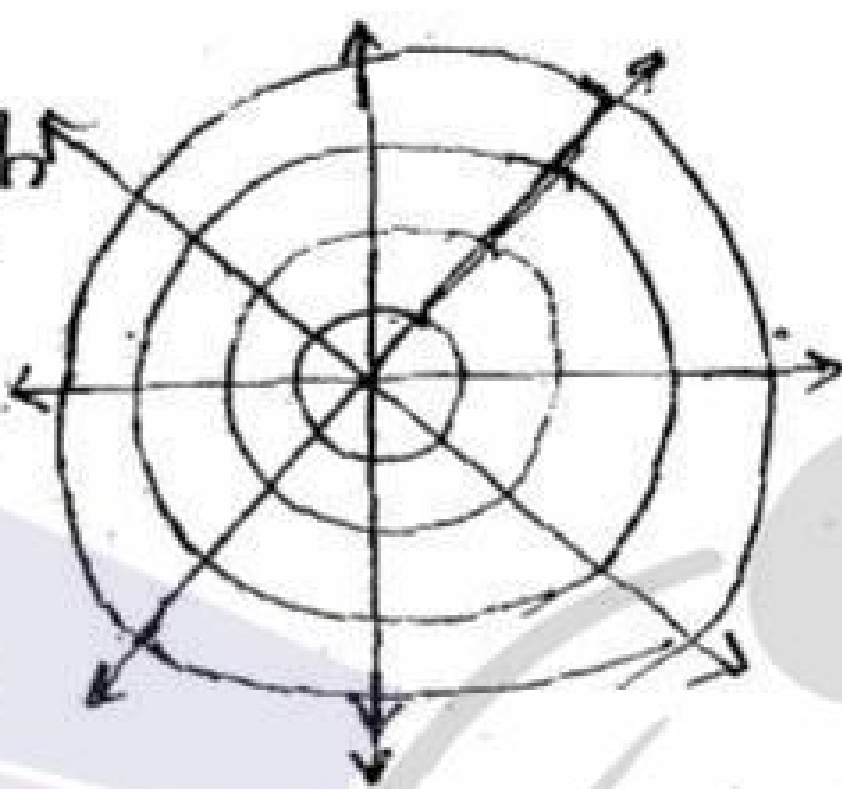
1) Consider a cause or source which has effects spread outward from the cause

eg:- (i) light from a bulb

(ii) air velocity from a punctured tyre

For all such phenomenon the strength dec. as the area of expansion inc. such that total outflow is same

The total outflow through any enclosing surface is always the same (constant) and this constant depends on central cause



$$\begin{aligned}\text{Total Outflow} &= \uparrow \text{Strength} \times \text{Area} \downarrow \\ &= \text{constant} \propto \text{Cause}\end{aligned}$$

The strength for all such phenomenon can be expressed as the constant per unit area of cause/area i.e. called as density.

NOTE:-

1) If the cause is a coulomb charge then the repulsive force or attractive force is called as electric flux and the strength is called as flux density (D) such that

$$\oint D \cdot dS = \psi_e \propto Q$$

total

$\Rightarrow$

$$\boxed{\oint D \cdot dS = Q}$$

If proportionality constant is 1 then this called Gauss law in integral form

NOTE 1-

→ If the surface is not completely enclosing then the effects are the partial flux crossing i.e.

$$\int \mathbf{D} \cdot d\mathbf{s} = \Psi_e \neq \Psi_e^{\text{total}}$$

This is not a Gauss law

→ Every vector closed surface is identified by a finite scalar volume

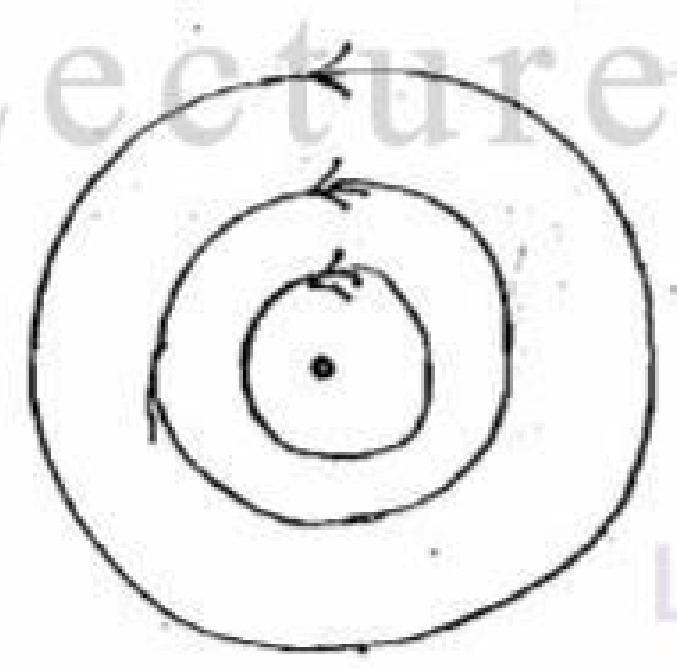
eg:-

$4\pi r^2$	<u>Sphere</u>	$\frac{4}{3}\pi r^3$
$2\pi r h$	<u>Cylinder</u>	$\pi r^2 h$
$6a^2$	<u>Cube</u>	a^3

→ Closed surface having direction while finite volume doesn't having direction

Circulation and curl :-

Cause or source →
Effects → around the cause



→ The total circulation in any closed length is always a constant and this constant depends on the central cause.

eg:- air velocity under the fan

$$\begin{aligned} \text{Total circulation} &= \text{Strength} \downarrow \times \text{length} \uparrow \\ &= \text{constant} \propto \text{Cause} \end{aligned}$$

$$\text{Strength} = \frac{\text{constant}}{\text{length}} \text{ or } \frac{\text{Cause}}{\text{length}}$$

→ Intensity

Cause or Source → I ampere current

Effects → Magnetic line field

Strength → Magnetic field intensity ($\vec{H}$)

$$\oint_{\text{closed}} \vec{H} \cdot d\vec{l} = I$$

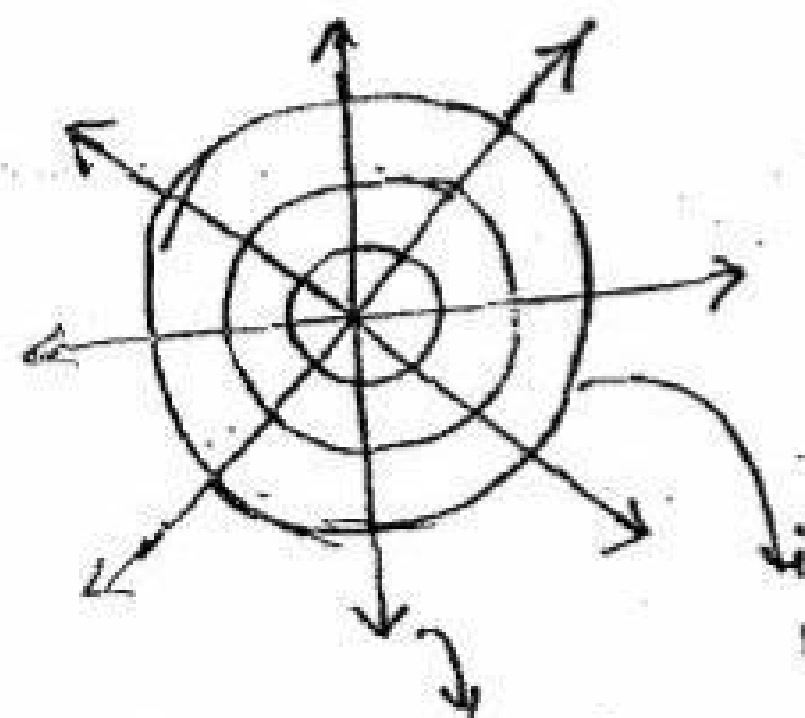
This is ampere's law in integral form

Note - 1 :-

Every closed line is identified with a finite area enclosed → vector in

$$\begin{aligned} 2\pi r & \xrightarrow{\text{circle}} \pi r^2 \\ 4a & \xrightarrow{\text{square}} a^2 \end{aligned}$$

Summary 1 :-



$$\psi_e = Q \quad \text{i.e.} \quad \text{outflow} = \text{Strength} \times \text{area} = \text{Cause}$$

→ Electric flux has the units of coulombs

is constant & direction of divergence

→ closed surface element

Strength around the cause $\vec{B} = \frac{d\phi}{ds} = \frac{c}{m^2}$

where ds = any closed surface element

Strength at the cause $= \frac{d\phi}{dv} = \frac{d}{dl} \left(\frac{d\phi}{ds} \right)$

$= \nabla \cdot \vec{B} = \text{Divergence of } \vec{B}$

$= \frac{\text{outflow}}{\text{volume}} = \rho_v$

$$\nabla \cdot \vec{B} = \rho_v$$

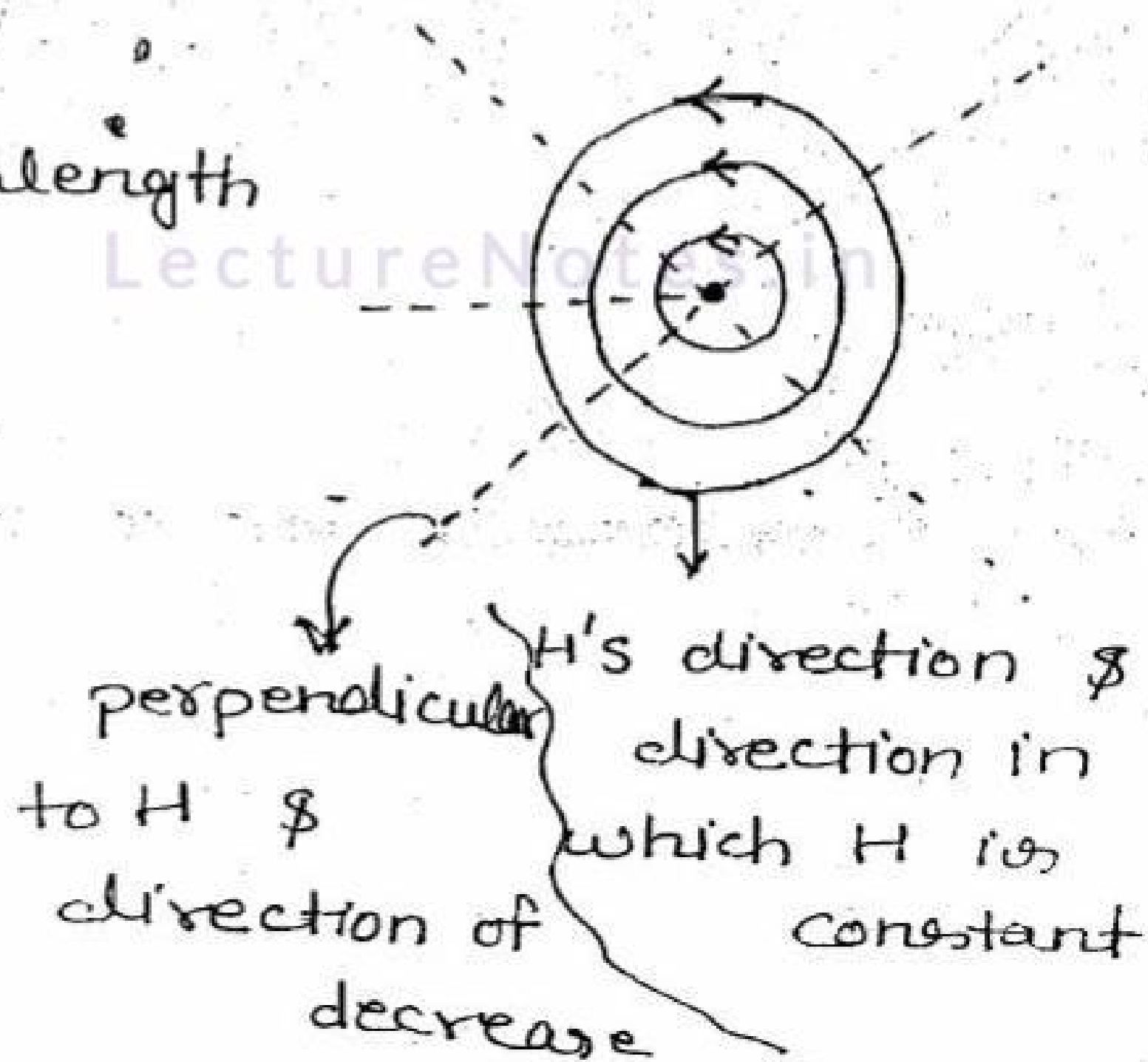
→ Gauss law in point form

→ The $(\cdot)$ dot product signifies that $\vec{B}$'s change or derivative exists only when we move in direction of $\vec{B}$. This is called as directional derivative.

→ The $(\cdot)$ dot product indicates a surface (vector) derivative resulting in volume (scalar) derivative.

Summary 2:-

Total Circulation = Strength $\times$ length
= current



$$|OA| = |OB| \cos \phi$$

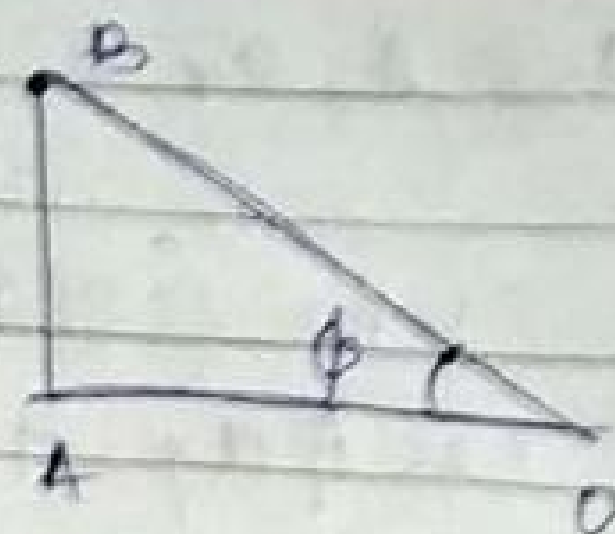
$$|\vec{r}_A| = \cos \phi$$

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{AB}$$

$$a_y = \cos \phi a_f + \sin \phi (-a_\phi)$$

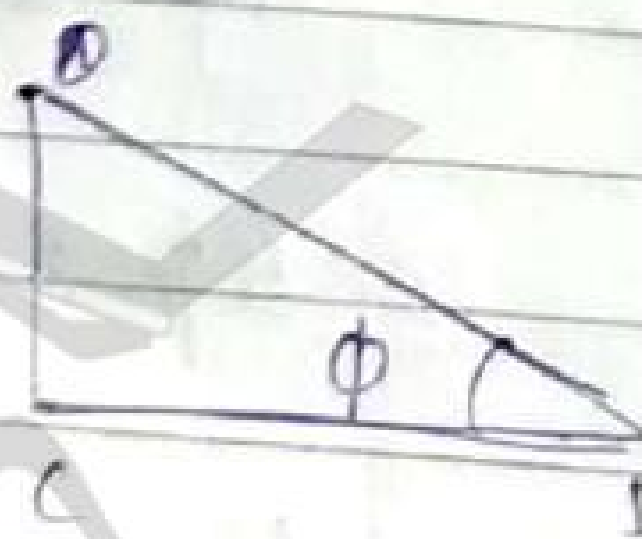
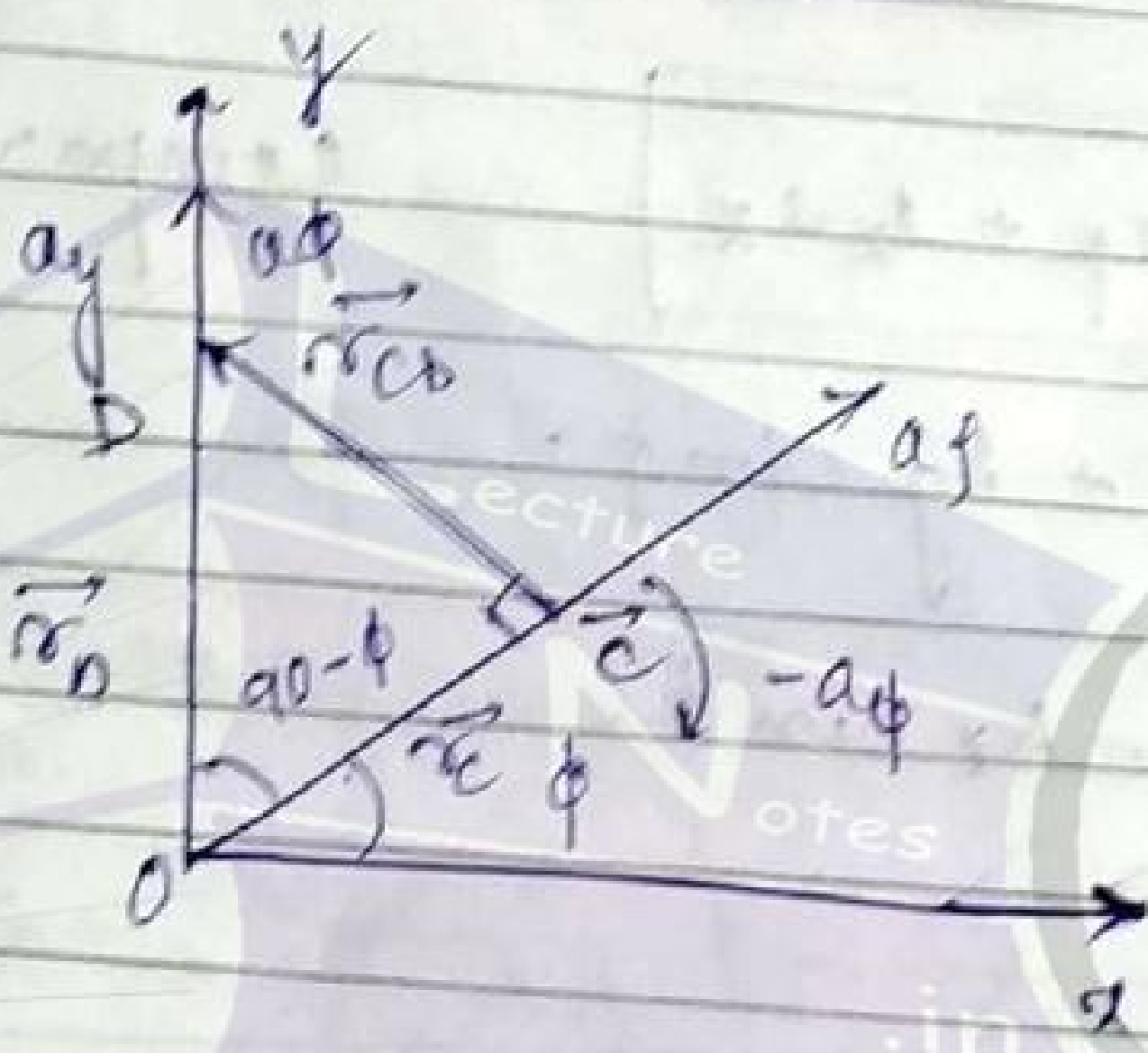
$$a_y = \cos \phi a_f - \sin \phi a_\phi$$



$$\sin \phi = \frac{AB}{OB}$$

$$|\vec{r}_{AB}| = |\vec{r}_{AB}| \sin \phi$$

$$|\vec{r}_{AB}| = \sin \phi$$



$$\vec{OB} = \vec{OC} + \vec{CB}$$

$$\vec{r}_B = \vec{OB} = |\vec{OB}| a_y$$

$$\vec{r}_B = a_y$$

$$\vec{r}_B = \vec{r}_C + \vec{r}_{CB}$$

$$a_y = |\vec{r}_C| a_f + |\vec{r}_{CB}| a_\phi$$

$$a_y = \sin \phi a_f$$

$$\cos \phi = \frac{|CB|}{|OB|}$$

$$CB = \cos \phi$$

$$\sin \phi = \frac{OC}{OB}$$

$$OC = \sin \phi$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$= A_x (\cos \phi \hat{a}_f - \sin \phi \hat{a}_\phi) + A_y (\sin \phi \hat{a}_f + \cos \phi \hat{a}_\phi) + A_z \hat{a}_z$$

$$= A_x \cos \phi \hat{a}_f - A_x \sin \phi \hat{a}_\phi + A_y \sin \phi \hat{a}_f + A_y \cos \phi \hat{a}_\phi + A_z \hat{a}_z$$

$$\vec{A} = (A_x \cos \phi + A_y \sin \phi) \hat{a}_f + (A_y \cos \phi - A_x \sin \phi) \hat{a}_\phi + A_z \hat{a}_z$$

$$\vec{A} = A_f \hat{a}_f + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

comparing both eqⁿ.

$$A_f = A_x \cos \phi + A_y \sin \phi$$

$$A_\phi = A_y \cos \phi - A_x \sin \phi$$

$$A_x = A_z$$

$$\begin{bmatrix} A_f \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\vec{A}(x, y, z) \longleftrightarrow \vec{A}(f, \phi, z)$$

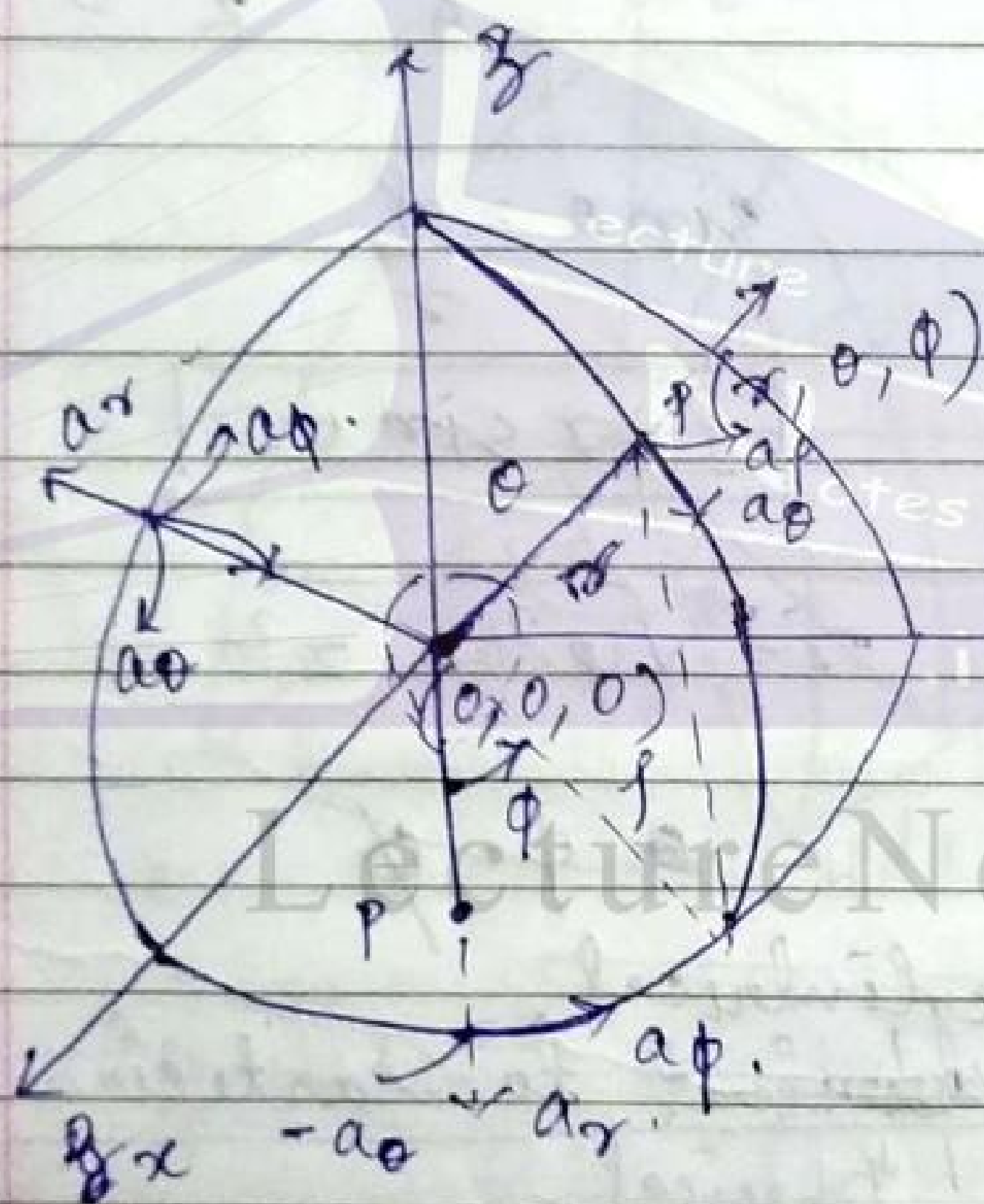
$$a_x = \cos \phi a_r - \sin \phi a_\phi \quad (\text{if given})$$

$$\left| a_y \cdot a_\phi = \frac{\sin \phi}{\cos \phi} \right| \Rightarrow (\sin \phi a_r + \cos \phi a_\phi) \cdot a_\phi = \sin \phi$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix}$$

$$\vec{r} = r \hat{e}_r \Rightarrow$$

Spherical coordinate : (r, θ, ϕ)



$$\begin{cases} 0 \leq r \leq \infty \\ 0 \leq \theta \leq \pi \\ 0 \leq \phi \leq 2\pi \end{cases}$$

- r is the position vector from the origin to the point P .

$$P = (r, \theta, \phi)$$

θ = co-altitude angle

$$\vec{r} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

- It is the angle measured from the z-axis to the position vector (R) is called co-altitude angle (θ).

~~Range~~

- ϕ is the azimuthal angle measured from the x-axis

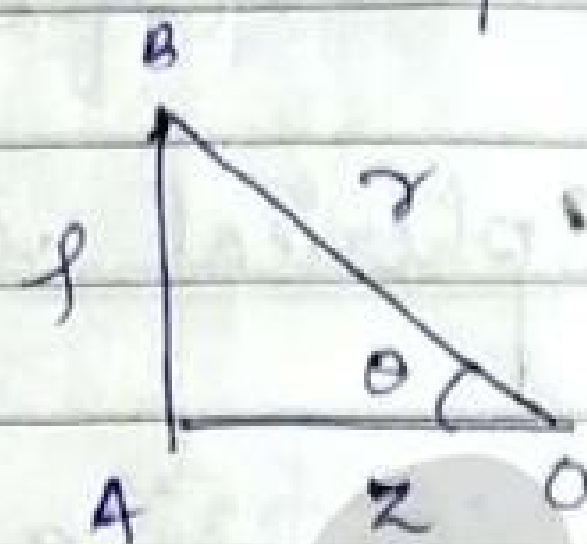
22.01.19

Conversion from cylindrical to spherical

$$(\rho, \phi, z) \rightarrow (r, \theta, \phi)$$

$\triangle OAB$

$$\cos \theta = \frac{z}{r}$$



$$z = r \cos \theta$$

$$\sin \theta = \frac{\rho}{r}$$

$$\tan \theta = \frac{\rho}{z}$$

$$\rho = r \sin \theta$$

$$\theta = \tan^{-1} \left(\frac{\rho}{z} \right)$$

$$r = \sqrt{\rho^2 + z^2}$$

$$\cancel{r} = \cancel{\tan^{-1} \left(\frac{\cancel{\rho}}{\cancel{z}} \right)}$$

$$\phi = \phi$$

Conversion from ~~spherical~~ cylindrical to cartesian

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

Cartesian to spherical

$$(x, y, z) \rightarrow (r, \theta, \phi)$$

$$x = r \cos \phi$$

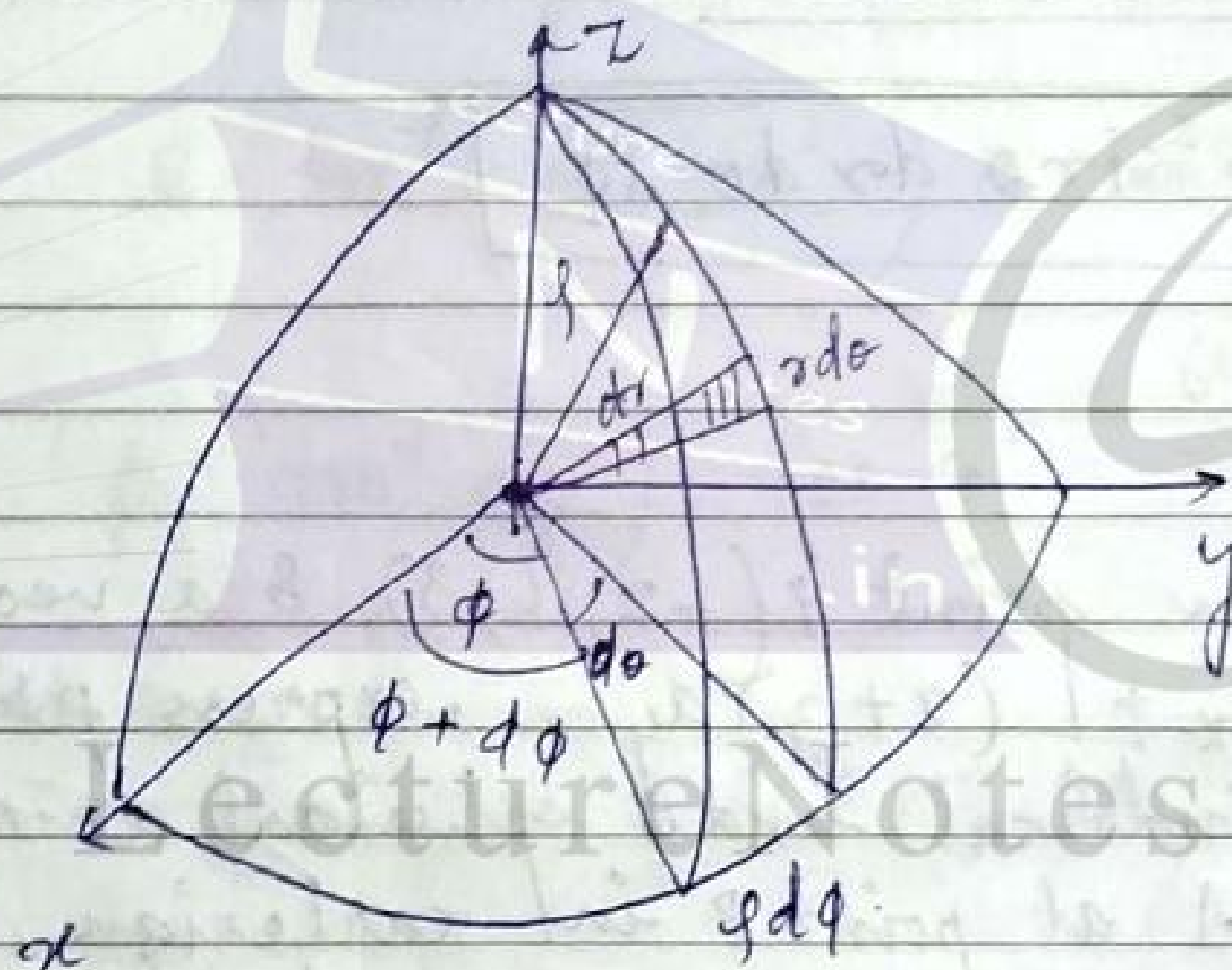
$$\boxed{x = r \sin \theta \cos \phi}$$

$$y = r \sin \phi \quad \boxed{y = r \sin \theta \sin \phi}$$

$$\boxed{z = r \cos \theta}$$

$$(r, \theta, \phi) \rightarrow (x, y, z)$$

vector calculus for spherical co-ordinate:



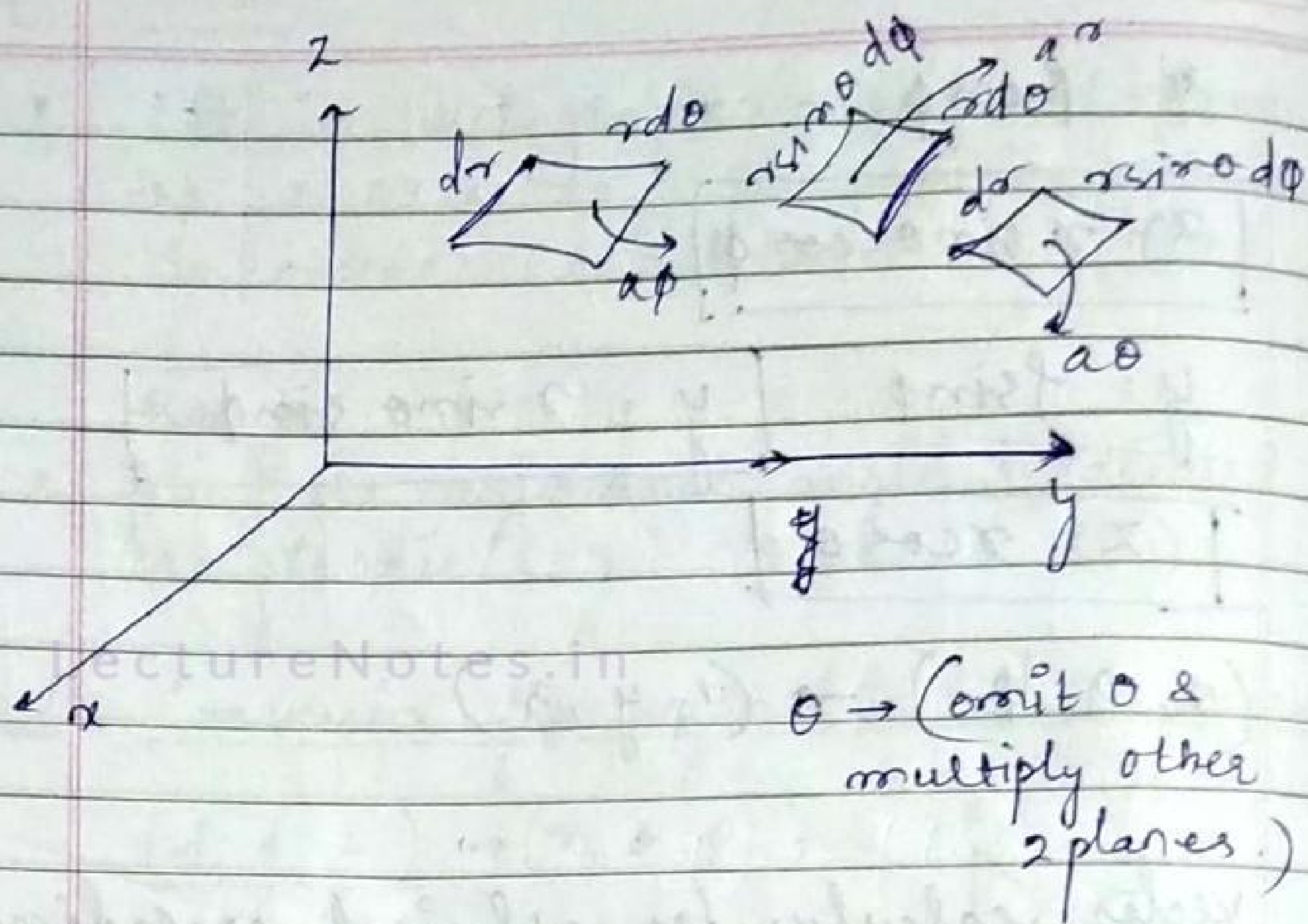
① Differential length

$$\boxed{dl = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin \theta d\phi \hat{a}_\phi}$$

② Differential surface

$$ds_1 = r dr d\theta d\phi$$

$$= r^2 \sin \theta d\theta d\phi \hat{a}_r$$



③ Differential volume :

$$dv = r^2 \sin \theta \, dr \, d\theta \, d\phi$$

23.01.19

Q) A given point $P(-2, 6, 3)$ & a vector $\vec{A} = y\hat{a}_x + (x+z)\hat{a}_y$, express P & $\vec{A}$ in cylindrical and spherical co-ordinate. Evaluate $\vec{A}$ at point P in cartesian, cylindrical & spherical coordinate.

A) It is in cartesian system

$$1) P(x, y, z) \rightarrow P(r, \theta, \phi) \\ P(r, \theta, \phi)$$

for P

$$\text{given: } x = -2, y = 6, z = 3$$

$$P(x, y, z) \rightarrow P(r, \theta, \phi)$$

$$\boxed{\rho = \sqrt{x^2 + y^2}} = \sqrt{(-2)^2 + (6)^2} = \sqrt{4 + 36} = \sqrt{40} = 6.32$$

$$\boxed{\phi = \tan^{-1} \frac{y}{x}}$$

$$= \tan^{-1} \left(\frac{6}{-2} \right) = 108.4^\circ$$

$$\boxed{z = z} \quad z = 3$$

$$P(x, y, z) \rightarrow P(r, \theta, \phi)$$

$$r = \sqrt{x^2 + y^2 + z^2} = 7$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right) = \tan^{-1} \left(\frac{\sqrt{(-2)^2 + (6)^2}}{3} \right) = 64.5^\circ$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right) = 108.4^\circ$$

for $\vec{A}$

$$\vec{A} = y a_x + \theta(x + z) a_y$$

$$\vec{A} \bigg|_{P(-2, 6, 3)} = 6 a_x + (-2 + 3) a_y$$

$$\boxed{\vec{A} = 6 a_x + a_y} \rightarrow \text{cartesian}$$

for cylindrical:

$$\vec{A} = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = A_x \cos\phi + A_y \sin\phi$$

$$A_r = y \cos\phi + (x+z) \sin\phi$$

$$= r \sin\phi \cos\phi + (r \cos\phi + z) \sin\phi$$

$$= 6.32 \sin(108.4^\circ) \cos(108.4^\circ) +$$

$$\{ 6.32 \times \cos(108.4^\circ) + 3 \} \sin(108.4^\circ)$$

$$= -0.9487$$

$$A_\phi = -\sin\phi A_x + \cos\phi A_y$$

$$= -y \sin\phi + (x+z) \cos\phi$$

$$= -r \sin\phi \sin\phi + (r \cos\phi + z) \cos\phi$$

$$= -r \sin^2\phi + r \cos^2\phi + z \cos\phi$$

$$= -6.32 \sin^2(108.4^\circ) + 6.32 \cos^2(108.4^\circ) +$$

$$3(\cos(108.4^\circ))$$

$$= -6$$

$$\therefore \boxed{\vec{A} = -0.94 a_r - 6.0 a_\phi}$$

cylindrical

for spherical :

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

(r, θ, ϕ)

$$(7, 0.45, 108.4)$$

Imp

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_\theta \\ A_\phi \end{bmatrix}$$

ex

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_\theta \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -0.94 \\ -6 \\ 0 \end{bmatrix}$$

$$\begin{aligned} A_r &= \sin\theta \times (-0.94) + 0 \\ &= \sin(64.5) \times (-0.94) \\ &= -0.85 \end{aligned}$$

$$\begin{aligned} A_\theta &= \cos\theta (-0.94) \\ &= \cos(64.5) (-0.94) \end{aligned}$$

$$A_\phi = -6$$

$$\therefore \vec{A} = -0.85 \vec{a}_x - 0.4 \vec{a}_y - 6 \vec{a}_\phi$$

$$Q) \vec{A} = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} \vec{a}_x - \frac{yz}{\sqrt{x^2+y^2+z^2}} \vec{a}_y$$

Transform the $\vec{A}$ into spherical & cylindrical coordinate.

As spherical coordinate

$$\vec{A} = A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = A_x \sin\theta \cos\phi + A_y \sin\theta \sin\phi + A_z \cos\theta$$

$$A_x = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}, \quad r = \sqrt{x^2+y^2+z^2}$$

$$= \frac{r \sin\theta}{r} = \sin\theta \quad x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$A_y = 0, \quad A_z = \frac{-yz}{\sqrt{x^2+y^2+z^2}} = \frac{-\sqrt{r^2 \sin^2\theta \cos^2\phi} \cdot r \sin\theta \sin\phi}{r^2 \sin^2\theta \cos^2\phi + r^2 \sin^2\theta \sin^2\phi}$$

$$A_z = \frac{-r \sin\theta \sin\phi \cos\theta}{r} = -\sin\theta \sin\phi \cos\theta = -\sin\theta \cos\theta \sin\phi$$

$$A_z = -r \sin \theta \sin \phi \cos \theta$$

$$\therefore A_r = \sin \theta \sin \theta \cos \phi - r \sin \theta \cos \theta \sin \phi \cos \theta$$

$$A_r = \sin^2 \theta \cos \phi - r \sin \theta \cos^2 \theta \sin \phi$$

$$A_\theta = A_r \cos \theta \cos \phi + A_y(0) - \sin \theta A_z$$

$$= \sin \theta \cos \theta \cos \phi + r \sin^2 \theta \cos \theta \sin \phi$$

$$A_\phi = -A_r \sin \phi - 0 - 0$$

$$= -\sin \theta \cdot \sin \phi$$

$$\vec{A} = (\sin^2 \theta \cos \phi - r \sin \theta \cos^2 \theta \sin \phi) \hat{a}_r$$

$$+ (\sin \theta \cos \theta \cos \phi + r \sin^2 \theta \cos \theta \sin \phi) \hat{a}_\theta$$

$$- \sin \theta \sin \phi \hat{a}_\phi$$

cylindrical :

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = \cos \phi A_x + A_y \sin \phi + 0$$

$$= \cos \phi A_x$$

$$= \cos \phi \sin \theta$$

$$A_\theta = -\sin \phi A_x$$

$$A_z = A_z$$

$$= -r \sin \theta \sin \phi \cos \theta$$

$$\therefore \vec{A} = A_\phi \hat{a}_\phi + A_\theta \hat{a}_\theta + A_z \hat{a}_z$$

$$= \cos$$

$$a) \vec{A} = \frac{f}{\sqrt{f^2 + z^2}} \left[\cos \phi \hat{a}_r - \sin \phi \hat{a}_\phi - z \sin \phi \hat{a}_z \right]$$

into spherical.

$$\vec{A} = \frac{f \cos \phi}{\sqrt{f^2 + z^2}} \hat{a}_r - \frac{f \sin \phi}{\sqrt{f^2 + z^2}} \hat{a}_\phi - \frac{z \sin \phi}{\sqrt{f^2 + z^2}} \hat{a}_z$$

$$\left\{ \begin{array}{l} f = r \sin \theta \\ z = r \cos \theta \\ r = \sqrt{f^2 + z^2} \end{array} \right.$$

$$A_r = \frac{f \cos \phi}{\sqrt{f^2 + z^2}} = \frac{r \sin \theta \cos \phi}{r} = \sin \theta \cos \phi$$

$$A_\phi = -\frac{f \sin \phi}{\sqrt{f^2 + z^2}} = -\frac{r \sin \theta \sin \phi}{r} = -\sin \theta \sin \phi$$

$$A_z = \frac{-\rho z \sin \phi}{\sqrt{\rho^2 + z^2}} = \frac{-\cancel{r} \sin \theta \cancel{r} \cos \theta \sin \phi}{\cancel{r}}$$

$$= -\sin \theta \cos \theta \sin \phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$A_r = \sin \theta A_\rho + \cos \theta A_z$$

$$= \sin \theta \cos \phi \sin \theta + \cos \theta \cdot (-\sin \theta \cos \theta \sin \phi)$$

$$= \sin^2 \theta \cos \phi - \cos^2 \theta \sin \theta \sin \phi$$

$$A_\theta = \cos \theta A_\rho - A_z \sin \theta$$

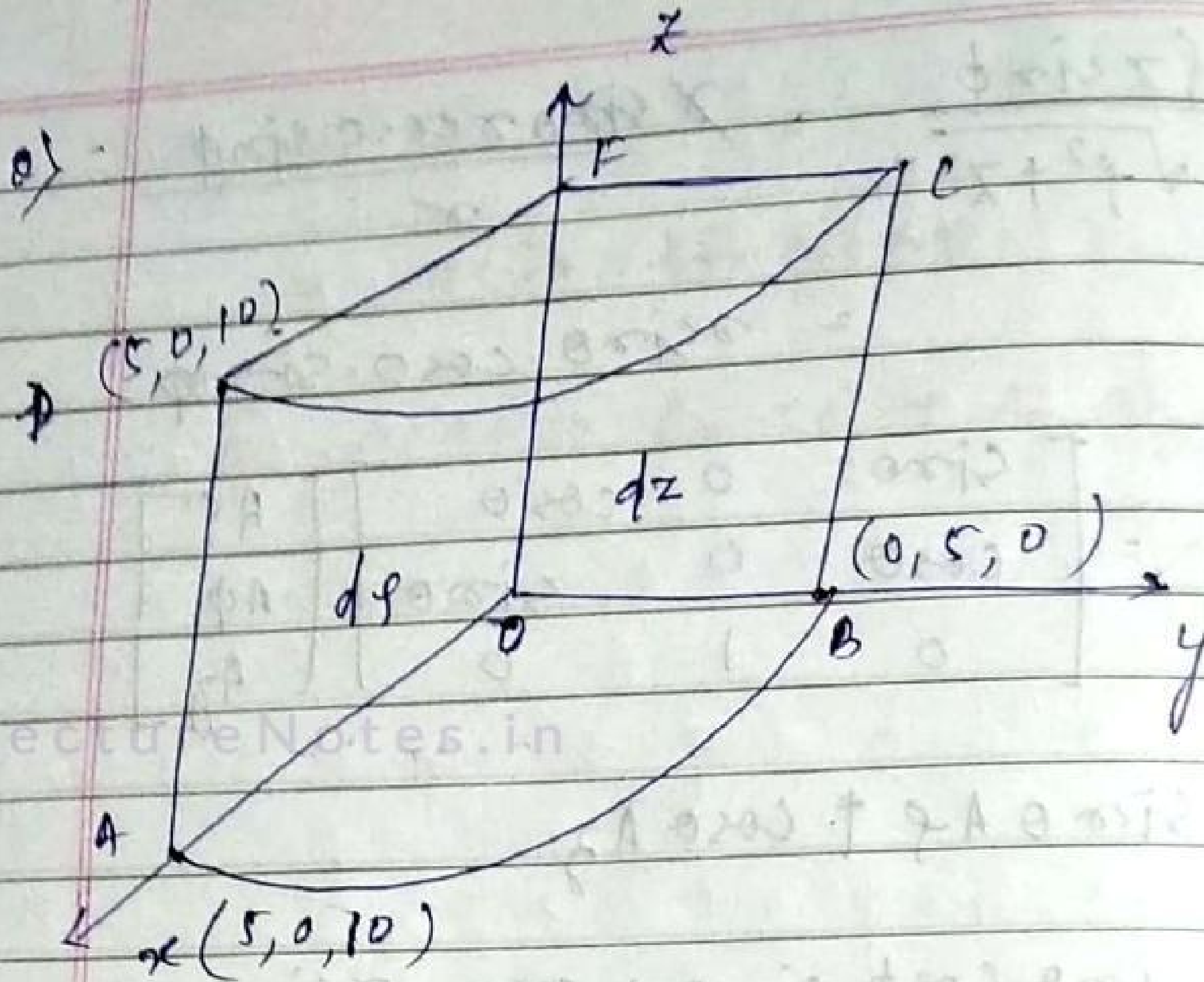
$$= \sin \theta \cos \phi \cos \theta - (-\sin \theta \cos \theta \sin \phi \sin \theta)$$

$$A_\phi = A_\phi$$

$$= \sin \phi \sin \theta$$

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

$$= (\sin^2 \theta \cos \phi) \hat{a}_r + (\sin \theta \cos \phi \cos \theta - \sin \theta \cos \theta \sin \phi) \hat{a}_\theta + (\sin \phi \sin \theta) \hat{a}_\phi$$



consider the object shown in the figure & calculate.

- (i) length of BC
- (ii) length of CD .
- (iii) surface area $ABCD$.
- (iv) S.A of ABO .
- (v) S.A of $ADFD$
- (vi) volume.

As $A(5, 0, 0) \rightarrow A(5, 0, 0)$

cartesian to cylindrical.

$$\left\{ \begin{array}{l} r = \sqrt{x^2 + y^2} = \sqrt{5^2} = 5 \\ \phi = \tan^{-1} \frac{0}{0} = 0 \\ z = z = 0 \end{array} \right\} (r, \phi, z)$$

$$B(0, 5, 0) \rightarrow B(5, \pi/2, 10)$$

$$C(0, 5, 10) \rightarrow C(5, \pi/2, 10)$$

$$D(5, 0, 10) \rightarrow D(5, 0, 10)$$

(i) length of BC (along z axis).

$$d\vec{l} = dr \hat{a}_r + r d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$d\vec{l} = dz \hat{a}_z$$

$$BC = \int_0^{10} d\vec{l} = [z]_0^{10} = 10$$

(ii) length of CD (along y-axis).

$$d\vec{l} = dr \hat{a}_r + r d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$d\vec{l} = r d\phi \hat{a}_\phi$$

$$CD = \int_{\phi=0}^{\pi/2} r d\phi = 5 \int_0^{\pi/2} d\phi = 9.05$$

(iii) surface area of ABCD.

$$dS = r d\phi dz = 5 \int_0^{\pi/2} d\phi \int_0^{10} dz$$

(iv) S.A of ABO

$$ds = \int_0^r f dr \int_0^{\pi/2} d\phi$$

v) S.A of AOED

$$ds = \int f dr dz$$

$$= r \int dr \times \int_0^{10} dz$$

$$\boxed{\text{S.A. of AOED} = 5 \times 10 = 50}$$

vi) $dv = \int f dr d\phi dz$ (volume)

$$= \int_0^r f dr \int_0^{\pi/2} d\phi \int_0^{10} dz$$

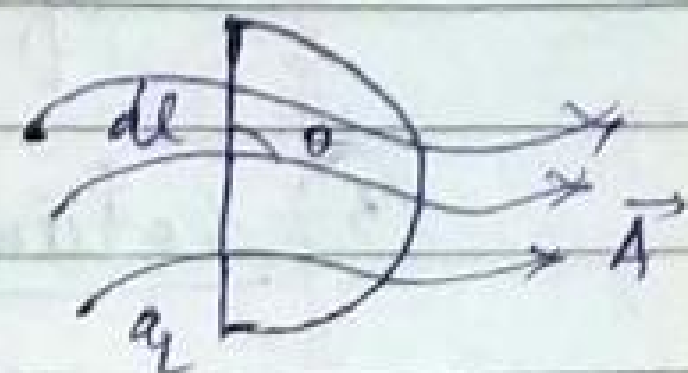
$$= \frac{f^2}{2} \int_0^5 \times \theta \int_0^{\pi/2} \times z \int_0^{10}$$

$$\boxed{V = 125 \frac{\pi}{2} = 62.5 \pi}$$

CHAPTER - 2

1) Line Integral : It is defined as the integral of the tangential component of A around the curve L .

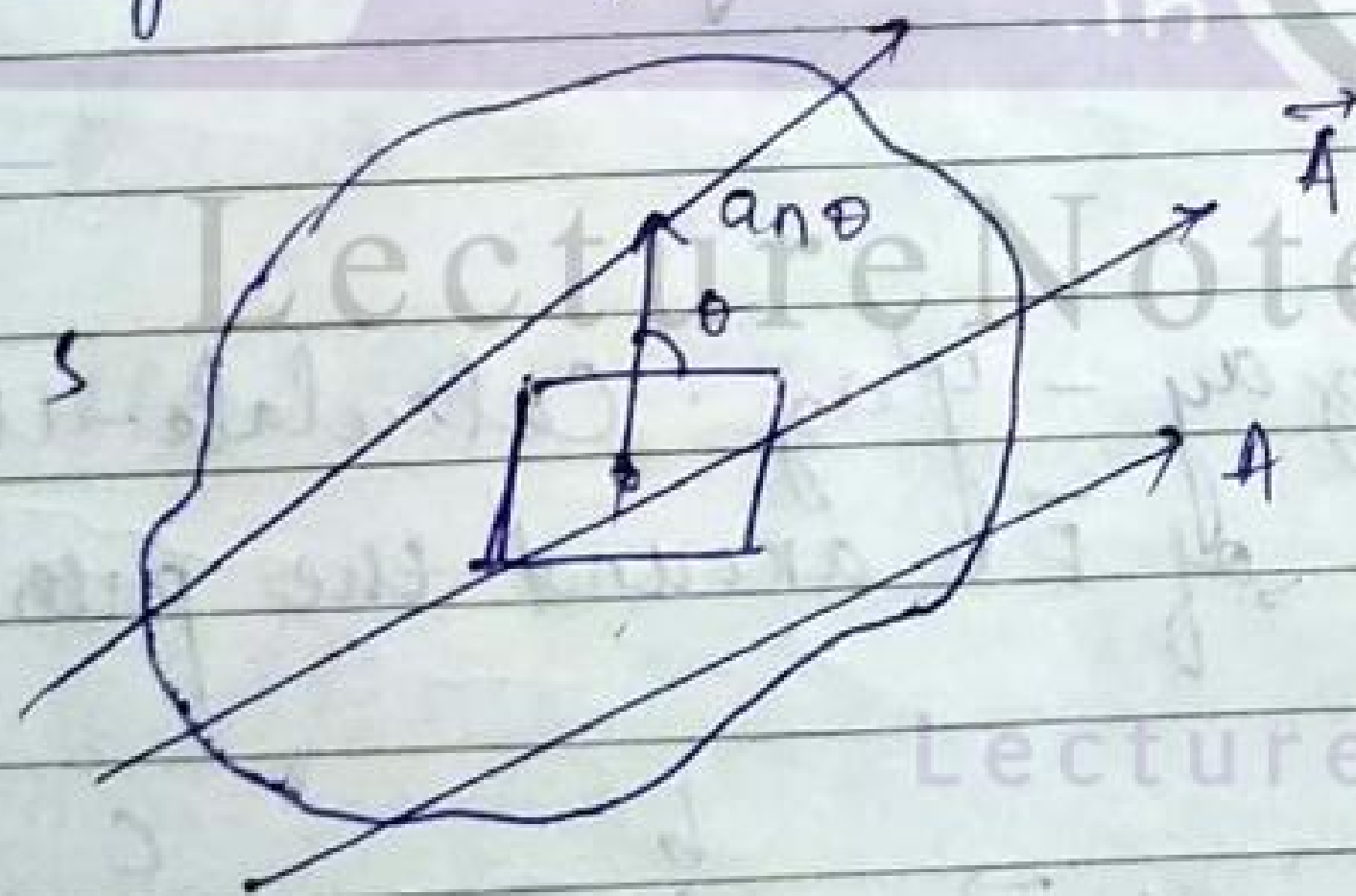
$$\int_L A \, dl = \int_a^b |A| \cos \theta \cdot dl$$



* If the path is a closed curve, then $\oint_L \vec{A} \cdot d\vec{l}$ is called as circulation of A around the path L .

* If $\oint_L \vec{A} \cdot d\vec{l} = 0 \rightarrow$ it is called as conservative field / lamellar field.

② Surface Integral :



$$\int_S \vec{A} \cdot d\vec{s} = \int_S |A| \cos \theta \cdot ds = \int_S \vec{A} \cdot \vec{a}_n ds$$

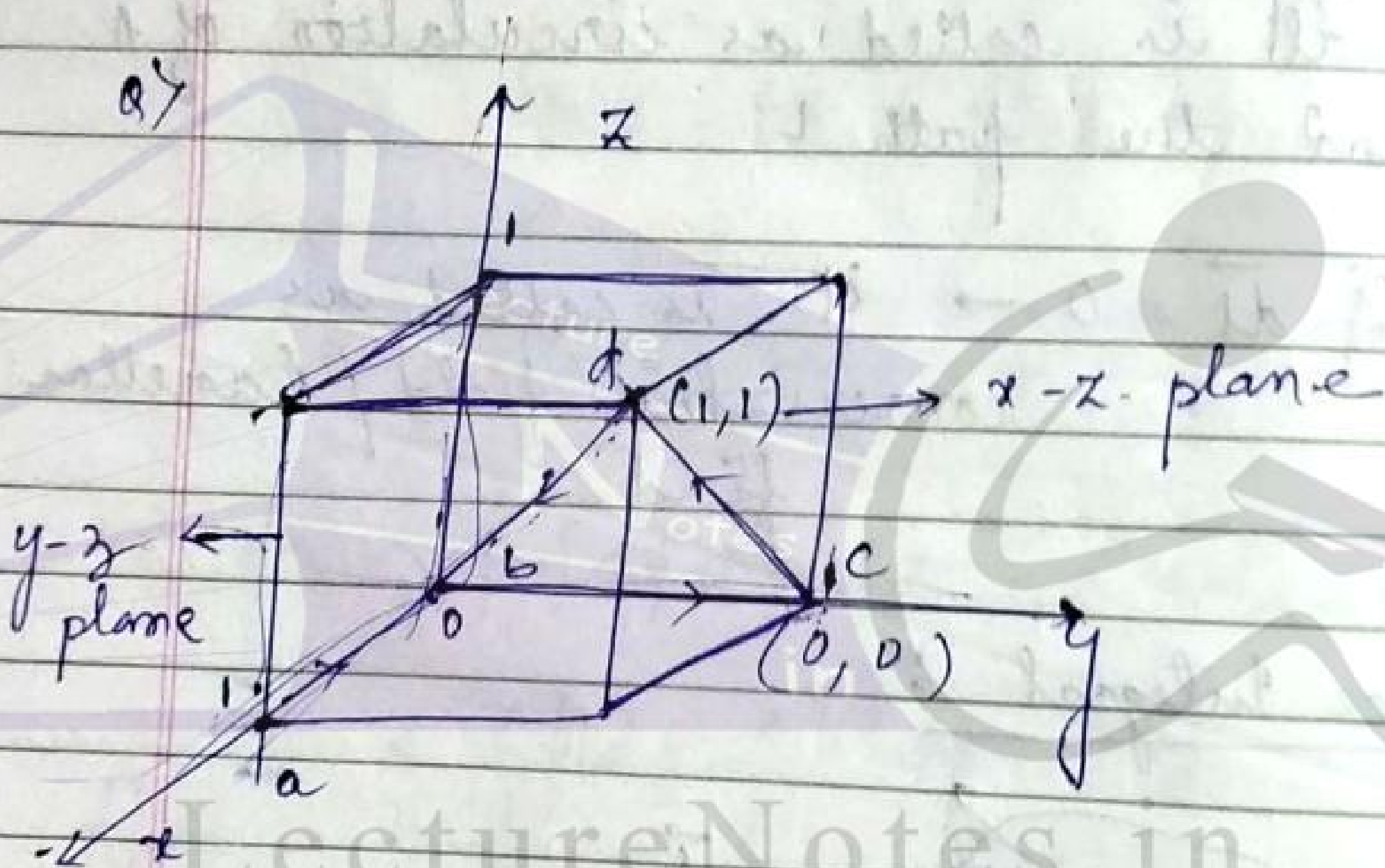
$\psi = \int_S \vec{A} \cdot d\vec{s} =$ flux of $\vec{A}$ through S .

$$\boxed{\Psi_{\text{net outward}} = \oint_S \vec{A} \cdot d\vec{s}}$$

for a close surface, the surface integral will refer as the net outward flux of $\vec{A}$ from S .

③ Volume Integral.

$$\boxed{\int_V f_v dv}$$



$\vec{F} = x^2 \hat{a}_x - xz \hat{a}_y - y^2 \hat{a}_z$. Calculate the circulation of $\vec{F}$ around the path shown.

$$\oint_L \vec{F} \cdot d\vec{l} = \int_{ab} \vec{F} \cdot d\vec{l} + \int_{bc} \vec{F} \cdot d\vec{l} + \int_{cd} \vec{F} \cdot d\vec{l} + \int_{da} \vec{F} \cdot d\vec{l}$$

segment ab.

$$y=0, z=0$$

$$\therefore \vec{F} = x^2 \vec{a}_x \quad \vec{dl} = dx \vec{a}_x$$

$$\vec{F} \cdot d\vec{l} = x^2 \cdot dx$$

$$\therefore \int_a^b \vec{F} \cdot d\vec{l} = \int_1^0 x^2 \cdot dx = \left[\frac{x^3}{3} \right]_1^0 = -\frac{1}{3}$$

segment bc : $(x=0, z=0)$ y-axis

$$\vec{F} = -y^2 \vec{a}_z \quad d\vec{l} = dy \vec{a}_y$$

$$\vec{F} \cdot d\vec{l} = -y^2 \vec{a}_z \cdot dy \vec{a}_y = 0$$

$$\int_a^b \vec{F} \cdot d\vec{l} = \int_0^0 0 = 0$$

segment cd :

$$y=1, \quad x=z$$

$$\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - \vec{a}_z$$

$$d\vec{l} = dx \vec{a}_x + dz \vec{a}_z$$

$$\vec{F} \cdot d\vec{l} = (x^2 \vec{a}_x - xz \vec{a}_y - \vec{a}_z) \cdot (dx \vec{a}_x + dz \vec{a}_z)$$

$$= x^2 dx - dz$$

$$\vec{F} \cdot d\vec{l} = x^2 \cdot dx - dz = (x^2 - 1) dx$$

$$\int \vec{F} \cdot d\vec{l} = \int_1^0 (x^2 - 1) \cdot dx$$

segment da is $(1,1)(0,0)$

$$x=1, \quad z=y \Rightarrow dy=dz$$

$$\vec{F} = x\vec{a}_x - z\vec{a}_y - y^2\vec{a}_z \quad d\vec{l} = dy\vec{a}_y + dz\vec{a}_z$$

$$\vec{F} \cdot d\vec{l} = -z dy - y^2 dz$$

$$= -y dy - y^2 dy$$

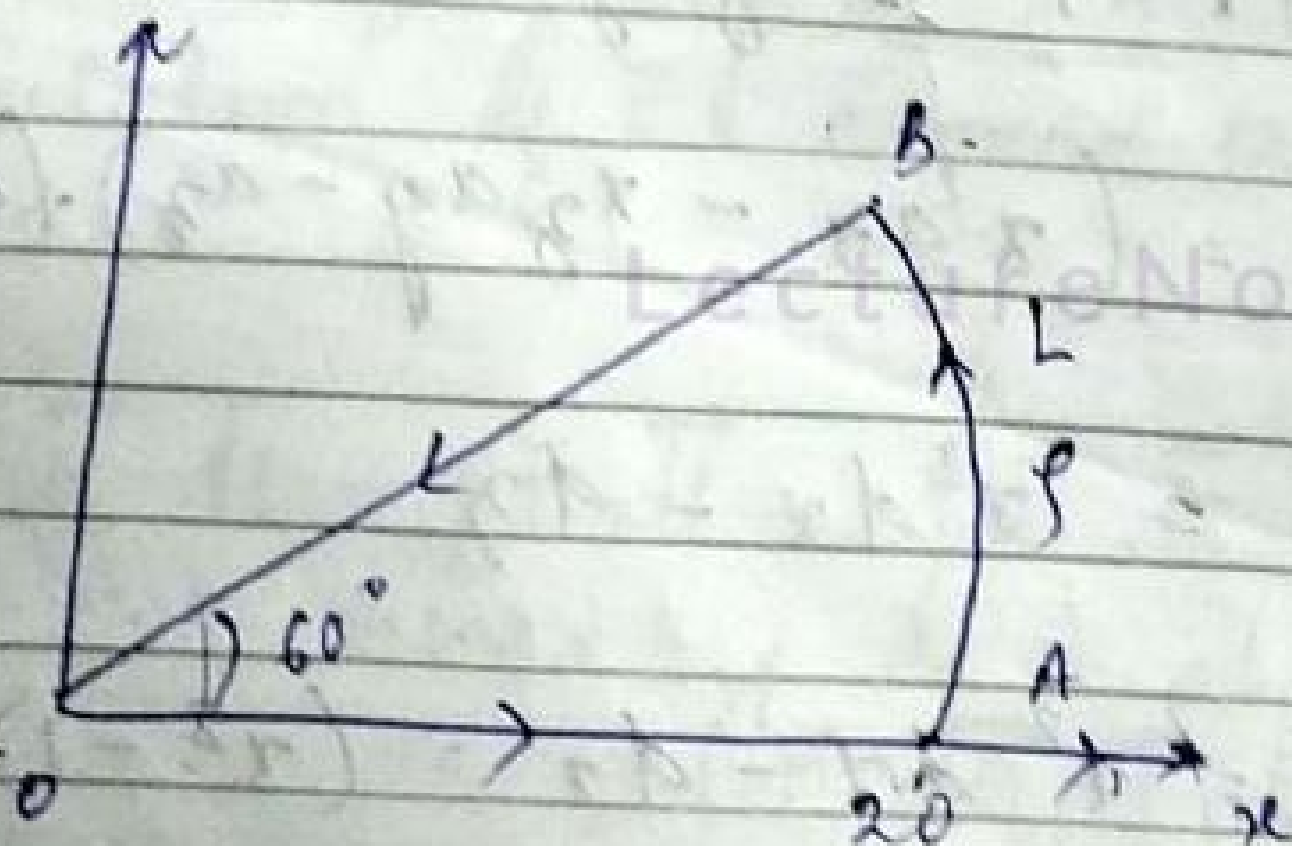
$$= dy(-y - y^2)$$

$$= \int_1^0 \vec{F} \cdot d\vec{l} = \int_1^0 (-y - y^2) \cdot dy = \left[-\frac{y^2}{2} - \frac{y^3}{3} \right]_1^0$$

$$\therefore \oint \vec{F} \cdot d\vec{l} = -\frac{1}{3} + 0 - \frac{2}{3} + \frac{5}{6} = -1/6$$

$$\oint_{abcd} \vec{F} \cdot d\vec{l} = -\frac{1}{6}$$

30.01.19



calculate the circulation of $\vec{A} = \cos\phi \vec{a}_\phi + \sin\phi \vec{a}_z$ around l : $0 \leq \phi \leq \pi/2$

Spherical :-

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Workbook - 1

$$\vec{B} = \rho \cdot z \cos^2 \phi \hat{a}_z$$

$$\rho_v = ? \quad \text{at } (1, \pi/4, 3)$$

$$\nabla \cdot B = \rho_v$$

$$\nabla \cdot B = \frac{\partial (\rho z \cos^2 \phi)}{\partial z}$$

$$= \rho \cos^2 \phi = 1 \cdot \frac{1}{2} = 0.5 \text{ C/m}^3$$

$$5. \vec{B} = \rho \cdot z \cdot \cos^2 \phi \hat{a}_\rho$$

$$\nabla \cdot B = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \cdot \rho z \cos^2 \phi)$$

$$= \frac{1}{\rho} z \cos^2 \phi \cdot 2\rho = 2 \cdot 3 \cdot \frac{1}{2} = 3 \text{ C/m}^3$$

$$4. \vec{F} = \rho \hat{a}_\rho + \rho \sin^2 \phi \hat{a}_\phi - z \hat{a}_z$$

$$\nabla \cdot \vec{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \cdot \rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho \cdot \sin^2 \phi) + \frac{\partial}{\partial z} (-z)$$

$$= 2 + 2 \sin \phi \cos \phi - 1$$

$$= 1 + \sin 2\phi$$

Ans - D

Ques - GATE 2012

$$\vec{A} = k r^n \hat{a}_r \quad n = ?$$

$$\text{If } \nabla \cdot A = 0$$

Soln:-

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 k r^n)$$

$$= \frac{k}{r^2} (n+2) r^{n+1} = 0$$

$$\Rightarrow n = -2, \text{ Ans}$$

Curl of a vector $\vec{A}$:-

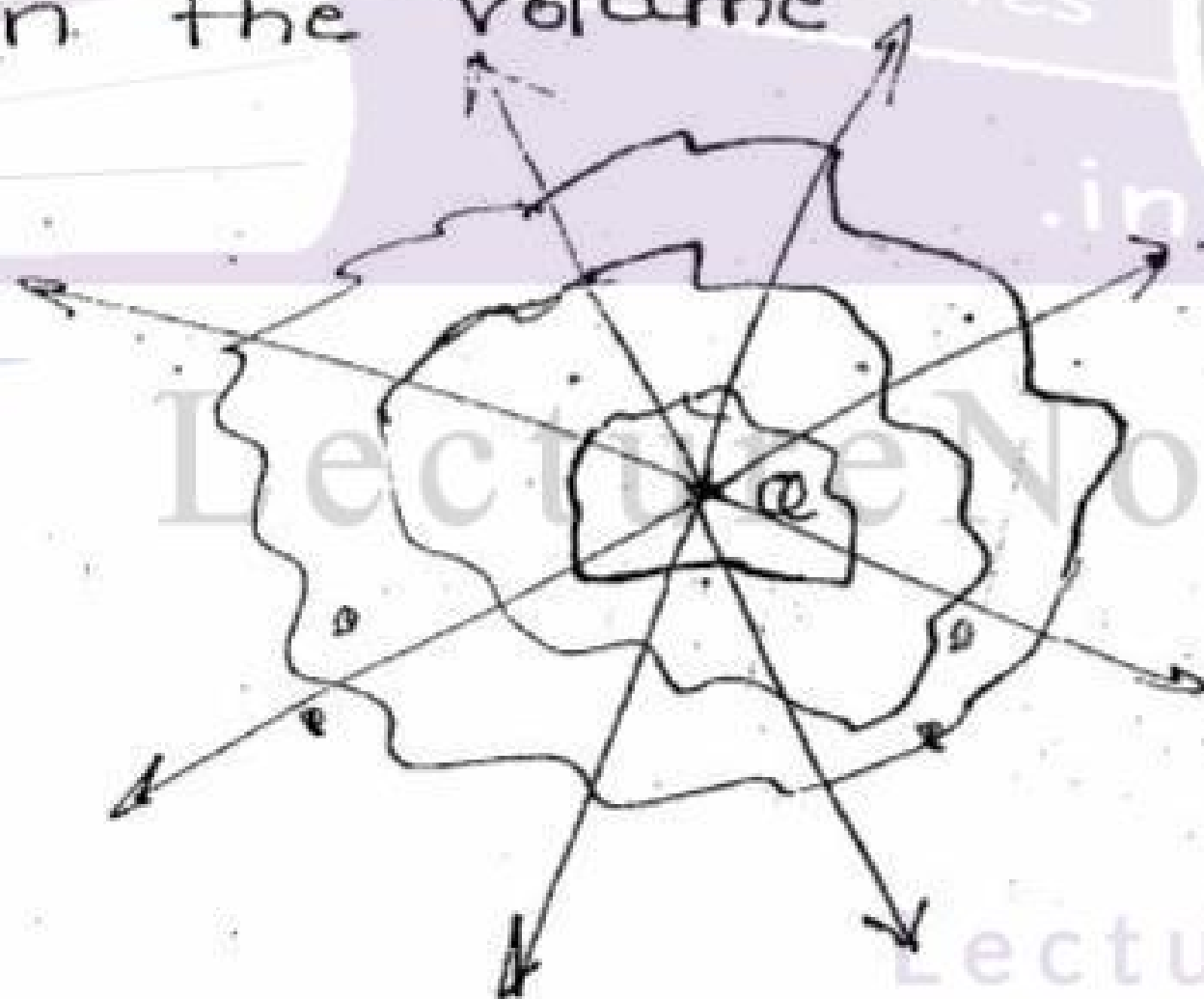
$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

Static Electric Fields :-

Gauss Law :-

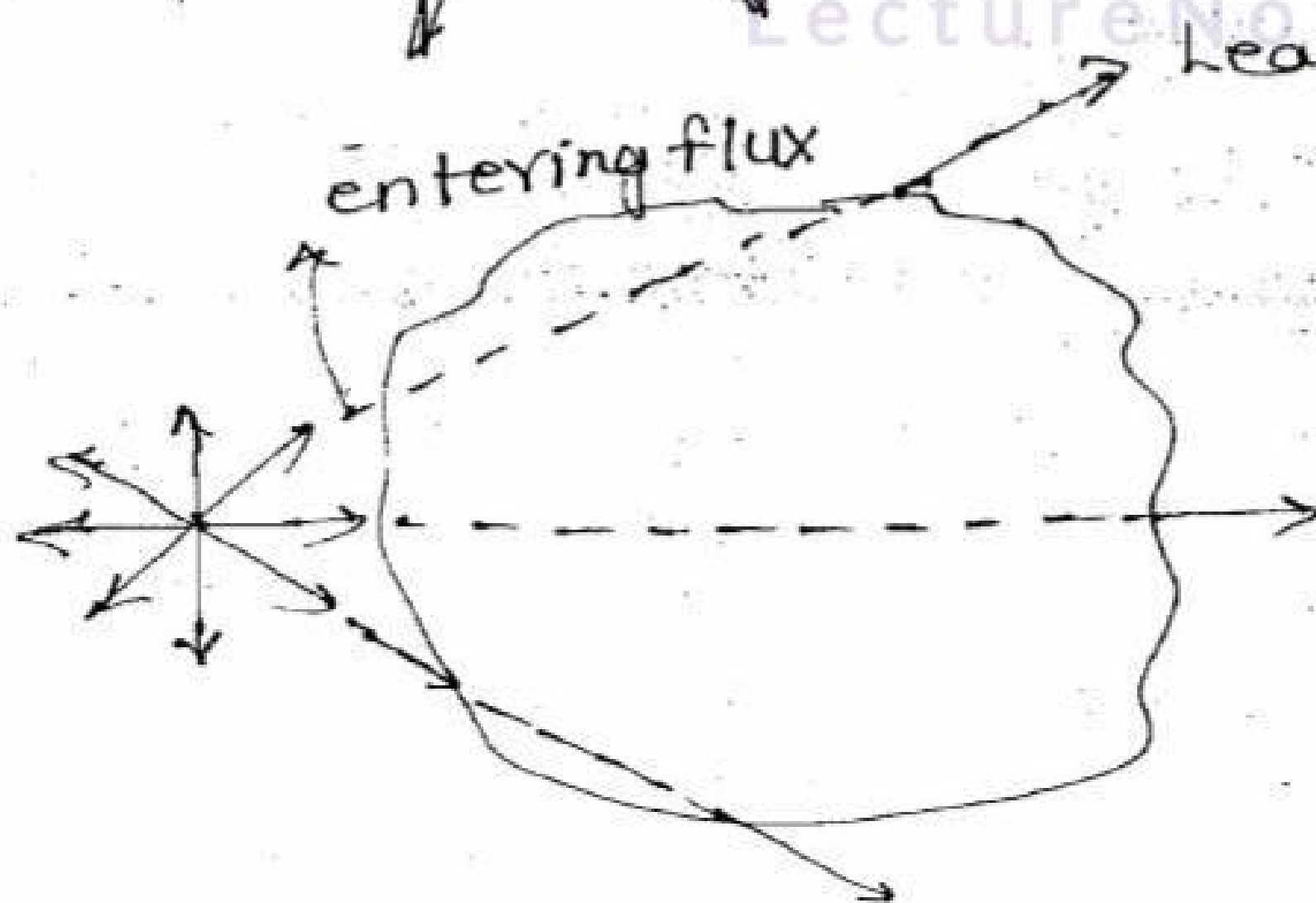
total effects

The net electric flux leaving any closed surface is always equal to the charge enclosed in the volume cause



Flux leaving

$$\oint \vec{E} \cdot d\vec{s} = q$$



entering flux

leaving flux

$$\oint \vec{E} \cdot d\vec{s} = 0$$

The word electric flux means the attractive or repulsive force on any test charge placed in the electric field. Hence electric field or electric flux or lines of force physically represent the direction in which a test charge moves away when placed in the field.

→ When the surface is closed the total outflow or flux leaving is independent of density and area. i.e.

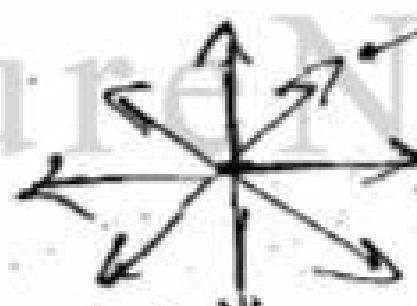
$$\oint \mathbf{E} \cdot d\mathbf{s} = Q$$

→ If the charge is outside the effects still exists but net effects is zero

entering flux = leaving flux

$$\oint \mathbf{E} \cdot d\mathbf{s} = 0$$

→ If the surface is not completely enclosing the flux crossing depends on the density of the flux and the surface area of consideration.



This is crossing flux

$$\int \mathbf{E} \cdot d\mathbf{s} = \psi_c$$

↓
Not Gauss law

Application of Gauss law! -

1) Electric field strength of a point charge! -

Consider a concentric symmetrical surface such that E is same everywhere



$$\oint \vec{D} \cdot d\vec{s} = Q \quad \begin{array}{l} r = \text{constant} \\ a_r \text{ const directed} \end{array}$$

$$\Rightarrow D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$\Rightarrow \boxed{D = \frac{Q}{4\pi r^2} a_r} \quad \text{C/m}^2$$

→ Coulomb had a different measure of field strength i.e. the force measured & he related this with charge measure using ϵ . This is called as intensity

$$\vec{E} = \frac{\vec{F}}{Q_1} = \frac{\vec{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2} a_r$$

↓
Intensity
(N/C)

$$\Rightarrow \boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon r^2}}$$

This is Coulomb's law

Static Magnetic Fields:

Biot - Savart's Law (Ampere's law for current elements):

It is derived from Ampere's law & consider a dl length I (DC current) carrying as a basic cause of H field

$I dl$ — (Amp-m) is basic cause of H field

$$\vec{H} = \frac{I dl \times a_r}{4\pi r^2}$$

↓
Intensity
(Amp/m).

→ $\vec{H}$ direction = I flow $\times$ Radial vector direction [cross] from current

→ Lorentz had a different measure of field strength i.e. density measure which is physically force per basic cause

$$\vec{B} = \mu \vec{H} = \frac{\mu I dl}{4\pi r^2} \times d\vec{r} = \frac{\vec{F}}{I dl} = \frac{\text{Newton}}{\text{Amp-m}}$$

$$\vec{F} = q (\vec{v} \times \vec{B})$$

$$\Rightarrow dF = dq \left(\frac{d\vec{r}}{dt} \cdot \vec{B} \cos\theta \right) = I dl B$$

$$\Rightarrow B = \frac{F}{I dl}$$

Note:-

$$\rightarrow D \cdot E = \frac{\text{Col}}{\text{m}^2} \times \frac{\text{Newton}}{\text{Col}} = \frac{\text{Newton}}{\text{m}^2}$$

= Electric pressure at that point

$$\rightarrow B \cdot H = \frac{\text{Newton}}{\text{Amp-m}} \times \frac{\text{Amp}}{\text{m}} = \frac{\text{Newton}}{\text{m}^2}$$

= Magnetic Pressure

$$\rightarrow \frac{\text{Newton} \times \text{m}}{\text{m}^2 \times \text{m}} = \frac{\text{Joules}}{\text{m}^3} = \text{Energy density}$$

$$\rightarrow \frac{1}{2} D \cdot E = \frac{1}{2} \epsilon E^2 = \text{Energy density in electric field}$$

$$\rightarrow \frac{1}{2} B \cdot H = \frac{1}{2} \mu H^2 = \text{Energy density in H field}$$

Workbook

$$Q \rightarrow (0, 0, 3)$$

$$E \text{ at } C(4, 0, 0)$$

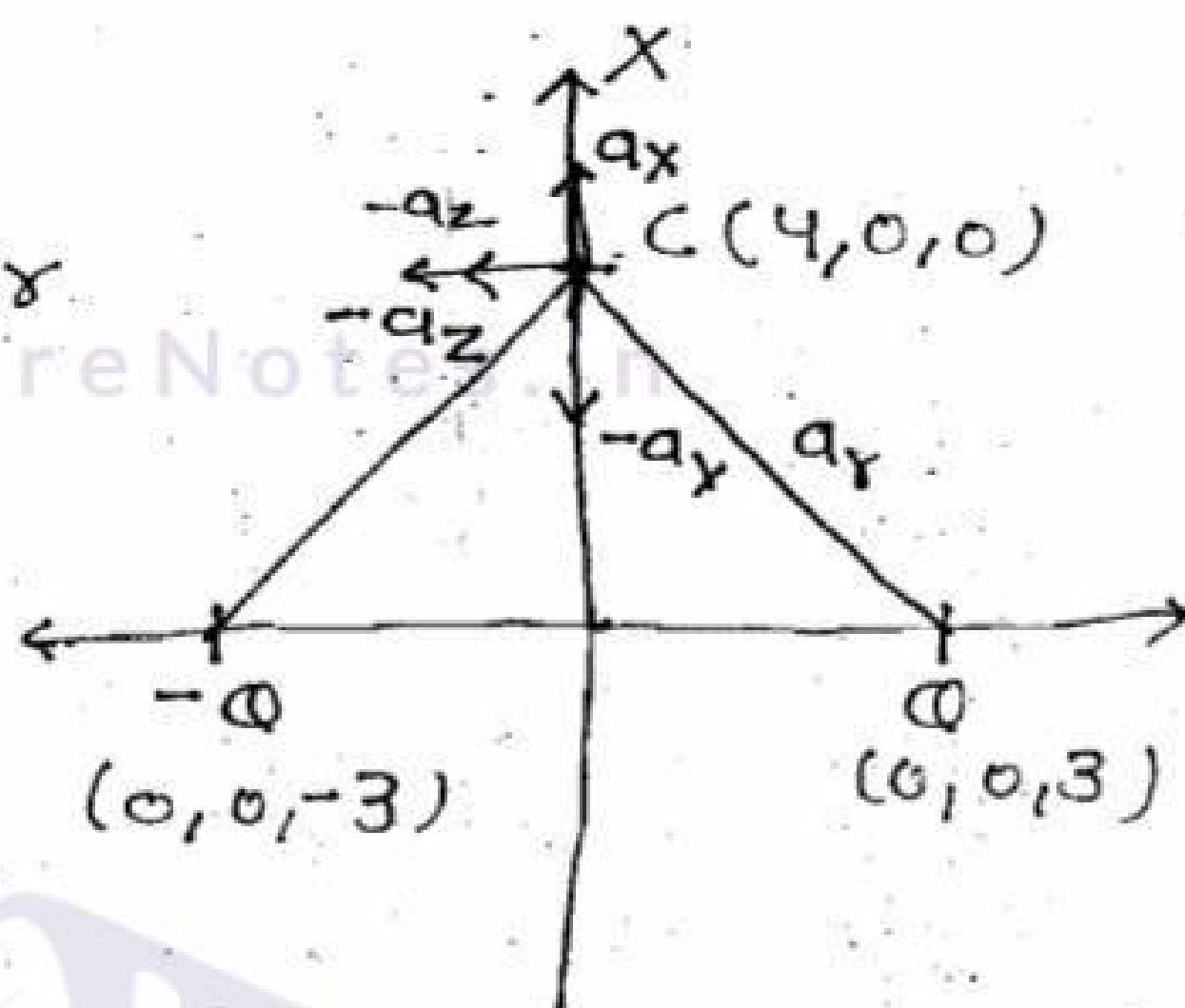
$$-Q \rightarrow (0, 0, -3)$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} a_r$$

$$E_1 = (a_x, -a_z)$$

(Q)

$$= \frac{4a_x - 3a_z}{5} \quad (0, 0, -3)$$



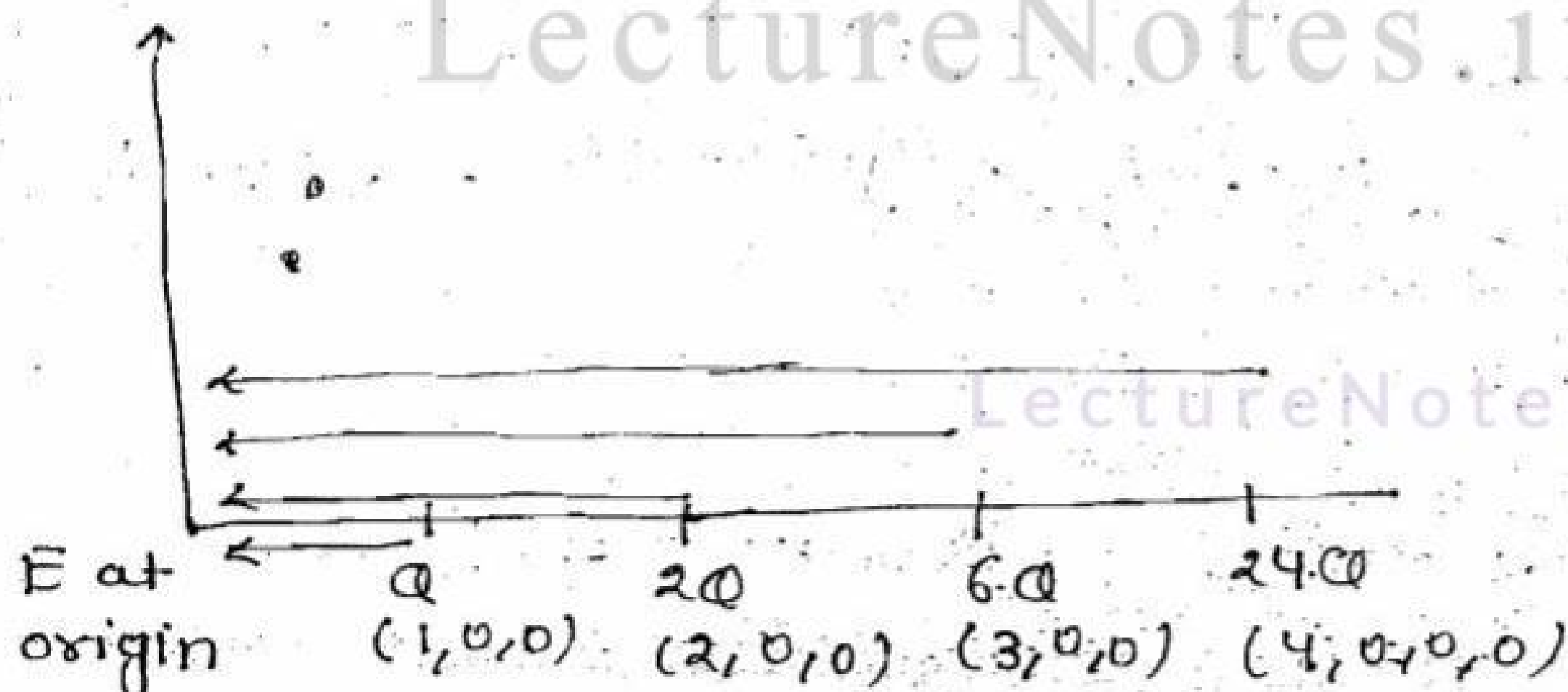
$$E_2 = (-a_x, -a_z)$$

(-Q)

$$= \frac{-4a_x - 3a_z}{5}$$

$$E_{\text{total}} = -a_z$$

(direction)



$$E_T = E_1 + E_2 + E_3 + E_4$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} a_r$$

$$a_r = -a_x$$

$$\vec{E}_T = \left(\frac{Q}{4\pi\epsilon(1)^2} + \frac{2Q}{4\pi\epsilon(2)^2} + \frac{6Q}{4\pi\epsilon(3)^2} + \frac{24Q}{4\pi\epsilon(4)^2} \right) \hat{a}_x$$

$$= \frac{Q}{4\pi\epsilon} \left(1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} \right) (-\hat{a}_x)$$

Net flux leaving = $\oint \vec{D} \cdot d\vec{s} = Q = \text{charge enclosed}$

Sphere $\rightarrow$ Centre = origin

radius = 6m

$$2C \rightarrow (4, 8, 3) \rightarrow d_1 = \sqrt{4^2 + 8^2 + 3^2} > 6$$

$$8C \rightarrow (2, -1, -3) \rightarrow d_2 = \sqrt{2^2 + 1^2 + 3^2} < 6$$

$$-12C \rightarrow (-4, 0, 1) \rightarrow d_3 = \sqrt{4^2 + 1^2} < 6$$

Net flux leaving = $-4C$

Centre = $(2, -3, 2)$

$$d_1 = \sqrt{2^2 + 11^2 + 1^2} > 6 \rightarrow \text{out}$$

$$d_2 = \sqrt{0^2 + 2^2 + 5^2} < 6 \rightarrow \text{in}$$

$$d_3 = \sqrt{6^2 + 3^2 + 1^2} > 6 \rightarrow \text{out}$$

Net flux leaving = $8C$

Net flux leaving = $Q = -\rho_L d$

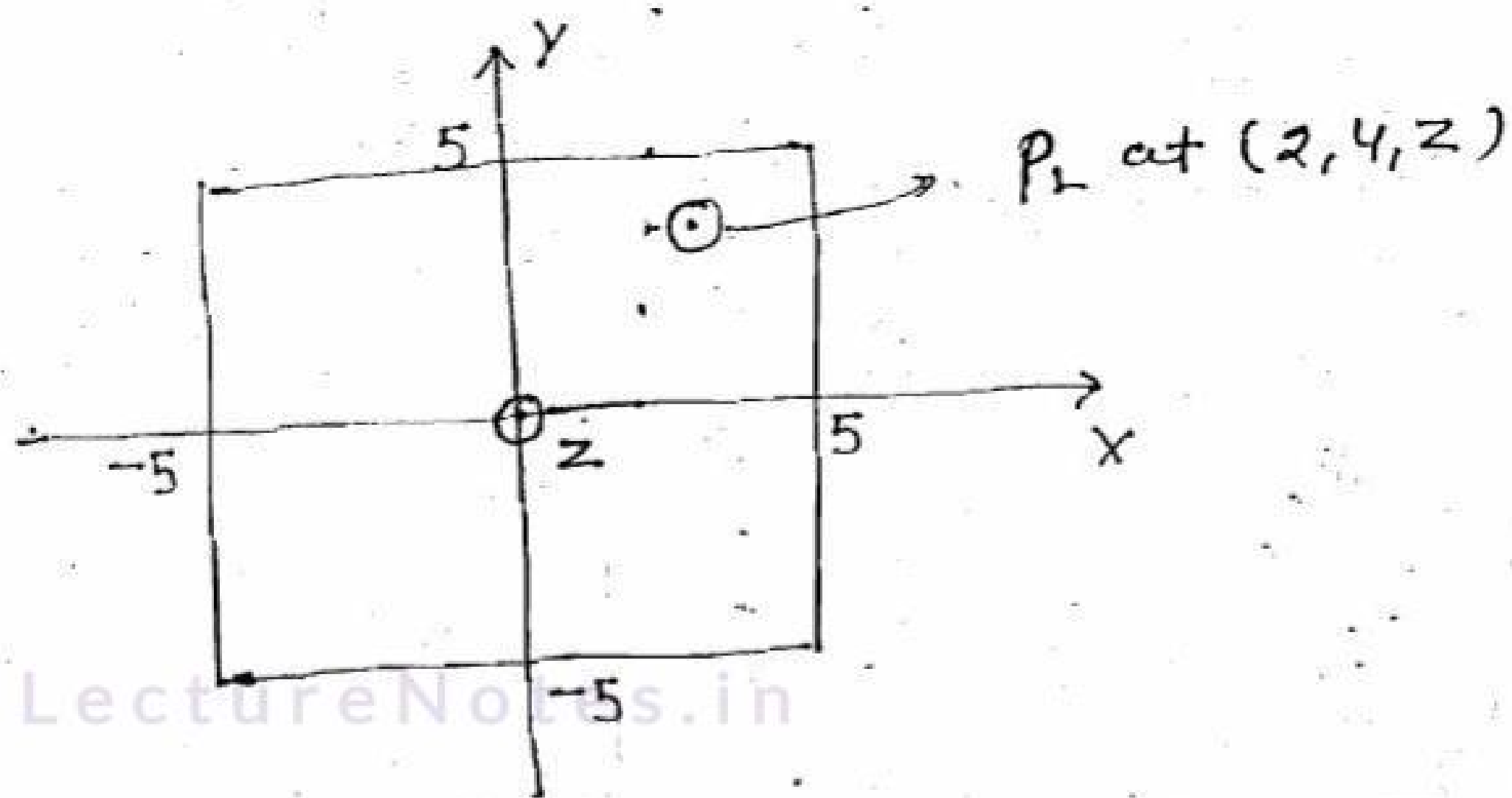
$$\rho_L = 15 \text{ nC/m}$$

at $x=2, y=4$ for all z

closed surface = $-5 < x < 5$

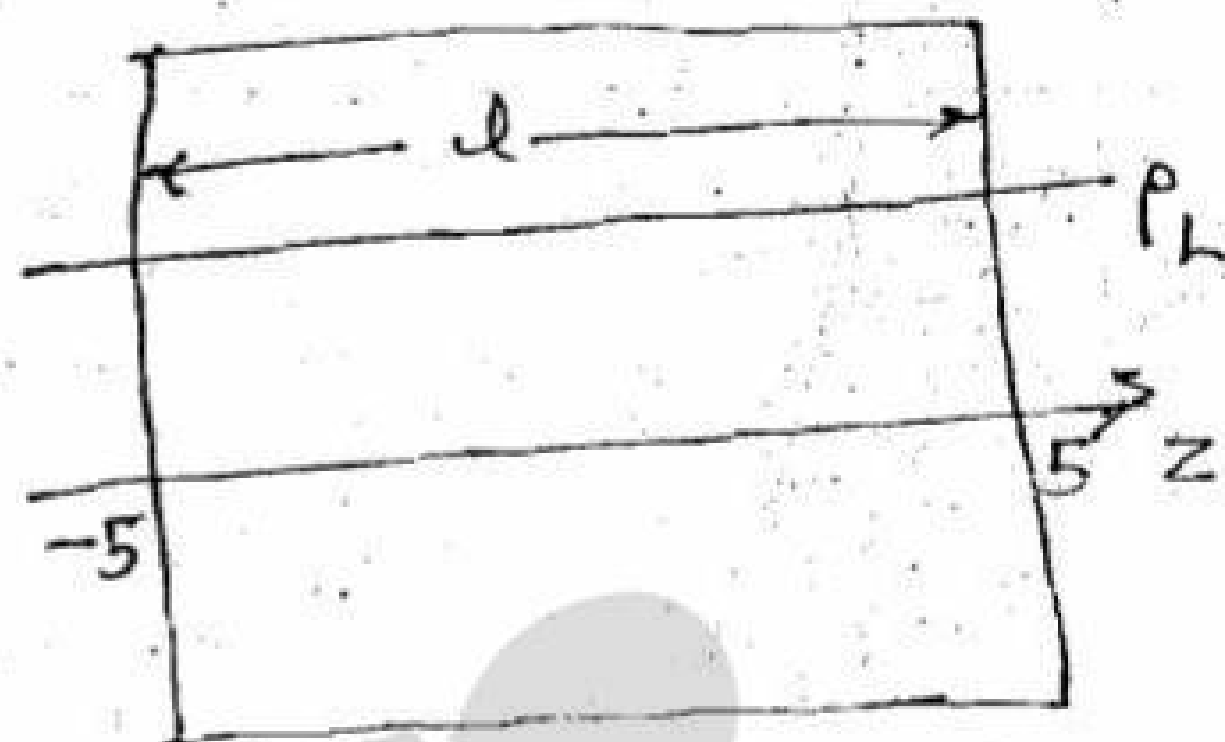
$-5 < y < 5$

$-5 < z < 5$



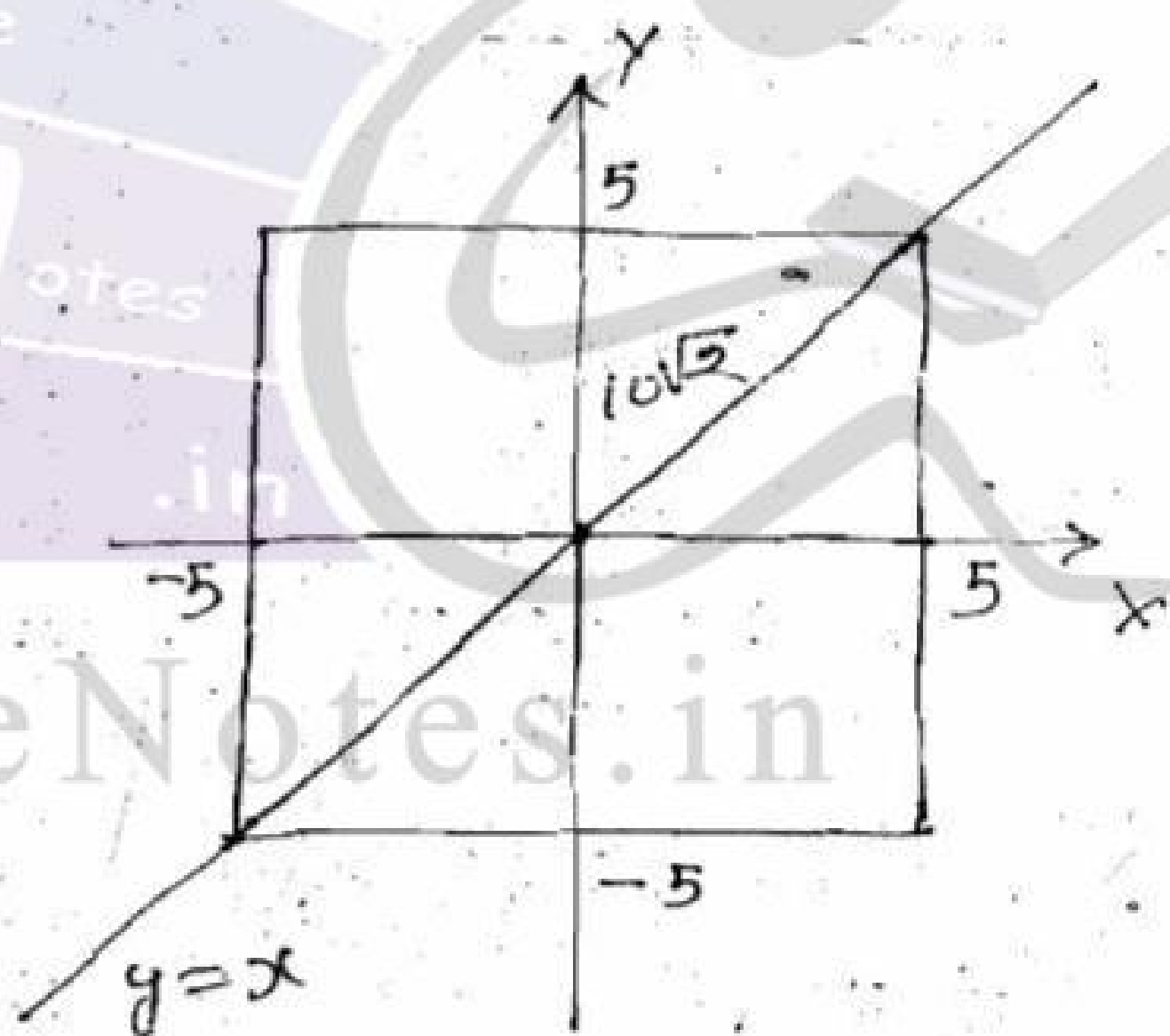
$$Q = 15 \frac{nC}{m} \times 10$$

$$= 150 nC, \text{ Ans}$$



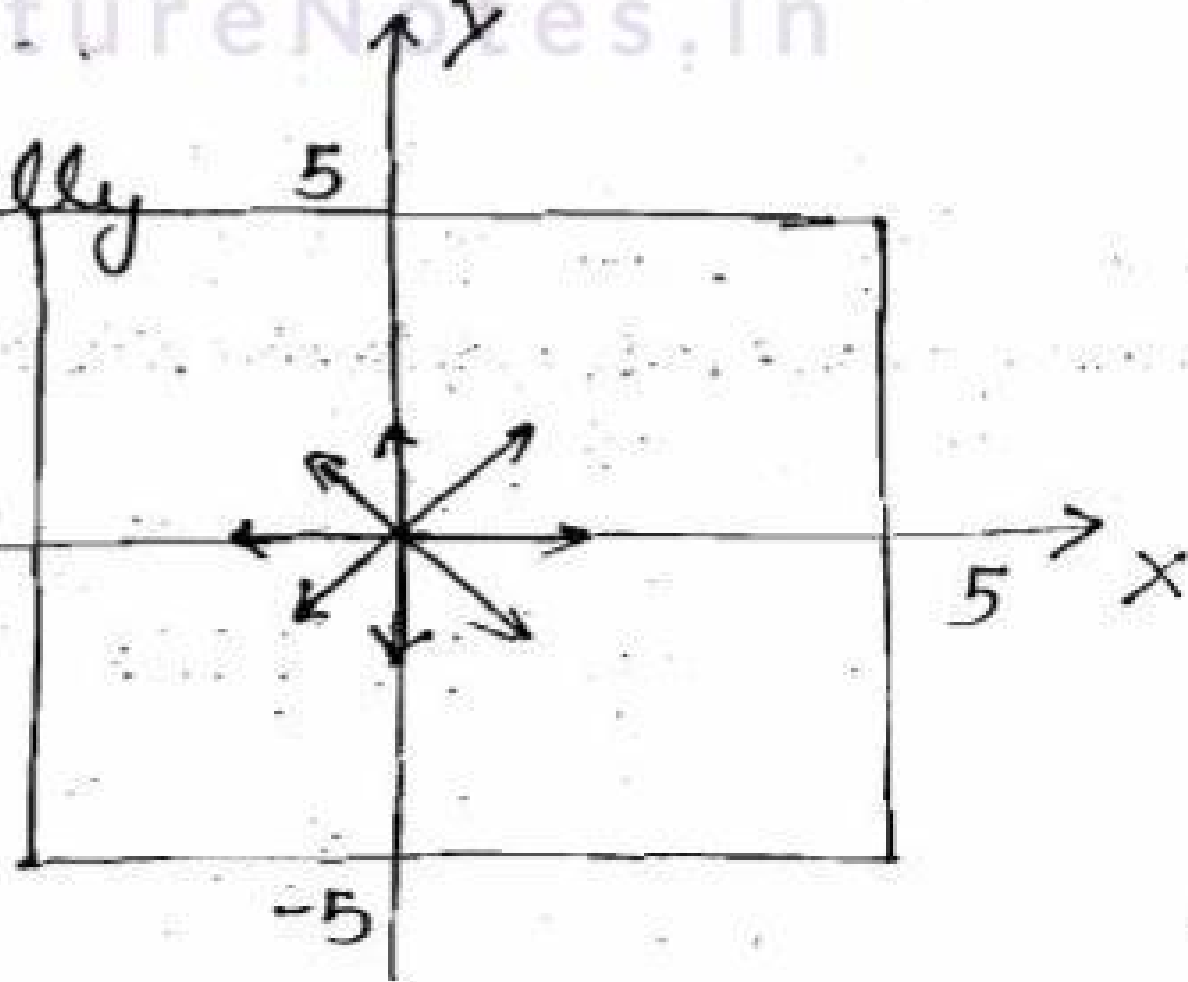
$$Q = 15 \times 10\sqrt{2}$$

$$= 150\sqrt{2}$$



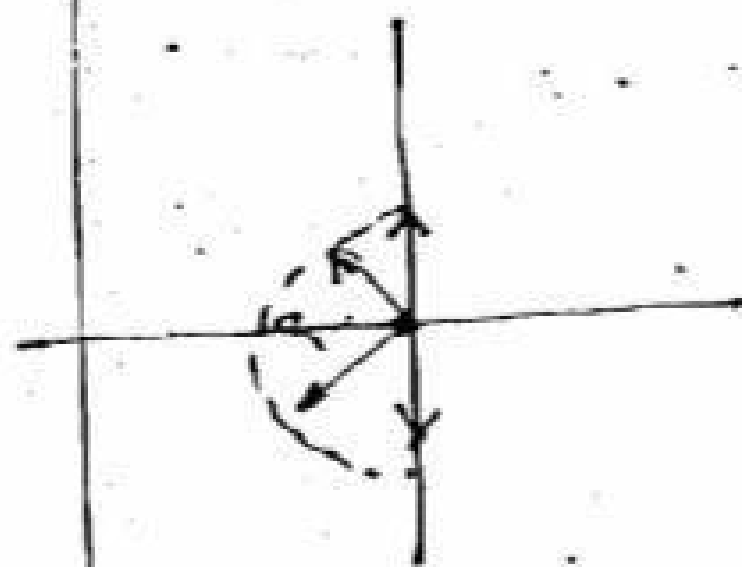
$$Q = 6 nC$$

$6 nC \rightarrow$ flux symmetrically
crosses 6 sides of
cube. Inc-flux
cross per 1 side $= 5$



$z = 5m$ -plane

$3nC, 141-2$



LectureNotes.in

Note:-

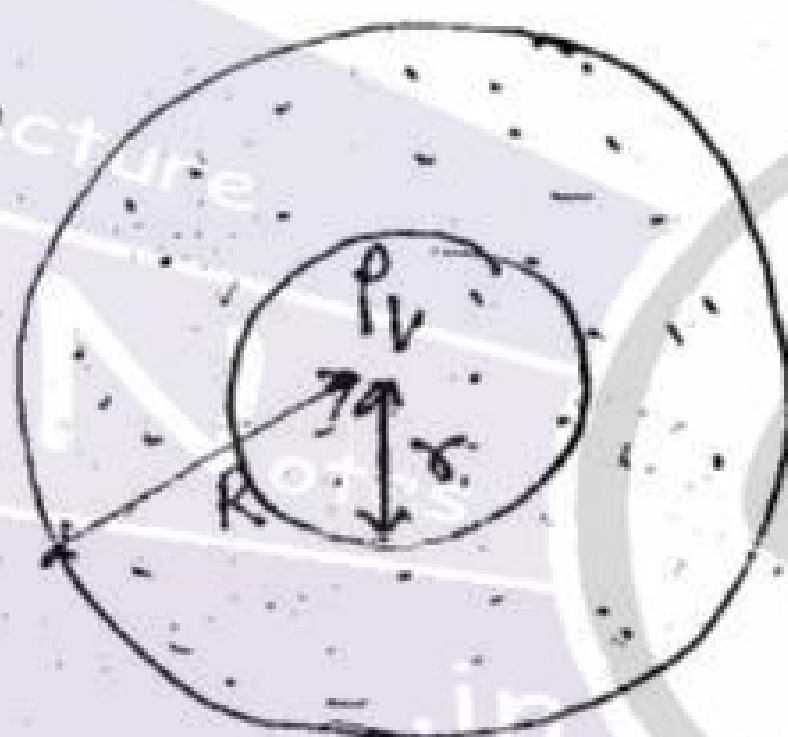
A point charge and a infinite sheet near by has half the flux crossing through

$$\oint \mathbf{E} \cdot d\mathbf{S} = Q$$

Constant

$$\mathbf{E} \cdot S = Q$$

$$\Rightarrow \mathbf{E} = \frac{Q}{S}$$



$$= \frac{\rho_v \frac{4\pi r^3}{3}}{4\pi r^2} = \frac{\rho_v r}{3}$$

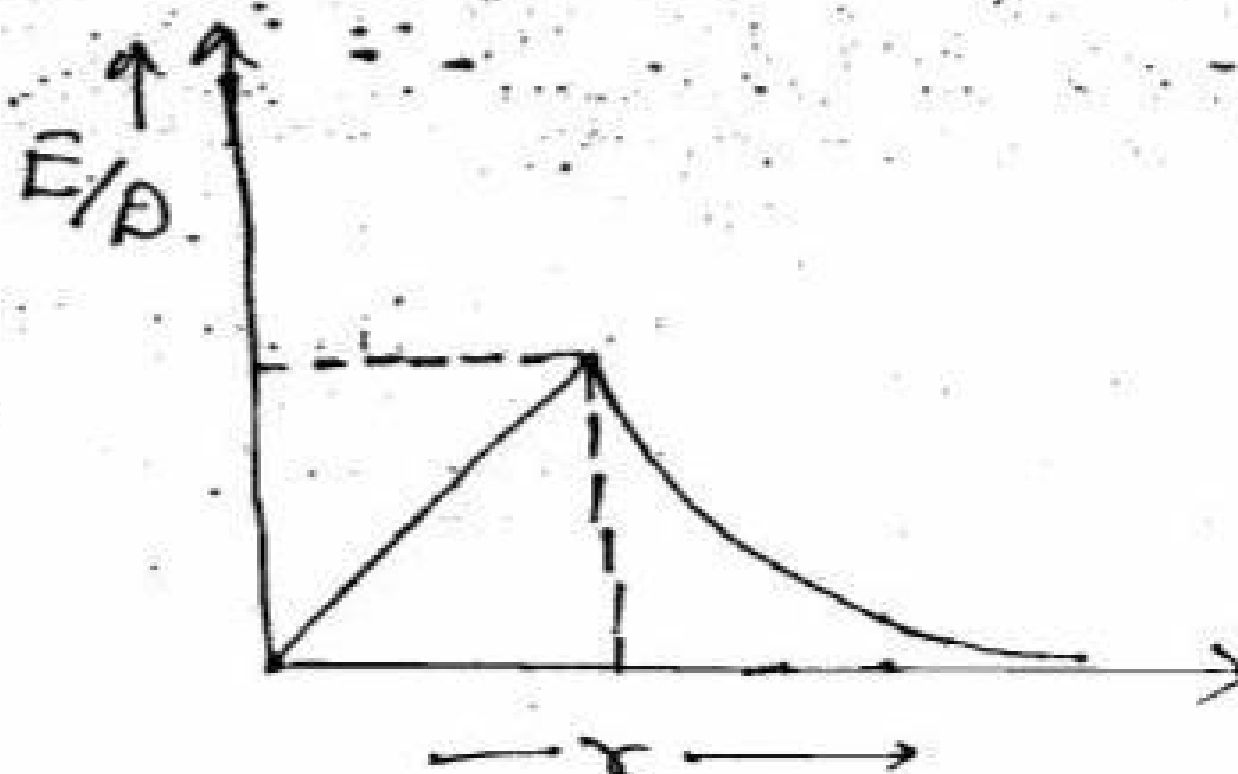
$$\mathbf{E} = \frac{\rho_v r}{3\epsilon}$$

$$\boxed{\mathbf{E}(\frac{R}{2}) = \frac{\rho_v R}{6\epsilon}}$$

$$\mathbf{E} \cdot S = Q$$

$$\Rightarrow \mathbf{E} = \frac{Q}{S}$$

$$= \frac{\rho_v \frac{4\pi R^3}{3}}{4\pi r^2}$$



$$\phi = \frac{\rho_v R^3}{3\epsilon_0}$$

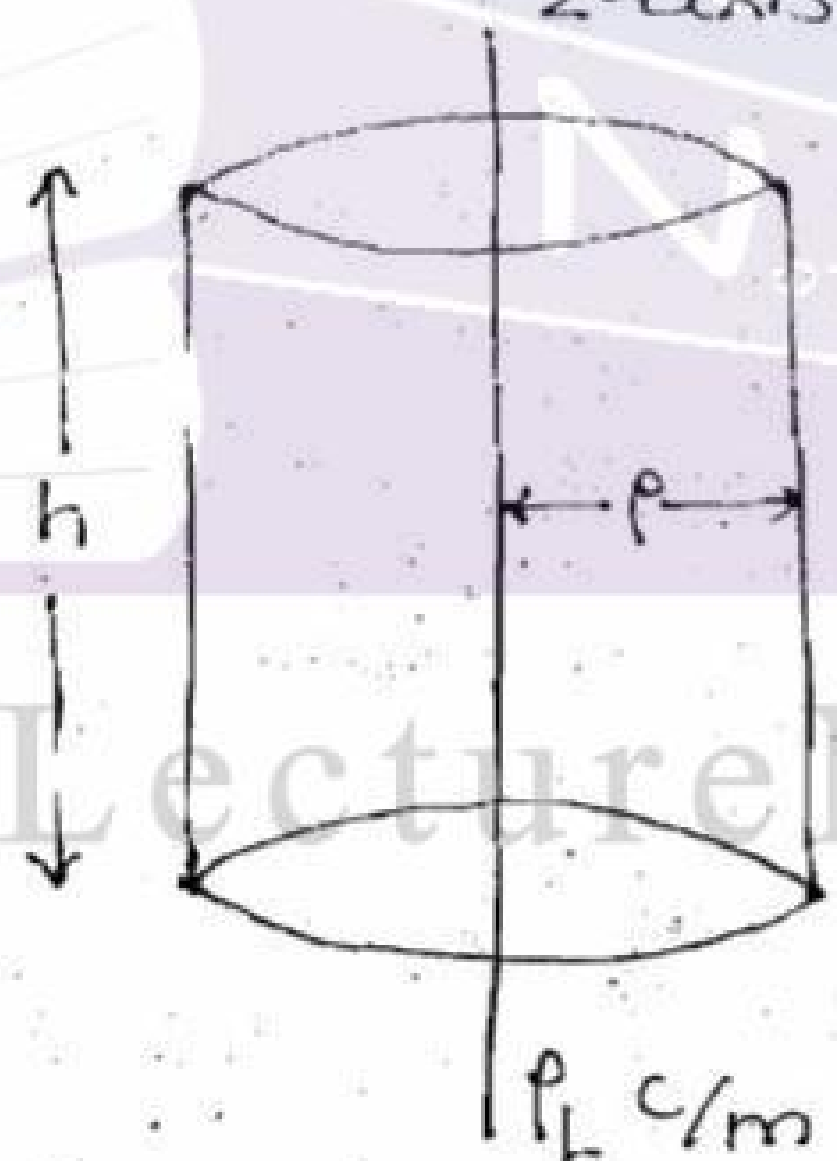
$$E = \frac{\rho_v R^3}{3\epsilon_0 r^2}$$

$$E(2R) = \frac{\rho_v R}{12\epsilon_0}$$

Gauss Law Application - 2 :-

Electric field strength of an infinite length ρ_L C/m line charge :-

Consider a concentric symmetric Gaussian surface which is a cylinder around the charge



$$\oint \rho \cdot d\mathbf{s} = Q$$

$\downarrow$
 ρ directed
 Constant

$$\rho \cdot S = Q$$

$$\Rightarrow \rho = \frac{Q}{S}$$

$$\Rightarrow \rho = \frac{\rho_L \cdot h}{2\pi r h}$$

$$\Rightarrow \rho = \frac{\rho_L}{2\pi r} q_r$$

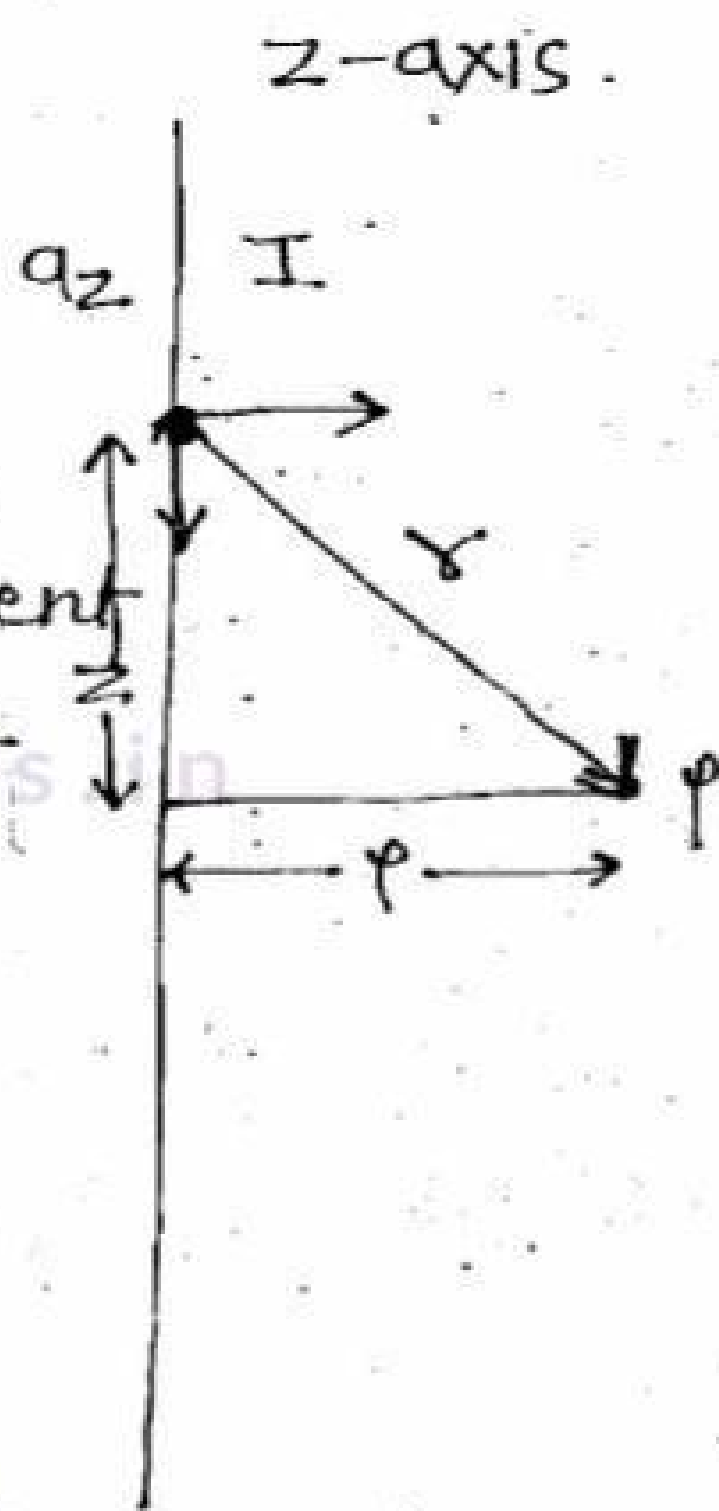
$$\Rightarrow E = \frac{\rho_L}{2\pi\epsilon_0 r} q_r$$

Magnetic field strength of an infinite length I

carrying wire:-

$$H = \frac{I d\mathbf{l} \times \mathbf{a}_r}{4\pi r^2}$$

Consider a small $d\mathbf{l}$ length I current element at the z point height above the point



$$d\mathbf{l} = dz \mathbf{a}_z$$

$$r = \sqrt{\rho^2 + z^2}$$

$$\mathbf{a}_r = \frac{\mathbf{r}}{|\mathbf{r}|}$$

$$= \frac{\rho \mathbf{a}_\rho - z \mathbf{a}_z}{\sqrt{\rho^2 + z^2}}$$

$$dH = \frac{I dz \mathbf{a}_z \times \left(\frac{\rho \mathbf{a}_\rho - z \mathbf{a}_z}{\sqrt{\rho^2 + z^2}} \right)}{4\pi (\rho^2 + z^2)}$$

$$H = \int_{z=-\infty}^{\infty} dH$$

$$\Rightarrow H = \frac{I\rho}{4\pi} \int_{z=-\infty}^{\infty} \frac{dz (\mathbf{a}_z \times \mathbf{a}_\rho)}{(\rho^2 + z^2)^{3/2}}$$

$$\text{Put } z = \rho \tan \theta$$

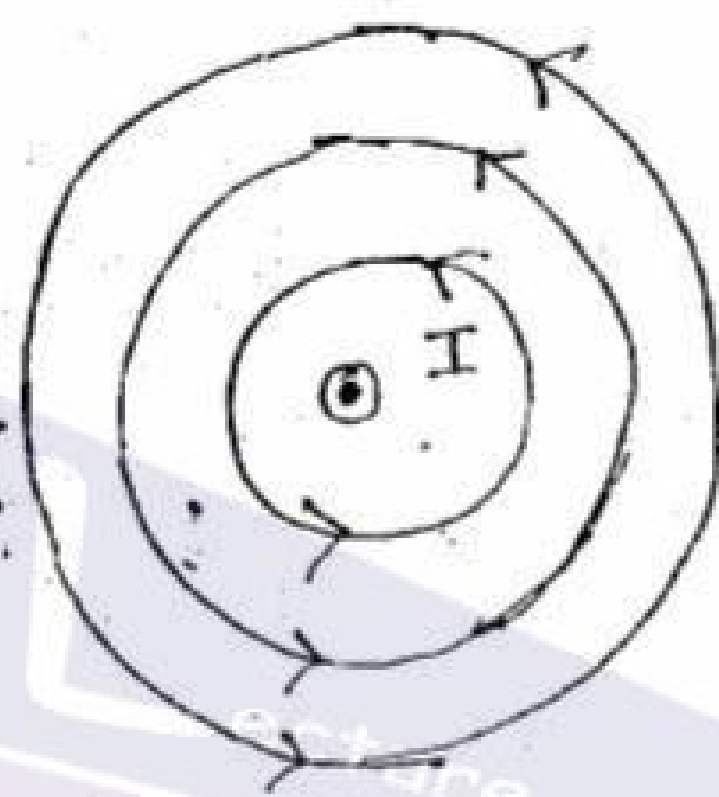
$$dz = \rho \sec^2 \theta d\theta$$

$$(\rho^2 + z^2)^{3/2} = \rho^3 \sec^3 \theta$$

$$H = \frac{I\rho}{4\pi} \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho \sec^2 \theta d\theta}{\rho^3 \sec^3 \theta} = \frac{I}{4\pi\rho} \int_{-\pi/2}^{\pi/2} \cos \theta \cdot d\theta$$

$$\Rightarrow H = \frac{I}{2\pi\rho} (a_z \times a_\rho) = \frac{I}{2\pi\rho} a_\phi$$

Note :-



$$\oint H \cdot d\ell = I$$

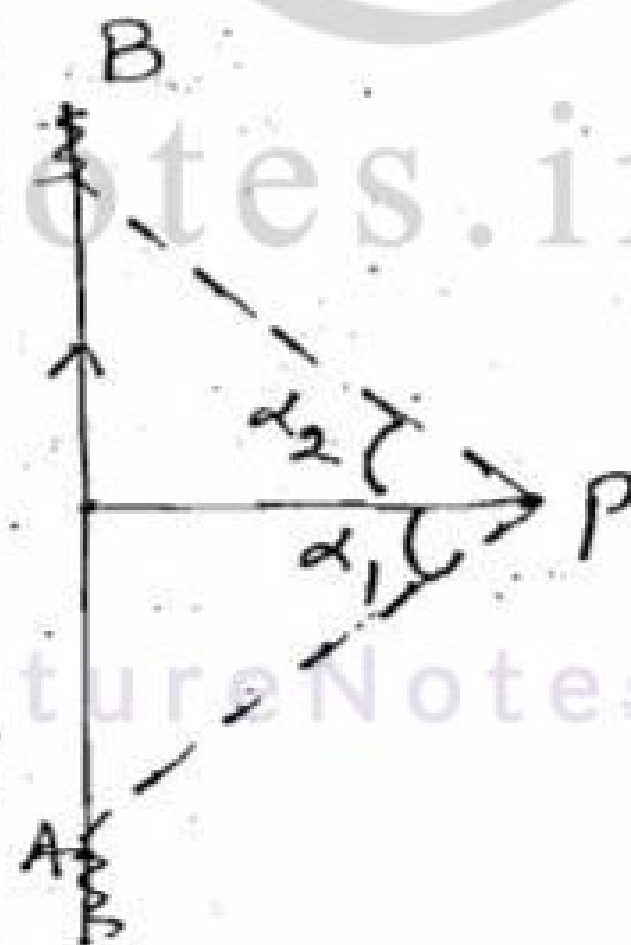
$H \cdot \text{Length} = \text{current}$

$$H = \frac{I}{2\pi\rho} a_\phi$$

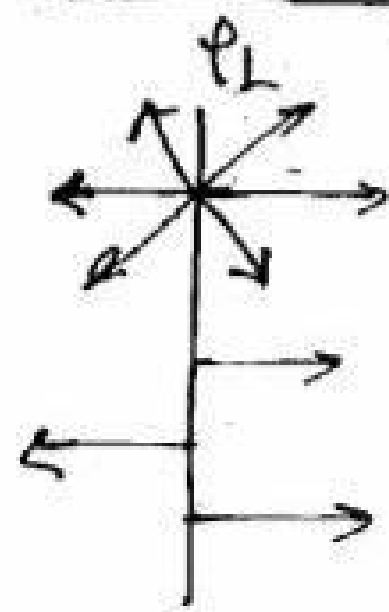
Extension :-

H-field of finite length I wire :-

$$H = \frac{I}{4\pi\rho} (\sin \alpha_1 + \sin \alpha_2) a_\phi$$



Summary :-



$\rho_L \rightarrow$ Line charge

$E \rightarrow$ Field

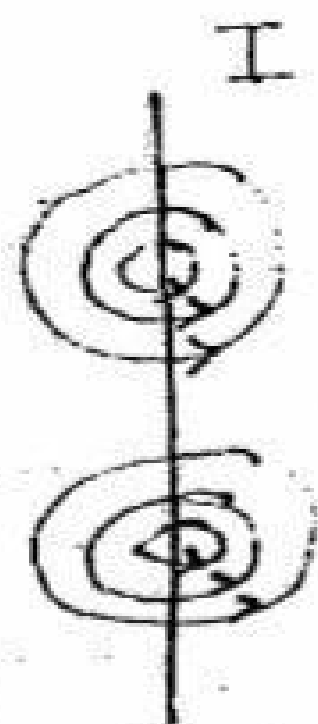
$$\rho = \frac{\rho_L}{2\pi\rho} a_\rho$$

$$E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho$$

I carrying H-field

$$H = \frac{I}{2\pi r} a_\phi$$

$$B = \frac{\mu I}{2\pi r} a_\phi$$



7:-

consider a dl length
on the charge and
apply the $\frac{q}{4\pi\epsilon r^2} a_r$

Field calculation

$$Q = \rho_L dx$$

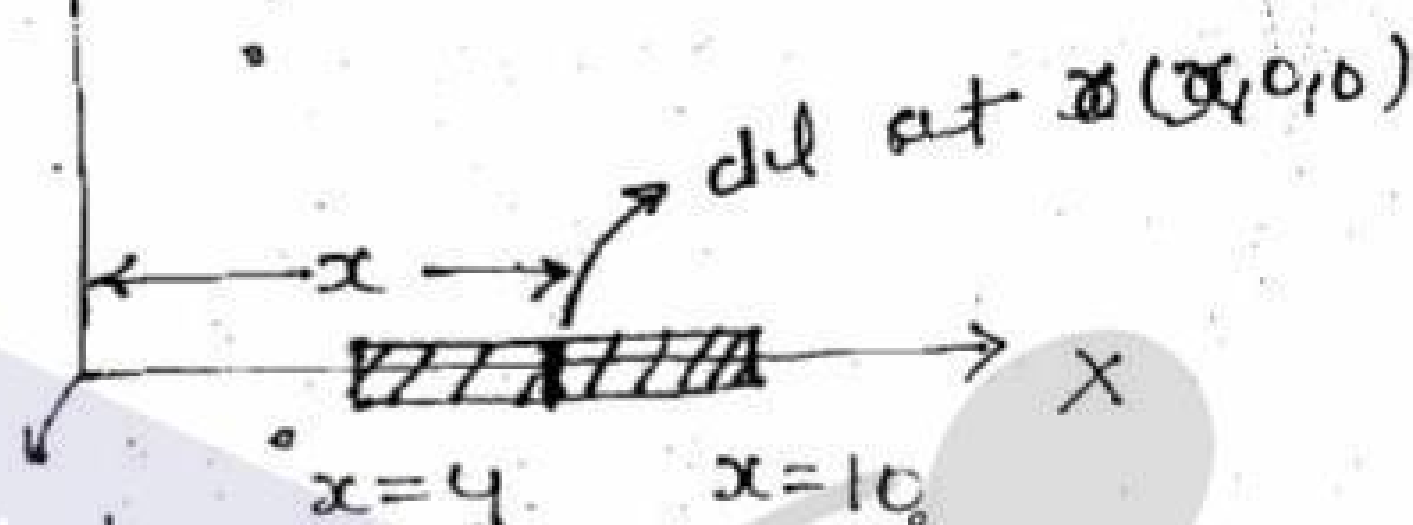
$$r = x$$

$$a_r = -a_x$$

$$dE = \frac{\rho_L dx}{4\pi\epsilon x^2} (-a_x)$$

$$E = \frac{\rho_L}{4\pi\epsilon} \int_{x=4}^{10} \frac{dx}{x^2} (-a_x)$$

$$= \frac{\rho_L}{4\pi\epsilon} \left(-\frac{1}{x} \right)_4^{10} (-a_x)$$



$$H = \frac{8}{4\pi \cdot 3} \left(1 + \frac{4}{5} \right)$$

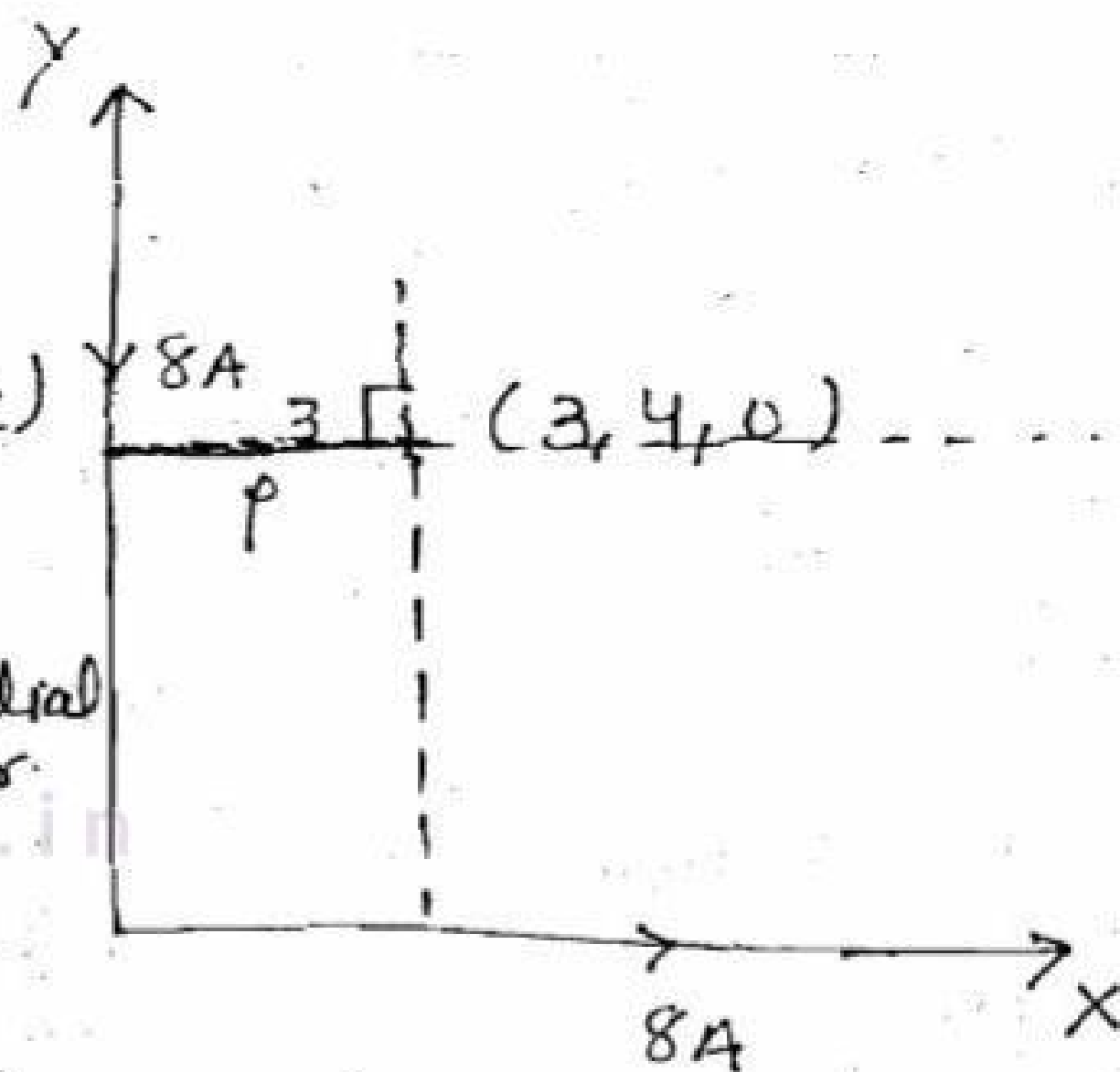
(Y-axis)

$$(-a_y \times a_x)$$

$$a_\phi = a_z \times a_\rho$$

$$H\text{-direction} = I \times \text{Radial direction}$$

$$= \frac{8}{4\pi \cdot 3} \times \frac{9}{5} (a_z)$$



$$H = \frac{8}{4\pi \cdot 4} \left(\frac{3}{5} + 1 \right) (a_x \times a_y)$$

(X-axis)

$$= \frac{8}{16\pi} \times \frac{8}{5} (a_z)$$

$$H = \frac{2}{\pi} a_z$$

(total)

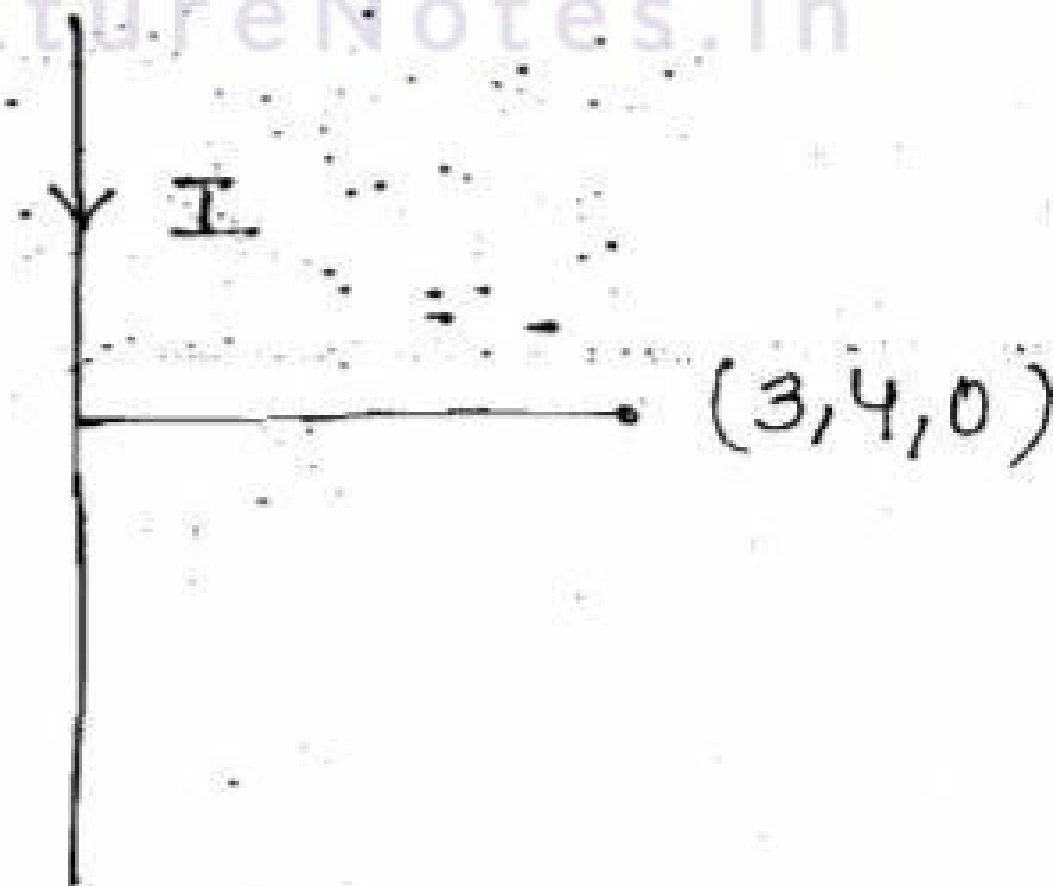


ques:- Repeat the above problem if current is only entire y-axis

Soln:-

$$H = \frac{8}{2\pi \cdot 3} a_z$$

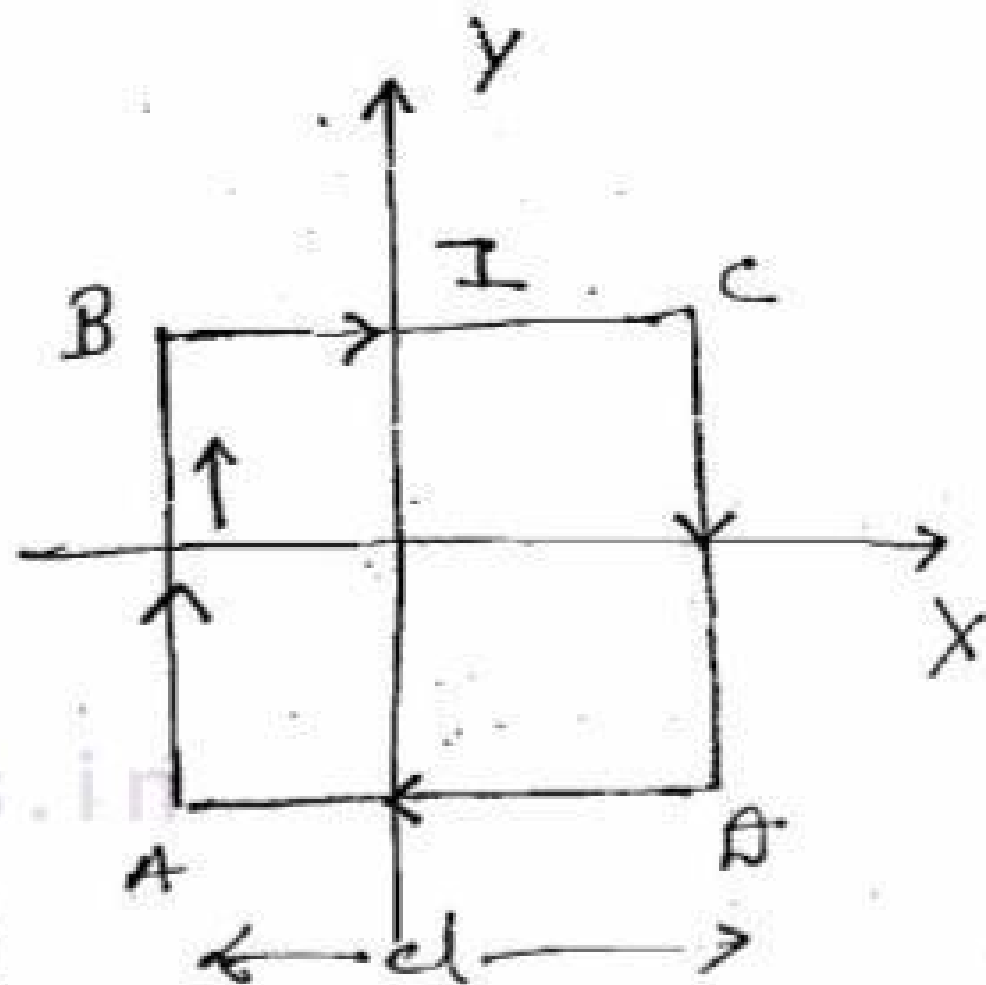
$$= \frac{1.33}{\pi} a_z$$



19

$$H_{AB} = a_y \times a_x = -a_z$$

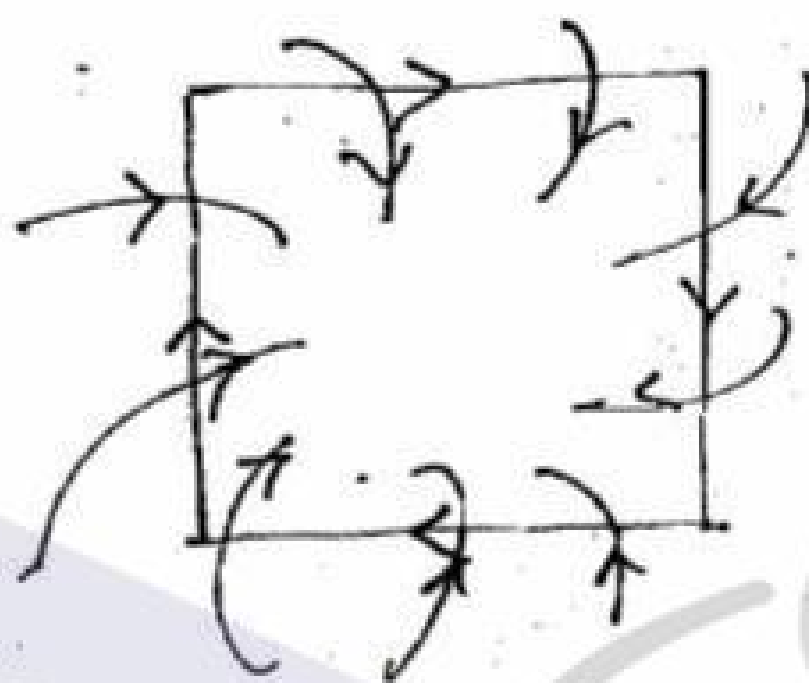
$$H_{BC} = a_x \times -a_y = -a_z$$



Note:-

→ For any symmetric current distribution H field at geometric centre is always max

→ For any symmetric charge distribution E field at the geometric centre is always zero



$$H = \frac{I}{4\pi \cdot \frac{d}{2}} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \times 4 (-a_z)$$

$$= \frac{2\sqrt{2} I}{\pi d} (-a_z) = 0.9 \frac{I}{d} (-a_z)$$

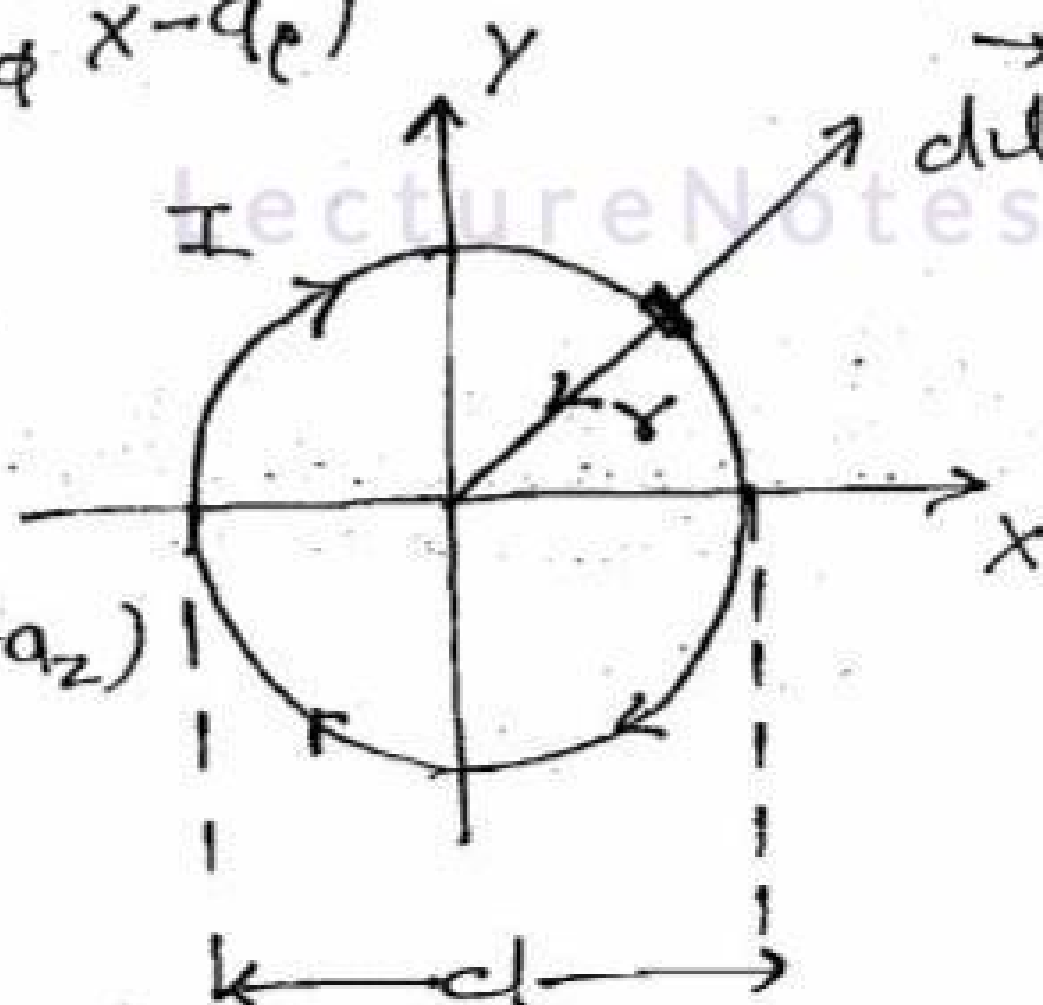
20

$$dH = \frac{I \cdot d\vec{l} \times (-a_\phi \times -a_\rho)}{4\pi r^2}$$

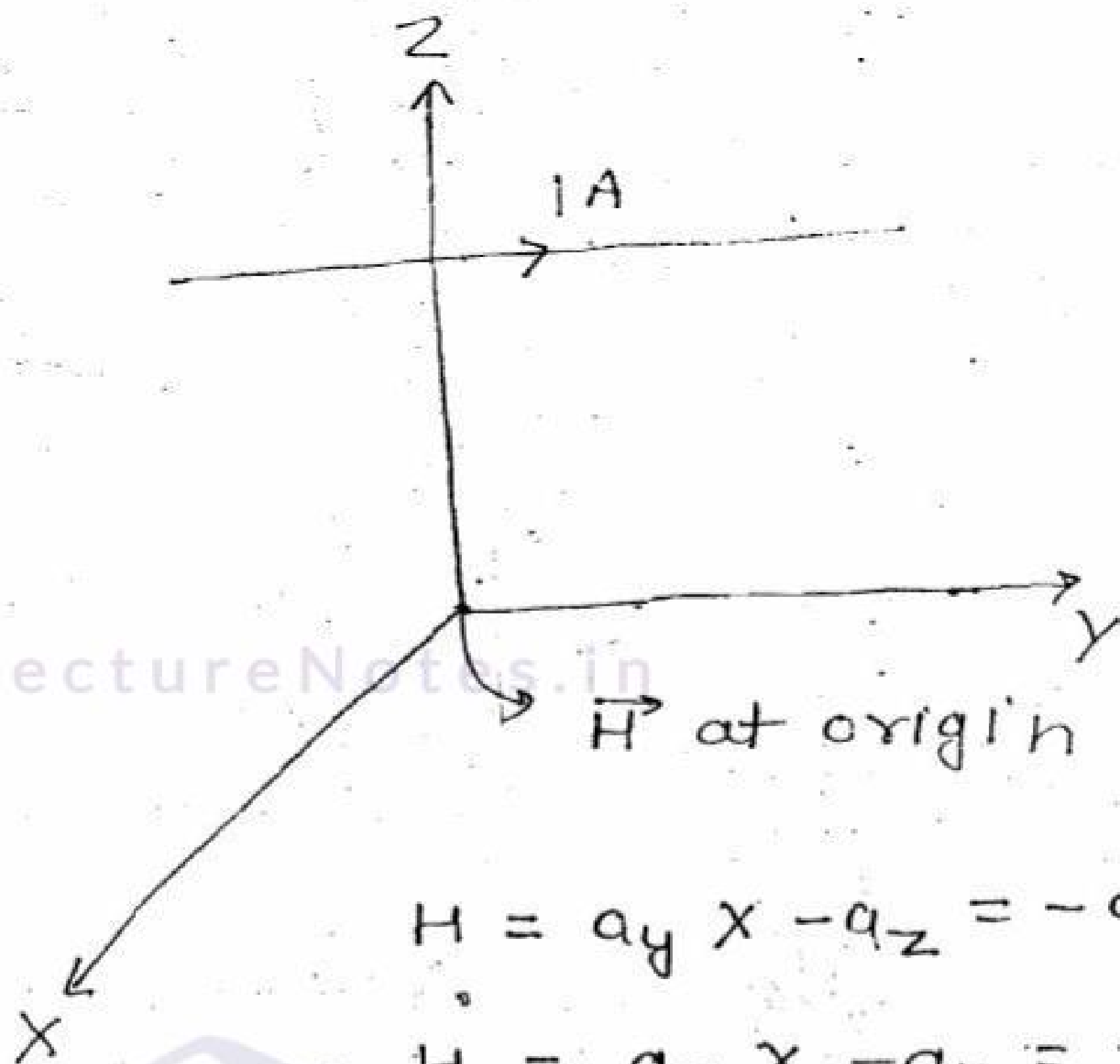
$$H = \int dH$$

$$= \frac{I 2\pi r}{4\pi r^2} (-a_z)$$

$$= \frac{I}{2r} = \frac{I}{d} (-a_z)$$



22 :-



$$H = a_y x - a_z = -a_x$$

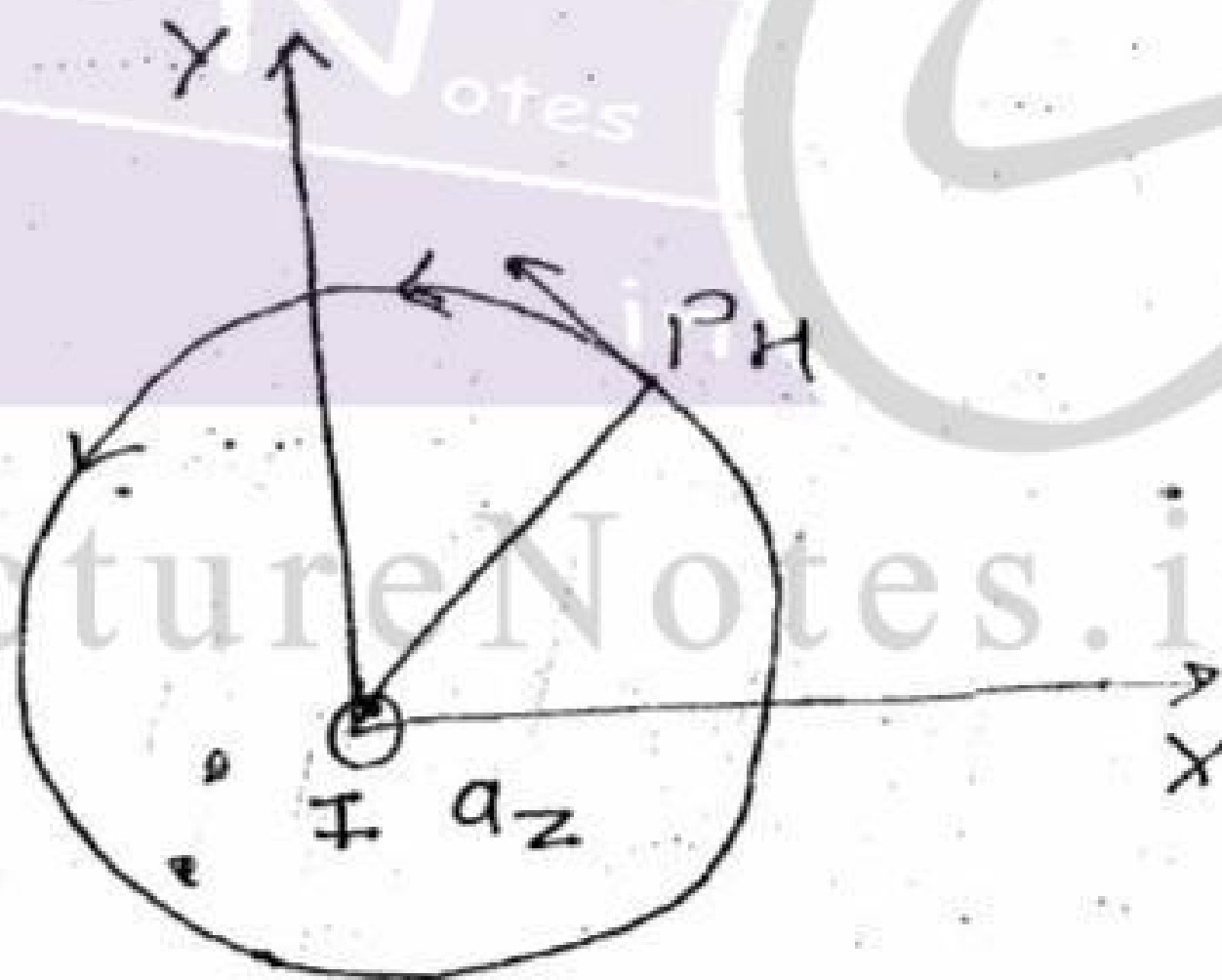
$$H = a_x x - a_y = -a_z$$

Ans - d

23.

$$H_d = a_z \times a_p$$

Ans - (c)



Lecture - 5

21.

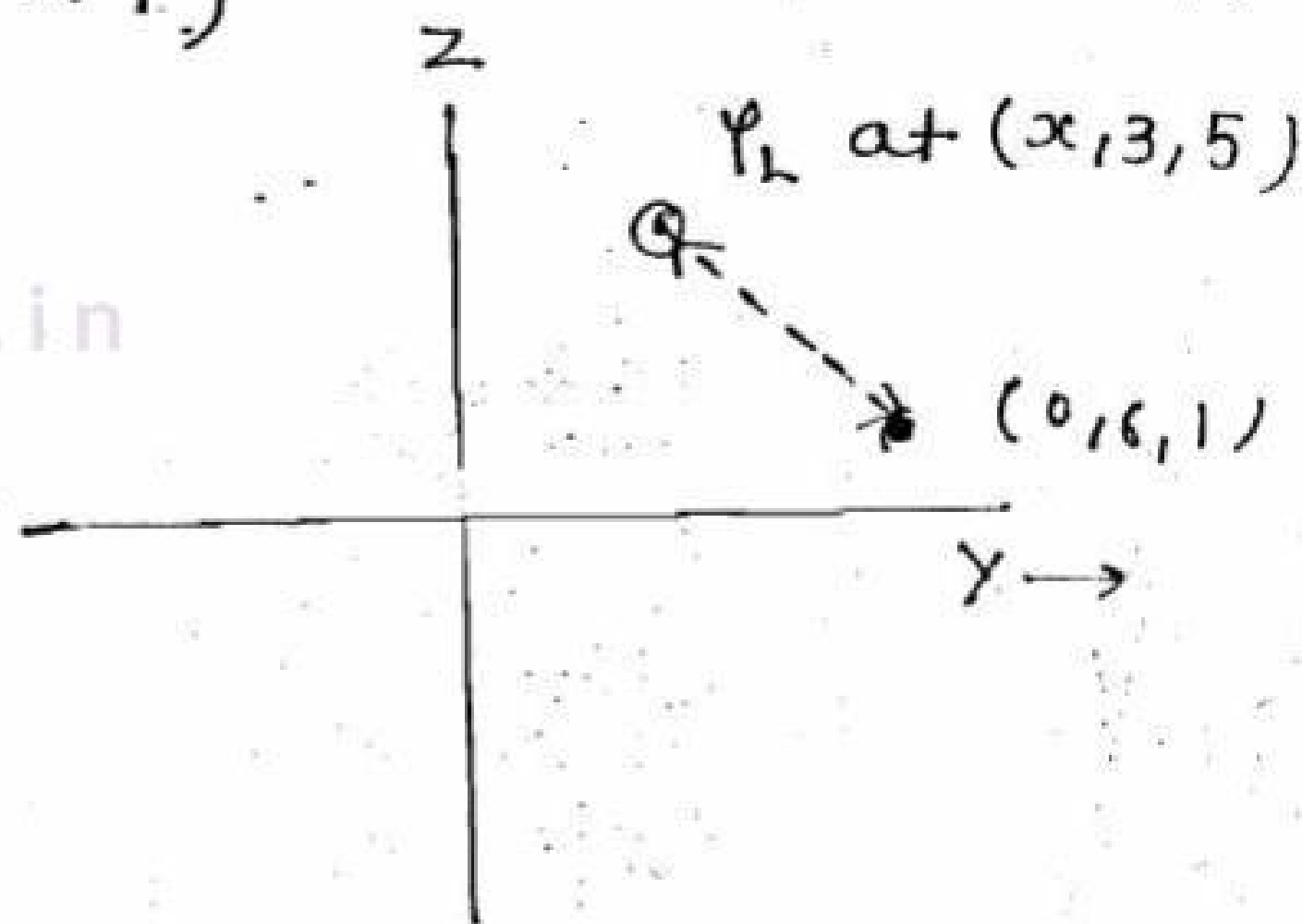
ρ_L at $y=3$, $z=5$ for all x

Given $\vec{E}$ at $(0, 6, 1)$

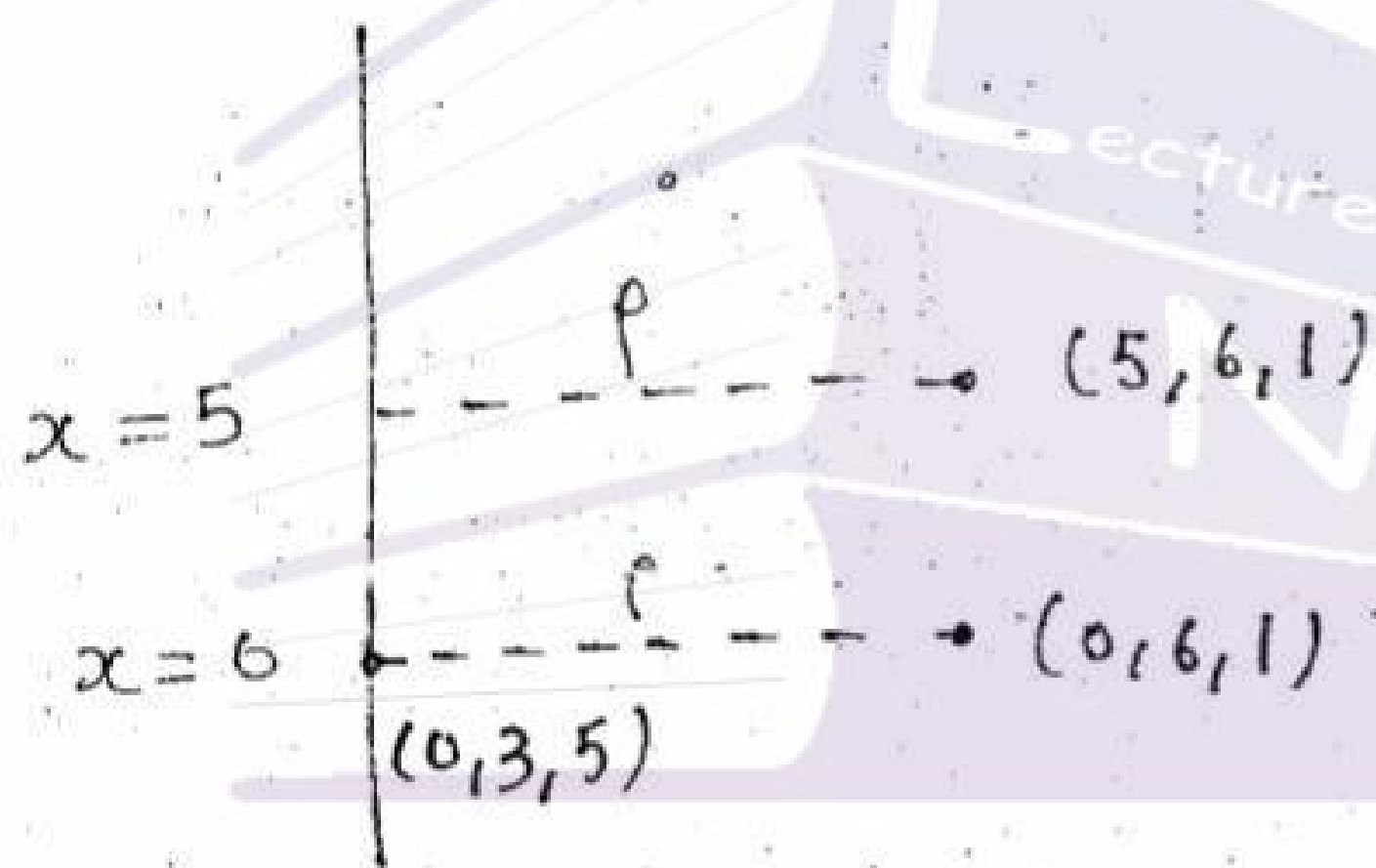
$\vec{E} = ?$ at $(5, 6, 1)$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon\rho} \hat{q}_\rho$$

$$E \propto \frac{1}{\rho}$$



ρ_L at $(x, 3, 5)$



For both points ρ is same

$$\rho = \sqrt{3^2 + 4^2} = 5$$

Note:-

In general z -axis $\rightarrow (0, 0, z)$

Any point (x, y, z)

$$\rho = \sqrt{x^2 + y^2}$$

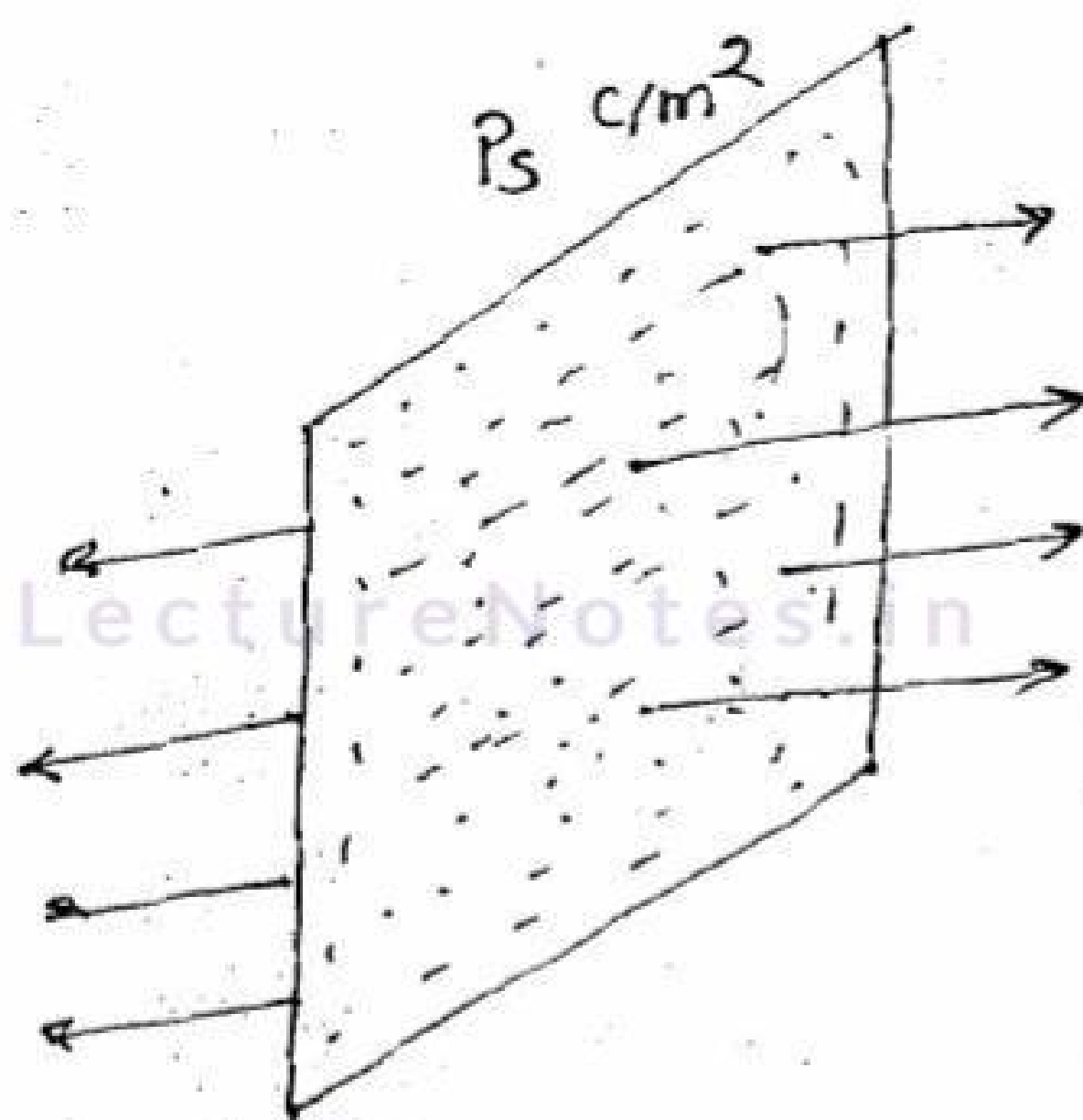
Any line parallel to x -axis $\rightarrow (x, a, b)$

Any point $\rightarrow (x, y, z)$

$$\rho = \sqrt{(y-a)^2 + (z-b)^2}$$

Electric field strength of an infinite sheet of

ρ_s C/m² charge density :-



→ The electric field is completely normal to the sheet and the field lines are parallel to themselves

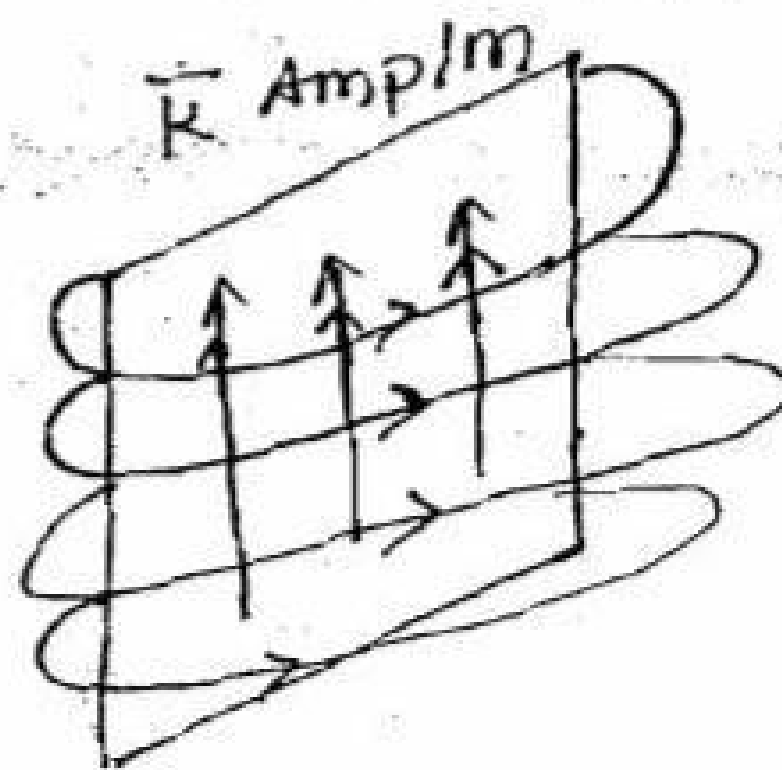
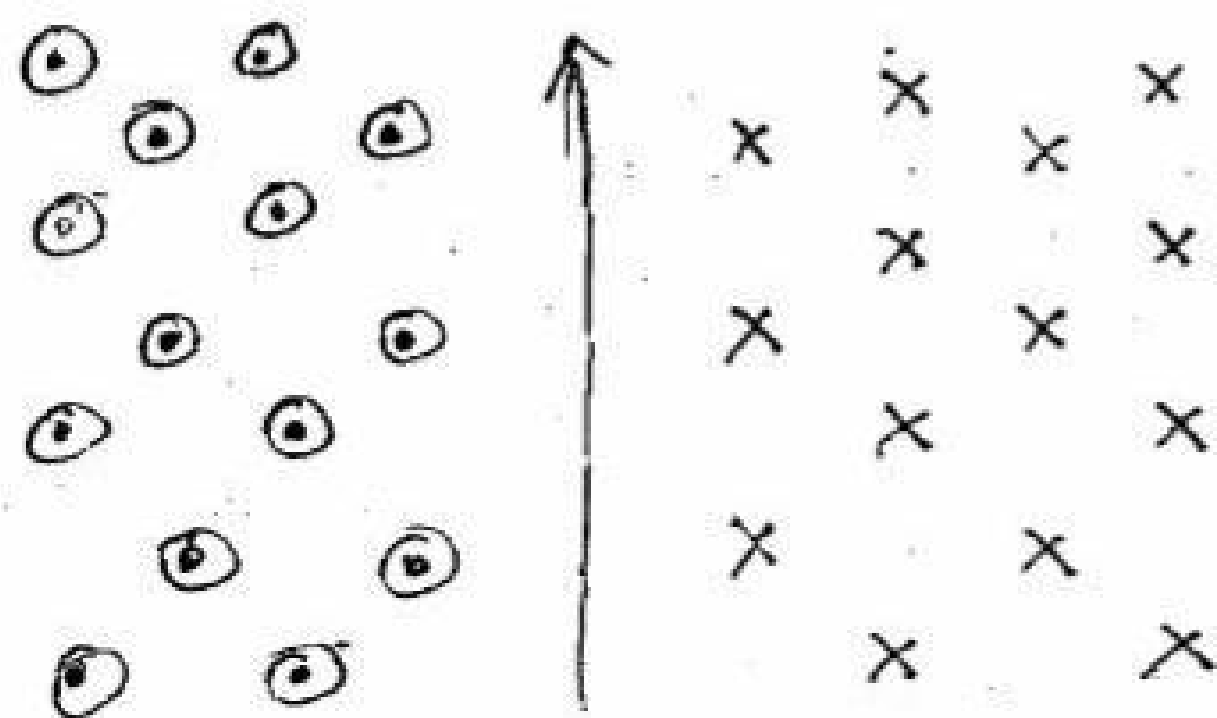
→ The flux density is same everywhere. The field is uniform.

$$\rightarrow D = \frac{\rho_s}{2} a_N$$

$$\rightarrow E = \frac{D}{\epsilon} = \frac{\rho_s}{2\epsilon} a_N \Rightarrow \boxed{E = \frac{\rho_s}{2\epsilon} a_N}$$

Magnetic field strength of an infinite sheet of

ρ_s C/m² charge density :-



→ The magnetic field is completely normal to the current and the field lines are parallel to themselves and to the sheet

→ The flux density is same everywhere i.e. field is uniform

→ $H = \frac{\vec{k} \times a_N}{2} \text{ Amp/m}$

and $B = \frac{\mu \vec{k} \times a_N}{2}$

24.

$a_N = \pm a_z$

At $z=0$, $a_N = -a_z$

$$E = \frac{\rho_s}{2\epsilon} a_N$$

$$= \frac{20 \times 10^{-9}}{2 \times \frac{1}{36\pi \times 10^9}} (-a_z)$$

$z=10$

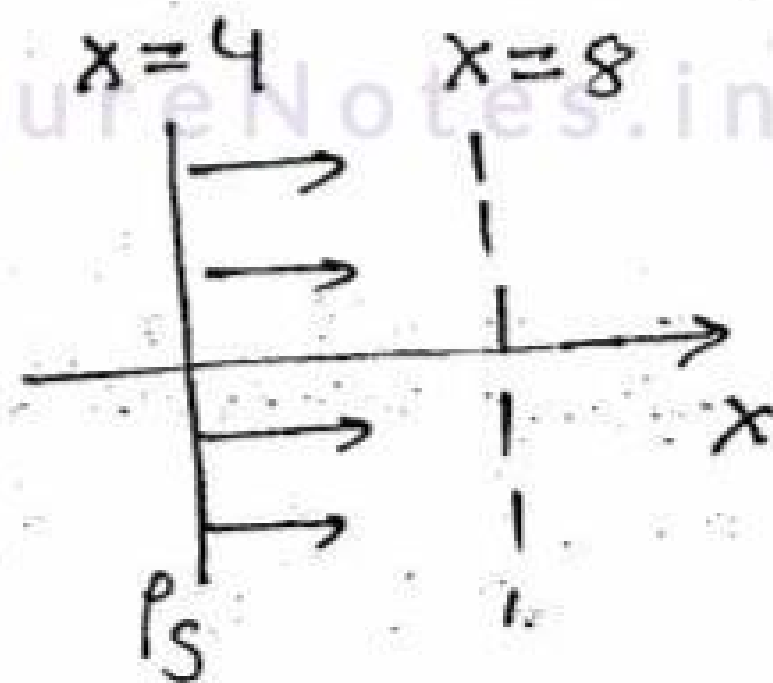
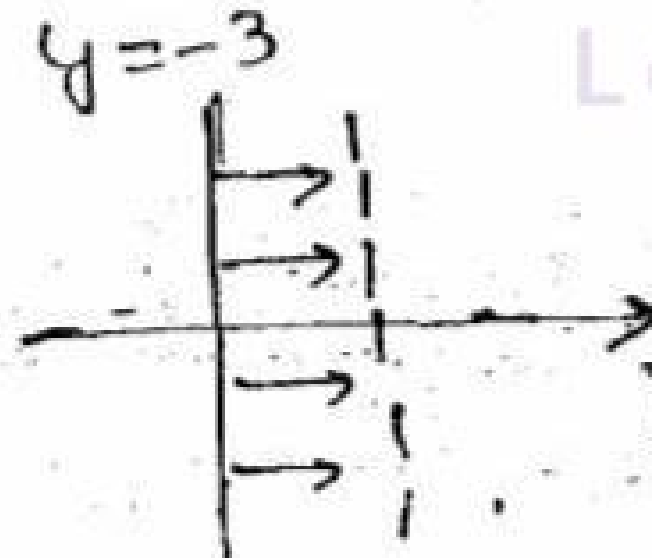
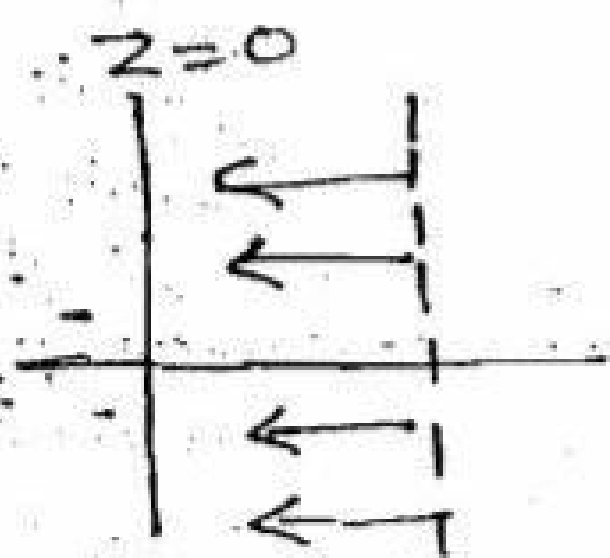
$\rho_s = 20 \text{ nC/m}^2$

25.

$x=4$ $\rho_{s1} = 18 \text{ nC/m}^2 \rightarrow E_1 = \pm a_x$

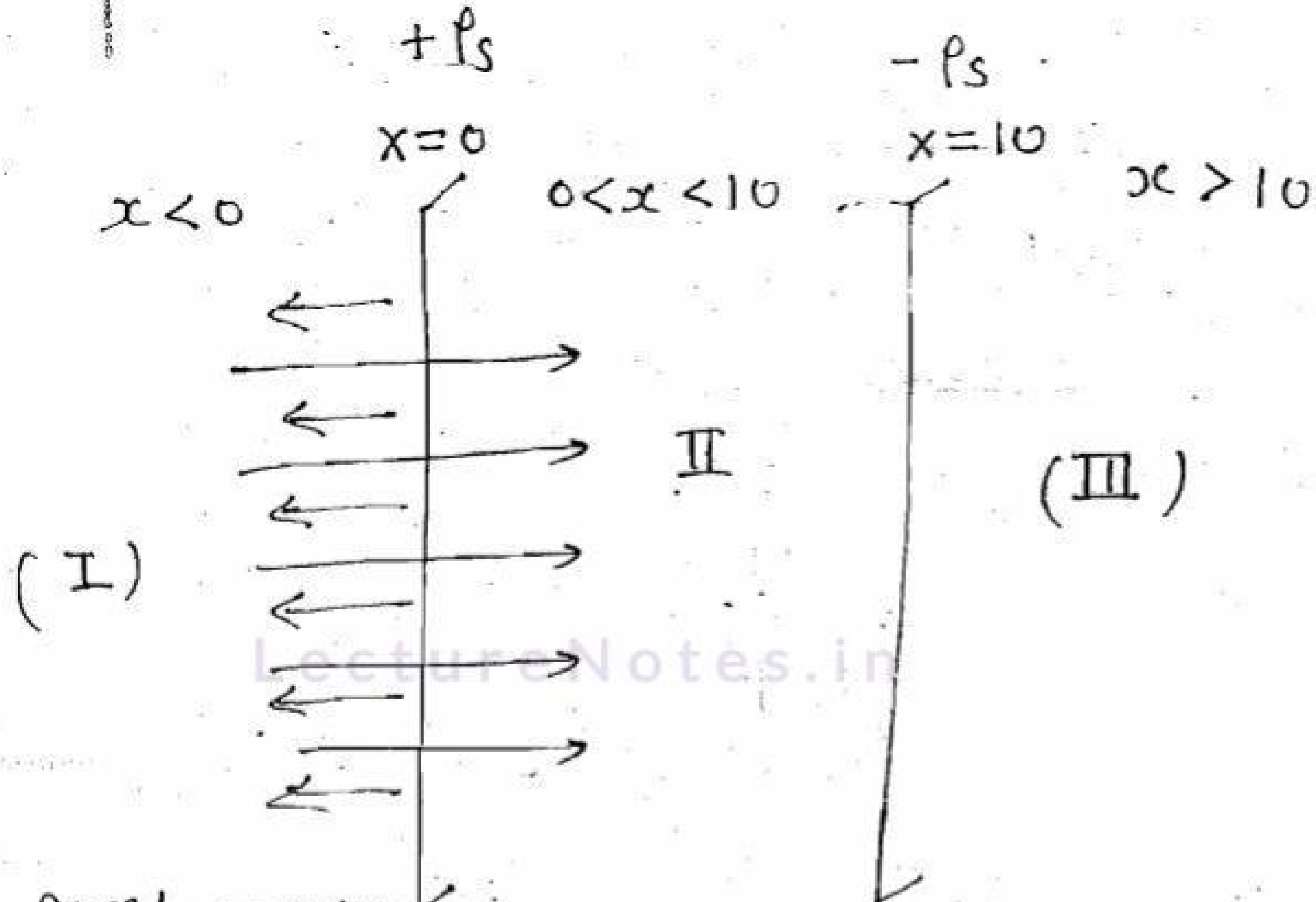
$y=-3$ $\rho_{s2} = 9 \text{ nC/m}^2 \rightarrow E_2 = \pm a_y$

$z=0$ $\rho_{s3} = -24 \text{ nC/m}^2 \rightarrow E_3 = \pm a_z$



$$E_3 = \frac{24}{2\epsilon} (-a_z) \quad E_2 = \frac{9}{2\epsilon} a_y \text{ nN/C} \quad E_1 = \frac{18}{2\epsilon} a_x \text{ nN/C}$$

$$E_T = \frac{3}{2\epsilon} (6a_x + 3a_y - 8a_z) \text{ nN/C}$$



In first region

$$E_1 = \frac{\rho_s}{2\epsilon} (-a_x)$$

$$E_2 = \frac{\rho_s}{2\epsilon} (a_x)$$

$$\vec{E}_{\text{Net}} = 0$$

In third region

$$\vec{E}_{\text{Net}} = 0$$

In second region :-

$$E_1 = \frac{\rho_s}{2\epsilon} a_x$$

$$E_2 = \frac{\rho_s}{2\epsilon} a_x$$

$$E = \frac{\rho_s}{\epsilon} a_x \quad 0 < x < 10$$

$$= 0 \quad \text{elsewhere}$$

$$\vec{K} = 30 a_z \text{ mA/m} \rightarrow \gamma = 0 \text{ plane} \Rightarrow zx \text{ plane}$$

$$H = \frac{\vec{K}}{2} \times a_N$$

$$\text{at } y = 2$$

$$a_N = \pm a_y$$

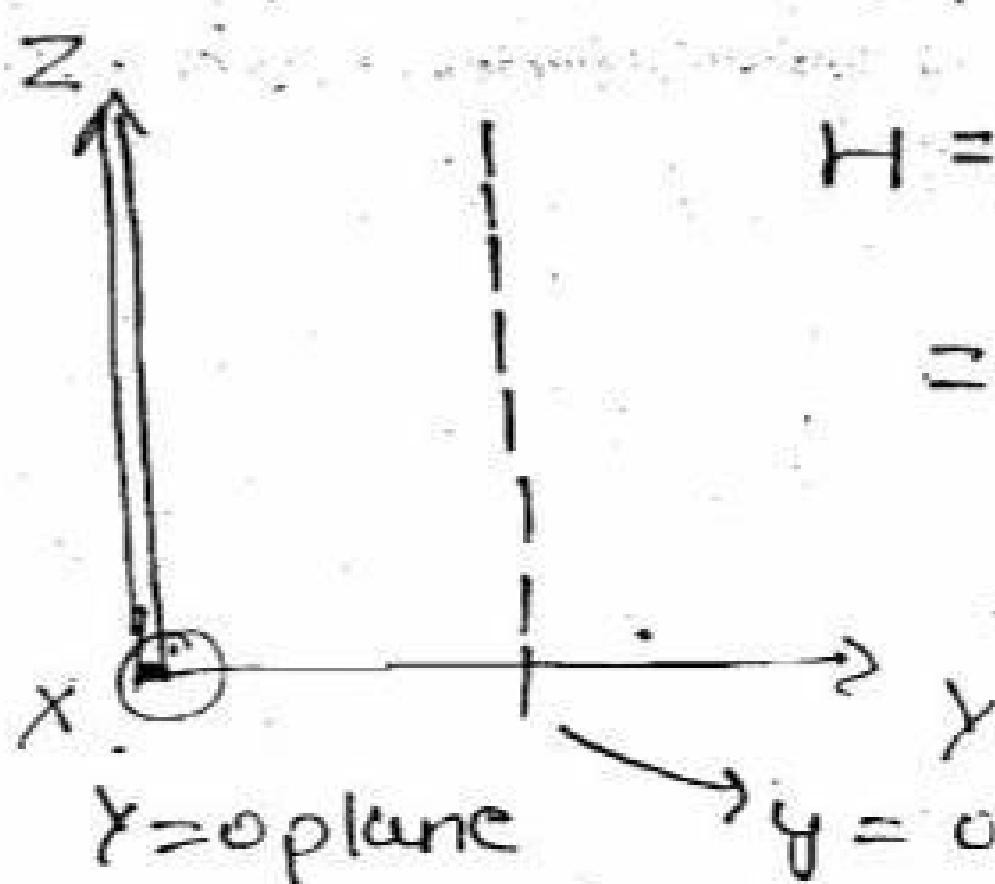
$$\text{for } y = 20$$

$$a_N = a_y$$

$$H = \frac{30}{2} a_z \times a_y$$

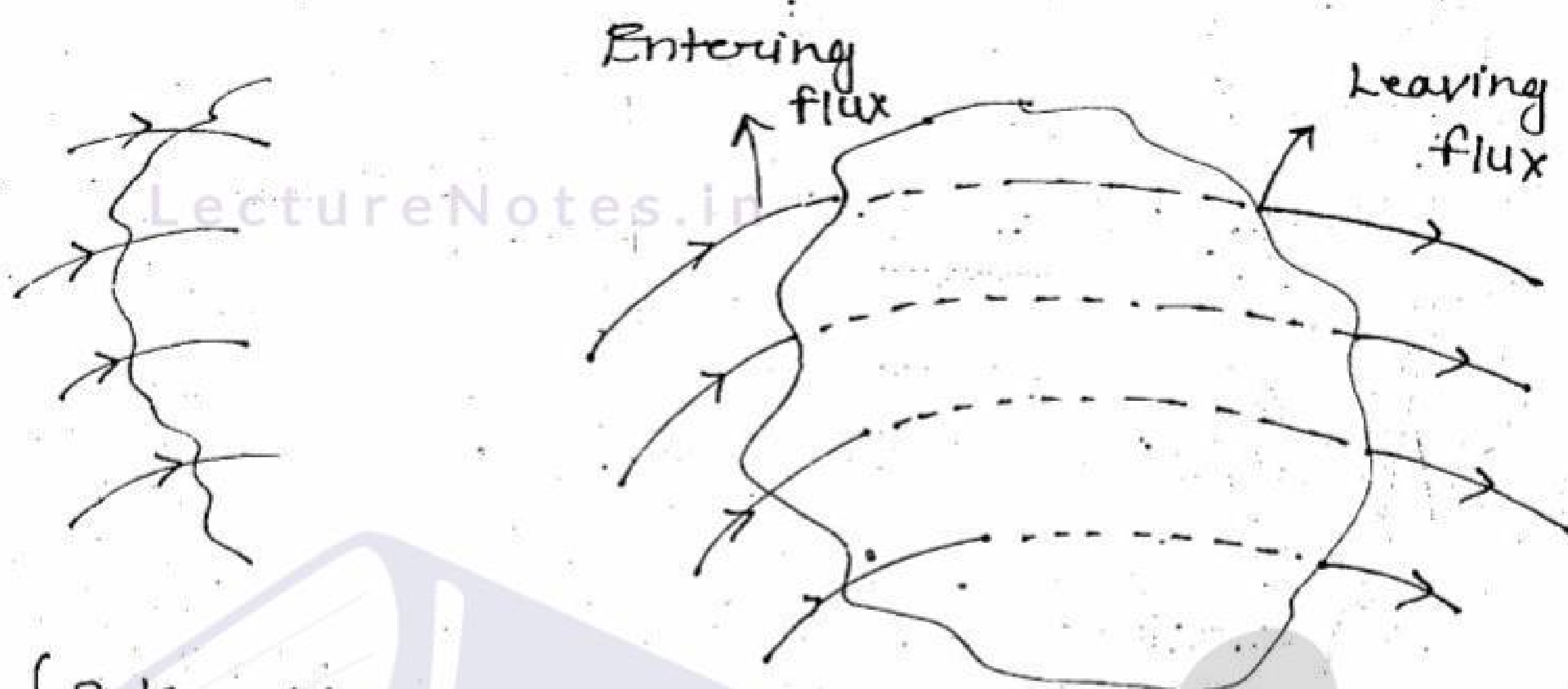
$$= -15 a_x \text{ mA/m}$$

Ans-(A)



Closed surface Integral of B — Maxwell's III Equation:-

→ Magnetic field always forms closed lines around the current and has no starting / ending point for flux lines i.e. No sources/sinks exists for B flux lines



$$\int \mathbf{B} \cdot d\mathbf{s} = \psi_m$$

This is not Maxwell's equation

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

This is Maxwell III equation

→ For any closed surface

Entering flux = Leaving flux

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

B field is solenoidal (No divergence)

$$\nabla \cdot \mathbf{B} = 0$$

Note :-

$$\left[\begin{array}{l} \oint \mathbf{B} \cdot d\mathbf{s} = 0 \rightarrow q = 0 \\ \phantom{\oint \mathbf{B} \cdot d\mathbf{s} = 0} \rightarrow +q, -q \end{array} \right] \Rightarrow \boxed{\oint \mathbf{B} \cdot d\mathbf{s} = 0}$$

Always

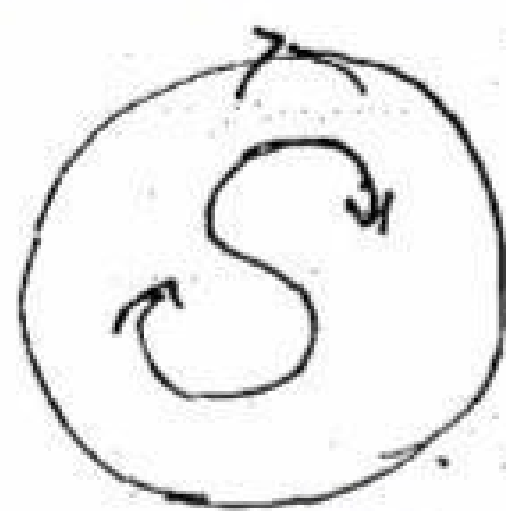
→ By comparison, equals of charge in E fields do not exist in B fields i.e. every cause of B field is a dipole, i.e. Magnetic monopoles don't exist.

eg:- Bar Magnet, N/S - Dipole.

best example:- I. current carrying wire

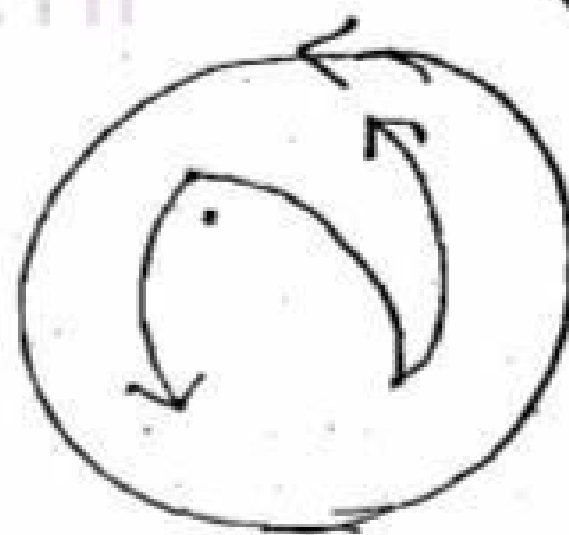
→ I always flows only when both the polarities exist and are connected hence it is always treated as magnetic dipoles.

→ I always flows in closed circuits and every closed I wire is a magnetic dipole



Clockwise

→ South pole



Anticlockwise

→ North pole

→ cause - current - closed

Entering current = Leaving current
(Junction)

Effect - B field - closed around the cause

Entering flux = Leaving flux (closed surface)

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

→ KCL in Magnetic Field

Potential, Gradient, closed line Integral in E field:-

Potential is a ^{scalar} ~~square~~ measure of E field strength in terms of energy at any point in the field.

V (Volts)
Potential at any point = $\frac{\text{Work done by the charge to reach the point}}{\text{charge}}$

$$V = \frac{W}{Q} = \frac{\text{Joules}}{\text{Coulomb}}$$

$$\Rightarrow \text{Work} = \text{Force} \cdot \text{displacement}$$

$$\Rightarrow dW = \vec{F} \cdot d\vec{l} = Q \vec{E} \cdot d\vec{l}$$

= Work done in field / force direction,

= Work done on the charge

→ Work is done on the charge when it moves in the field direction and hence the charge acquires energy

$$V = \frac{W}{Q} = - \int \vec{E} \cdot d\vec{l} \rightarrow \text{Scalar potential function of space}$$

$$V_{AB} = - \int_{\text{Ref B}}^{\text{Ref A}} \vec{E} \cdot d\vec{l} = \text{Potential diff b/w A \& B}$$

If ref. B has $V_B = 0$, then V_A is called as absolute potential at A.

Potential function of a point charge :-

$$V = - \int \vec{E} \cdot d\vec{l} \quad \text{Spherical coord.}$$

$$= - \int \frac{Q}{4\pi\epsilon_0 r^2} a_r dr a_r = \frac{Q}{4\pi\epsilon_0 r}$$

$$\Rightarrow \boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$$

$\vec{E} \rightarrow$ Intensity $\sim \frac{1}{r^2}$ decrease

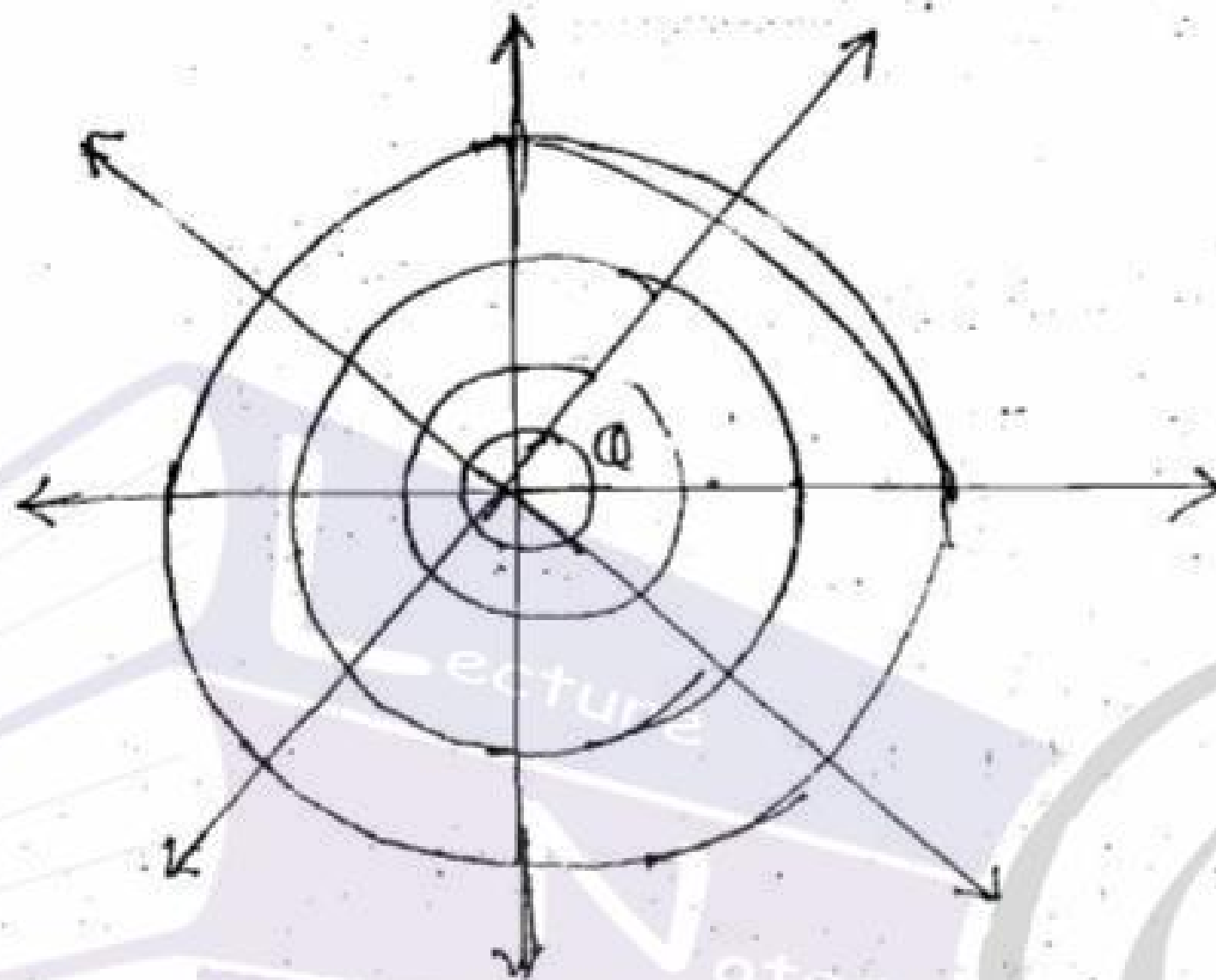
$\vec{a}_r$ directed vector

$V \rightarrow$ Potential $\sim \frac{1}{r}$ decrease

— scalar —

If $r = \text{constant}$ then $V = \text{constant}$

$\rightarrow$ concentric sphere $\rightarrow$ surface



$\rightarrow$ The family of concentric equipotential surfaces represent the potential distribution and variation of potential in the region and similar to vector intensity line

Potential function of a line charge:-

$$V = - \int \vec{E} \cdot d\vec{u}$$

$$= - \int \frac{\rho_L}{2\pi\epsilon\rho} a_\rho d\rho a_\rho$$

$$\Rightarrow V = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{1}{\rho}\right)$$

If $\rho = \text{constant} \Rightarrow V = \text{constant}$

Equipotential surfaces are concentric cylinders

$\vec{E}$ - Intensity - $\frac{1}{r}$ decrease

q_r directed vector

$V \rightarrow$ Potential - logarithmic decrease
- Scalar

Potential function of a sheet charge (Uniform):-

$$V = - \int \vec{E} \cdot d\vec{u}$$

$$\Rightarrow V = -Ed$$

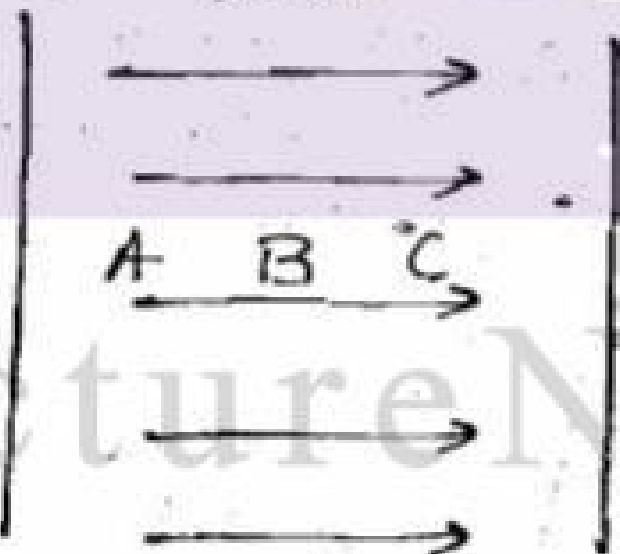
$$\Rightarrow E = -\frac{V}{d}$$

$\vec{E} \rightarrow$ Intensity - Uniform

V - Potential - linear decrease

eg:- Capacitor Plates

$\leftarrow 100V \rightarrow$

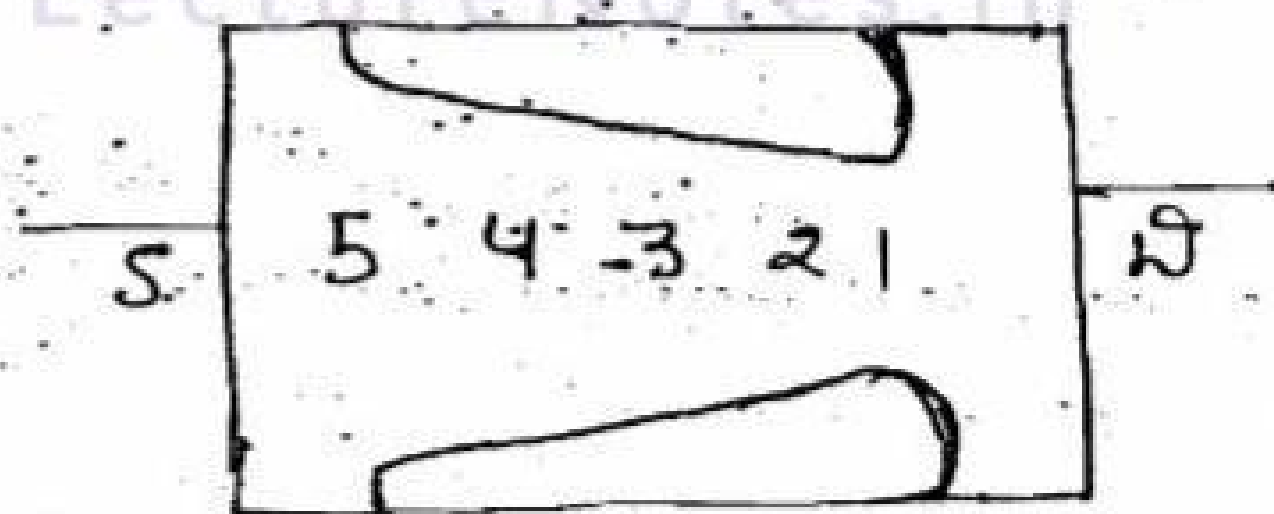


in E_A or E_B or E_C

$$= \frac{100}{10} = 10 \text{ kV/m}$$

$$V_A > V_B > V_C$$

FET channel



$$V = - \int \vec{E} \cdot d\vec{l} = - \int (y a_x + x a_y) \cdot (dx a_x + dy a_y)$$

$$V = - \int_{x=0}^4 y dx - \int_{y=0}^1 x dy$$

$$= - \int_0^4 \frac{x}{4} dx - \int_0^1 4y dy$$

$$= -\frac{1}{8} (16) - \frac{4}{2} \times 1$$

$$= -4V, \text{ Ans.}$$

Using the straight line path or equation

$$\frac{y-1}{1-0} = \frac{x-4}{4-0}$$

$$x = 4y$$

Note:-

In line integral,

$$\int f(x, y, z) dx$$

We have to transform y & z in terms of x using the known relationship

i.e. path of integration

$$\int f(x, y, z) dx = \int g(x) dx$$

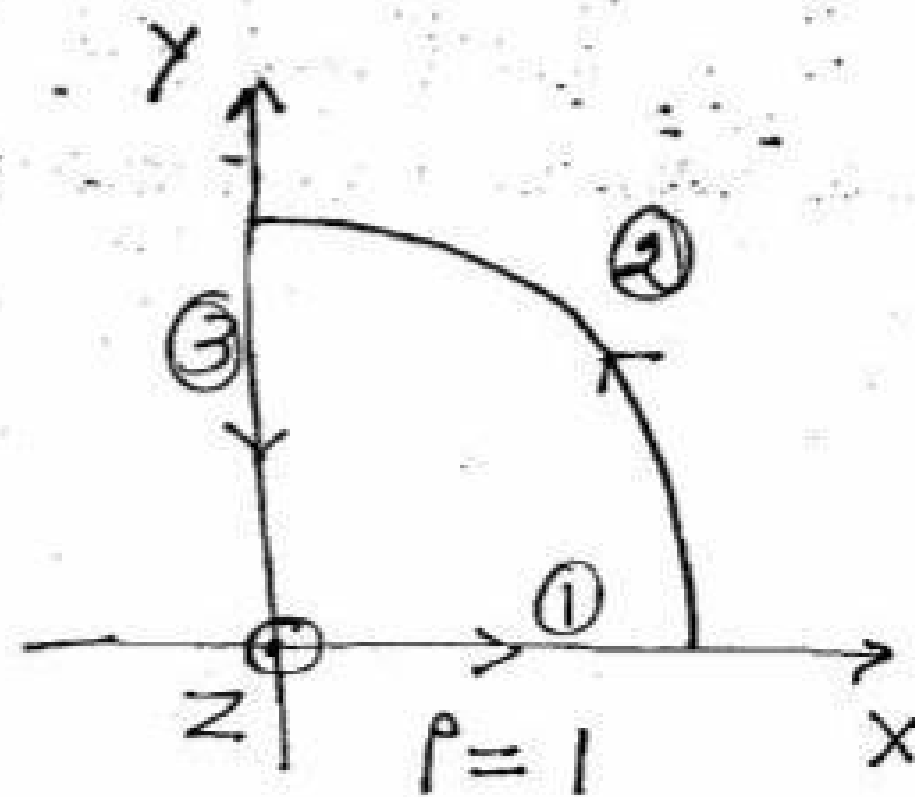
$$\vec{A} = 2\rho \cos\phi a_\rho$$

$$\oint \vec{A} \cdot d\vec{l} = 3 \rightarrow \int \vec{A} \cdot d\vec{l}$$

$$(1) \quad d\vec{l} = d\rho a_\rho$$

$$\phi = 0$$

$$A = 2\rho a_\rho$$



$$\int A \cdot d\mathbf{l} = \int_{\rho=0}^1 2\rho a_\rho d\rho \cdot a_\rho$$

$$= \rho^2 \Big|_0^1 = 1$$

2 → $d\mathbf{l} = \rho d\phi \underline{a}_\phi$ ($\therefore a_\phi \cdot a_\rho = 0$)

$$\int A \cdot d\mathbf{l} = 0$$

3 → $d\mathbf{l} = d\rho \underline{a}_\rho$
 ρ from 1 to 0

$$\phi = 90 \Rightarrow A = 0$$

So → $\vec{E} = 4a_x - 3a_y + 5a_z \rightarrow$ Uniform

$$W = q \int \vec{E} \cdot d\vec{l} = q E \cdot \mathbf{l}$$

Unit length
vector

$$= \frac{3a_x + 2a_y + 4a_z}{\sqrt{3^2 + 2^2 + 4^2}}$$

$$(6, 2, -3)$$

($\therefore$ 3m length vector)

$$W_q = 5 (4a_x - 3a_y + 5a_z) \cdot \frac{(3a_x + 2a_y + 4a_z) \times 2}{\sqrt{29}}$$

$$= \frac{260}{\sqrt{29}} \text{ J}$$

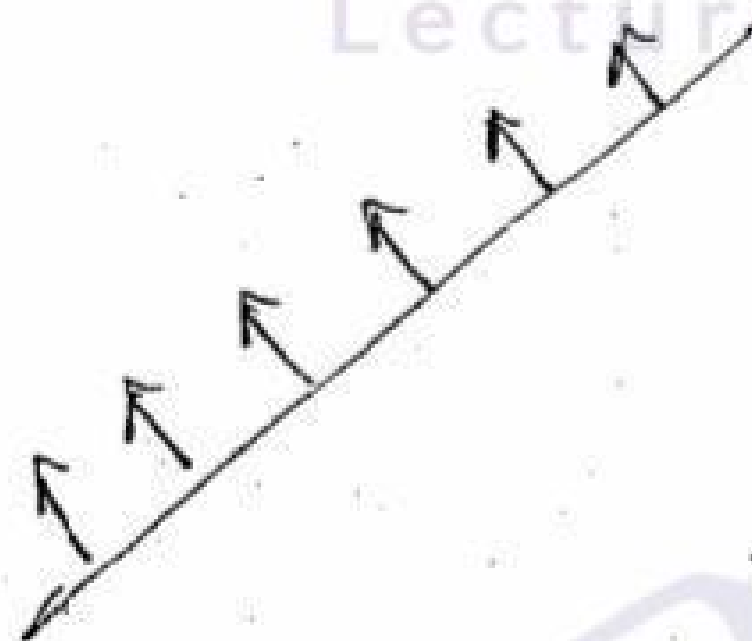
Potential Gradient :-

Scalar Surface equation (f) $\nabla f \rightarrow$ Vector (Normal) to the surface (direction)

eg:- Linear surface

$$f = 4x + 7y - 15z = 18$$

$$\nabla f = 4a_x + 7a_y - 15a_z$$



eg:- Non-linear surface

$$f = 4xy - 15x^2yz$$

$$\nabla f = (4y - 30xyz)a_x + (4x - 15xz^2)a_y + 15x^2y a_z$$



$$V = - \int \vec{E} \cdot d\vec{l}$$

$$dV = - \vec{E} \cdot d\vec{l} \cos \theta$$

$$\Rightarrow \boxed{\frac{dV}{dl} = -E \cos \theta}$$

Case - (1) :-

$$\text{If } \theta = 90^\circ \Rightarrow V = \text{constant}$$

$\rightarrow$ The direction of E field is always normal to equi-potential surfaces.

Case - (ii) :-

$$\text{If } \theta = 0^\circ/180^\circ \Rightarrow \left. \frac{dV}{dl} \right|_{\max} = |E|$$

→ The magnitude of E intensity is always the maximum rate of change of potential per unit length

If

Case - (iii) :-

$$\text{If } \theta = 0^\circ \Rightarrow |E| = - \left. \frac{dV}{dl} \right|_{\max}$$

→ The direction of E intensity is always the direction in which potential decreases by maximum

Note!:-

The operation is physically called as gradient

Hence $E = -\nabla V$ = potential gradient is field intensity.

→ Gradient signifies the maximum rate of change of scalar along with the direction of change.

eg:- Given $V = 4(x^2 - y^2)$ for all z

Find the equation of equipotential surface passing through $(3, 1, 4)$

soln:- V at $(3, 1, 4) = 4(9 - 1) = 32V$

All x & y when $V = 32V$ is the equipotential surface

$$\Rightarrow x^2 - y^2 = 8$$

Summary!:-

Equation of equipotential surface

OR

voltage function (V)

→ ∇V →

vector Intensity function (E)

which is

normal to

equi-potential surfaces

Mathematically, ∇V .

$$\nabla V = \frac{1}{h_1} \frac{\partial V}{\partial u} \underline{a}_u + \frac{1}{h_2} \frac{\partial V}{\partial v} \underline{a}_v + \frac{1}{h_3} \frac{\partial V}{\partial w} \underline{a}_w$$

eg:- If $V = \frac{4 \cos \theta}{r^2}$

Find $\vec{E}$ at $(2, \pi/4, \pi)$

Soln:- $\vec{E} = -\nabla V$

$$\nabla V = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{4 \cos \theta}{r^2} \right) \underline{a}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{4 \cos \theta}{r^2} \right) \underline{a}_\theta$$

$$= 4 \cos \theta \left(-\frac{2}{r^3} \right) \underline{a}_r + \frac{4}{r^3} (-\sin \theta) \underline{a}_\theta$$

$$\vec{E} = \frac{4}{r^3} (2 \cos \theta \underline{a}_r + \sin \theta \underline{a}_\theta) \Big|_{(2, \pi/4, \pi)}$$

$$= \frac{1}{2} (\sqrt{2} \underline{a}_r + \frac{1}{\sqrt{2}} \underline{a}_\theta)$$

(a) $V = 0$ $\nabla V = 6xy \underline{a}_x + (3x^2 - z) \underline{a}_y - 4 \underline{a}_z$
 $\neq 0$ at $(1, 0, -1)$

(b) $x^2y = 1$ in xy plane

$V = 3(1) - y(0)$

$V = 3 \rightarrow$ equipotential surface

(c) At $(2, -1, 4) \Rightarrow V = -8$

$V = 3(4)(-1) - (-1)4 = -8$

(d) Normal vector at $P = \frac{\nabla V}{|\nabla V|} \Big|_{(2, -1, 4)} = \frac{-12 \underline{a}_x + 8 \underline{a}_y - \underline{a}_z}{\sqrt{12^2 + 8^2 + 1}}$

$= (-0.83 \hat{x} + 0.55 \hat{y} + 0.07 \hat{z})$

Lecture - 6

Closed line integral of electric field = Maxwell's II Eqn:-

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

- Potential difference exists b/w 2 different points but b/w the same point potential difference is zero
- Potential is unique at a point at a time
- Work done in moving a charge in any closed path is always zero
- In any closed path energy acquired = energy lost
i.e. Energy acquired in field direction = energy lost opposite to field direction.
- Electric field is a conservative field & never forms closed loops.
i.e. Irrotational vector ($\nabla \times \mathbf{E} = 0$)
- Work done in moving a charge b/w two points is independent of path of consideration.
- This is KVL in electric fields.

Potential, Vector Potential and Ampere's Law in H field:-

- Potential is called as MMF or Magneto Motive force and is a scalar measure of magnetic field strength at any point in the field.

$$V_m = \text{MMF} = \int \vec{H} \cdot d\vec{l} = \text{Amp}$$

$$\mathbf{H} = \nabla V_m$$

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = I}$$

This is Ampere's law in Integral form

$$\nabla \times H = J$$

This is Ampere's law in point form

Statement:- effects

The net circulation of magnetic field in any closed line is always equal to the current crossing the surface enclosed.

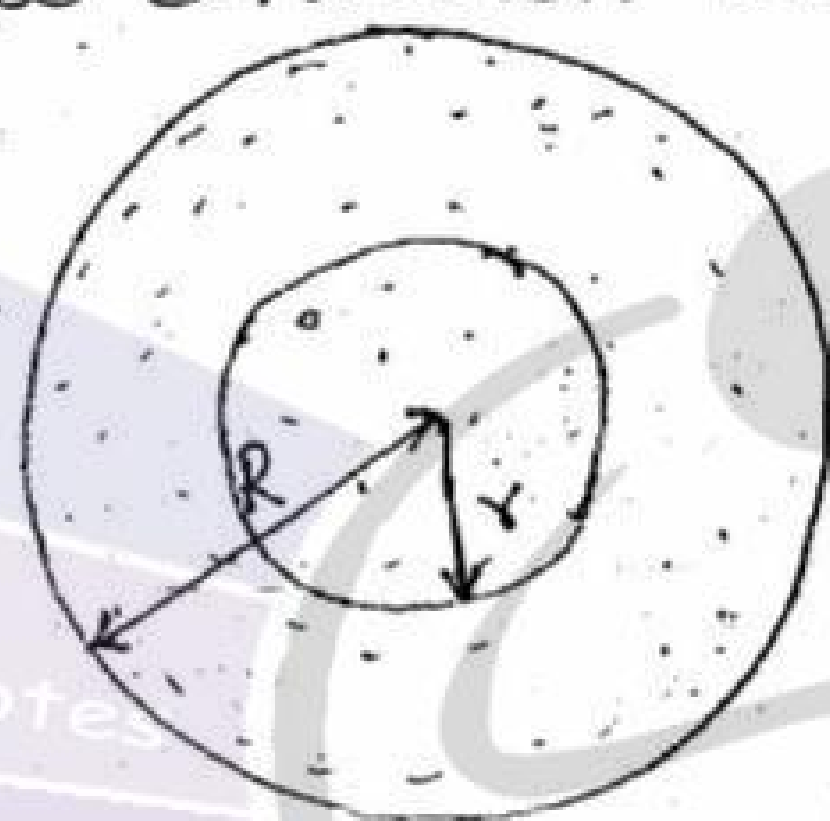
$$\begin{aligned} \text{Circulation} &= \text{Strength} \times \text{length} \\ &= \text{current} \end{aligned}$$

consider a closed ~~ring~~ line symmetrical & concentric with the I flow direction such that $H = \text{constant}$ everywhere

$$\oint H \cdot dl = I$$

↓
constant

$$H \cdot \text{length} = \text{current crossing } r < R$$



$$H = \frac{I \cdot \pi r^2}{\pi R^2} = \frac{I r}{2\pi R^2}$$

$$r > R$$

$$H = \frac{I}{2\pi r}$$

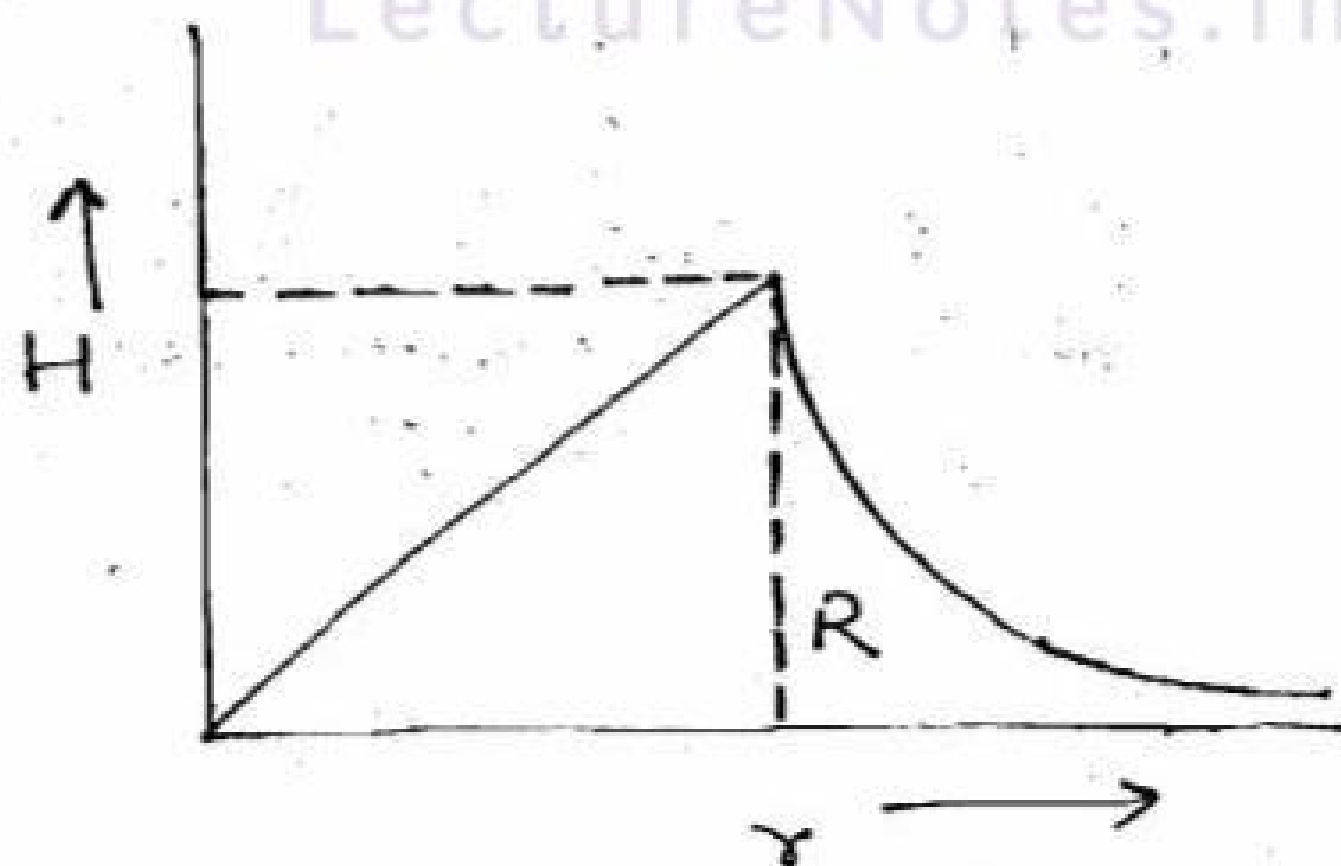
Note:-

Point $\rightarrow 1/r^2$

Line $\rightarrow 1/r$

Sheet $\rightarrow \text{Uniform}$

Solid $\rightarrow r$



Vector Potential :-

Limitation of Scalar Potential (MMF) :-

→ The definition of potential in electric field is consistent with its nature i.e. irrotational nature

$$E = -\nabla V$$

$$\nabla \times E = 0$$

$$\nabla \times (-\nabla V) = 0$$

$$\nabla \times (\text{Grad of Scalar}) = 0$$

→ A similar definition in magnetic field for MMF is not consistent with Ampere's law

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

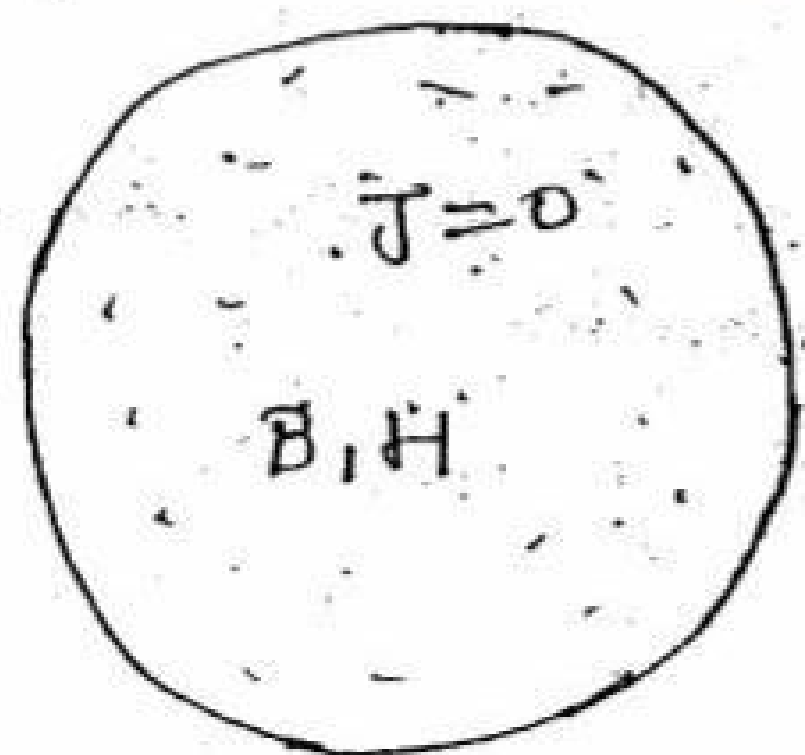
Hence MMF is defined and exists in only those regions where $J=0$

i.e. outside current flowing regions

i.e. free space conditions and outside conductor

eg:- (i) MMF b/w windings of solenoids

(ii) MMF in air gaps in machines



B, H, J

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Vector Potential $\vec{A}$:-

→ The basic definition of vector potential is in accordance to nature of magnetic field i.e. solenoidal nature. Hence if $\text{curl of } \vec{A} = \vec{B}$ then $\vec{A}$ is called as vector potential

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \cdot \vec{B} = 0$$

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$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\text{Div. (curl of vector)} = 0$$

→ The term vector potential has a units weber/m which physically signifies the work for current element

$$\vec{A} = \text{Weber/m} = \frac{\text{Joules}}{\text{Amp-m}}$$

Notes

$$= \frac{W}{I d\vec{l}}$$

and as $I d\vec{l}$ is a vector, $\vec{A}$ is a vector quantity.

Note:-

$$\frac{\text{Weber}}{\text{Second}} = \text{Volts}$$

$$\frac{\text{Weber}}{\text{second}} = \frac{\text{Joules}}{\text{Coulomb}}$$

$$\frac{\text{Weber}}{\text{m}} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$V = \frac{W}{Q} = \frac{Q}{4\pi\epsilon_0 r}$$

$$\vec{A} = \frac{W}{I d\vec{l}} = \frac{\mu I d\vec{l}}{4\pi r}$$

A's direction is along I flow direction & is a solenoidal vector.

Summary 1:-

→ Q - Coulomb - point charge - Basic cause
(Scalar) - E field

→ $I d\vec{l}$ - Amp-m - current element - H field
(Vector)

→ Q - $\rho_L dl$ - $\rho_S ds$ - $\rho_V dv$

→ $I d\vec{l}$ - $\vec{K} ds$ - $\vec{J} dv$

→ $\vec{E}$ - Intensity - Strength - Force - $\frac{F}{Q}$ - $\frac{1}{\epsilon}$

→ $\vec{B}$ - Density - Strength - Force - $\frac{\vec{F}}{I d\vec{l}}$
dependent

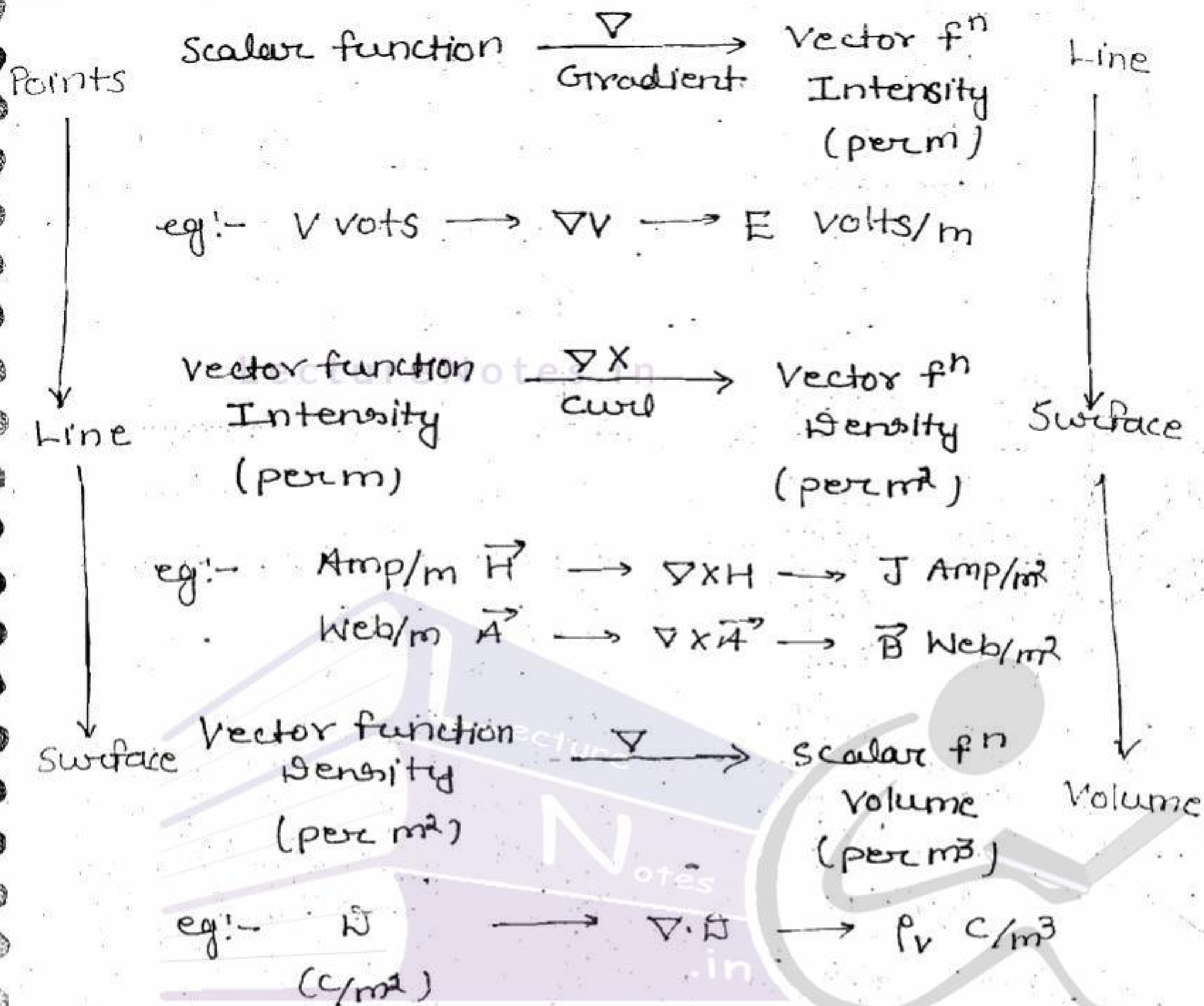
→ $\vec{D}$ - Density - strength - $\frac{dQ}{ds} = C/m^2$ - μ dependent

→ $\vec{H}$ - Intensity - Strength - $\frac{dI}{dl} = \text{Amp/m}$

→ V - Potential - strength - Work = W/Q → Scalar
(Volts)

→ $\vec{A}$ - Potential - strength - Work = $\frac{W}{I d\vec{l}}$
(Weber/m) → Vector.

Summary :-



OR

$$\oint \vec{A} \cdot d\vec{l} = \frac{\text{Weber}}{\text{m}} \times \text{m} = \text{Webers}$$

↓ flux

$$\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{s} = \int \vec{B} \cdot d\vec{s} = \psi_m$$

Stoke's theorem

$$\vec{V} = \nabla \times \vec{A}$$

C $\rightarrow$ closed line

S_c $\rightarrow$ Surface

(B)

$$\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{s} = \int \vec{V} \cdot d\vec{s}$$

Laplace / Poisson's Equations:-

→ They are second order differential equations relating volume charge and potential developed inside / outside the charge

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \cdot [\epsilon (-\nabla V)] = \rho_v$$

If $\epsilon = \text{constant}$ then

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \rightarrow \text{Poisson's Equation}$$

For $\rho_v = 0$, i.e. outside the charge, charge free region

$$\boxed{\nabla^2 V = 0} \rightarrow \text{Laplace Equation}$$

Note:-

In spite of being II order differential equation they always have a unique solution i.e. voltage is a single valued function of space

This is called as UNIQUE Uniqueness theorem

Mathematically

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

$$\nabla \cdot (\nabla V) = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial V}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_3 h_1}{h_2} \frac{\partial V}{\partial v} \right) + \frac{\partial}{\partial w} \left(\frac{h_1 h_2}{h_3} \frac{\partial V}{\partial w} \right) \right]$$

Extension:-

$\vec{A}$ vs $\vec{J}$ in Magnetic Fields

$$\nabla \times \vec{H} = \vec{J}$$

$$\Rightarrow \nabla \times \left(\frac{\vec{B}}{\mu} \right) = \vec{J}$$

$$\Rightarrow \nabla \times (\nabla \times \vec{A}) = \mu \vec{J}$$

$$\Rightarrow \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}$$

$$\Rightarrow \boxed{\nabla^2 \vec{A} = -\mu \vec{J}}$$

For current free region

$$\boxed{\nabla^2 \vec{A} = 0}$$

$$V = -\frac{6r^5}{\epsilon_0}$$

$$1. \nabla^2 V = -\rho_v / \epsilon$$

$$2. Q = \int \rho_v dv$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \left(-\frac{6r^5}{\epsilon} \right) \right) = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot 6 \cdot 5 \cdot r^4) = \rho_v$$

$$\frac{30}{r^2} 6 \cdot r^5 = \rho_v$$

$$\Rightarrow \rho_v = 180 r^3$$

$$Q = \int_{r=0}^1 \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} 180 r^3 \cdot r^2 \sin \theta \, dr \, d\theta \, d\phi$$

$$= 180 \frac{\pi^6}{6} \Big|_0^1 (-\cos \theta)_0^{\pi} (\phi)_0^{2\pi} = 120\pi C, \text{ Ans.}$$

36. $\rho_v = -10^{-8} (1 + 10\rho)$

V on $\varphi = 5\text{cm}$

Given $\rho = 2\text{cm}$ $V \& E = 0$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\begin{aligned} \nabla^2 V &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \cdot \frac{\partial V}{\partial \rho} \right) = \frac{10^{-8} (1 + 10\rho)}{\frac{1}{36\pi \times 10^9}} \\ &= 360\pi (1 + 10\rho) \end{aligned}$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \int (\rho + 10\rho^2) d\rho$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \left(\frac{\rho^2}{2} + \frac{10\rho^3}{3} \right)$$

$$\begin{aligned} \Rightarrow V &= 360\pi \int \left(\frac{\rho}{2} + \frac{10\rho^2}{3} \right) d\rho \\ &= 360\pi \left(\frac{\rho^2}{4} + \frac{10\rho^3}{9} \right) \Big|_{2 \times 10^{-3}}^{5 \times 10^{-2}} \end{aligned}$$

$$\Rightarrow \boxed{V = 0.5V}$$

37. $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho_v}{\epsilon}$

$$= 20 \cdot 3 \cdot 2 \cdot x + 10 \cdot 4 \cdot 3 \cdot y^2 \Big|_{(2,0)} = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \boxed{\rho_v = -240\epsilon_0}$$

38 ϕ — 1 dimension function — satisfying — Laplace

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\frac{\partial \phi}{\partial x} = C_1 = \text{Rate of change is constant}$$

$$\frac{\phi_2 - \phi_1}{2d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\phi = C_1 x + C_2$$

→ Linear equation

Ans — B

39.

$$V = \sinh x \cdot \cos ky \cdot e^{pz}$$

$$\nabla^2 V = 0$$

$$\frac{e^x + e^{-x}}{2} = \sinh x$$

$$\nabla^2 V = 1V - k^2 V + p^2 V = 0$$

$$\Rightarrow \boxed{k = \sqrt{1+p^2}}$$

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Boundary Conditions for Electric fields

→ If a field is spread out into two different medium and known the field in one region. The field in the adjacent region can be calculated under two conditions

Case - (I) :-

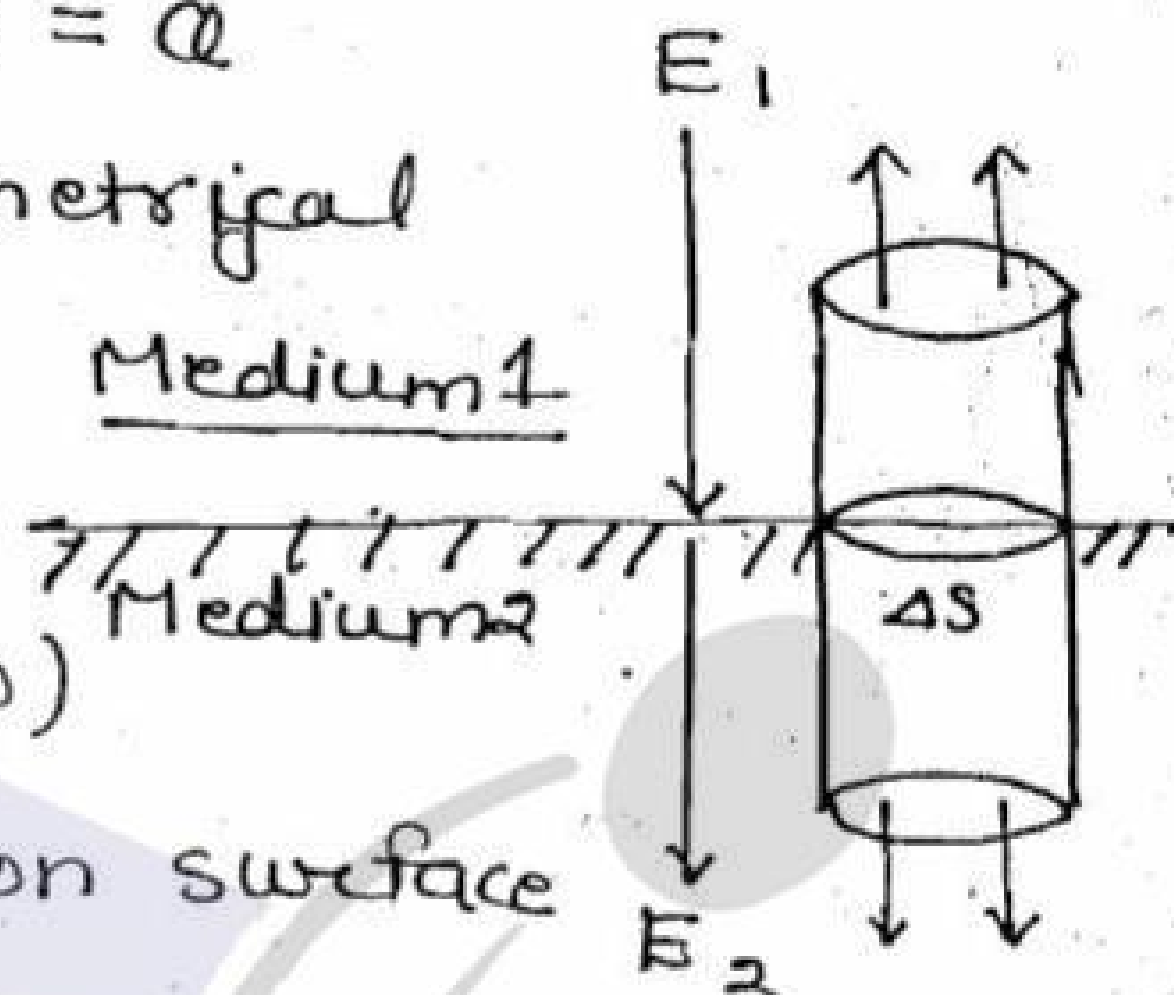
Electric field is normal to the boundary.

Using Gauss law $\oint \mathbf{B} \cdot d\mathbf{s} = Q$

consider a surface symmetrical in both the medium

$$\boxed{B_2 \Delta S - B_1 \Delta S = 0}$$

$\therefore Q = 0$



If charge is present on surface then

$$\boxed{B_2 \Delta S - B_1 \Delta S = \rho_s \Delta S}$$

$$B_{n1} = B_{n2}$$

$$B_{n2} - B_{n1} = \rho_s$$

Statement :-

The normal components of electric flux density is same on either side but otherwise discontinuous when a surface charge density exists on the boundary

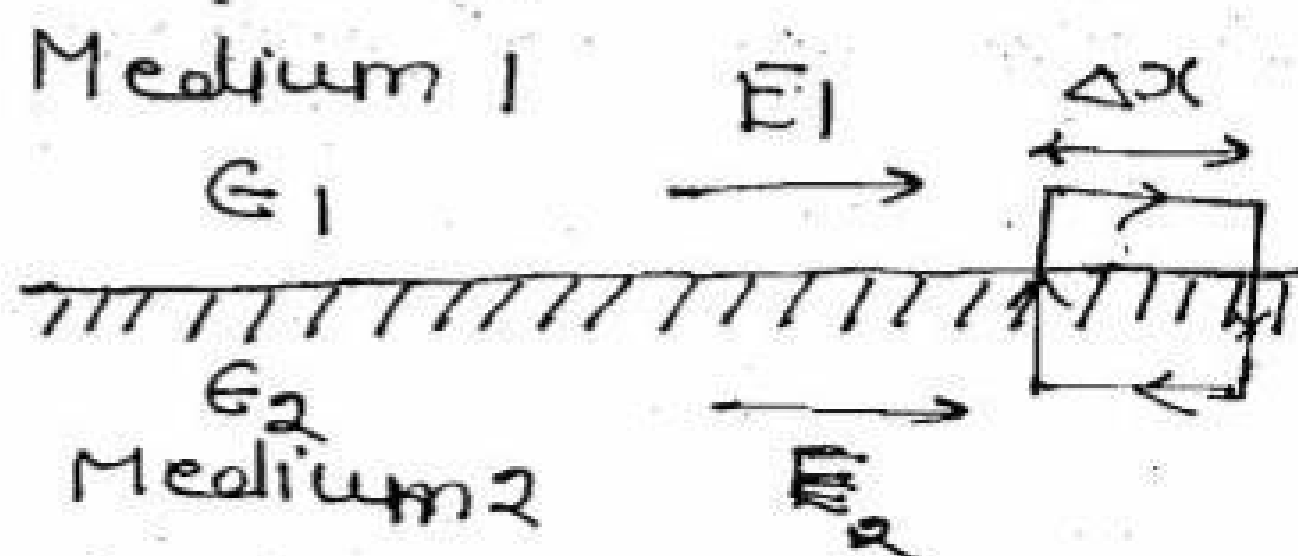
Case - (II) :-

Electric field is ^{tangential} ~~parallel~~ to boundary :-

Using $\oint \mathbf{E} \cdot d\mathbf{l} = 0$

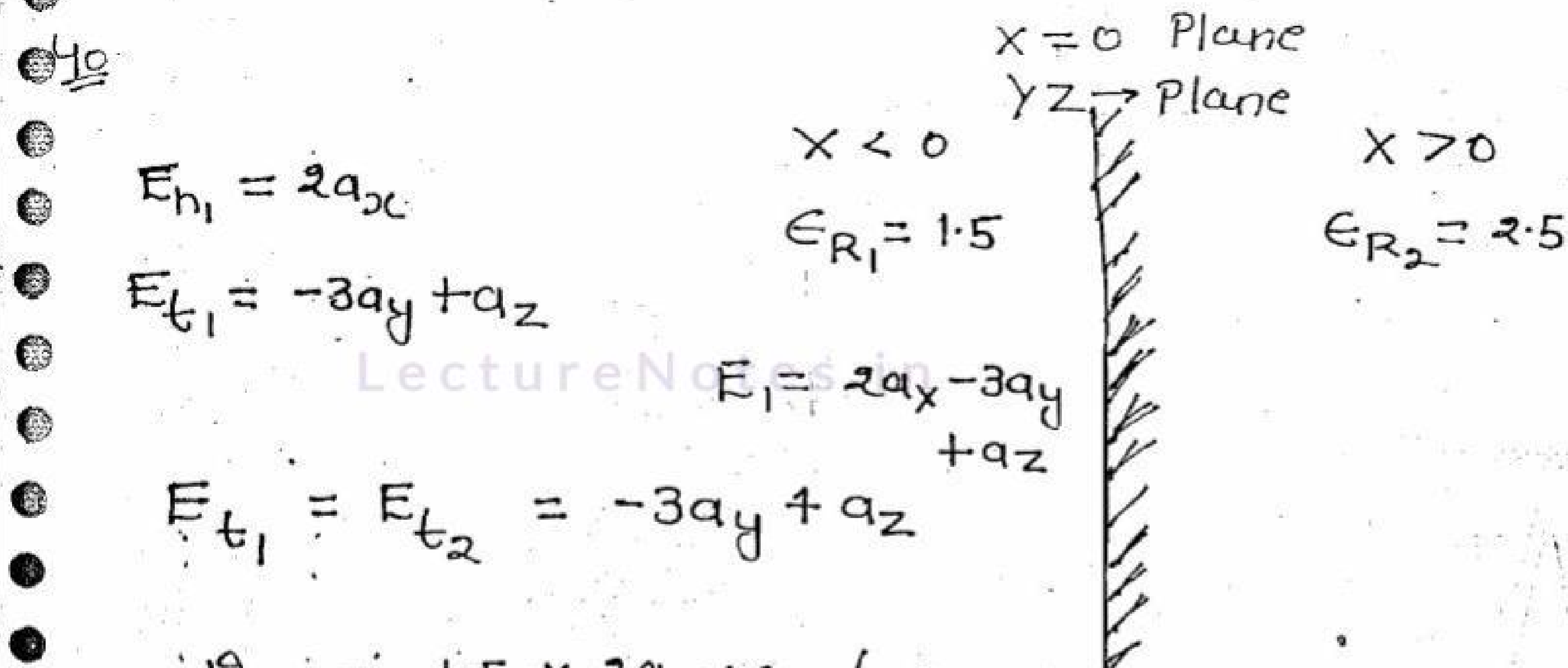
$$E_1 \Delta x - E_2 \Delta x = 0$$

$$\Rightarrow \boxed{E_1 = E_2}$$



$$E_{t1} = E_{t2}$$

The tangential components of electric field intensity is always continuous



$$\rho_{n1} = 1.5 \times 2a_x \times \epsilon_0 \quad (\rho_{n1} = \epsilon_R \times E_n)$$

$$= 3a_x \epsilon_0 = \rho_{n2}$$

$$\Rightarrow \rho_{n2} = 3\epsilon_0 a_x$$

$$E_{n2} = \frac{3\epsilon_0 a_x}{2.5 \epsilon_0} = 1.2 a_x$$

$$\rho_1 = \epsilon_0 (3a_x - 4.5a_y + 1.5a_z)$$

$$\rho_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$$

$$\boxed{E_2 = 1.2a_x - 3a_y + a_z} \quad \underline{\text{Ans}}$$

Note:-

→ The value of vector dec. then its projection dec. i.e. moves away from normal and moves towards the surface

→ The projection of normal component is dec. in E_2 when compare to E_1 . Hence the field is shifting away from the normal. It can be verified with inc. tangential component when Comparing ρ_1 & ρ_2

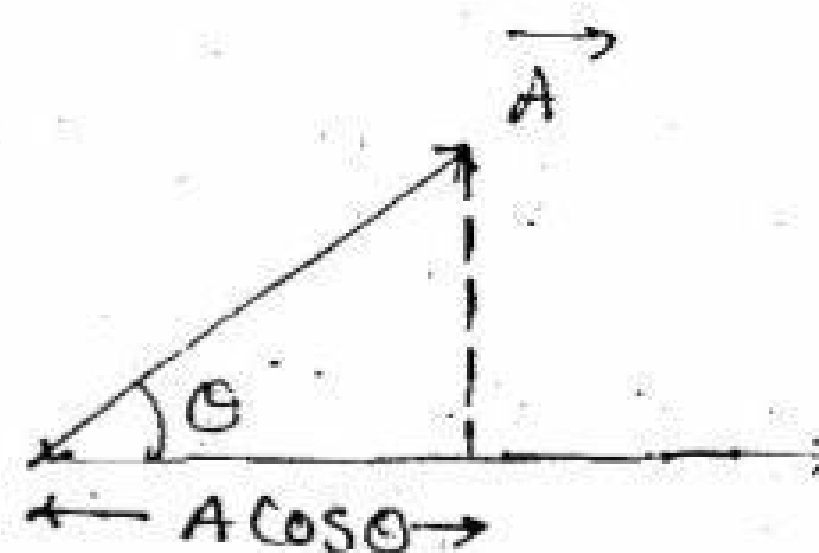
Sur

A's scalar projection on B

$$= A \cos \theta$$

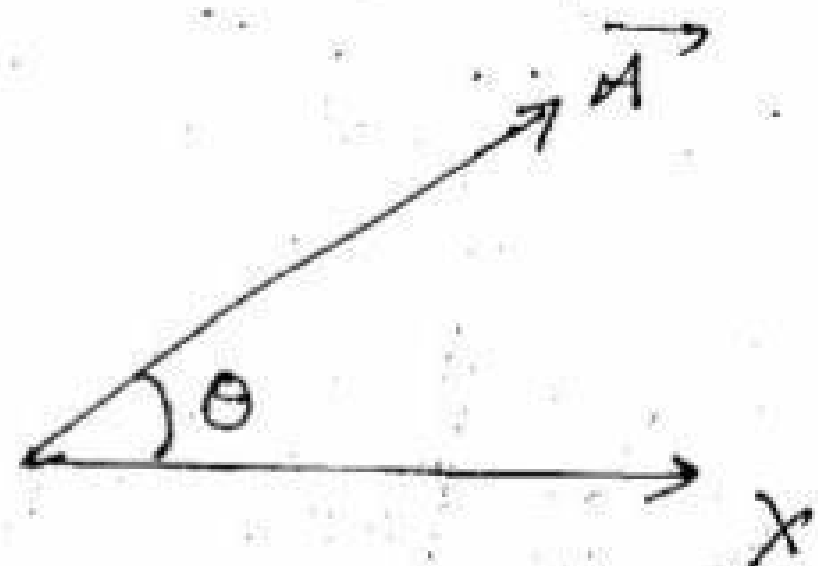
$$= \frac{A \cdot B}{|B|}$$

$$= \frac{A \cdot B}{|B|}$$



A's projection in X

$$= \frac{A \cdot a_x}{|a_x|}$$



A's vector projection on B

$$= (A \cos \theta) \cdot \hat{B}$$

$$= \frac{(A \cdot B)}{|B|} \hat{B}$$

$$= \frac{(A \cdot B)}{|B|^2} \vec{B}$$

41.

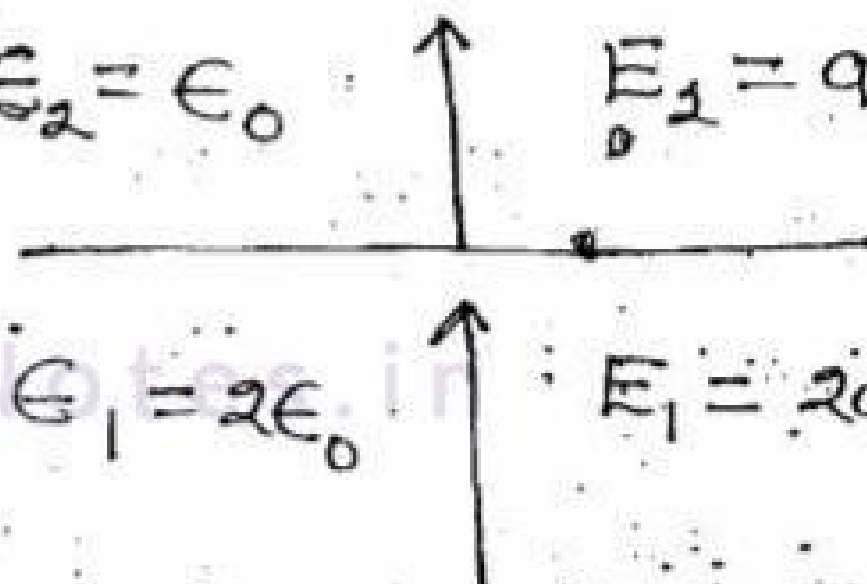
$$19_2 - 19_1 = p_s$$

$$1 \cdot \epsilon_0 - 2 \cdot 2 \epsilon_0 = p_s$$

$$p_s = -3\epsilon_0$$

$$E_2 = \epsilon_0$$

$$E_1 = q_x$$



42.

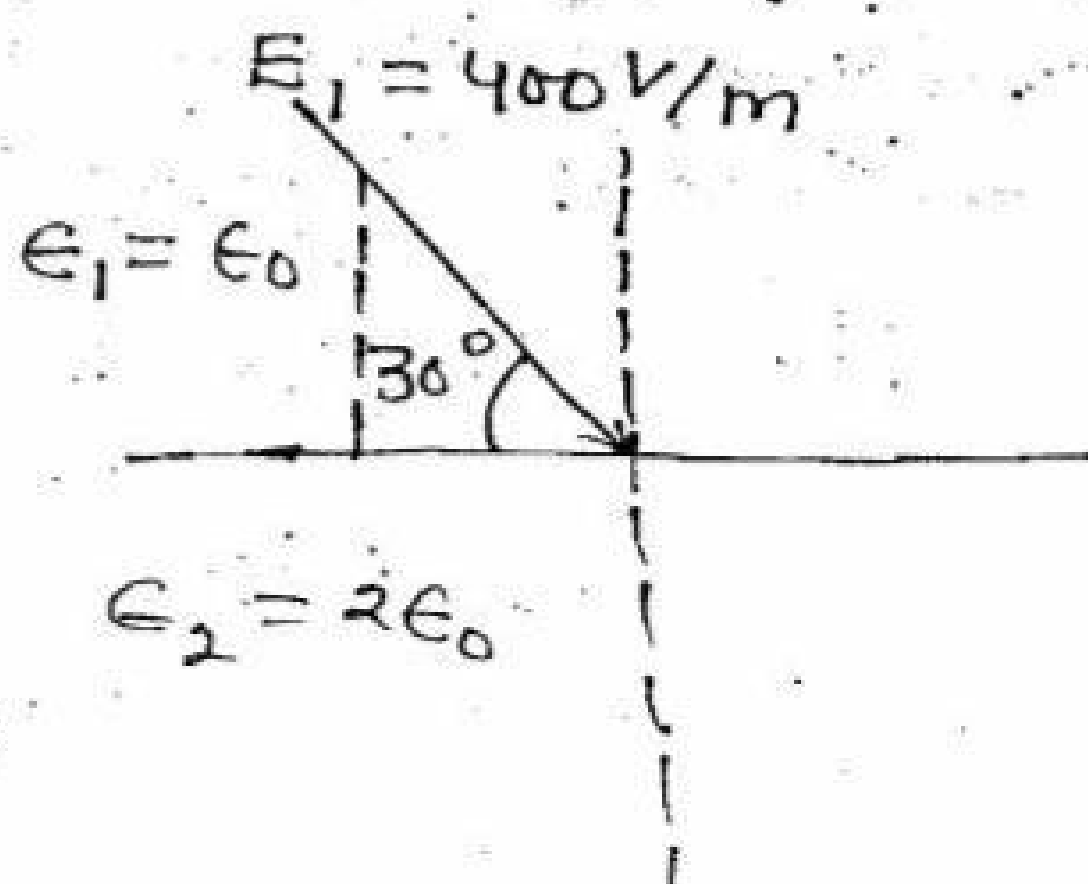
$$E_{t1} = E_{t2}$$

$$E_1 \cos 30^\circ = E_{t2}$$

$$\Rightarrow E_{t2} = 200\sqrt{3}$$

$$\epsilon_0 E_1 \sin 30^\circ = 20 \epsilon_0 E_{n2}$$

$$\Rightarrow E_{n2} = 10$$



Ohm's law and Continuity Equation:

$$\int (\nabla \cdot \mathbf{J}) dV = \oint \mathbf{J} \cdot d\mathbf{s} = I = \frac{dQ}{dt} = \frac{d}{dt} \int \rho_V dV = \int \frac{\partial \rho_V}{\partial t} dV$$

$$\text{i.e. } I = \frac{dQ}{dt} = \frac{d}{dt} \int \rho_V dV = \int \frac{\partial \rho_V}{\partial t} dV$$

$$\Rightarrow I = \oint \mathbf{J} \cdot d\mathbf{s} = \int (\nabla \cdot \mathbf{J}) dV$$

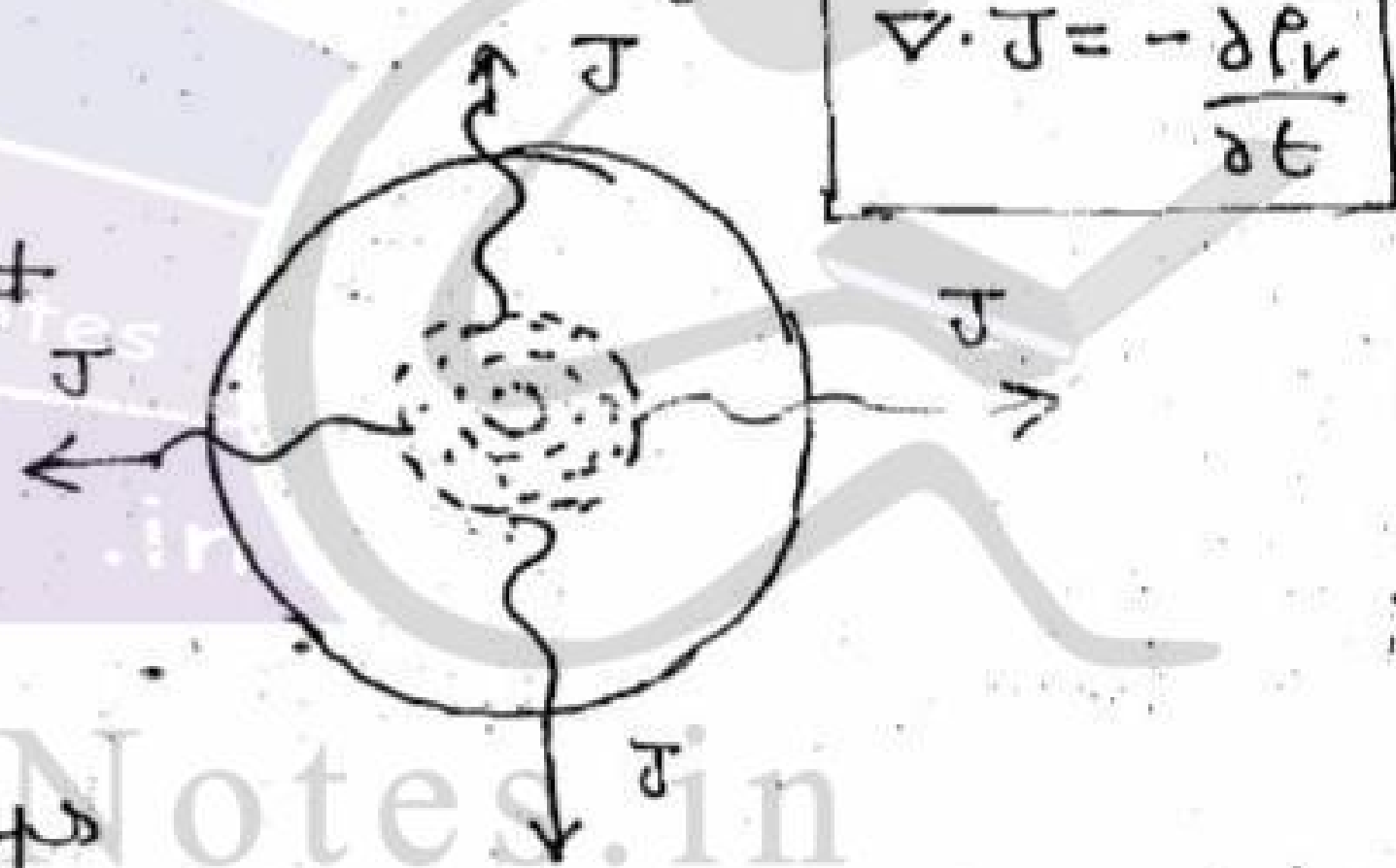
Divergence theorem

$$\boxed{\nabla \cdot \mathbf{J} = \frac{\partial \rho_V}{\partial t}}$$

→ Current has outflow and divergence at a rate depending on decrease of volume charges with time

→ Practically
Entering current = Leaving current

charge can neither be created nor be destroyed



$$\left. \begin{array}{l} \oint \mathbf{J} \cdot d\mathbf{s} = 0 \\ \nabla \cdot \mathbf{J} = 0 \end{array} \right\} \rightarrow \text{Always}$$

→ For a uni-directional current flow

$$\frac{\partial J}{\partial x} = \frac{\partial \rho_V}{\partial t}$$

$$\partial J = \partial \rho_V \frac{\partial x}{\partial t}$$

Integrating on both sides

$$J = \rho_V v_d$$

where $v_d = \frac{\partial x}{\partial t}$ = drift velocity

$$V_f = \text{free velocity} = \sqrt{\frac{2qV}{m}}$$

= vacuum tubes, CRO

= few $10^6 - 10^7$ m/s

$$V_f \propto \sqrt{E}$$

where $V_d = \frac{\Delta x}{\Delta t}$ = drift velocity

= Charge in a material (bulk)

= few cm/sec

$$V_d \propto E \Rightarrow V_d = \mu E$$

↓
mobility of the particle

$$J = \rho_v V_d$$

$$J = \rho_v \mu E$$

$$\Rightarrow J = \sigma E \rightarrow \text{Ohm's law in point form}$$

where σ = conductivity (mho/m) = $\rho_v \mu$

= ability to allow current

= available free carriers $\times$ mobility of carriers

Case-(1) :-

$\sigma = \infty \rightarrow$ very good conductor

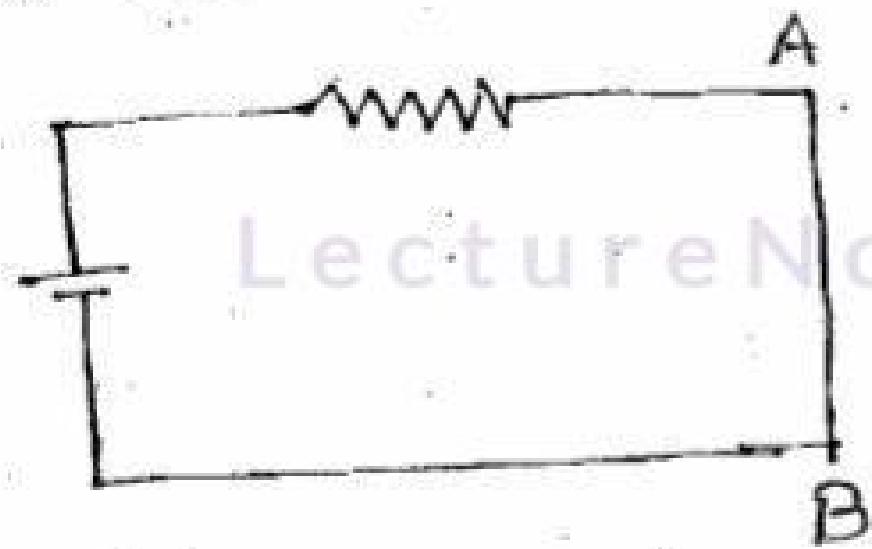
$$\frac{J}{\sigma} = E = 0$$

$\rightarrow$ Electric field cannot exist in a very good conductor

→ $V = \int \mathbf{E} \cdot d\mathbf{l} = \text{constant}$;

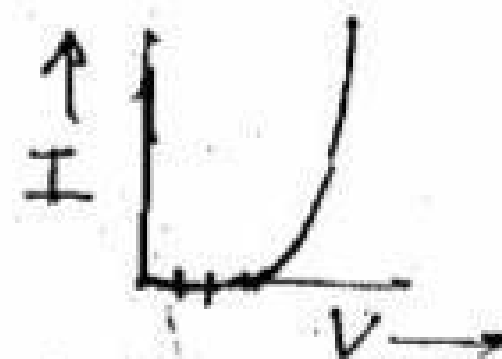
conductor is always an equi-potential region

→ Potential difference or voltage cannot exist in a very good conductor only current can flow but voltage difference good conductor cannot exist



$V_{AB} = 0$ but current flows

$V \uparrow$ $V_{AB} = 0$ $I_{AB} \uparrow$



eg: → (i) diode current after cutting voltage

(ii) Accumulation — does not exist — only flow exists

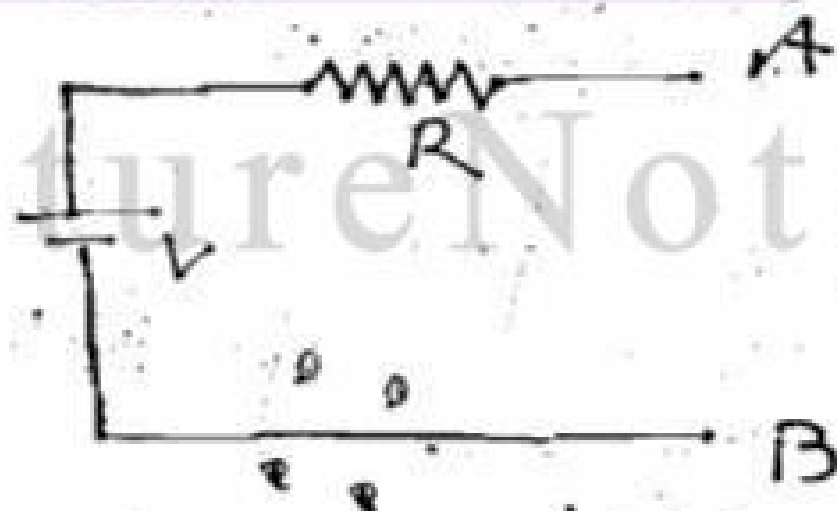
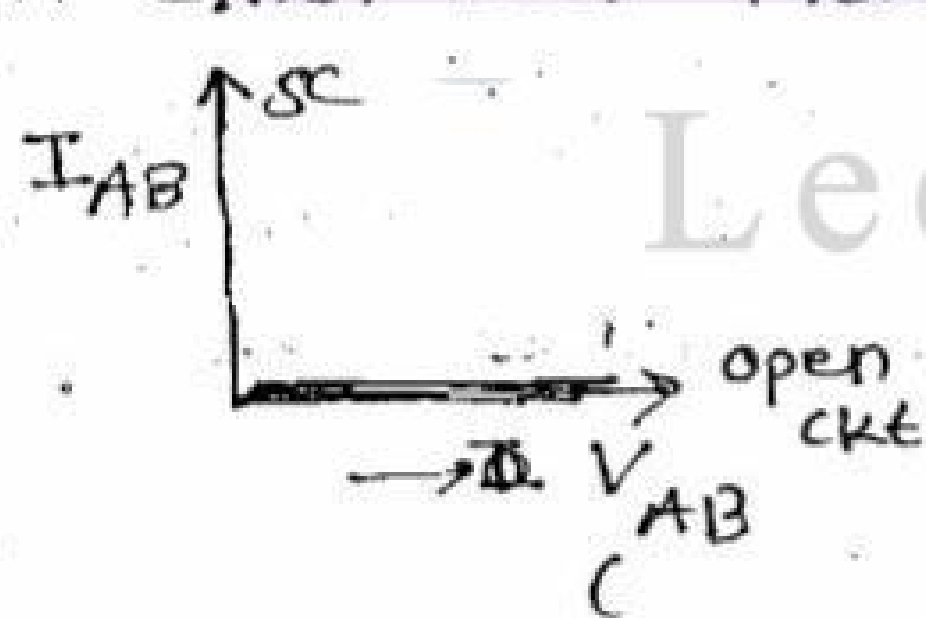
Case — (ii) :—

$\sigma = 0$ very good dielectric

$J = 0$

→ current cannot exist in a very good dielectric

→ Voltage or potential difference or Electric field can exist but flow current exist



→ Accumulation can exist but flow can't exist

$$\nabla \cdot \mathbf{J} = - \frac{\partial \rho_v}{\partial t}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\nabla \cdot (\sigma \mathbf{E}) = - \frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma (\nabla \cdot \mathbf{E}) = - \frac{\partial \rho_v}{\partial t}$$

The ρ_v is time solution is

$$\rho_v(t) = \rho_{v0} \cdot e^{-\frac{t}{\epsilon}}$$

Note:-

Every charge density exponentially spread on the medium at a rate depending on $\frac{\epsilon}{\sigma}$ that called the relaxation time

$$\frac{\epsilon}{\sigma} = \text{Relaxation time} = \frac{\text{Farad}}{m \times \frac{mho}{m}} = \text{ohms} \times \text{farad}$$

Boundary Conditions at conductor surfaces: — = Second

(1) $E_{t1} = E_{t2}$ (General)

As $\sigma = \infty$ along the conductor

$$\frac{J}{\sigma} = E = 0 \text{ along the surface}$$

$$\Rightarrow E_{\text{tang.}} = 0$$

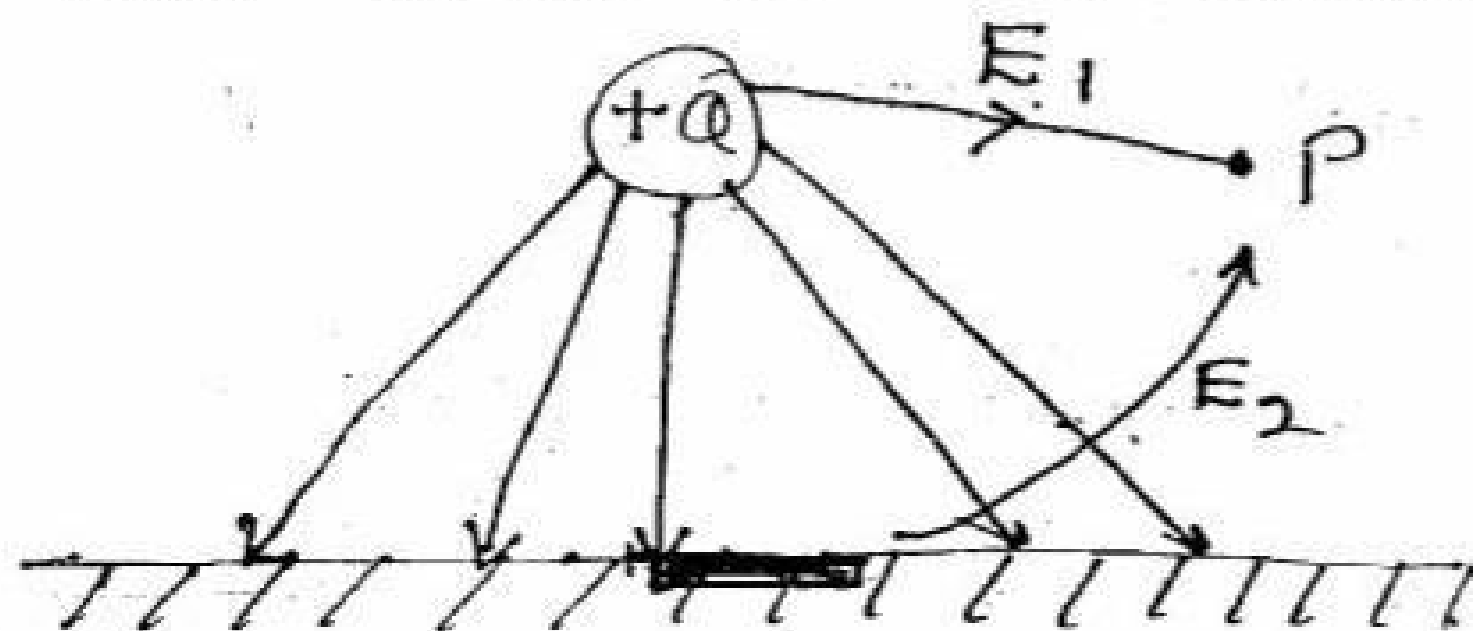
(11) $\rho_{n2} - \rho_{n1} = \rho_s$ (General)

Electric field can be normal to a conductor which depends on ρ_s on the surface

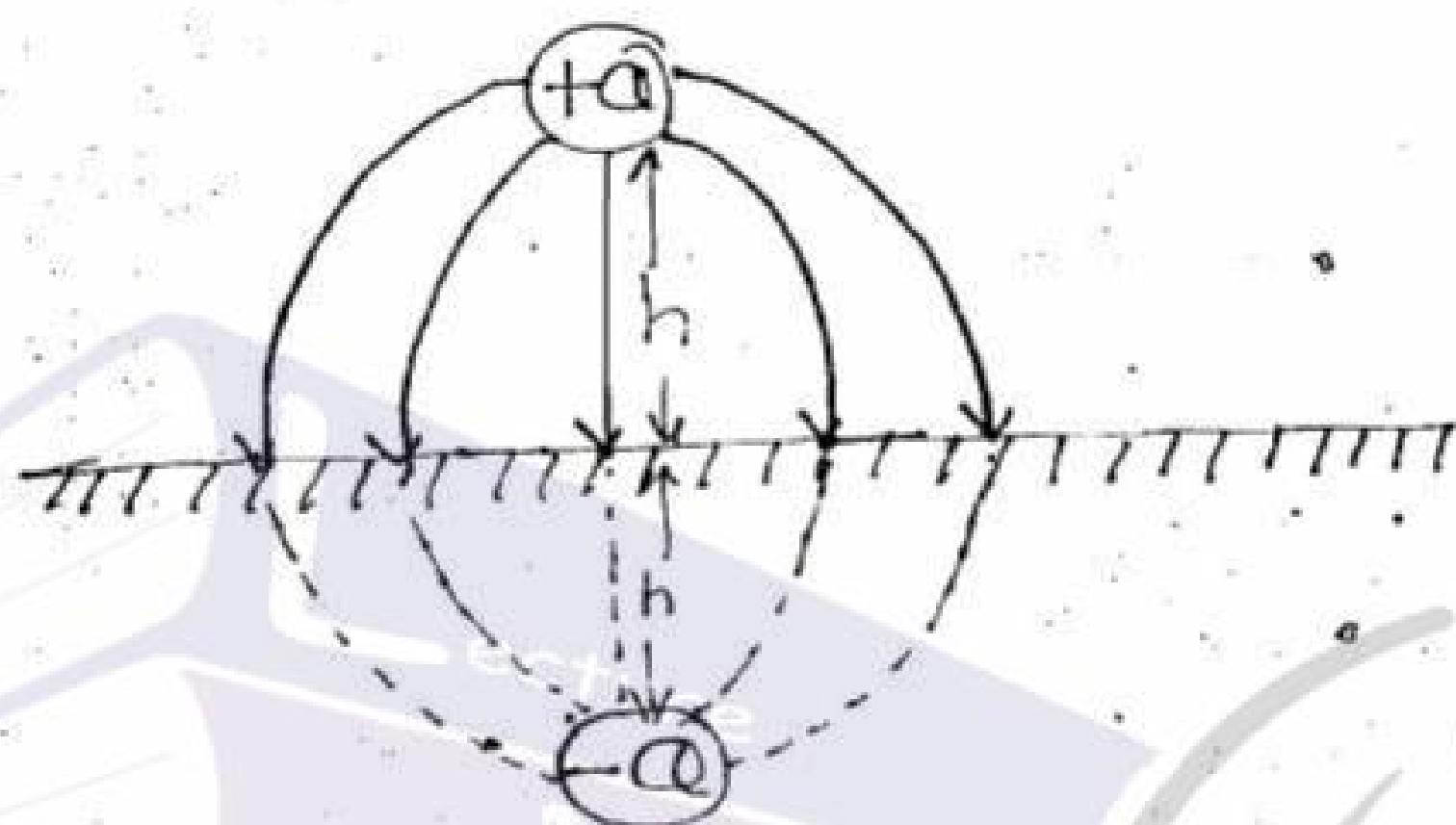
$$\rho_{\text{normal}} = \rho_s$$

Conductors, Induction, Method of Images:-

→ When a charge is placed near a conductor surface the field is as shown below



- The field has tangential component which displaces the free carriers. Hence they are periodically accumulated all the other along the conductor surface. This is called as induced charge.
- The resultant field at any point is the vector sum of field due to actual charge and induced charge.
- This field is such that the tangential components are removed and has only normal components as shown below.



- The field appears to be a dipole field with negative charge called as image charge below the conductor surface.

Summary:-

- Every charge and its induction effects are represented by a image charge below the conductor obeying all rules of light and optics.

44. $D_1 = 2(q_x - \sqrt{3}q_z) \text{ C/m}^2 = P_s$

$P_s = |D_n| = 2\sqrt{1+3} = 4$

45. $E_n = 2 \text{ V/m}$

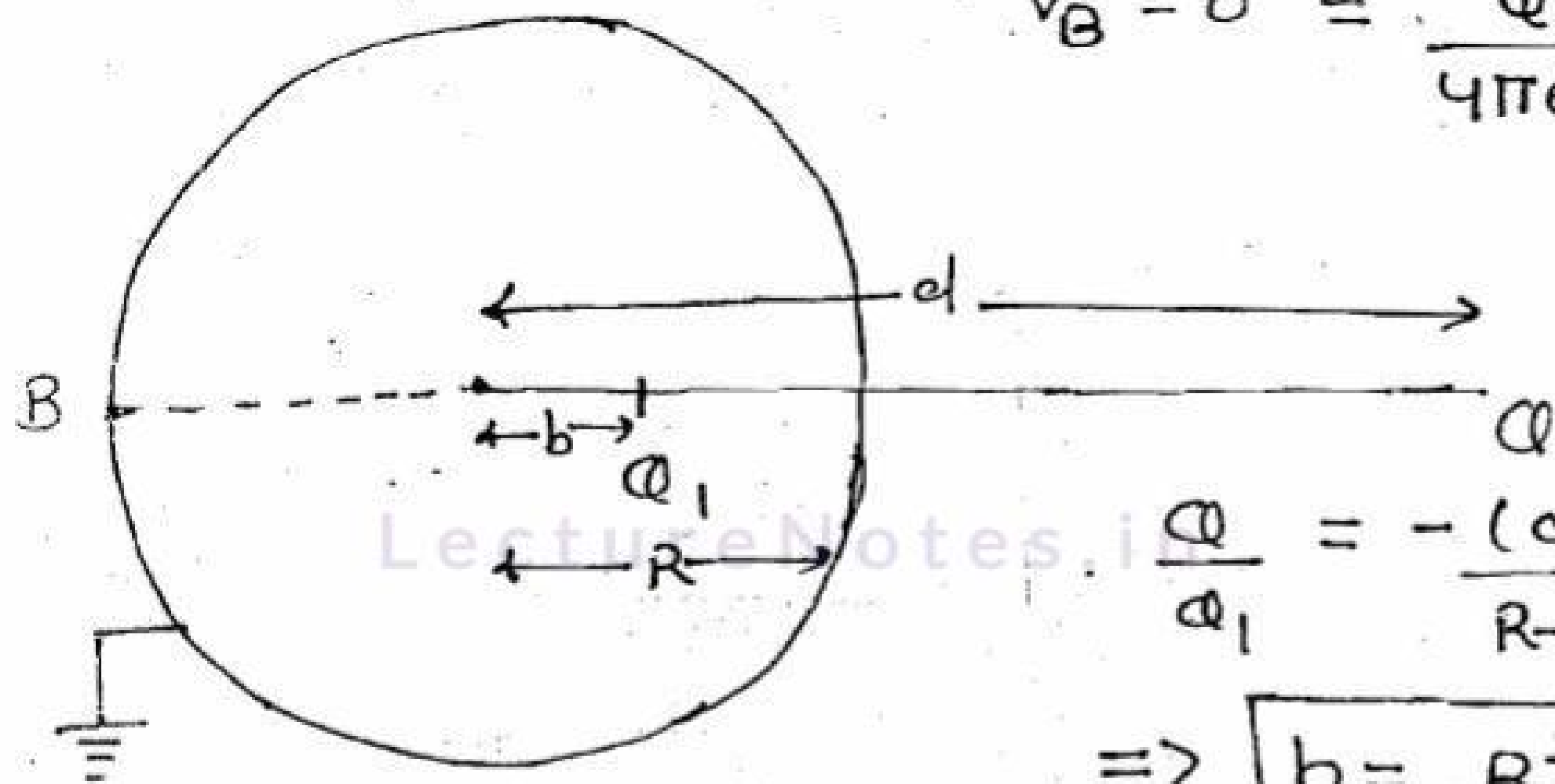
$D_n = 80\epsilon_0 E_n = P_s$

$\Rightarrow P_s = 80 \times 8.8 \times 10^{-12} \times 2$

$= 1.41 \times 10^{-9} \text{ C/m}^2$

$$V_A = 0 = \frac{Q}{4\pi\epsilon(d-R)} + \frac{Q_1}{4\pi\epsilon(R-b)}$$

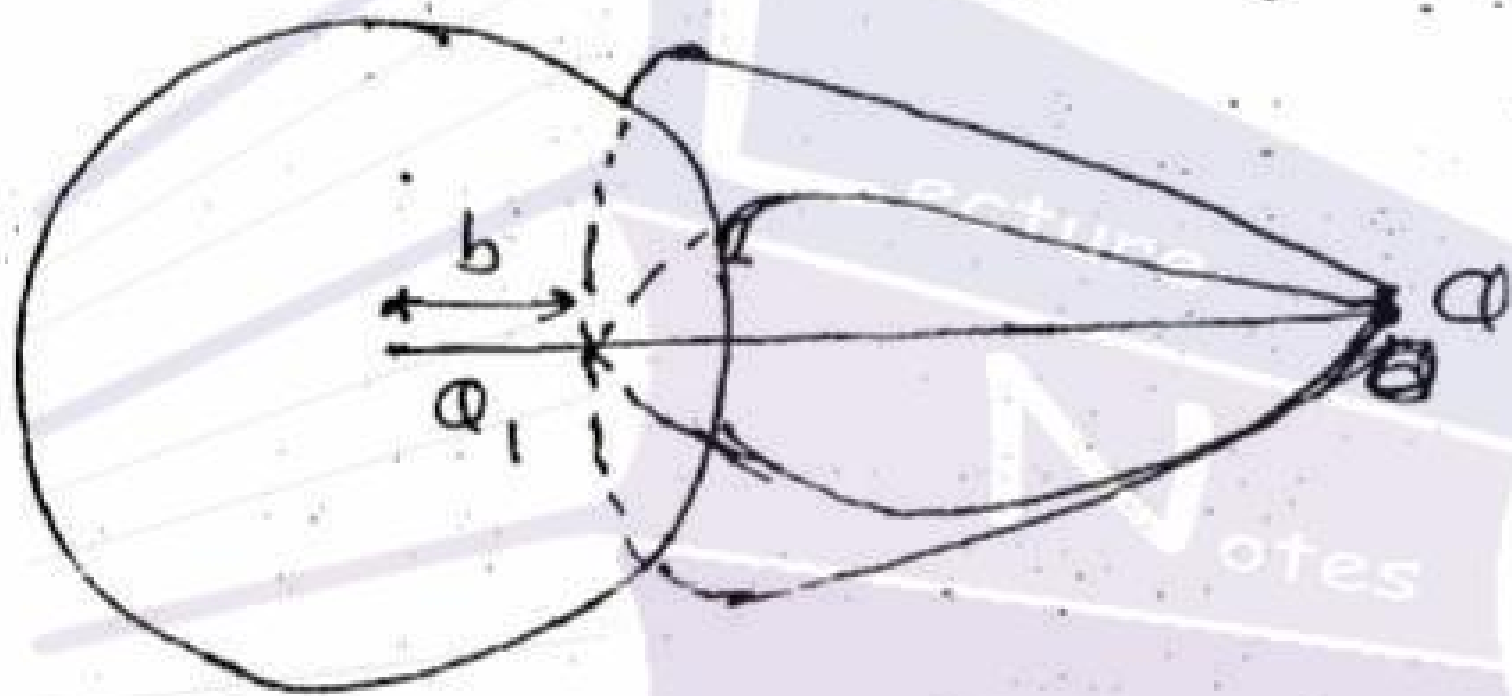
$$V_B = 0 = \frac{Q}{4\pi\epsilon(d+R)} + \frac{Q_1}{4\pi\epsilon(R+b)}$$



$$\frac{Q}{Q_1} = -\frac{(d-R)}{R-b} = -\frac{(d+R)}{(R+b)}$$

$$\Rightarrow \boxed{b = \frac{R^2}{d}} \quad \& \quad \boxed{Q_1 = -\frac{R}{d}Q}$$

Note:-



7. $4\mu C$, Ans

Energy Density in Electric fields ($\frac{dW_E}{dv} = \frac{1}{2}\epsilon E^2$) :-

Consider $Q_1, Q_2, Q_3, \dots, Q_n$ point charges assemble in a region

Total energy of the Electric field = Total Energy expended in assembling the charges

$$W_1 = 0$$

$$W_2 = -Q_2 V_{21}$$

$$W_3 = -Q_3 V_{31} - Q_3 V_{32}$$

$$W_4 = -Q_4 V_{41} - Q_4 V_{42} - Q_4 V_{43}$$

$\vdots$

$$W_n = -Q_n V_{n1} - Q_n V_{n2} - \dots - Q_n V_{nn-1}$$

V_{21} = Potential at 2nd due to 1st charge

$$Q_2 V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\epsilon V_{21}} = Q_1 V_{12}$$

Substitute subscripts can be interchange without change in meaning of value.

$$W_1 = 0$$

$$W_2 = -Q_1 V_2$$

$$W_3 = -Q_1 V_{13} - Q_2 V_{23}$$

$$W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34}$$

$$W_n = -Q_1 V_{1n} - Q_2 V_{2n} - \dots - Q_{n-1} V_{n-1,n}$$

$$W_E = W_1 + W_2 + W_3 + \dots + W_n$$

total energy

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 - \dots - Q_n V_n$$

$$\Rightarrow W_E = -\frac{1}{2} \sum_{i=1}^n Q_i V_i$$

For a continuous charge distribution

$$W_E = -\frac{1}{2} \int \rho_v V dv = -\frac{1}{2} \int (\nabla \cdot \mathbf{D}) V dv = \int \frac{1}{2} \mathbf{D} \cdot (-\nabla V) dv$$

$$\Rightarrow W_E = \int \frac{1}{2} (\mathbf{D} \cdot \mathbf{E}) dv$$

$$\frac{dW_E}{dv} = \text{Energy density}$$

= Strength of energy at any point

$$= \frac{1}{2} (\mathbf{D} \cdot \mathbf{E}) = \frac{1}{2} \epsilon E^2$$

Extension:-

$$\frac{dW_H}{dv} = \text{Magnetic Energy density}$$

$$= \frac{1}{2} \mathbf{B} \cdot \mathbf{H} = \frac{1}{2} \mu H^2$$

Capacitors and Inductors:-

→ Capacitance is the ability to confine Electric field in a finitely small region:

$$C = \text{Farad} = \frac{\oint \mathbf{D} \cdot d\mathbf{s}}{\int \mathbf{E} \cdot d\mathbf{l}} = \frac{\epsilon \oint \mathbf{E} \cdot d\mathbf{s}}{\int \mathbf{E} \cdot d\mathbf{l}} = \frac{Q}{V}$$

→ It is the ratio with charge utilized to the potential developed by the charge.

Specific Geometries:-

- Parallel Plates
- Cocentric cylinders
- Cocentric sphere

Inductance:-

Inductance is the ability to confine H field in a finitely small region

$$L = \text{Henry} = \frac{\oint \mathbf{B} \cdot d\mathbf{s}}{\oint \mathbf{H} \cdot d\mathbf{l}} = \frac{\mu \oint \mathbf{H} \cdot d\mathbf{s}}{\oint \mathbf{H} \cdot d\mathbf{l}} = \frac{\Psi_m}{I}$$

→ It is the ratio of the flux developed to the current utilized by the flux

Specific Geometries:-

- Solenoids
- Cocentric cylinder
- Toroids

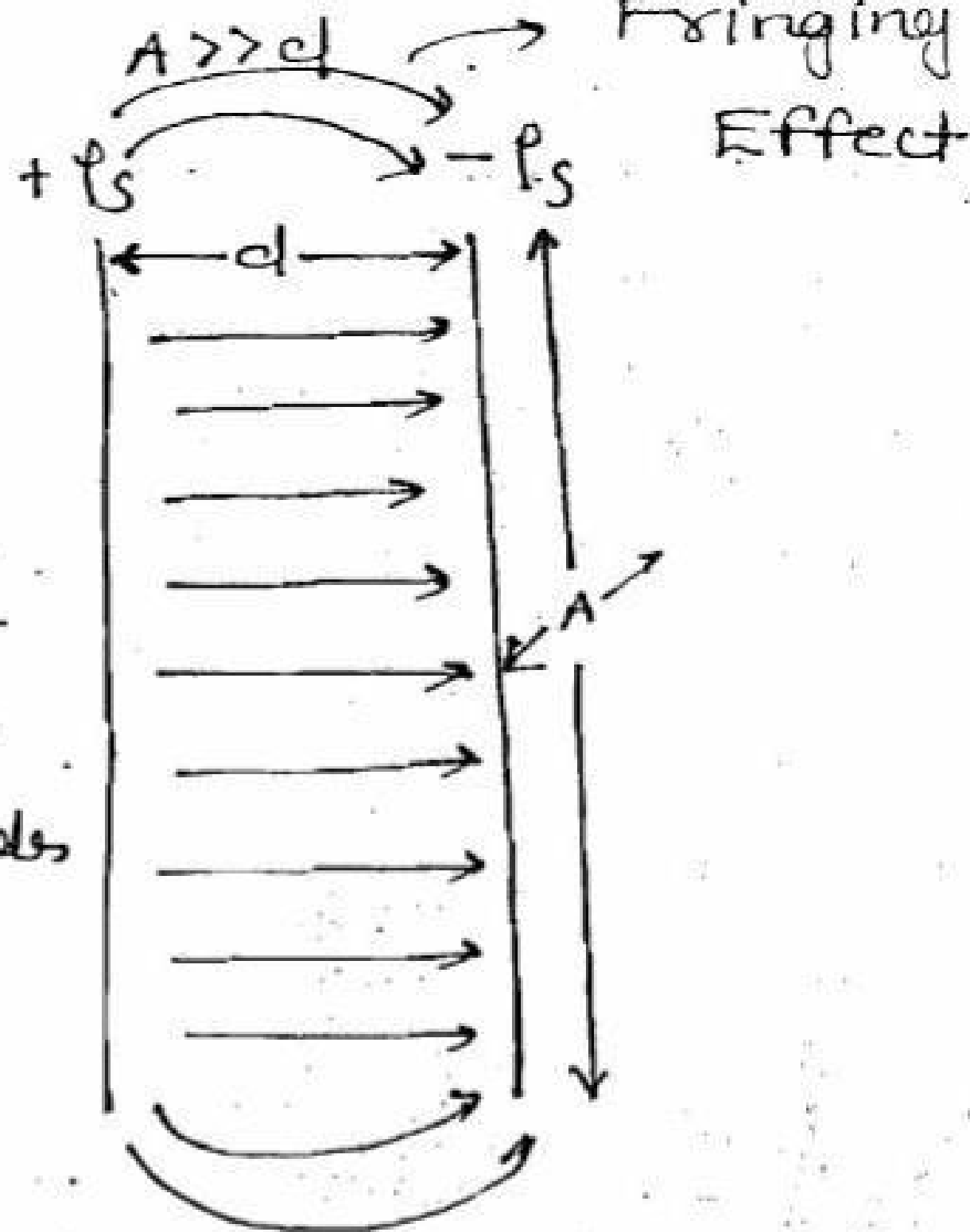
Parallel Plate Capacitors:-

Parallel Plate Capacitors:-

$$E = \frac{\rho_s}{\epsilon}$$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\epsilon}} = \frac{\epsilon A}{d}$$

Capacitance is independent of Q or V and always depends on Area and distance or physical dimensions



$$W_E = \frac{1}{2} \epsilon E^2 (Ad)$$

$$= \frac{1}{2} \frac{\epsilon A}{d} (Ed)^2$$

$$= \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV$$

Multiple Dielectrics in Capacitors:-

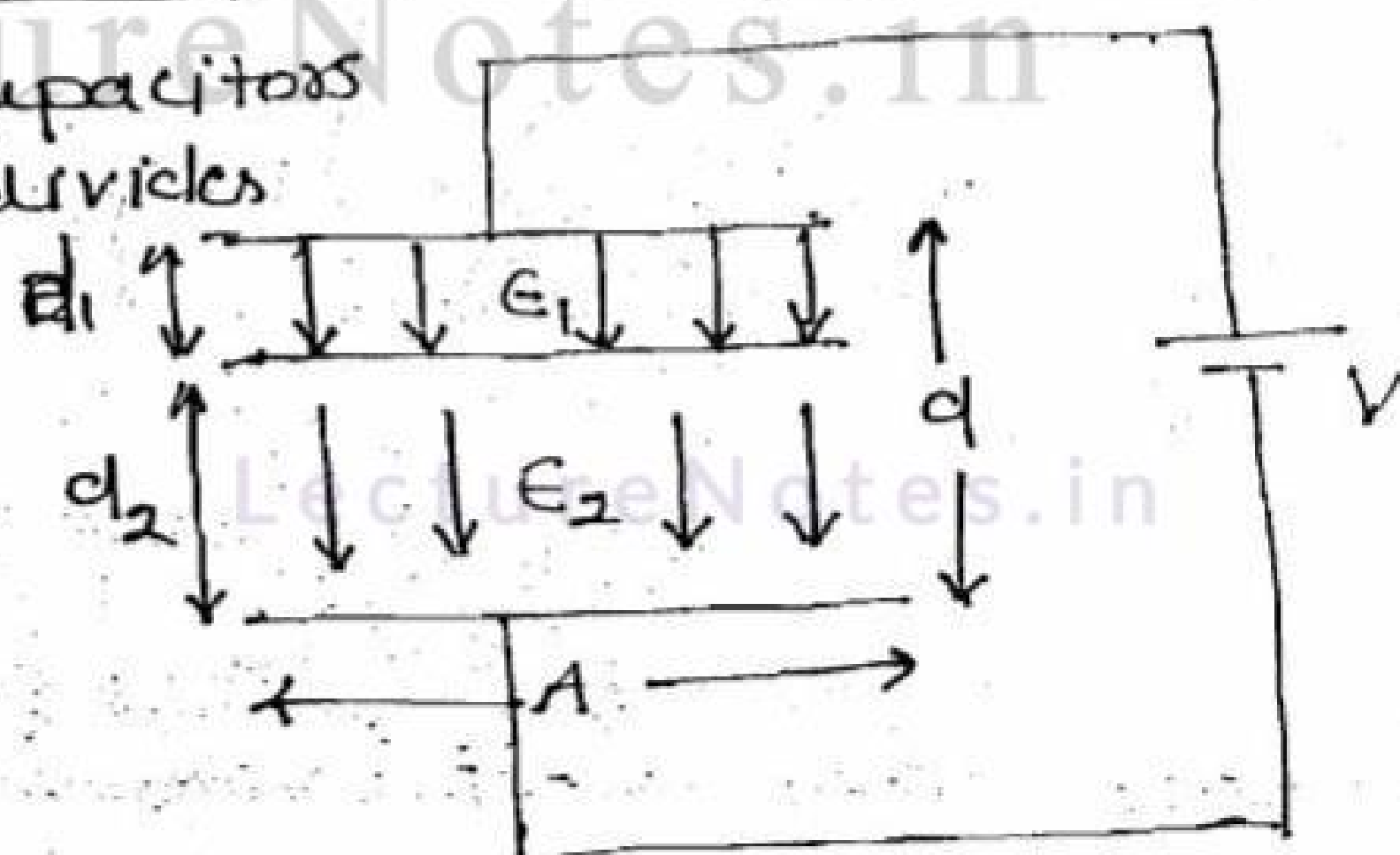
Case-(1):-

Equal areas of cross-section unequal width of the dielectrics:-

They are two capacitors in series as voltage dividers

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$



$$\theta_1 = \theta_2 \Rightarrow \epsilon_1 E_1 = \epsilon_2 E_2 \Rightarrow \frac{\epsilon_1 V_1}{d_1} = \frac{\epsilon_2 V_2}{d_2}$$

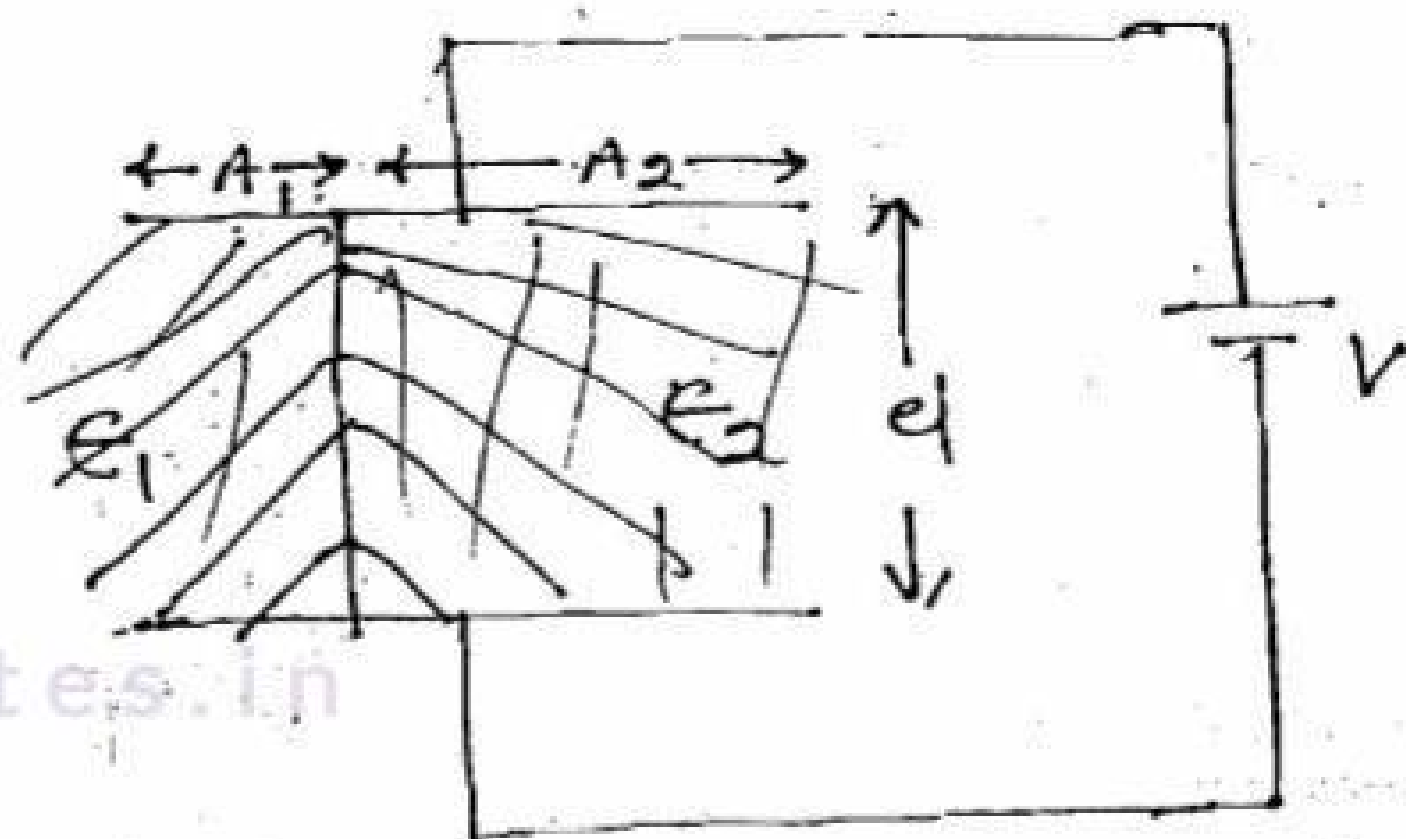
$$\Rightarrow \boxed{\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \left(\frac{d_1}{d_2} \right)}$$

Case-(II)

Unequal areas of cross-section and equal width of the dielectrics :-

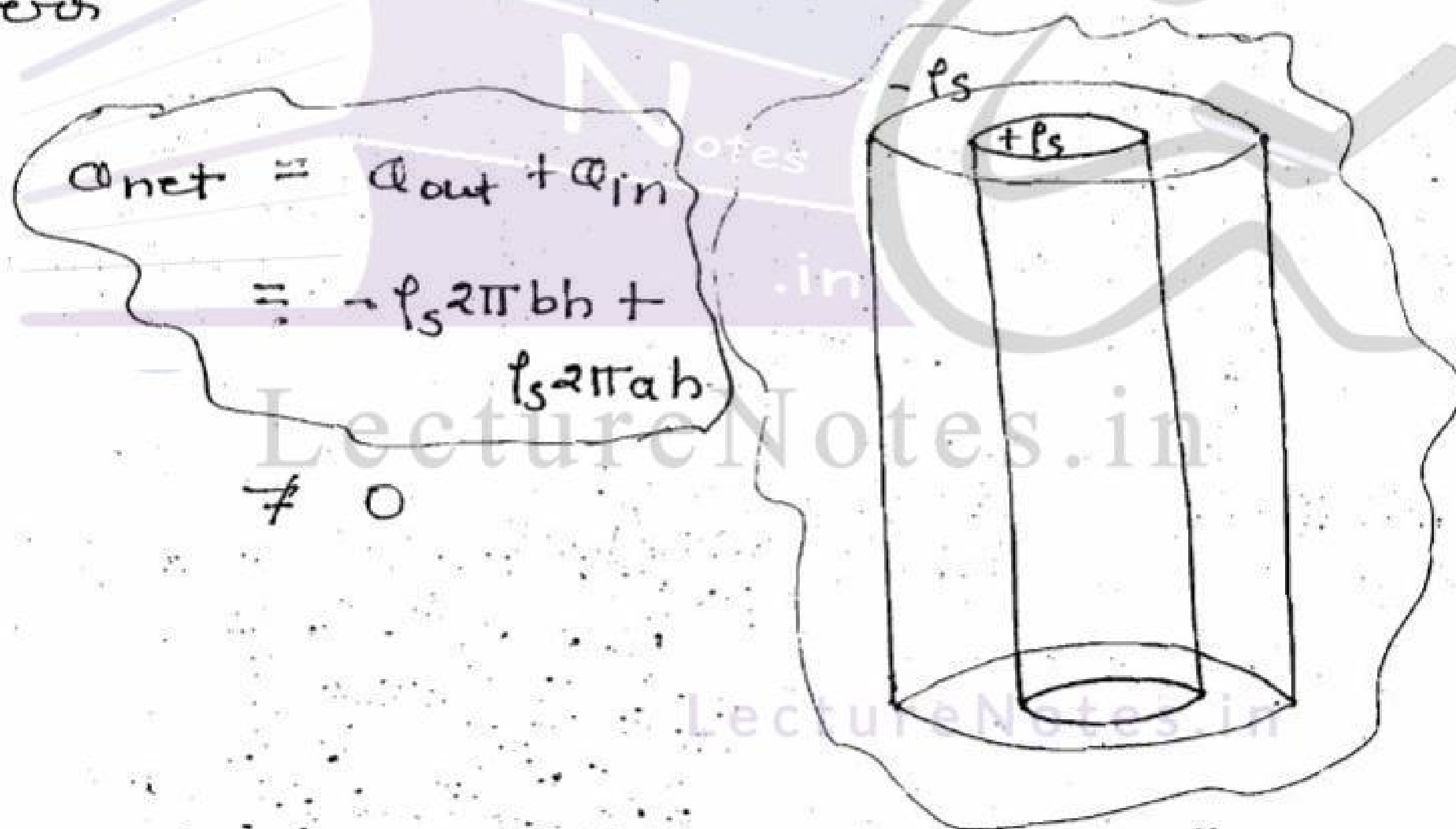
$$C_{eq} = C_1 + C_2$$

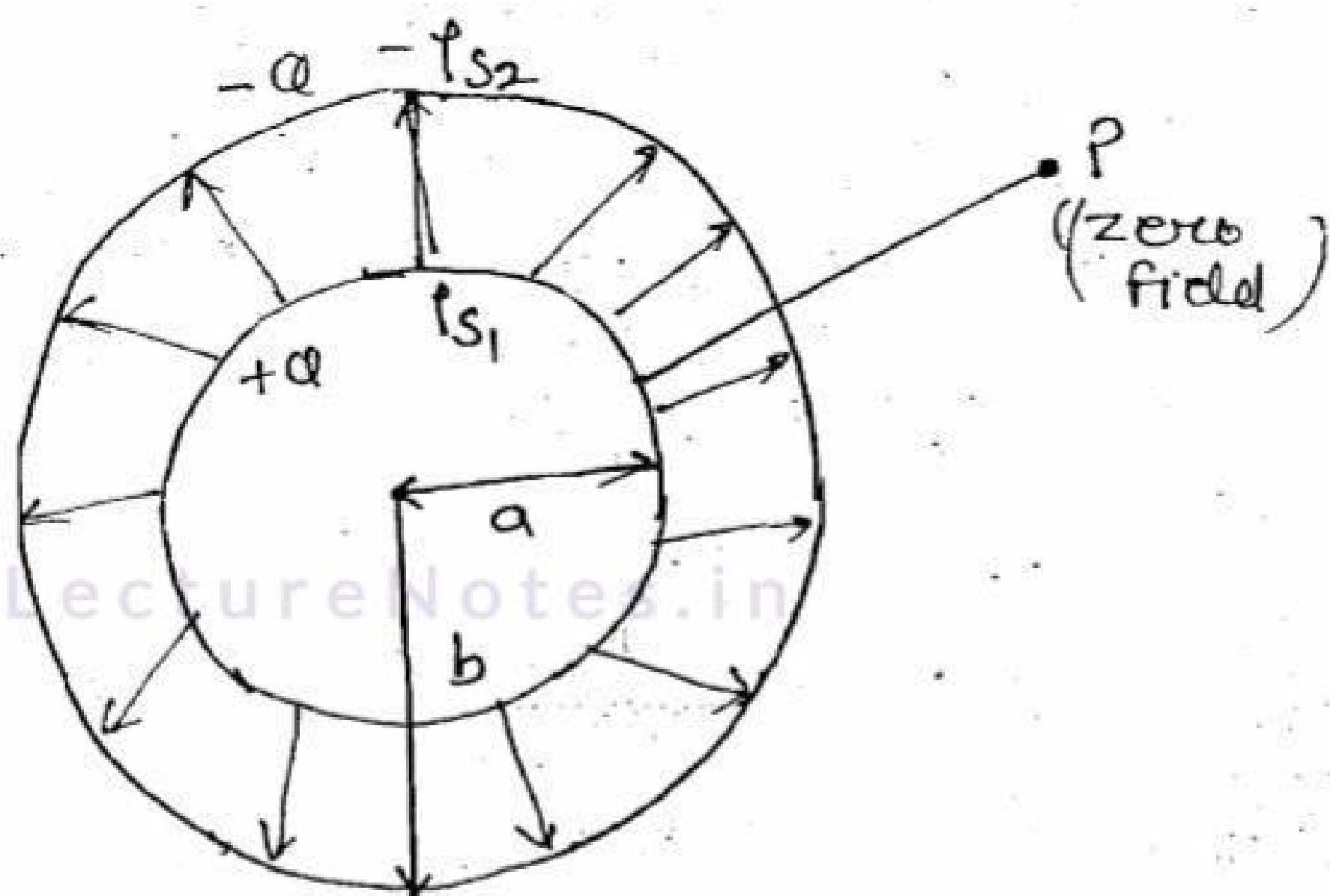
$$= \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$



Cocentric Cylinder Capacitance :-

Two concentric cylinders with equal and opposite charge density may not have confined flux but two equal and opposite charges on concentric cylinders has flux confined only b/w the cylinders





$$\lambda \propto \rho_s \propto \frac{1}{r}$$

→ The field of a sheet of cylindrical charge obeys same geometry as a line charge

$$C = \frac{Q}{V} = \frac{\rho_l h}{\frac{\rho_l}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow C = \frac{2\pi\epsilon h}{\ln(b/a)}$$

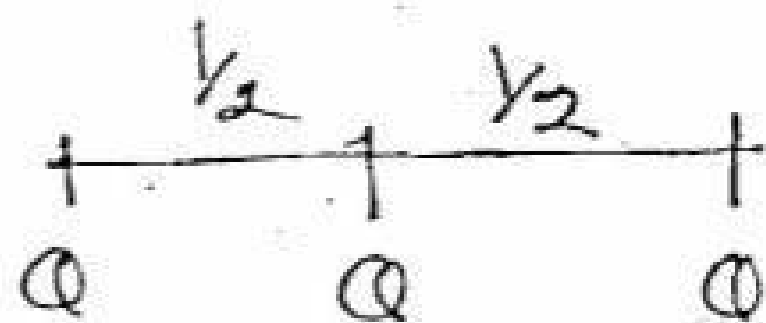
Extension:-

Cocentric sphere capacitance

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Q}{4\pi\epsilon} \left(\frac{1}{a} - \frac{1}{b} \right)} = \frac{4\pi\epsilon}{\left(\frac{1}{a} - \frac{1}{b} \right)}$$

$$W_{E1} = 0 + 0 \cdot \frac{Q}{4\pi\epsilon_1 \frac{1}{2}}$$



$$+ 0 \cdot \frac{Q}{4\pi\epsilon_1 \frac{1}{2}} + 0 \cdot \frac{Q}{4\pi\epsilon_1 \cdot 1} = \frac{5Q^2}{4\pi\epsilon_1}$$

$$W_{E2} = \frac{5Q^2}{8\pi\epsilon_1} = \frac{W_{E1}}{2}$$

Note! —

$$W_E \propto QV \propto \frac{1}{r}$$

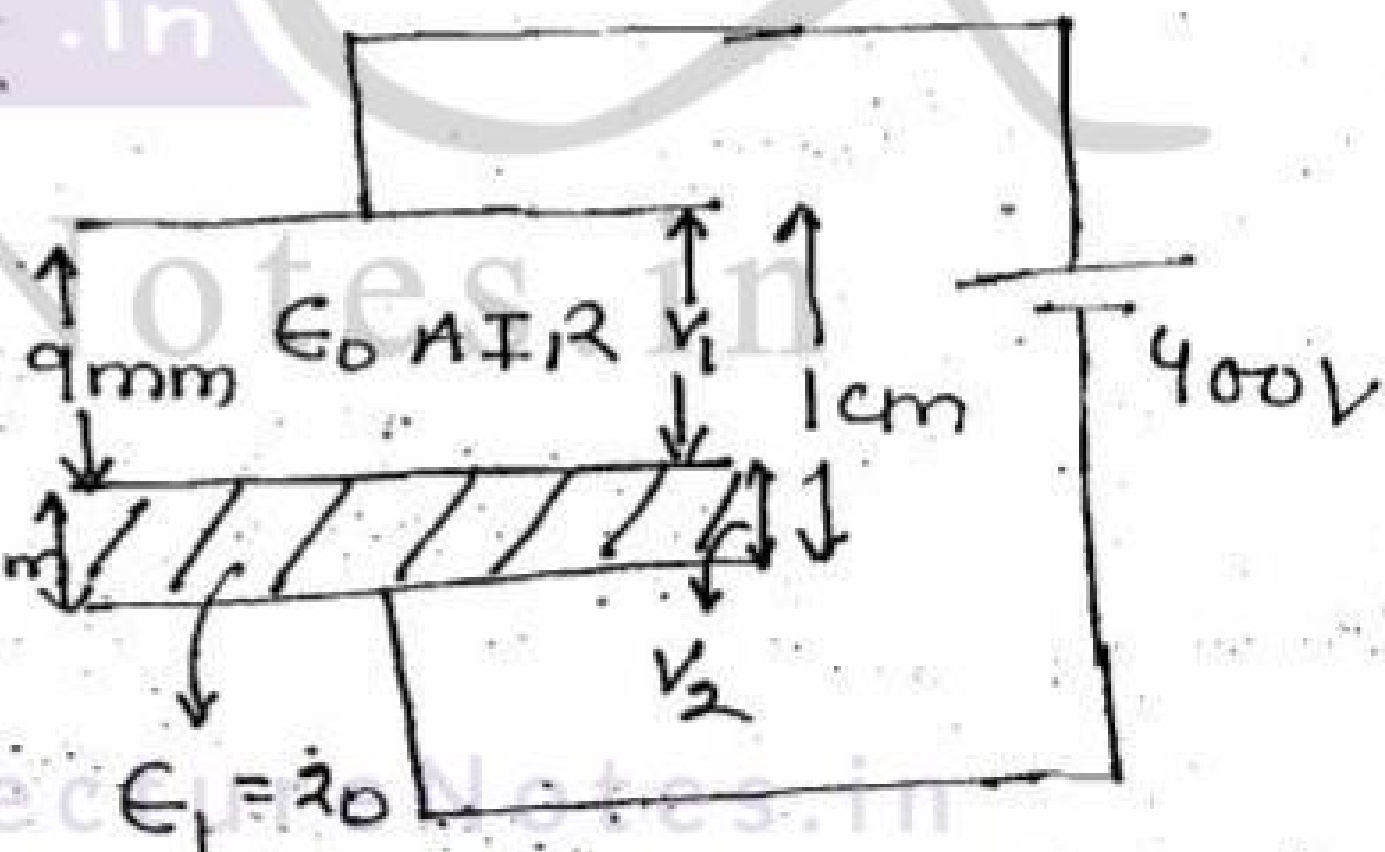
$$\frac{1}{2} (J \cdot A) = \frac{\text{Amp}}{m^2} \times \frac{\text{Joules}}{\text{Amp} \cdot m} = \frac{\text{Joules}}{m^3}$$

$$\Rightarrow \frac{1}{2} (J \cdot A) = \frac{1}{2} ((\nabla \times H) \cdot A) = \frac{1}{2} (H (\nabla \times A)) = \frac{1}{2} B \cdot H$$

$$E_{AIR} = \frac{V_{AIR}}{9mm}$$

$$V_1 + V_2 = 400 \text{ (1)}$$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \left(\frac{d_1}{d_2} \right) \text{ (2)}$$

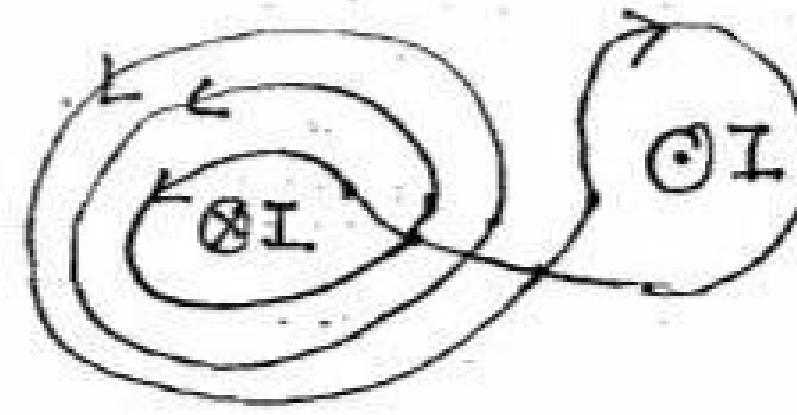


$$\frac{V_1}{V_2} = \frac{20\epsilon_0}{\epsilon_0} \times \frac{9mm}{1mm} = 180$$

$$V_1 = 180V_2$$

$$\text{Ans} \rightarrow V_1 = 44 \text{ kV/m}$$

$$\begin{aligned} \text{Circulation} &= \oint H \cdot dl = I \\ &= I + I + I - (-I) \\ &= 4I, \text{ Ans.} \end{aligned}$$



Lecture - 8

Time Varying Fields and Maxwell's Equation

Static Fields - Maxwell's Equations:-

Integral Form

(i) $\oint \mathbf{D} \cdot d\mathbf{s} = Q$

(ii) $\oint \mathbf{E} \cdot d\mathbf{l} = 0$

(iii) $\oint \mathbf{B} \cdot d\mathbf{s} = 0$

(iv) $\oint \mathbf{H} \cdot d\mathbf{l} = I$

Point form

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Note:-

- The surface and Integral and Divergence expressions are consistently the same in static / time varying fields
- The line integral and curl expression are modified in time varying fields to explain AC voltages and currents

Open Integrals (Not Maxwell Equations):-

1. $\int \mathbf{D} \cdot d\mathbf{s} = \psi_e = \text{coulombs}$

2. $\int \mathbf{E} \cdot d\mathbf{l} = V = \text{EMF} = \text{volts}$ ← time

3. $\int \mathbf{B} \cdot d\mathbf{s} = \psi_m = \text{Webers}$

4. $\int \mathbf{H} \cdot d\mathbf{l} = \psi_m = \text{MMF} = \text{Amps}$

Maxwell 2nd Equation and Faraday's Law:-

Faraday's Law Statement:-

EMF or voltage is induced even in a closed conductor when the magnetic flux crossing the surface changes with time.

i.e. Rate of change of magnetic flux is equal to induced EMF

$$\oint E \cdot dl = V = - \frac{d\psi_m}{dt} \quad \text{due to Lenz's law}$$

$$= - \frac{d}{dt} \int B \cdot ds$$

$$= \int - \frac{\partial B}{\partial t} \cdot ds$$

Lenz's law:-

The induced EMF ^{always} opposes the basic changing flux (Cause)

$$\oint E \cdot dl = \int - \frac{\partial B}{\partial t} \cdot ds$$

$$\oint E \cdot dl = V = - \frac{d\psi_m}{dt} = - \frac{d}{dt} \int B \cdot ds = \int - \frac{\partial B}{\partial t} \cdot ds \leftarrow$$

→ Apply Stoke's theorem = $\int \nabla \times E \cdot ds$

$$\boxed{\nabla \times E = - \frac{\partial B}{\partial t}}$$

Modified Maxwell's Second Equation

Note:-

Potential is unique at a time at a point in a space but changes with time and hence the modification

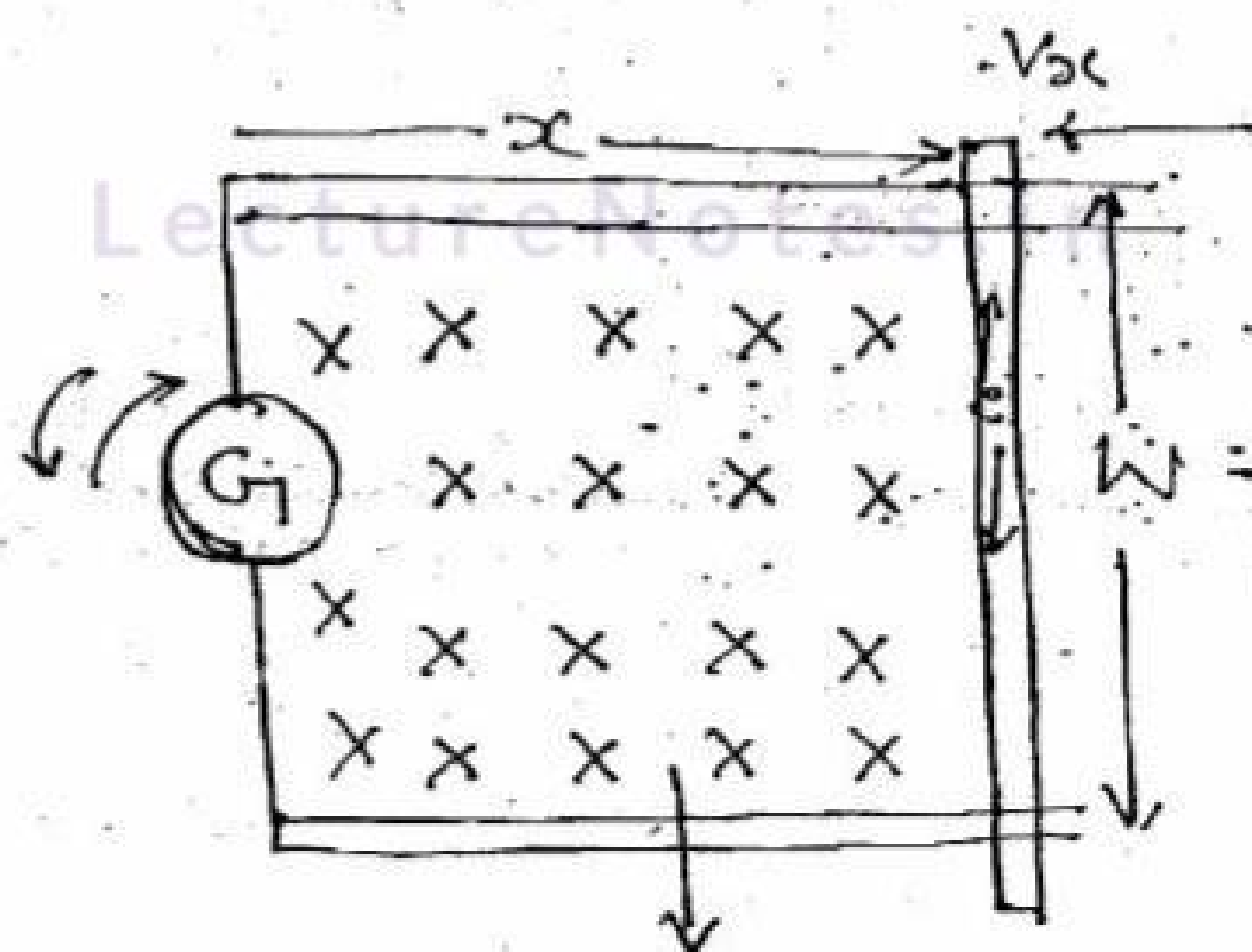
Sliding Rail Experiment:-

$$V = - \frac{d\psi_m}{dt}$$

$$\Rightarrow V = - \frac{d}{dt} (B \cdot A)$$

$$\Rightarrow \boxed{V = - B \cdot W \frac{dx}{dt}}$$

$$\Rightarrow \boxed{V = - B \cdot W \cdot V_{sc}}$$



Static Uniform
B field

Note:-

- When a conducting rod is moving in a magnetic field a force exists on the electron obeying Lorentz Law. This displaces the electrons towards one side.
- This accumulation is called as induced voltage.
- As the rod moves to and fro the voltage polarity changes. This is called as AC voltage.
- The already displaced e^- opposes another further coming e^- 's. This is called as Lenz's Law.

$$F_y = q (V_x \times B_z) = qE$$

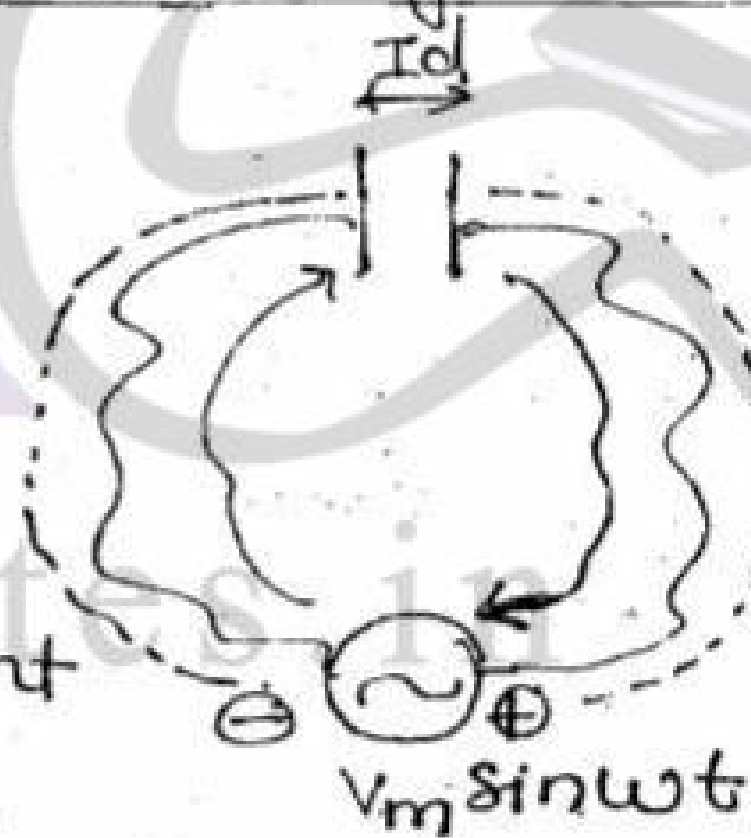
$$\Rightarrow \frac{dx}{dt} B \sin \theta = \frac{V}{W} \quad \text{fc}$$

$$\Rightarrow V = \frac{d}{dt} (x BW) = \frac{d}{dt} (BA) \quad (\theta = 90^\circ)$$

$$\Rightarrow V = \frac{d\psi_m}{dt}$$

Maxwell's IV Equation and Inconsistency of Ampere's Law:-

- When a capacitor is connected with AC harmonic voltage there is a current flowing in the wire and plates obeying Ohm's Law like any linear element.



$$I = C \frac{dV}{dt}$$

$$I = C \cdot W \cdot V_m \sin(\omega t + 90^\circ)$$

$$(e^{j90} = j) \rightarrow \text{Phase} \quad \text{Phase}$$

$$I = j\omega C V_m \sin \omega t$$

$$V = \left(\frac{1}{j\omega C} \right) I$$

- There is no current flowing b/w plates as there is dielectric. Hence the circuit is not closed. But Ampere's Law states that current flows only

closed circuits. Hence Maxwell's modified Ampere's law as

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_c + I_d$$

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \mathbf{J}_d$$

Using equation of continuity

$$\nabla \cdot \mathbf{J}_d = \frac{\partial \rho_v}{\partial t} = \frac{\partial (\nabla \cdot \mathbf{D})}{\partial t} = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t} \Rightarrow$$

$$\Rightarrow \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \quad \& \quad I_d = \int \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$

Maxwell IV Equation is

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_c + \int \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$

$$\Rightarrow \nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$$

where I_c = conduction current
OR
= Moving $\bar{e}$'s

→ When an AC voltage applied to the capacitor plate the plates are alternatively charge and discharge. This continues takes place as the polarity changes. Hence this self establish continuity

→ As ρ_s on the plates changes with ρ time, ρ b/w the plates changes with time. Hence $\frac{\partial \mathbf{D}}{\partial t}$ is

$$\frac{\text{C/m}^2}{\text{second}} = \text{Amp/m}^2 = \mathbf{J}_d$$

This is also a format of current

This is called as Displacement current density.

Summary 1:—

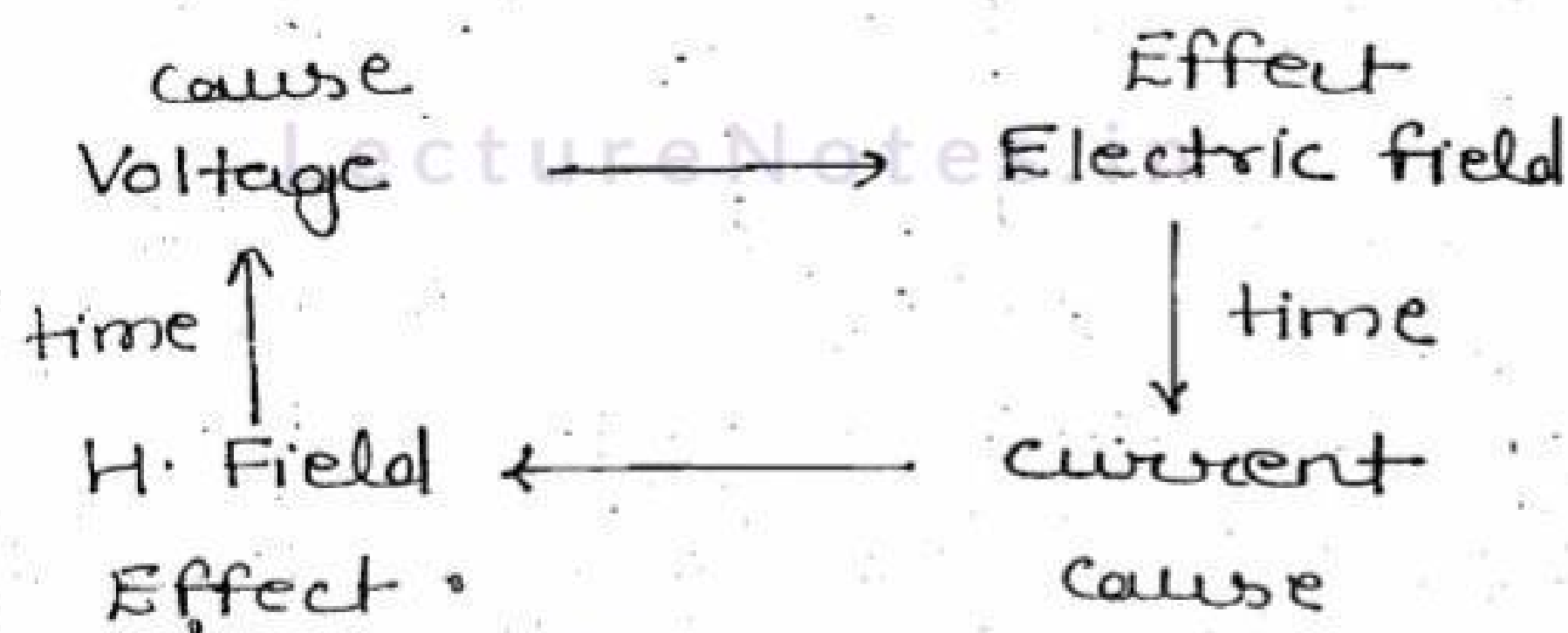
$$\oint \mathbf{E} \cdot d\mathbf{l} = \int -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} + I_c$$

- A time varying magnetic flux, is a cause of voltage
- A time varying electric flux is a format of current

$$\frac{\text{Weber}}{\text{second}} = \text{Volts}$$

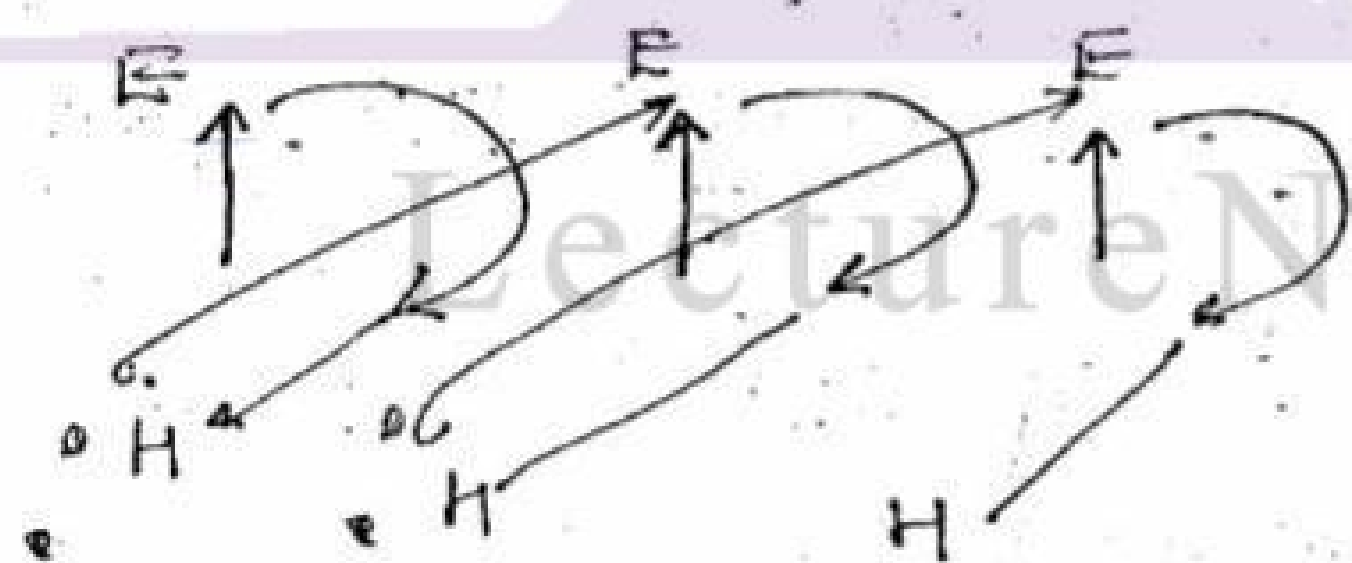
$$\frac{\text{Coulomb}}{\text{Sec}} = \text{Amp}$$



Summary 2:-

$$\begin{aligned} \text{Space Varying Electric field} &\leftarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \text{time Varying B field} \\ \nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} + \vec{J}_c \end{aligned}$$

- A time varying E field produces a space varying orthogonal H field and vice-versa



$$\text{H field} \xrightleftharpoons[\text{Space}]{\text{time}} \text{E Field}$$

- Accumulation leads to flow and flow leads to accumulation sustaining each other

Summary 3:-

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}_c$$

→ $\mu = \text{Henry/m}$, $\epsilon = \text{Farad/m}$, $\sigma = \text{mho/m}$ →

For E to transform to H and vice-versa the material and its permitting abilities are also important.

i.e. Material constitutes decides the E/H dynamics in the medium

→ $\vec{E}$ - Volts/m, $\vec{H}$ - amps/m, ∇ - perm

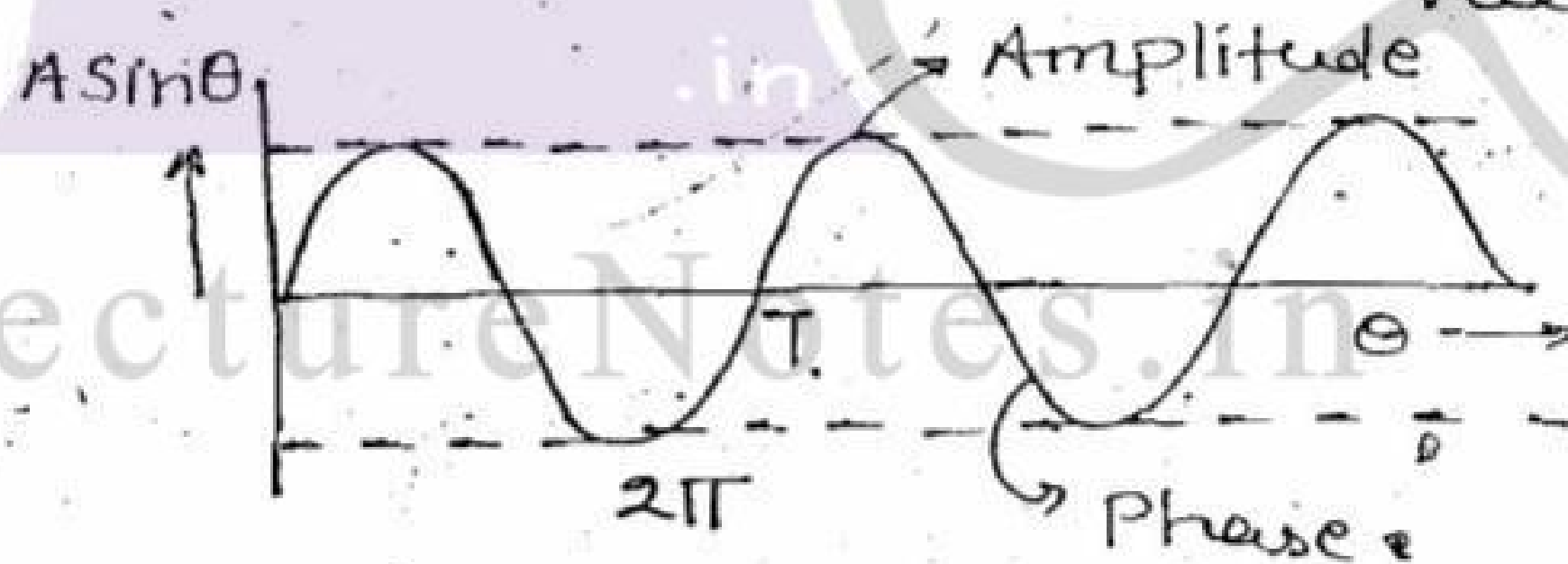
Note:-

→ For sustain oscillation or ever existing phenomenon are produce when the time derivative of E and H is always back the same function and such a function called as Harmonic function.

Harmonic Functions:-

They are 2 dimension quantities and having 3 formats

→ $A \sin \theta$	} Amplitude → Domain 1 → Peak value	
→ $A \cos \theta$		Phase → Domain 2 → Instantaneous value
→ $A e^{j\theta}$		



Property 1:-

The phase should be a linear function of the variable

(i) $\theta \propto t$ → time Harmonic

⇒ $\theta = \omega t$ $\omega = \text{Phase shift constant per unit time}$

$= \frac{2\pi}{T} \text{ rad/second}$

(ii) $\theta \propto z$ → Space Harmonic

⇒ $\theta = \beta z$ $\beta = \text{Phase shift constant per unit length} = \frac{2\pi}{\lambda} \text{ rad/}$

Property 2 :- The derivative of every harmonic has to be back the same function shifted orthogonally by 90°

$$A \sin \omega t \xrightarrow{\text{I-derivative}} \omega A \sin(\omega t + 90^\circ) \xrightarrow{\text{II-der}} \omega^2 A \sin(\omega t + 180^\circ)$$

$$\swarrow \text{III-derivative} \\ \omega^3 A \sin(\omega t + 270^\circ)$$

$$A e^{j\omega t} \xrightarrow{\text{I-der}} \omega A e^{j(\omega t + 90)} \xrightarrow{\text{II-der}} \omega^2 A e^{j(\omega t + 180)}$$

$$(\because j^3 = -j = e^{j270})$$

$$(-1 = e^{j180})$$

$$\downarrow \text{III-der} \\ \omega^3 A e^{j(\omega t + 270)} \\ (\because j = e^{j90})$$

→ All harmonics obey the basic property the second order derivative is back the same function

→ They satisfied the differential equation

$$\partial^2 - M^2 = 0$$

$$\partial^2 + M^2 = 0$$

eg:- E/H field equation in free space

Electro magnetic wave equation in free space

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

Taking ∇ on both sides

$$\nabla \times (\nabla \times H) = \epsilon \frac{\partial}{\partial t} (\nabla \times E)$$

$$-\nabla^2 H = \epsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial H}{\partial t} \right)$$

$$\boxed{\begin{aligned}\nabla^2 H &= \epsilon \mu \frac{\partial^2 H}{\partial t^2} \\ \nabla^2 E &= \epsilon \mu \frac{\partial^2 E}{\partial t^2}\end{aligned}}$$

Space Harmonic $\nearrow$ $\nwarrow$ Time Harmonic

These are called as E/H wave equations in free space

eg-(2):- V/I Equations in LC circuits

$$I = C \frac{dV}{dt}$$

$$V = -L \frac{dI}{dt} = -L \frac{d}{dt} \left(C \frac{dV}{dt} \right)$$

$$\Rightarrow V = -LC \frac{d^2 V}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2 V}{dt^2} = -\frac{1}{LC} V}$$

and $\boxed{\frac{d^2 I}{dt^2} = -\frac{1}{LC} I}$

By comparison

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Types of Exponential Functions:-

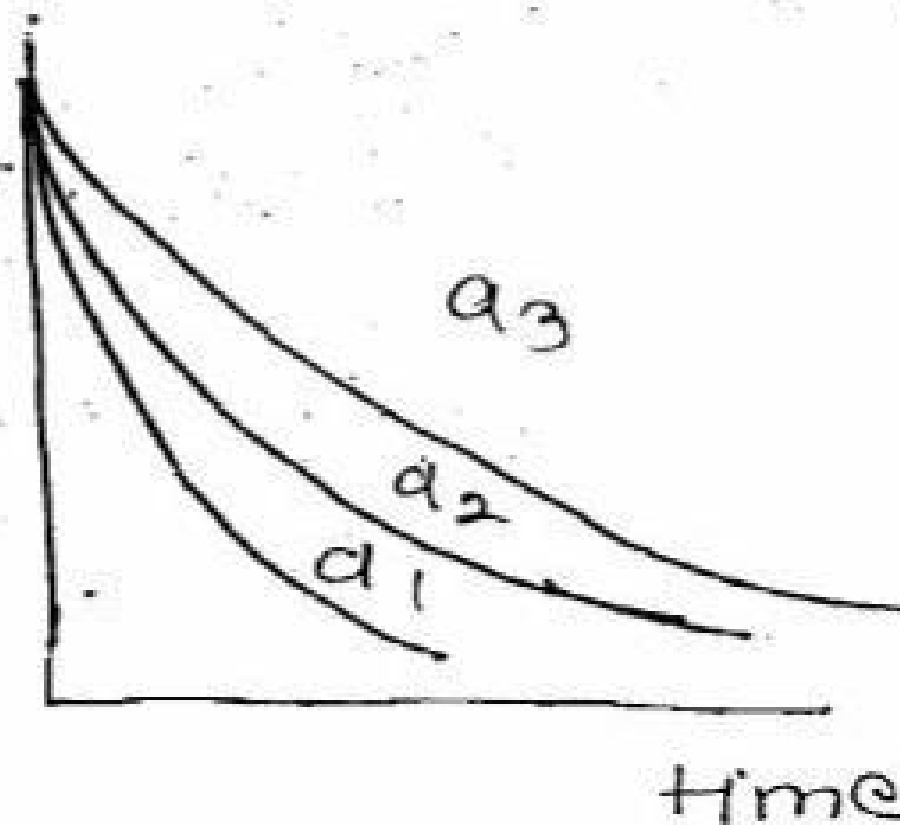
$\rightarrow e^{-kt}$ vs t

(1) Case - (1) :-

$k = a = \text{+ve real no.}$ e^{-kt}

$$a_1 > a_2 > a_3$$

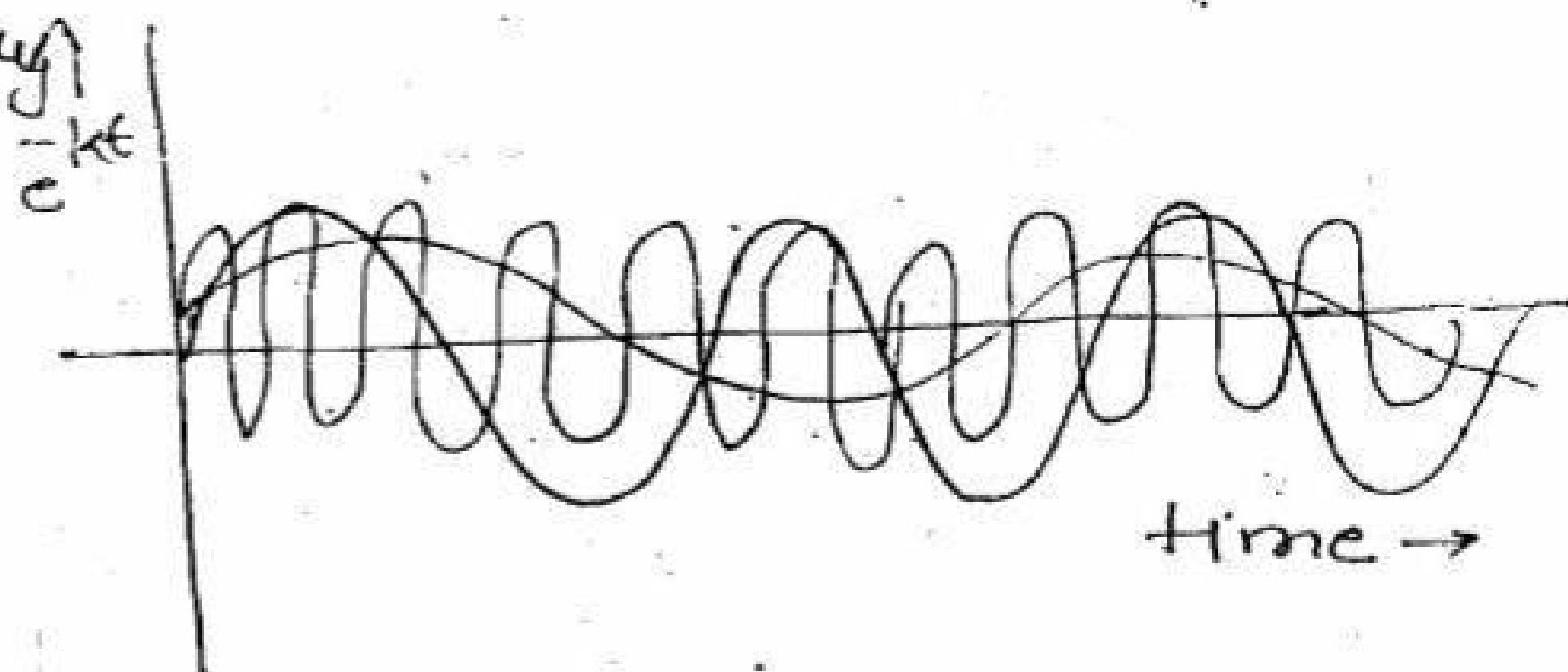
$\rightarrow$ Real Exponential



ii) Case - (ii) :-

$k = j\omega$ = purely imaginary
no.

ω = Phase Shift
Constant



iii) Case - (iii) :-

$k = a + j\omega$

$(e^{-at}) (e^{-j\omega t})$

$\downarrow$
 $A \cdot e^{j\theta}$



Note :-

j stands for an orthogonal shift from domain 1 to a second independent domain 2

eg:- (i) east and north displacements

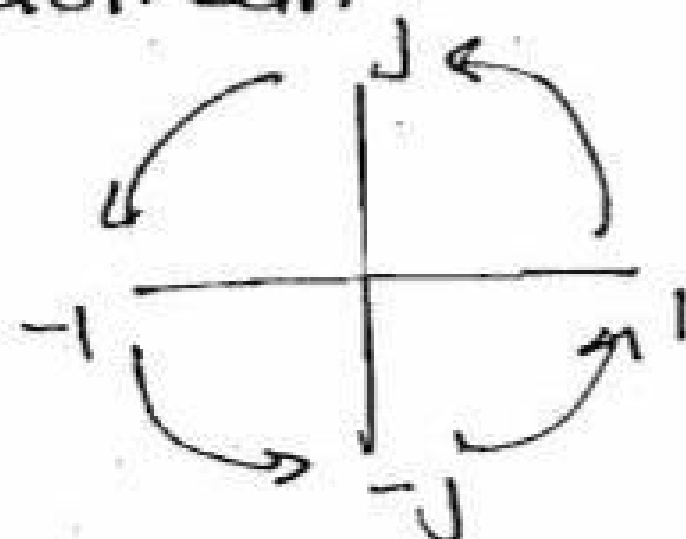
(ii) Resistance and reactance of a impedance

(iii) Amplitude and phase of a harmonic

A scaling of j in domain 1 is a shift of 90° in domain 2

eg:- a) $j\omega e^{-j\omega t} \longrightarrow \omega e^{j(\omega t + 90^\circ)}$

(ii) s-domain:



$$1 \times j = j = 1 \angle 90^\circ$$

$$j \times j = -1 = 1 \angle 180^\circ$$

$$-1 \times j = -j = 1 \angle 270^\circ$$

$$-j \times j = 1 \angle 360^\circ$$

EM Propagation in General Materials:- (σ, ϵ, μ)

Using Maxwell's Equation

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

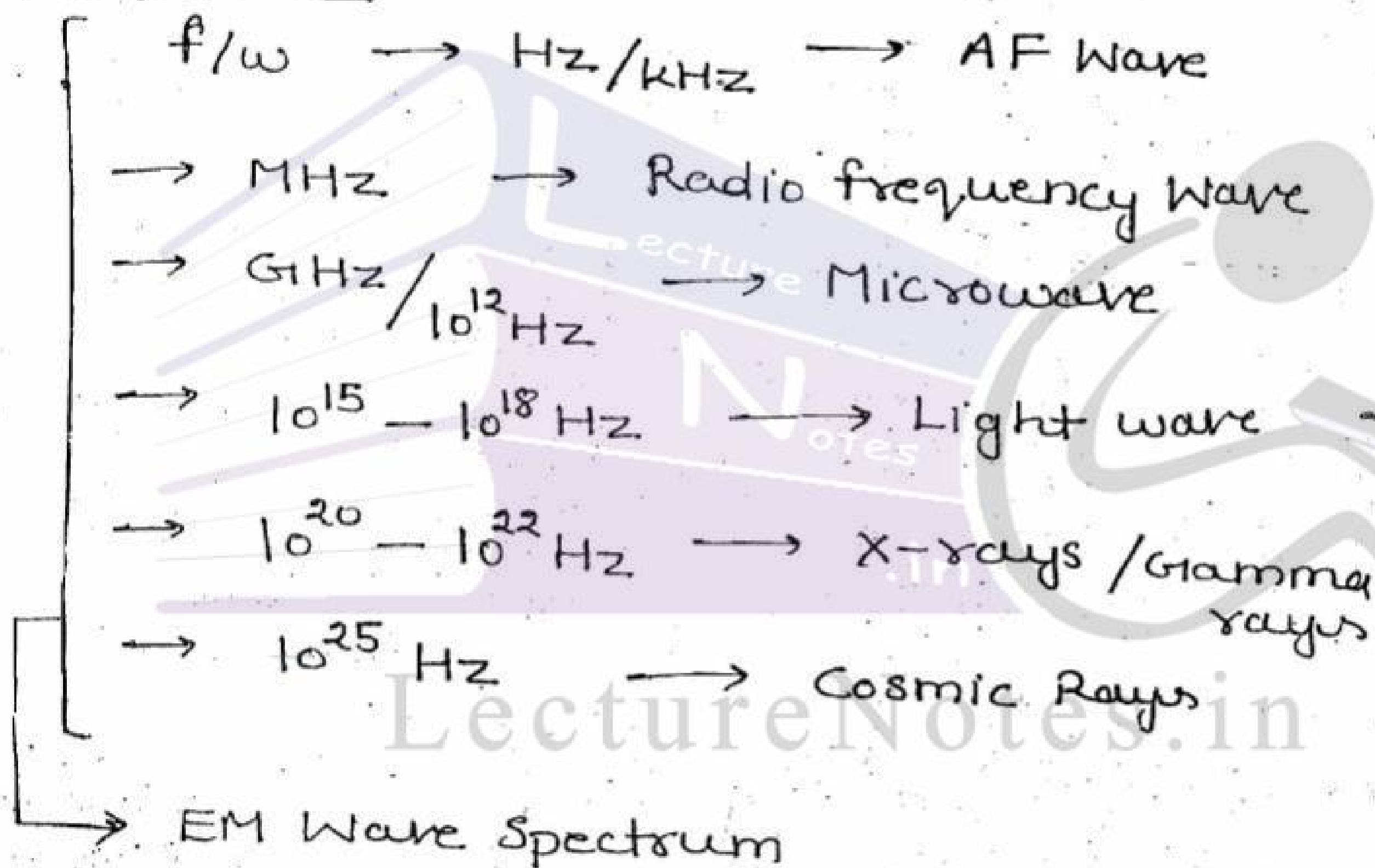
→ Every EM wave needs a time harmonic source at one end

$$E_s = E_0 e^{j\omega t}$$

$$H_s = H_0 e^{j\omega t}$$

ω = frequency in rad/s

Range of ω :-



$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t} = \sigma E_0 e^{j\omega t} + \epsilon j\omega E_0 e^{j\omega t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} = (\sigma + j\omega\epsilon) E$$

$$\Rightarrow \nabla \times H = (\sigma + j\omega\epsilon) E \quad \text{--- (1)}$$

$$\nabla \times E = -j\omega \mu H \quad \text{--- (2)}$$

$$\nabla \times E = \mu \frac{\partial H}{\partial t}$$

$$= -\mu j\omega H_0 e^{j\omega t} = -j\omega\mu H$$

$$\nabla \times E = -j\omega\mu H \quad \text{--- (1)}$$

Eq-(1) & (11) are called as Maxwell Equations for time Harmonic source

Take H from (11) & put in (1)

LectureNotes.in

$$H = \frac{\nabla \times E}{-j\omega\mu}$$

$$\Rightarrow \nabla \times \left(\frac{\nabla \times E}{-j\omega\mu} \right) = (\sigma + j\omega\epsilon) E$$

$$\Rightarrow \nabla \left(\underbrace{\nabla \cdot E}_{=0} \right) - \nabla^2 E = -j\omega\mu (\sigma + j\omega\epsilon) E$$

For a charge free region

$$\nabla^2 E = j\omega\mu (\sigma + j\omega\epsilon) E \quad \text{--- (11)}$$

Similarly

$$\nabla^2 H = j\omega\mu (\sigma + j\omega\epsilon) H \quad \text{--- (1V)}$$

Space

time

LectureNotes.in

These are called as Helmholtz's Propagation Equations.

Note:-

LectureNotes.in

→ If the source is the time harmonic then the effect is space harmonic propagation in the medium.

$$\nabla^2 E = j\omega\mu (\sigma + j\omega\epsilon) E \quad \text{--- (III)}$$

$$\nabla^2 H = j\omega\mu (\sigma + j\omega\epsilon) H \quad \text{--- (IV)}$$

Consider ∇ in one dimension dimension of z only
 E is in x direction, H is in y -direction

Consider $\sqrt{j\omega\mu (\sigma + j\omega\epsilon)} = \gamma$

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \quad \text{--- (III)}$$

$$\frac{\partial^2 H_y}{\partial z^2} = \gamma^2 H_y \quad \text{--- (IV)}$$

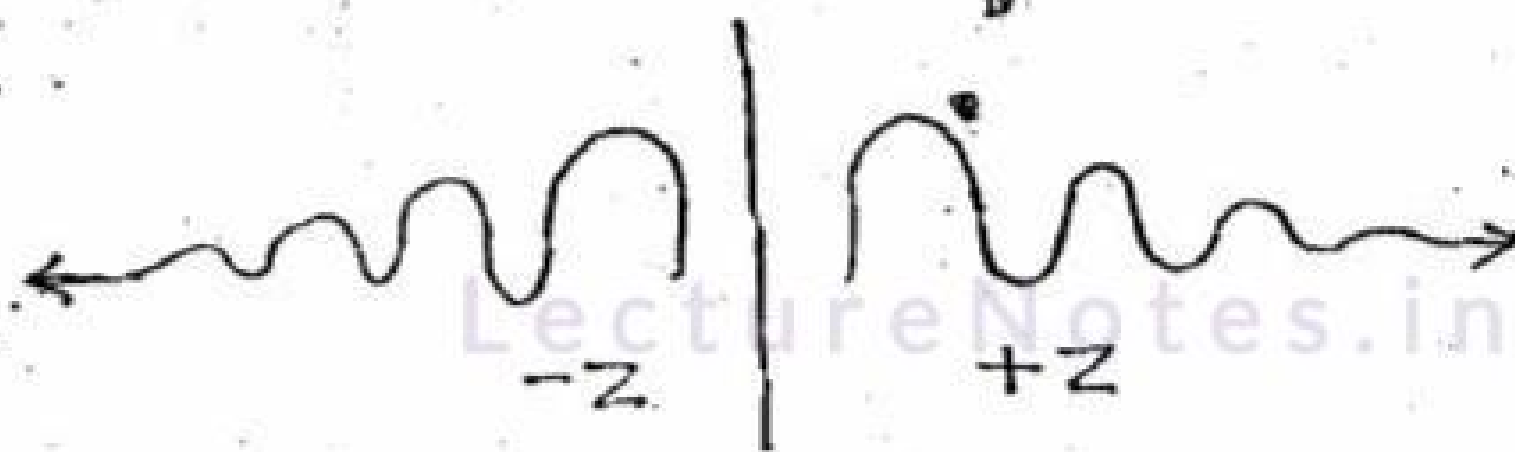
The $E(z)$ and $H(z)$ solution is.

$$E(z)_x = (c_1 e^{-\gamma z} + c_2 e^{\gamma z}) a_z \quad \text{--- (V)}$$

$$H(z)_y = (c_3 e^{-\gamma z} + c_4 e^{\gamma z}) a_y \quad \text{--- (VI)}$$

$\downarrow$ $\downarrow$
 $+z$ $-z$

The solution physically represent two waves travelling on either sides from a planar source and both decaying. This is called as uniform plane wave



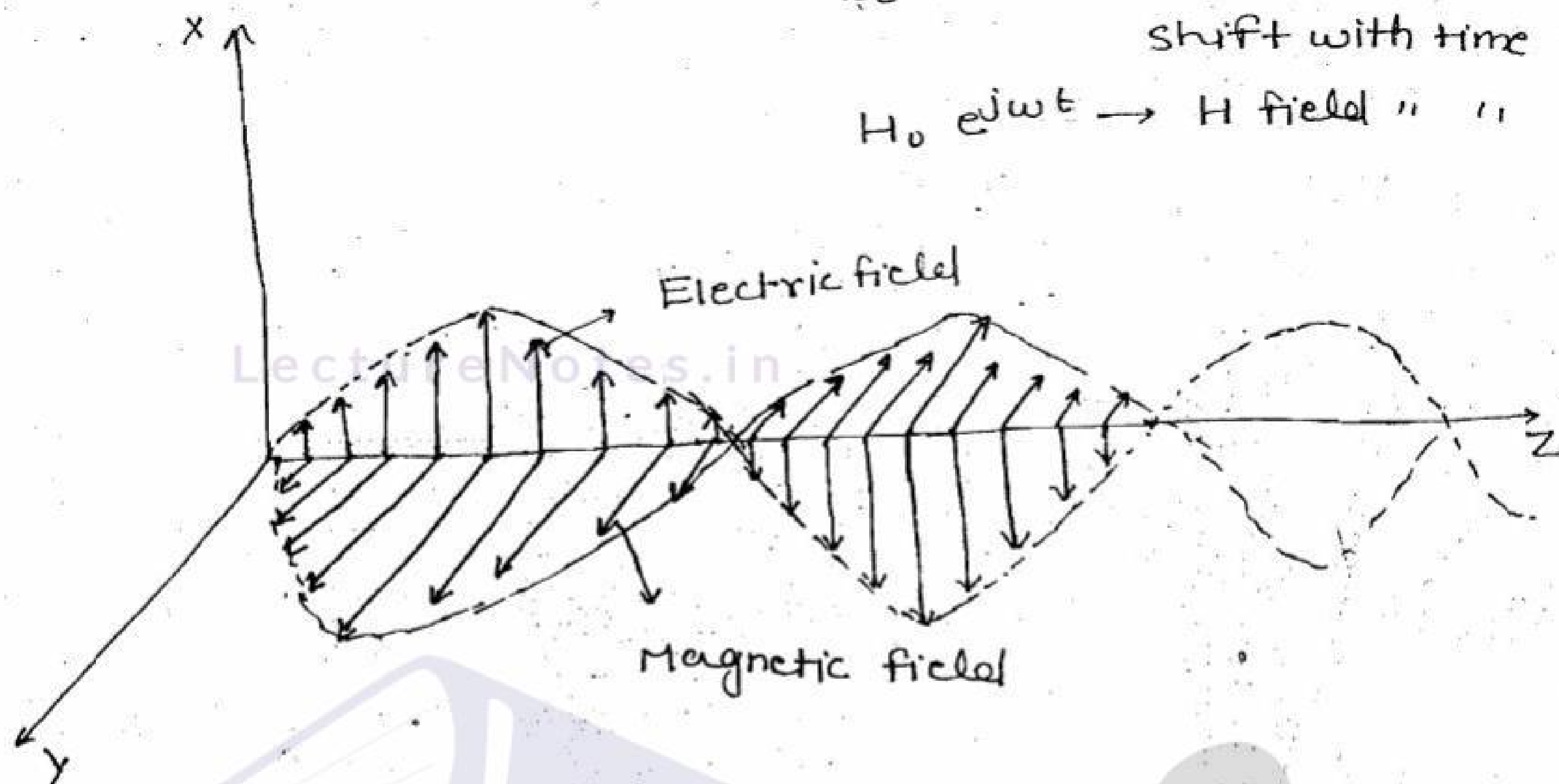
Considering only the $+z$ solution and c_1 & c_3 being the initial source values. The final E/H wave solution is

$$E(z, t)_x = (E_0 e^{j\omega t} e^{-\gamma z}) a_x$$

$$H(z, t)_y = (H_0 e^{j\omega t} e^{-\gamma z}) a_y$$

The solution is a product solution of time and space harmonics.

$\vec{E}_0 e^{j\omega t} \rightarrow$ electric field shift with time
 $H_0 e^{j\omega t} \rightarrow$ H field " "



- Electric and H field perpendicular and shifts simultaneously with time
- Cross product result in transverse and dot product result in longitudinal.

Propagation constant (γ) :-

- γ is the constant of the exponential with z that decides the course of propagation
- i.e. γ is called as propagation constant

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

$$E(z, t) = (\vec{E}_0 e^{-\alpha z} \cdot e^{j\omega t} e^{-j\beta z}) a_x$$

$$E(z, t)_x = (\vec{E}_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} a_x$$

$$H(z, t)_y = (H_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} a_y$$

Field direction

Amplitude Phase

Note-1:-

Every EM wave is harmonic whose amplitude exponentially decays at α rate

α = attenuation constant

Note-2:-

Every EM wave is a harmonic whose phase independently but linearly changes with time and space.

Note-3:-

The harmonic E & H have direction and obeying the basic transverse nature with propagation

$$\begin{array}{ccc} E & \times & H \\ (\text{V/m}) & & (\text{A/m}) \end{array} = \text{Propagation constant}$$

Intrinsic Wave Impedance (η) :-

$$E \Rightarrow H$$

$$\eta = \frac{E}{H} = \frac{\text{Volts/m}}{\text{Amp/m}} = \text{ohm}$$

When E is transformed to H & vice-versa the rate of transformation is called as intrinsic impedance of the medium.

$$\nabla \times E = -j\omega\mu H$$

$$\frac{\partial}{\partial z} \times E_{ax} = -j\omega\mu H_{ay}$$

$$\frac{\partial E}{\partial z} = -j\omega\mu H$$

$$\text{As } E = E_0 e^{-\gamma z}$$

$$\Rightarrow -\gamma E = -j\omega\mu H$$

$$\Rightarrow \frac{E}{H} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\Rightarrow \boxed{\frac{E}{H} = \sqrt{\frac{j\omega\mu}{1 + j\omega\epsilon}}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{1 + j\omega\epsilon}} = \text{complex} = R + jX$$

The transformation has a resistance which is a loss (α) in transformation and a reactance which leads to phase shift (β)

$$\gamma = \alpha + j\beta$$

$$\eta = \underset{\uparrow}{R} + j\underset{\uparrow}{X}$$

Workbook-2

$$1) \quad V = -\frac{d\psi_m}{dt} = -\frac{d}{dt} \left(\frac{1}{3} t^3 \right) = -\frac{1}{3} \cdot 3t^2 \Big|_{t=3s} = 9V$$

$$\Rightarrow \boxed{\lambda = -1}$$

$$(2) \quad V = -\frac{d\psi_m}{dt} = -\frac{d}{dt} (BA) = -\pi r^2 \frac{dB}{dt}$$

$$= -\pi \times (0.1)^2 \times 10 \left[-\sin(120\pi t) \cdot 120\pi \right]$$

$$= 118 \sin(120\pi t) \quad \text{Ans}$$

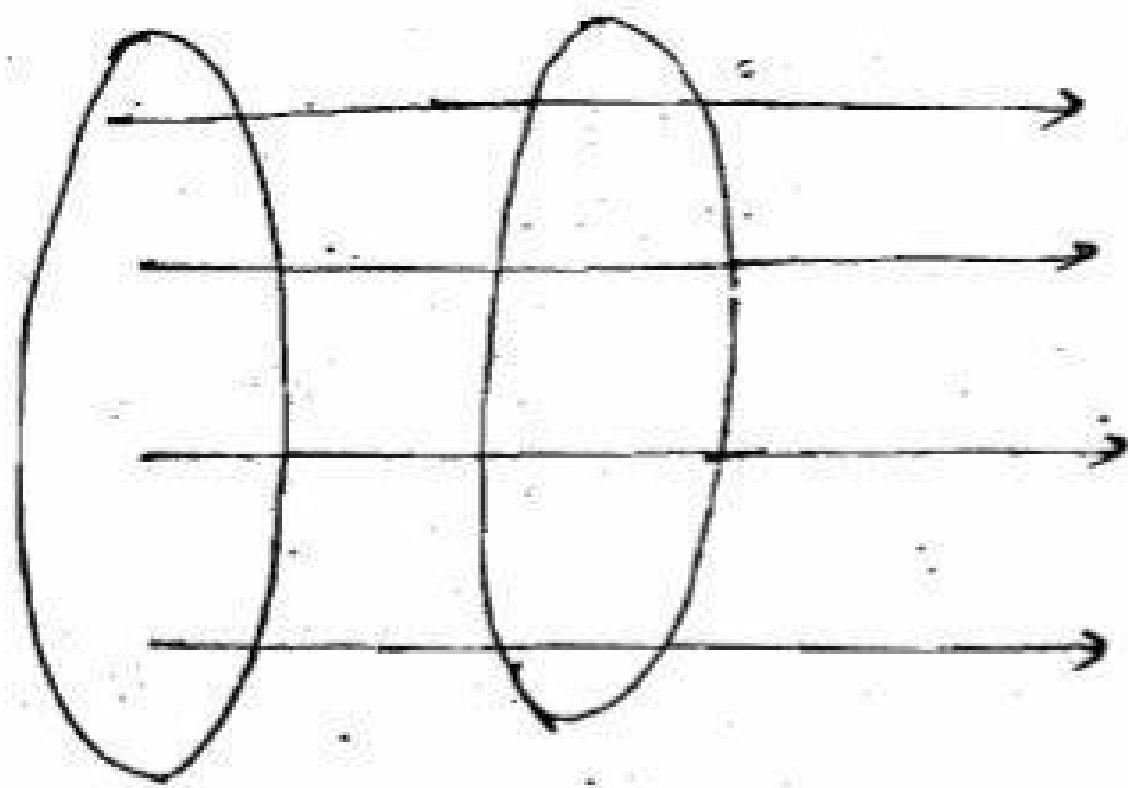
3) $B \rightarrow$ Linearly decrease with time $B(t) = -kt$

$\rightarrow$ Uniform decrease with space

$$\bar{V} = -\frac{d\psi_m}{dt} = -\frac{d}{dt} (BA) = -A(-k) = \text{constant} = 19c \text{ voltage}$$

$\rightarrow$ Upto open circuit E flows and emf is induced

$\rightarrow$ Induced emf converts into current in coil 1

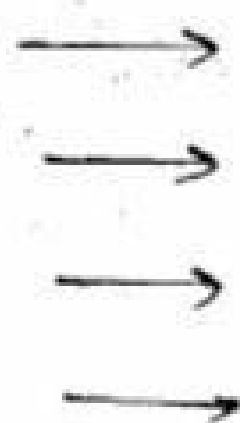


Ans - D

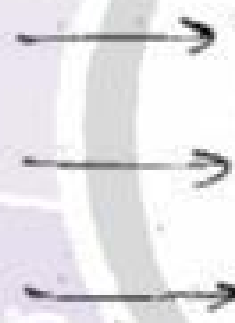
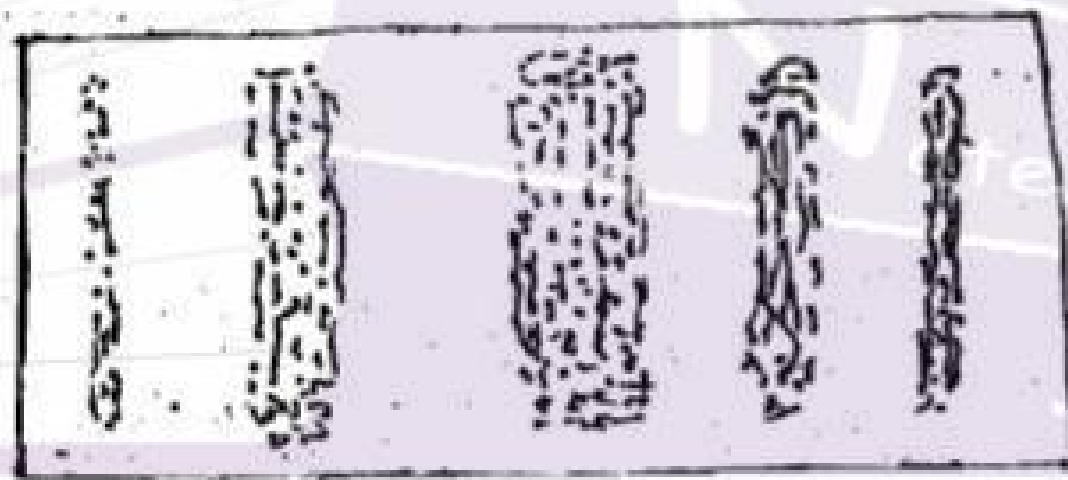
$$V=0 \quad V \neq 0$$

$$I \neq 0 \quad I=0$$

Note:-



DC current



AC current

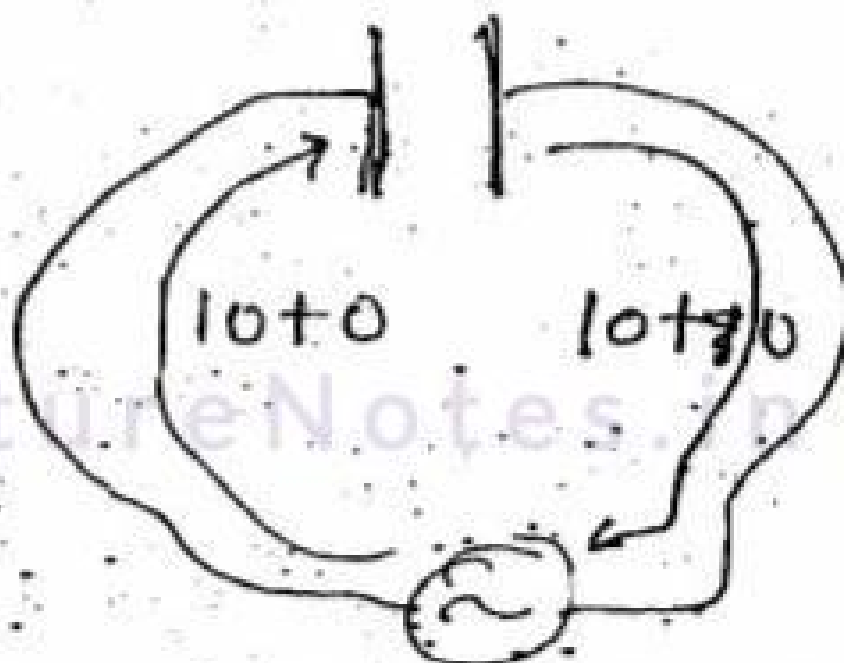
$$I_c = I_d$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_c + I_d \neq 2I_d$$

or
 $2I_c$

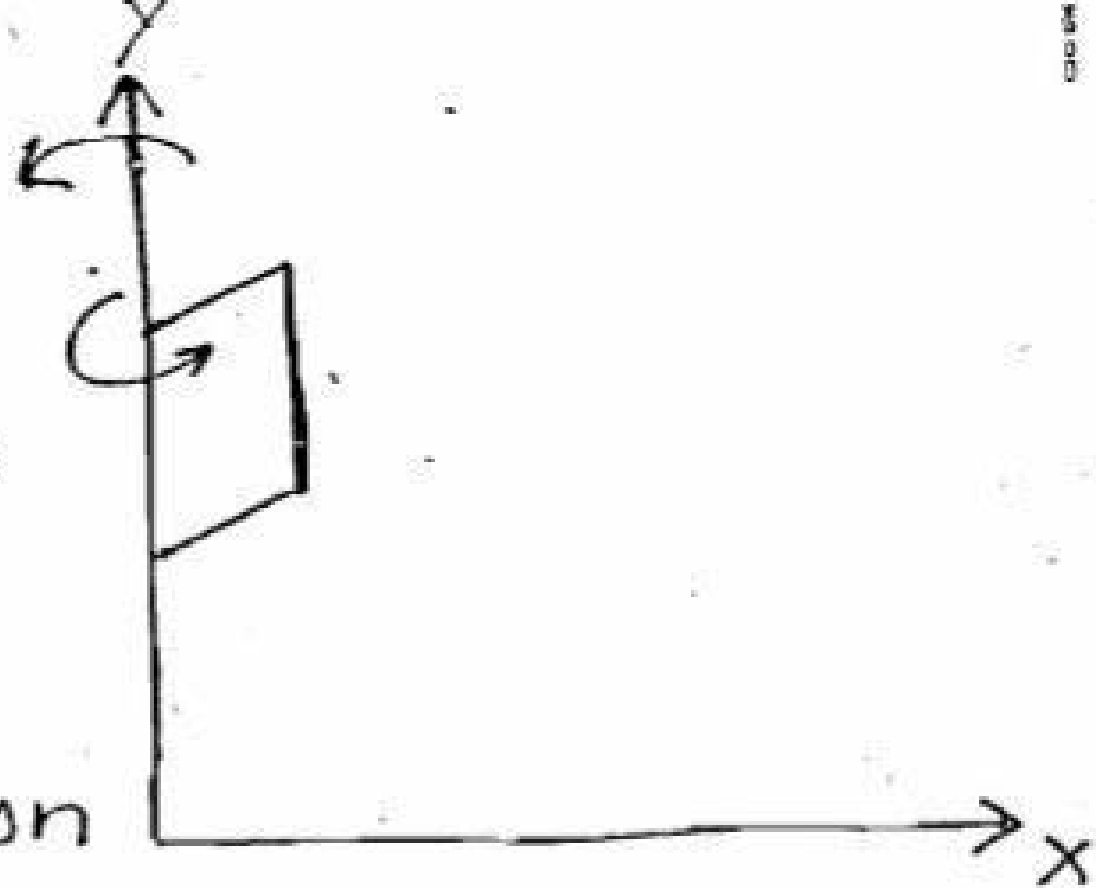
$$J_d = \epsilon \frac{\partial E}{\partial t}$$

$$I_d = \epsilon A \cdot j\omega E = j\omega \frac{\epsilon A}{d} V$$



Ans - (a)

$$\vec{B(t)} \times \vec{v}$$



Uniform and Harmonic
in time

Transformer Action

involved

AC Voltage and harmonic $B(t)$ being

$$V = - \frac{d\psi_m}{dt} = - \frac{d(BA)}{dt} = - \underbrace{B \frac{dA}{dt}}_{\text{Rotation}} - A \frac{dB}{dt}$$

$$B(t) = \cos(\omega t) \rightarrow \text{AC voltage}$$

Ans - c)

Radiation $\rightarrow$ EM waves $\rightarrow$ generate due to cross product

$$\gamma = \alpha + j\beta$$

$$\text{Amplitude} = |E|$$

$$|E| = E_0 e^{-\alpha z} = \frac{1}{e} E_0 \quad \text{when } z = 20\text{m}$$

$$\Rightarrow \alpha z = 1 \Rightarrow \alpha = \frac{1}{20}$$

$$\theta \propto z \Rightarrow \theta = \beta z$$

$$\Rightarrow \beta = \frac{\pi/6}{20\text{m}} = \frac{\pi}{120}$$

$$\boxed{\gamma = \frac{1}{20} + j \frac{\pi}{120}}$$

$|E| \rightarrow 20\%$ of initial value

$$|E| \rightarrow 20 \xrightarrow{E_0} 100 \rightarrow z = 5m$$

$$|E| \rightarrow 40 \xrightarrow{100} z = ?$$

It is not a linear function

$$|E| = E_0 e^{-\alpha z}$$

$$\Rightarrow 20 = 100 e^{-\alpha 5}$$

$$\Rightarrow e^{5\alpha} = 5$$

LectureNotes.in

$$\Rightarrow 5\alpha = \ln 5$$

$$\Rightarrow \alpha = \frac{\ln 5}{5}$$

$$40 = 100 e^{-\alpha z}$$

$$\Rightarrow e^{\alpha z} = \frac{100}{40} = 2.5$$

$$\Rightarrow \alpha z = \ln 2.5$$

$$\Rightarrow z = \frac{5 \ln 2.5}{\ln 5} = 2.8, \text{ Ans.}$$

Note:-

Change is depending on function value

Case-(1) :-

$$|E| = E_0 e^{-\alpha z}$$

EM Wave
Propagation
in free
space

$\frac{3}{2}$

$$\begin{array}{l} E_0 = 100 \\ \downarrow \\ z = \frac{1}{\alpha} \rightarrow \frac{100}{e} = 37 \\ \downarrow \\ z = \frac{1}{\alpha} \rightarrow \frac{100}{e^2} = 13 \\ \downarrow \\ z = \frac{1}{\alpha} \rightarrow \frac{100}{e^3} = 4 \end{array}$$

Case - (i) :-

EM Wave Propagation in free space $\left[\sigma = 0, \epsilon = \epsilon_0 \right]$
 $\mu = \mu_0$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0} = \text{purely imaginary}$$

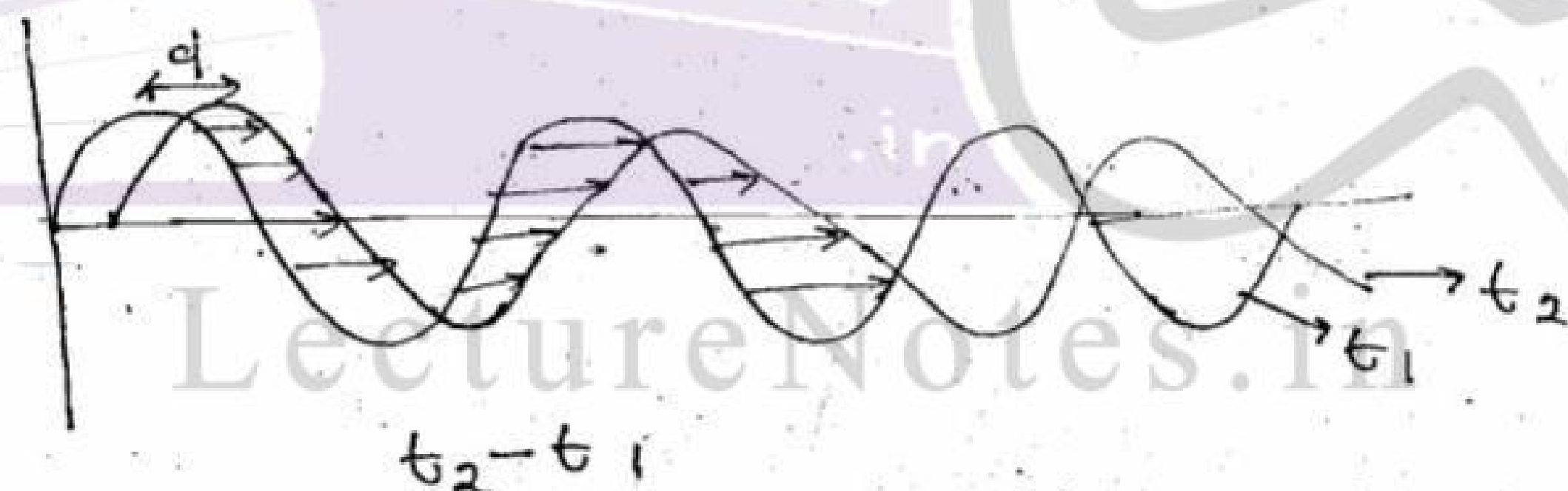
$\Rightarrow \alpha = 0$ No attenuation of EM wave in free space

$\beta = \omega\sqrt{\mu_0\epsilon_0}$ = only phase shift and propagation constant exists

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \text{real}$$

$$= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^9} = 120\pi = 377$$

Phase Velocity (V_p) :-



The distance travelled by any in-phase point per unit time is called as phase velocity.

$$V_p = \frac{d}{t}$$

$$= \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\Rightarrow V_p = 3 \times 10^8 \text{ m/s}$$

Case - (ii) :-

EM Wave Propagation in Ideal dielectrics

→ Ideal dielectrics = Perfect / lossless dielectrics

$$\rightarrow \sigma = 0, \epsilon = \epsilon_0 \epsilon_R, \mu = \mu_0$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0\epsilon_R}$$

= purely imaginary

$\Rightarrow \alpha = 0$ (i.e. No attenuation of EM wave in ideal dielectrics)

$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_R}$ = only phase shift and property exist

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_R}} = \text{real}$$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi \times 10^9}{\epsilon_R}} = \frac{120\pi}{\sqrt{\epsilon_R}} = \frac{377}{\sqrt{\epsilon_R}}$$

Phase Velocity (V_p) :-

$$V_p = \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_R}}$$

$$\Rightarrow \boxed{V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} \text{ m/s}}$$

$$E(x, t) = 25 \sin(\omega t + 4x) a_y$$

By comparison, propagation direction $\rightarrow -a_x$

$$\beta = 4 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{\pi}{2}$$

$$f = \frac{3 \times 10^8}{\frac{\pi}{2}} = \frac{600}{\pi} \text{ MHz}$$

Method-1:-

$$\nabla \times \vec{E} = -j\omega\mu H = -\mu \frac{\partial H}{\partial t}$$

$$H = \frac{-1}{\mu} \int (\nabla \times \vec{E}) dt$$

$$\nabla \times \vec{E} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

Method 2:-

$$\vec{E}(x,t) = 25 \sin(\omega t + 4x) a_y$$

Step-1:-

$$\frac{|\vec{E}|}{|\vec{H}|} = |\eta|$$

Step-2:-

when η is real $\vec{E}$ & $\vec{H}$ have the same harmonic and phase

Step-3:-

$$\frac{\vec{E} \times \vec{H}}{\text{div}} = \frac{P_{\text{avg}}}{\text{div}} \quad \left(\begin{array}{l} \text{Basic Transverse} \\ \text{Nature} \end{array} \right)$$

$$\vec{H}(x,t) = \left(\frac{25}{120\pi} \right) \sin(\omega t + 4x) (-a_x \times a_y)$$

$$= \left(\frac{-25}{120\pi} \right) \sin(\omega t + 4x) a_z$$

$$\vec{E}(y,t) = 25 \sin(10^8 t - y) a_z$$

const

free
space

lossless
dielectrics

$$\omega = 10^8$$

$$\beta = 1$$

$$V_p = \frac{\omega}{\beta} = 10^8 \neq 3 \times 10^8$$

$$V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} = 10^8$$

$$\Rightarrow \epsilon_R = 9$$

Propagation direction $\rightarrow a_y$

$$f = \frac{10^8}{2\pi}$$

$$\lambda = \frac{1}{\frac{1}{2\pi}} 2\pi$$

$$H(y, t) = \left(\frac{25}{\frac{120\pi}{\sqrt{4}}}} \right) \sin(10^8 t - y) (a_y \times a_z)$$

$$= \left(\frac{5}{8\pi} \right) \sin(10^8 t - y) a_x$$

1.

$$H = (0.5 e^{-0.1x}) \cos(10^6 t - 2x) a_z$$

$$\omega = 10^6 \text{ rad/second}$$

$$\beta = 2 = \frac{2\pi}{\lambda} \Rightarrow \lambda = 3.14$$

Propagation $\rightarrow +x$

$\rightarrow$ Polarization means E field direction.

Note:-

Polarization direction is always E field orientation and direction

2. Free Space

$$E = 50 \sin(10^7 t + \beta z) a_y$$

$$\rightarrow \beta = \frac{10^7}{3 \times 10^8} = \frac{1}{30} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 60\pi = 188\text{m}$$

$$\rightarrow k = \beta = \frac{1}{30} = 0.033$$

$\rightarrow$ Free space, no attenuation

$$H = 0.5 e^{-0.1z} \sin(10^6 t - 2x) a_z$$

$$v_p = \frac{10^6}{2} = 5 \times 10^5$$

$$V = 0.1 + j2$$

direction $\rightarrow +a_x$

$$\epsilon_r = 81$$

$$\omega = 6\pi \times 10^8, \beta = ?$$

$$V = \frac{3 \times 10^8}{\sqrt{81}} = \frac{6\pi \times 10^8}{\beta}$$

$$\Rightarrow \beta = 18\pi$$

$$\epsilon_r = 8, \mu_r = 2$$

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = 120\pi \sqrt{\frac{2}{8}} = 188 \Omega$$

$$H = 0.1 \cos(\omega t - \beta z) a_x$$

$$E = (0.1 \times 377) \cos(\omega t - \beta z) (a_x \times a_z)$$

$$= -37.7 \cos(\omega t - \beta z) a_y$$

Ans - B

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} = \frac{2\pi}{\lambda}$$

$$\lambda \propto \frac{1}{\sqrt{\epsilon}}$$

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{\epsilon_0 \epsilon_r}{\epsilon_0}} = \frac{2}{1} \Rightarrow \epsilon_r = 4$$

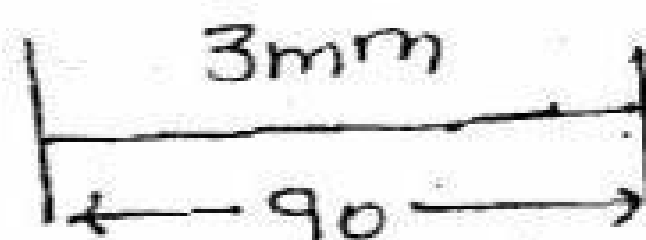
$$E = 20 e^{-j5z} a_x - 25 e^{-j5z} a_y$$

$$H = \left(\frac{20}{120\pi} \right) e^{-j5z} a_y - 25 e^{-j5z} (-a_x)$$

$$= \left(\frac{25}{120\pi} \right) e^{-j5z} a_x + \left(\frac{20}{120\pi} \right) e^{-j5z} a_y$$

19

$$\beta = \frac{\theta}{z} = \frac{2\pi}{\lambda}$$



$$= \frac{\pi/2}{3\text{mm}} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 12\text{mm}$$

$$\lambda f = 12 \times 10^{-3} \times 10 \times 10^9 = \frac{3 \times 10^8}{\sqrt{\epsilon_R}}$$

$$\Rightarrow \epsilon_R = 6.25$$

20

$$E = A \cos\left(\omega t - \frac{\omega}{c} z\right) a_y$$

$$\frac{\omega}{c} = \beta$$

$$H = \left[\frac{A}{\sqrt{\mu/\epsilon}} \right] \cos\left(\omega t - \frac{\omega}{c} z\right) (-a_x)$$

$$= -A \sqrt{\frac{\epsilon}{\mu}} \cos\left(\omega t - \frac{\omega}{c} z\right) a_x$$

Ans-19

21

$$\nabla^2 E + k^2 E = 0$$

$$e^{-\gamma z} + e^{\gamma z}$$

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\text{if } \gamma = j\beta$$

$$e^{-j\beta z} + e^{j\beta z}$$

$$(1 \pm z)$$

$$\nabla^2 E + \beta^2 E = 0$$

$$(\beta^2 + m^2 = 0)$$

Ans-C

22

$$E(x, z, t)_y = 25 \sin(\omega t - 3x + 4z) a_y$$

$$\text{Unit Prop. direction} = \frac{3a_x - 4a_z}{5}$$

$$\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5 = \frac{2\pi}{\lambda}$$

Note -

$$\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial z^2} = \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(z, t) = E_0 e^{j\omega t} \cdot e^{-j\beta z}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(x, z, t) = E_0 e^{j\omega t} \cdot e^{-j\beta x \cdot x} \cdot e^{-j\beta z \cdot z}$$

It is a product solution of time, x & z harmonics

$$E(x, z, t) = 25 \sin(\omega t - 3x + 4z) a_y$$

$$H(x, z, t) = \left(\frac{25}{120\pi} \right) \sin(\omega t - 3x + 4z)$$

$$= \frac{(3a_x - 4a_z)}{5} \times a_y$$

$$= \left(\frac{25}{120\pi} \right) \sin(\omega t - 3x + 4z) \left(\frac{3a_z + 4a_x}{5} \right)$$

$$E(x, z, t)_y$$

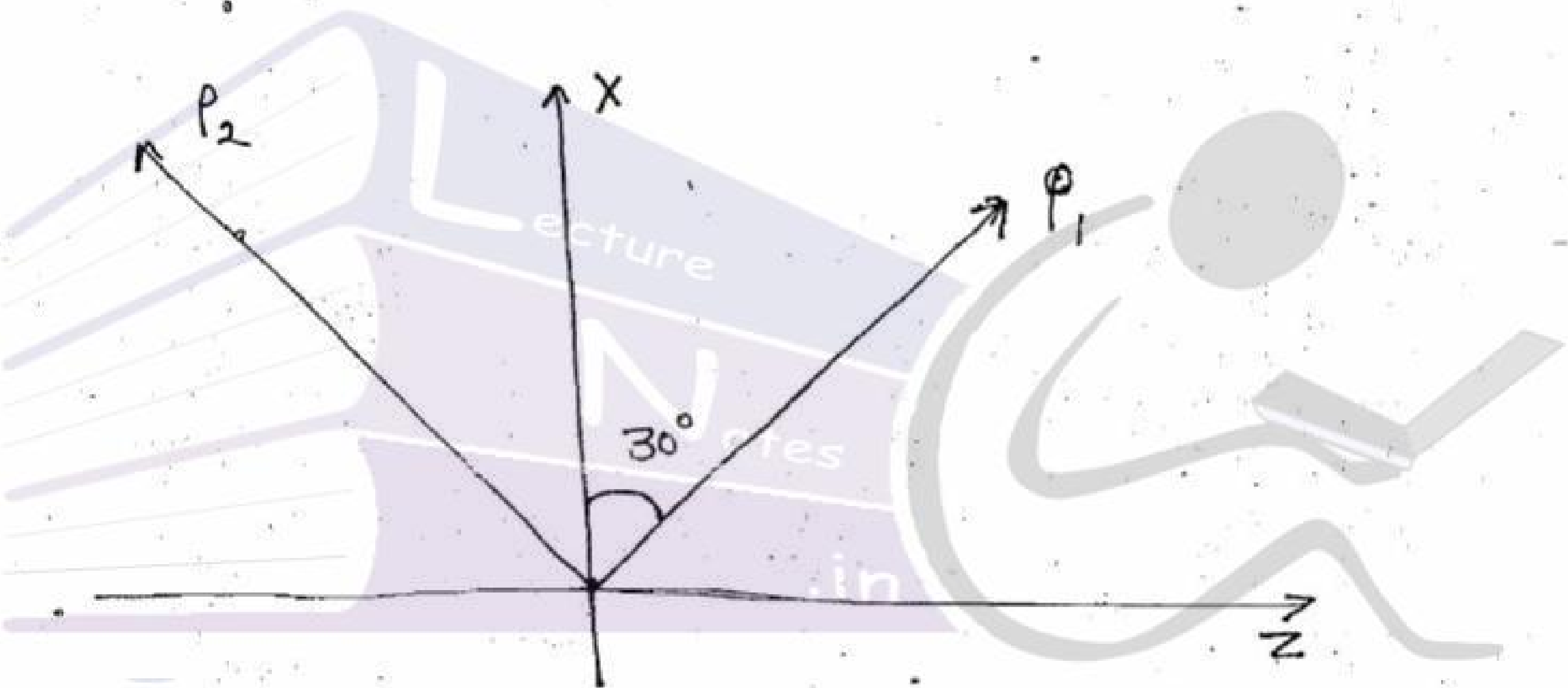
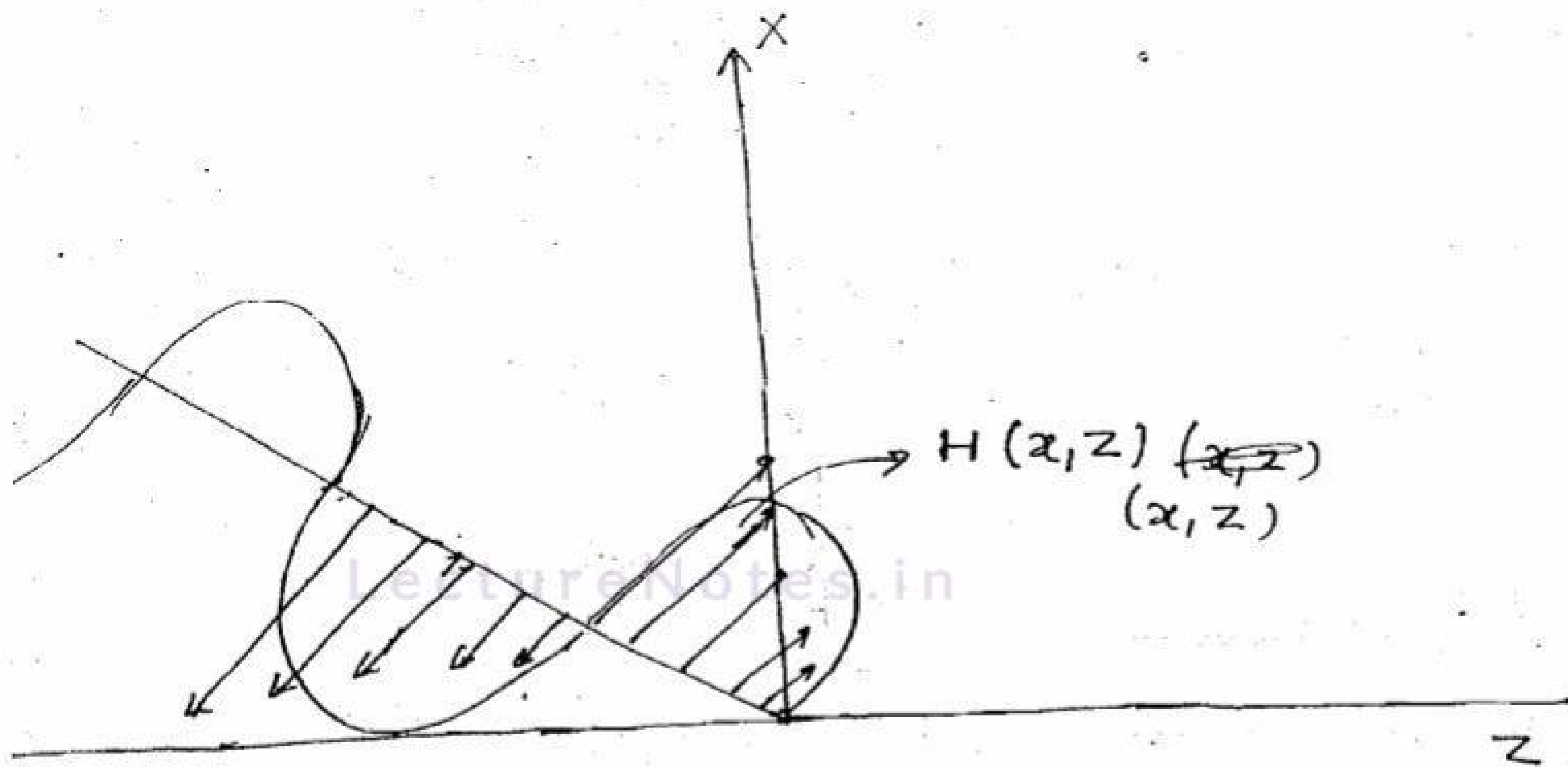
$$H(x, z, t)(x, z)$$

$$\text{Prop. direction} \rightarrow 3a_x - 4a_z$$

$$H \text{ direction} \rightarrow 4a_x + 3a_z$$

$$E \text{ direction} \rightarrow a_y \quad (\because \text{dot product} = 0)$$

$$\boxed{E \perp H \perp P} \rightarrow \text{Basic Transverse Nature}$$



90° to y -axis $\Rightarrow$ zx Plane

30° to the x -axis

$$E(x, z, t) = E_0 e^{j\omega t} e^{-j\beta_x x} e^{\pm j\beta_z z}$$

$$\tan 30 = \frac{\beta_z}{\beta_x} = \frac{1}{\sqrt{3}}$$

$$\beta_x = \sqrt{3} \beta_z$$

$$\beta = \frac{2\pi}{\lambda} = \sqrt{\beta_x^2 + \beta_z^2} = \sqrt{(\sqrt{3} \beta_z)^2 + \beta_z^2}$$

Ans - (a)

$$\Rightarrow \beta_z = \frac{\pi}{\lambda} \quad \beta_x = \frac{\sqrt{3}\pi}{\lambda}$$

$$\beta = 280\pi = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{1}{140}$$

$$f = 14 \times 10^9$$

$$\lambda f = 10^8 = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} \Rightarrow \epsilon_R = 9$$

LectureNotes.in

$$\frac{E_p}{3} = \frac{120\pi}{\sqrt{9}} \Rightarrow E_p = 120\pi, \text{ Ans}$$



LectureNotes.in

LectureNotes.in

Lecture - 10

Note:-

$R, L, G, C \rightarrow$ Primary constant $-\frac{V}{I}$ Waves
|

Transmission
line

$\mu, \sigma, \epsilon \rightarrow$ Material constant $-\frac{E}{H}$ Waves
|

Materials

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad Y = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Note:- (1)

$$\mu = \frac{|\gamma\eta|}{\omega}$$

$$\sigma = \text{Real} \left[\frac{\gamma}{\eta} \right]$$

$$\epsilon = \frac{\text{Imag} \left[\frac{\gamma}{\eta} \right]}{\omega}$$

Note 2:-

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \text{Complex} = \frac{E_x}{H_y} = |\eta| \angle \theta$$

64

$$= \left| \frac{E_x}{H_y} \right| \angle \theta$$

- In general material E_x and H_y are orthogonal in directions but have a phase shift or time delay which is η 's phase

Example :- $E \rightarrow \sin(\omega t - \beta z) a_x$
 $H \rightarrow \sin(\omega t - \beta z - \theta) a_y$

Note 3:-

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right)}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right)}$$

Loss Tangent or Dissipation Factor:-

- $\frac{\sigma}{\omega\epsilon}$ decides the propagation aspects α or β of the EM waves

If $\frac{\sigma}{\omega\epsilon} \gg 1 \Rightarrow$ very good conductor

α is very large $\Rightarrow$ EM wave propagation is very tough

- EM waves cannot easily propagate in good conductors

- The propagation aspects depends on $\frac{\sigma}{\omega\epsilon}$ and hence on frequency - ω also

eg:- $\frac{\sigma}{\omega\epsilon} \gg 1$ For human body upto light frequency

$\frac{\sigma}{\omega\epsilon} \ll 1$ For human body beyond X-ray frequency

eg:- $\frac{\sigma}{\omega\epsilon} \gg 1$ For earth upto MHz only

$\frac{\sigma}{\omega\epsilon} \ll 1$ for earth beyond microwaves

summary:-

The term $\frac{\sigma}{\omega\epsilon} = \left| \frac{J_c}{J_d} \right|$ = loss tangent and hence decides the behaviour of the material

$J_c \gg J_d \Rightarrow$ good conductor

$J_d \gg J_c \Rightarrow$ good dielectric

$$\frac{J_c}{J_d} = \frac{\sigma E}{\epsilon \frac{\partial E}{\partial t}} = \frac{\sigma E}{j\omega\epsilon E} = \frac{\sigma}{\omega\epsilon}$$

Case - (III) :-

EM Wave Propagation in very good conductors ($\sigma \gg \omega\epsilon$)

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \simeq \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{\omega\mu\sigma} \angle 90^\circ = \sqrt{\omega\mu\sigma} \angle 45^\circ$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} \simeq \sqrt{\frac{\omega\mu}{\sigma}} \angle 90^\circ$$

$$= \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$R = X = \sqrt{\frac{\omega\mu}{2\sigma}}$$

Note (1) :-

→ η 's real part (R) = Cause of attenuation (α)

η 's Imaginary part (X) = Cause of phase shift (β)

η 's phase = 45° means E_x & H_y are delayed in time by 45°

Skin Depth or Depth of penetration :-

$$|E| = E_0 e^{-\alpha z}$$

$$E_0 = 100$$

$$z = \frac{1}{\alpha}$$

$$\frac{1}{e}$$

$$37$$

$$z = \frac{2}{\alpha}$$

$$\frac{1}{e^2}$$

$$13$$

$$\frac{1}{e^3}$$

$$\frac{1}{e^3}$$

$$4$$

→ The first $\frac{1}{e}$ distance travelled by the wave is where majority of the wave amplitude is attenuated. This distance is called as skin depth. Hence skin depth is the distance travelled by the wave where the wave leakage to $\frac{1}{e}$ times (37%) of the initial value.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{2}{\omega \mu}} \cdot \frac{1}{\sigma} = \frac{1}{\sigma R_s}$$

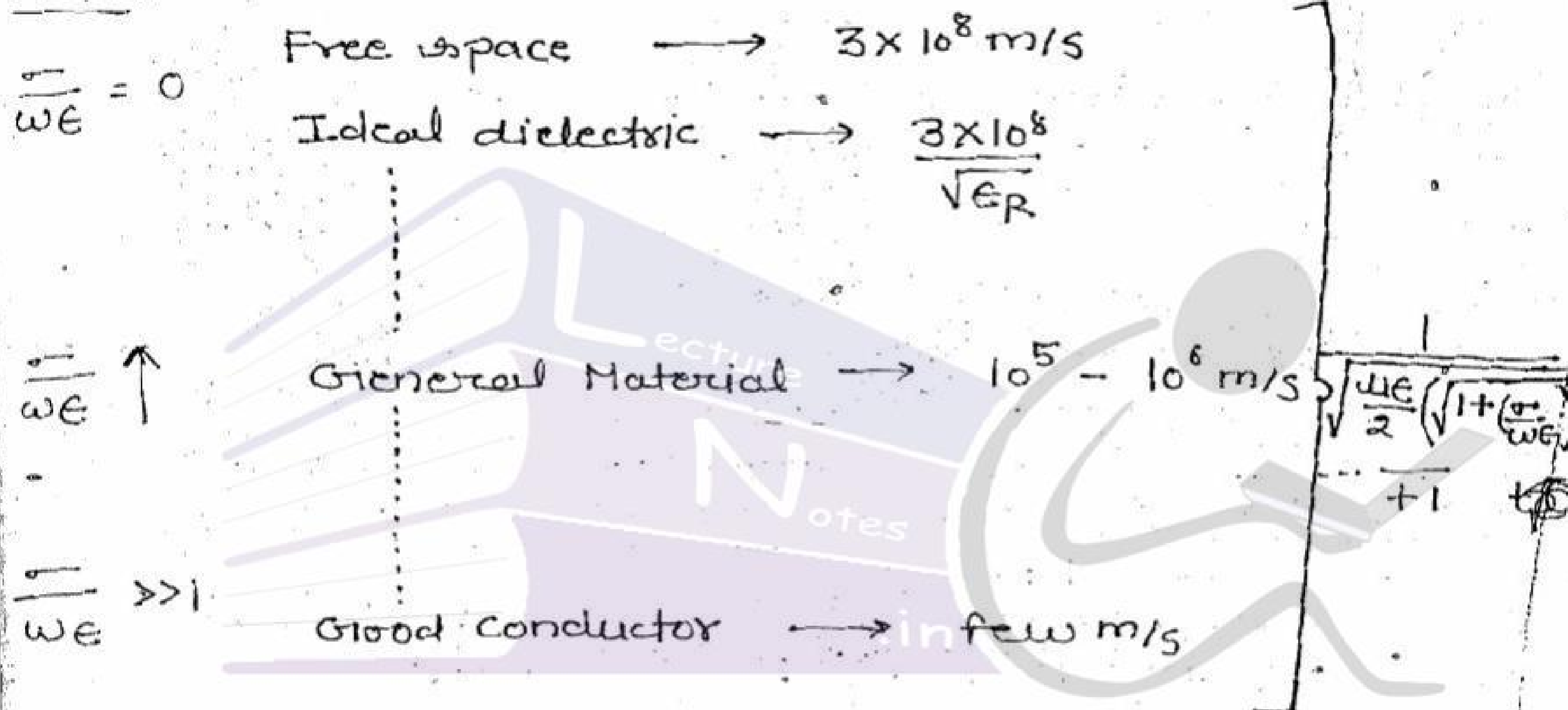
where $R_s = \eta$'s real part = skin resistance

15. $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega \delta} = \omega \delta = 2\pi \times 2 \times 10^5 \times 4 \times 10^{-6}$
 $= 1.6\pi \text{ m/s} \approx 5 \text{ m/s}$

16. $\omega' = 4\omega$ $\delta = \sqrt{\frac{2}{\omega \mu \sigma}} \Rightarrow \delta' = \frac{\delta}{2}$

$V_p' = \omega' \delta' = 4\omega \frac{\delta}{2} = 2V_p = 10 \text{ m/s}$

Note:-



$V_p = \frac{\omega}{\sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} + 1 \right)}}$

17. $\frac{\sigma}{\omega \epsilon} = 1.73$

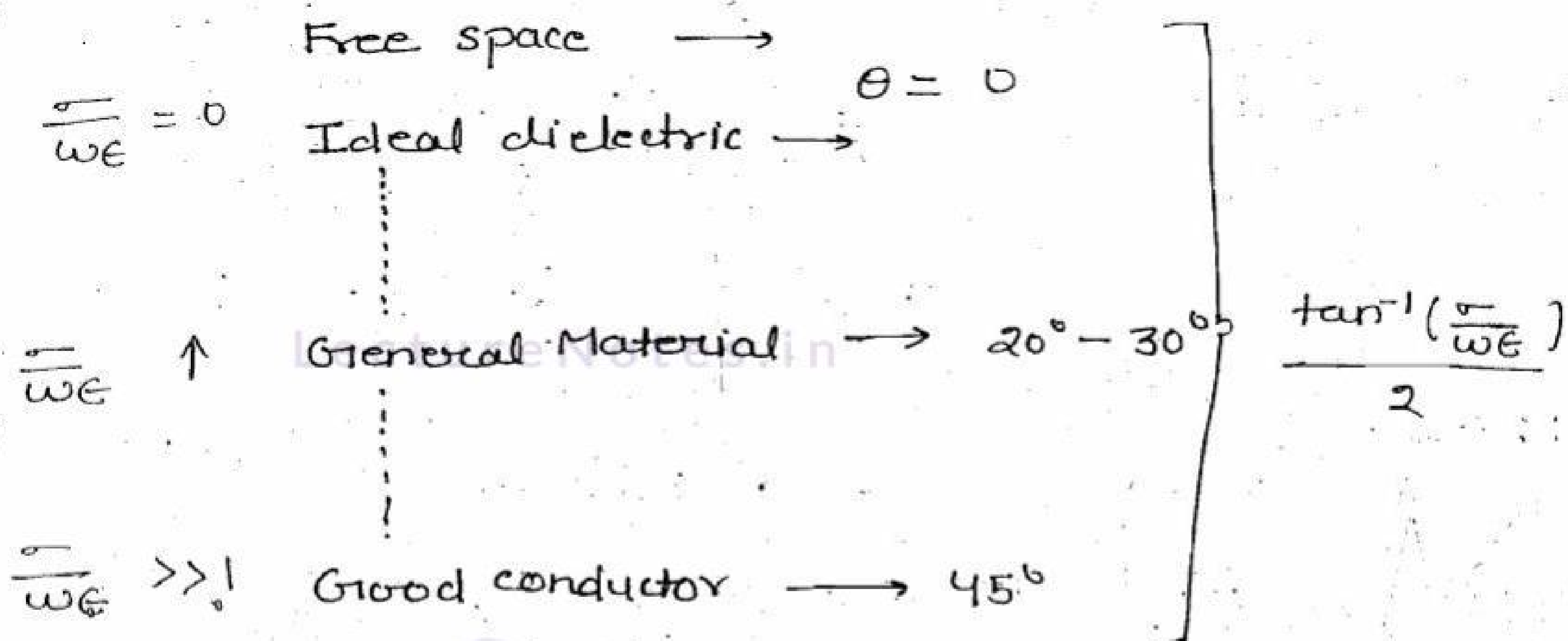
η 's phase $= \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}} = \sqrt{\frac{\omega \mu \angle 90}{K \left[\tan^{-1} \left(\frac{\omega \epsilon}{\sigma} \right) \right]}}$

$= \sqrt{K \left[90 - \tan^{-1} \left(\frac{\omega \epsilon}{\sigma} \right) \right]}$

η 's phase $= \frac{90 - \tan^{-1} \left(\frac{\omega \epsilon}{\sigma} \right)}{2} = \frac{\tan^{-1} \sqrt{3}}{2} = \frac{60}{2} = 30^\circ$

Note:-

η 's phase = E_x to H_y Phase



28.

$$J_c = J_d$$

$$\frac{\sigma}{\omega\epsilon} = 1$$

$$\Rightarrow \frac{\sigma}{2\pi f\epsilon} = 1$$

$$f = \frac{10^{-2}}{2\pi \times 4 \times 10^9} = 45 \text{ MHz}$$

29.

$$\frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\sigma}{\omega\epsilon}$$

$$\Rightarrow I_d = \frac{I_c \omega\epsilon}{\sigma} = \frac{1 \times 2\pi \times 50 \times 8.8 \times 10^{-12}}{58}$$

Ans - (B)

30.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$= R + jX$$

Ans - D

31.

$$\eta = 0.02 \angle 45^\circ$$

Ans - (a)

32.

lossy dielectric, $\frac{\sigma}{\omega\epsilon} \ll 1$

$$\begin{aligned} Y &= \sqrt{j\omega\mu \left(\frac{1}{j\omega\epsilon} + j\omega\epsilon \right)} = \sqrt{j\omega\mu - j\omega\epsilon \left(1 + \frac{\sigma}{j\omega\epsilon} \right)} \\ &= j\omega\sqrt{\mu\epsilon} \left(1 + \frac{\sigma}{j2\omega\epsilon} \right) = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon} \end{aligned}$$

Power, Power density and Poynting Vector :-

→ Energy that can be transformed into other formats is said to be a power format

$$\frac{\text{Joules}}{\text{second}} = \text{Watts}$$

Using Maxwell's IV Equation

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Take $\cdot E$ on both sides

$$(\nabla \times H) \cdot E = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$\Rightarrow -\nabla \cdot (E \times H) + H \cdot (\nabla \times E) = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$\Rightarrow -\nabla \cdot (E \times H) - \sigma E^2 = \epsilon \frac{\partial E}{\partial t} \cdot E + \mu \frac{\partial H}{\partial t} \cdot H$$

$$= \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right)$$

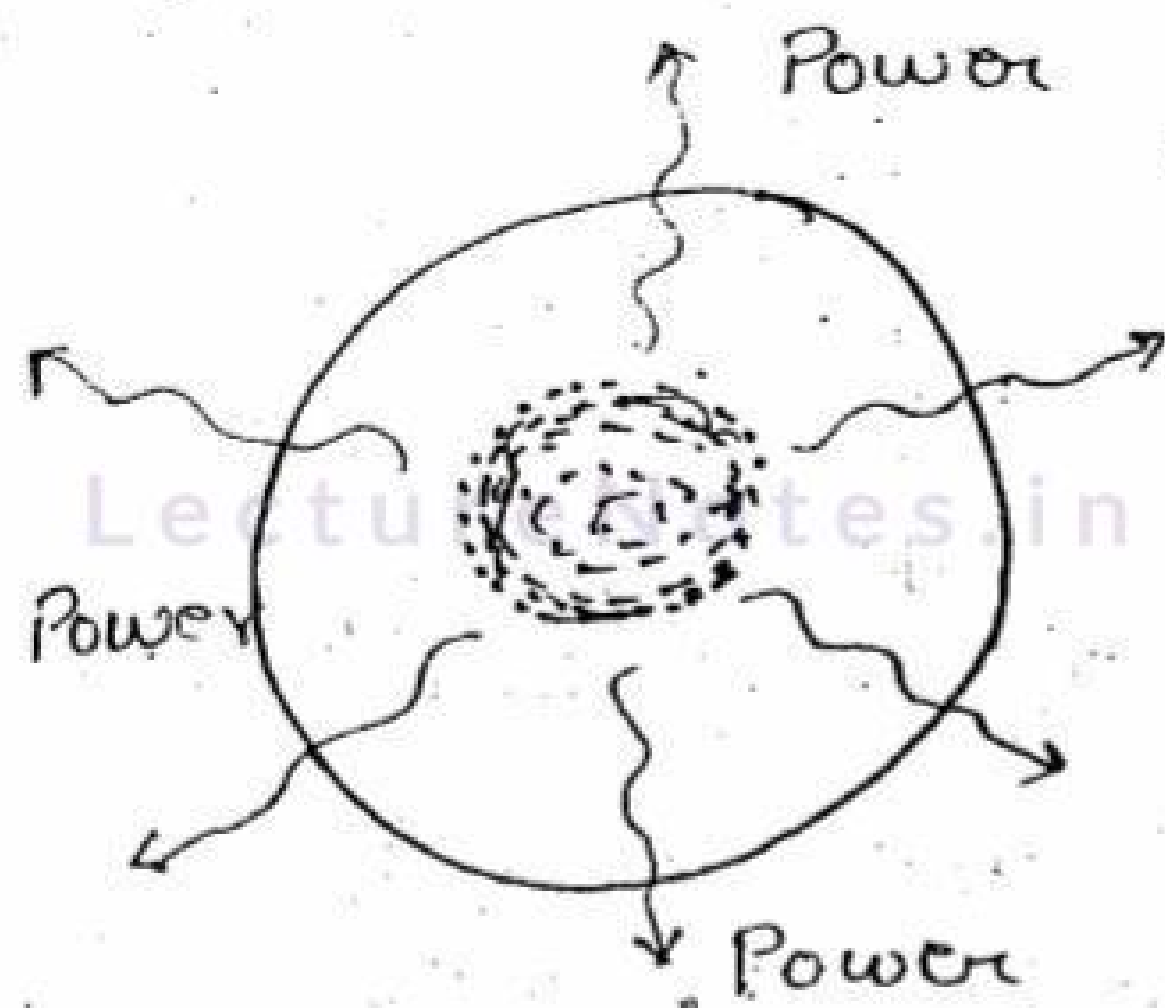
Take $\int dv$ on both sides

$$\int \nabla \cdot (E \times H) dv + \int \sigma E^2 dv = -\frac{\partial}{\partial t} \int \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dv$$

Using Divergence theorem

$$\oint (E \times H) \cdot dS + \int \sigma E^2 dv = \text{Rate of decrease of } E/H$$

Energy inside a volume



The LHS is a EM wave power crossing the surface

Hence $E \times H$ = Power density of EM Wave
 = Strength of power of EM Wave
 = Poynting Vector of the EM Wave
 = It has the direction of Propagation

$$E(z,t)_x \times H(z,t)_y = P(z,t)_z$$

= Power density at a time at a point in space

= Instantaneous Poynting Vector

$P(z)_{avg}$ = Time Average Poynting Vector

$$= E_{RMS} \cdot H_{RMS} = \frac{E_0 e^{-\alpha z}}{\sqrt{2}} \cdot \frac{H_0 e^{-\alpha z}}{\sqrt{2}}$$

$$P(z)_{avg} = \frac{1}{2} E_0 H_0 e^{-2\alpha z} = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z} = \frac{1}{2} \eta H_0^2 e^{-2\alpha z}$$

Note 1:-

The average power decays at 2α rate exponentially in the medium

→ This power lost in the wave is acquired by the medium as ohmic power

$$J \cdot E \text{ or } -E^2 \text{ Watts/m}^3$$

Note 2:-

Alternatively

$$P(z)_{avg} = \frac{1}{2} (E \times H^*)$$

* conjugate operation means average power is independent of harmonic and its phase and depends only on amplitude.

> In free space

$$P_{avg} = \frac{1}{2} E_0 H_0 = \frac{1}{2} \frac{E_0^2}{\eta} = \frac{1}{2} \eta H_0^2$$

Workbook:-

33. $P_{avg} = \frac{1}{2} \eta H_0^2 (-a_y)$

$$H_0 = 0.1$$

$$= \frac{1}{2} \cdot 120\pi \cdot (0.1)^2 (-a_y) \rightarrow \frac{B}{\mu_0}$$

34.

$$W = \int P_{avg} \cdot ds$$

$$= \int \frac{1}{2} \frac{E_0^2}{\eta} a_x \cdot ds \quad \begin{matrix} \nearrow \\ x = \text{constant} \end{matrix} a_x$$

$$= \frac{1}{2} \cdot \frac{60 \cdot 60}{120\pi} \pi \times 4^2 = 240W$$

35

C

36 (1) $H = 50 \sin(\omega t - \beta z) a_x + 150 \sin(\omega t - \beta z) a_y$

$$P_{avg} = \frac{1}{2} \eta H_0^2 (a_z)$$

$$H = 50 \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}} \right) \sqrt{10}$$

$$= (50\sqrt{10}) \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}} \right)$$

$$P_{avg} = \frac{1}{2} \cdot 120\pi \cdot (50\sqrt{10})^2 a_z$$

$$H_0 = 50\sqrt{10}$$

$$\begin{aligned} \text{(ii)} \quad H &= 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y \\ &= 50 \sin(\omega t - \beta z) a_x (1 + 3j) \\ &= 50 (1 + 3j) \sin(\omega t - \beta z) a_x \end{aligned}$$

Note:-

Orthogonal field components in time ^{or} ~~and~~ space have same effect

$$\text{(ii)} \quad H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$$

$$H = 50 \sin(\omega t - \beta z) (a_x + 3j a_y)$$

$$P_{avg} = \frac{1}{2} (E \times H^*)$$

$$E = (50\eta) \sin(\omega t - \beta z) (-a_y) + (150\eta) \cos(\omega t - \beta z) (a_x)$$

$$= (50\eta) \sin(\omega t - \beta z) (3j a_x - a_y)$$

$$P_{avg} = \frac{1}{2} \cdot 50 \cdot 120\pi (3j a_x - a_y) \times 50 (a_x - 3j a_y)$$

$$= \frac{1}{2} \cdot 120\pi \cdot 50^2 (a_z + 9a_z)$$

$$= \frac{1}{2} \cdot 120\pi (50\sqrt{10})^2 a_z$$

$$\underline{\underline{37.}} \quad \underline{\underline{E}} = (a_x + ja_y) e^{jkz - j\omega t}$$

$$H = \frac{k}{\omega \mu} (a_y + ja_x) e^{jkz - j\omega t}$$

$$P_{avg} = \frac{1}{2} (a_x + ja_y) \times \frac{k}{\omega \mu} (a_y - ja_x)$$

$$= \frac{k}{2\omega \mu} (a_z - a_z) = 0 \rightarrow \text{Null vector}$$

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Conventional :-

$$\underline{\underline{1.}} \quad E = 100 (ja_x + 2a_y - ja_z) e^{j\omega t}$$

$$H = (-a_x + ja_y + a_z) e^{j\omega t}$$

$$P_{avg} = \frac{1}{2} (E \times H^*) = \frac{1}{2} \cdot 100 (ja_x + 2a_y - ja_z) \times (-a_x - ja_y + a_z)$$

$$= 50 \begin{vmatrix} a_x & a_y & a_z \\ j & 2 & -j \\ -1 & -j & 1 \end{vmatrix}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ j & 2 & -j \\ -1 & -j & 1 \end{vmatrix}$$

$$= 50 (3a_x + 3a_z)$$

$$|P_{avg}| = 150\sqrt{2}$$

$$P_{avg} \text{ direction} = (a_x + a_z)$$

LectureNotes.in

$$\text{Unit } P_{avg} \text{ direction} = \frac{a_x + a_z}{\sqrt{2}}$$

Wave Polarization:-

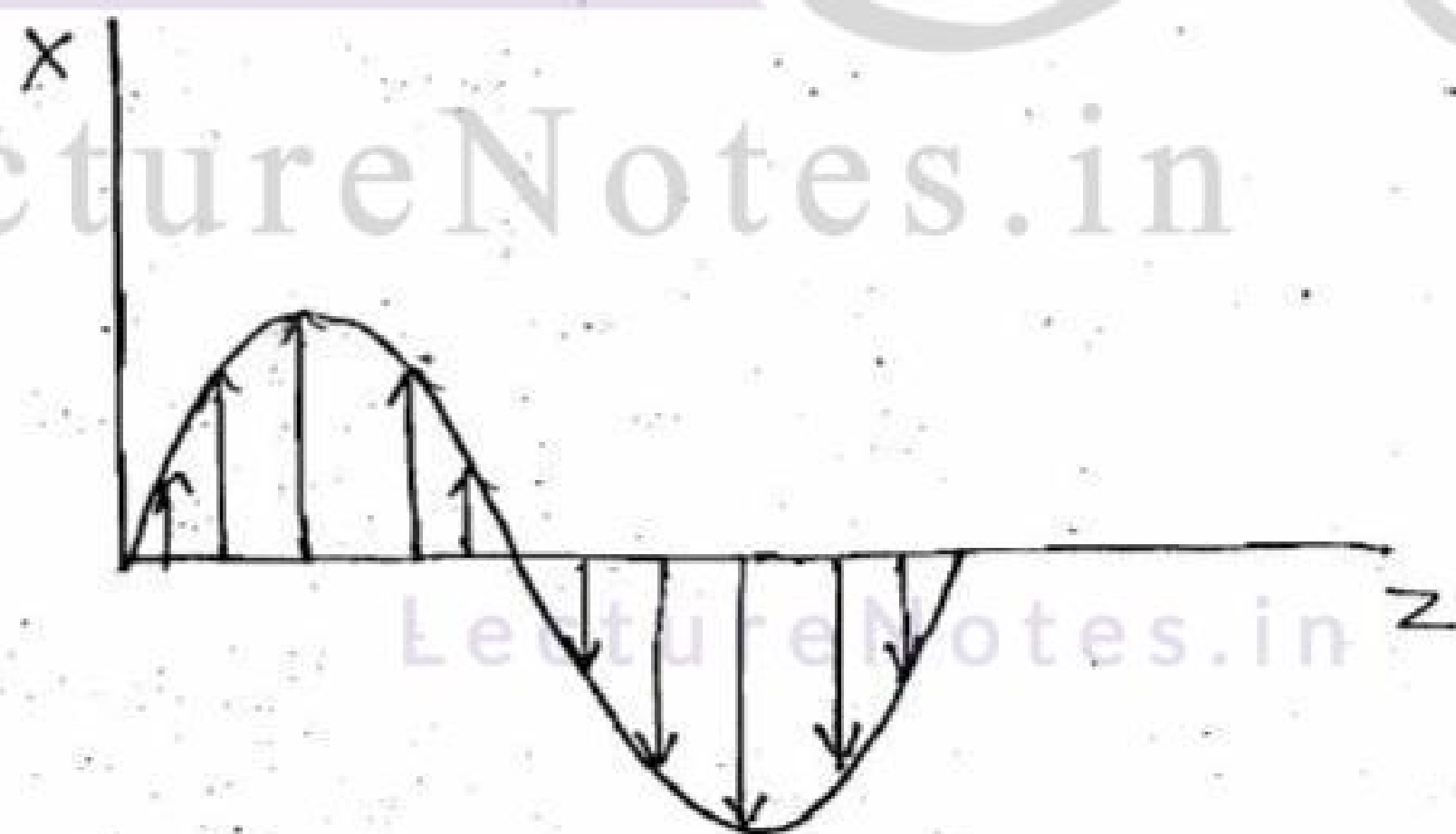
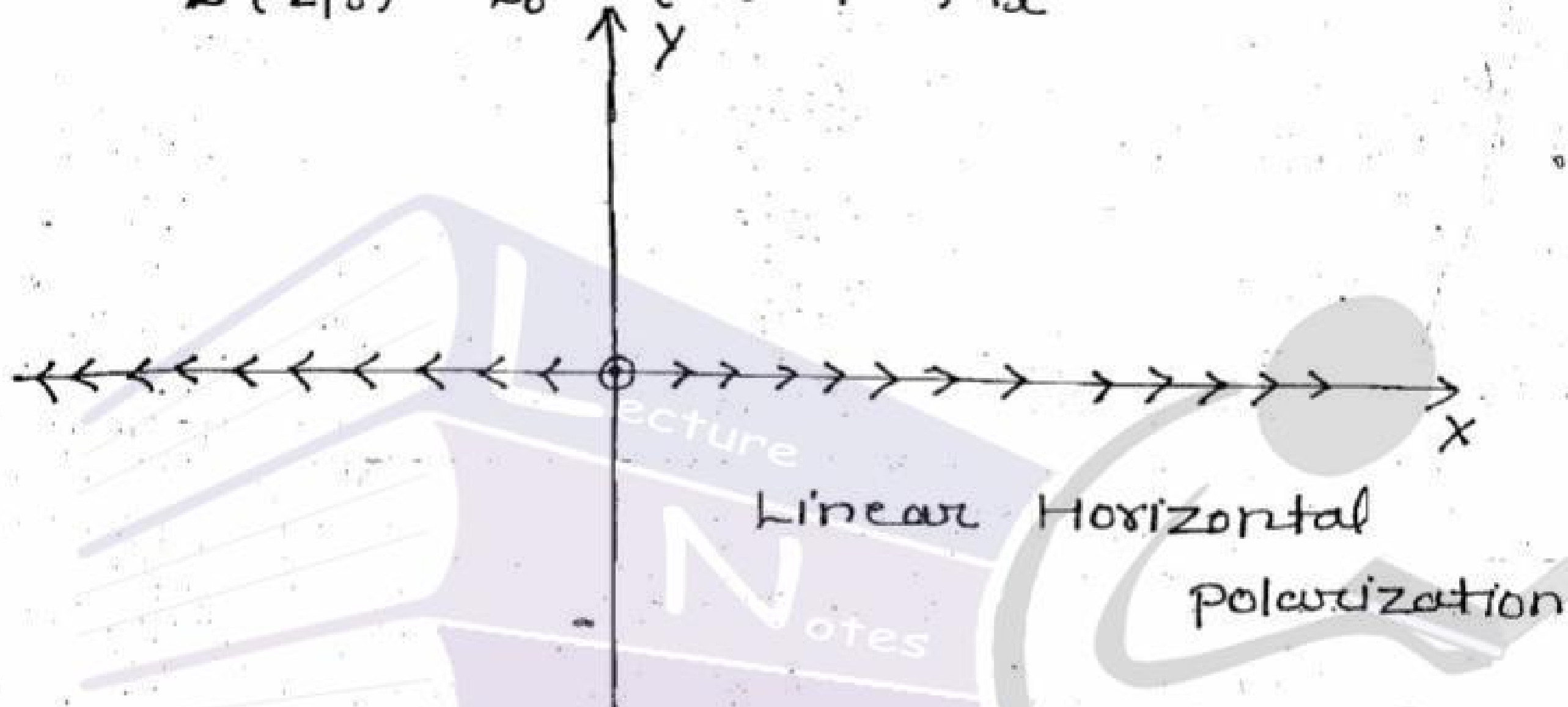
It is the electric field orientation & the possible planar components of the E field satisfying the basic transverse nature

EM Wave is propagating along z-axis in the following cases:-

Case (I):-

E field is oriented in x-direction

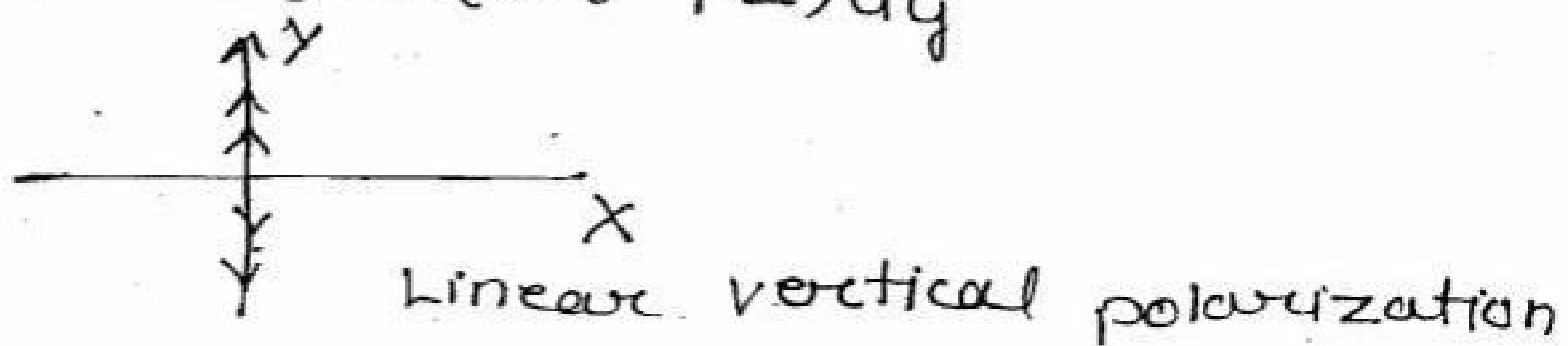
$$E(z,t) = E_0 \sin(\omega t - \beta z) a_x$$



Case - (II):-

E field is oriented in y-direction

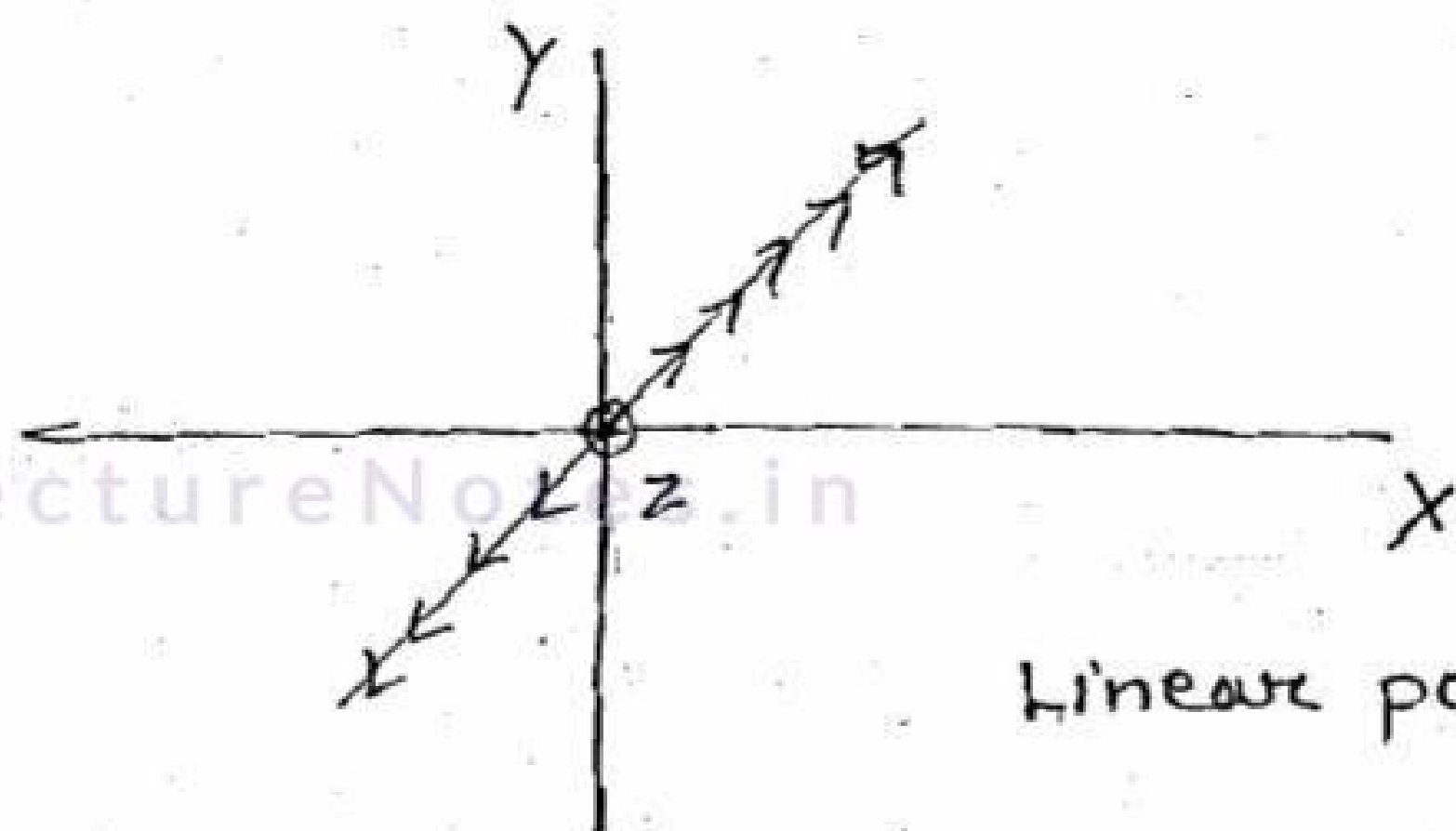
$$E(z,t) = E_0 \sin(\omega t - \beta z) a_y$$



Case - (III) :-

$\vec{E}$ field is oriented in x & y directions

$$\vec{E}(z, t) = E_1 \sin(\omega t - \beta z) \hat{a}_x + E_2 \sin(\omega t - \beta z) \hat{a}_y$$



Note! -

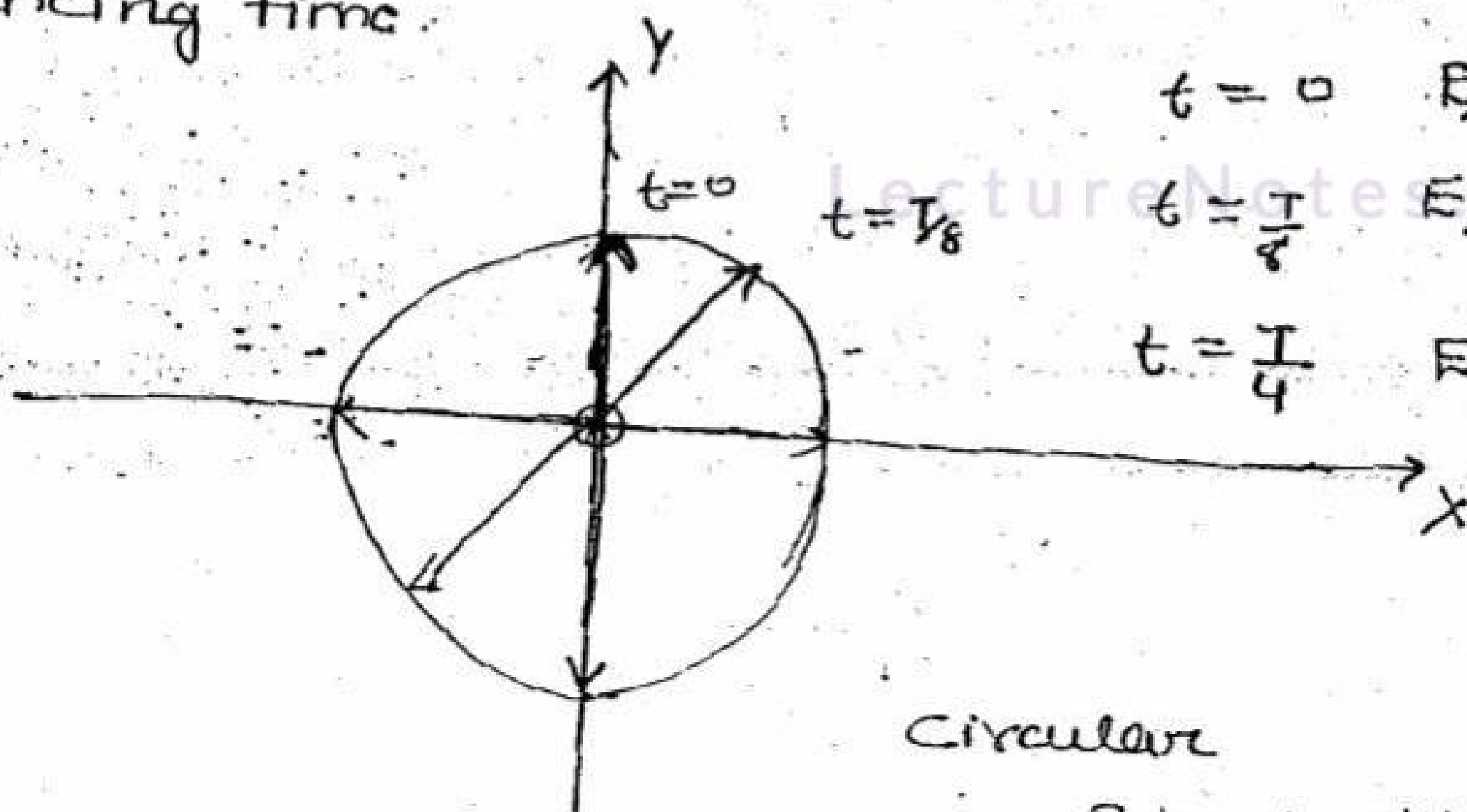
If the EM waves has single $\vec{E}$ field components or two planar components both inphase the wave is said to be linearly polarized

Case - (IV) :-

$\vec{E}$ field is oriented in x & y directions

$$\vec{E}(z, t) = E_0 \sin(\omega t - \beta z) \hat{a}_x + E_0 \cos(\omega t - \beta z) \hat{a}_y$$

→ Polarization is identified by studying the track of the $\vec{E}$ field on the $z=0$ plane for various advancing time.



$$t=0 \quad E_x=0, E_y=E_0$$

$$t=\frac{T}{8} \quad E_x=\frac{E_0}{\sqrt{2}}, E_y=\frac{E_0}{\sqrt{2}}$$

$$t=\frac{T}{4} \quad E_x=E_0, E_y=0$$

Circular Polarization.

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Summary:-

If the EM Waves has two planar components of E field both out of phase at 90° and equal amplitude, the wave is circularly polarised.

Note:-

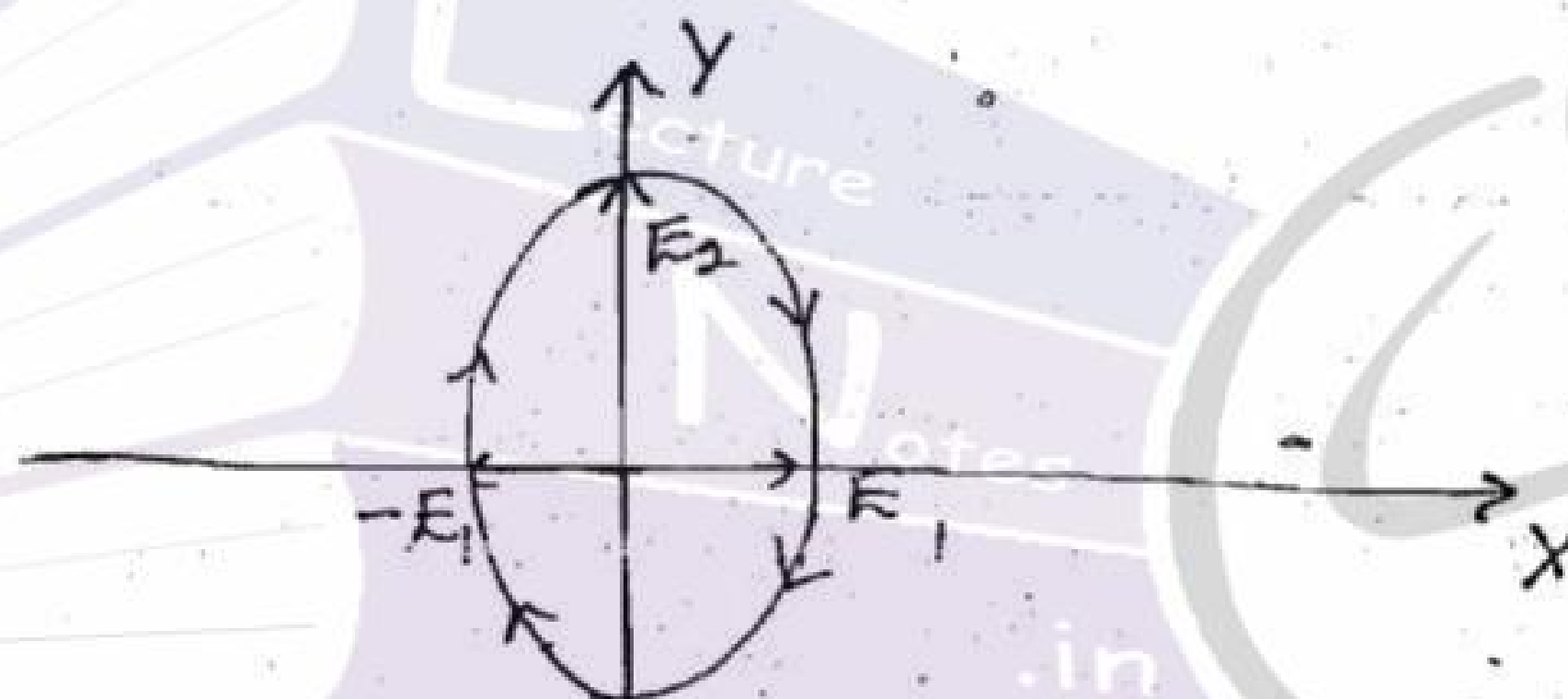
Sense of rotation left or right, if the left hand thumb points towards propagation direction and the closed finger along advancing time, the wave is left circularly polarised

eg:- out of paper - propagation clockwise time advancement - left circular

Case-(V):-

E field is oriented in x & y directions

$$E(z,t) = E_1 \sin(\omega t - \beta z) a_x + E_2 \cos(\omega t - \beta z) a_y$$

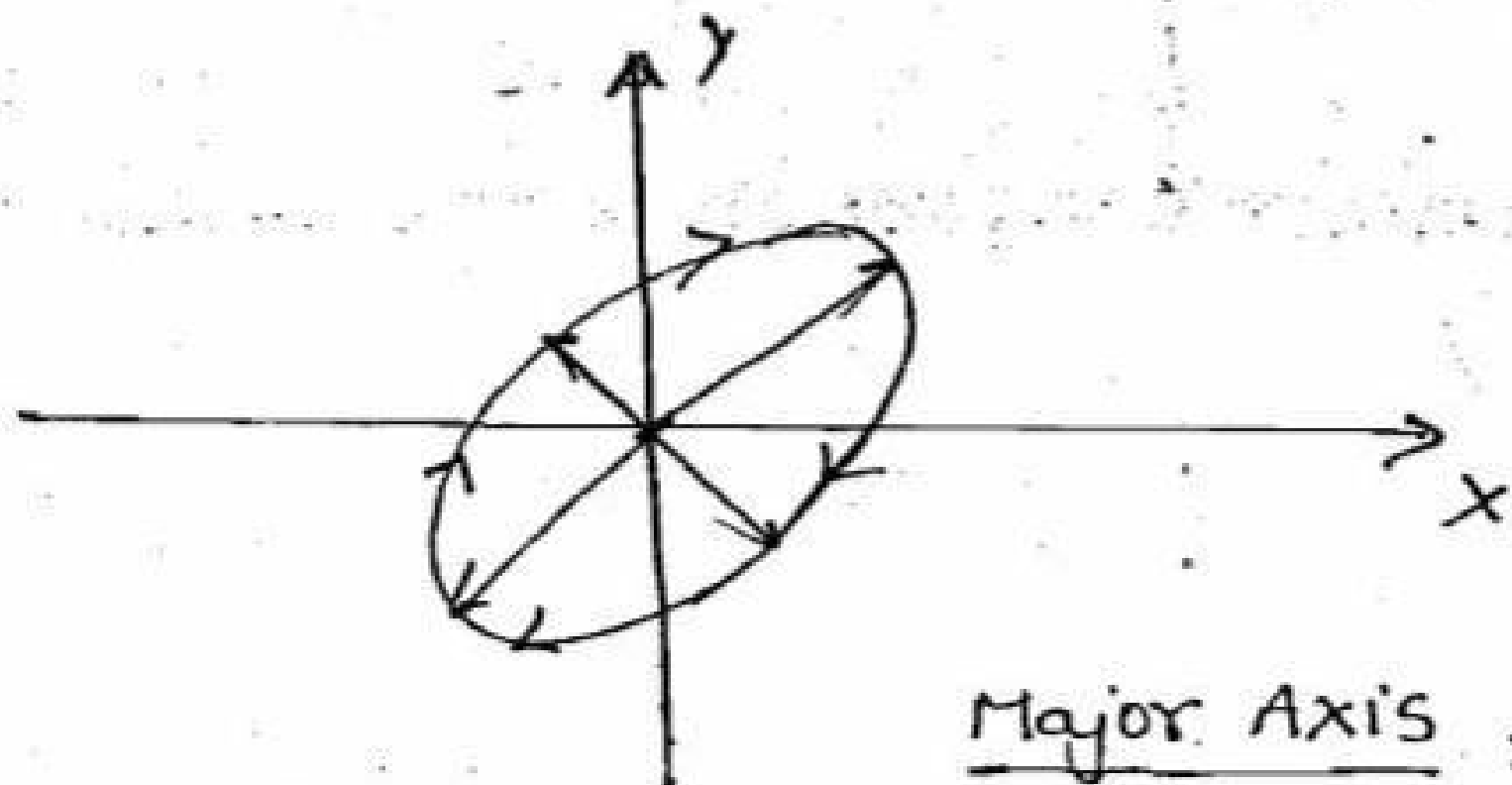


→ Elliptical polarization

Case-(VI):-

E field is oriented in x & y direction

$$E(z,t) = E_1 \sin(\omega t - \beta z) a_x + E_2 \sin(\omega t - \beta z + \theta) a_y$$



Major Axis = AR = $[1, \infty)$

Minor Axis

↙ Circle ↘
Linear

Workbook :-

i) $\vec{E} = 25 \sin(\omega t + 4x) (a_y + 6a_z)$

→ Linear

ii) $\vec{E} = 25 \sin(\omega t + 4x) a_y + 25 \cos(\omega t + 4x) a_z$

→ Circular

iii) $\vec{E} = 25 \sin(\omega t + 4x + 60^\circ) a_y + 25 \cos(\omega t + 4x + 60^\circ) a_z$

→ Circular

iv) $\vec{E} = 25 \sin(\omega t + 4x) a_y + (25\sqrt{2}) \sin(\omega t + 4x + 45^\circ) a_z$

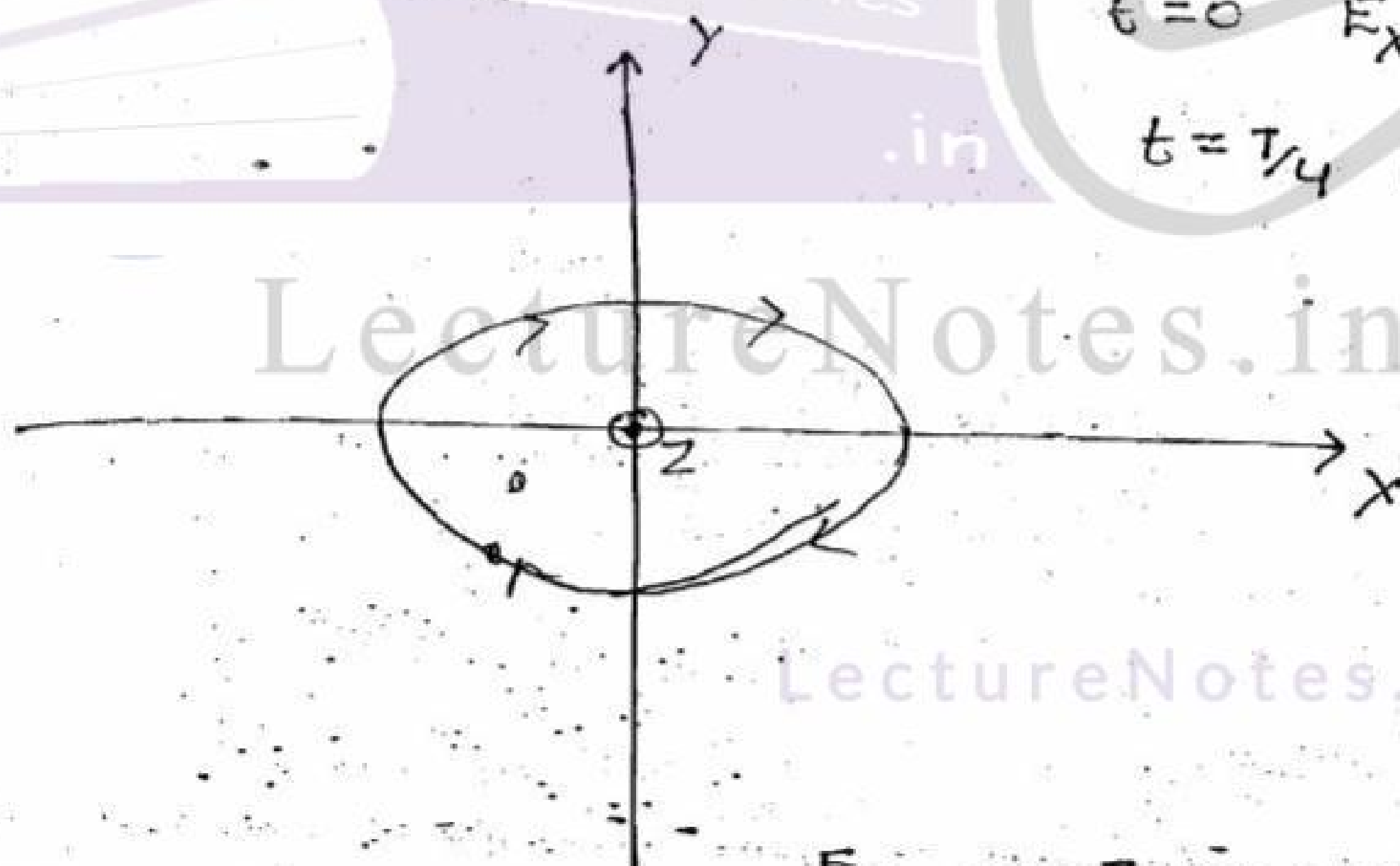
→ Elliptical

v) Elliptical

vi) Circular

vii) $\theta = 60^\circ \rightarrow$ elliptical

viii) $\theta = 90^\circ$



$t=0 \quad E_x = E_1 \neq E_y = 0$

$t=T/4 \quad E_x = 0 \quad E_y = E_2$

Summary:-

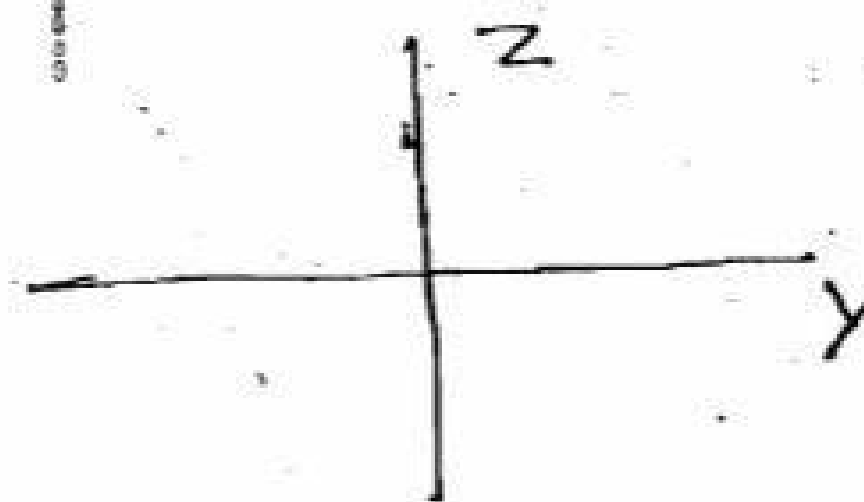
E_x	E_y	Prop.	Polarization
$\cos$	$-\sin$	z	Left
$\cos$	$\sin$	z	Right
$\cos$	$-\sin$	$-z$	Right
$-\sin$	$\cos$	z	Right

$\left. \begin{array}{l} E_x \rightarrow E_1 \cos \omega t \\ E_y \rightarrow E_2 \sin \omega t \end{array} \right\} e^{jKz}$

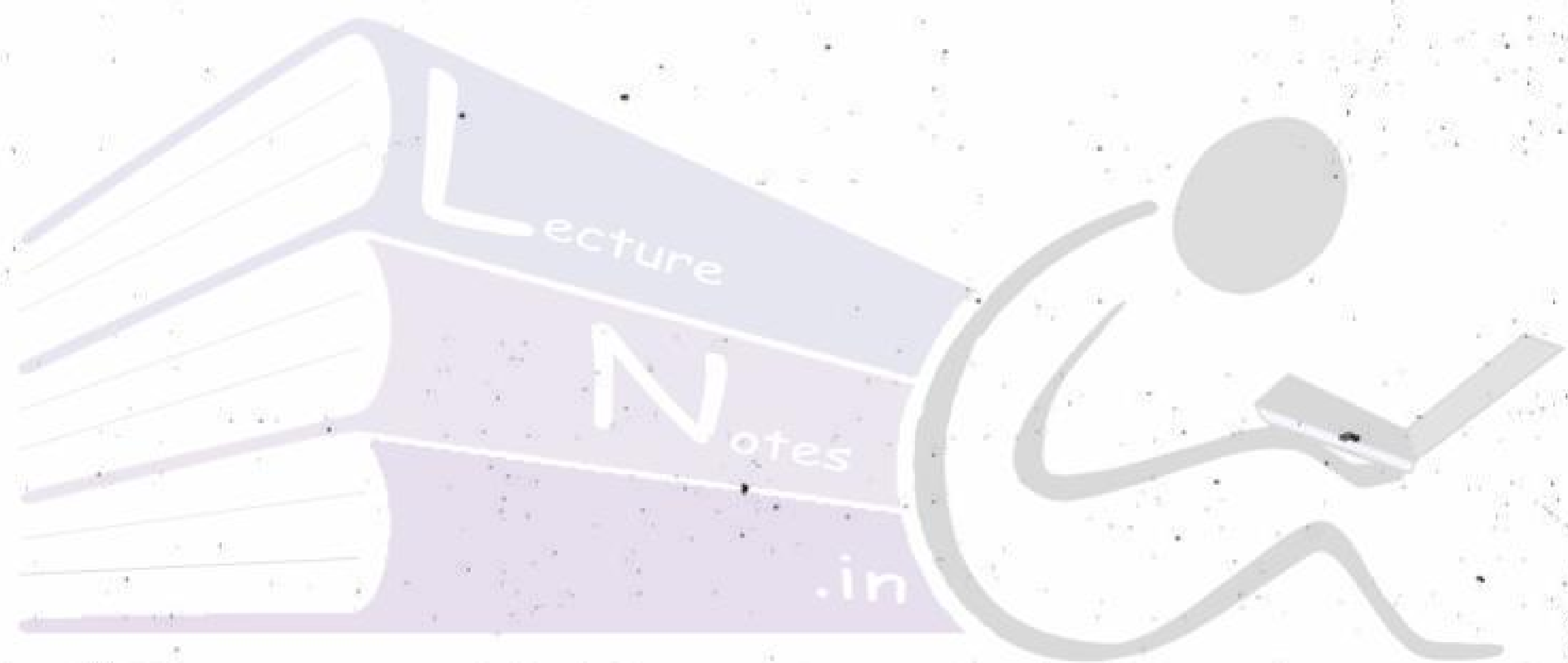
$$(dy + jdz) e^{j\theta}$$

$\downarrow$
 $\sin$
 $\downarrow$
 $\cos$

$\downarrow$
 $\cos$
 $\downarrow$
 $-\sin$



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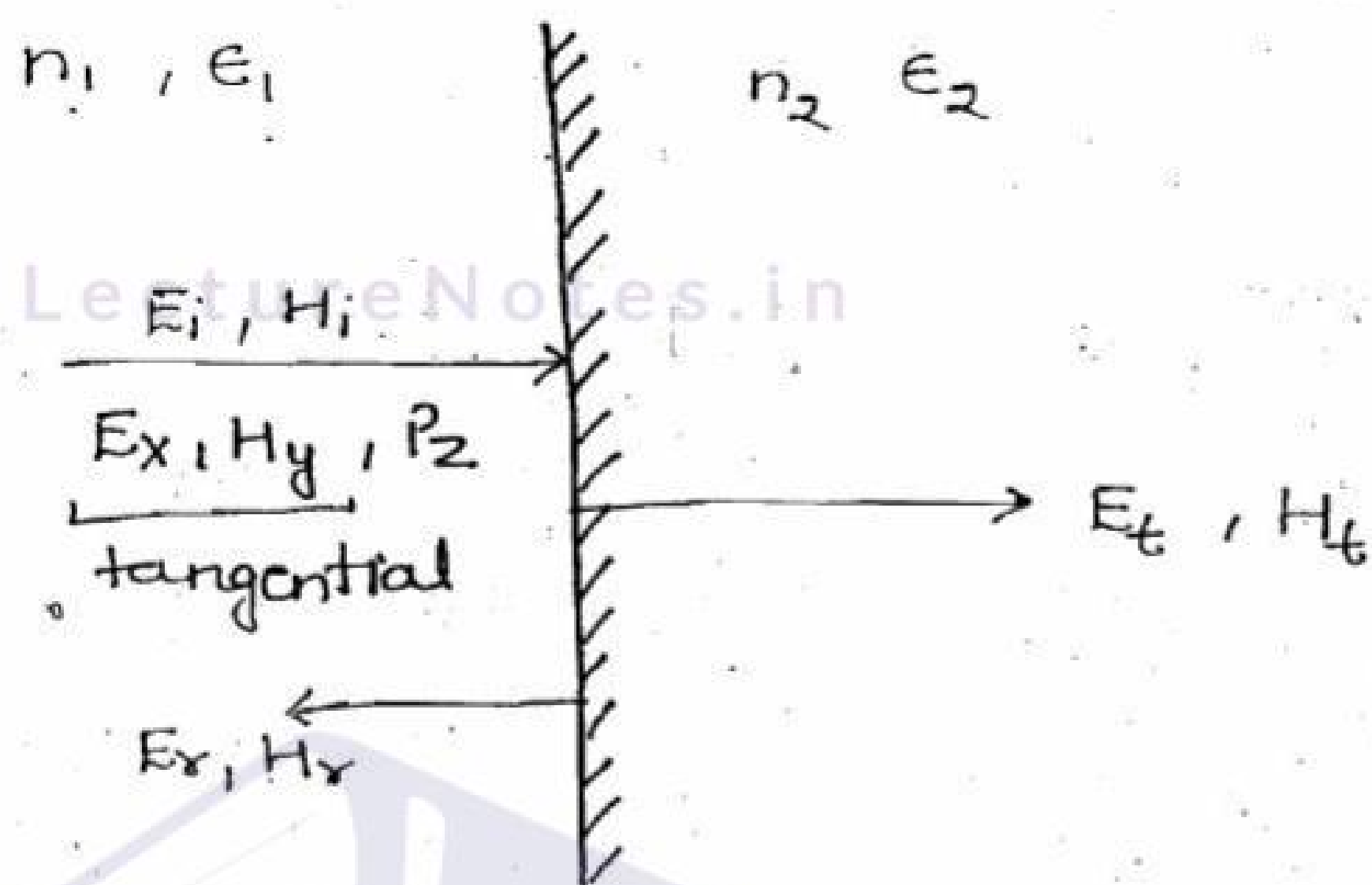
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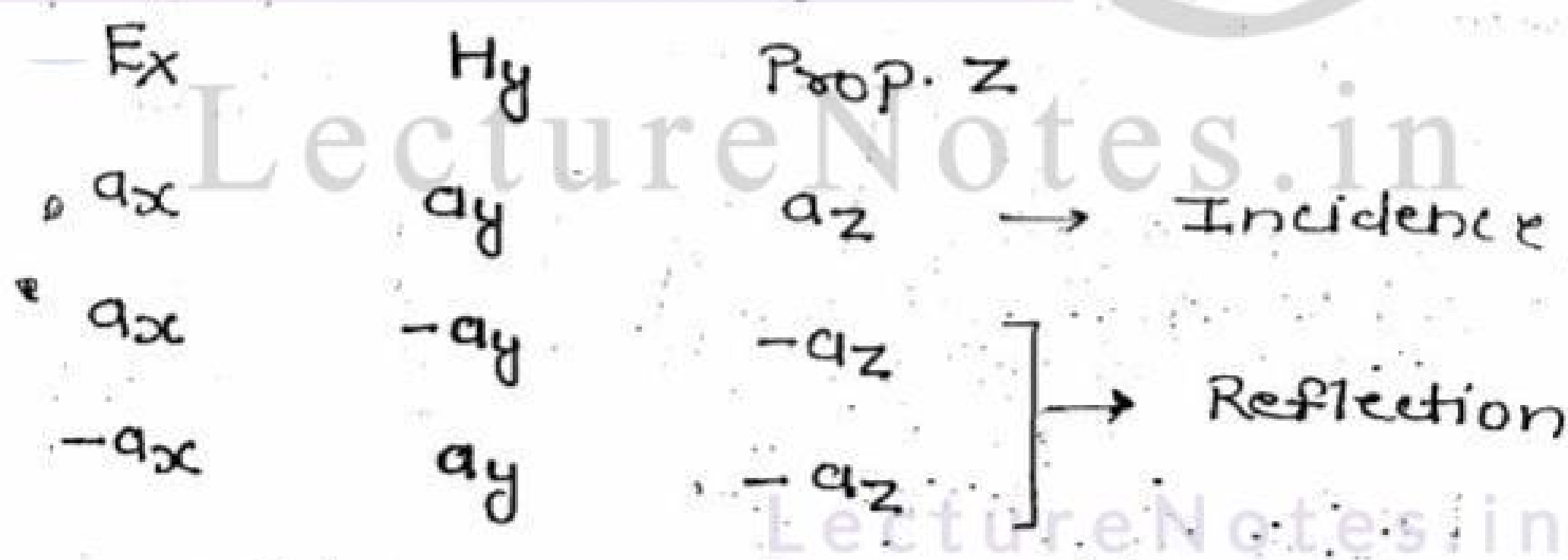
Lecture-11

Reflections / Transmissions of EM Waves.

- (I) Normal Incidence (II) Dielectric - Dielectric
 $z < 0$ $z = 0$ (XY Plane)



- (I) $E_t > E_i$ or $E_t < E_i$ → It depends on ϵ_1 & ϵ_2
 (II) $H_t > H_i$ if $E_t < E_i$
 (III) $H_t < H_i$ if $E_t > E_i$
 (IV) $P_t < P_i$ → Always



→ When reflections occur either E or H negates along with propagation direction but not both

$$E_i = \eta_1 H_i$$

$$E_t = \eta_2 H_t$$

$$E_r = -\eta_1 H_r$$

→ For normal incidence the E/H fields are tangential.

$$E_{t1} = E_{t2} \rightarrow E_i + E_r = E_t$$

$$H_{t1} = H_{t2}$$

$$E_i + E_r = E_t$$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow \boxed{1 + \Gamma = \tau} \quad (1)$$

$$\text{where } \Gamma = \frac{E_r}{E_i}$$

= Reflection coefficient
for the E fields

$$\tau = \frac{E_t}{E_i} = \text{Transmission coefficient
for the E fields}$$

$$H_i + H_r = H_t$$

$$\frac{E_i}{n_1} - \frac{E_r}{n_1} = \frac{H_t}{n_2}$$

$$E_i - E_r = \frac{n_1}{n_2} E_t$$

$$\boxed{\Gamma = \frac{n_2 - n_1}{n_2 + n_1}}$$

Note :-

$$\Gamma = \frac{n_1 - n_2}{n_1 + n_2} = -\Gamma_E$$

$$\Gamma_P = \Gamma_E \cdot \Gamma_H = -\Gamma_E^2 = -\Gamma_H^2$$

$$1 + \Gamma = \tau$$

Always - Either E or H or Power

(iv) If $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ & $\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

If $\epsilon_1 > \epsilon_2 \Rightarrow \Gamma_E = +ve \quad \tau_E > 1$

$$E_t > E_i$$

$$\Rightarrow \Gamma_H = -ve, \quad \tau_H < 1$$

$$H_t < H_i$$

$$\Gamma_P = -ve, \quad \tau_P < 1 \rightarrow \text{Always}$$

Workbook! -

12.

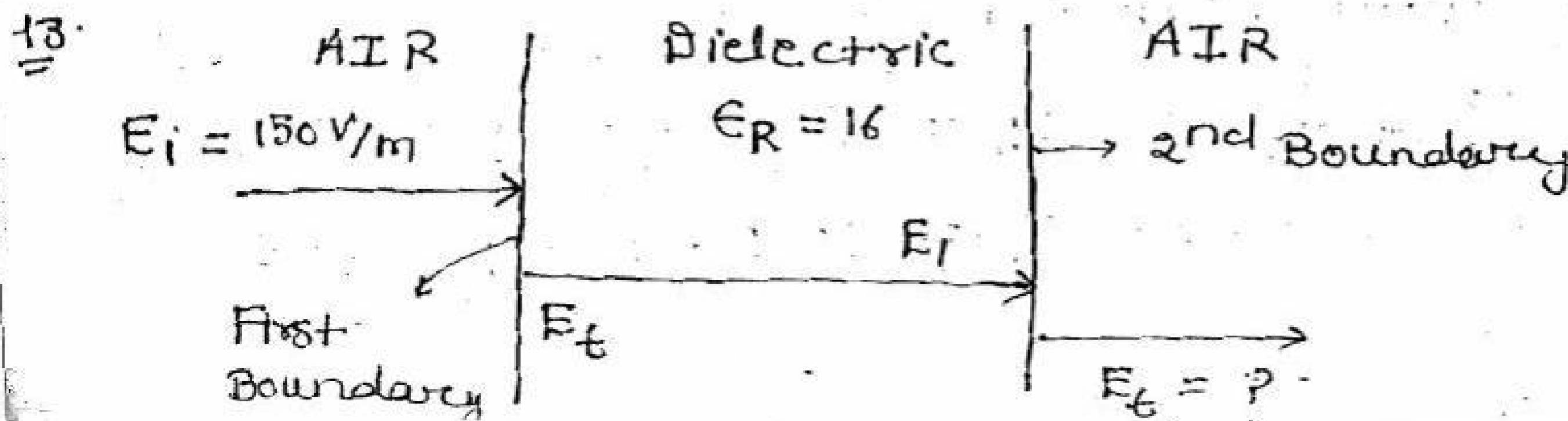
Air - Dielectric $\tau_P = ?$

$$\frac{\epsilon_0}{4\epsilon_0}$$

$$\Gamma_E = \frac{\sqrt{\epsilon_0} - \sqrt{4\epsilon_0}}{\sqrt{\epsilon_0} + \sqrt{4\epsilon_0}} = \frac{1-2}{1+2} = -\frac{1}{3}$$

$$\Gamma_H = +\frac{1}{3} \quad \Gamma_P = -\frac{1}{9}$$

$$\Rightarrow \tau_P = 1 - \frac{1}{9} = \frac{8}{9}$$



At the 1st boundary

$$\Gamma_E = \frac{1 - \sqrt{16}}{1 + \sqrt{16}} = -\frac{3}{5}$$

$$\tau_E = 1 - \frac{3}{5} = \frac{2}{5} = \frac{E_t}{E_i} \Rightarrow E_t = \frac{2}{5} \times 150 \text{ V/m} \\ = 60 \text{ V/m}$$

E_i at the 2nd Boundary = 60 V/m

$$\Gamma_E = \frac{\sqrt{16} - \sqrt{1}}{\sqrt{16} + \sqrt{1}} = \frac{3}{5}$$

$$\tau_E = 1 + \frac{3}{5} = \frac{8}{5} = \frac{E_t}{E_i}$$

$$E_t = \frac{8}{5} \times 60 = 96 \text{ V/m}$$

$$\Gamma_E = \frac{1 - \sqrt{4}}{1 + \sqrt{4}} = -\frac{1}{3}$$

$$\tau_E = 1 - \frac{1}{3} = \frac{2}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{2}{3} E_0 \cos(\omega t - 2\beta z) a_y$$

$$(\beta' = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_R} = 2\beta)$$

Note:-

$\alpha, \beta, \gamma, \eta, V_p, \lambda \rightarrow$ Propagation aspects

& material aspects and change as ϵ, μ changes

$\omega \rightarrow$ source aspect / time aspect and is same for every material.

$$\Gamma_E = -\frac{1}{3}$$

$$H_i = \cos(10^8 t - \beta z) a_y$$

$$E_i = 120\pi \cdot \cos(10^8 t - \beta z) a_y$$

$$\frac{10^8}{\beta} = 3 \times 10^8 \Rightarrow \beta = \frac{1}{3}$$

$$E_y = \left(-\frac{1}{3} \cdot 120\pi\right) \cos\left(10^8 t + \frac{z}{3}\right) a_x$$

16.

$$\frac{E_t}{E_y} = \frac{E_t}{E_i} \frac{E_y}{E_i} = \frac{\tau}{\Gamma} = \frac{1 + \Gamma}{\Gamma} = -2$$

$$\Gamma = -\frac{1}{3} = \frac{1 - \sqrt{\epsilon_2/\epsilon_1}}{1 + \sqrt{\epsilon_2/\epsilon_1}} \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 4$$

48. EM Wave reflections at conductors:-

$$\Gamma_E = \frac{n_2 - n_1}{n_2 + n_1}$$

$$n_1 = 120\pi = 377$$

$$n_2 = \sqrt{\frac{j\omega\mu}{\sigma}} = 0$$

→ EM wave power is completely reflected at conductors

$$\Gamma_E = \frac{E_y}{E_i} = 1$$

$$-E_i + E_y = E_t = 0$$

$$\tau_E = 0$$

$$\Gamma_H = \frac{H_y}{H_i} = 1$$

Ans-cc)

$$H_i + H_y = H_t = H_{\max}$$

$$\tau_H = 2$$

47.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad (\text{if } \sigma = 0) \text{ real}$$

$$= \sqrt{\frac{\mu^*}{\epsilon^*}} \quad (\text{if } \sigma \neq 0) \text{ complex}$$

$$\mu = \epsilon$$

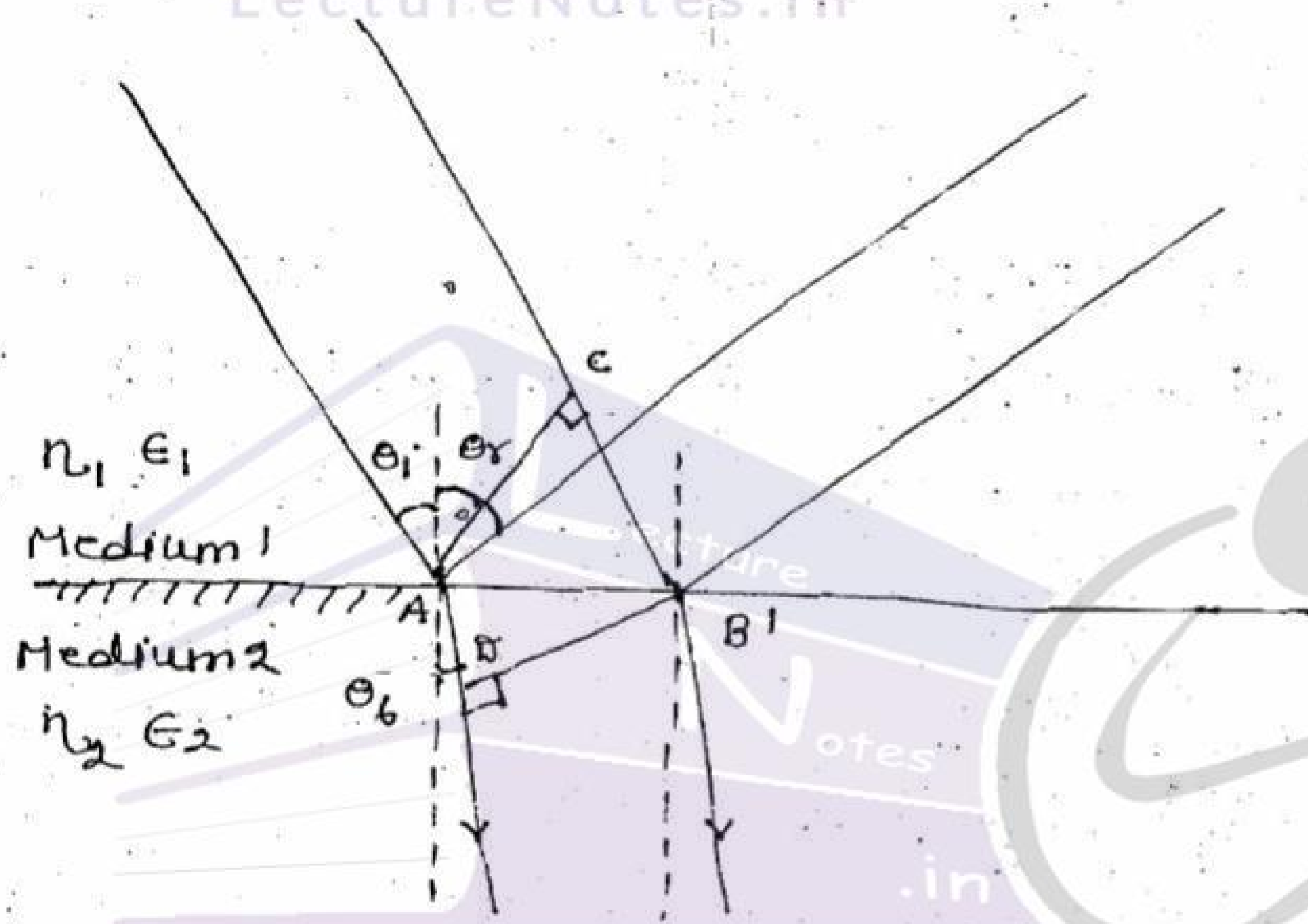
$$\eta = \sqrt{\frac{\mu^*}{\epsilon^*}} = 1$$

$$\Gamma = \frac{1 - 377}{1 + 377}$$

$$\Rightarrow \Gamma \approx -1$$

Ans - (c)

Oblique (Inclined) Incidence :-



- A/C are in phase points in medium 1
- CB is the extra distance travelled in medium 1
- B/D are in phase points in medium 2
- AD is the extra distance travelled in medium 2

$$\frac{CB}{AD} = \frac{v_1}{v_2} = \text{Refracting Index of med. 2 w.r.t med. 1}$$

$$\frac{AB \cos(90 - \theta_i)}{AB \cos(90 - \theta_t)} = \frac{1}{\sqrt{\mu_0 \epsilon_1}} \sqrt{\mu_0 \epsilon_2}$$

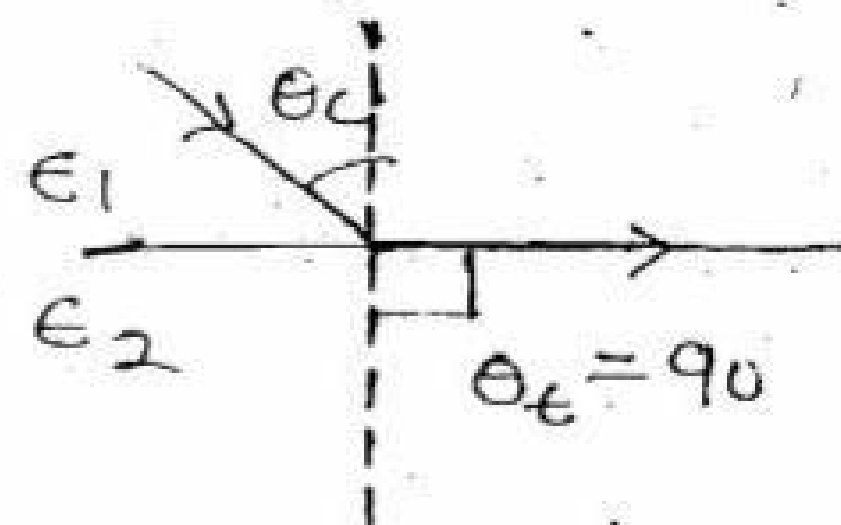
$$\boxed{\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

→ Refracting Index

Snell's 1st law $\rightarrow \boxed{\theta_i = \theta_r}$

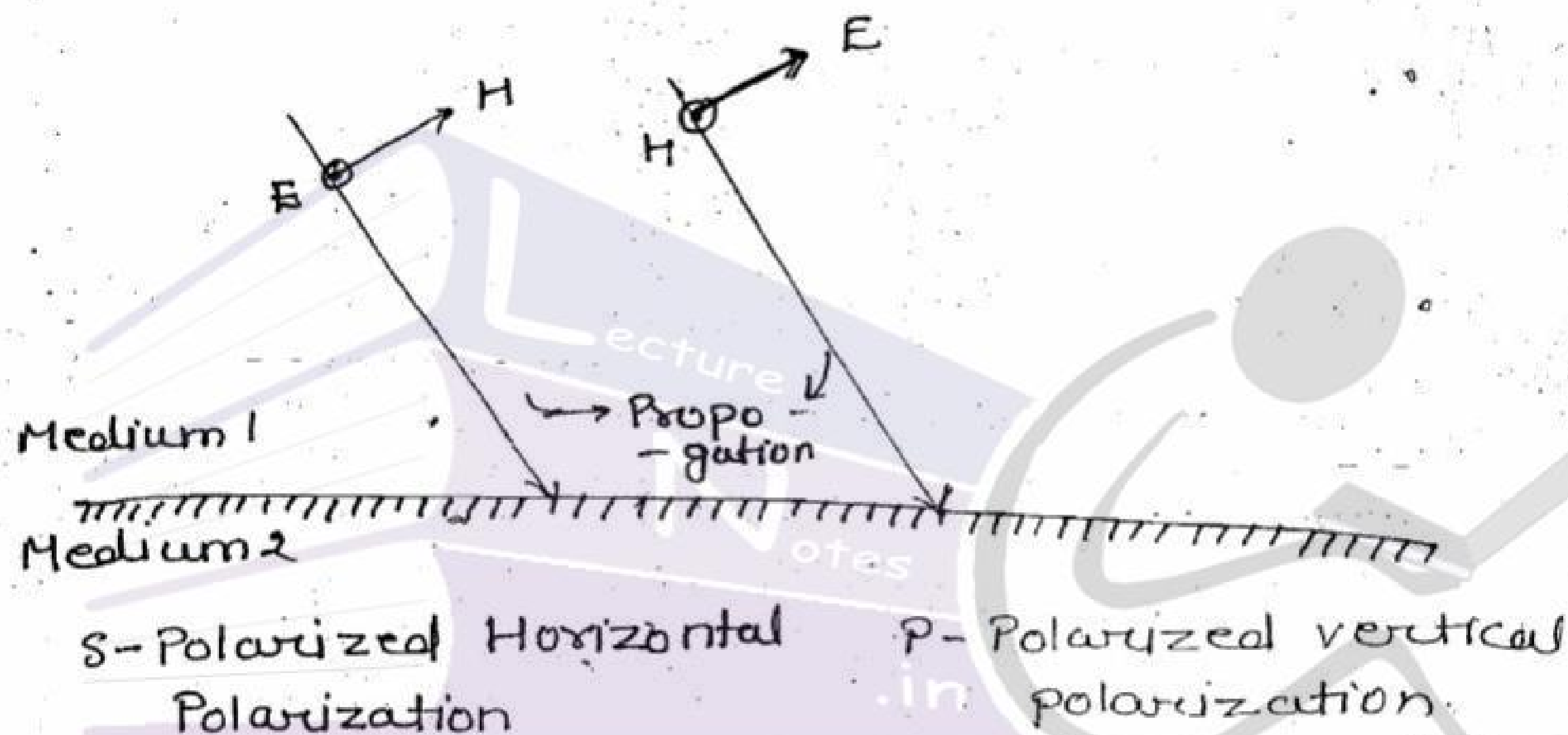
If $\theta_t = 90^\circ$ then $\theta_i = \theta_c = \text{critical angle}$

For all $\theta_i > \theta_c$ complete reflections and zero transmission takes place

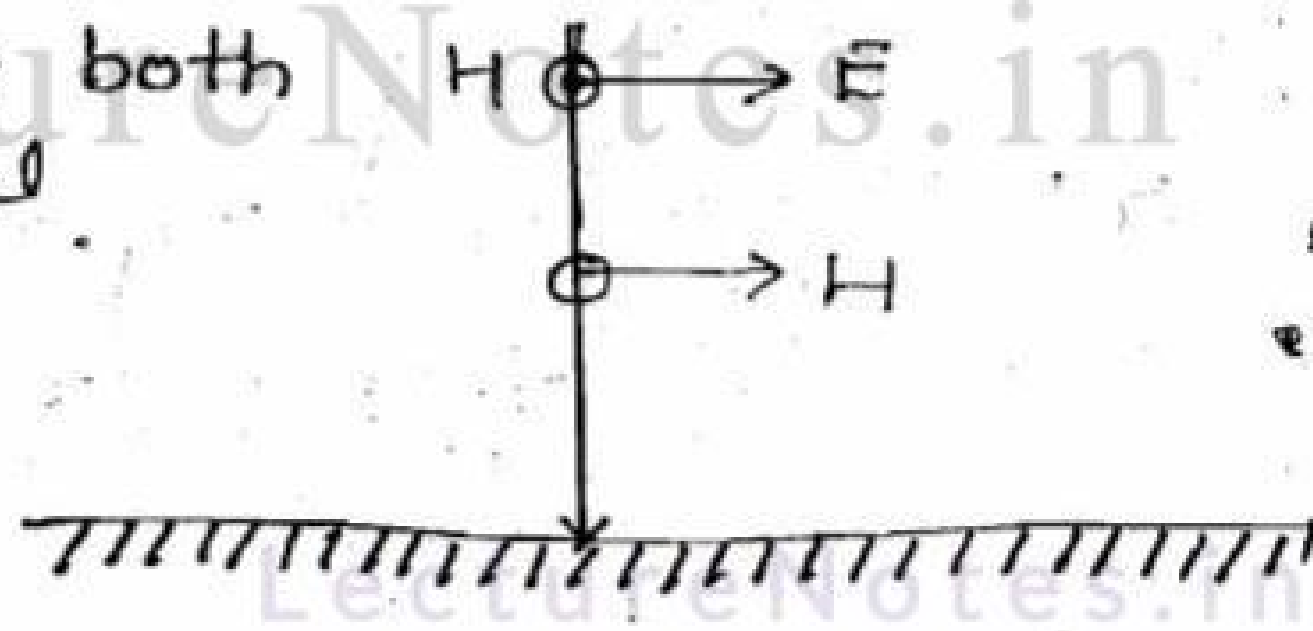


$$\boxed{\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

S & P Polarized Waves:-



In normal incidence both fields are Horizontal



Applying the Boundary Conditions,

$$E_{t1} = E_{t2}$$

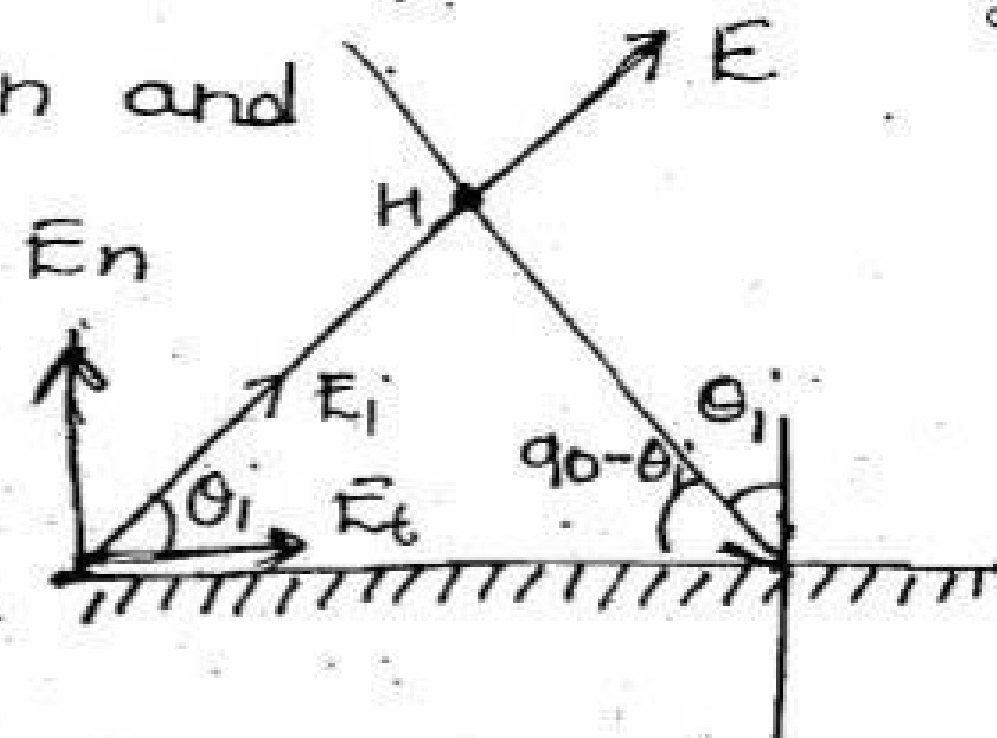
For s-polarized waves

$$E_i + E_r = E_t$$

$$\boxed{1 + \Gamma_s = \tau_s} \quad \text{--- (1s)}$$

Note:-

The angle b/w propagation and normals is the same angle b/w fields and surface



For p-polarised waves

$$E_i \cos \theta_i + E_r \cos \theta_r = E_t \cos \theta_t$$

$$E_i + E_r = E_t \frac{\cos \theta_t}{\cos \theta_i}$$

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$$1 + \Gamma = \tau \frac{\cos \theta_t}{\cos \theta_i} \quad \text{--- } 1p$$

Similarly using $H_{t1} = H_{t2}$

$$\Gamma_s = \frac{n_2 \sec \theta_t - n_1 \sec \theta_i}{n_2 \sec \theta_t + n_1 \sec \theta_i}$$

$$\Gamma_p = \frac{n_2 \cos \theta_t - n_1 \cos \theta_i}{n_2 \cos \theta_t + n_1 \cos \theta_i}$$

Note:-

$\Gamma_s \neq \Gamma_p$ in oblique incidence

but when $\theta_i = \theta_r = \theta_t = 0$ then

$$\Gamma_s = \Gamma_p = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\text{As } n_1 = \sqrt{\frac{\mu_0}{\epsilon_1}} \quad n_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$$

$$\Gamma_s = \frac{\sqrt{\epsilon_1} \cos \theta_i - \sqrt{\epsilon_2} \cos \theta_t}{\sqrt{\epsilon_1} \cos \theta_i + \sqrt{\epsilon_2} \cos \theta_t}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$$

$$\cos \theta_t = \sqrt{1 - \frac{\epsilon_1}{\epsilon_2} \sin^2 \theta_i}$$

$$\Gamma_s = \frac{\cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}$$

$$\Gamma_p = \frac{\frac{\epsilon_2}{\epsilon_1} \cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}{\frac{\epsilon_2}{\epsilon_1} \cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}$$

If $\Gamma_p = 0$ i.e. zero reflections & complete transmission

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2 \theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\sin^2 \theta_i \left(1 - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2\right) = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2 \theta_i = \frac{\frac{\epsilon_2}{\epsilon_1} \left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}{\left(1 + \frac{\epsilon_2}{\epsilon_1}\right) \left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}$$

$$\sin \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\tan \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

This $\theta_i = \theta_B$ is called as Brewster angle.
for p-polarised wave

$$\text{If } \Gamma_s = 0$$

$$\cos^2 \theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\Rightarrow \epsilon_2 = \epsilon_1$$

Brewster angle and zero reflections cannot occur for s-polarized waves.

Summary:-

zero transmission, complete reflections

θ_c = critical angle

zero reflections, complete transmission

θ_B = Brewster angle

$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$2. \tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\epsilon_1 > \epsilon_2$ is a required condition

3. No such medium restriction

No such polarization restriction

4. Exists only for p-polarized waves

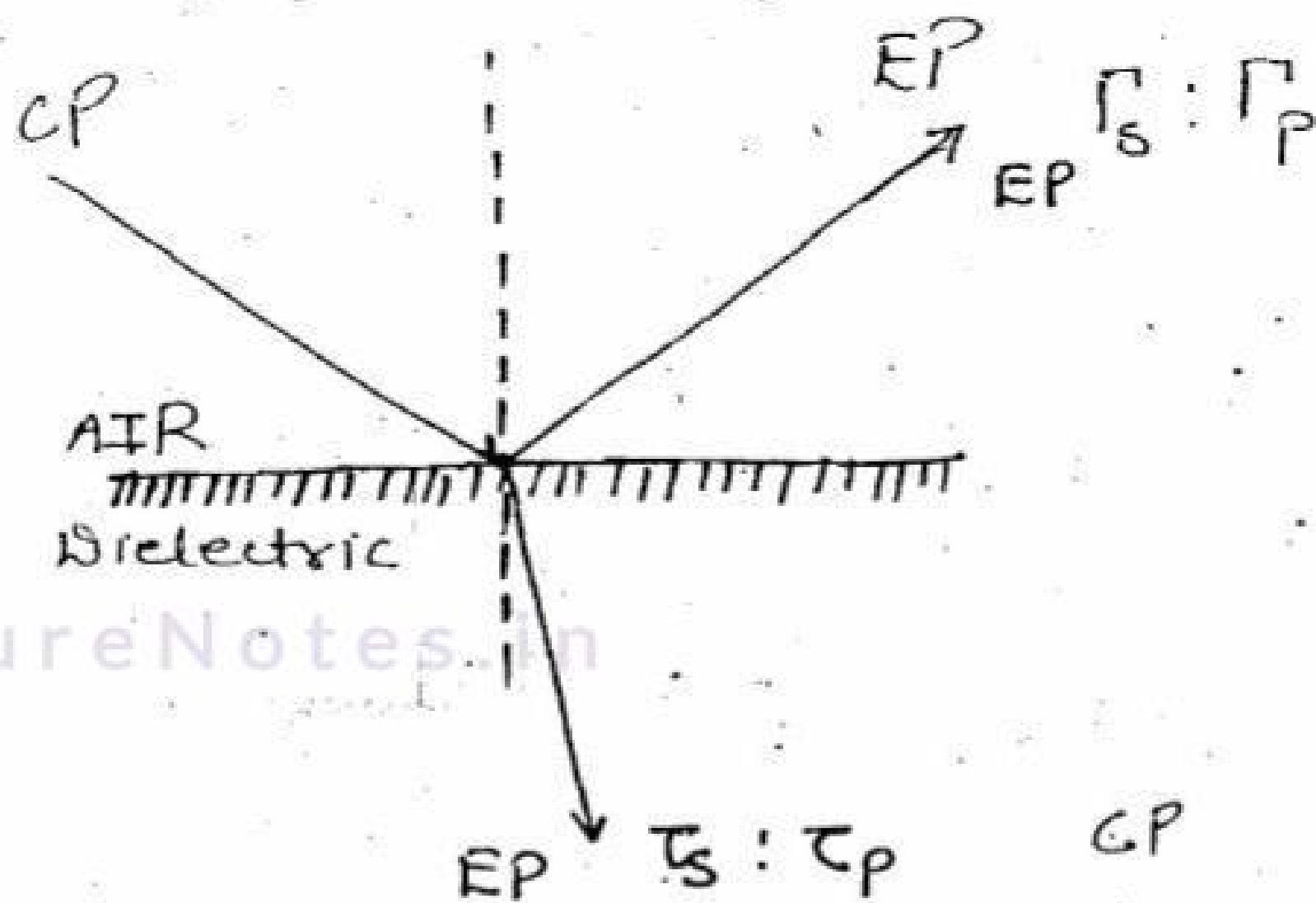
For all $\theta_i > \theta_c$

→ the same effect

5. At $\theta_i = \theta_B$ only this occurs

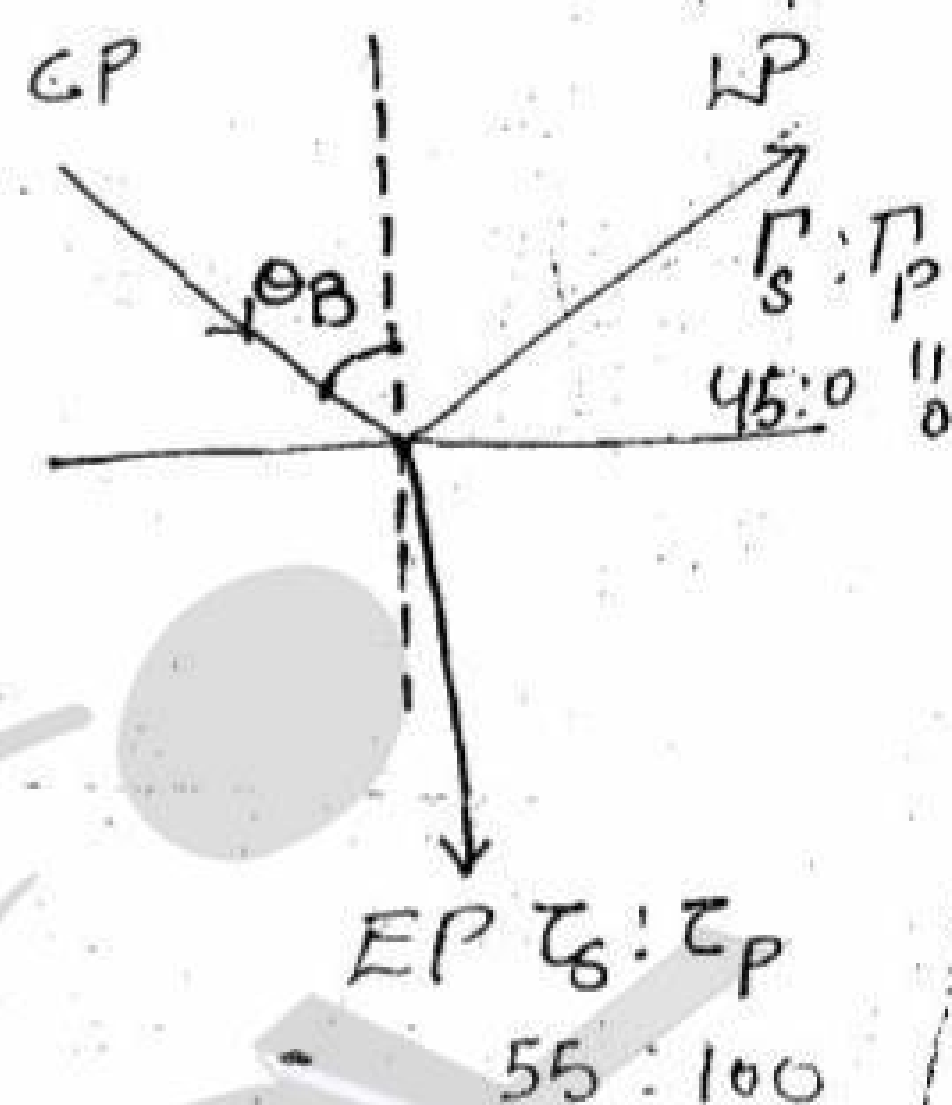
19.

RCP $\rightarrow$ S+P



$$\tan \theta_B = \tan 60^\circ = \sqrt{\frac{\epsilon_R \epsilon_0}{\epsilon_0}}$$

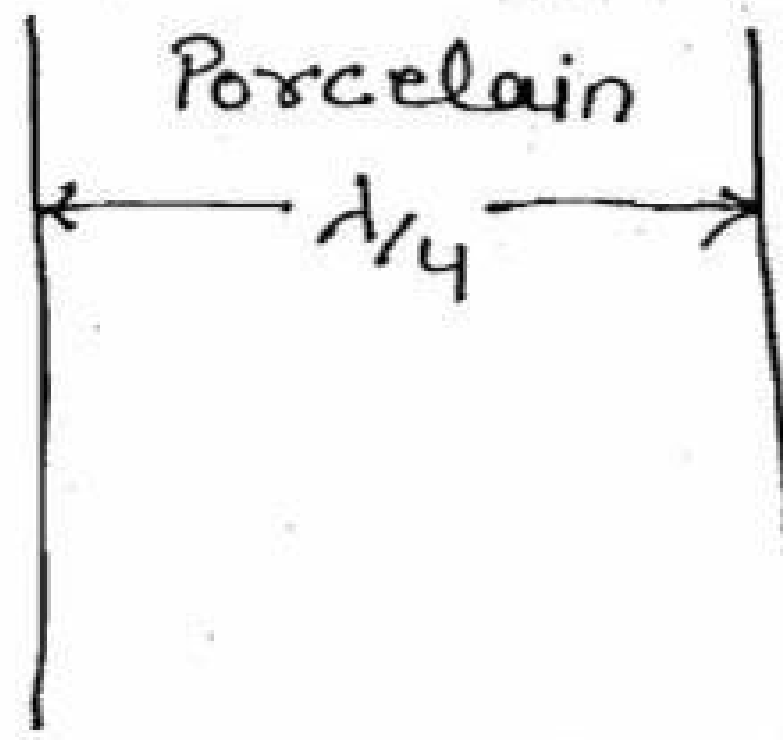
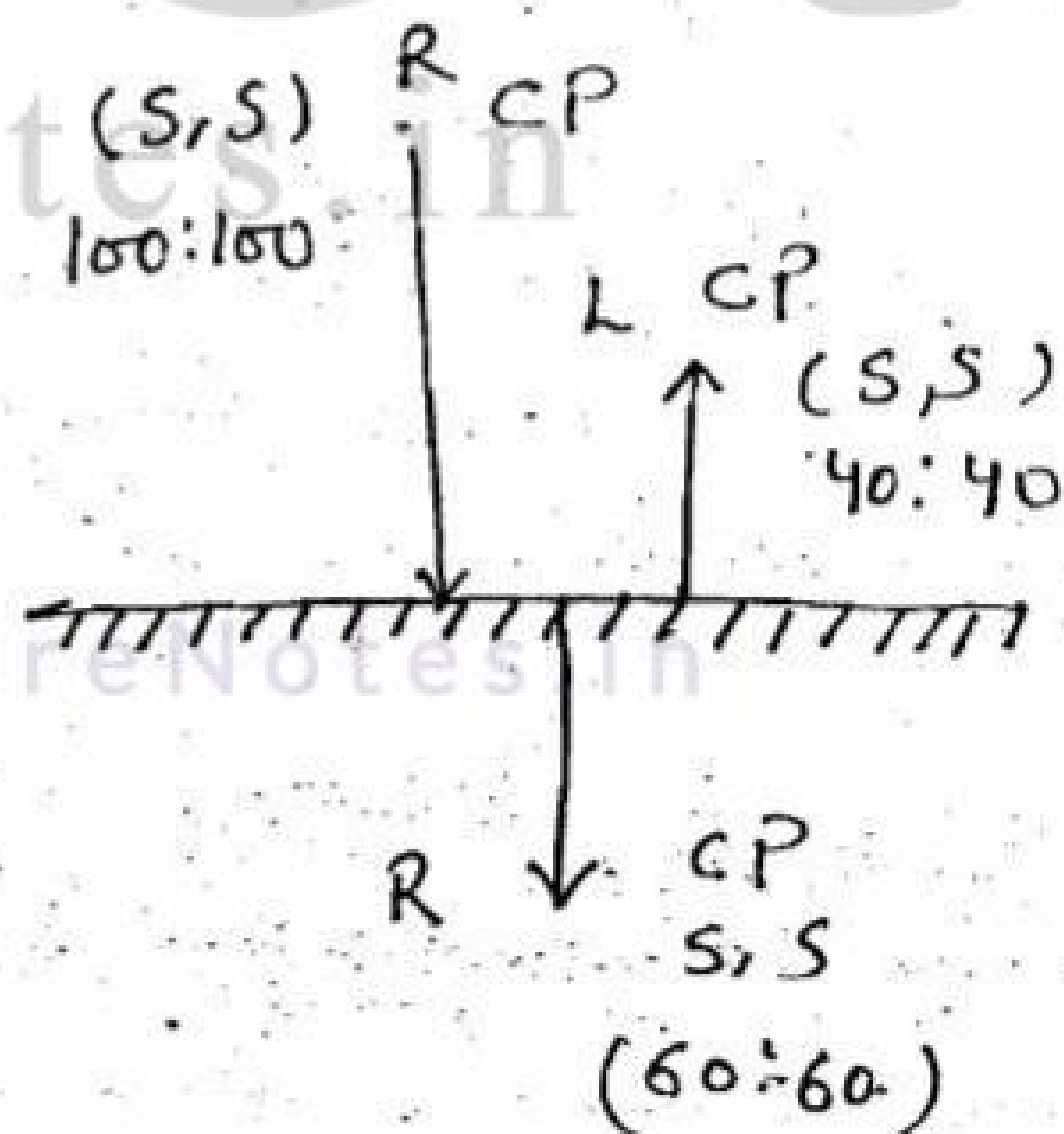
$$\Rightarrow \epsilon_R = 3$$



Linearly Polarised

a b

LectureNotes.in



$$\text{thickness} = \frac{\lambda}{4} = \left(\frac{3 \times 10^8}{\sqrt{\epsilon_R} \times 10^9 \times 4} \right)$$

53.

μ_R $\left\{ \begin{array}{l} J_b \rightarrow \text{Bond current density} \\ \rightarrow \text{Due to spinning/revolving electrons} \\ \text{constituting a magnetic dipole.} \end{array} \right.$

$\sim [J_c \rightarrow \text{conduction current} - \text{moving electron}]$

$\in [J_d \rightarrow \text{displacement current} \rightarrow \text{changing Electric flux}]$

41.

$$\vec{E} = 3 \sin(\omega t - \beta z) \hat{a}_x + 6 \sin(\omega t - \beta z + 75^\circ) \hat{a}_y$$

$$E_0 = \sqrt{3^2 + 6^2} = 3\sqrt{5}$$

$$P_{avg} = \frac{1}{2} \frac{E_0^2}{\eta} \hat{a}_z$$

$$= \frac{1}{2} \frac{(3\sqrt{5})^2}{120\pi} \hat{a}_z \approx 58 \text{ mW}$$

OR

$$\vec{E} = 3 \sin(\omega t - \beta z) (\hat{a}_x + 2e^{j75^\circ} \hat{a}_y)$$

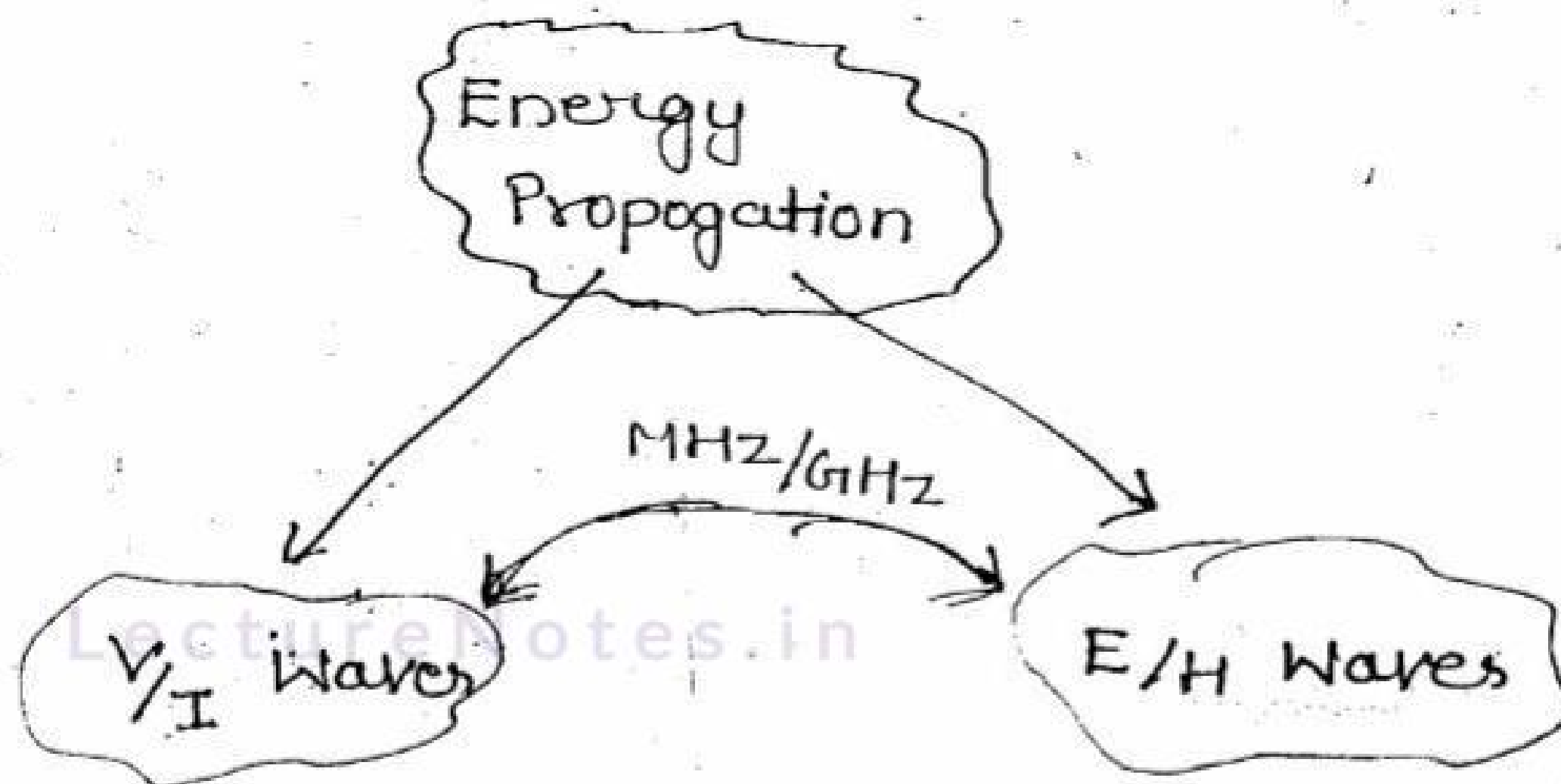
$$\vec{H} = \left(\frac{3}{120\pi} \right) \sin(\omega t - \beta z) (\hat{a}_y - 2e^{j75^\circ} \hat{a}_x)$$

$$P_{avg} = \frac{1}{2} \cdot 3 (\hat{a}_x + 2e^{j75^\circ} \hat{a}_y) \times \left(\frac{3}{120\pi} (\hat{a}_y - 2e^{-j75^\circ} \hat{a}_x) \right)$$

$$= \frac{1}{2} \frac{3 \cdot 3}{120\pi} (\hat{a}_z + 4\hat{a}_z)$$

$$= \frac{1}{2} \frac{(3\sqrt{5})^2}{120\pi} \hat{a}_z$$

TRANSMISSION LINES



$$\rightarrow \epsilon = \infty$$

$\rightarrow$ Conductors

$\rightarrow$ Wired or point to point

$\rightarrow$ Extremely low frequency

$$f < \text{kHz}$$

$$\rightarrow \epsilon = 0$$

$\rightarrow$ Free space

$\rightarrow$ Broadcast wireless

$\rightarrow$ Extremely high frequency

$$f > 10^{12} \text{ Hz}$$

$$V_s$$

$$I_s$$

ω
Source

$$R, L, G, C$$

Primary
constant

Transmission
line

$$V(x)$$

$$I(x)$$

$$Y = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Propagation

$E_s \rightarrow \omega$ source $\rightarrow \mu, \sigma, \epsilon$ Material constant $\rightarrow$ Medium $\rightarrow E(z)$
 $H_s \rightarrow H(z)$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

LectureNotes.in

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Primary constants:-

Resistance:-

→ It is the resistance of the conducting material all along the length of the line.

$$\rightarrow R = \rho \frac{l}{A} \quad R \cdot d \cdot d$$

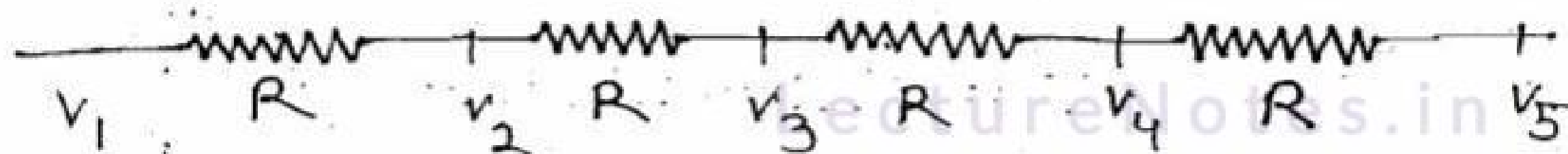
It is distributed all along the length

$$R = \frac{R}{l} = \text{ohm/m}$$

↓
Primary constant

→ It is the resistance at any instance on the line

→ It appears in series along the line

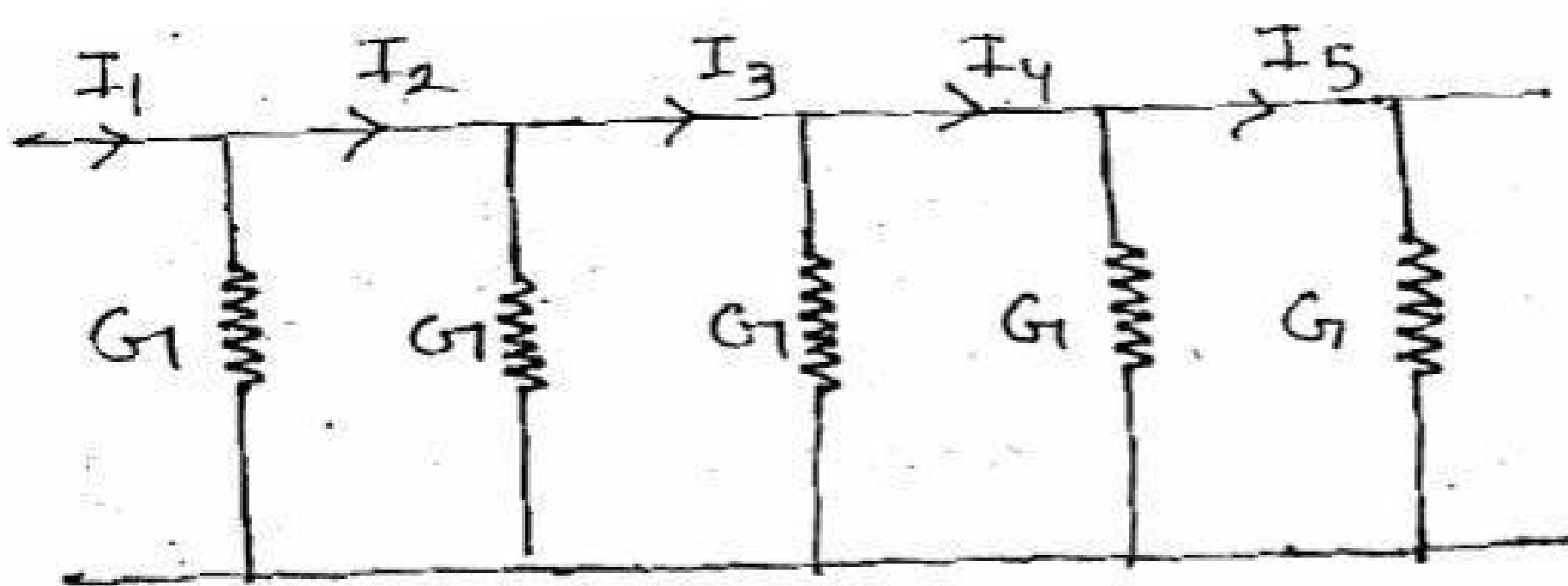


→ It causes a voltage decay all along the line

Conductance:-

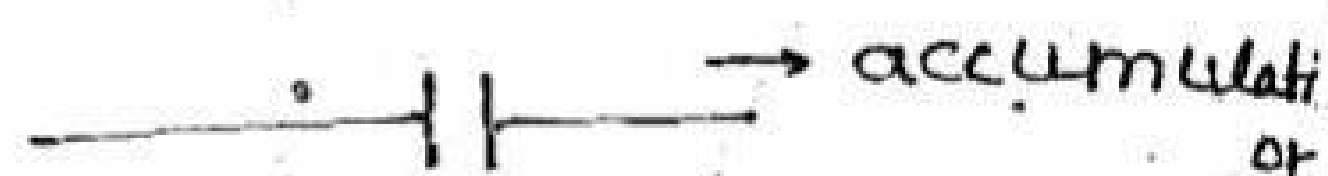
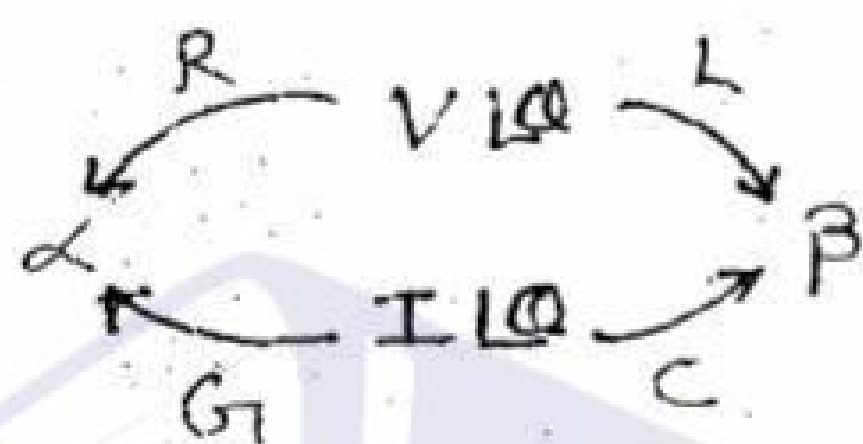
→ It is the conductance of the insulating material b/w the lines

→ It appears in shunt b/w the lines



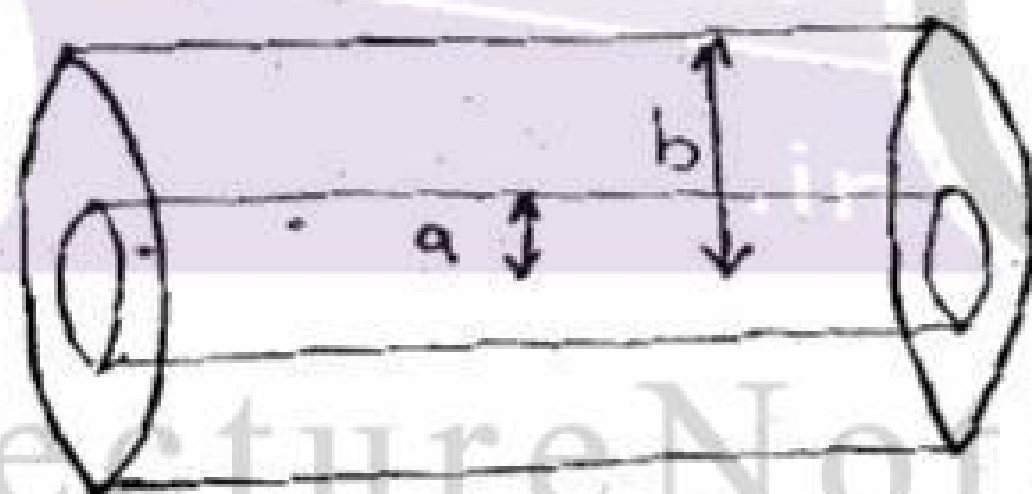
- It causes current decay
- It is distributed all along the length

$$G = \frac{G_1}{l} = \frac{mho}{m}$$

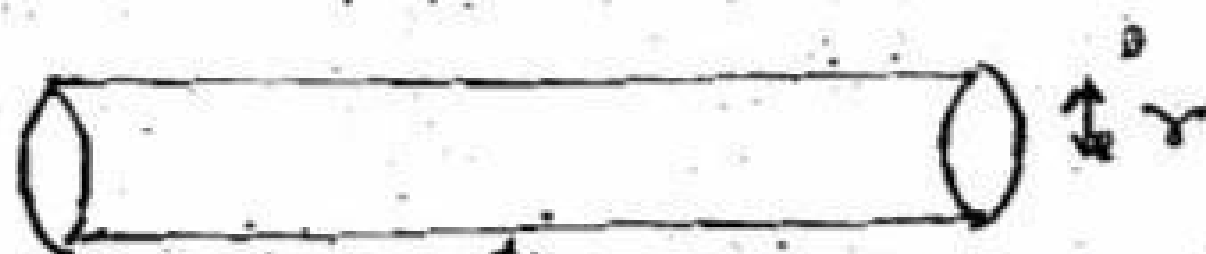


Capacitance:-

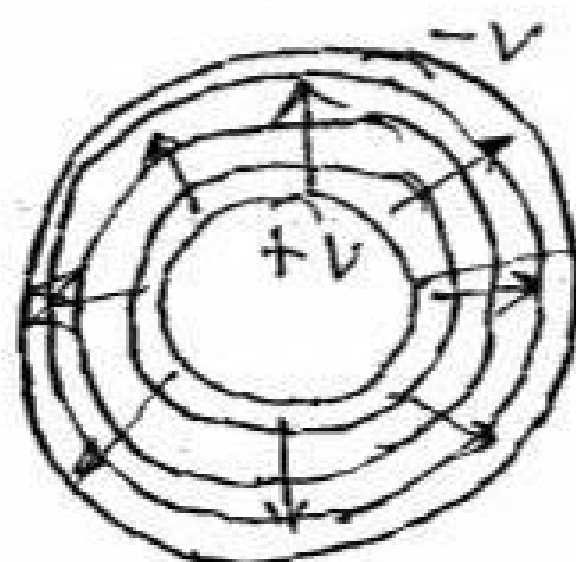
- It is due to the voltages on the line there is a surrounding E field and hence capacitance



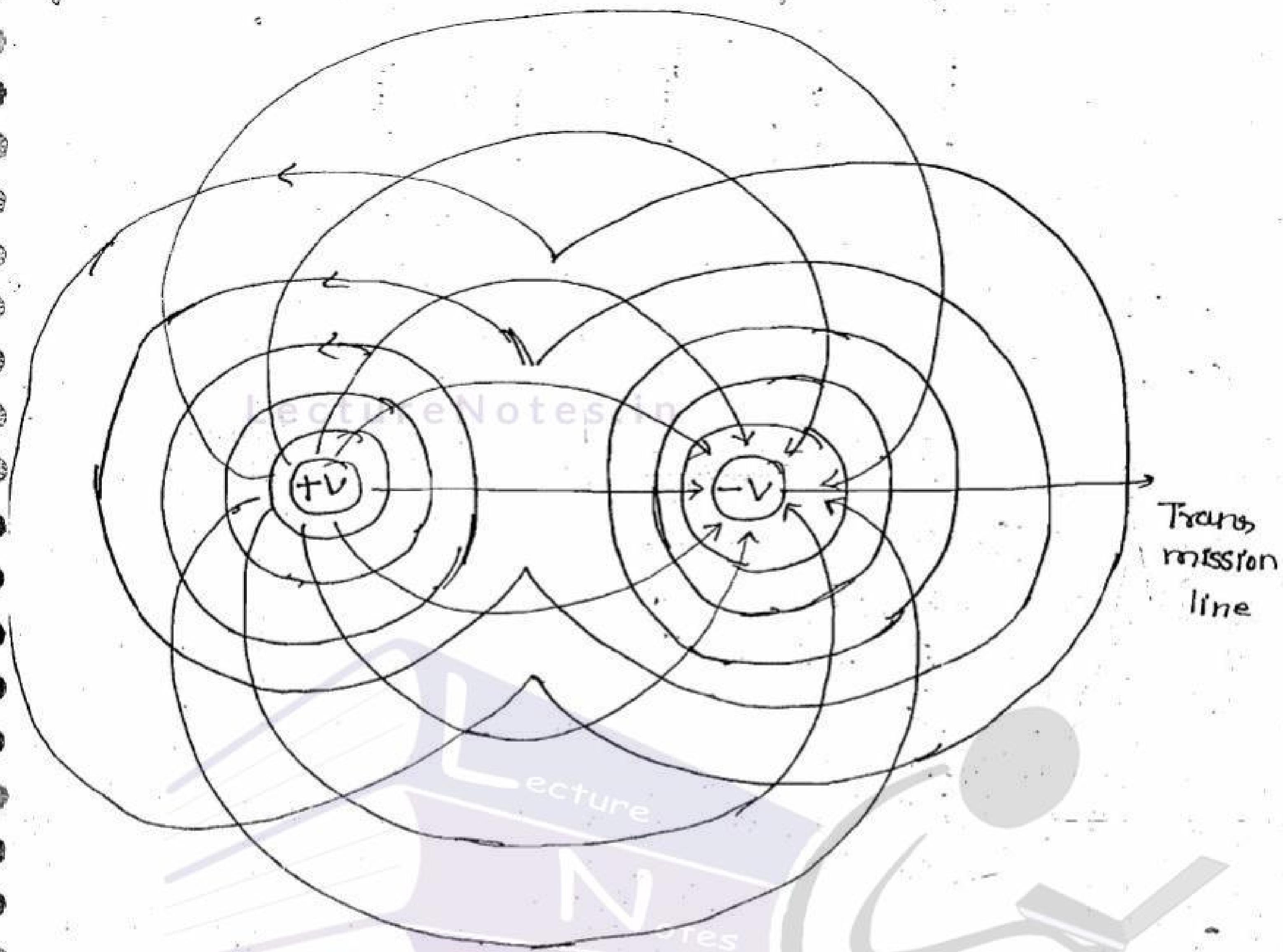
→ Co-axial cable



→ Parallel Wire.



→ Transmission line.



$$C = \frac{2\pi\epsilon_0 l}{\ln(b/a)} \rightarrow \text{Co-axial cable}$$

$$C = \frac{\pi\epsilon_0 l}{\ln(D/r)} \rightarrow \text{Parallel wire}$$

$\rightarrow C \propto l \rightarrow$ capacitance is distributed

$$\rightarrow C = \frac{C}{l} = \frac{\text{Farads}}{\text{m}}$$

$\rightarrow$ It appears in shunt b/w the cables

Inductance:-

→ It is due to the currents on the line there is a surrounding H-field and hence inductance

→ $L = \frac{\mu l \ln(b/a)}{2\pi}$ → co-axial cable

$L = \frac{\mu l}{\pi} \cdot \ln(R/r)$ → parallel wire

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→ L is distributed i.e. $L \propto l$

$$L = \frac{L}{l} \frac{\text{Henry}}{\text{m}}$$

Note:-

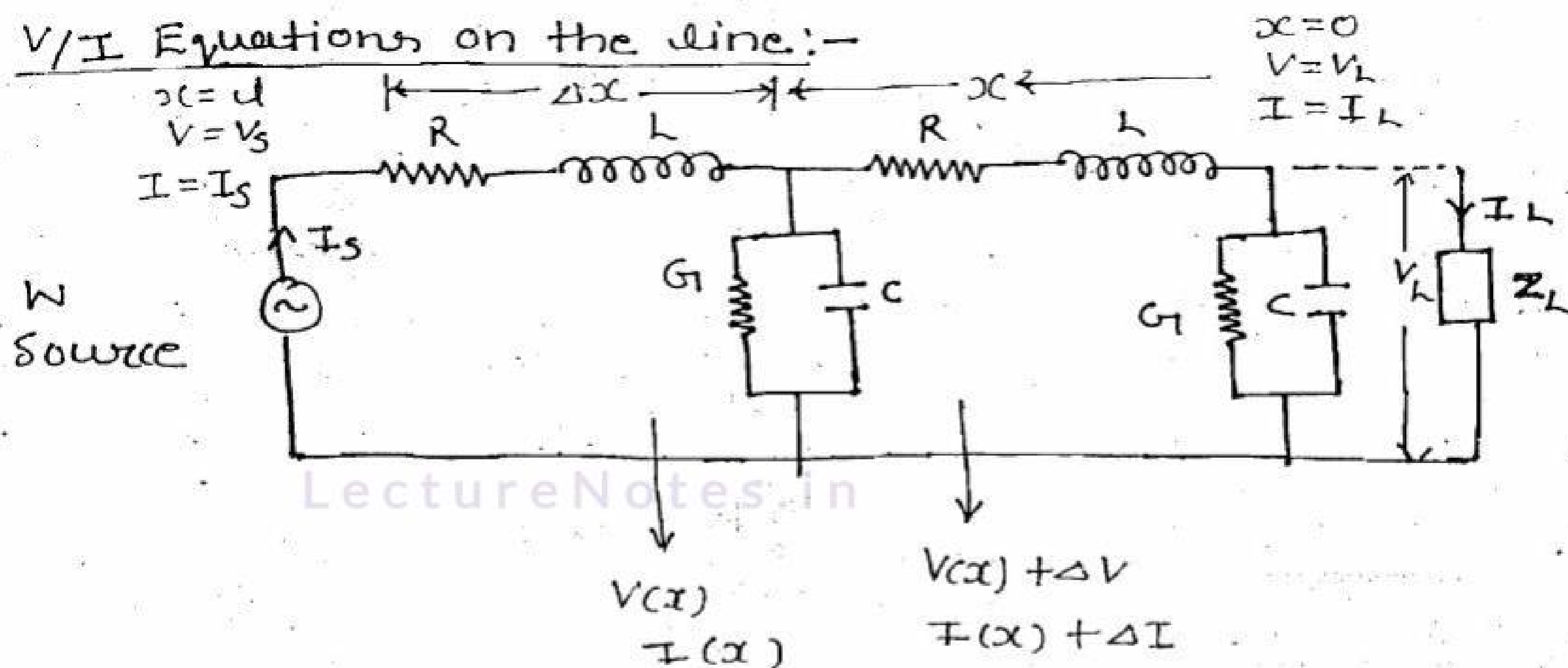
$$LC = \frac{\text{Distributed } L}{l} \times \frac{\text{Distributed } C}{C} = \mu \epsilon$$

For any transmission line and for any geometry.

LectureNotes.in

LectureNotes.in

V/I Equations on the line:-



$$Z_{\text{series of } \Delta x \text{ length}} = (R + j\omega L) \cdot \Delta x$$

$$Y_{\text{shunt of } \Delta x \text{ length}} = (G + j\omega C) \cdot \Delta x$$

$$\Delta V = I \cdot (R + j\omega L) \cdot \Delta x$$

$$\frac{\Delta V}{\Delta x} = \frac{dV}{dx} = (R + j\omega L) I \quad \text{--- (I)}$$

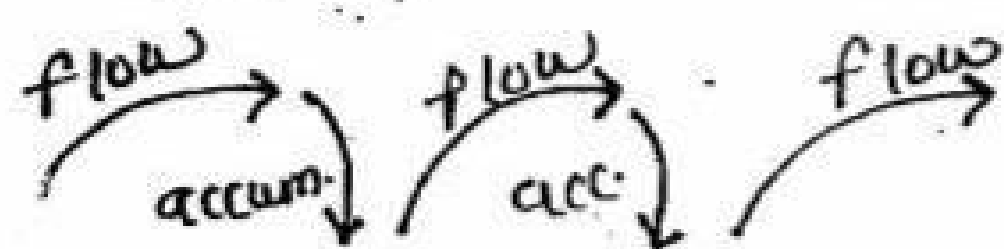
$$\frac{\Delta I}{\Delta x} = \frac{dI}{dx} = (G + j\omega C) V \quad \text{--- (II)}$$

Take I from (I) & put in (II)

$$\frac{d}{dx} \left[\left(\frac{1}{R + j\omega L} \right) \frac{dV}{dx} \right] = (G + j\omega C) V$$

$$\frac{d^2 V}{dx^2} = (R + j\omega L) (G + j\omega C) \quad \text{--- (III)}$$

$$\frac{d^2 I}{dx^2} = (R + j\omega L) (G + j\omega C) \quad \text{--- (IV)}$$



→ meaning of Harmonic

Note:-

If the source is time harmonic at one end the effect is space harmonic all along the length of the line.

$$\text{Let } \sqrt{(R+j\omega L)(G+j\omega C)} = Y \text{ (per cm)}$$

Eq-(III) & (IV) can be written as Unit $\downarrow$

$$\frac{d^2 V}{dx^2} - Y^2 V = 0$$

$$\frac{d^2 I}{dx^2} - Y^2 I = 0$$

The V/I solution on the line is

$$V(x) = c_1 e^{-Yx} + c_2 e^{Yx} \text{ --- (V)}$$

$$I(x) = c_3 e^{-Yx} + c_4 e^{Yx} \text{ --- (VI)}$$

Applying initial conditions, $(x=0, V=V_L, I=I_L)$

$$V_L = c_1 + c_2 \text{ --- (VII)}$$

$$I_L = c_3 + c_4 \text{ --- (VIII)}$$

Using eq-(I) in eq-(V)

$$\frac{dV}{dx} = -Y c_1 e^{-Yx} + Y c_2 e^{Yx} = (R+j\omega L) I$$

Put $x=0$

$$\begin{aligned} \frac{c_2 - c_1}{\sqrt{(R+j\omega L)(G+j\omega C)}} &= \frac{(R+j\omega L) I_L}{\sqrt{(R+j\omega L)(G+j\omega C)}} \\ &= \sqrt{\frac{R+j\omega L}{G+j\omega C}} I_L = I_L Z_0 \end{aligned}$$

$$\text{where } Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$I_L Z_0 = c_2 - c_1 \text{ --- (IX)}$$

$$\text{Similarly } \frac{V_L}{Z_0} = c_4 - c_3 \text{ --- (X)}$$

$$\boxed{c_1 = \frac{V_L - I_L Z_0}{2} \quad c_2 = \frac{V_L + I_L Z_0}{2}}$$

$$C_3 = \frac{I_L - V_L/Z_0}{2}, \quad C_4 = \frac{I_L + V_L/Z_0}{2}$$

$$V(x) = \frac{V_L}{2} \left[\left(1 - \frac{Z_0}{Z_L}\right) e^{-\gamma x} + \left(1 + \frac{Z_0}{Z_L}\right) e^{\gamma x} \right]$$

$$V(x) = \frac{V_L (Z_L + Z_0)}{2Z_L} \left[e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-\gamma x} \right]$$

$$V(x) = V_0 \left(e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-\gamma x} \right)$$

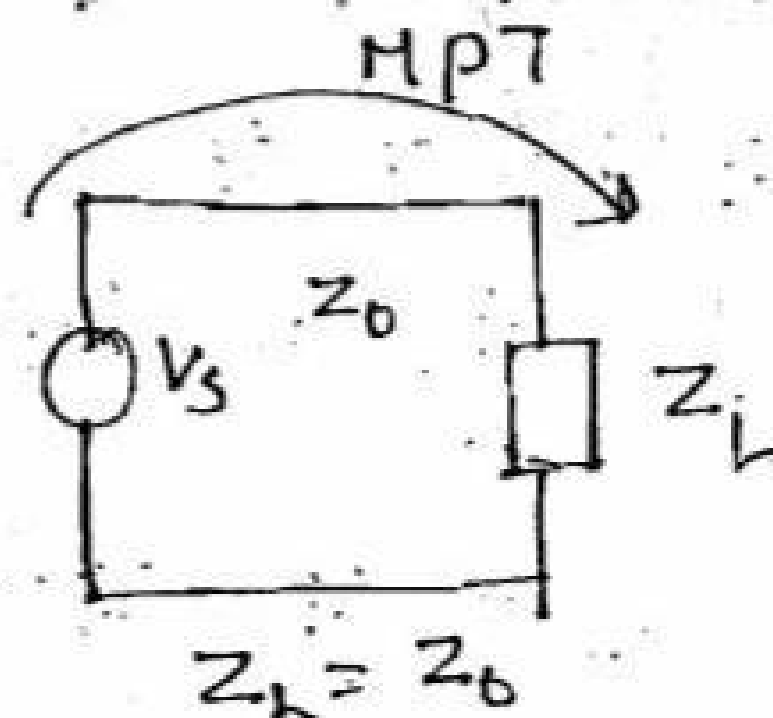
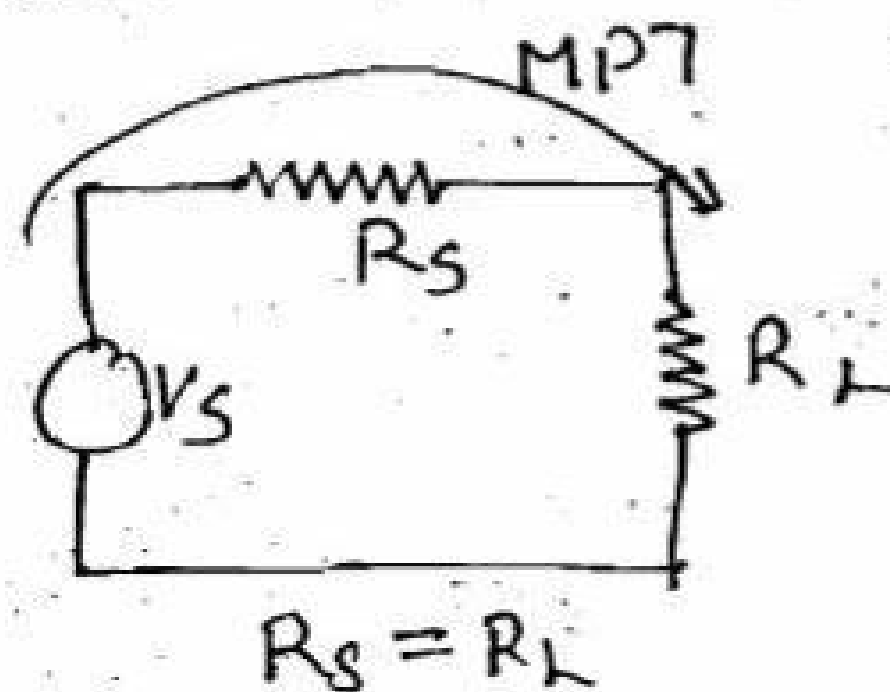
$$I(x) = \frac{I_L (Z_L + Z_0)}{2Z_0} \left[e^{\gamma x} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L} \right) e^{-\gamma x} \right]$$

$e^{\gamma x} \rightarrow -x$, Source-load forward wave

$e^{-\gamma x} \rightarrow +x$, load-source, reflected wave

→ Every transmission line has two waveforms i.e. the forward and reflected and the reflected wave does not exist in one single condition i.e. $Z_L = Z_0$. This is called as matched condition and this is a condition where complete source power is absorbed by the load.

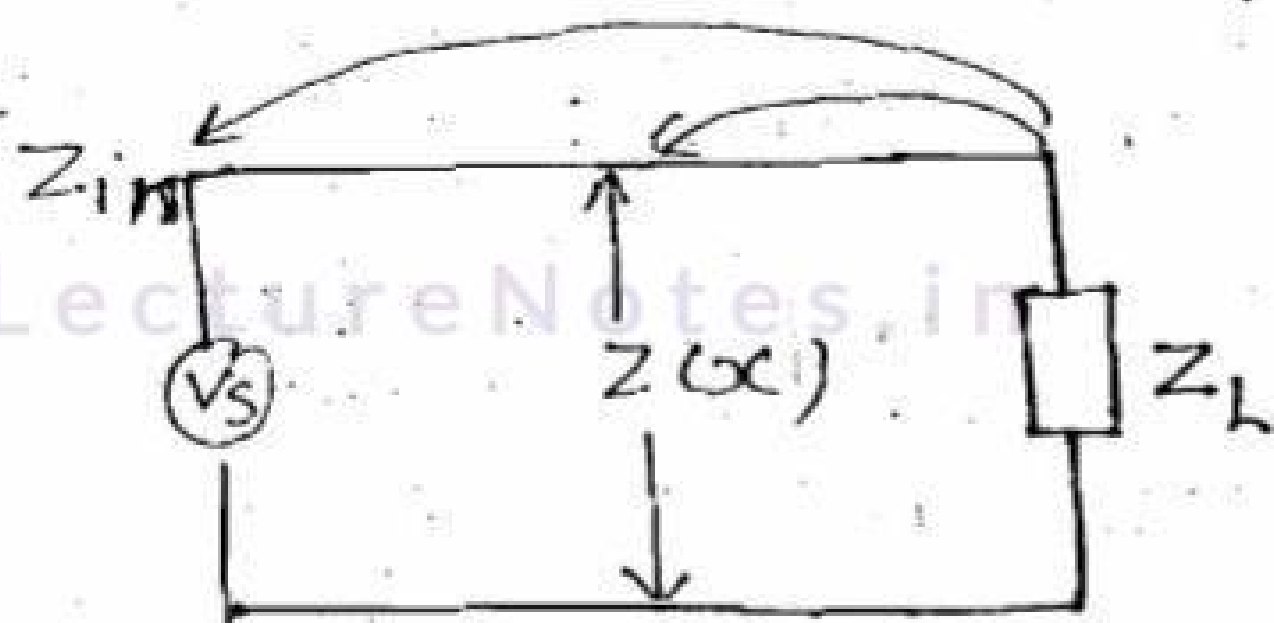
→ This is similar to max. power transfer in N/w or EM wave reflections b/w materials.



Note:-

- Z_0 is a unique impedance of the line with which if the line is terminated the reflections on the line and hence called as characteristic impedance

Impedance on line (z) and i/p impedance (Z_{in}):-



Note:-

Due to the loaded one end and primary constant in the line, impedance is different on different point of line such that

$$Z(x) = \frac{V(x)}{I(x)}$$

$$Z_{in} = Z(l)$$

$$V(x) = \frac{V_L}{Z_L} \left(\frac{(Z_L + Z_0)}{2} e^{\gamma x} + \frac{(Z_L - Z_0)}{2} e^{-\gamma x} \right)$$

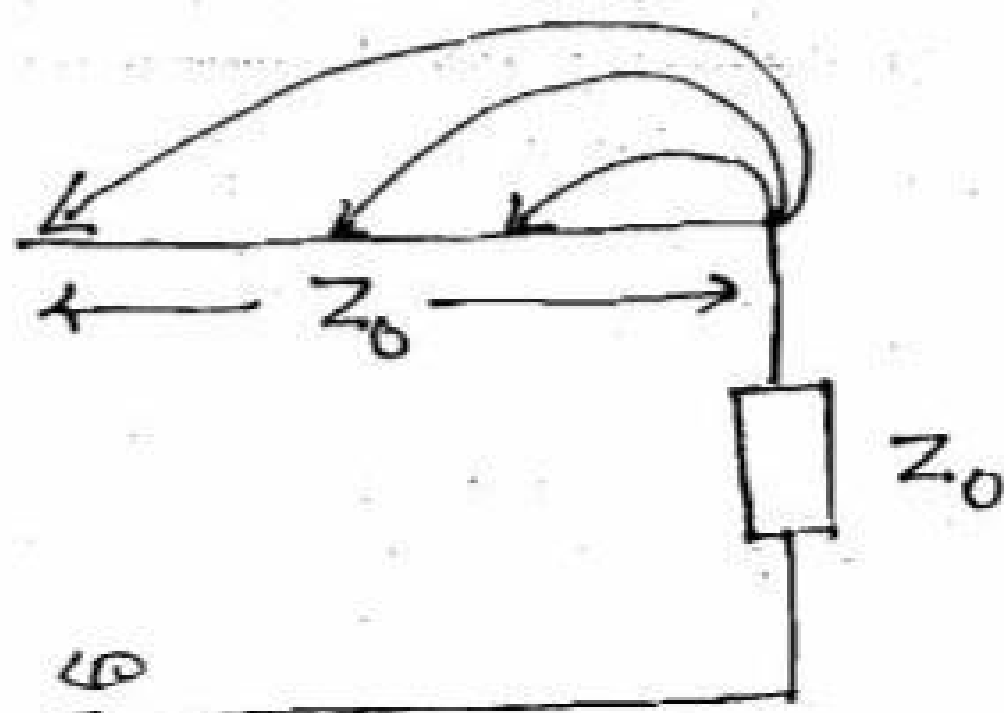
$$V(x) = V_L \cosh(\gamma x) + I_L Z_0 \sinh(\gamma x)$$

Similarly

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0} \sinh(\gamma x)$$

$$Z(x) = Z_0 \left[\frac{Z_L \cosh(\gamma x) + Z_0 \sinh(\gamma x)}{Z_0 \cosh(\gamma x) + Z_L \sinh(\gamma x)} \right]$$

If $Z_L = Z_0$ then $Z(x) = Z_0 = Z_L = Z_{in}$



Matched line

→ For a matched line the impedance anywhere on the line is also same Z_0

Note:-

Z_0 is that unique impedance with which if the line is terminated impedance on the line is also the same

Short circuit and open circuit lines:-

→ If $Z_L = 0 \Rightarrow$ SC line

$$Z_{in} = Z_{sc} = Z_0 \tanh(\gamma x)$$

→ If $Z_L = \infty \Rightarrow$ O.C line

$$Z_{in} = Z_{oc} = Z_0 \coth(\gamma x)$$

$$\Rightarrow Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}$$

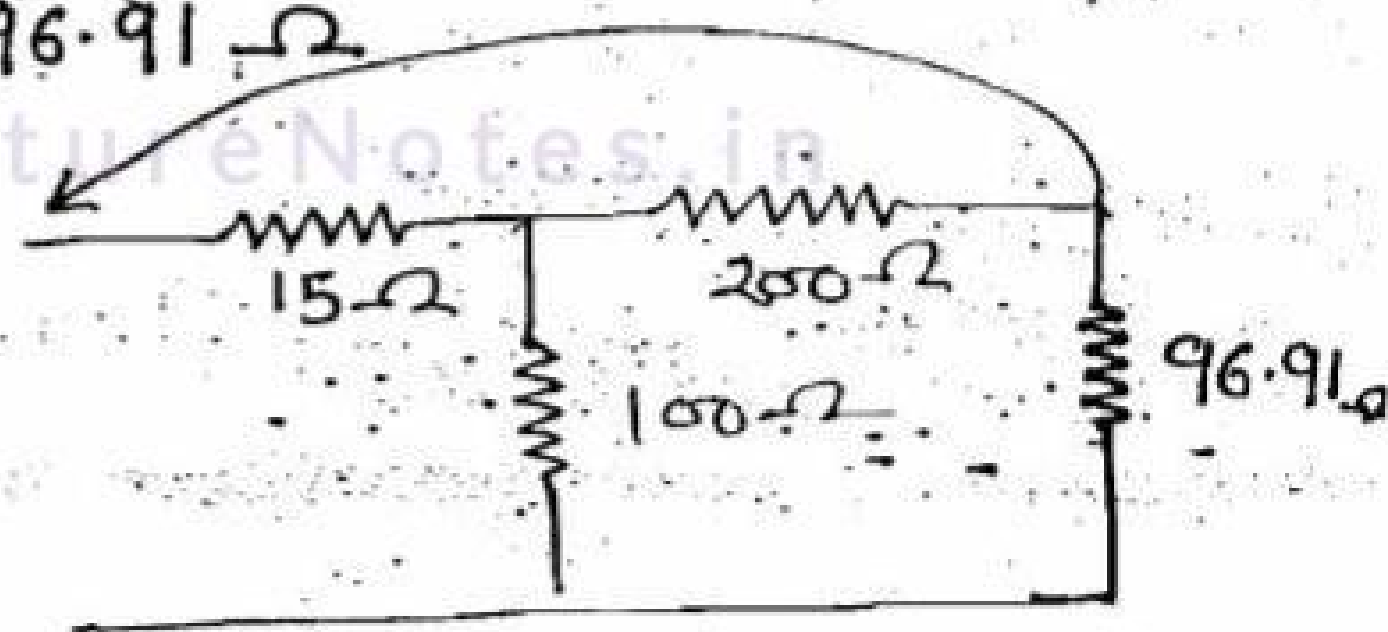
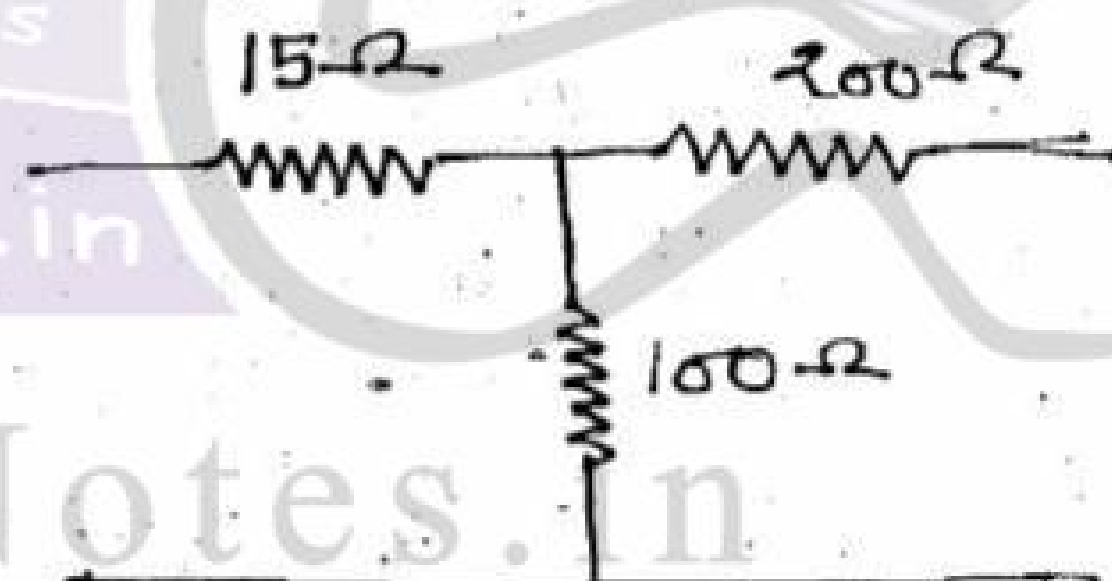
It is geometric mean of S.C and O.C i/p impedances.

eg:-

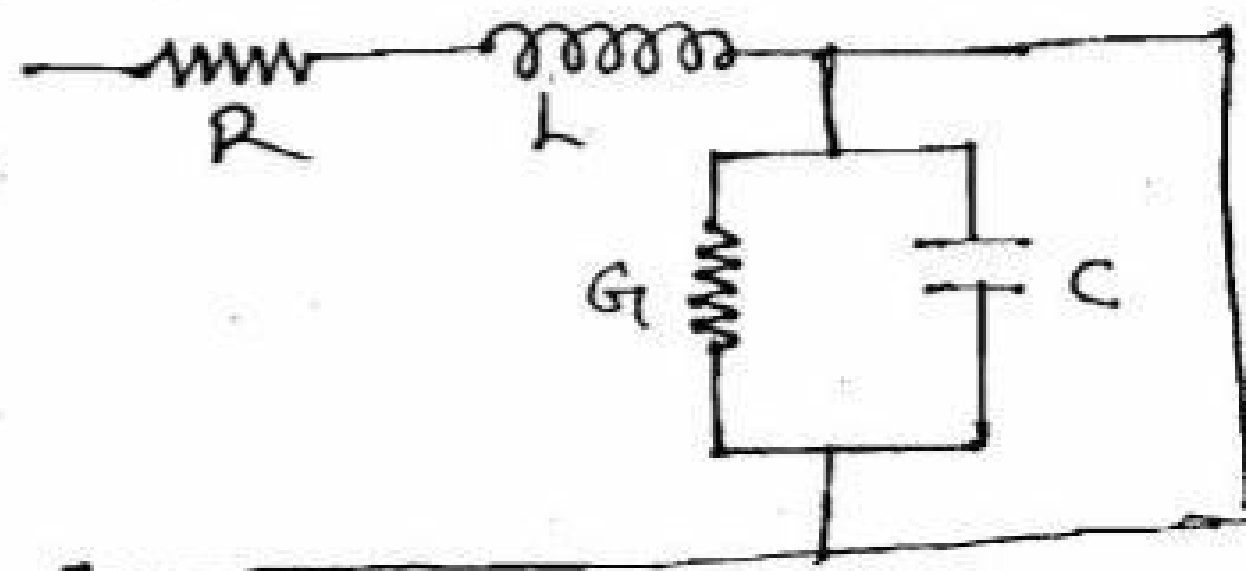
$$Z_{oc} = 115 \Omega$$

$$Z_{sc} = 15 + \frac{100 \cdot 200}{100 + 200} = 81.67 \Omega$$

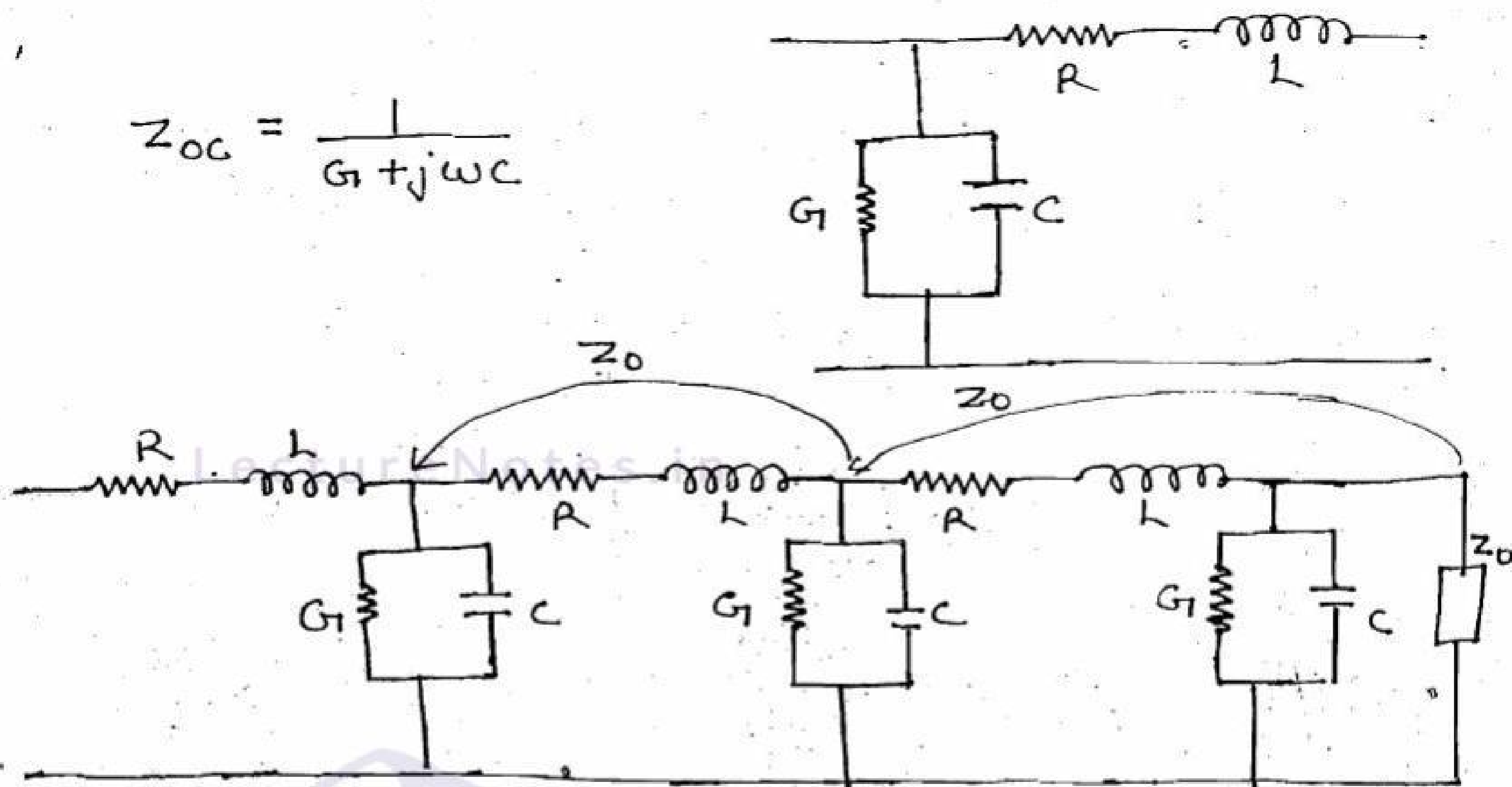
$$Z_0 = \sqrt{81.67 \times 115} = 96.91 \Omega$$



$$Z_{sc} = R + j\omega L$$



$$Z_{0C} = \frac{1}{G_1 + j\omega C}$$



Note:-

- Any N/w either discrete or continuous when terminated with its characteristic impedance the same impedance appears on other end
- For a transmission line whose every RLCG branch having Z_0 characteristic impedance when terminated with Z_0 at one end the loading effect appears to be the same anywhere on the line.
- Lossless line and distortionless line

$$Y = \sqrt{R/L}$$

Lossless line and distortionless line:-

$$Y = \sqrt{(R + j\omega L)(G_1 + j\omega C)} = \text{propagation constant} \\ = \alpha + j\beta$$

If $\alpha = 0$ everywhere on the line, the line is said to be lossless

$$(i) R = G_1 = 0$$

$$(ii) \left. \begin{array}{l} \omega L \gg R \\ \omega C \gg G_1 \end{array} \right\} \text{High frequency lines}$$

$$V = j\omega \sqrt{LC} \quad (\text{purely imaginary})$$

$$= j\beta$$

$$\beta = \omega \sqrt{LC}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_R}} = \frac{3 \times 10^8}{\sqrt{\epsilon_R}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} = \text{real}$$

$$= \sqrt{\frac{\mu_0 \ln(b/a) \ln(b/a)}{2\pi \cdot 2\pi \epsilon}}$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_R}} \cdot \frac{\ln(b/a)}{2\pi}$$

$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_R}}$$

→ Co-Axial cable

→ Z_0 is a design aspect of the line and depends on the cable dimensions but not on R, L, G, C or operational frequency of the line.

$$Z_0 = \frac{120 \ln(b/a)}{\sqrt{\epsilon_R}}$$

→ Parallel wire

Distortionless line:-

If the phase is linear the wave is said to be distortionless i.e. $\beta \propto \omega$ → For distortionless line

→ Equal Rise time = Full time is characteristic of distortionless line.

Series arm time constant = Shunt arm time constant

$$\frac{L}{R} = \frac{C}{G} \Rightarrow \boxed{LG = RC}$$

$$Y = \sqrt{R \left(1 + j\omega \frac{L}{R}\right) G \left(1 + j\omega \frac{C}{G}\right)}$$

$$= \sqrt{RG} \left(1 + j\omega \frac{L}{R}\right) = \text{Complex}$$

$$Z_0 = \sqrt{\frac{R \left(1 + j\omega \frac{L}{R}\right)}{G \left(1 + j\omega \frac{C}{G}\right)}} = \sqrt{\frac{R}{G}} = \text{Real}$$

Note:-

Every lossless line $\Rightarrow$ Distortionless $\nRightarrow$ lossless

$$R = G = 0$$

$$\beta = \omega \sqrt{LC}$$

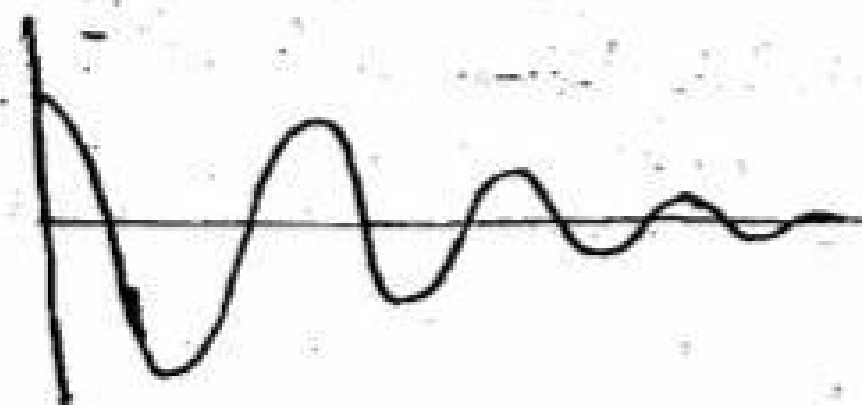
$$LG = RC$$

$$\beta \propto \omega$$

Workbook:-

$$1) \quad Z_{in} = Z_0 \left[\frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)} \right]$$

$$\cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

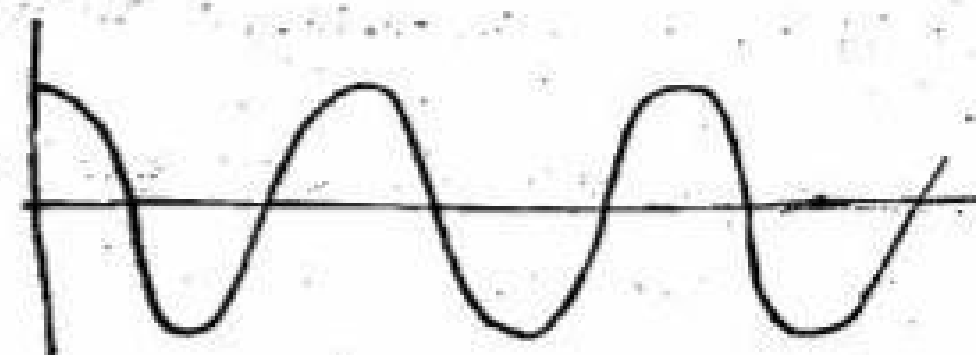


$$\sinh(j\theta) = j \sin \theta$$

$$\cosh(j\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$



$$= \cos \theta$$



$$Z_{in} = Z_0 \left[\frac{Z_L \cos \beta l + j Z_0 \sin \beta l}{Z_0 \cos \beta l + j Z_L \sin \beta l} \right]$$

$$(i) \quad l = \frac{\lambda}{8} \quad \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0}{Z_0 + j Z_L} \right]$$

$$(ii) \quad l = \lambda/4 \quad \beta l = \frac{\pi}{2} \quad Z_{in} = \frac{Z_0^2}{Z_L}$$

$$(iii) \quad l = \frac{\lambda}{2} \quad \beta l = \pi \quad Z_{in} = Z_L$$

$$Z_L = j50$$

$$Z_0 = 50 \Omega$$

$$Z_{in} = 50 \left[\frac{j50 + j50}{50 + j50} \right] = \infty \rightarrow o/c$$

$$\frac{Z_{in}}{(l/\lambda)} = \frac{50 \times 50}{j50} = -j50 \Omega$$

$$Z_{in} = Z_L = j50 \Omega$$

$$(l/\lambda)$$

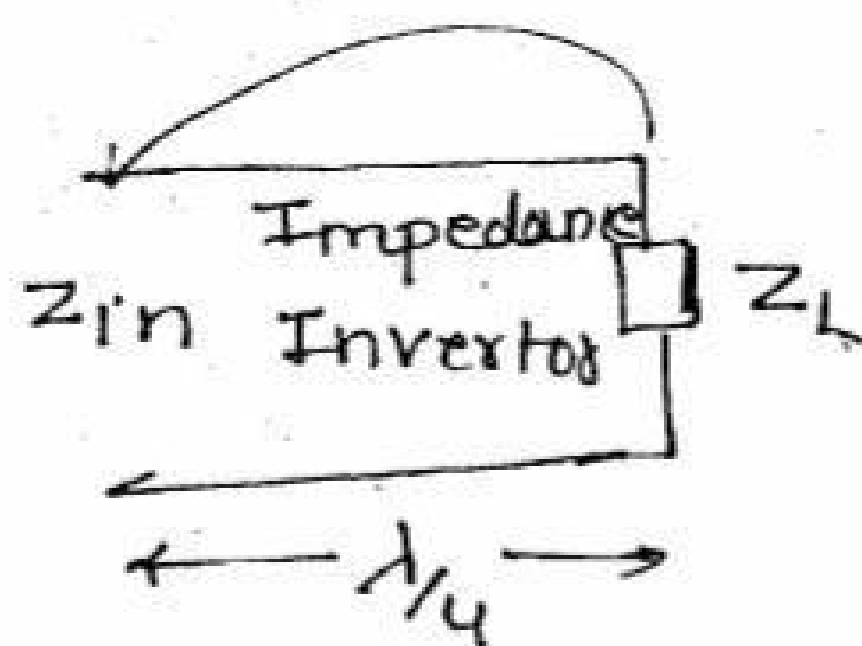
Note 1:-

→ βl is a critical aspect of the line that decides the nature and performance of the line and it is called as electrical path length.

→ length/ λ relationship is a critical aspect.

Note 2:-

$$l = \lambda/4 \quad Z_{in} = \frac{Z_0^2}{Z_L}$$



It's one end impedance has opposite nature to other end impedance hence called as impedance inverter

Note 3:-

$$l = \lambda/2$$

$$Z_{in} = Z_L$$

It is called as impedance reflector

Note 4:-

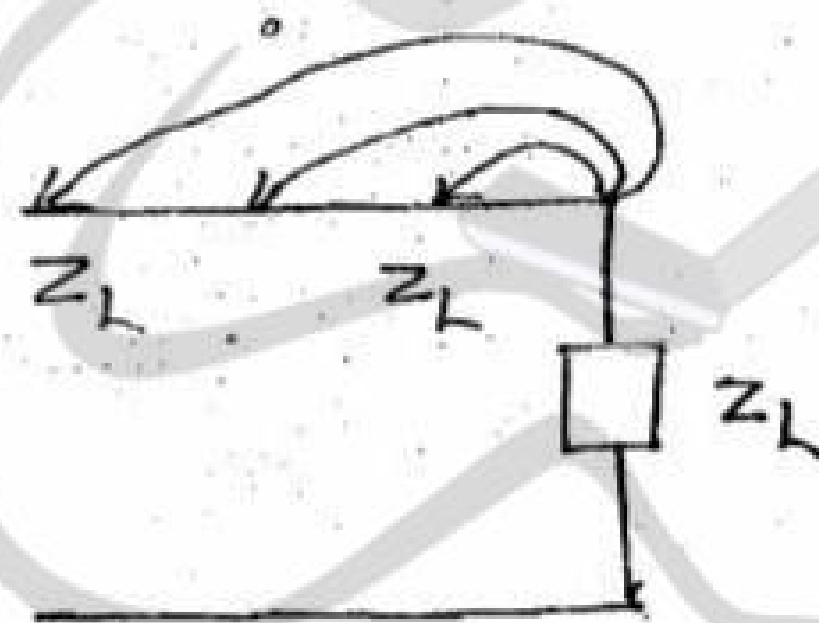
$$\text{If } l \rightarrow 0$$

$$\text{or } \lambda \rightarrow \infty$$

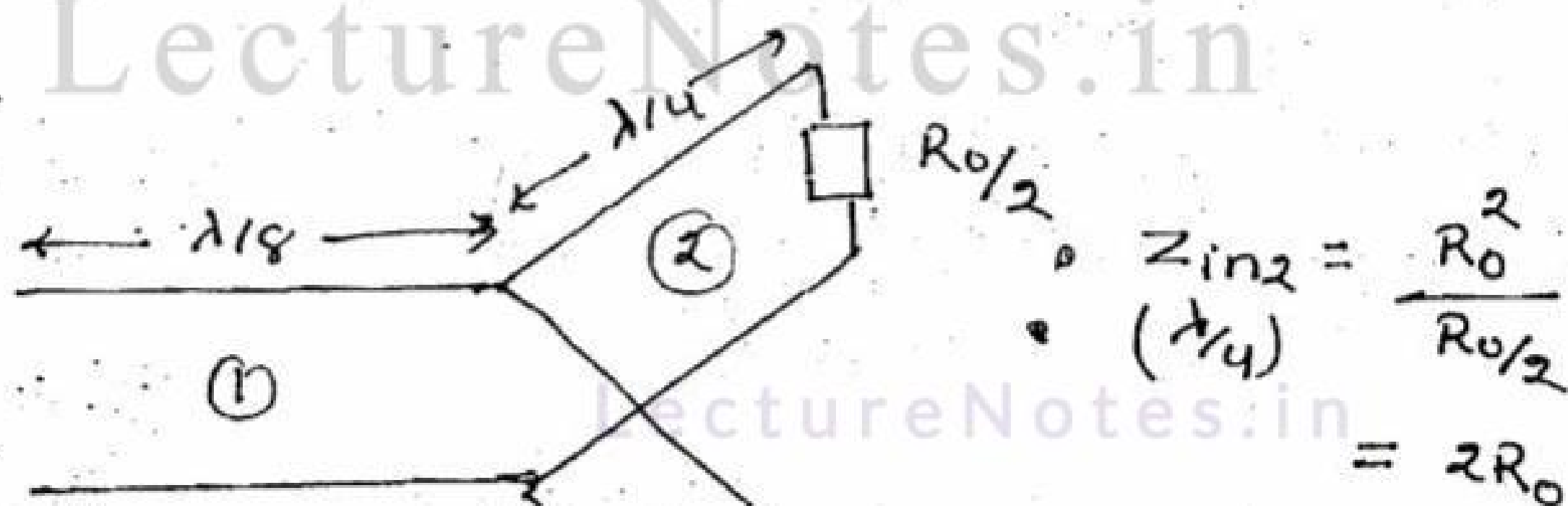
$$\text{where } \beta l \rightarrow 0$$

$$Z(x) = Z_L$$

For electrically short line the wire is a simple connecting wire or a short ckt.



LectureNotes.in



$$Z_{in1}(\lambda/8) = Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right]$$

$$Z_{in2}(\lambda/4) = \frac{R_0^2}{R_0/2} = 2R_0$$

$$Z_{in3}(\lambda/2) = 2R_0$$

$$Z_L = Z_{in2} \parallel Z_{in3}$$

$$= R_0 \left[\frac{R_0 + jR_0}{R_0 + jR_0} \right]$$

$$= R_0$$

When $Z_{in2} = \frac{R_0^2}{2R_0} = \frac{R_0}{2}$ (all Interchanged)

$$Z_{in3} = \frac{R_0}{2} \quad (\lambda/2)$$

$$Z_{in1} = Z_0 \left[\frac{Z_{L1} + jZ_0}{Z_0 + jZ_{L1}} \right] = R_0 \left[\frac{R_0/4 + jR_0}{R_0 + jR_0/4} \right] \quad (\lambda/8)$$

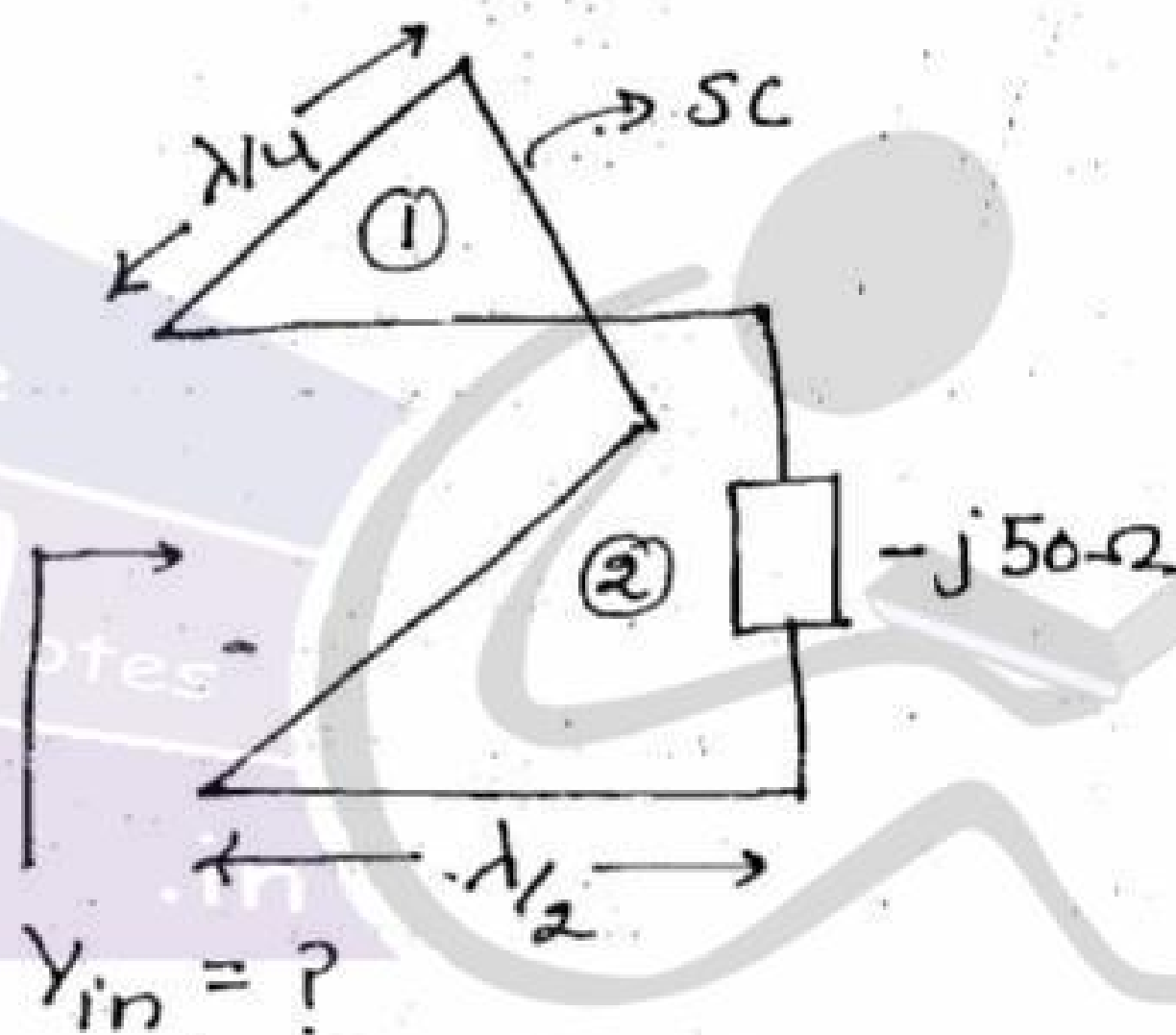
$$= R_0 \left[\frac{1 + j4}{4 + j} \right]$$

$$Y_{in} = Y_{in1} + Y_{in2}$$

$$Y_{in1} = \frac{Z_L}{Z_0^2} = 0 \quad (\lambda/4)$$

$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{-j50} \quad (\lambda/2)$$

$$= j0.02$$



$Y_{in} = ?$

$$Z_{in} = (Z_{in1} || Z_{in2})$$

$$= (\infty || -j50) = -j50$$

$$Y_{in} = j0.02 \Omega$$

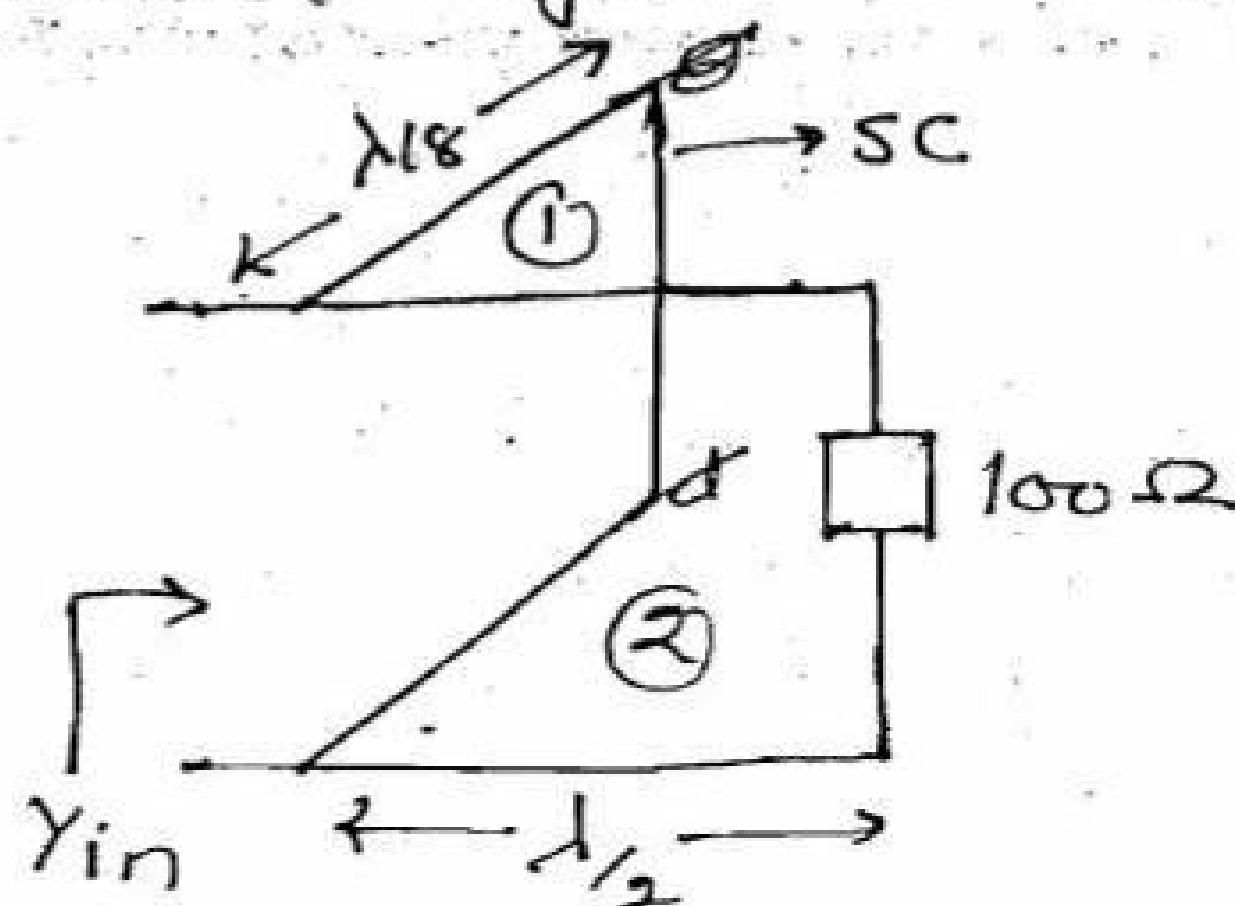
Stub $\Rightarrow$ SC line of finite length

$$Y_{in} = Y_{in1} + Y_{in2}$$

$$Z_{in1} = Z_0 \left[\frac{0 + jZ_0}{Z_0 + j0} \right] \quad (\lambda/8)$$

$$= jZ_0 = j50 \Omega$$

$$Y_{in1} = \frac{1}{j50} = -j0.02$$



$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{100} = 0.01$$

$$Y_{in} = Y_{in1} + Y_{in2} = 0.01 - j0.02 \text{ S}$$

8.

$$l = \lambda/4$$

$$Z_{in1} = R_0 \left[\frac{0 + jR_0}{R_0 + j0} \right]$$

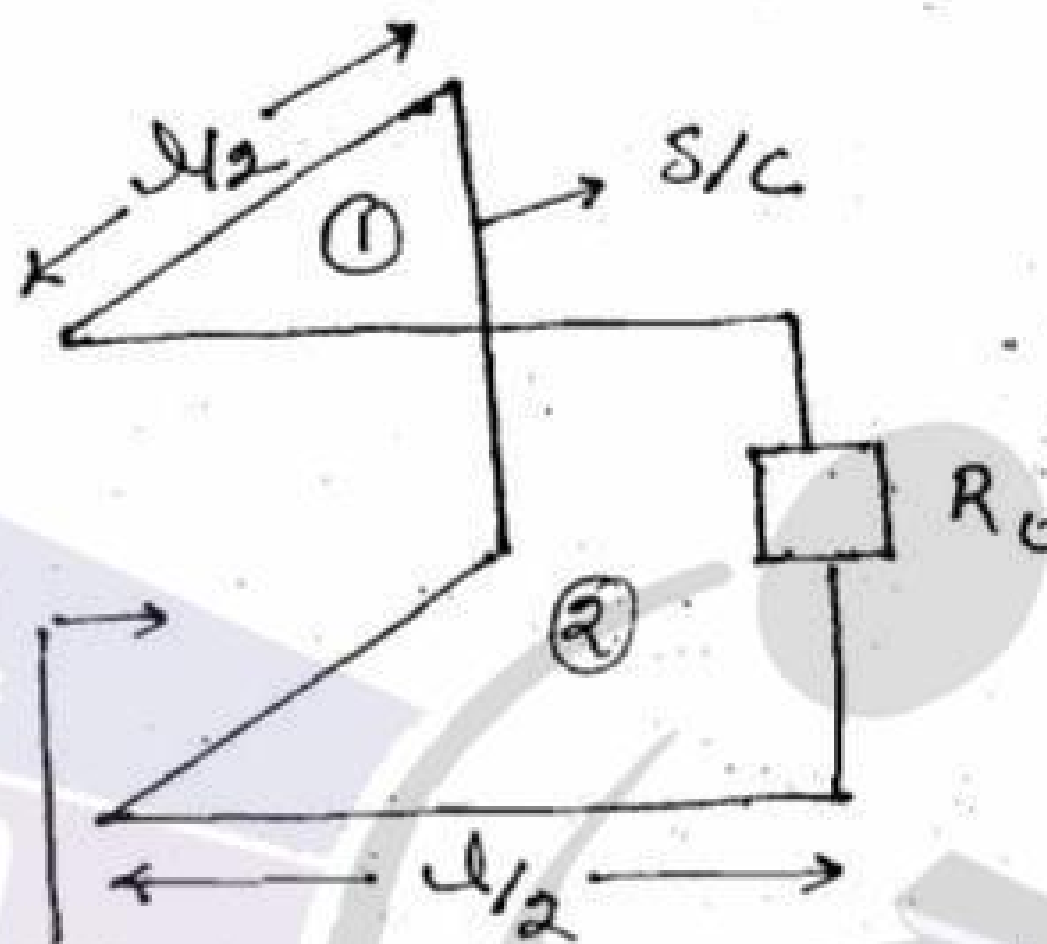
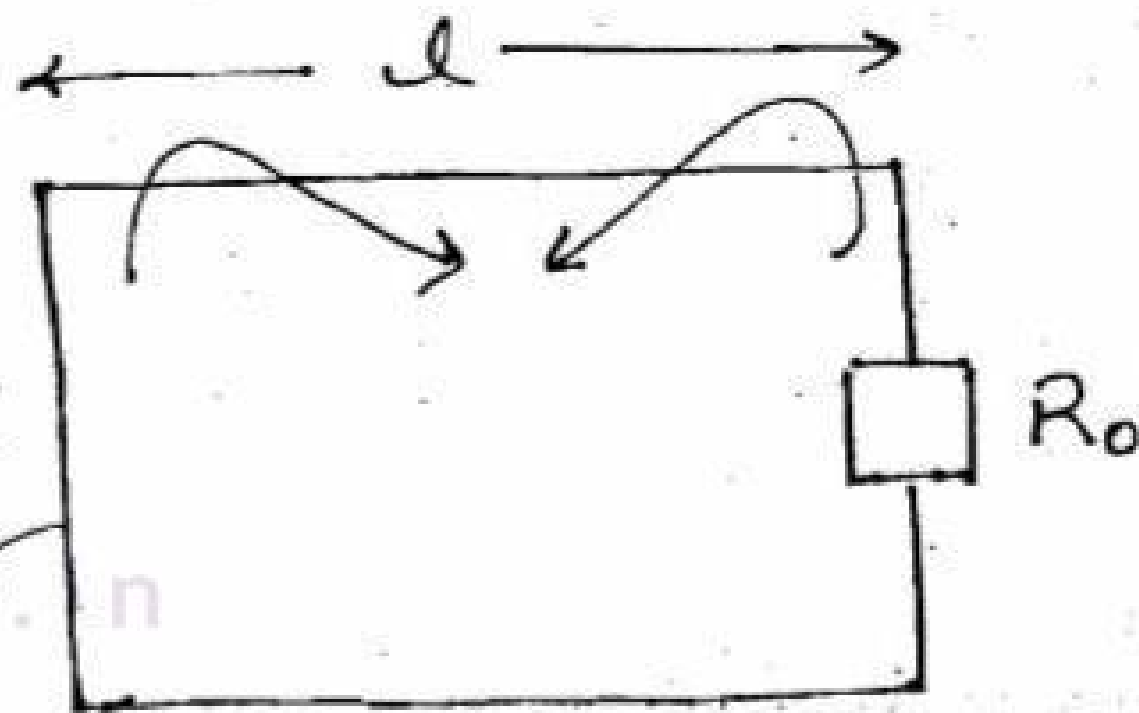
$$= jR_0$$

$$Z_{in2} = R_0$$

$$Z = \frac{jR_0 \cdot R_0}{jR_0 + R_0} = \frac{jR_0}{j+1}$$

$$l = \lambda \quad Z = R_0$$

$$l = \lambda/8 \quad Z = SC$$



9.

$$V_p = 2 \times 10^8 \quad f = 10 \text{ MHz} \quad \lambda = 20 \text{ m} \quad \leftarrow \begin{matrix} l = \lambda/2 \\ l = 10 \text{ m} \end{matrix}$$

$$Z_{in} = Z_L = 30 - j40 \Omega$$

10.

$$f' = 5 \text{ MHz} \quad \lambda' = 40 \text{ m} \quad l = \frac{\lambda'}{4}$$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50 \times 50}{30 - j40} = 30 + j40$$

11.

$$Z_{in} = j60 = Z_{sc} = Z_0 \tanh(j\beta l) = jZ_0 \tan \beta l$$

$$\beta l = \frac{\pi}{4}$$

$$= jZ_0$$

$$Z_0 = 60 \Omega$$

$$f' = 2f$$

$$12 \text{ MHz} \rightarrow 24 \text{ MHz}$$

$$\lambda' = \lambda/2$$

$$\beta l = 2\beta l = \pi/2$$

$$\therefore Z_{in} = Z_{sc} = jZ_0 \tan \pi/2 = \infty$$

$$f = 12 \text{ MHz}$$

$$Z_{in} = j60 = j\omega L \Rightarrow L = \frac{60}{2\pi \times 12 \times 10^6} = \frac{2.5}{\pi} \mu\text{H}$$

Summary:

$$Z_{sc} = jZ_0 \tan \beta l$$

$$0 < \beta l < \frac{\pi}{2}$$

$$0 < l < \lambda/4$$

Z_{in} is inductive

$$\frac{\pi}{2} < \beta l < \pi$$

$$\lambda/4 < l < \lambda/2$$

Z_{in} is capacitive

$$Z_{oc} = -jZ_0 / \tan \beta l$$

Z_{in} inductive $\rightarrow Z_{in}$ is capacitive

Z_{in} is capacitive $\rightarrow Z_{in}$ is inductive

one end of a line is S.C or O.C then the other end is purely reactive

Q. (a) $Z_{in} = -jZ_0$ For s/c line $\lambda/8$ length X

✓ (b) $Z_{in} = \pm j\infty$ for SC line $1/4$ length

$$Z_{sc} = j Z_0 \tan \beta l$$

$$Z_{in} = \frac{Z_o^2}{Z_h}$$

$X_{CC}) \quad Z_{in} = -jZ_0 \rightarrow \text{o/c line} \rightarrow \lambda/2 \text{ length}$

$$Z_{in} = Z_L$$

✓ (d). $Z_{in} = Z_o$

$$\frac{1}{15} \quad Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC}{C^2}} = \frac{\sqrt{\mu_0 \epsilon_0 \epsilon_r}}{C} = \frac{\sqrt{\epsilon_r}}{v_p \cdot C}$$

16. (a) $\alpha = \text{even}$, $\beta = \text{odd}$ X

(b) $\alpha = \text{constant}$ $\beta = k\omega$ ✓

c.c) $\frac{h}{c} = \text{constant} = \frac{R}{G}$

(d) Matched line ($Z_L = Z_0$) $\times$

17. $\beta d = \omega \sqrt{LC} d = 108^\circ$

Reflection Coefficient:-

$$V(x) = \underbrace{V_0 e^{j\beta x}}_{V_f} + \underbrace{V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}}_{V_r}$$

$$\frac{V_r}{V_f} = \text{Reflection coefficient for the voltages anywhere on the line} = \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j2\beta x}$$

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$$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}$$

$$\text{At } x=0 \quad \Gamma_{\text{at load}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \text{complex}$$

$$= |\Gamma| \angle \theta = \left| \frac{V_r}{V_f} \right| \angle \theta$$

$|\Gamma|$ $\rightarrow$ It is the ratio of reflected voltage to the forward voltage amplitude.

$\angle \theta$ $\rightarrow$ It is the time delay or phase difference b/w the reflected and forward voltage.

Γ is the measure of mismatch b/w the expected impedance and the actual impedance Z_L at the load.

$\Gamma(x)$ is the measure of mismatch b/w the expected impedance Z_0 and the actual impedance $Z(x)$ anywhere on the line.

Note:-

$$(i) \quad \Gamma_I = \frac{Z_0 - Z_L}{Z_0 + Z_L} = -\Gamma_V$$

If V_f and I_f they are in phase

$$V_0 e^{j\beta x} + V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}$$

$$I_0 e^{j\beta x} - I_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}$$

then V_r and I_r are out of phase by 180°

(ii)

$$0 \leq |\Gamma| \leq 1$$

Matched line Complete Reflection (Mismatch)

Lecture - 13 Notes

Four cases of complete mismatch on a line ($|\Gamma| = 1$):

Case - (1) :-

$$Z_L = jR_0 = \text{Inductive load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$|\Gamma| = 1$$

$$V_f \rightarrow \sin$$

$$V_r \rightarrow \cos$$

$$\Gamma = \frac{jR_0 - R_0}{jR_0 + R_0} = \frac{j-1}{j+1}$$

$$= \frac{\sqrt{2} \angle 135^\circ}{\sqrt{2} \angle 45^\circ}$$

$$= 1 \angle 90^\circ = j$$

Case-(ii):-

$$Z_L = -jR_0 = \text{capacitive load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-j-1}{-j+1}$$

$$= \frac{\sqrt{2} \angle 45^\circ}{\sqrt{2} \angle 135^\circ} = -j$$

Note:-

Real Power dissipation in any resistive load is always zero.

Case-(iii):-

$$Z_L = 0 \Rightarrow \text{s.c load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{0 - Z_0}{0 + Z_0} = -1 = 1 \angle 180^\circ$$

$$V_f + V_r = 0$$

$$V_f \rightarrow \sin$$

$$V_r = -\sin$$

Voltage at s.c is always zero

Case-(iv):-

$$Z_L = \infty \Rightarrow \text{o.c load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1 \angle 0$$

$$V_f + V_r = V_{\max}$$

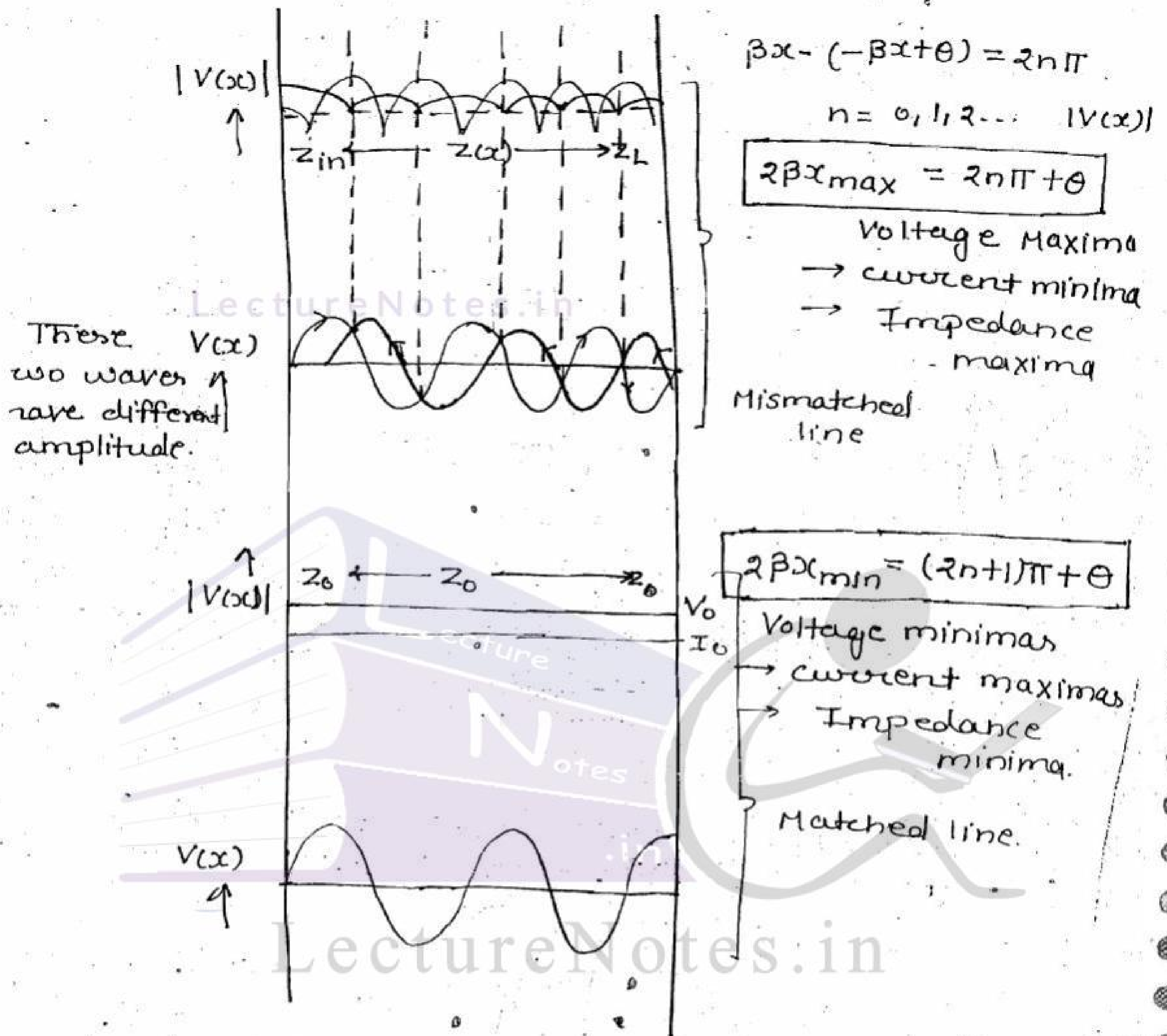
Voltage at o.c is always maximum

$$|\Gamma| = 1$$

$$V_f \rightarrow \sin$$

$$V_r = \sin$$

Standing Wave and SWR:-



$$V(x) = V_0 e^{j\beta x} + V_0 |\Gamma| e^{j\theta} e^{-j\beta x}$$

$$= V_0 e^{j\beta x} + V_0 |\Gamma| e^{j(-\beta x + \theta)}$$

The magnitude plot of a single harmonic matched line as a constant value straight line graph. The magnitude plot of two harmonics mismatched line as shown below:-

$$|V(x)| = V_{max} = V_0 + V_0 |\Gamma| \text{ when they are inphase}$$

$$\beta x - (-\beta x + \theta) = 2n\pi$$

$$|V(x)|$$

$$n = 0, 1, 2$$

$$2\beta x_{\max} = 2n\pi + \theta$$

Voltage Maxima Positions.

$$|V(x)| = V_{\min} = V_0 - V_0 |\Gamma| \text{ when they are out of phase}$$

$$2\beta x_{\min} = (2n+1)\pi + \theta$$

→ Voltage Minima

→ The magnitude plot of two harmonics travelling in opposite direction having periodic interference results in addition and cancellation leading to standing wave formation.

Note 1:-

The current also has identical standing wave pattern with the voltage maxima coincide with current minima.

Note 2:-

As the impedance is different at different point on line voltage and current distribution is also extremely spread on the line such that

$$Z_{\max} = \frac{V_{\max}}{I_{\min}}$$

$$Z_{\min} = \frac{V_{\min}}{I_{\max}}$$

Note 3:-

$$\left. \begin{array}{l} \beta x = \frac{2\pi}{\lambda} x \rightarrow \text{Periodic} - 2\pi \\ x \rightarrow \text{Periodic} - \lambda \end{array} \right\} \text{ Harmonic}$$

$$\left. \begin{array}{l} 2\beta x = \frac{2\pi}{\lambda/2} x \rightarrow \text{Periodic} - 2\pi \\ x \rightarrow \text{Periodic} - \lambda/2 \end{array} \right\} \text{ Standing wave}$$

Maxima to Maxima distance on standing wave is $\lambda/2$

Standing Wave Ratio (SWR):-

It can be VSWR or CSWR

$$= \frac{V_{\max}}{V_{\min}} \text{ or } \frac{I_{\max}}{I_{\min}}$$

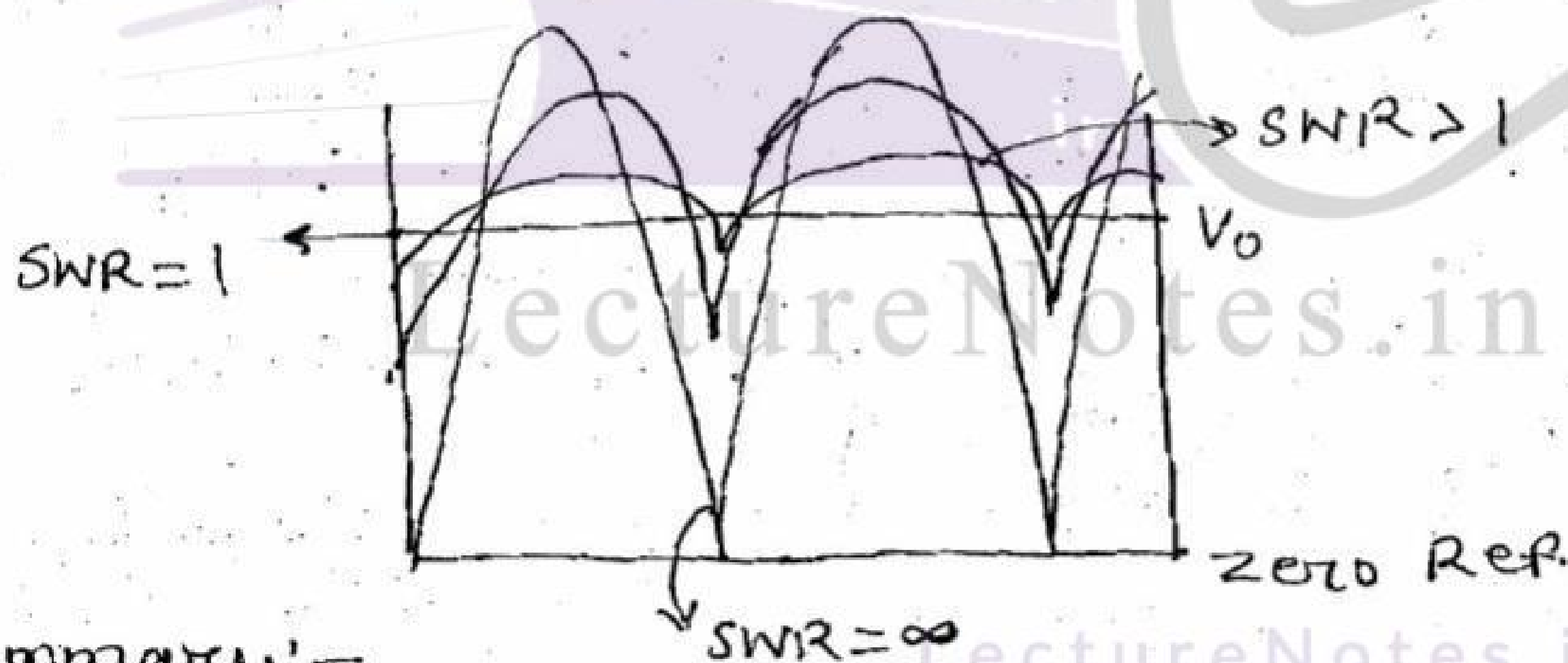
$$\boxed{\text{SWR} = \frac{V_0 + V_0 |\Gamma|}{V_0 - V_0 |\Gamma|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

$$0 \leq |\Gamma| \leq 1$$

$$\boxed{1 \leq \text{SWR} < \infty}$$

Matched line

Complete Mismatch



Summary:-

$$\left. \begin{array}{l} Z_0 \downarrow \uparrow \\ V_0 \uparrow \downarrow \\ I_0 \uparrow \downarrow \end{array} \right\} \text{SWR}$$

$$Z_{\max} = \frac{V_{\max}}{I_{\min}}$$

$$= \frac{V_0 + V_0 |\Gamma|}{I_0 - I_0 |\Gamma|}$$

SWR is a measure of mismatch on the line

$$Z_{\max} = Z_0 \cdot \text{SWR}$$

$$Z_{\min} = Z_0 / \text{SWR}$$

$$ESWR = 5 = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \frac{2}{3} \angle 0^\circ \text{ or } 180^\circ$$

$$n_2 + n_1 \rightarrow 120\pi \text{ (real)}$$

$$\frac{120\pi}{\sqrt{\epsilon_r}} \text{ (real)}$$

As $n_2 < n_1$

$$\Gamma = -\frac{2}{3} = \frac{n_2 - 120\pi}{n_2 + 120\pi} \Rightarrow n_2 = 24\pi$$

$$VSWR = 4$$

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$

$$\Gamma = \frac{Z_L - 50}{Z_L + 50} = \frac{3}{5} \angle 0^\circ \text{ or } 180^\circ$$

real → real

$|V(z)|$

4

load

V_{min} is exactly at the load, Z_{min} is at the load

$$Z_L < Z_0$$

$\Rightarrow \Gamma$ is negative

$$\Gamma = -\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L = 12.5 \Omega$$

→ 1st voltage maxima occurs at $\lambda/4$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = 2n\pi + \theta \Rightarrow \boxed{\theta = \pi}$$

Note:-

→ If Z_L, Z_0 are real
 $Z_L > Z_0$

Γ_0 is real & positive

Γ 's phase is 0

$$2\beta x_{\max} = 2n\pi + \theta \Rightarrow x_{\max} = 0$$

First voltage maxima exactly occurs

eg:- OC line where $\Gamma = +1$

→ If Z_L, Z_0 are real

$$Z_L < Z_0$$

Γ is real and negative

First voltage minima exactly occurs

eg:- SC line where $\Gamma = -1$

$$VSWR = 1 + \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$$

$$1 - \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$$

If Z_L, Z_0 are real & $Z_L > Z_0$

$$VSWR = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$1 - \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$VSWR = \frac{Z_L}{Z_0}$$

If Z_L, Z_0 are real

$$Z_L < Z_0$$

$$VSWR = ?$$

24.

$$\Gamma(\lambda/8) = ?$$

Γ

$$\Gamma \text{ at load} = 0.6 \angle 60^\circ$$

Γ

$$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-j2\beta x}$$

$$\Gamma(\lambda/8) = \Gamma(0) e^{-j2\beta \cdot \frac{\lambda}{8}}$$

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$$= 0.6 e^{j60} e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}}$$

$$= 0.6 e^{j60} e^{-j90}$$

$$= 0.6 \angle -30^\circ$$

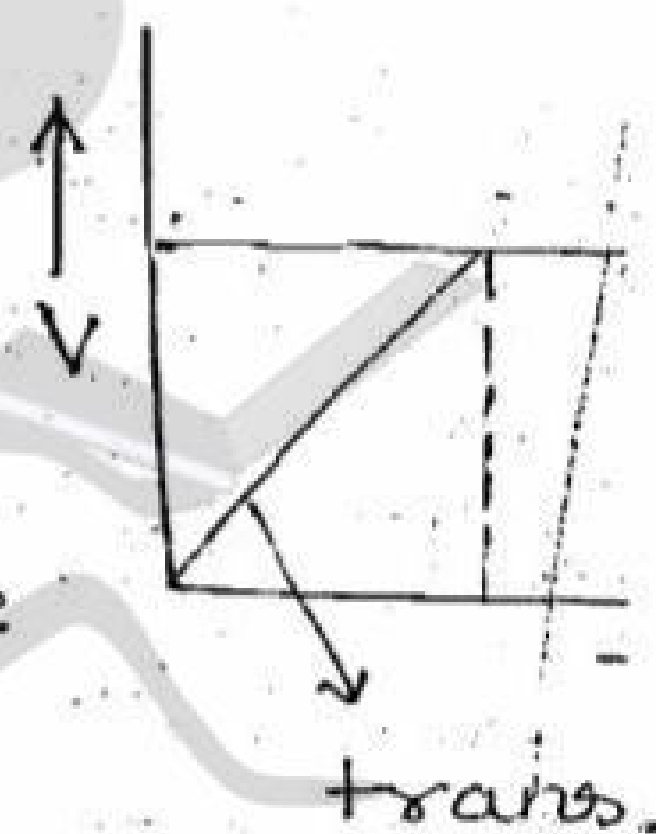
21. 1st
7cm
h=0

25.

30V D.C Battery

D.C Transients

During the transient state even DC has time variations and all AC fundamentals are applicable



$$V_f + V_r = \text{Adding}$$

$$\Gamma = \frac{10}{30} = \frac{Z_L - 50}{Z_L + 50}$$



$$\Rightarrow Z_L = 100 \Omega$$

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$$\overline{V_f} = \overline{V_r}$$

$$I_L = \frac{30}{Z_L} \Omega$$

(Steady state)

n=0

2. 2

$$= \frac{30}{100}$$

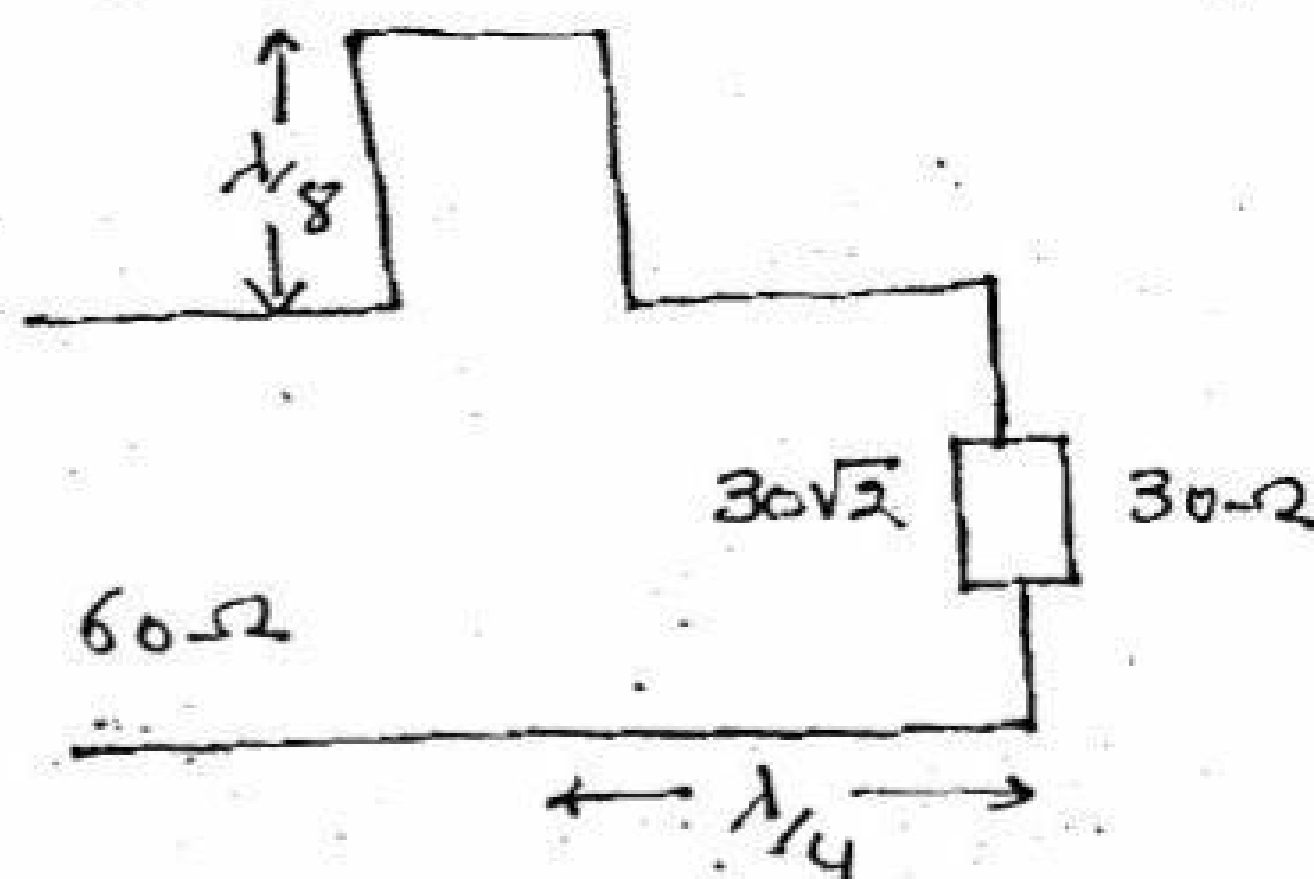
$$= 0.3 \text{ Amp.}$$

26.

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$= \frac{\sqrt{17} + 1}{\sqrt{17} - 1}$$

$$= 1.63$$



$$Z_{L1} = Z_{in2} + Z_{in3}$$

$$= 60 + j30 \Omega$$

$$Z_{in2} = j30 \Omega$$

$$Z_{in3} = \frac{(30\sqrt{2})^2}{30} = 60 \Omega$$

$$\Gamma = \frac{Z_{L1} - jZ_0}{Z_{L1} + jZ_0} = \frac{60 + j30 - 60}{60 + j80} = \frac{j}{j+4}$$

$$|\Gamma| = \frac{1}{\sqrt{17}}$$

27.

$$V_f = 25 \sin(\omega x - 75^\circ)$$

$$\Gamma = 0.6 \angle -30^\circ$$

$$V_r = ?$$

$$\frac{|V_r|}{|V_f|} = 0.6 \quad |V_r| = 15$$

$$V_r \text{ phase} = V_f \text{ phase} + \Gamma \text{ phase}$$

$$= -75^\circ - 30^\circ = -105^\circ$$

$$V_r = 15 \sin(-\omega x - 105^\circ)$$

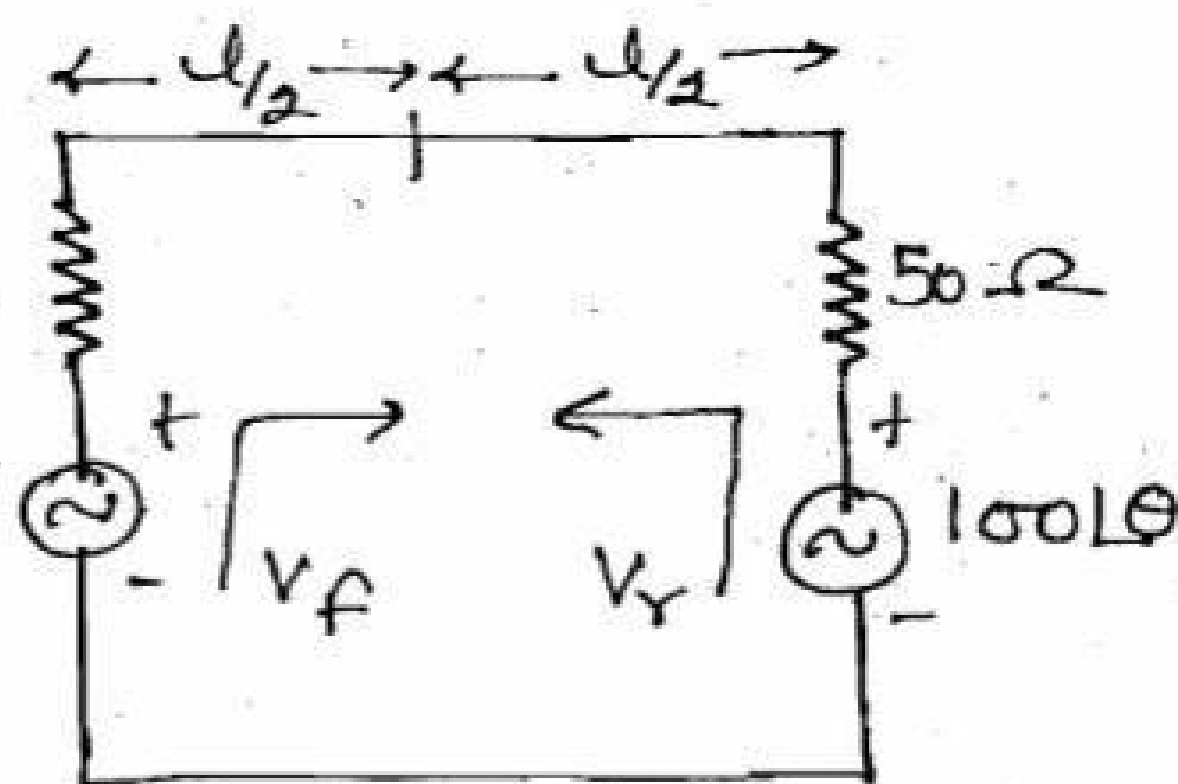
V_f & V_r are inphase
and add up at centre

I_f & I_r cancel
at the centre

$$|I_f| = |I_r|$$

$$\Rightarrow I_{\min} = 0$$

$$Z_{\max} = \frac{V_{\max}}{I_{\min}} = \infty$$



29. $VSWR = \frac{Z_L}{Z_0} = \frac{75}{50} = 1.5$

Smith Chart (Circle Diagrams):

→ It is used to calculate Γ and VSWR for a known Z_L & Z_0

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

→ It is used normalised or relative ratio of Z_L/Z_0 for its calculation

→ It is a rectangular graph or polar plot of Γ_r vs Γ_i having two families of circles

→ constant R circles

where $R = \text{Real} [Z_L/Z_0]$

→ constant X circles

$X = \text{Imag} [Z_L/Z_0]$

$$\Gamma_Y + j\Gamma_i = \frac{R+jX-1}{R+jX+1} = \frac{(R-1) + jX}{(R+1) + jX}$$

$$\left(\Gamma_Y - \frac{R}{R+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{R+1}\right)^2$$

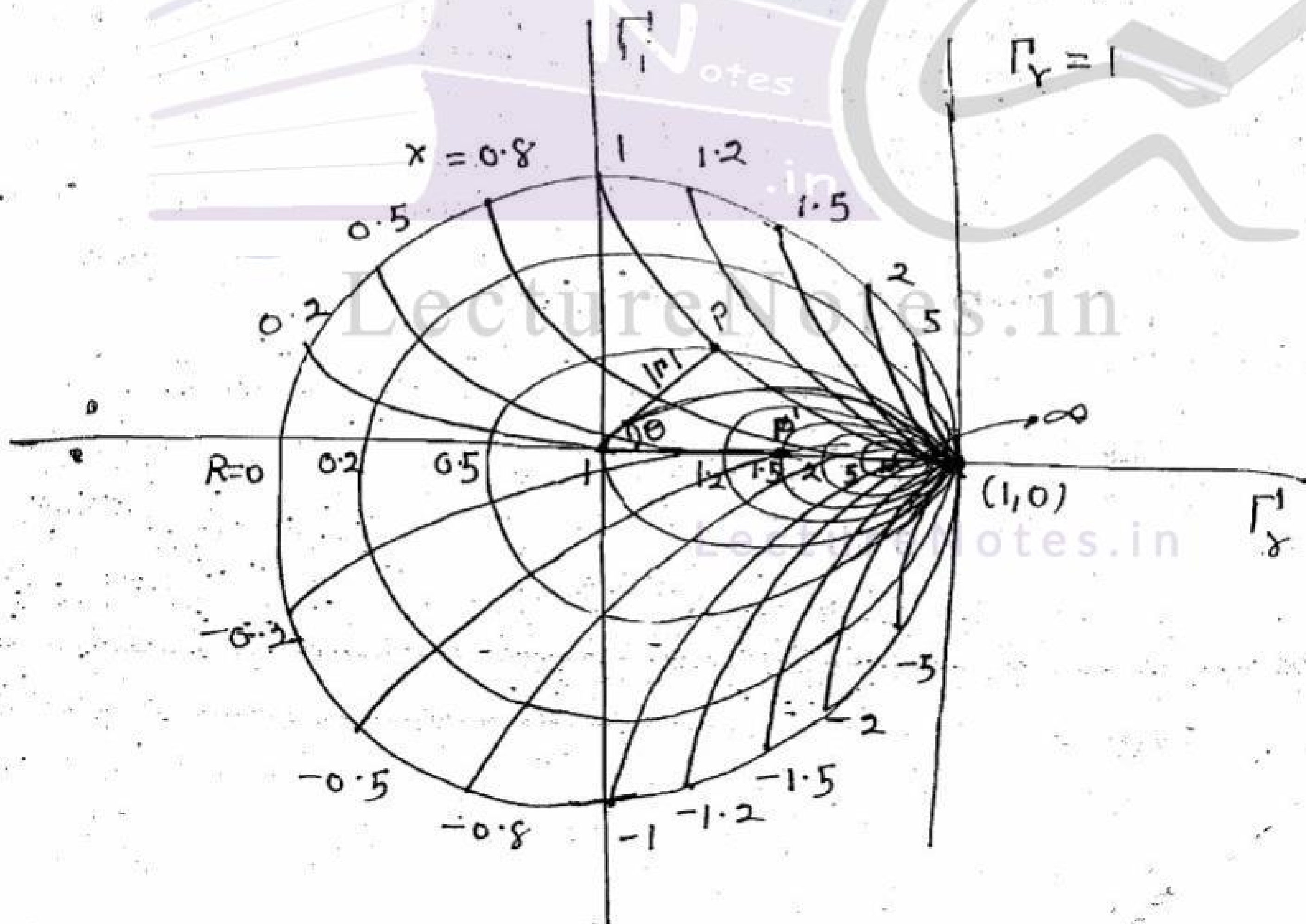
Constant R circles
Equation

$$(\Gamma_Y - 1)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

constant X circles
Equation

Properties of constant R circles:-

- (a) Centres - $\left(\frac{R}{R+1}, 0\right)$
- (b) Radius - $\left(\frac{1}{R+1}\right)$
- (c) They all pass through $(1, 0)$
- (d) Range $(0, \infty)$



1. The $R=0$ circle is the biggest possible R circle and it is periphery or boundary of Smith Chart.
2. All circles $R=0$ to 1 extend into the four quadrants the circles with $R>1$ confined in the 1st and IV quadrants
3. All the R -circles are concurrent with their centres on a straight line and have a common tangent $\Gamma_r = 1$ axis

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Properties of constant X -circles:-

- a) Centres - $(1, \frac{1}{X})$ (b) Radius - $(\frac{1}{X})$
- c) They all pass through $(1, 0)$
- d) Range $(-\infty, \infty)$
1. $x=0$ circle is the biggest possible X circle and it is Γ_r axis
2. All circles with $x=0$ to 1 extend into the first two quadrant but circles with $x>1$ confined in the 1st quadrant
3. All the x -circles are concurrent and have a common tangent with Γ_r axis

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Note:-

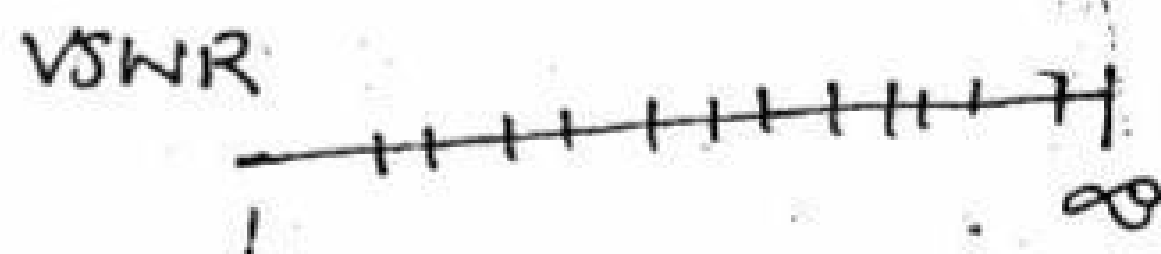
1. The centres of R circles lie on the common tangent of X -circle and vice-versa. Hence R and X constitute a pair of orthogonal families of circles.
2. A horizontal movement on the $x=0$ circle as R value increasing and a clockwise movement on the $R=0$ circle as X increasing inductively (trly)

Calculation of Γ :-

For a known Z_L and Z_0 Calculate

$\frac{Z_L}{Z_0} = R + jX$ and locate the corresponding point of intersection of R and X circle

- Join the point P to origin the OP length $= |\Gamma|$
- OP segments inclination with Γ_r axis is Γ 's phase
- Find $|\Gamma|$



Calculation of VSWR :-

- For a known $|\Gamma|$ identify a point P' on the true $|\Gamma|$ real axis such that OP' length $= |\Gamma|$. The R circle value at P' is VSWR

$$R + jX \Big|_{X=0} = \frac{Z_L}{Z_0} = \text{VSWR}$$

$X=0$
 Γ_r axis

Identify following points on the Smith's Chart :-

(I) $Z_L = jR_0$, $Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \underset{\substack{\downarrow \\ R=0}}{0} + j \underset{\substack{\downarrow \\ X=1}}{1}$$

$$\Gamma = j$$

(II) $Z_L = -jR_0$

$$Z_0 = R_0$$

bottom

(iii)

$$Z_L = 0$$

$$Z_0 = R_0$$

$$\frac{Z_L}{Z_0} = 0 + j0$$

↓ ↓
R X

$$\Gamma = -1$$

Extremely left

(iv)

$$Z_L = \infty$$

$$Z_0 = R_0$$

Extremely right

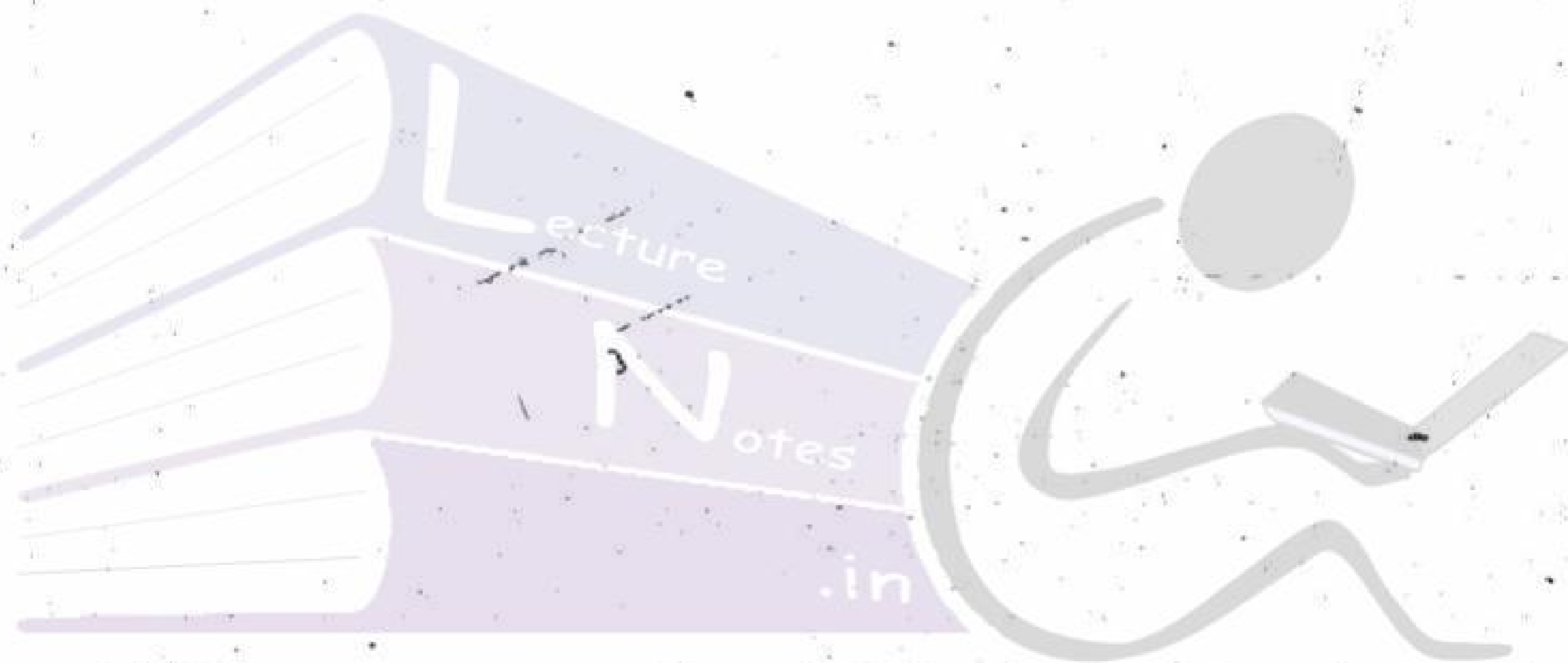
(v)

$$Z_L = R_0$$

$$Z_0 = R_0$$

→ centre

$$\Gamma = 0$$



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LectureNotes.in

Lecture - 14

Impedance Matching Technique:-

Active
(Bias)

CB Amp

CC Amp

Passive

Series

$\lambda/4$

Quarter wave
transformer

Shunt

Stub
matching

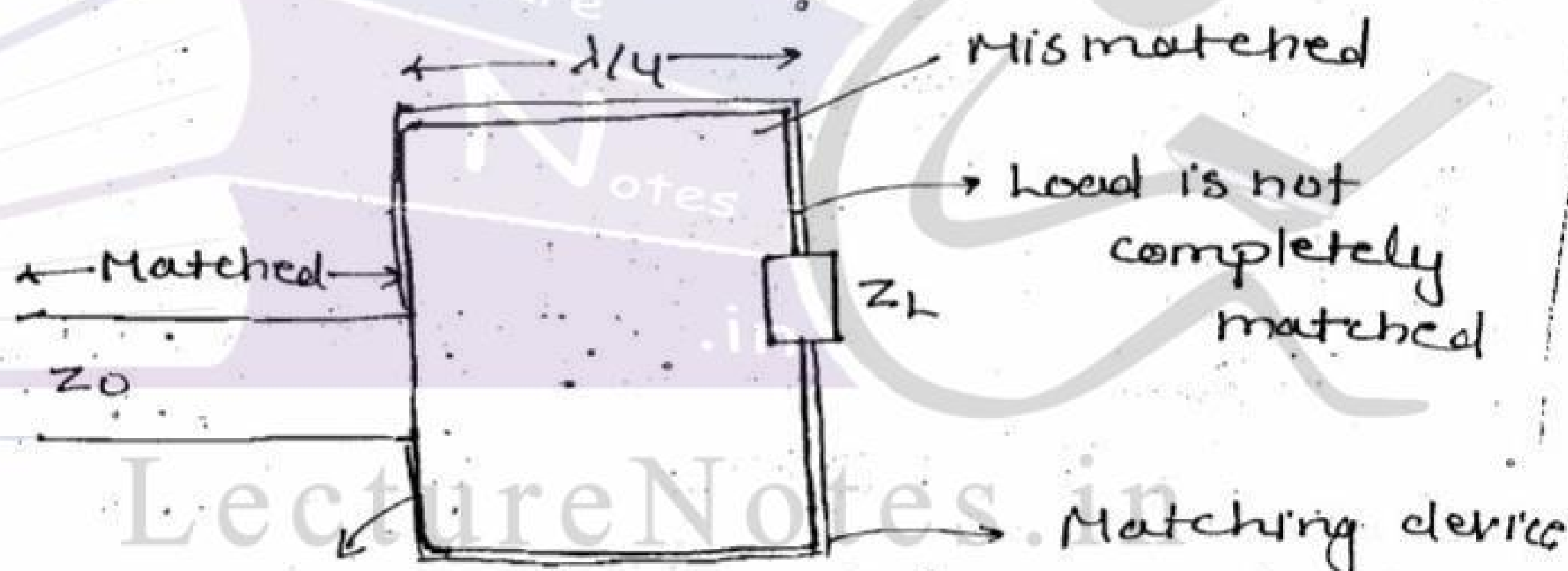
$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_r}}$$

= few 10Ω to 100Ω

$Z_L = \text{few}\Omega - k\Omega$

($Z_L \neq Z_0$
Most often)

Series ($\lambda/4$) Quarter wave transformer:-



The line is matched

The termination of Z_0 line $\hat{=}$ Z_{in} of $\lambda/4$ matching device

$$= \frac{Z_0^2}{Z_L} = Z_0$$

Design needs $\rightarrow$

$$Z_0' = \sqrt{Z_0 Z_L}$$

eg:- $50\Omega \rightarrow \text{line} \quad 100\Omega \rightarrow \text{load}$

$$\Gamma_{(load)} = \frac{100-50}{100+50} = \frac{1}{3}$$

Using a $\lambda/4$ transformer

$$Z_0' = \sqrt{50 \times 100} = 70\Omega$$

$$\Gamma_{(at load)} = \frac{100-70}{100+70} < \frac{1}{3}$$

Disadvantage:-

- 1) The technique is suited for a narrow range of frequencies as the length of the transformer is frequency dependent.

- 2) $Z_0' = \sqrt{Z_0 \cdot Z_L} \rightarrow$ The technique is suited only for lossless line and resistive load.

Shunt - Stub matching:-

- A stub is S.C or o.c line of a pre-calculated length, placed at a precalculated position such that the line is matched from the stub to the source

Design of a stub:-

Step-(1):-

- Identify a length l_s on the line from the load where the impedance has a real part Z_0

$$Z(l_s) = Z_0 \pm jX \quad \text{Matched} \quad \leftarrow l_s \quad \text{Stub matched}$$

$l_s =$ stub position

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \left(\sqrt{\frac{Z_L}{Z_0}} \right)$$

$$Z(l_s) = Z_0 \left(\frac{Z_L \cos \beta l_s + j Z_0 \sin \beta l_s}{Z_0 \cos \beta l_s + j Z_L \sin \beta l_s} \right) \quad \rightarrow \text{S.C}$$

$$Z(l_s) = \text{Real} \left[\frac{Z_L \cos \beta l_s + j Z_0 \sin \beta l_s}{Z_0 \cos \beta l_s + j Z_L \sin \beta l_s} \right] = 1$$

Step-(ii):-

At this identified position place an equal and opposite reactance in shunt such that cancel to the existing reactance i.e.

$$Z(l_s) = Z_0 \pm jX \quad \begin{matrix} \text{---} jX \\ \text{---} jX \end{matrix} \rightarrow \text{Stub}$$

$$Y(l_s) = Y_0 \pm jB \quad \begin{matrix} \text{---} jB \\ \text{---} jB \end{matrix} \rightarrow \text{Stub}$$

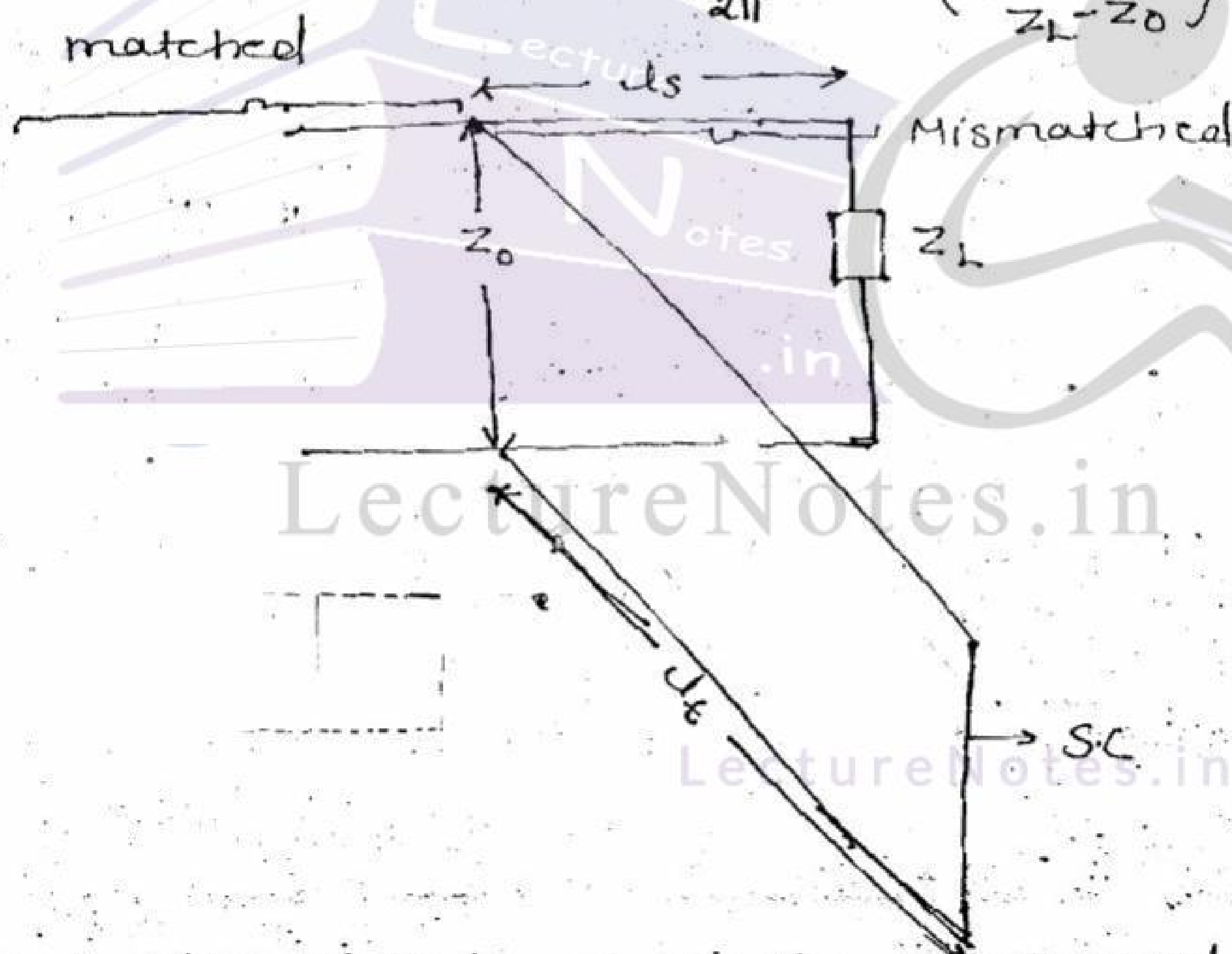
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Step-(iii):-

This reactance is realised from a SC or OC stub of a finite length l_t such that

$$Z_{sc} = jZ_0 \tan \beta l_t = -\text{Imag}[Z(l_s)]$$

$$l_t = \text{stub length} = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{\sqrt{Z_L Z_0}}{Z_L - Z_0} \right)$$



The junction impedance at the stub and the line purely real as Z_0 and hence matched from the stub to the source.

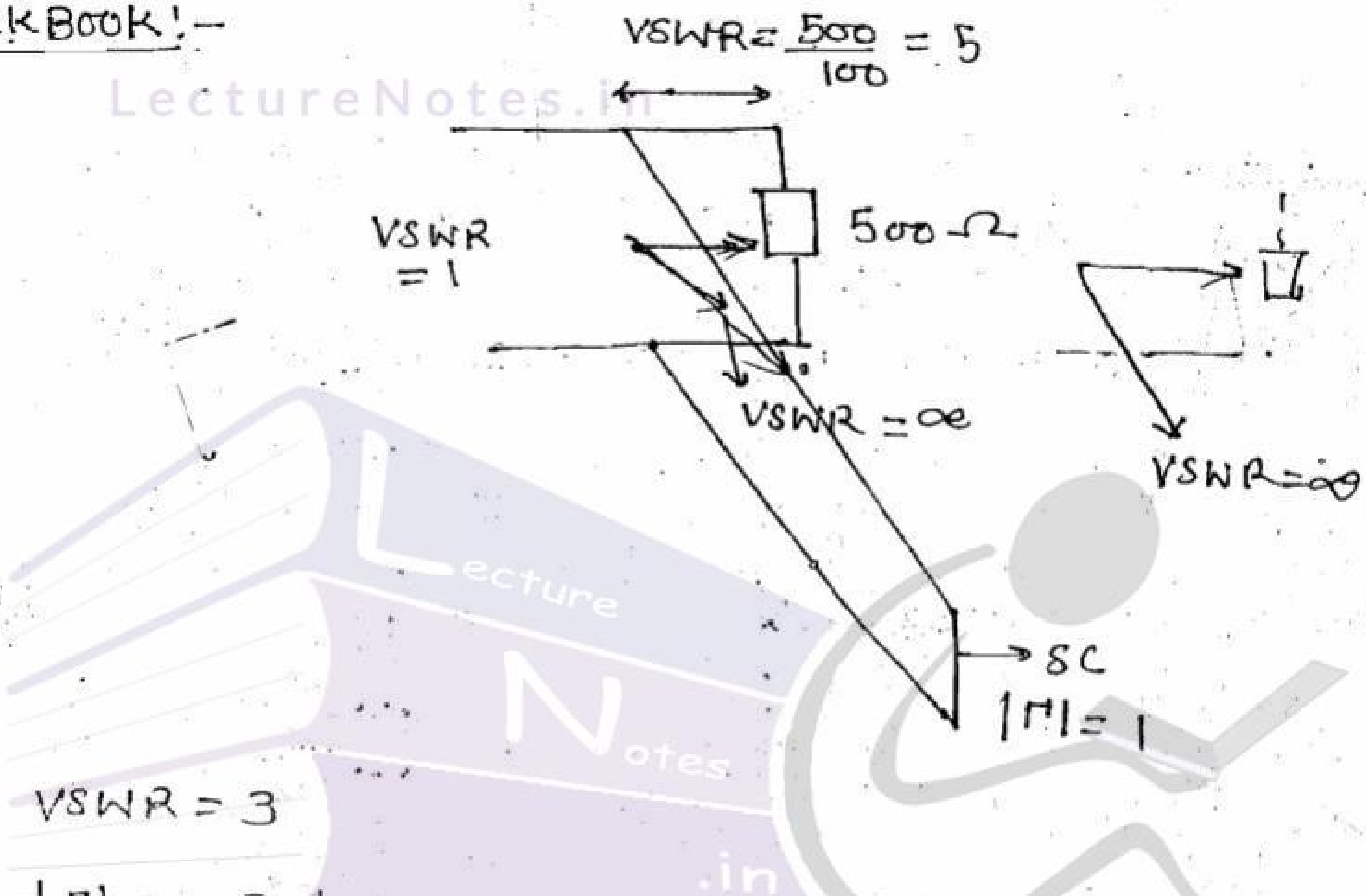
As the frequency on the line changes the stub length and stub position both have to be adjusted which is mechanically inconvenient.

however in SC stub it is easy to using movable shorts

NOTE:-

For wide range of frequencies we use double stub matching where ds_1 and ds_2 are two stub position fixed and dt_1 and dt_2 are varical.

WORKBOOK:-



$$|\Gamma|_V = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\Gamma_P = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 25\%$$

$$Z_0' = \sqrt{225 \times 250} = 240\ \Omega$$

$$Z_L = 75 - j40$$

$$Z_0 = 75$$

Note:-

Stub can never be placed at node

OC $\rightarrow$ Stubs $\rightarrow$ More difficult to realize

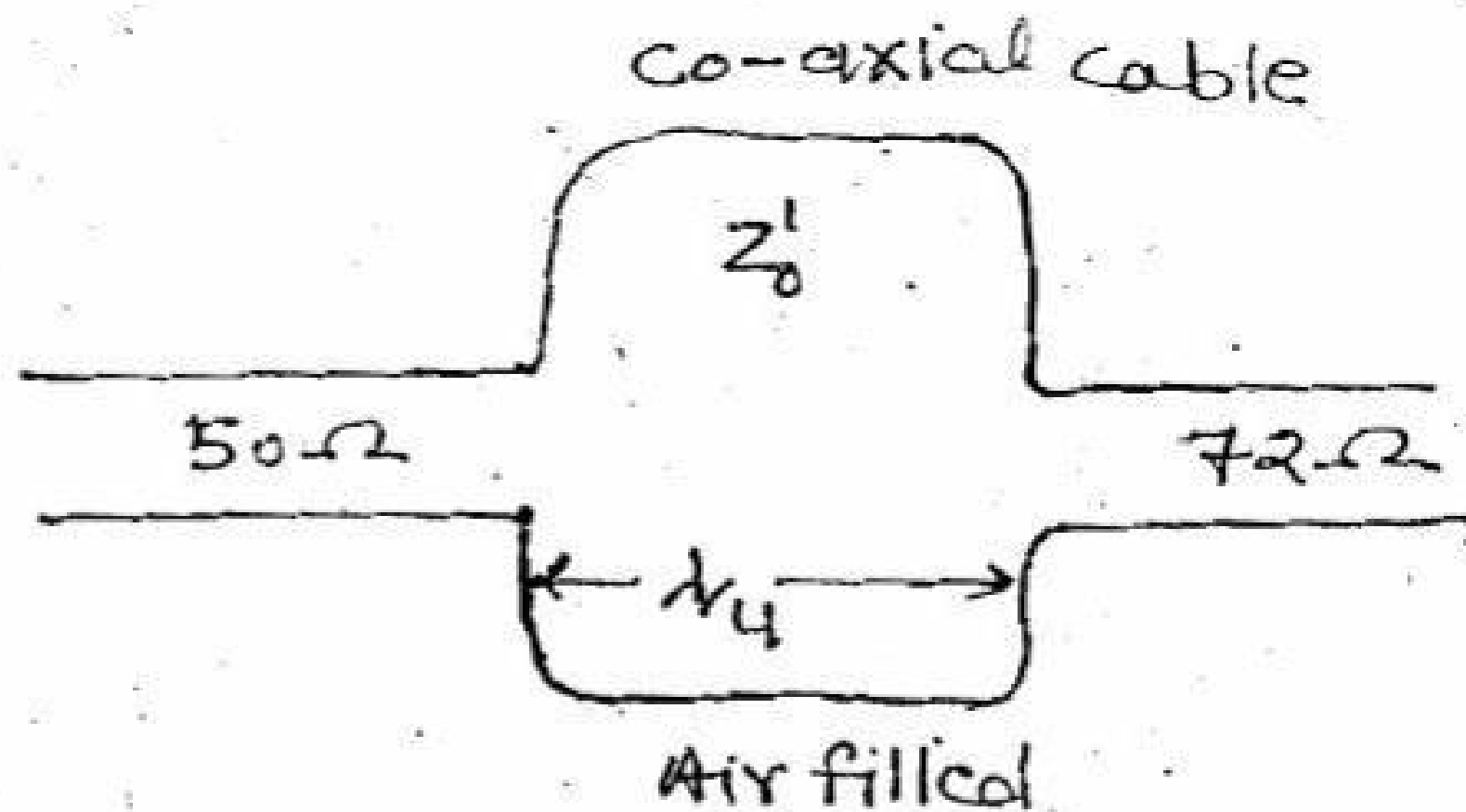
SC $\rightarrow$ Stubs

Ans-(a)

5. $\rightarrow \vec{D}$

36.

$$\begin{aligned} Z_0' &= \sqrt{50 \cdot 72} \\ &= 60 \Omega \\ &= \frac{60 \ln(b/a)}{\sqrt{\epsilon_r}} \end{aligned}$$



$\ln(b/a) = 1 \Rightarrow \frac{b}{a} = e = 2.71 \Rightarrow b = 27 \text{ mm}$

37. $0.5 - j0.3 = \frac{Z_L}{Z_0} = \text{Normalized value}$

Actual value = $(0.5 - j0.3) \times 50$
 $= 25 - j15$

38. Clocking on
 $R = \text{constant}$

$Z \text{ at } P = R + jX_1$

$Z' \text{ at } P' = R + jX_2$

with $X_2 > X_1$

$Z' \text{ at } P' = R + jX_1 + jX'$

Ans - (a)



inductive

39. (i) $\frac{50 + j50}{50} = 1 + j1$

$\downarrow R$

$\downarrow X$

(ii) $\frac{10 + j10}{50} = 0.2 + j0.2$

$\downarrow R$

$\downarrow X$

(iii) $\frac{25 - j50}{50} = 0.5 - j1$

$\downarrow R$

$\downarrow X$

40. (i) one revolution for 360° - length - $\lambda/2$

All impedances repeat for $\lambda/2$

- (II) Clockwise movement $x \uparrow$
towards generator $x \uparrow$

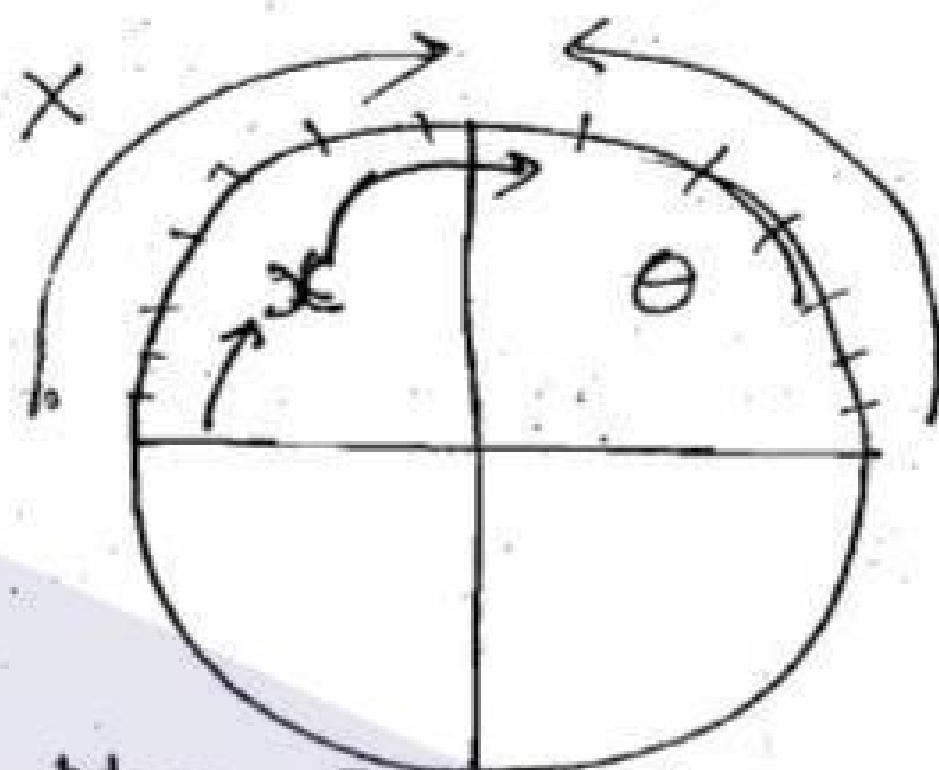
$$Z(x) = Z_0 \left[\frac{Z_L \cos \beta x + j Z_0 \sin \beta x}{Z_0 \cos \beta x + j Z_L \sin \beta x} \right]$$

$$= j Z_0 \tan \beta x$$

$x \uparrow$ $x \uparrow$

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(III) $\Gamma(x) = \Gamma(0) e^{-j2\beta x}$



Three scales

- $\rightarrow X$
- $\rightarrow \theta$
- $\rightarrow x$

Three scales on the Boundary

- (I) X - Reactance of the load
- (II) x - Length on the line
- (III) Γ 's phase

All the three are related scales

$\rightarrow$ one is sufficient

- (IV) Impedance (Wrong Statement)

Conventional :-

$$V(x) = \frac{V_L (Z_L + Z_0)}{2Z_L} \left[e^{Vx} + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-Vx} \right]$$

$$= V_L \cosh Vx$$

If $V = j\beta$ $V(x) = V_L \cos \beta x$ — (1)

OR

$$V(x) = V_L \cosh Vx + I_L Z_0 \sinh Vx$$

$$V(x) = V_L \cosh Vx$$

$$I(x) = I_L \cosh(Vx) + \frac{V_L}{Z_0} \sinh(Vx)$$

$$I(x) = \frac{V_L}{Z_0} \sinh(\gamma x) \quad \text{and} \quad \sin$$

$$\text{If } \gamma = j\beta \quad I(x) = \frac{jV_L}{Z_0} \sin \beta x$$

$$V_L = 12V$$

$$Z_0 = 60 \Omega$$

$$x = \lambda/4$$

$$d = \lambda/2$$

$$\text{if } \gamma = j\beta \quad V(x) = V_L \cos \beta x$$

$$Z_L = 100 - \frac{j}{\omega C} = 100 - j180 \Omega$$

$$= 100 - \frac{j}{2 \times \pi \times 10^8 \times \frac{1}{36 \pi \times 10^9}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{VSWR}$$

$$Z_{\max} = Z_0 (\text{SWR})$$

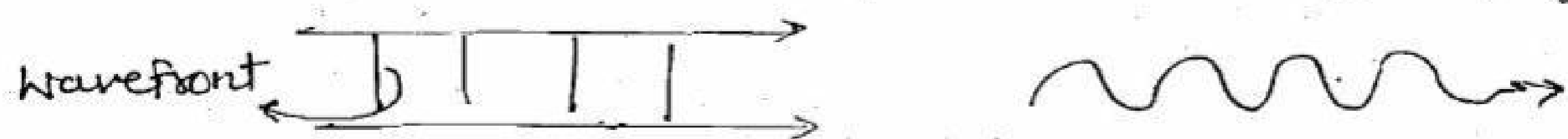
$$Z_{\min} = Z_0 / \text{SWR}$$

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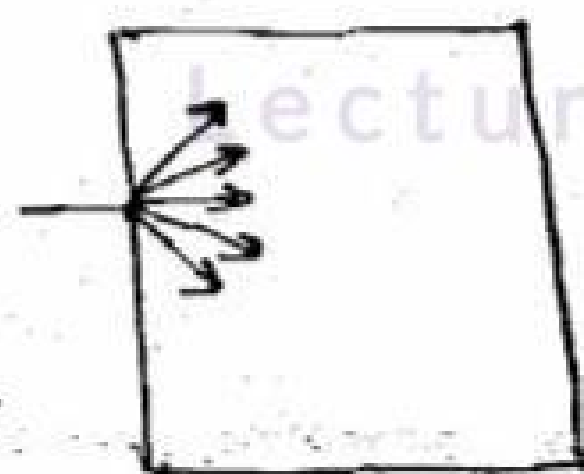
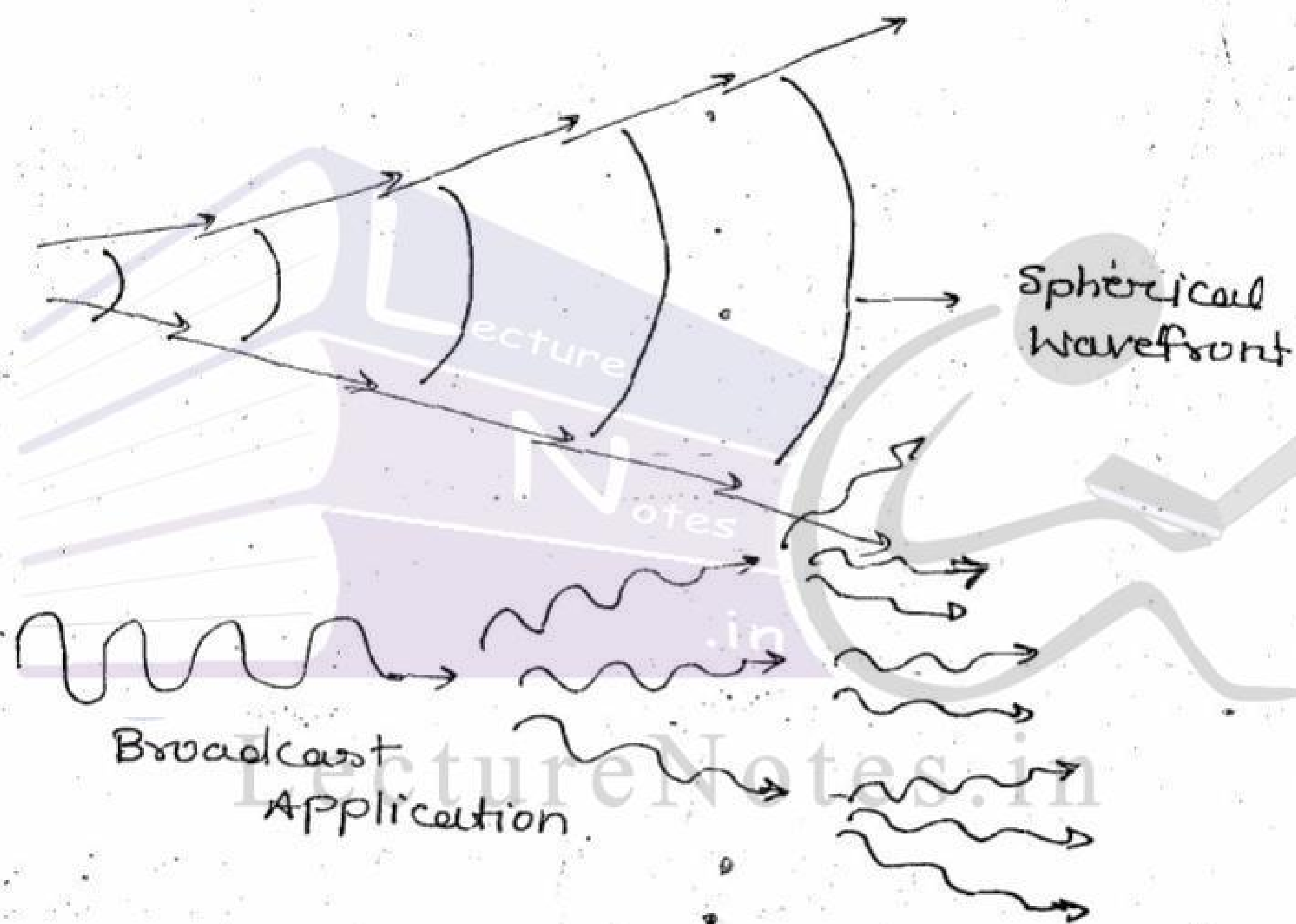
Wave Guides :-

Uniform Plane Wave $\rightarrow E(z,t)x, H(z,t)y$



Dispersive Beams $\rightarrow E(x,y,z,t)(x,y,z)$
 $H(x,y,z,t)(x,y,z)$

LectureNotes.in



Scattering
Diffraction
Diffusion

- All practical EM waves are dispersive in nature and hence they obey Huygen's wave principle that every ray is a source of secondary emission. This is the cause of diffusion, diffraction and scattering properties of EM Waves

→ This is an advantage in broadcast application but a serious limitation in point to point communication. Hence waveguide are used to restrict the wave with a specific bound

$$\left. \begin{array}{l} E(x, z, t) \\ H(x, z, t) \end{array} \right\} \begin{array}{l} \text{one dimension restriction in } x \\ \text{one dimension propagation in } z \end{array}$$

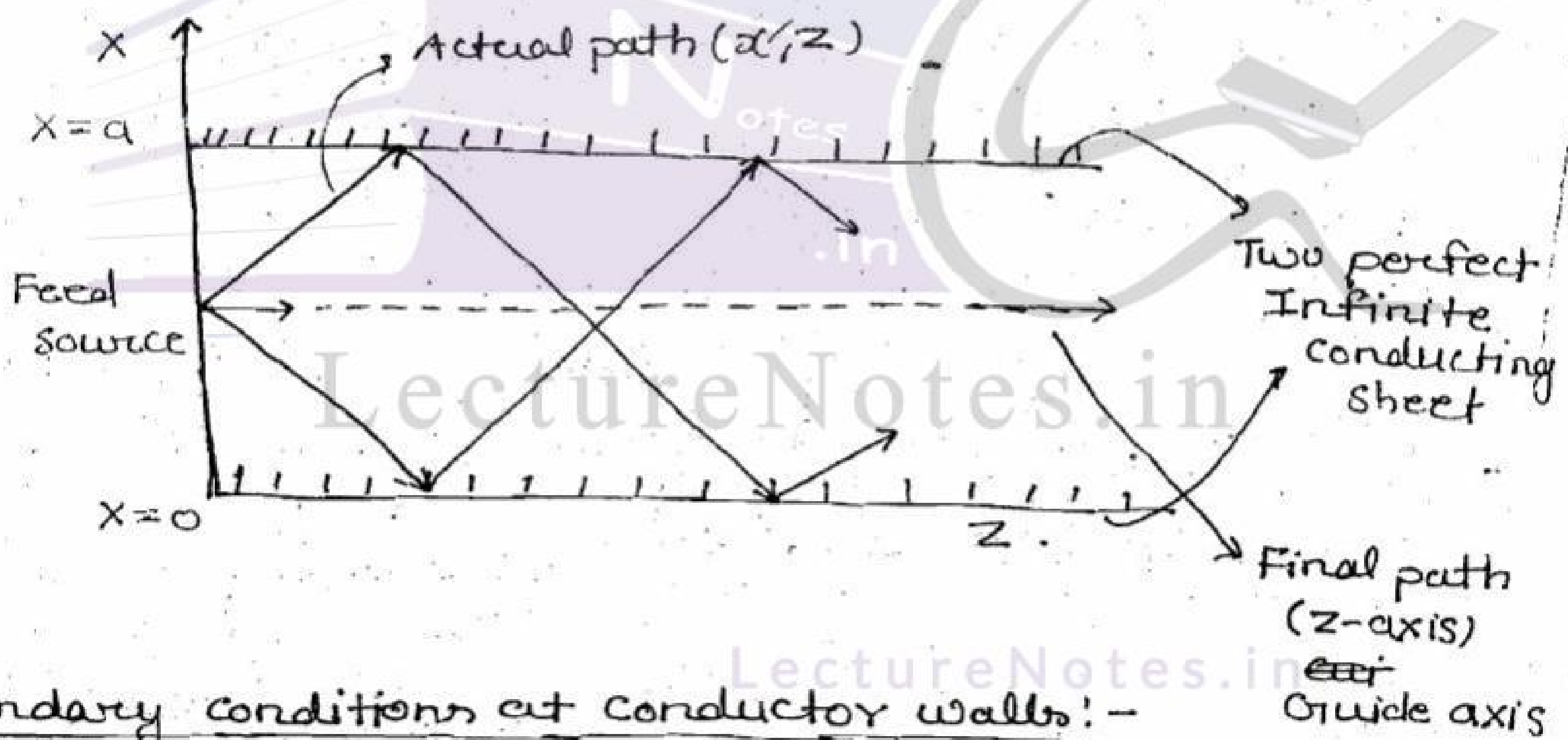
↓
Parallel plane waveguide

eg:- earth and ionosphere guiding

$$\left. \begin{array}{l} E(x, y, z, t) \\ H(x, y, z, t) \end{array} \right\} \begin{array}{l} \text{2 dimension restriction in } x \\ \text{1 dimension propagation in } z \end{array}$$

↓
Rectangular Waveguide

Parallel Plane Waveguides:-



Boundary conditions at conductor walls:-

$$E_{\text{tang}} = 0 \text{ at } x=0, x=a$$

$$E(x)_{\text{tang}} = 0 \text{ at } x=0, x=a$$

$$E(x)_y \text{ \& } E(x)_z = 0 \text{ at Guide walls}$$

Propagation along the Guide Axis:-

Using $\nabla^2 E = \gamma^2 E$ Helmholtz's Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \epsilon E$$

Let $E(z)$ be any natural harmonic $e^{-\bar{\gamma}z}$ as the z side is un-restricted

where $\bar{\gamma} = \gamma_z =$ Propagation constant in guide axis.

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\gamma^2 + \omega^2 \mu \epsilon) E$$

$$\text{where } \bar{\gamma}^2 + \omega^2 \mu \epsilon = \gamma_x^2$$

where $\gamma_x =$ Propagation constant in x side or restricted side

$$\frac{\partial^2 E}{\partial x^2} = -\gamma_x^2 E$$

The $E(x)$ solution is also harmonic

$$E(x) = C_1 \sin(\gamma_x x) + C_2 \cos(\gamma_x x)$$

The restricted side propagation has to be trigonometric harmonic only

Applying the Boundary conditions at $x=0$

$$E(0)_{\text{tang}} = 0 + C_2 = 0$$

$$\Rightarrow C_2 = 0$$

Note:-

The tangential E field harmonic has to be a 'sin' only in the restricted side at $x=a$

$$E(a)_{\text{tang}} = C_1 \sin(\gamma_x a) = 0$$

$$V_x = \frac{m\pi}{a}$$

$$m = 0, 1, 2, 3, \dots$$

Note:-

The restricted side propagation constant can take only discrete solutions but not any continuous value

$$\text{Finally } \bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

Concept 1:-

(f_c) cut off frequency of the guide

$$\text{If } \left(\frac{m\pi}{a}\right)^2 > \omega^2 \mu \epsilon$$

then $\bar{\gamma} = \bar{\alpha} + j0$ No propagation along the guide axis

$$\text{If } \omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2$$

$$\omega > \frac{m\pi}{a\sqrt{\mu \epsilon}}$$

$$\text{then } \bar{\gamma} = 0 + j\bar{\beta}$$

The wave travels along the guide axis without attenuation.

$$\omega > \frac{m\pi c}{a}$$

Note:-

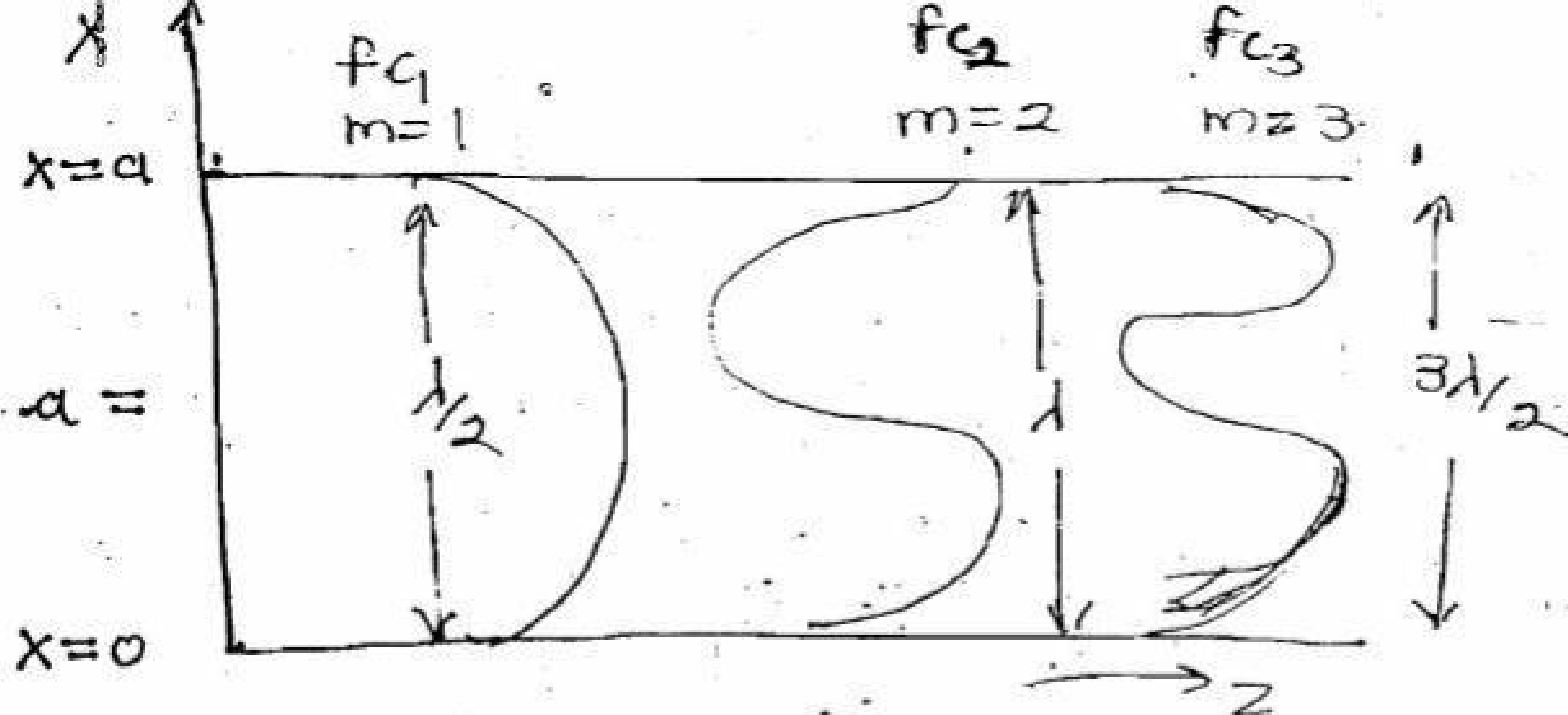
Every waveguide has a minimum cut-off frequency below which there cannot be propagation i.e. there is a max. wavelength above which there cannot be propagation and this wavelength is comparable to the guide dimensions.

$$\text{Hence } \omega_c > \frac{m\pi c}{a}, f_c > \frac{mc}{2a}, \lambda_c > \frac{2a}{m}$$

$$\Rightarrow \omega_c = \frac{m\pi c}{a}$$

$$f_c = \frac{mc}{2a}$$

$$\lambda_c = \frac{2a}{m}$$



At exact cut-off frequency where $\bar{V} = 0$, the wave resonates b/w guide walls and oscillates b/w the walls for 'm' such frequencies.

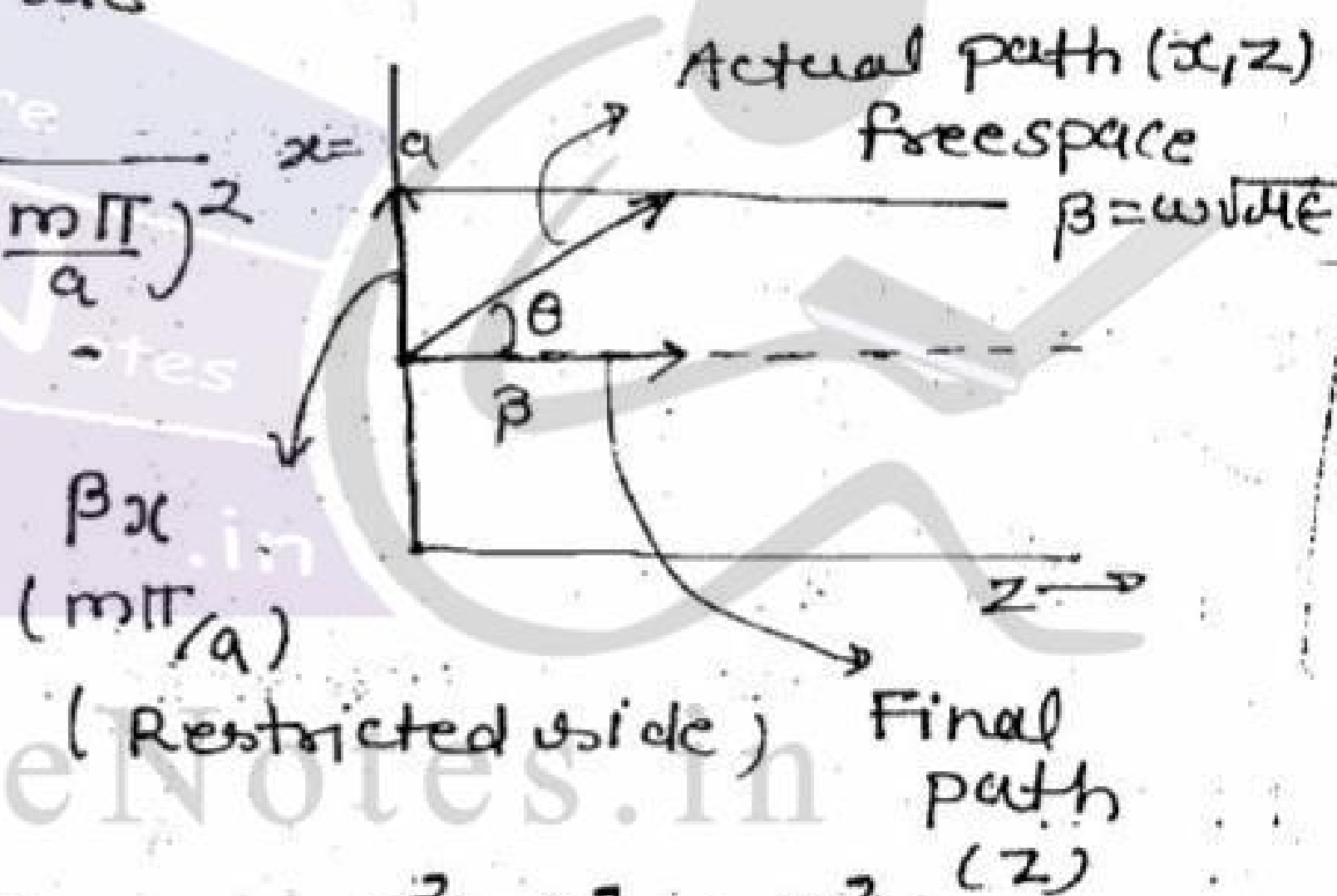
Concept 2 :-

Wave Angle or Tilt Angle :-

$$\bar{V} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

$$= j \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$



$$\beta^2 = \beta_x^2 + \bar{\beta}^2$$

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

Phase velocity along the guide axis

$$\bar{V}_p = \frac{\omega}{\bar{\beta}}$$

$$= \frac{\omega}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}$$

$$\bar{V}_p =$$

$$= \frac{1}{\sqrt{\mu \epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$=$$

$$= \frac{\frac{1}{\sqrt{\mu \epsilon}}}{\sqrt{1 - \left(\frac{m\pi}{a\omega \sqrt{\mu \epsilon}}\right)^2}}$$

$$\bar{V}_p = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\bar{\beta} = \beta \cos \theta$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi}{\lambda} \cos \theta$$

$$\Rightarrow \bar{\lambda} = \frac{\lambda}{\cos \theta}$$

$$\Rightarrow \bar{\lambda} f = \frac{cf}{\cos \theta}$$

$$\bar{V}_p = \frac{c}{\cos \theta}$$

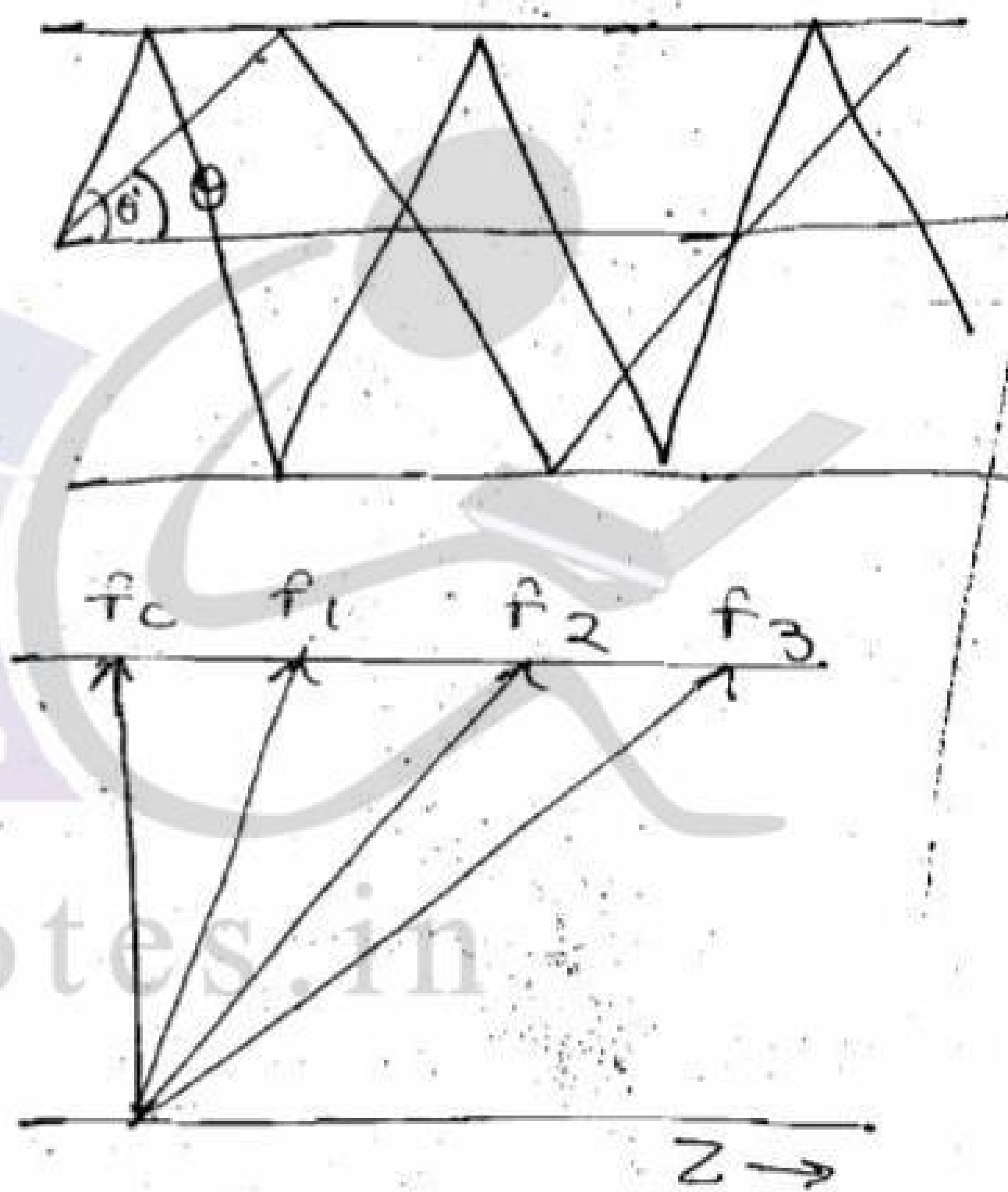
By comparison

$$\sin \theta = \frac{f_c}{f}$$

Note:-

Every frequency has a unique tilt angle and unique velocity inside the guide so that two different frequencies never overlap to each other.

$$f_3 > f_2 > f_1 > f_c$$



Concept 3:-

Group Velocity ($\bar{V}_g$) :-

$$\bar{V}_p = \frac{c}{\cos \theta} \Rightarrow \bar{V}_p > c \rightarrow \text{Always}$$

In linear conditions, $\beta \propto \omega$
velocity is phase velocity $V_p = \frac{\omega}{\beta}$

eg:- lossless EM Waves in free space
lossless VI Waves in transmission line

$$\beta = \omega \sqrt{\mu\epsilon}$$

$$\beta = \omega \sqrt{LC}$$

But in dispersive condition β is not linear with ω

eg:- $V_g = \frac{d\omega}{d\beta}$ = Group velocity

$$\bar{\beta} = \sqrt{\omega^2 \mu\epsilon - \left(\frac{m\pi}{a}\right)^2} \rightarrow \text{dispersive condition}$$

β is not linear with ω

$$\frac{d\bar{\beta}}{d\omega} = \frac{1}{\sqrt{\omega^2 \mu\epsilon - \left(\frac{m\pi}{a}\right)^2}} \cdot 2\omega\mu\epsilon$$

$$= \frac{\mu\epsilon}{\sqrt{\mu\epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{\sqrt{\mu\epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu\epsilon}}\right)^2}}$$

$$= \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\bar{V}_g = c \cos\theta$$

$$\bar{V}_g < c \rightarrow \text{Always}$$

$$\boxed{\bar{V}_g \cdot \bar{V}_p = c^2} \rightarrow \text{Always}$$

Concept 4:-

Modes of operation

Concept 4 :-

Modes of Operation :-

Note :-

Mode stands for physical connection of field which can be axial or longitudinal feed

→ The no. of field connection decides the integer m of that mode and thus m decides the cut-off freq. of the mode

→ The mode can be identified from field such that m stands for no. of half cycles b/w the guide walls ~~and~~ or no. of maxima b/w the guide walls.



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Transverse Electric (TE), TM and TEM Modes: -

$$E(x, z, t) \quad (x, y, z)$$

$$H(x, z, t) \quad (x, y, z)$$

Using Maxwell's equation

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times H = j\omega\epsilon E$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \nabla \times E$$

→ All y derivatives are zero

→ All z derivatives are back the same function with $\bar{y}$ scaling

$$\nabla \times E = -j\omega\mu H$$

$$\bar{y} E_y = -j\omega\mu H_x$$

$$-\bar{y} E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$\nabla \times H = j\omega\epsilon E$$

$$\bar{y} H_y = j\omega\epsilon E_x$$

$$-\bar{y} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} = j\omega\epsilon E_z$$

$$H_x = -\frac{\nabla E_y}{j\omega\mu}$$

$$-\nabla \left(-\frac{\nabla E_y}{j\omega\mu} \right) - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\nabla^2 E_y - j\omega\mu \frac{\partial H_z}{\partial x} = -\omega^2\mu\epsilon E_y$$

$$(\nabla^2 + \omega^2\mu\epsilon) E_y = j\omega\mu \frac{\partial H_z}{\partial x}$$

$$(I) \quad \frac{\partial H_z}{\partial x} = \frac{\nabla_x^2}{j\omega\mu} E_y$$

$$(II) \quad \frac{\partial H_z}{\partial x} = -\frac{\nabla_x^2}{\nabla} H_x$$

$$(III) \quad \frac{\partial E_z}{\partial x} = -\frac{\nabla_x^2}{\nabla} H_y$$

$$(IV) \quad \frac{\partial E_z}{\partial x} = -\frac{\nabla_x^2}{\nabla} E_x$$

Note:-

The axially directed field components determine the complete standing of the wave.

If $E_z = 0 \rightarrow$ physical connection

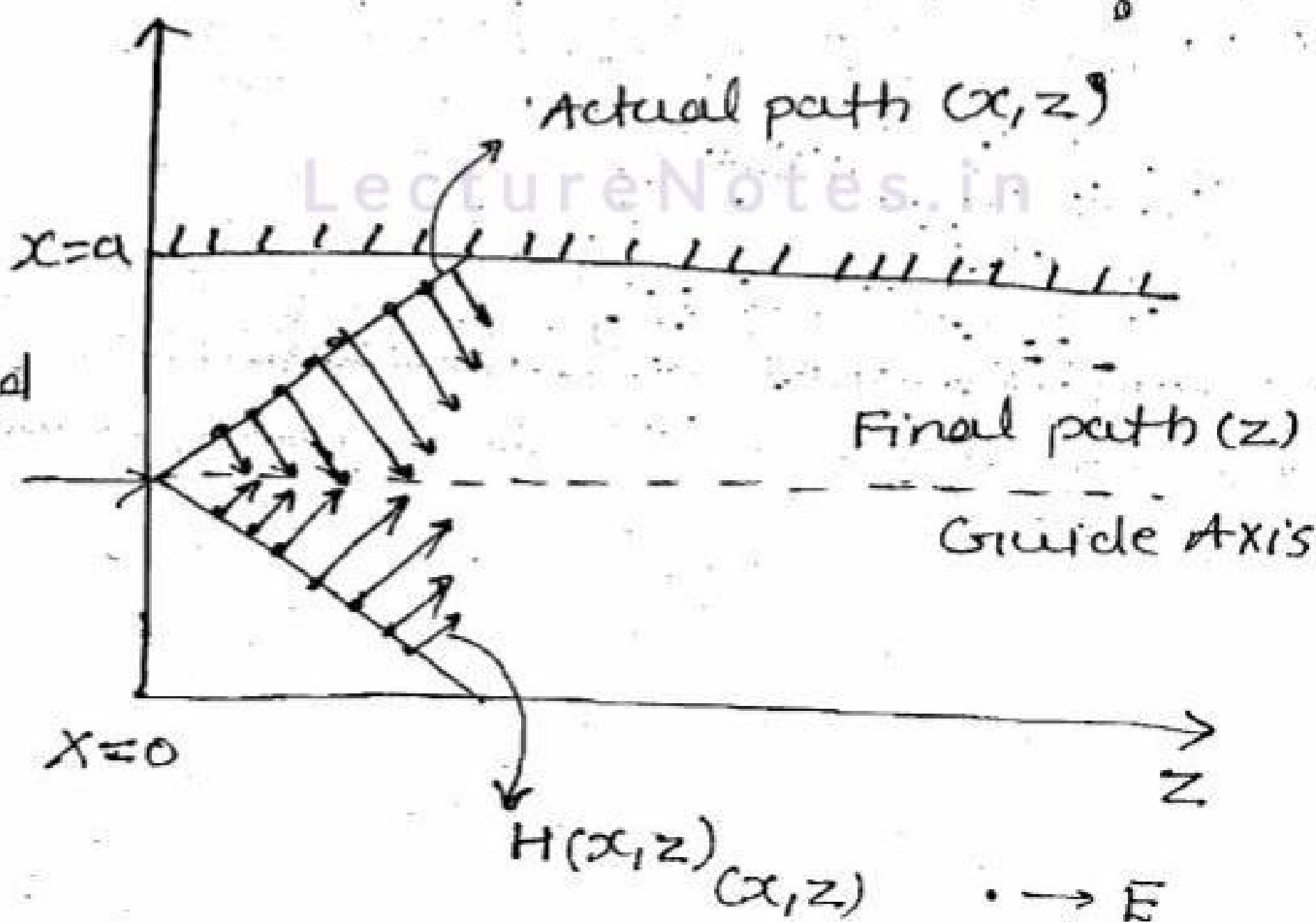
then $E_x = H_y = 0$ The waves becomes

$E(x, z, t) \rightarrow y$

$H(x, z, t) \rightarrow (x, z)$

$E \perp$ Guide Axis

The wave is called as TE wave



If $H_z = 0$, then $H_x = E_y = 0$
 the wave becomes $H(x, z, t)_y$
 $E(x, z, t)_{x, z}$

$$\boxed{H \perp \text{Guide Axis}}$$

The wave is called as TM Waves

$$\boxed{E \perp H \perp \text{Propagation}}$$

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TE Wave solutions in Parallel-Plane Waveguides.

The waves has

$$(E_x = E_z = H_y = 0)$$

$$E(x, z, t)_y = E_{y0} \cdot \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$E(x, z, t)_x = H_{x0} \cdot \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H(x, z, t)_z = H_{z0} \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_z$$

It is a product (AND) solution of time, x & z

Harmonics

$$\frac{E(t)}{H(t)} \rightarrow e^{j\omega t} \text{ Source harmonic}$$

$$\frac{E(z)}{H(z)} \rightarrow e^{-\gamma z} \text{ Natural Harmonic}$$

$$\frac{E(x)}{H(x)} \rightarrow \text{Trigonometric Harmonic}$$

$$\frac{E(x)_{\text{tang}}}{H(x)} \rightarrow \text{Sin}^2 \text{ Harmonic}$$

$$\frac{\partial H_z}{\partial x} = () H_x$$

$$\frac{\partial H_z}{\partial x} = () E_y$$

$$E(x)_y \text{ or } E(x)_z$$

$$\text{or } E(x)_z$$

If $m=0$ the TE wave does not exist and hence $m=1$ is the least value required for propagation

TM Wave Solutions in Parallel Plane Waveguides:-

$$(\text{ } H_z = H_x = E_y = 0)$$

$$H(x, z, t)_y = H_{y0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

$$E(x, z, t)_x = E_{x0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$E(x, z, t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_z$$

If $m=0$, $E_z=0$

The wave becomes $E(z, t)_x$

$H(z, t)_y$

This wave is called as ~~TEM waves~~ TEM Waves with $E_z = H_z = 0$

Properties of TEM Waves:-

- i) It has $m=0$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
i.e. Propagates only along guide axis i.e. $\theta=0$
→ Always
- ii) It has $f_c=0$ i.e. No cut off frequency
- iii) It has $m=0$ i.e. No multi-mode connection mechanism

Summary:-

$\vec{E}(x, t)$ → Wave at f_c , $\theta = 90^\circ$, $\bar{\gamma} = 0$

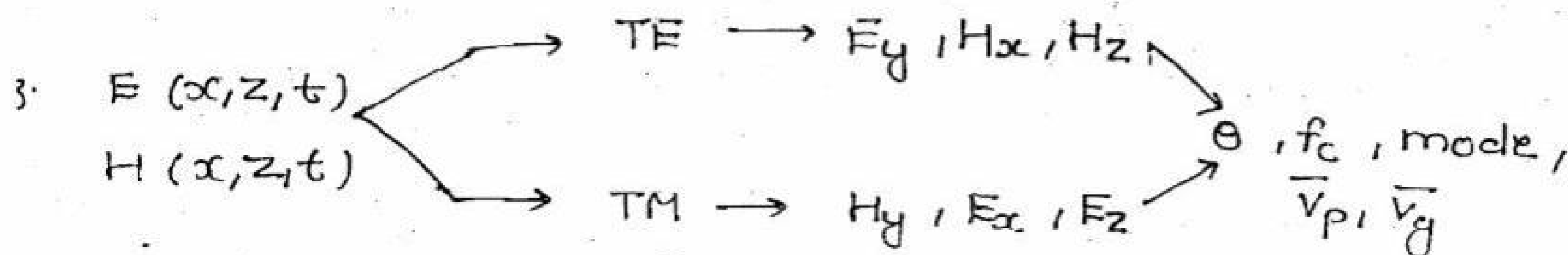
$H(x, t)$

($f_{c1}, f_{c2} \dots f_{cm}$ exists)

$\vec{E}(z, t)_x$

$H(z, t)_y$

→ TEM wave, $\theta = 0^\circ$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
 $m=0$, $f_c=0$



Waveguides (Workbook) :-

i) $f = 22 \text{ GHz}$ $\sin \theta = \frac{f_c}{f}$

$f_c = \frac{mc}{2a}$

$f_{c1} = \frac{1.38 \times 10^8}{2.25 \times 10^{-2}} = 6 \text{ GHz}$

1st Mode -
6 GHz $\rightarrow \infty$

1st Mode $\sin \theta = \frac{6}{22}$

11nd Mode
12 GHz $\rightarrow \infty$

III Mode $\sin \theta = \frac{18}{22}$

111rd Mode
18 GHz $\rightarrow \infty$

ii) $f = 40 \text{ GHz}$

$f_c = 36 \times 10^9 = \frac{3.3 \times 10^8}{2 \times a} \Rightarrow a = 1.25 \text{ cm}$

iii) $\sin \theta = \frac{36}{40} = \frac{f_{c3}}{f_3} = \frac{f_{c7}}{f_7} = \frac{84}{f_7}$

3rd Mode $\rightarrow 36$

7th Mode $\rightarrow 84$

$f_7 = 93.3 \text{ GHz}$

iv) $f = 2 \text{ GHz}$

$f_{c1} = \frac{1.3 \times 10^8}{\sqrt{9} \cdot 2 \times 3 \times 10^{-2}} = \frac{5}{3} = 1.67 \text{ GHz}$

1st $\rightarrow 1.67 \rightarrow \infty$
2nd $\rightarrow 3.34 \rightarrow \infty$

$\sin \theta = \frac{5/3}{2} = \frac{5}{6}$

Note:-

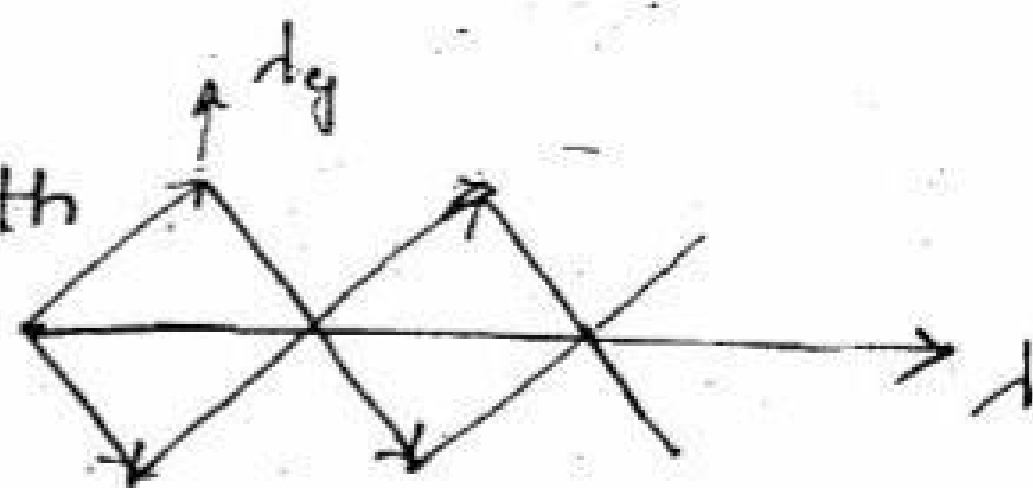
1. $\bar{V}$, $\bar{B}$, $\bar{A}$, $\bar{V}_p$, $\bar{V}_g$, TRANSVERSE $\rightarrow$ w.r.t guide axis

$\eta_{TE} = \frac{E_{trans}}{H_{trans}} = \frac{E_y}{H_x} = \frac{E_{total}}{H_{total} \cos \theta} = \frac{120\pi}{\sqrt{1 - (\frac{f_c}{f})^2}}$

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_{total} \cos \theta}{H_{total}} = 120 \pi \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\lambda = \frac{\lambda}{\cos \theta} = \lambda_g = \text{Guide Wavelength}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$



$$1 - \left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

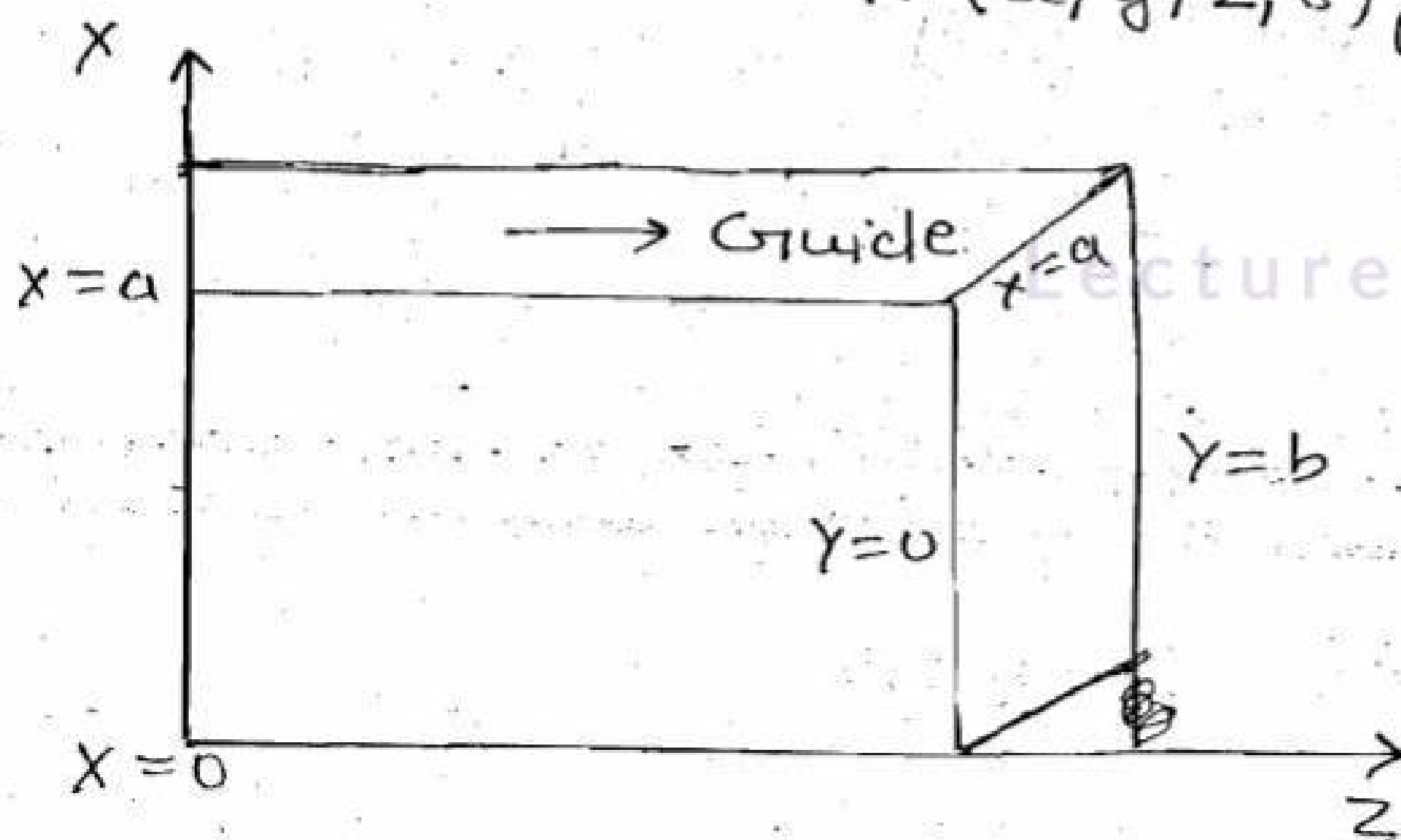
$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$\frac{c}{f}$
free space
wavelength

$\frac{2a}{m}$
cut-off wavelength

Rectangular Waveguides:-

The waves are $E(x, y, z, t)$ (x, y, z)
 $H(x, y, z, t)$ (x, y, z)



→ It is a single conductor hollow structure used to confine EM wave in 2 dimension using 4 walls

$$x=0 \quad x=a$$

$$y=0 \quad y=b$$

It has $V_x = \frac{m\pi}{a}$

$$V_y = \frac{n\pi}{b}$$

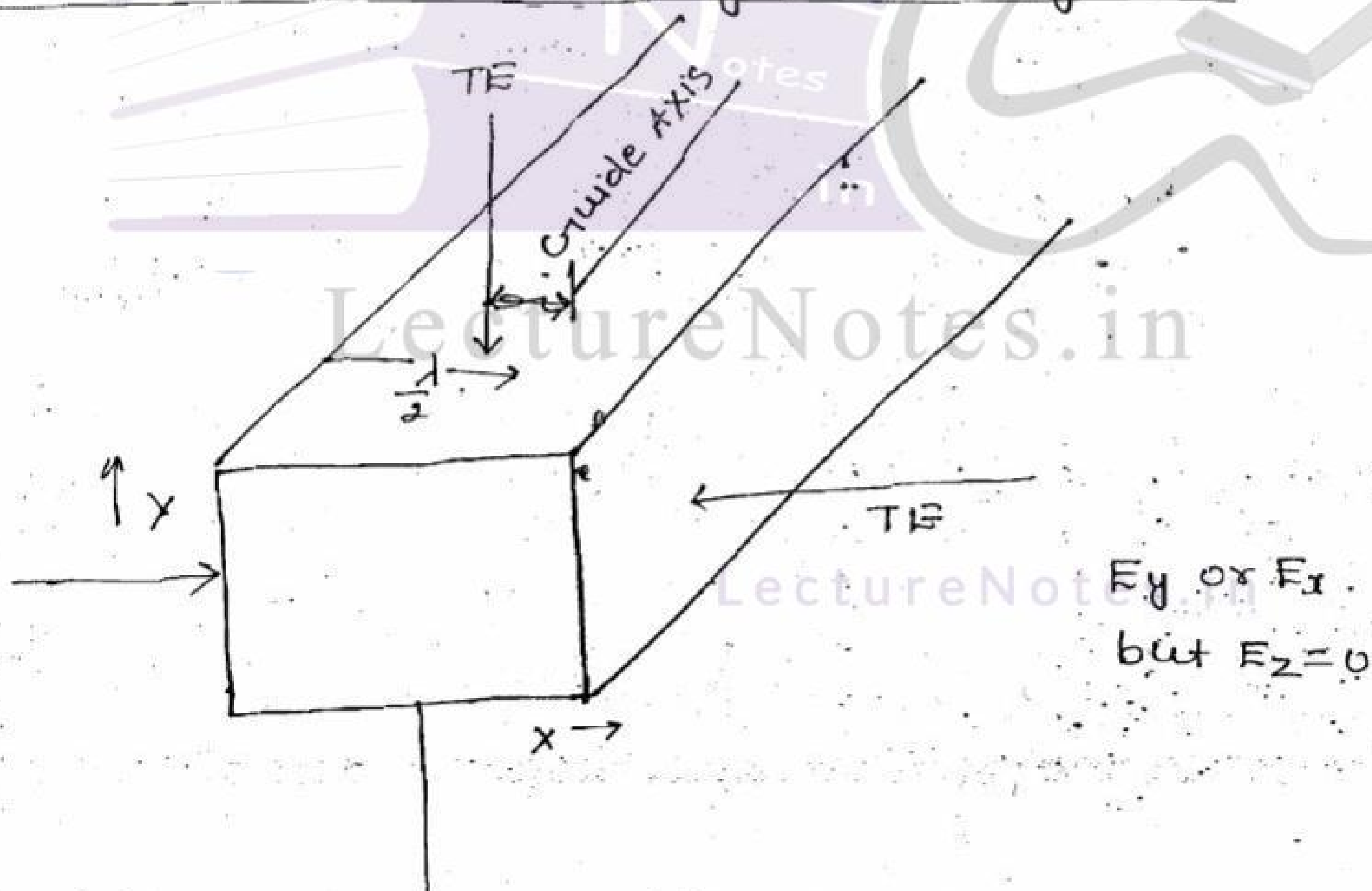
Applying $\nabla^2 E = -\omega^2 \mu \epsilon E$

$$\bar{V} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} - \omega^2 \mu \epsilon$$

$$\omega_c = \text{cut-off frequency} = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \right) c$$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$$

Modes and Fields in Rectangular Waveguides:-



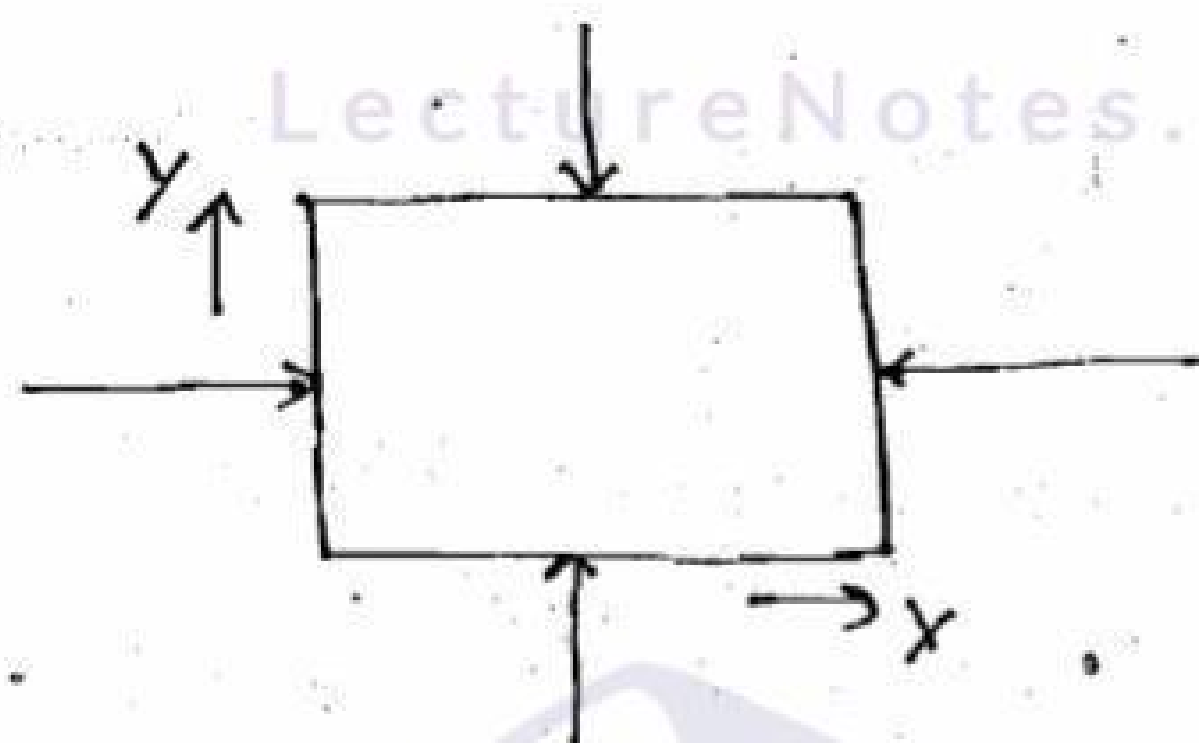
A horizontal or vertical field always gives a suitable E field but not along the guide axis

The no. of out of phase feed connections decides the integers m and n .

The modes are always designated as TE_{mn}
 H_x & H_y but $H_z = 0$

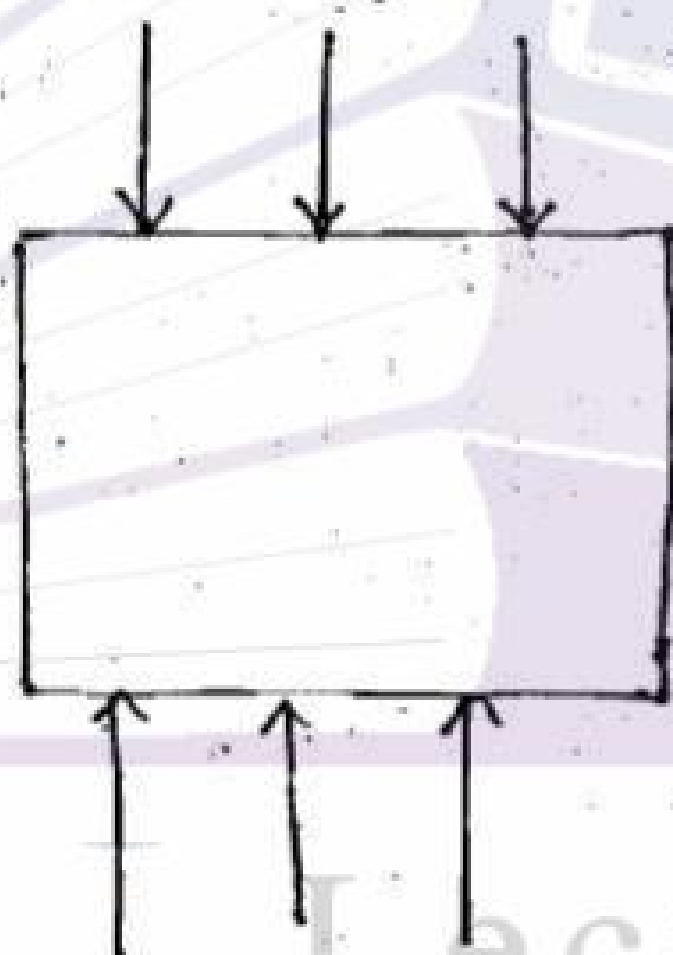
A lateral or axial feed always gives a suitable H field but not along the guide axis

(i)



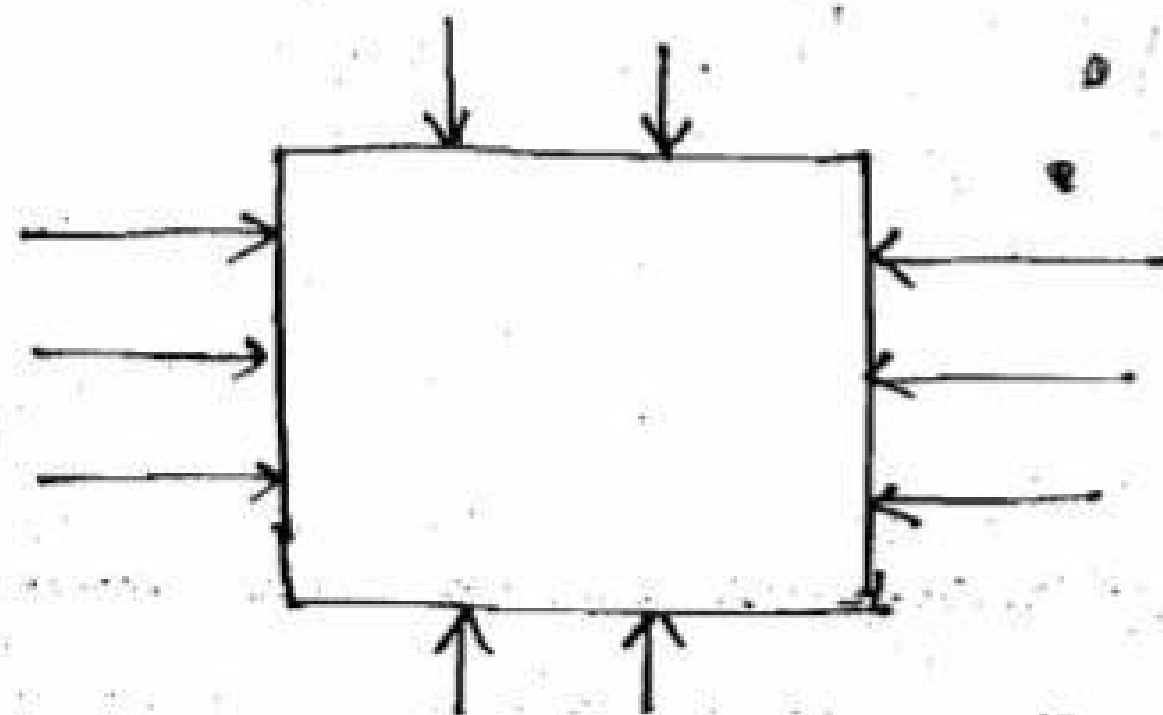
TE_{11}

(ii)



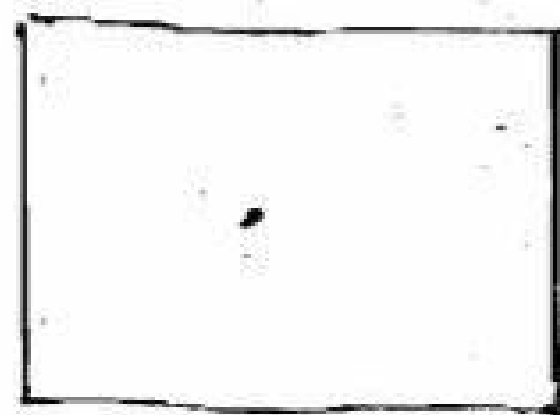
TE_{03}

(iii)

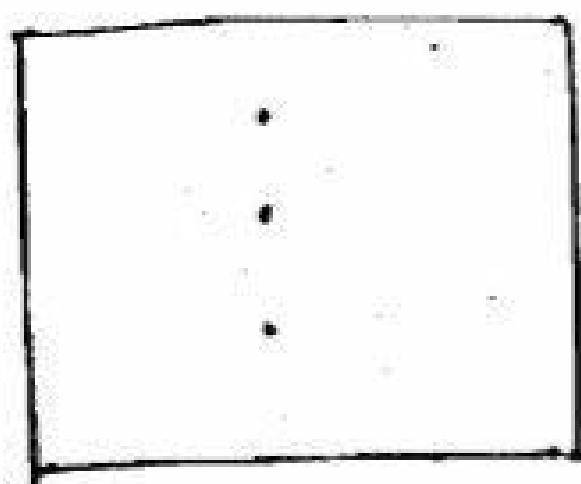


TE_{23}

(iv)



TM_{11}



TM₁₃



TM₄₂

Note:-

TM_{m0} or TM_{0n} modes are even sort and do not exist in rectangular waveguides.

Such a connection mechanism doesn't exist.

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$$

For both TE & TM are same. So TE₇₄, TM₇₄ are same cutoff. called degenerate mode.

Two different modes having the same f_c but different connection mechanism) are said to be degenerate modes.

iii) For $m=1$, $n=0$

or $m=0$, $n=1$

i.e. TE₁₀ & TE₀₁ mode have the least f_c

$$f_c (TE_{10}) = \frac{c}{2a}$$

$$f_c (TE_{01}) = \frac{c}{2b}$$

If $a > b$ TE₁₀ is dominant mode

$a < b$ TE₀₁ is dominant mode

- Broadside dimensions decides the dominant mode
- Narrowside dimensions decides the maximum operable voltage and power handling ability

TM Wave Solutions in Rectangular Waveguide ($H_z=0$):-

The wave is $E(x, y, z, t)$ (x, y, z)

$H(x, y, z, t)$ (x, y)

$$E(x, y, z, t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E(x, y, z, t)_x = E_{x0} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x, y, z, t)_y = E_{y0} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H(x, y, z, t)_x = H_{x0} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H(x, y, z, t)_y = H_{y0} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

It is a product solution of x, y, z, t harmonics

$$E(t)/H(t) \rightarrow e^{j\omega t} \rightarrow \text{Source harmonic}$$

$$E(z)/H(z) \rightarrow e^{-\gamma z} \rightarrow \text{Natural Harmonic}$$

$$H/E(x \text{ or } y) \rightarrow \text{Trigonometric harmonic}$$

$$\left. \begin{array}{l} E(x)_y \text{ or } E(x)_z \\ E(y)_x \text{ or } E(y)_z \end{array} \right\} \text{Tangential harmonic} \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{'sin'} \\ \text{Harmonics} \end{array}$$

If $m=0$ & $n \neq 0$ or $m \neq 0$ & $n=0$

TM waves donot exist i.e. they are even sent modes

Wave Solutions in Rectangular Waveguide ($E_z=0$):-

The wave is $E(x, y, z, t)$ (x, y)

$$H(x, y, z, t) (x, y, z)$$

$$H(x, y, z, t)_z = H_{z0} \cdot \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E(x, y, z, t)_x = E_{x0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x, y, z, t)_y = E_{y0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H(x, y, z, t)_x = H_{x0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H(x, y, z, t)_y = H_{y0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

If $m \neq 0$ & $n = 0$

The TE_{m0} wave exists $E(x, z, t)_y$
 $H(x, z, t) (x, z)$

If $m = 0$ & $n \neq 0$

The TE_{0n} wave exists $E(y, z, t)_x$
 $H(y, z, t) (y, z)$

If $E_z = H_z = 0$ the wave cannot exist

$$\text{or } m = n = 0$$

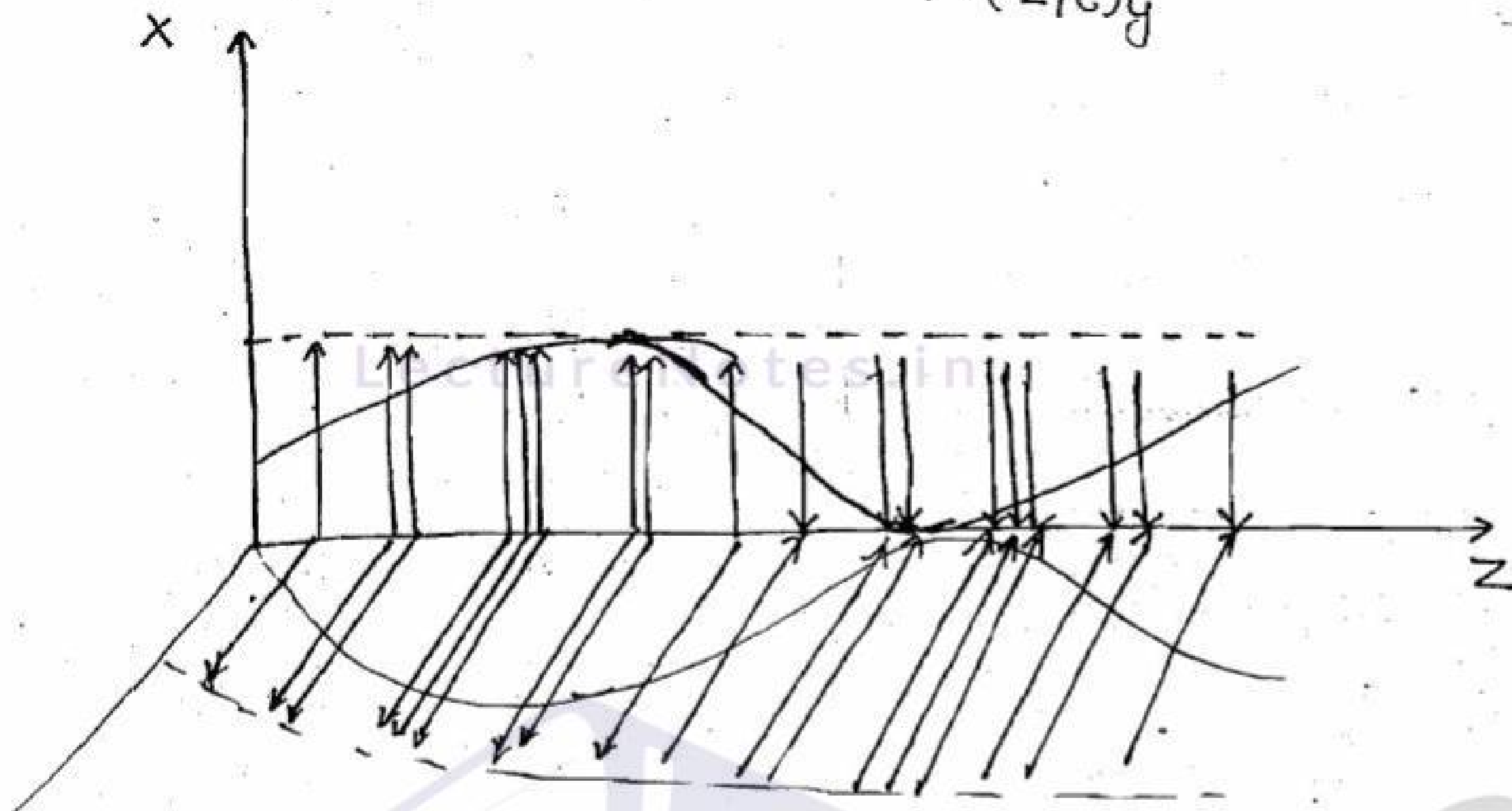
i.e. TEM waves do not exist in rectangular waveguide.

In general TEM waves do not exist in all single conductor guides

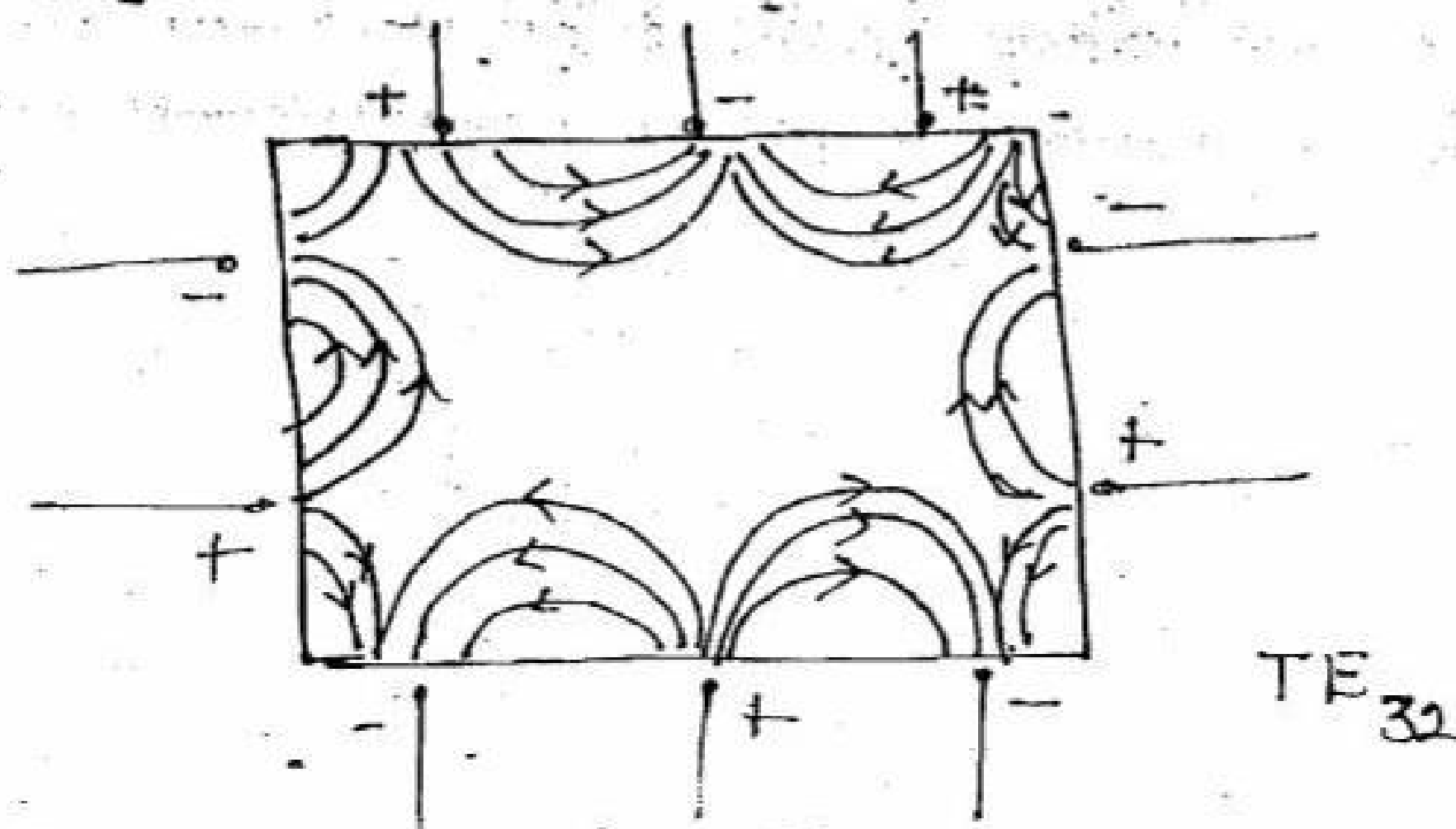
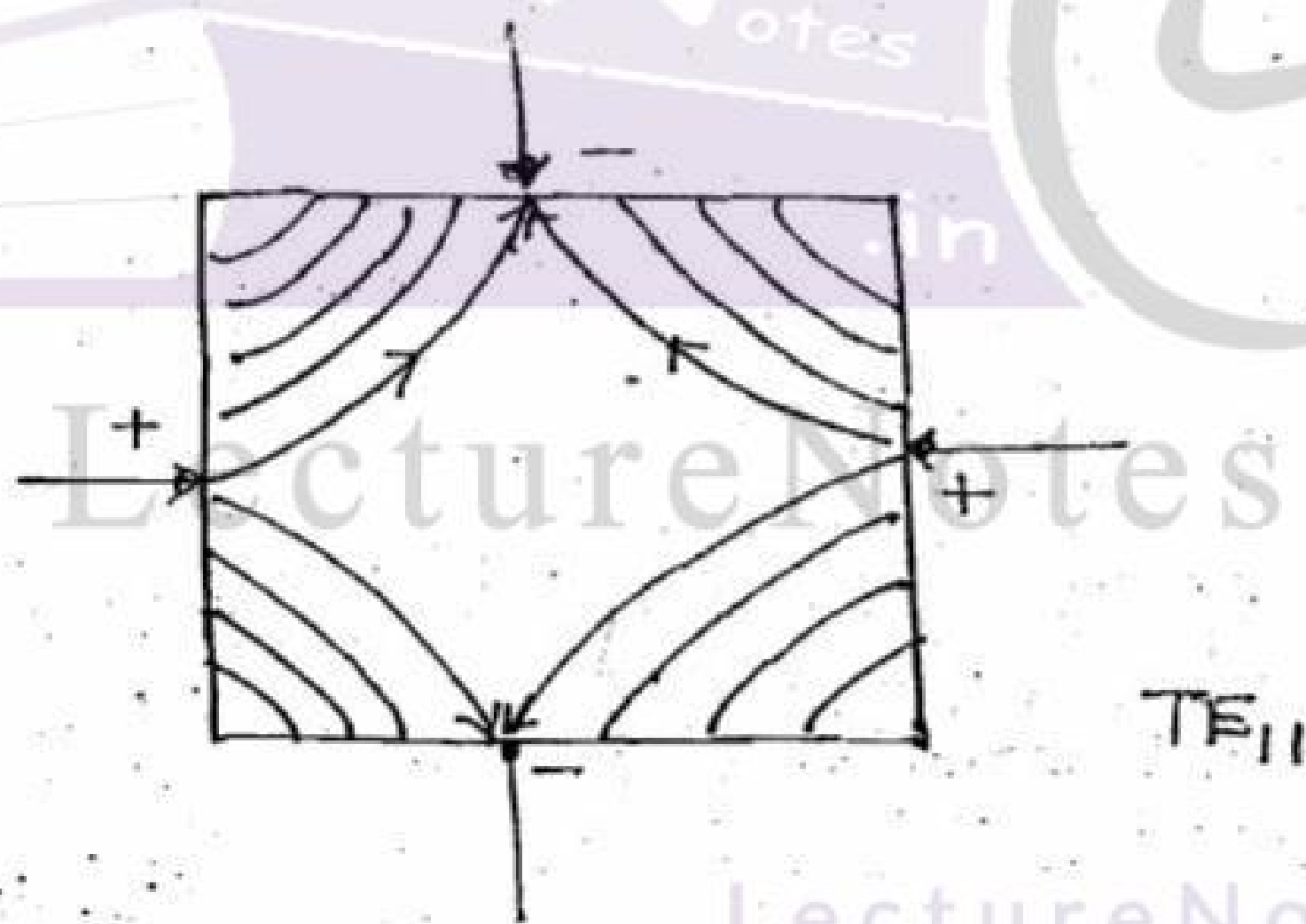
TEM needs two distinct conductors for its existence.

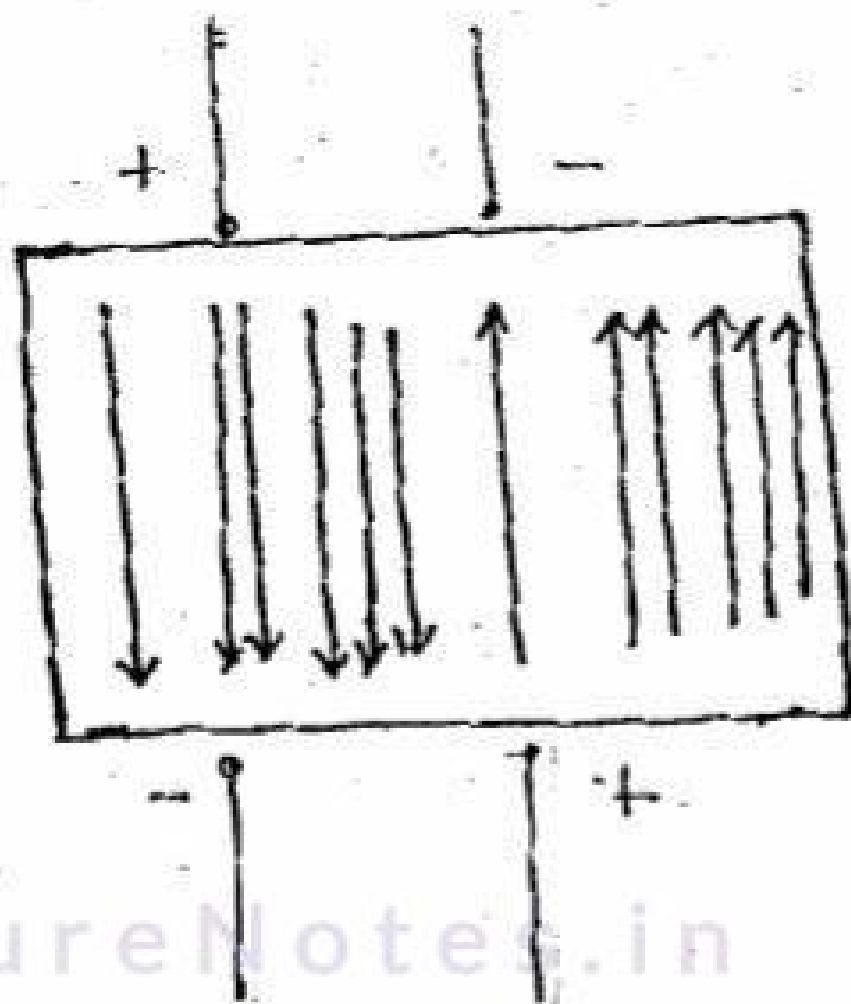
Field line Representations of guided waves:-

(i) Uniform plane wave - $E(z,t)x$
 $H(z,t)y$



TE Waves:- (E_x, E_y)

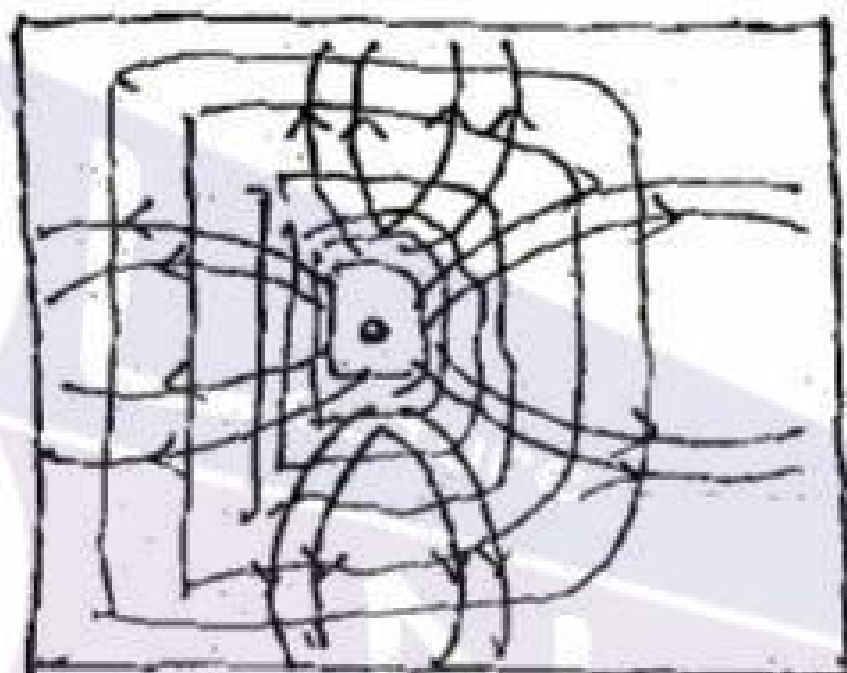




TE_{20}

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TM Waves (E_x, E_y, E_z):-

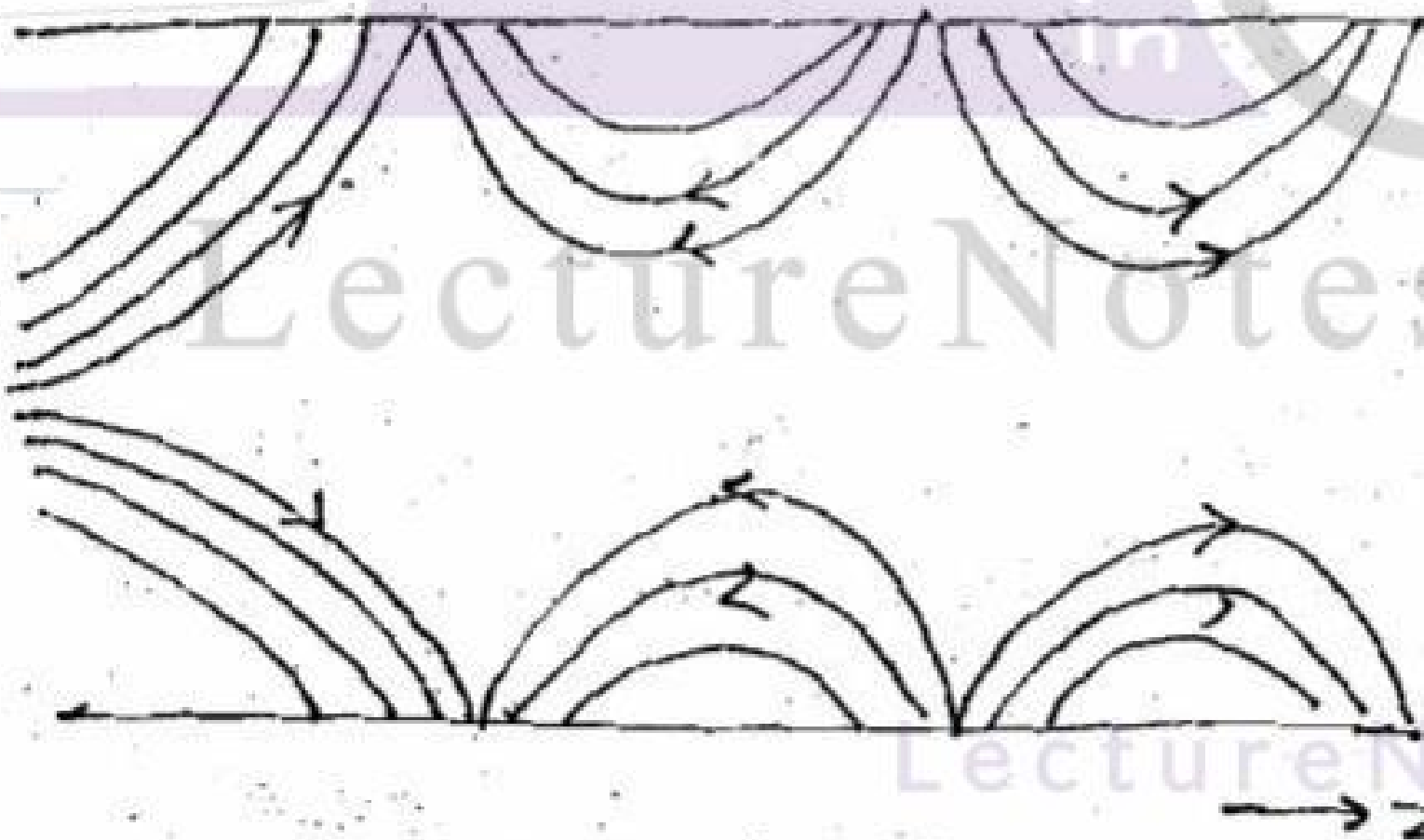


TE

TM_{11}

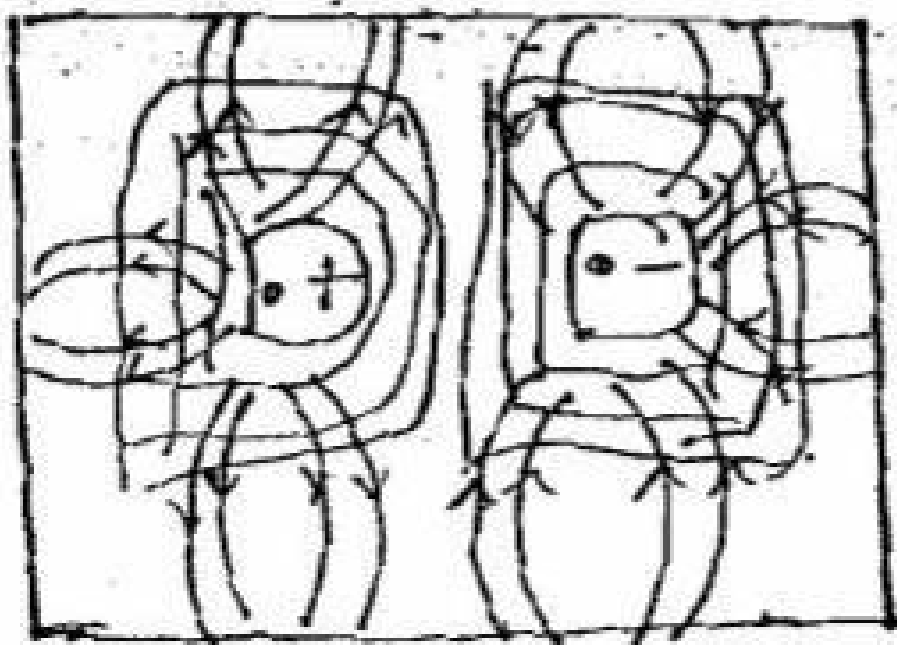
otes

in



LectureNotes.in

TM_{21} :-



TM_{21}

With a guide axis out of board and both E & H fields traced on the board the wave is called as TEM waves else TE or TM

eg:- Co-axial cable and all other low frequency Transmission line. So TEM waves is low frequency energy transfer format

TM/TE $\rightarrow$ High frequency transmission waveguide

Workbook:-

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$$\frac{3 \times 10^8}{2.5 \times 10^9} = 12 \text{ cm}$$

$$2a = 20 \text{ cm}$$

$$\lambda_g =$$

$$6. \quad f_c = \frac{1 \times 3 \times 10^8}{\sqrt{4} \times 2 \times 3 \times 10^{-2}}$$

$$= 2.5 \text{ GHz}$$

$$v_p > c$$

$$f_c = 10 \times 10^9 = \frac{1 \times 3 \times 10^8}{2 \times a}$$

$$a = 1.5 \text{ cm}$$

$$a \cdot b > 2$$

$$a > b$$

$$9. \quad \eta_{TE} = \frac{120 \pi}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{377}{\sqrt{1 - \left(\frac{10}{30}\right)^2}}$$

$$= 400 \Omega$$

$$f_c = \frac{c}{2a} = 10 \text{ GHz}$$

$$E(x, z, t)_y$$

Not depends on y

$$\Rightarrow n = 0$$

$$TE_{m0}$$

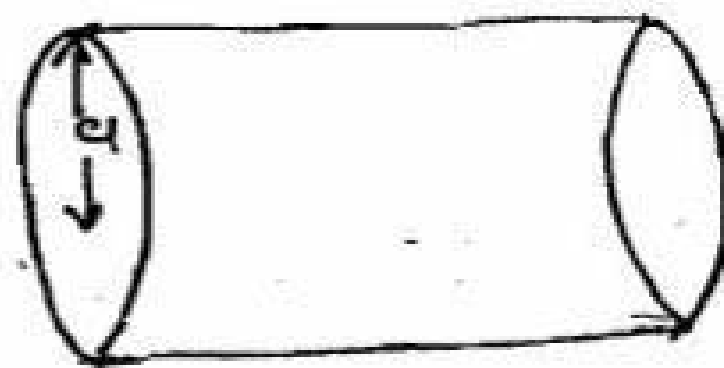
$$TE_{20}$$

$$\sin\left(\frac{2\pi}{a} x\right)$$

$$\sin(\omega t - \beta z)$$

TEM exists $\rightarrow$ single conductor

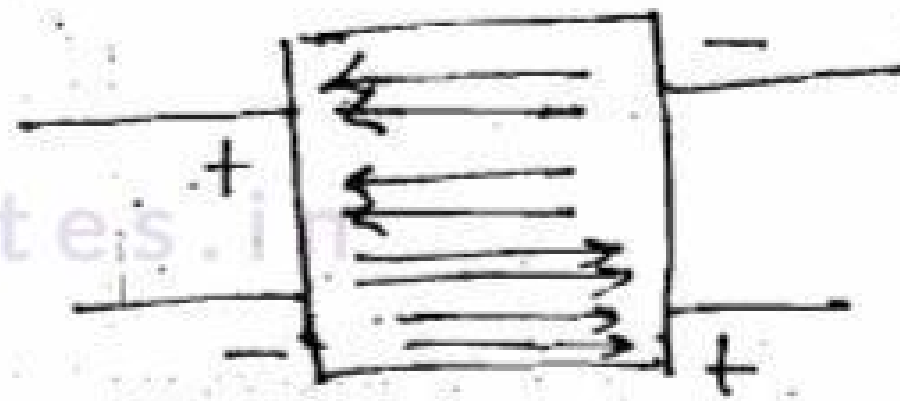
(B.)



12. No cut off freq. $\rightarrow$ TEM exists

Ans-A

13. TE_{02}



14. $f_c = \frac{3 \times 10^8}{2 \times 2 \times 10^{-2}} = \frac{3 \times 10^8}{2 \times \sqrt{\epsilon_r} \times 1 \times 10^{-2}} \Rightarrow \epsilon_r = 4$

15. Note! -

Parallel waveguide

TEM $\rightarrow (0 - \infty)$

$TE_1/TM_1 \rightarrow (f_1 - \infty)$

$TE_2/TM_2 \rightarrow (f_2 - \infty)$

15. ① $m=1, n=0$
 ② $m=0, n=1$
 ③ $m=1, n=1$ } $f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$
 4x3 cm

① $\rightarrow TE_{10}$ $f_{c1} = \frac{c}{2a} = 3.75 \text{ GHz} (3.75 \rightarrow \infty)$

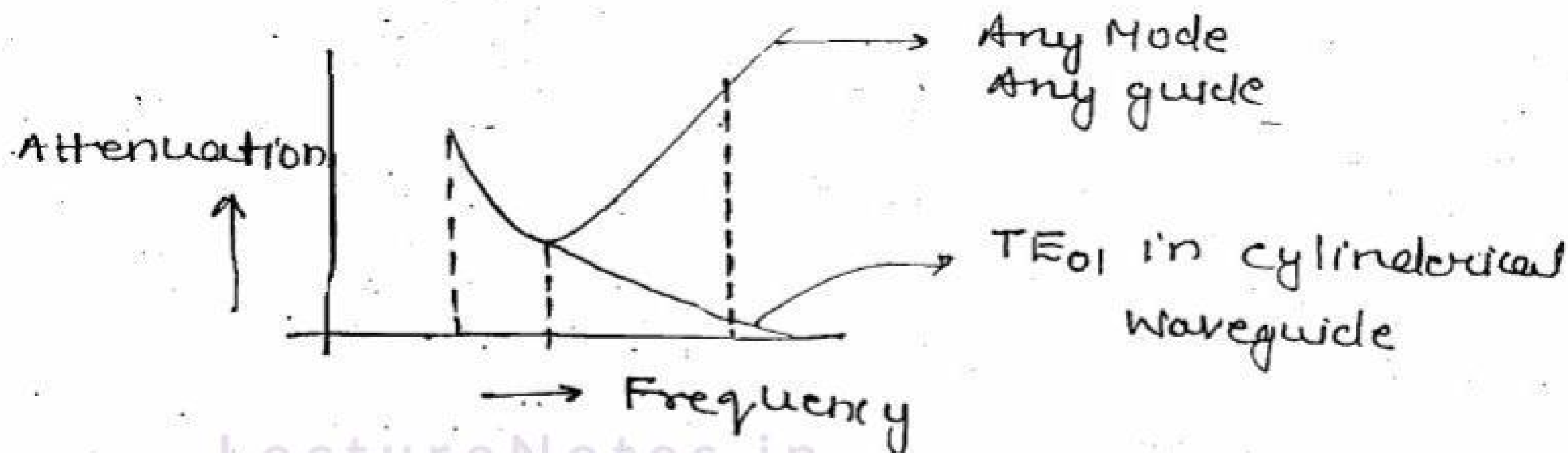
TE_{01} $f_{c2} = \frac{c}{2b} = 5 \text{ GHz} (5 \text{ GHz} \rightarrow \infty)$

$\frac{TE_{11}}{TM_{11}}$ $f_{c3} = 6.25 \text{ GHz} (6.25 \text{ GHz} \rightarrow \infty)$

From (3.75-5) GHz there is strictly single mode operation.

Note:-

Practically Graph :-



With practical non-ideal conducting walls, higher freq. are not preferred in lower modes

16. $\rightarrow D$

17. $\rightarrow TE_{10}$ only

$$f_c < f < f_{c_{TE_{01}}}$$

$$\frac{c}{2a} < \frac{c}{\lambda} < \frac{c}{2b}$$

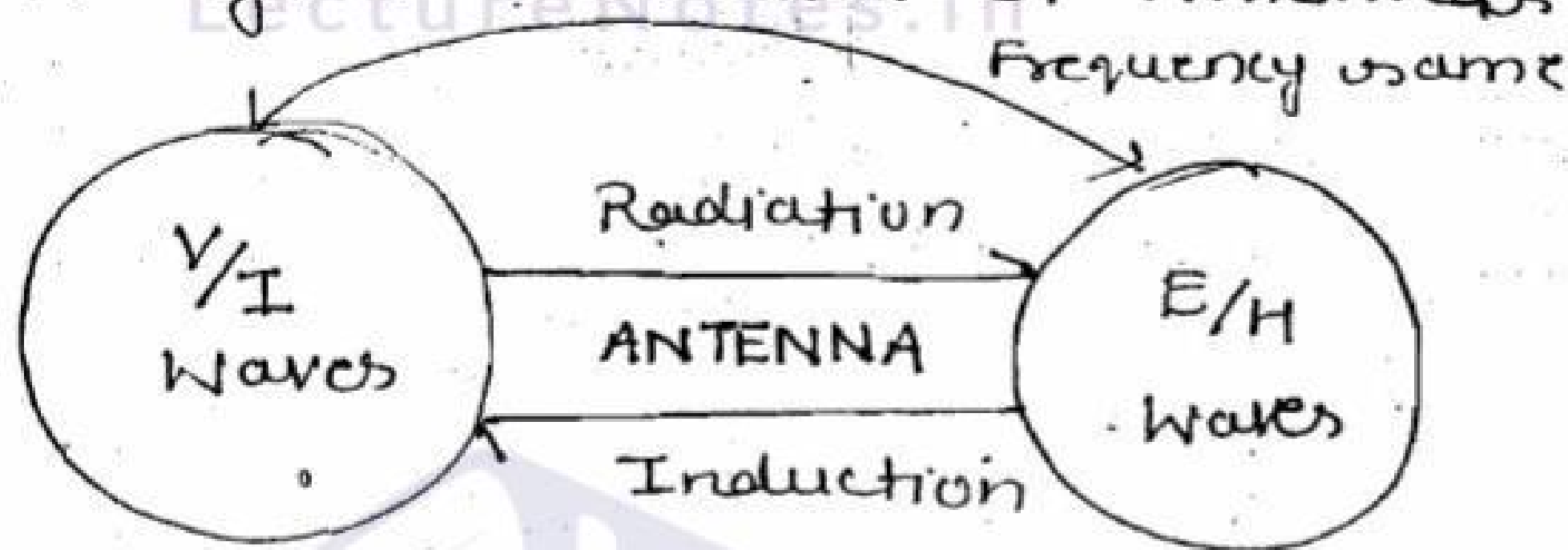
$$2a > \lambda > 2b$$

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:- ANTENNAS :-

- Hertzian Dipole / Halfwave Dipole
- Basic Terms and Definitions → 50%
- Antenna Arrays
- Friis - Free Space Propagation
- Study / classification of Antennas



Hertzian Dipole as a Basic Radiating Element :-

It is a dl length $I(t) = I_m \sin \omega t$ (oscillatory or Harmonic) current element with $dl \ll \lambda$

$$(i) \quad A(t) = \frac{\mu I(t) dl}{4\pi r}$$

$$(ii) \quad B(t) = \mu H(t) = \nabla \times \vec{A}$$

$$(iii) \quad \nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$$(iv) \quad E(t) = \frac{1}{\epsilon} \int (\nabla \times H) dt$$

Every harmonic current produces a time/space harmonic E & H around it. This EM wave generation is called as radiation.

Expression for Radiated fields of Hertzian Dipole :-

$$\left\{ \begin{array}{l} \vec{E}(\gamma, \theta, \phi, t)_{\theta} = \left(\frac{I_m dl \sin \theta \cdot \omega}{4\pi \epsilon c^2 \gamma} \right) \sin \omega t \cdot e^{-j\beta \gamma} a_{\theta} \\ \vec{H}(\gamma, \theta, \phi, t)_{\phi} = \left(\frac{I_m dl \sin \theta \omega}{4\pi c \gamma} \right) \sin \omega t \cdot e^{-j\beta \gamma} a_{\phi} \end{array} \right.$$

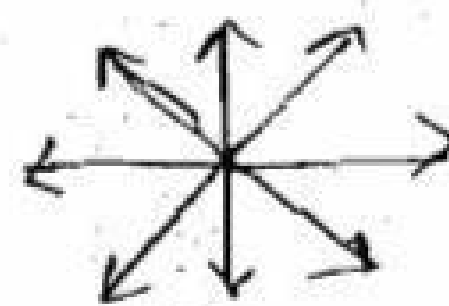
$$W \rightarrow E(z,t)_x = E_0 e^{j\omega t} e^{-j\beta z} \dots a_x$$

Properties of Radiated Waves:-

- i) They travel radially outward and hence their amplitudes decrease as $\frac{1}{r}$ due to their power density decreases as $\frac{1}{r^2}$

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→ Amplitude dec. but does not attenuate.



$$E_\theta \times H_\phi = P_r$$

$$\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \frac{1}{\epsilon \sqrt{\frac{1}{\mu \epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \Omega$$

- (ii) The amplitude of the radiated wave is not omni-directional and depends on θ and ϕ
i.e. Radiation is directive in nature.

- (iii) The amplitude of the radiated wave always depends on $\left(\frac{dI}{d\lambda}\right)$ ratio
i.e. frequency decides radiated power

Total Power Radiated from a Hertzian Dipole:-

$$W_r = \oint P_{avg} \cdot dS$$

$$= \oint_{\text{Sphere}} \frac{1}{2} \frac{E_0^2}{\eta} a_r \cdot dS \cdot a_r$$

$$W_r = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{I_m d \sin \theta \omega}{4\pi \epsilon c^2 r} \right)^2 \frac{1}{120\pi} r^2 \sin \theta d\theta d\phi$$

$$\left(\omega = \frac{2\pi c}{\lambda} \right)$$

$$W_r = I_{rms}^2 \cdot 80 \pi^2 \left(\frac{dl}{\lambda} \right)^2$$

The expression is similar to a resistor dissipating power as heat i.e. every antenna radiates power in the form of EM wave

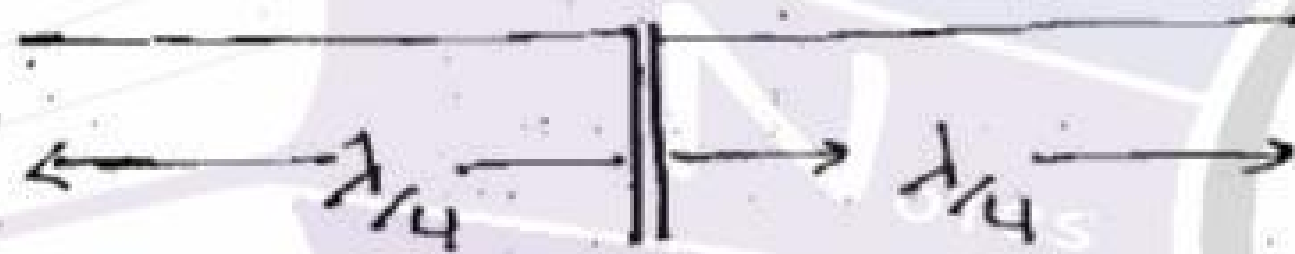
$$R_r = \text{Radiation resistance of Hertzian Dipole}$$

$$= 80 \pi^2 \left(\frac{dl}{\lambda} \right)^2$$

→ R_r is a measure of radiated power for a given input current i.e. it should be as possible for a practical antenna

Halfwave dipole as a fundamental Antenna:-

→ A centre feed $\lambda/2$ - dipole is a transmission opened out by $\lambda/4$ on either sides



→ The length of antenna is strictly depends on the frequency of operation

<u>FM</u>	<u>GSM</u>
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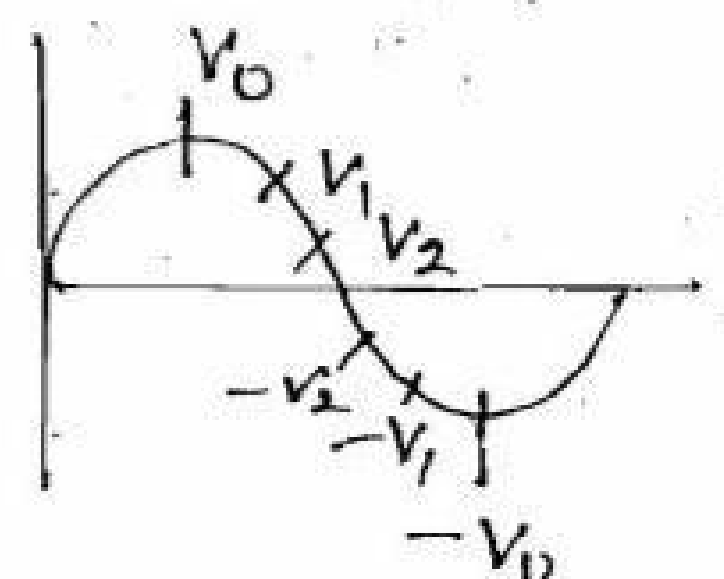
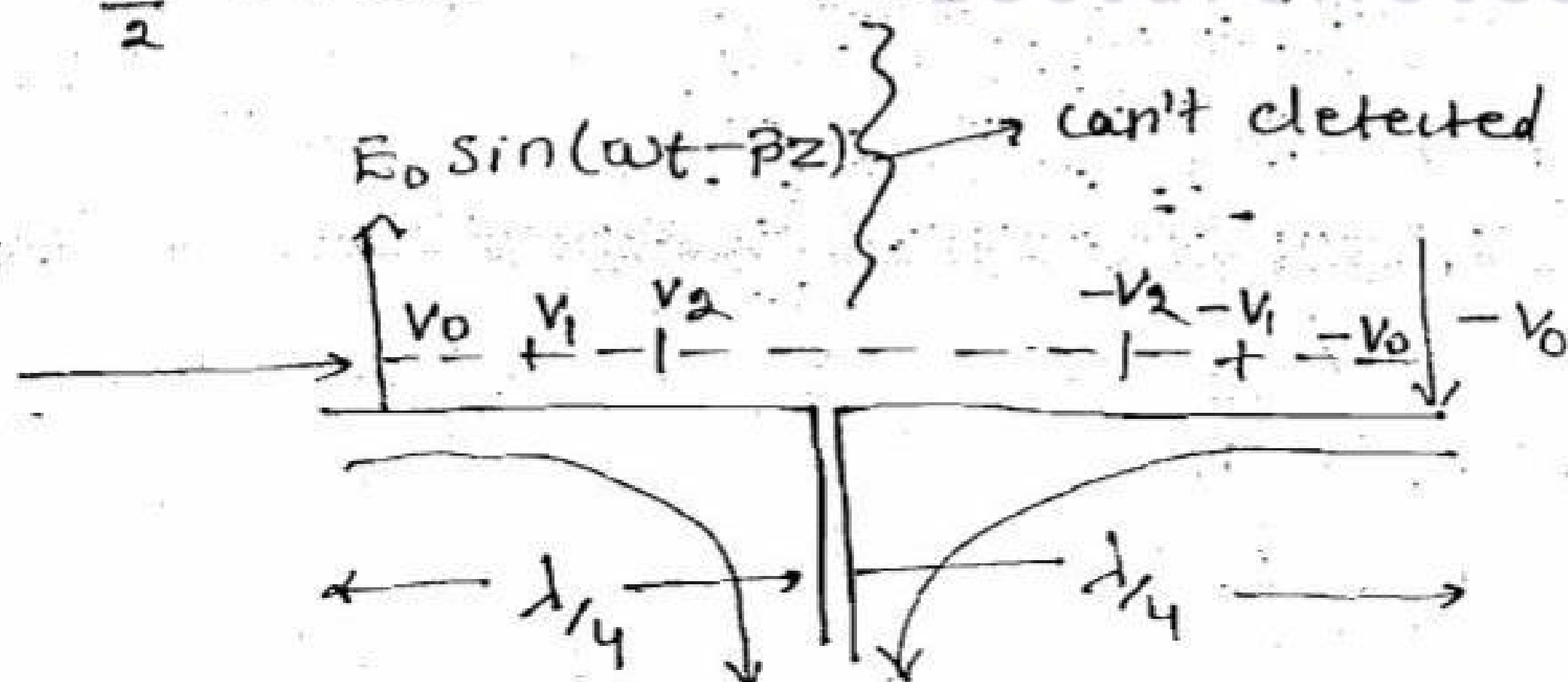
$$f = 100 \text{ MHz}$$

$$f = 1800 \text{ MHz}$$

$$\lambda = 3 \text{ m}$$

$$\lambda = \text{few cm}$$

$$\frac{\lambda}{2} = 1.5 \text{ m}$$



Note:-

When an EM waves having right frequency and right polarization travels along the axis of the antenna, it induced V_0 voltage at one end and progressively decrease voltage such that it is $-V_0$ at the other end. Hence the word half wave dipole.

> These voltages drive a current towards centre which is maximum in the line such that

$$\text{Asymptotic } \left\{ \begin{array}{l} |V(z)| = V_0 \sin \beta z \\ |I(z)| = I_0 \cos \beta z \end{array} \right\} \rightarrow z = -\frac{\lambda}{4} \text{ to } \frac{\lambda}{4}$$

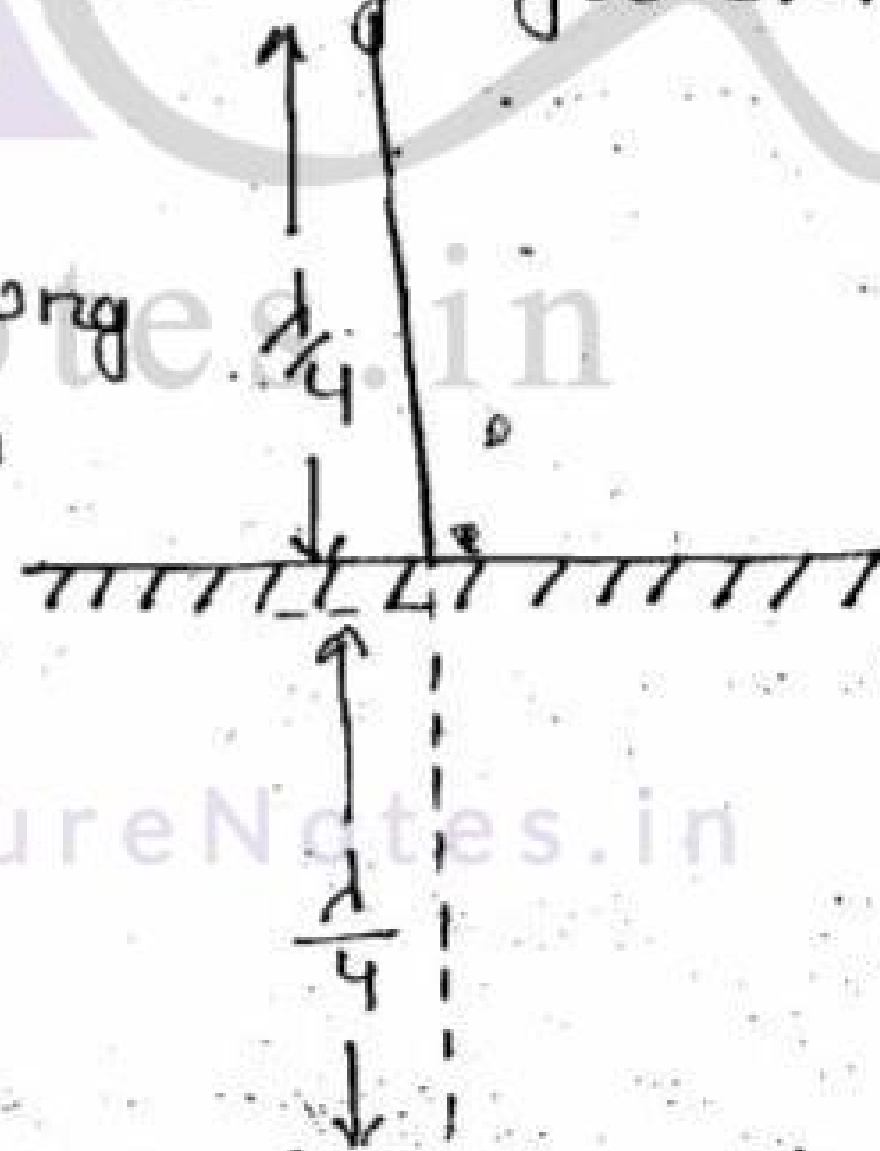
$\frac{V}{I}$ distribution

* $V_0 = -ve$ at other end bec. of phase diff = 180°

Quarterwave Monopole as a Practical Antenna:-

→ It is a low frequency vertically grounded single wire of $\lambda/4$ length.

→ It is a base feed and along with its image works like a harmonic dipole.



Summary:-

A halfwave dipole is an array of Hertzian dipoles with dl from $-\lambda/4$ to $\lambda/4$ and

$$I_m = |I(z)| = I_0 \cos \beta z$$

Asymptotic

Radiation Expressions for Halfwave Dipole:-

$$E(r, \theta, \phi, t)_\theta = \left(\frac{60 I_0}{r} \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right) \sin\omega t \cdot e^{-j\beta r} a_\theta$$

$$H(r, \theta, \phi, t)_\phi = \left(\frac{I_0}{2\pi r} \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right) \sin\omega t \cdot e^{-j\beta r} a_\phi$$

Total power radiated from Halfwave Dipole

$$W_r = \int \frac{1}{2} \frac{E_0^2}{\eta} \cdot dV$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{60 I_0}{r} \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right)^2 \cdot \frac{1}{120\pi} r^2 \sin\theta d\theta d\phi$$

$$= I_{rms}^2 (73)$$

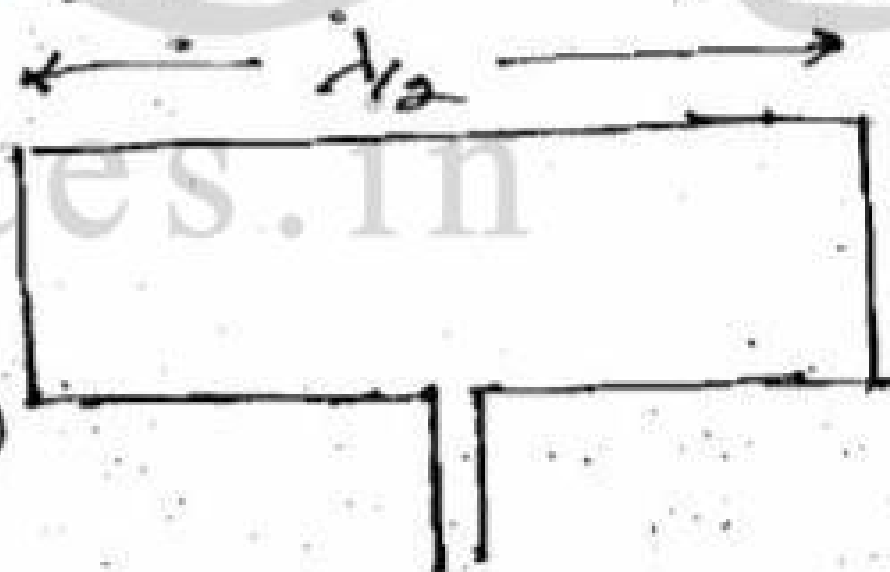
R_r for a Halfwave Dipole = 73Ω

R_r for quarterwave monopole = 36.5Ω

R_r for a folded dipole = $2^2 \times 73 \Omega = 292 \Omega$

R_r for n -Dipoles = $n^2 \times 73$

($n^2 \times 73$)



Basic Terms and Definitions:-

1) Isotropic Antenna:-

It radiates power in all directions uniformly. Its E field pattern is independent of θ and ϕ

eg:- Broadcast antenna

(II) Radiation Power Density :-

It is the strength of the radiated EM wave in any direction at any distance from the antenna

$$\begin{aligned}\frac{\text{Power}}{\text{Area}} \text{ or Watts/m}^2 &= \frac{dW_r}{ds} \\ &= \text{Poynting vector of EM wave} \\ &= \frac{1}{2} \frac{E_0^2}{\eta} (r, \theta, \phi) \\ &= V(r, \theta, \phi)\end{aligned}$$

mp III) Radiation Power Intensity :-

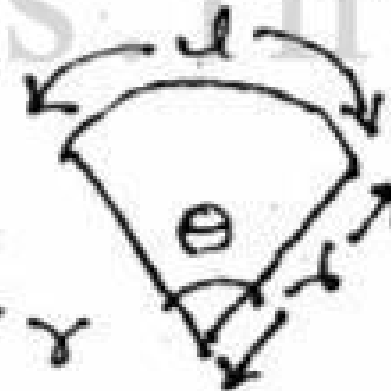
It is the strength of the radiated EM wave in any direction from the antenna

$$\begin{aligned}\frac{\text{Power}}{\text{direction}} &= \frac{\text{Power}}{\text{Solid angle}} \\ &= \text{with steradian} = \frac{dW_r}{d\Omega}\end{aligned}$$

$$\Omega = \theta r$$

If $\theta = 1$ then $\Omega = r$

If $\theta = 6.28$ radian then $\Omega = 6.28 r$



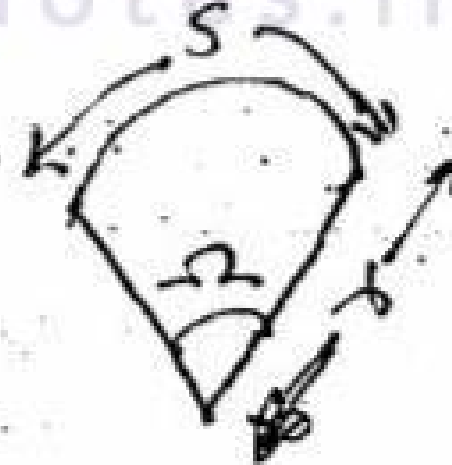
$$S = \Omega r^2$$

$$\Omega = 1 \text{ steradian}$$

$$S = r^2$$

$$\Omega = 12.56 \text{ steradian}$$

$$\text{Total surface area} = 12.56 r^2$$



Any small incremental area, $ds = r^2 d\Omega = r^2 \sin\theta \cdot d\theta \cdot d\phi$
 $d\Omega = \sin\theta d\theta d\phi$

$$\frac{dW_r}{d\Omega} = \frac{dW_r}{ds} \cdot \frac{ds}{d\Omega}$$

$$= V(r, \theta, \phi) r^2 = \psi(\theta, \phi)$$

eg:- Isotropic Antenna

$$\frac{dW_r}{ds} = \frac{W_r}{4\pi r^2} = V_{avg}$$

$$\frac{dW_r}{d\Omega} = \frac{W_r}{4\pi} = \psi_{avg}$$

IV) Gain of an Antenna:-

$G_D \rightarrow$ Directive gain

$G_P \rightarrow$ Power gain

$D \rightarrow$ Directivity

(a) Directive Gain (G_D):-

The radiation intensity of the antenna in a given direction to the radiation intensity of isotropic antenna

$$G_D = \frac{\psi(\theta, \phi)}{\psi_{avg}} = \frac{4\pi \psi(\theta, \phi)}{W_r} = \frac{4\pi \psi(\theta, \phi)}{\int \psi(\theta, \phi) d\Omega}$$

$G_D > 1$ or $G_D < 1 \rightarrow$ depends on direction

(b) Power Gain:-

$$G_P = \frac{4\pi \cdot \psi(\theta, \phi)}{W_N}$$

W_r = Total o/p power

W_N = Total i/p power

$$G_p = \frac{4\pi \psi(\theta, \phi)}{W_r} \cdot \frac{W_r}{W_N}$$

$$= G_p \times \text{Efficiency of Radiation}$$

$$\text{Efficiency} = \frac{W_r}{W_r + W_d} = \frac{R_r}{R_r + R_d}$$

$$\text{Directivity} = G_p|_{\max}$$

$$D \geq 1$$

→ Always

$D = 1$ for isotropic antenna

Radiation Pattern of an Antenna:

It is a polar plot of radiation intensity indicating various directions around the antenna where radiation is finately strong

eg:- $|E| = \frac{k \sin \theta}{r}$

$$\psi = \psi_0 \sin^2 \theta$$

$$\theta = 0^\circ / 180^\circ$$

$$|E| = 0 \rightarrow \text{Null points}$$

→ θ_{np}

HPBW

Half Power
Beam width

BWFN

Beam
Width
Full
Nulls

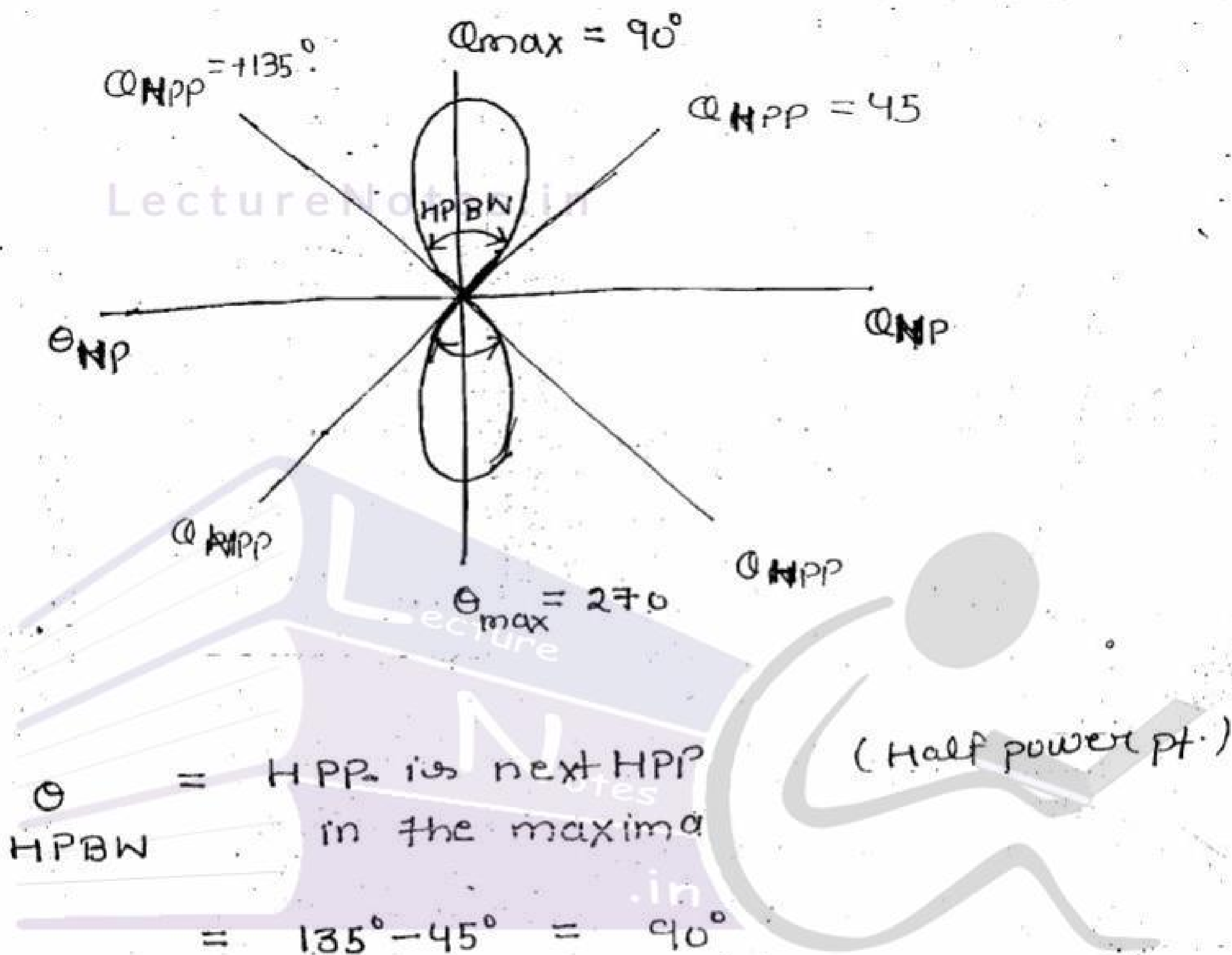
$$\theta = 90^\circ / 270^\circ$$

$$|E| = E_{\max}$$

$$\theta_{\max}$$

$$\theta = 45^\circ / 135^\circ / 225^\circ / 315^\circ$$

$$|E| = \frac{E_{\max}}{\sqrt{2}}$$



The antenna considered has a ϕ independent pattern and hence a circular top view of the beam (for all ϕ)

In general

$$\theta_{HPBW} \times \phi_{HPBW} = \Omega_A \text{ Beam solid angle}$$

$$= \text{steradian} = \text{radian}^2$$

In circular

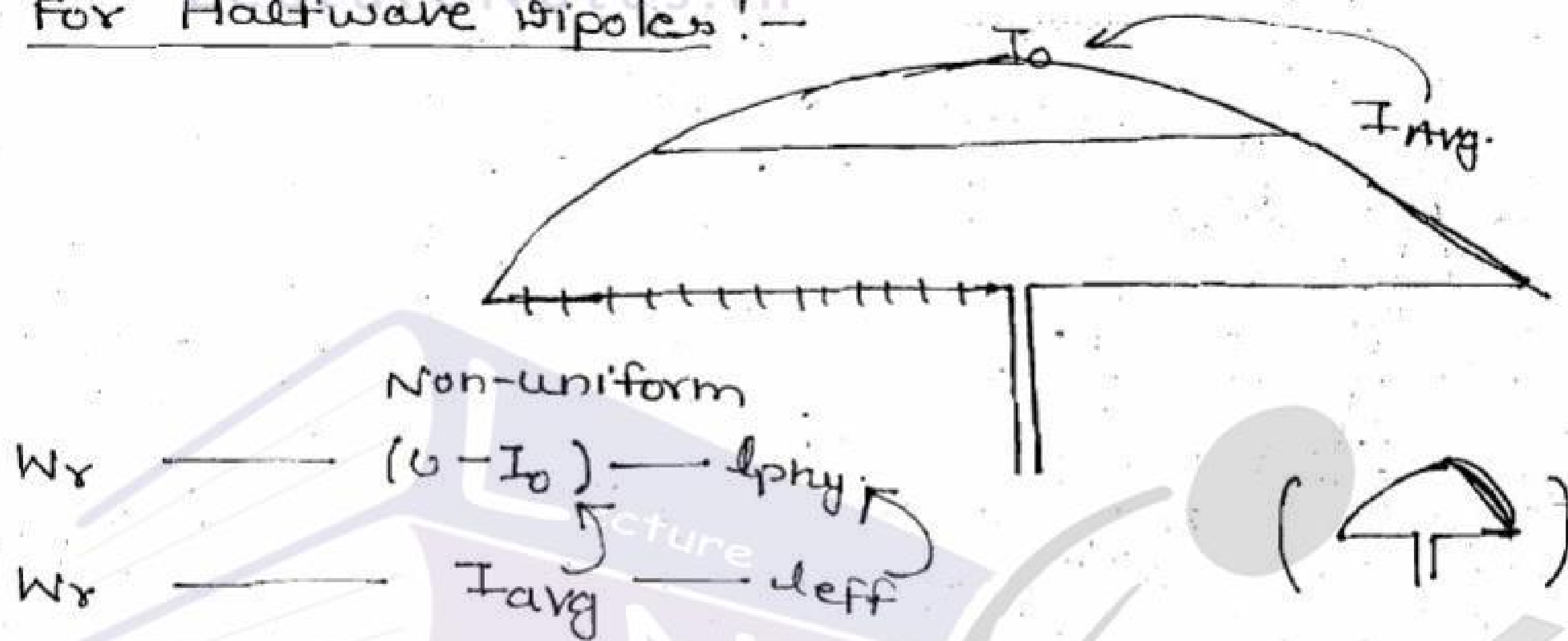
$$\left(\theta_{HPBW} \right)^2 = \Omega_A$$

$$\Omega \propto \frac{1}{\Omega_A}$$

$$\Rightarrow \boxed{\Omega = \frac{4\pi}{\Omega_A}}$$

Q (VI) Effective length of an antenna:-

(a) For Halfwave dipoles:-



Note:-

→ An antenna radiates W_r power over its physical length and non-uniform currents everywhere. Effective length is a length required to radiate same power assuming uniform currents everywhere.

$$I_{avg} = \frac{1}{\pi} \int_0^{\pi} I_0 \sin t \, dt = \frac{2I_0}{\pi}$$

$$\boxed{l_{eff} = \frac{2l_{phy}}{\pi}}$$

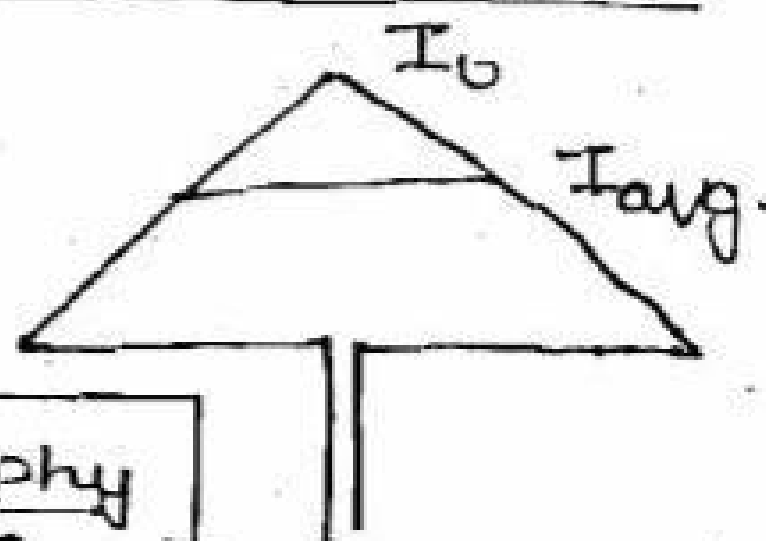
(b) For electrically short dipoles ($l_{phy} < \frac{\lambda}{10}$):-

$$|I(z)| \propto z$$

linear current

$$I_{avg} = \frac{I_0}{2}$$

$$\boxed{l_{eff} = \frac{l_{phy}}{2}}$$



c) For Hertzian Dipole $\left(l_{\text{phy}} < \frac{\lambda}{25} \right)$:-

$$l_{\text{phy}} = l_{\text{eff}} = dl$$

$$I_{\text{avg}} = I_m \rightarrow \text{Uniform currents}$$

$$\text{For Hertzian Dipole } R_r = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$$

$$\text{For any Dipole, } R_r = 80\pi^2 \left(\frac{l_{\text{eff}}}{\lambda} \right)^2$$

$$\begin{aligned} \text{For Electrically short Dipole } R_r &= 80\pi^2 \left(\frac{l}{2\lambda} \right)^2 \\ &= 20\pi^2 \left(\frac{l}{\lambda} \right)^2 \end{aligned}$$

$$\text{Monopoles } R_r = 10\pi^2 \left(\frac{l}{\lambda} \right)^2$$

$$\begin{aligned} \text{Monopoles over conducting earth } R_r &= 10\pi^2 \left(\frac{2h}{\lambda} \right)^2 \\ &= 40\pi^2 \left(\frac{h}{\lambda} \right)^2 \end{aligned}$$

$$\text{For halfwave dipole } = 80\pi^2 \left(\frac{2l}{\pi\lambda} \right)^2$$

$$= 320\pi^2 \left(\frac{l}{\lambda} \right)^2 = 80\Omega$$

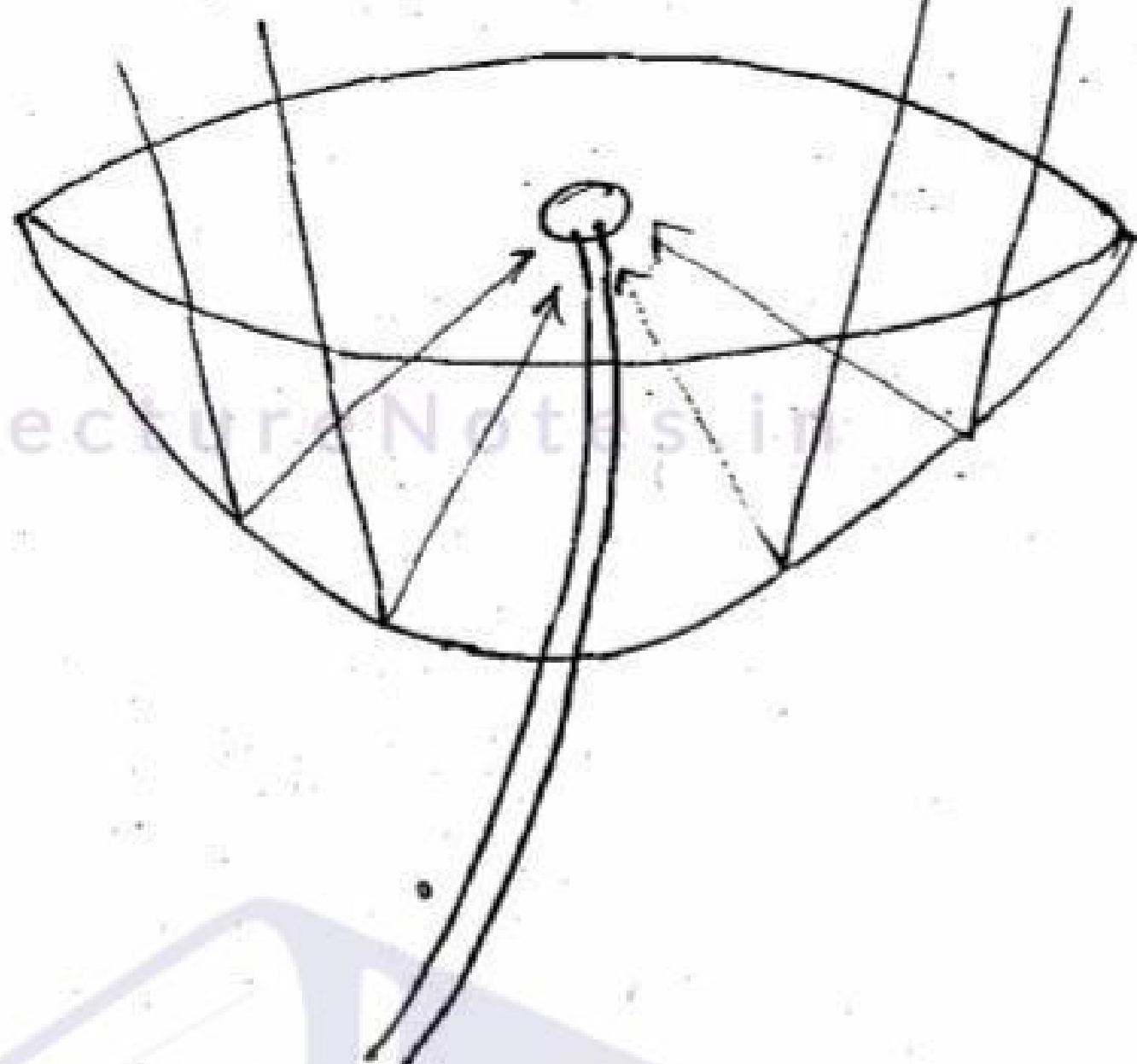
vii) Capture Area or Effecting Area (A_c) :-

In microwave antenna $l = \text{few cm}$

$\lambda = \text{few } 1000\text{km}$

$$A_c = \text{Capture Area} = \frac{\text{Power induced}}{\text{Poynting Vector}}$$

$$A_c = \frac{\lambda^2}{4\pi} G$$



Workbook :-

$$\theta \rightarrow G_{\theta} |_{\max} \rightarrow \psi(\theta, \phi) \rightarrow U(r, \theta, \phi) \rightarrow E(r, \theta, \phi)$$

$$\text{Hertzian Dipole, } |E| = \frac{k \sin \theta}{r^2 \eta}$$

$$U(r, \theta, \phi) = \frac{1}{2} \frac{k^2 \sin^2 \theta}{r^2 \eta}$$

$$\psi(\theta, \phi) = \frac{1}{2} \frac{k^2 \sin^2 \theta}{\eta}$$

$$G_{\theta} = 4\pi^2 \frac{1}{2} \frac{k^2 \sin^2 \theta}{\eta}$$

$$\int_0^{\pi} \int_0^{2\pi} \frac{1}{2} \frac{k^2 \sin^2 \theta}{\eta} \sin \theta \, d\theta \, d\phi = 2 \sin^2 \theta$$

$$\int_0^{\pi} \sin^3 \theta \, d\theta = \frac{4}{3}$$

$\Leftarrow$

$$= \frac{3}{2} \sin^2 \theta$$

$$D = G D_{\max} = 1.5$$

For Halfwave Dipole, $D = 1.63$

$\frac{4-\lambda}{2}$ Dipole

$$W_r = 4 I_{rms}^2 \cdot 73 = 4 \left(\frac{0.5}{\sqrt{2}} \right)^2 \cdot 73 = 36.5 \text{ Watts}$$

$$D = \frac{4\pi}{\Omega_A}$$

$$10 \log D = 44 \text{ dB}$$

$$\Rightarrow D = 10^{4.4}$$

$$\Rightarrow D = \frac{4\pi}{(\theta_{HPBW})^2}$$

$$10^{4.4} = \frac{4 \times 3.14 \times (57)^2}{(\theta_{HPBW})^2}$$

$$\text{Efficiency} = \frac{R_{\text{radiation}}}{R_r + R_d}$$

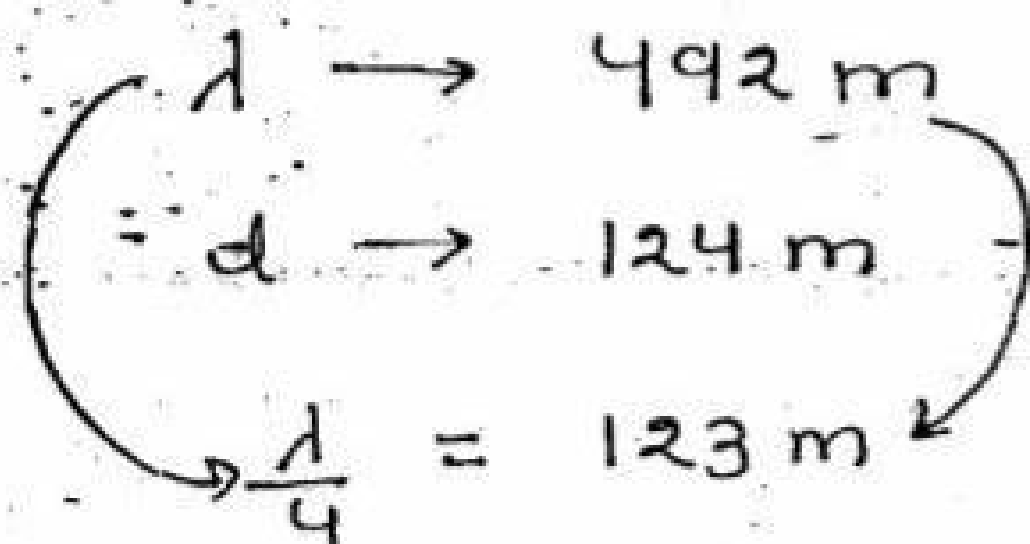
R_r for $\lambda/8$ Dipole

$$R_r \lambda/2 = 73 \Omega$$

$$R_r \lambda/4 = 36.5 \Omega$$

$$R_r \lambda/8 = 18.25 \Omega$$

$$\text{Efficiency} = \frac{18.25}{19.75} = 89\%$$



$$R_r = 36.5 \Omega$$

$$10 \log 4 = 6 \text{ dB}$$

$$G = 4$$

$$W_{IN} = 1 \text{ mW}$$

lossless

$$W_r = W_{IN}$$

→ Ans - (B)

(B)