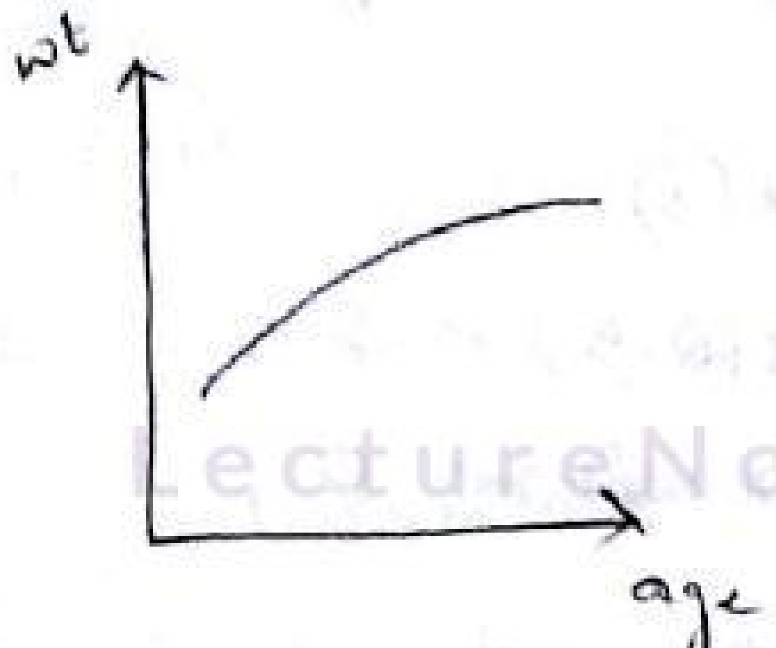
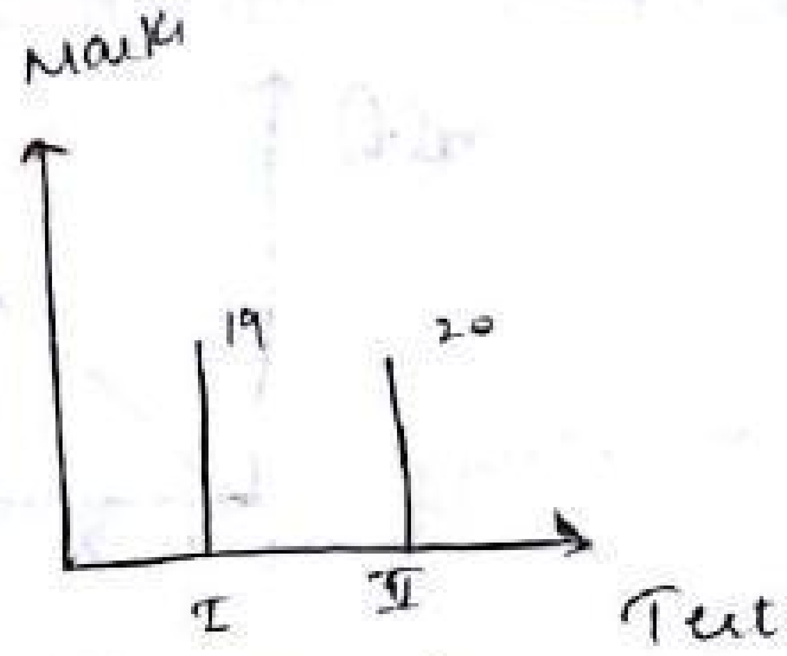
9/2/17

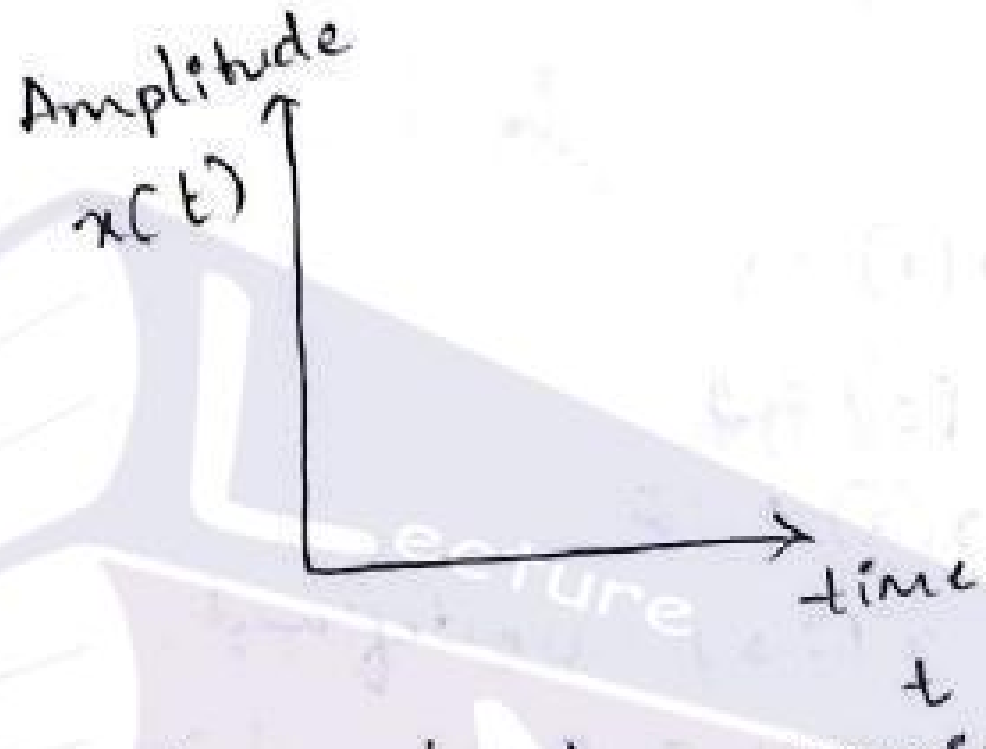
Signal is graphical Representation which conveys information in a secured form Represented interms of One (or) More variables.



Continuous



discreate

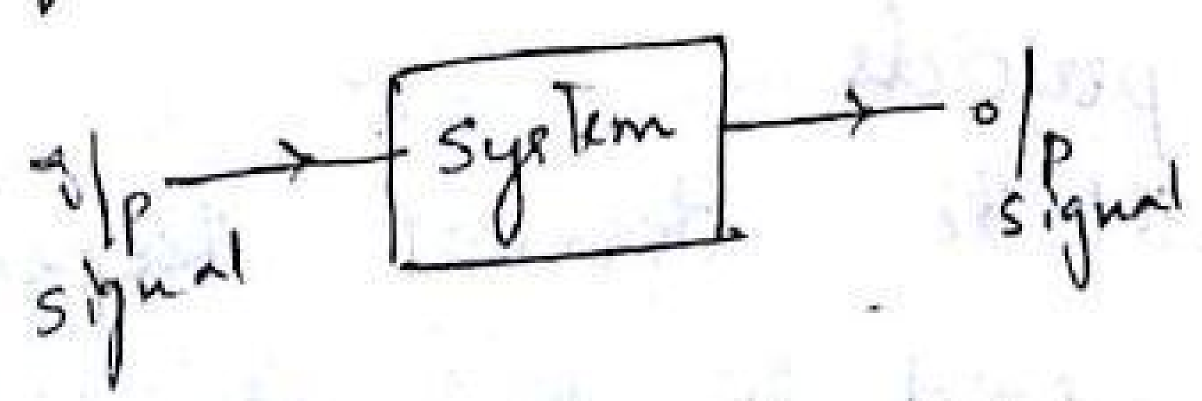


Time (t) independent variable
Amplitude (x) dependent variable. It depend on time, so we Represent as x(t)

Audio v(t)
Image (x,y)
3D Image D(x, y, z)
Video D(x, y, z, t)

System

System is an entity which takes i/p manupulat as a function of variable and gives o/p.



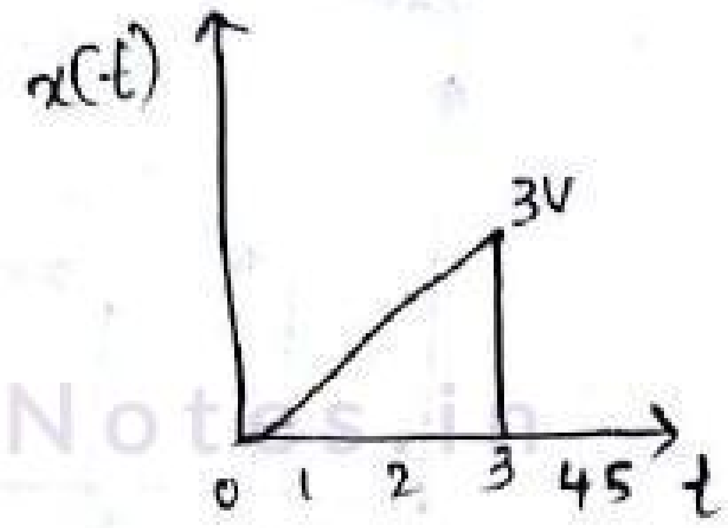
NOTE:

System is a Circuit.

Classification of Signal

i) Continuous and Discrete Signal

Signal whose amplitude exist at every instant of time is said to be continuous signal.

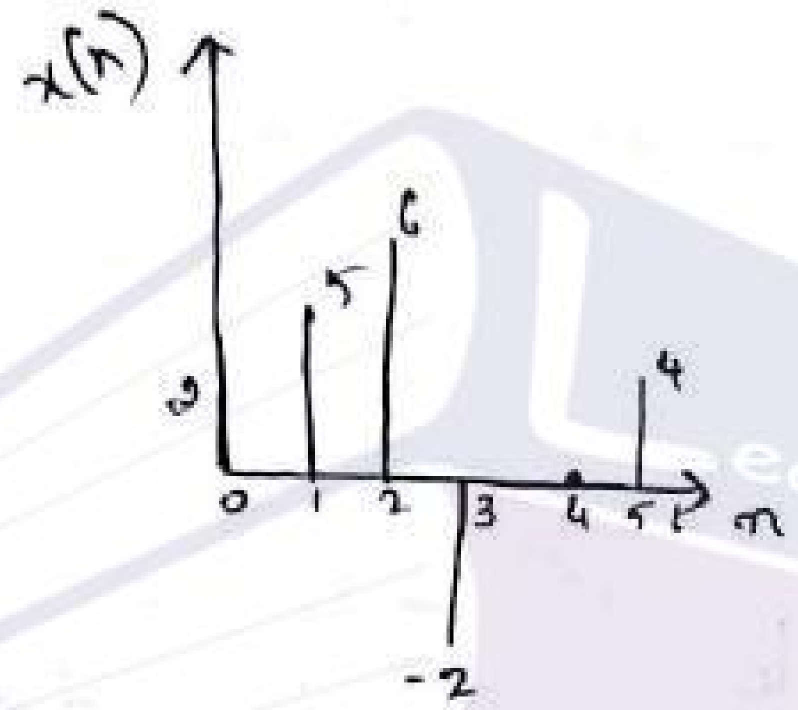


$x(3) = 3$

$x(2.5) = 2.5$

$x(4) = 0$

Signal whose amplitude exist at perticular instant of time is said to be discrete signal



$x(1) = 5$

$x(0) = 2$

$x(3) = -2$

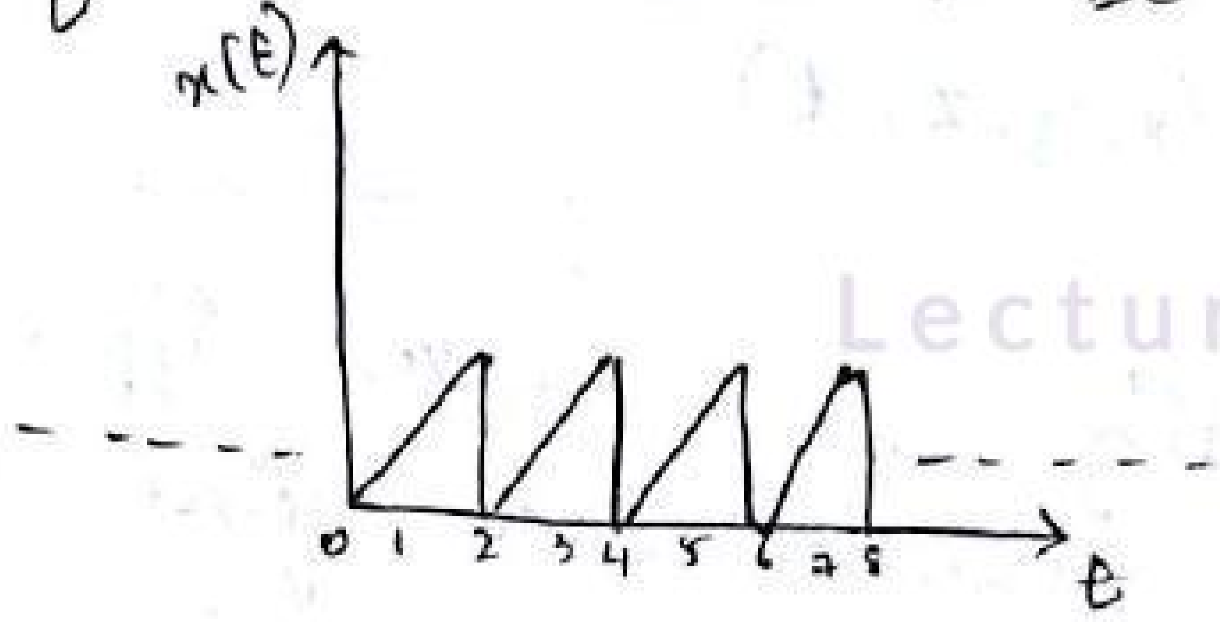
$x(2.8) =$ undefined

$x(4) = 0$

In discrete in between the time instant amplitude is undefined.

ii) periodic and Non-periodic Signal

Signal whose amplitude repeats at Regular interval of time is said to be periodic signal.



$x(2) = 4$ $\quad x(2) = x(4)$

$x(4) = 4$ $\quad x(3) = x(5)$

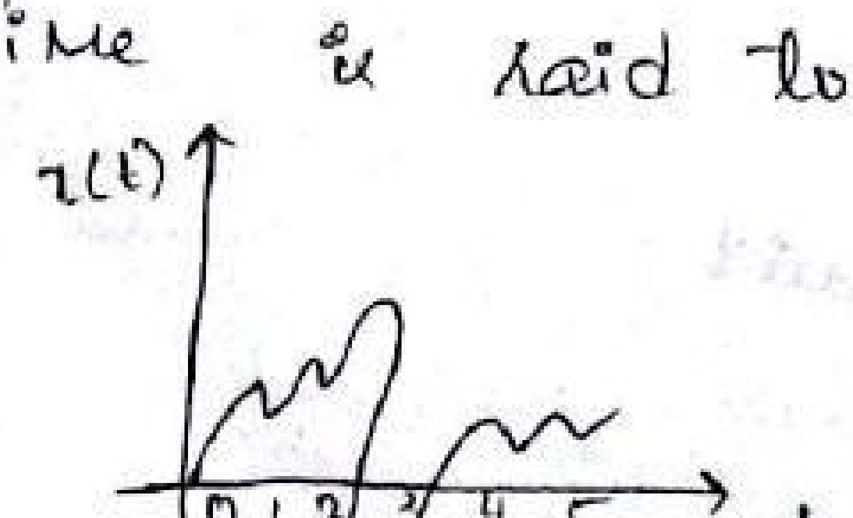
$x(6) = 4$ $\quad x(t) = x(t+2)$

$x(t) = x(t+T)$

$x(n) = x(n+N)$

T, N are Time periods.

Signal whose amplitude doesnot Repeats at any interval of Time is said to be Non-periodic Signal



$x(t) \neq x(t+T)$

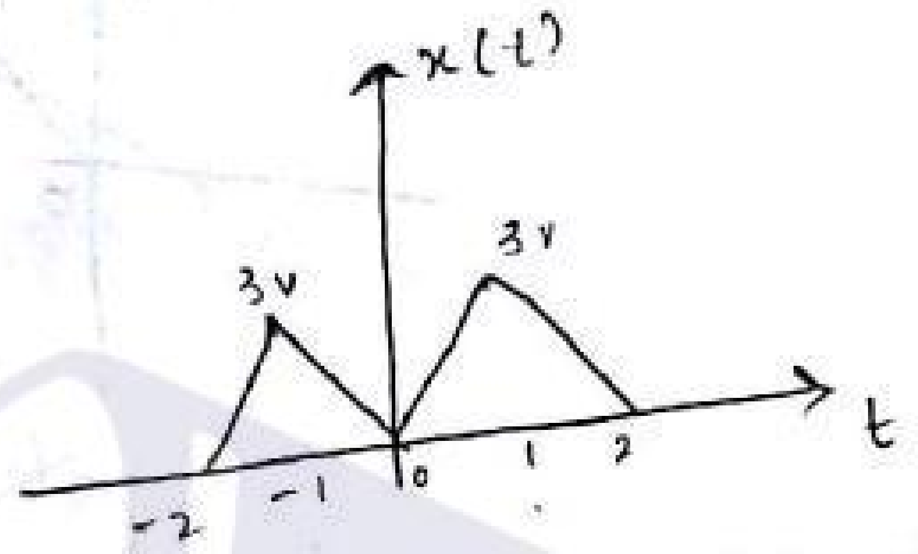
iii) Deterministic and Random Signal

Signal whose amplitude can be predicated before its actual occurance is called Deterministic signal. eg:- ECG

Signal whose amplitude cannot be predicated before its actual occurance is said to be Random signal.

iv) Even and odd signal

Signal which has same amplitude both in positive and -ve time is said to be even signal.

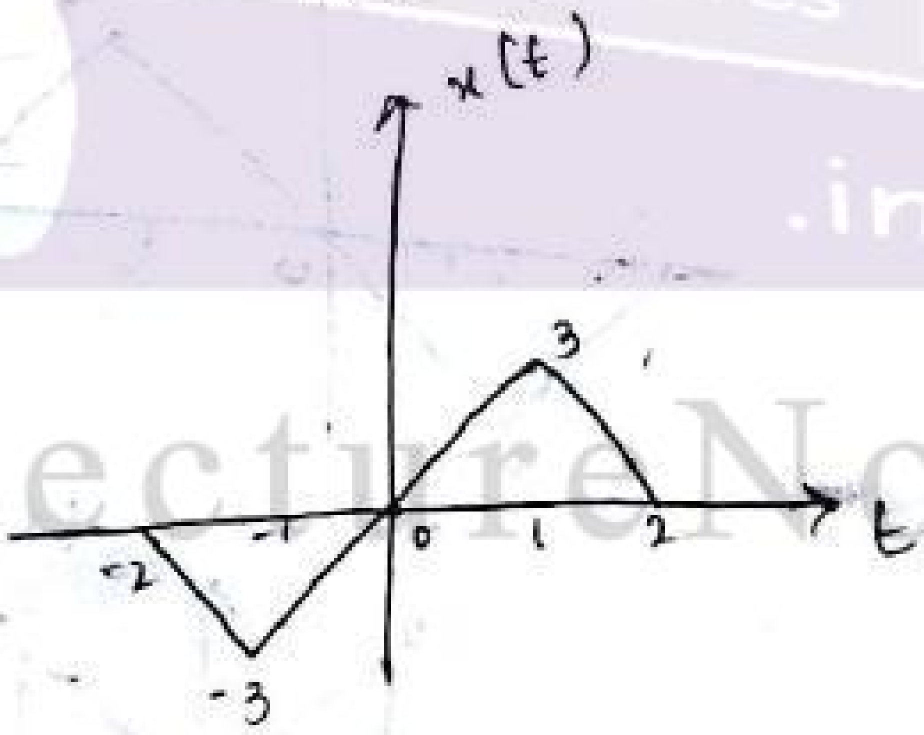


$x(1) = 3$
$x(-1) = 3$

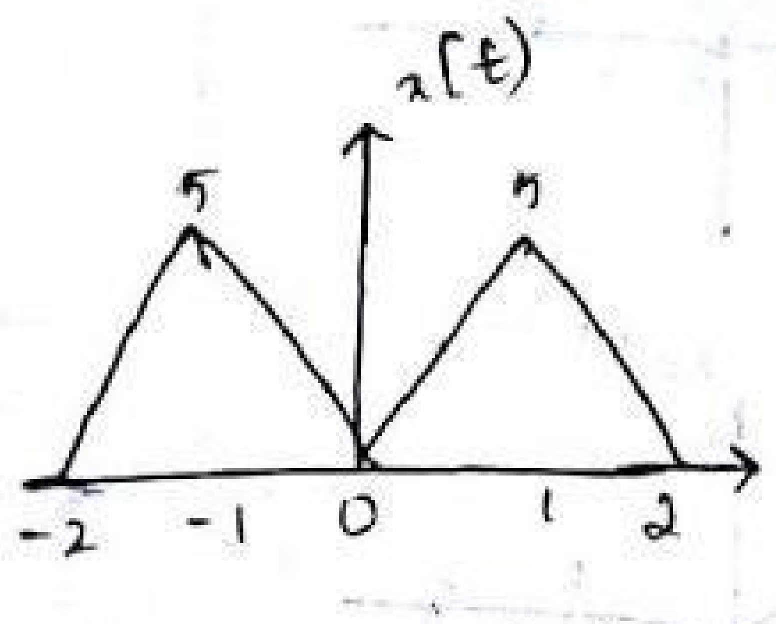
$x(1) = x(-1)$

$\boxed{x(t) = x(-t)}$

Signal which has opposite amplitude both in positive and -ve time is said to be odd signal.



$x(1) = 3$
$x(-1) = -3$
$x(1) = -x(-1)$
$\boxed{x(t) = -x(-t)}$



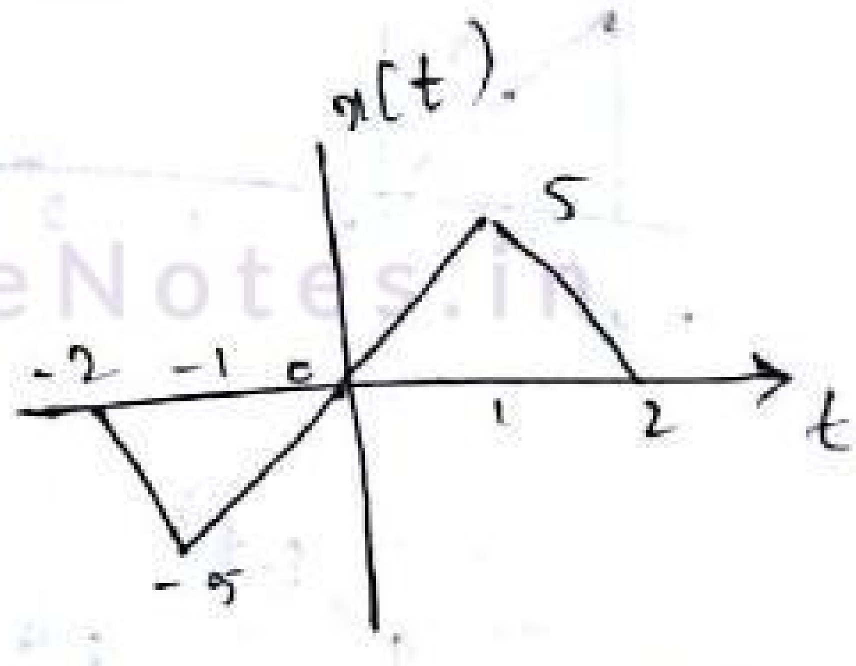
$x(1) = x(-1)$

$x(t) = x(-t)$

$x(1) + x(-1) = \cancel{2(x_1)}\ 2x(1)$

$x(t) + x(-t) = 2x_e(t)$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \rightarrow (1)$



$x(1) = -x(t)$

$x(t) = -x(-t)$

$x(1) - x(-1) = 2x(1)$

$x(t) - x(-t) = 2x_o(t)$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

(2)

Add (1) & (2)

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

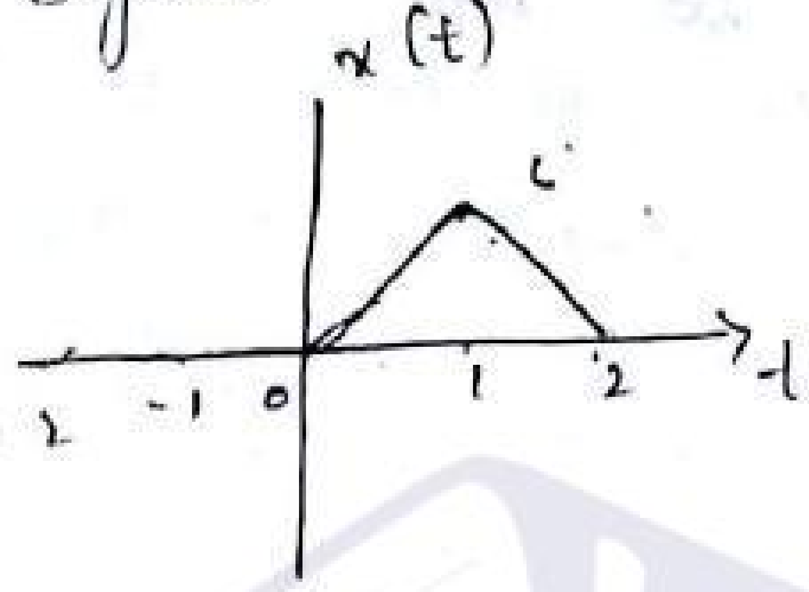
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

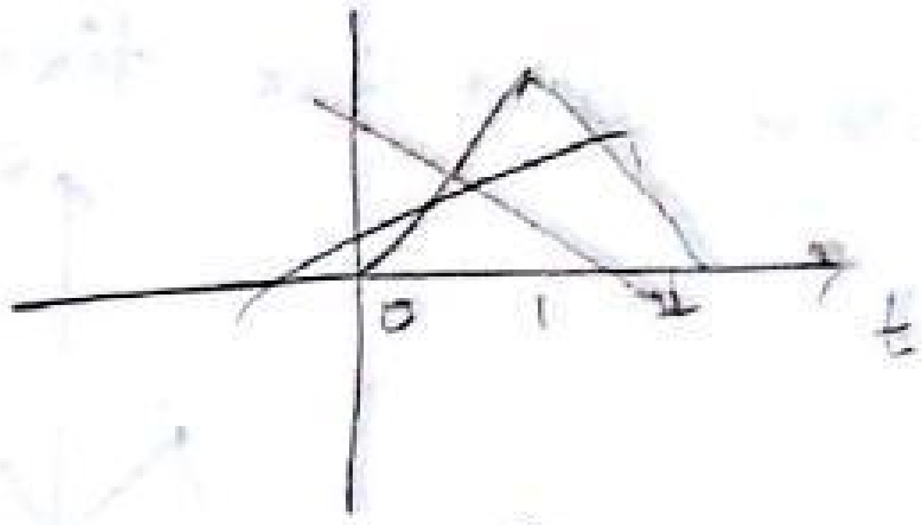
$$x_e(t) + x_o(t) = x(t)$$

Any Signal is Combination of even & odd

(1) find even and odd Component for the given Signal

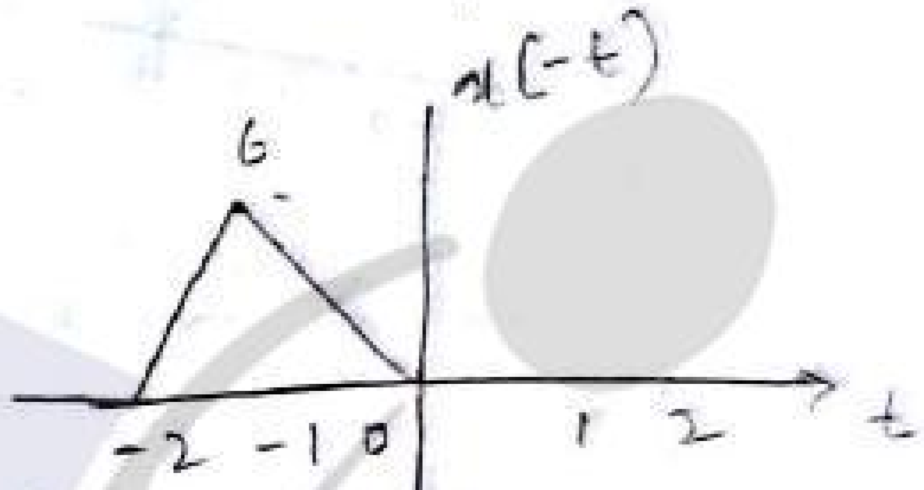
→

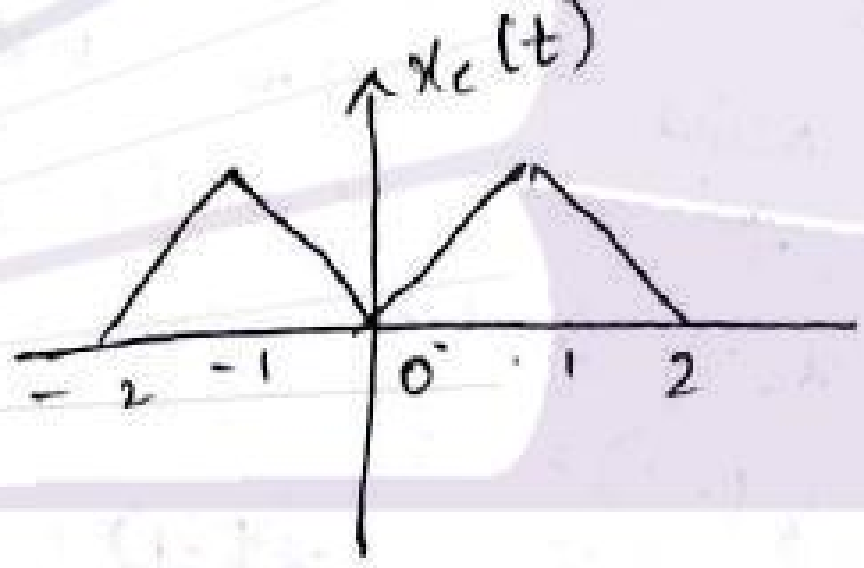


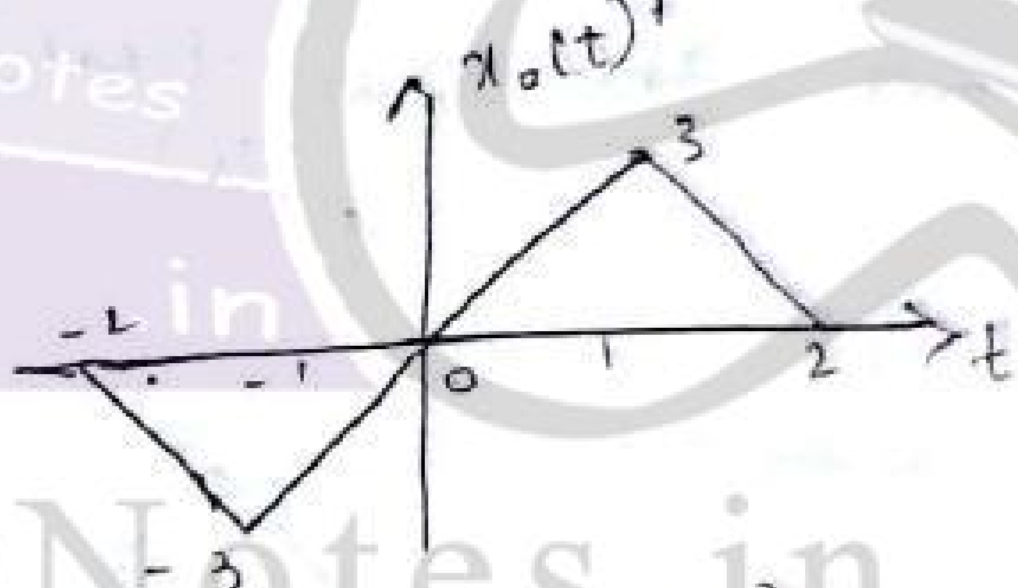


$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

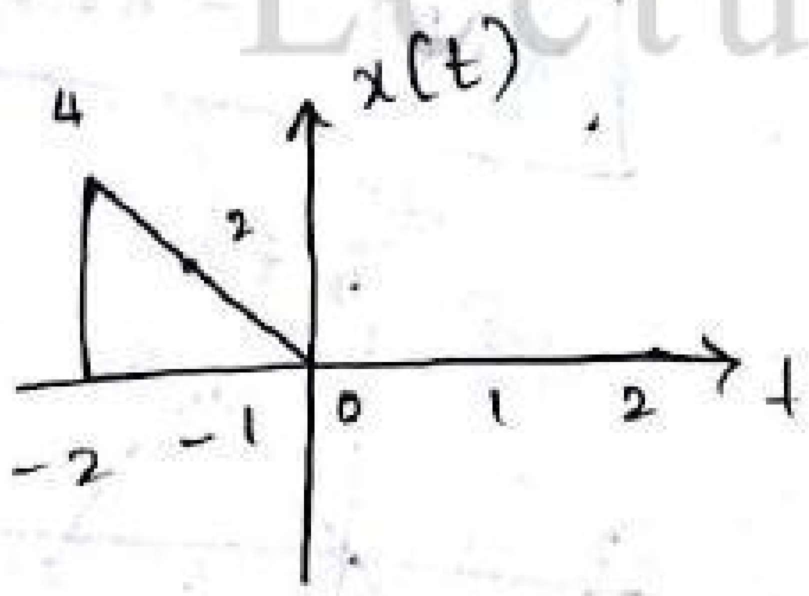
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

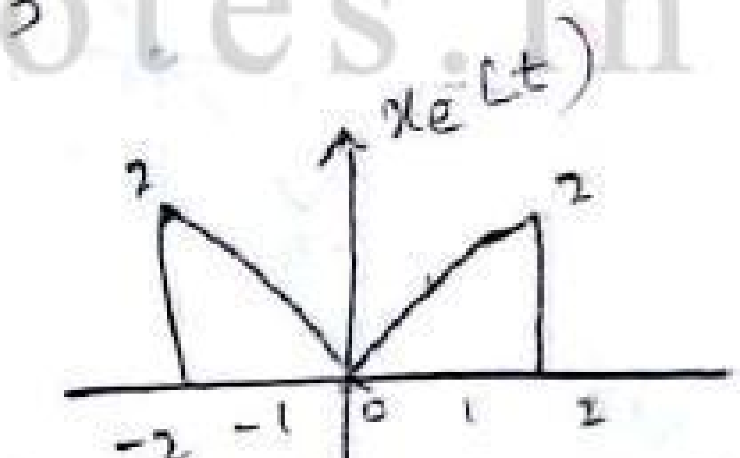


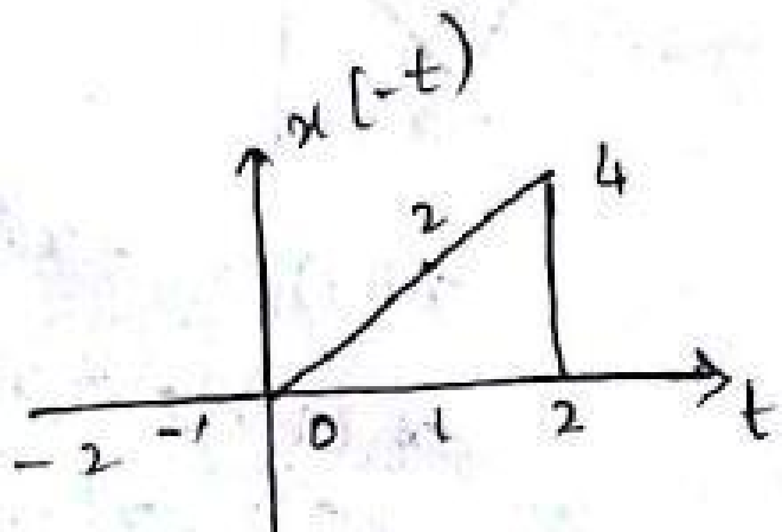


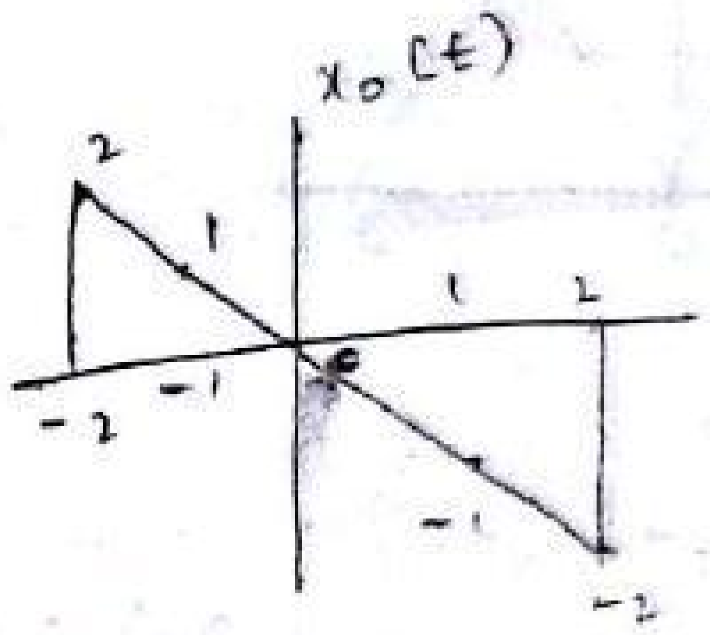


(2)









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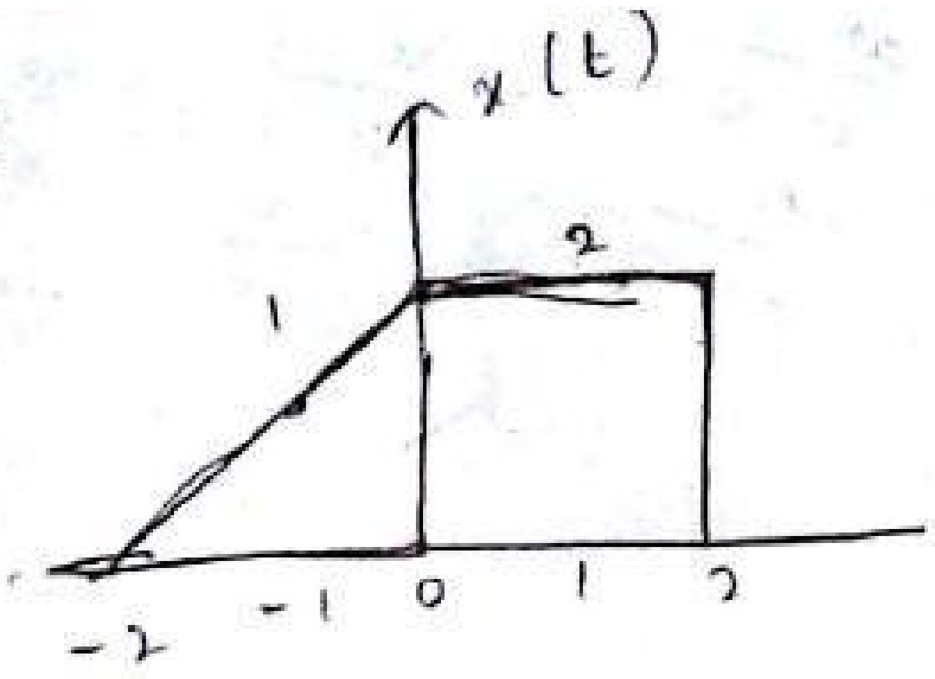

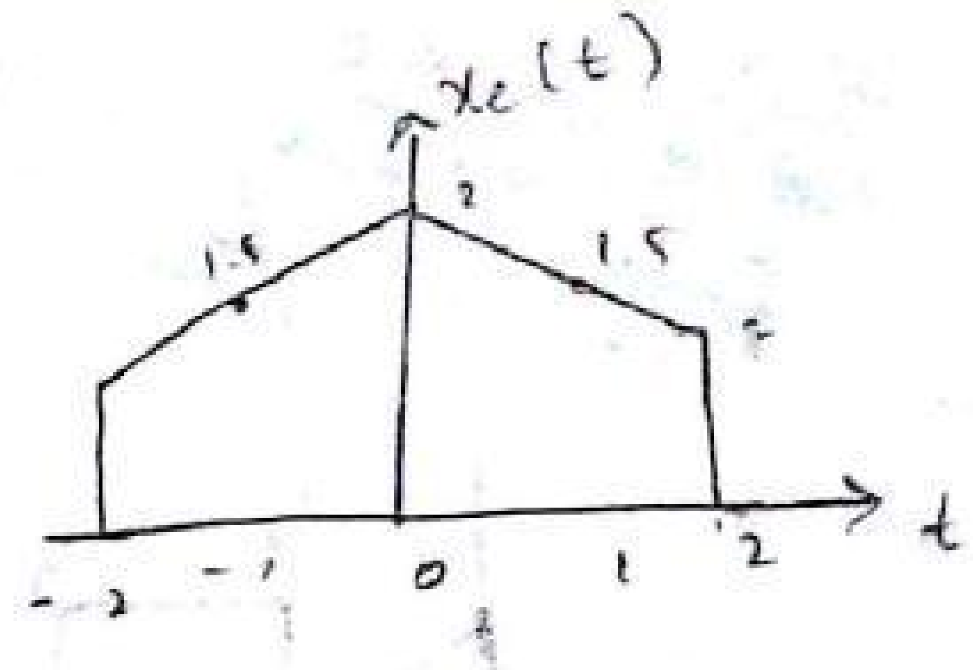

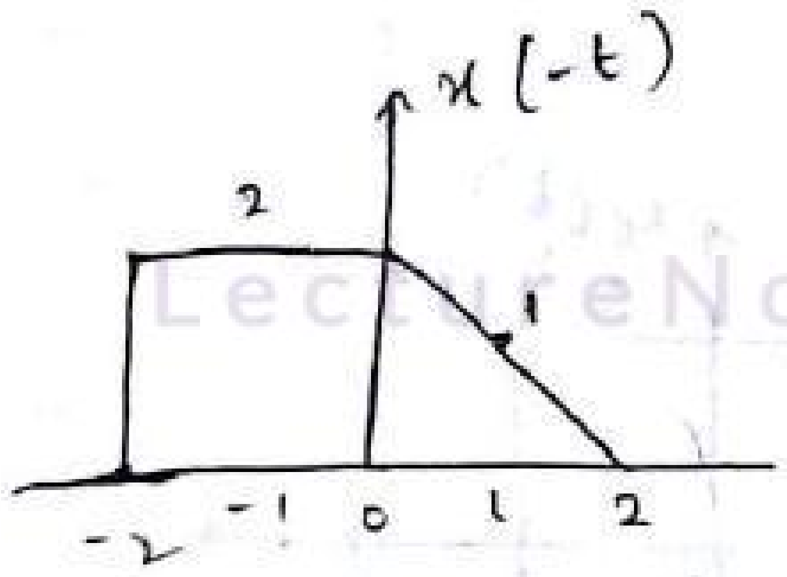

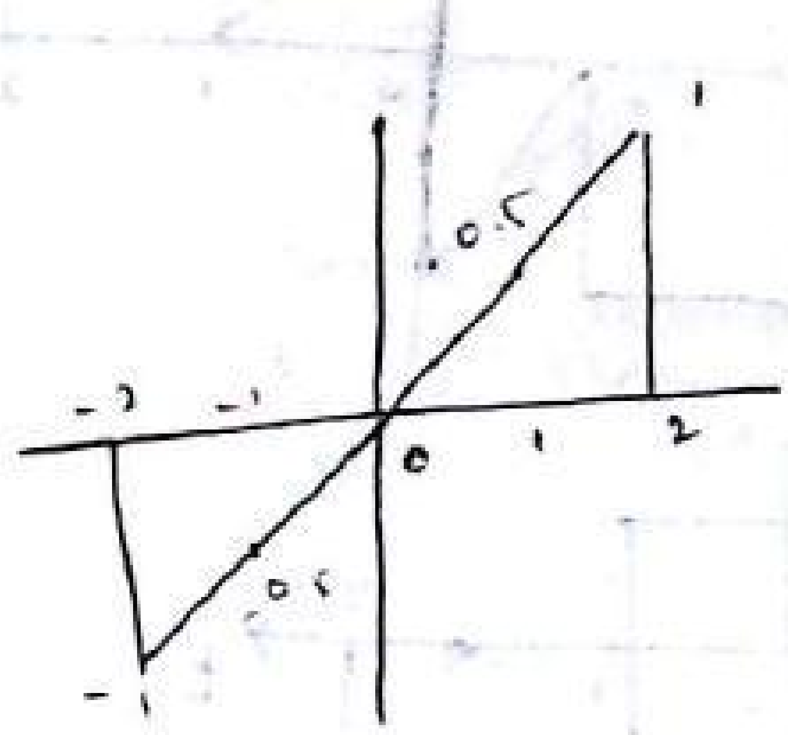

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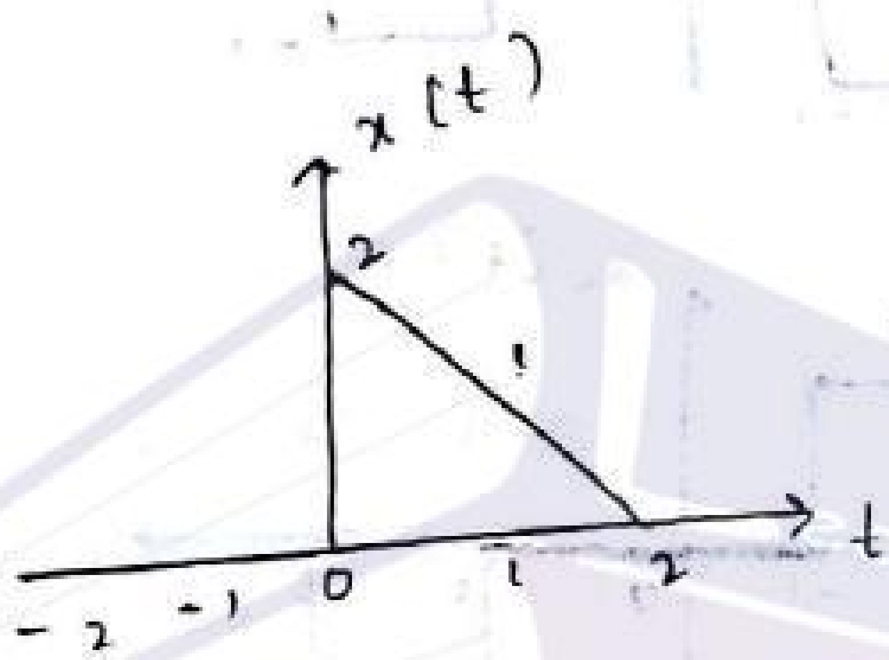

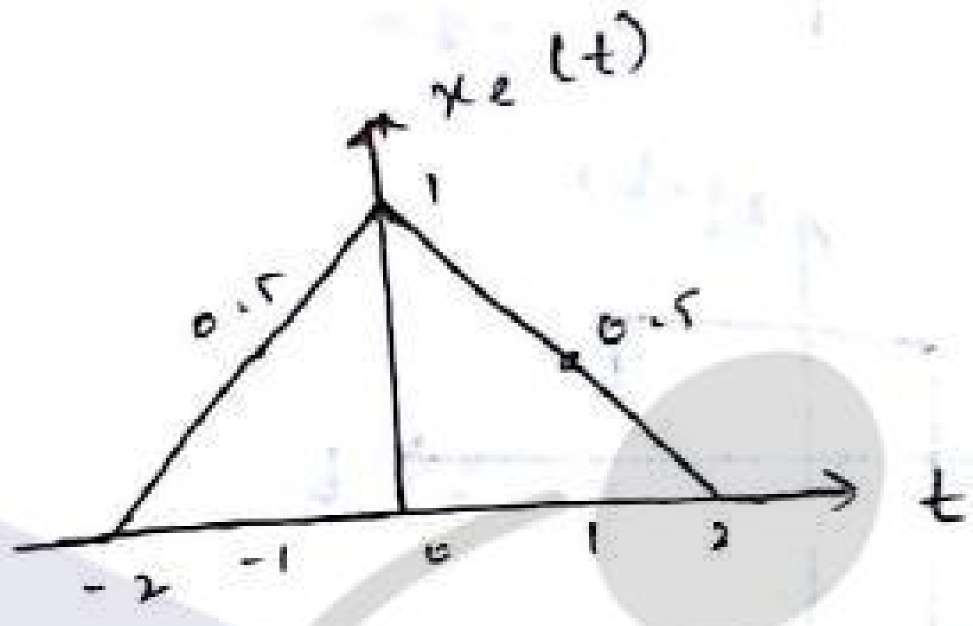

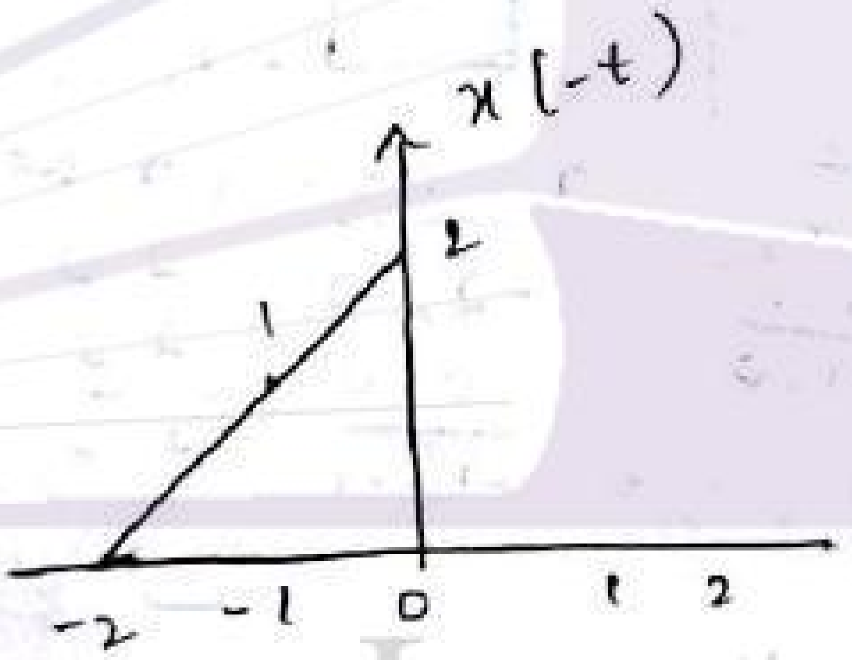

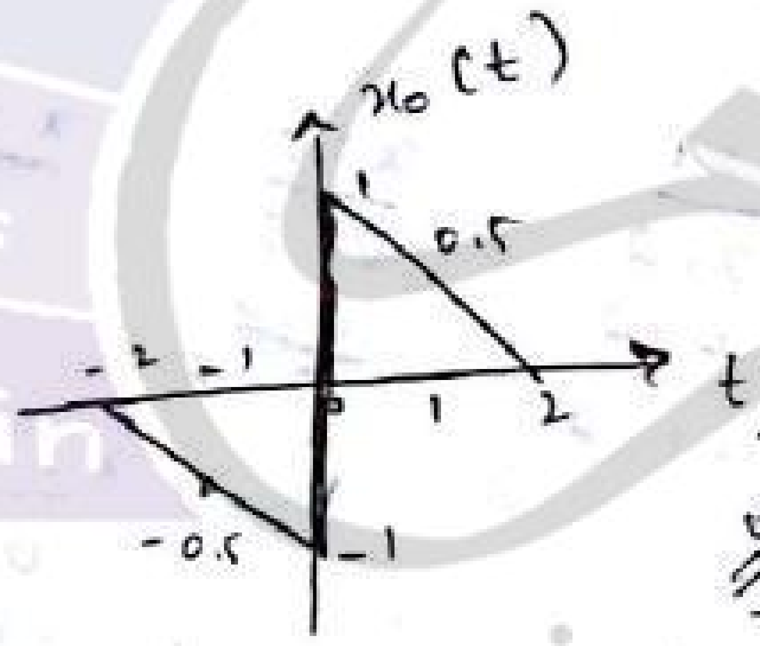

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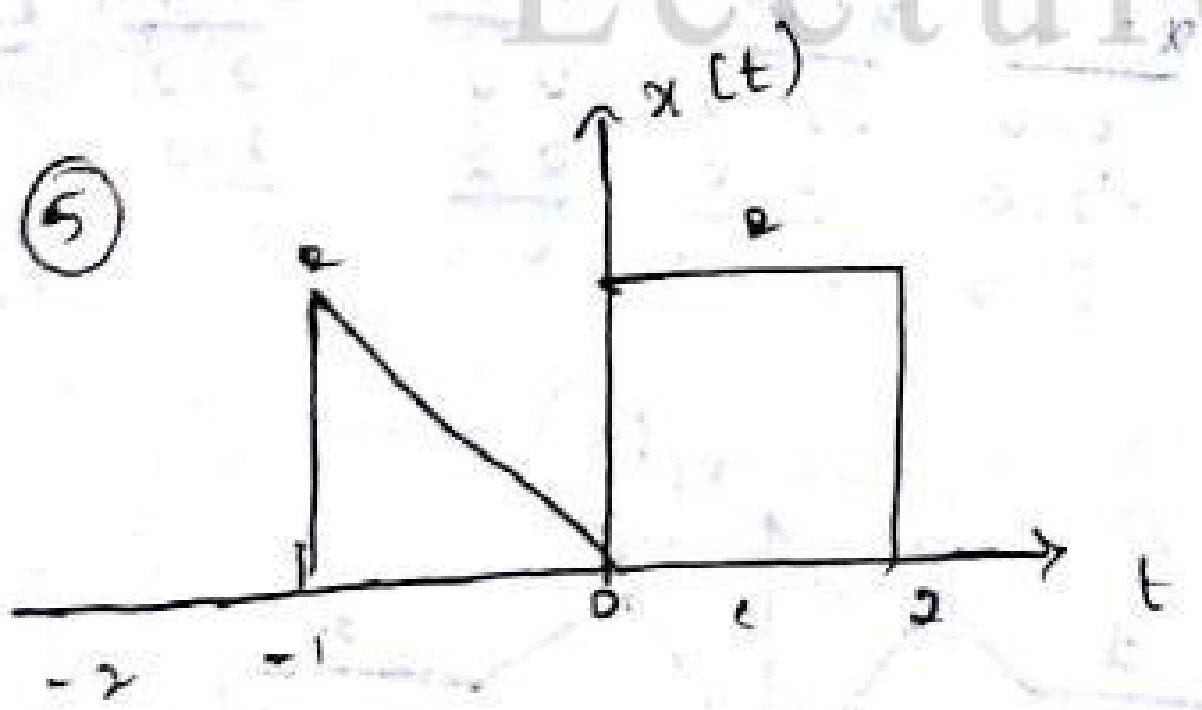
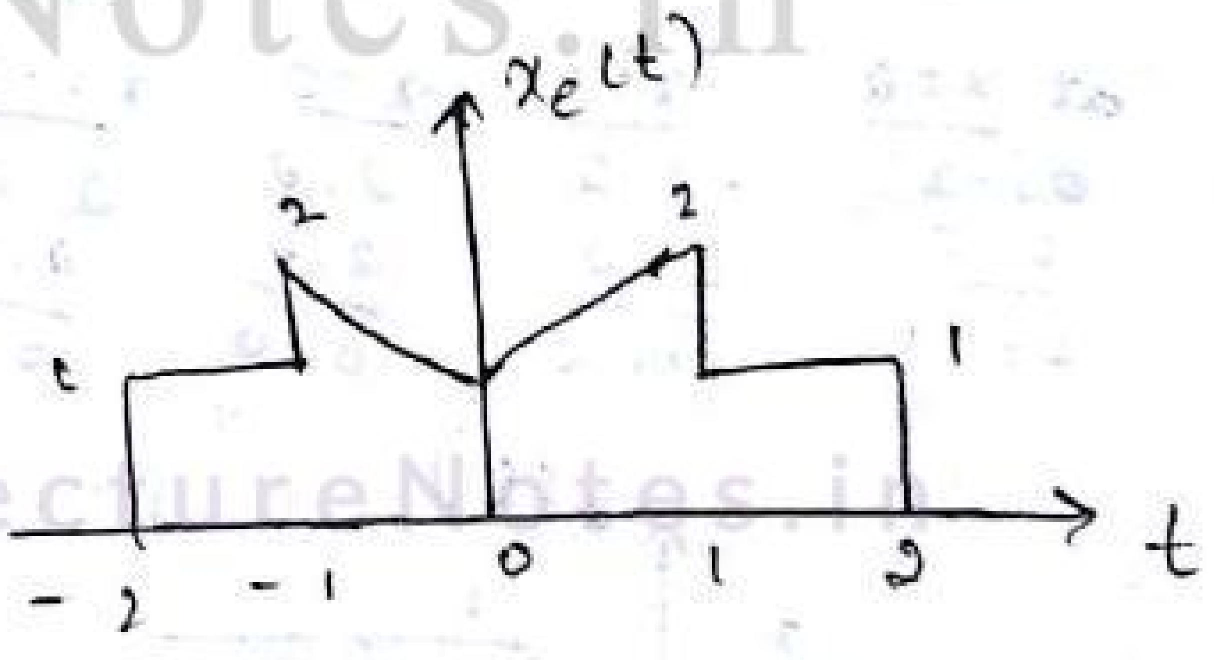
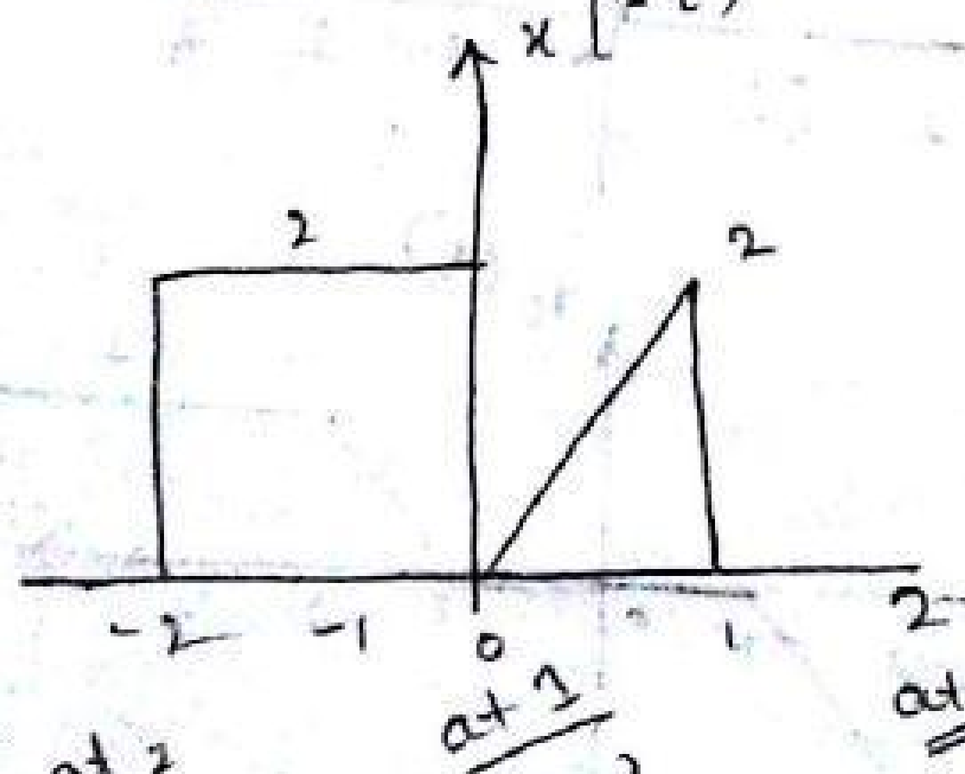






even

at 2: 0,2 / 0,0 / 0,1

at 1: 2,2 / 0,2 / 1,2

at 0: 2,0 / 0,2 / 1,1

at -1: 2,0 / 2,2 / 2,1

at -2: 0,0 / 2,0 / 1,0

odd

at 2
0, 2
0, 0
0, 1

at 1
2, 2
0, 2
1, 0

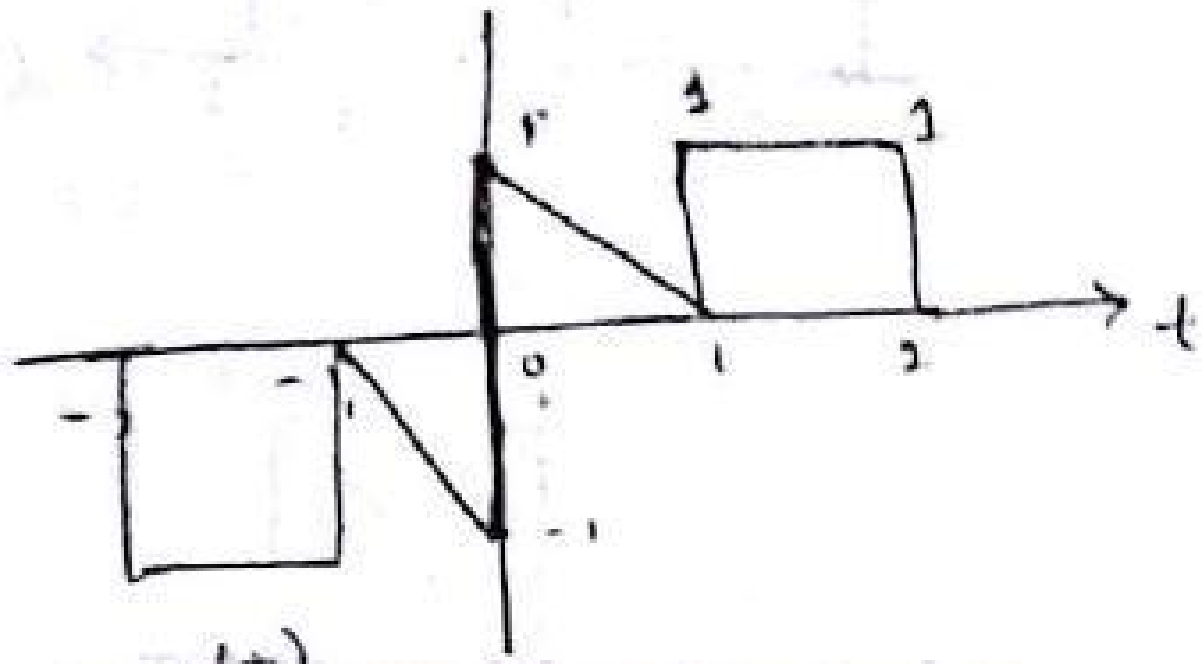
at 0
2, 0
0, 2
1, -1

at -1
2, 0
2, 2
0, -1

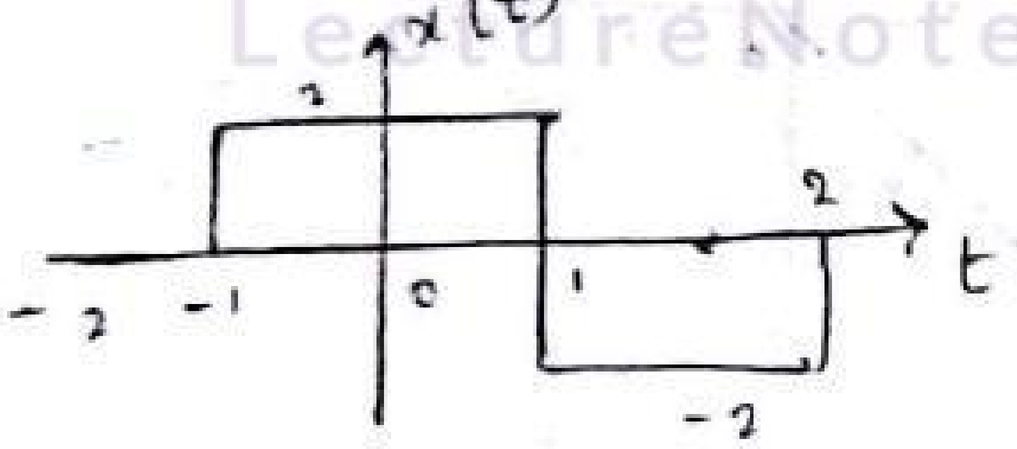
at -2
0, 0
2, 0
-1, 0

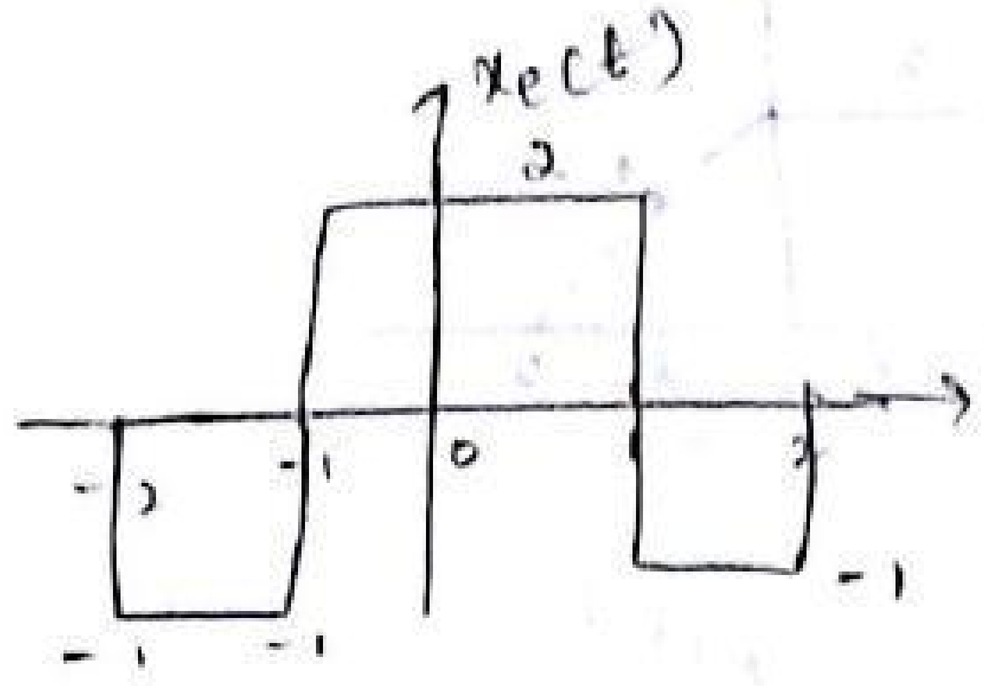
at 1.5
2, 2
0, 0
1, 1

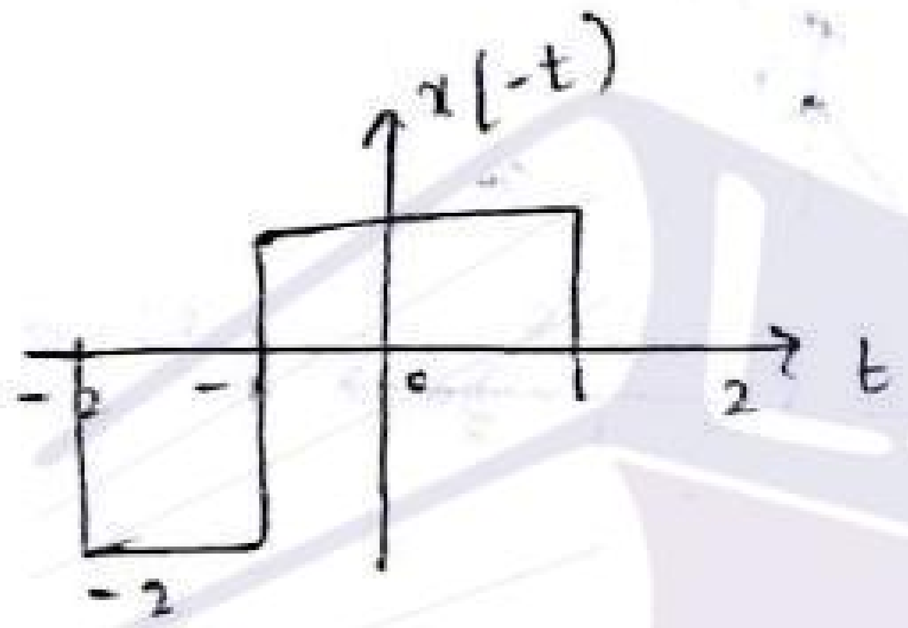
at 0.5
2
1
0.5

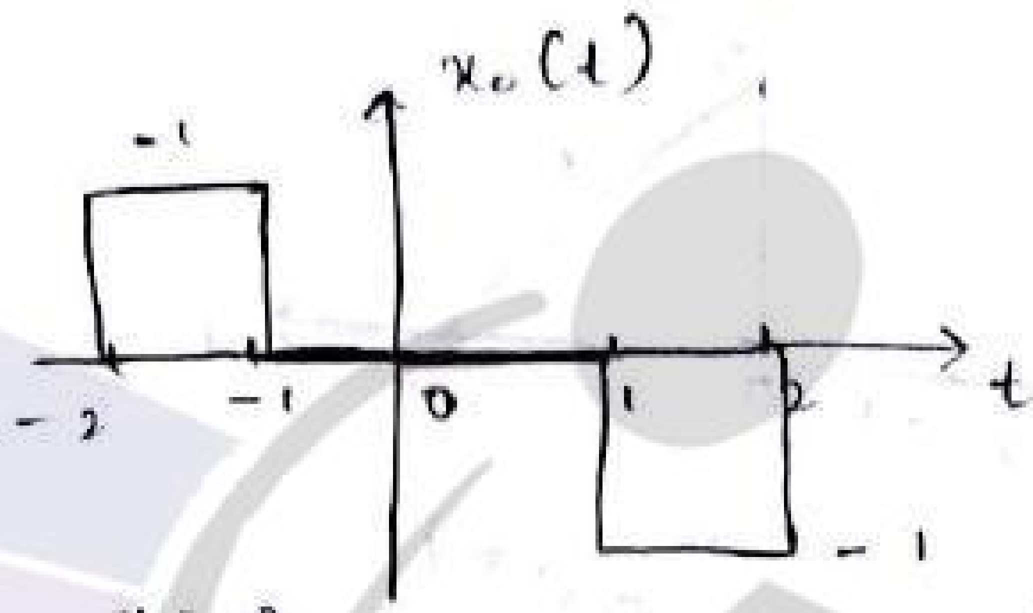


(6)









even

at x=2
0, -2
0, 0
0, -1

x=1
-2, 2
0, 2
-1, 2

x=0
2, 2
2, 2
2, 2

x=-1
2, 0
2, -2
2, -1

x=-1.5
0, 0
-2, -2
-1, -1

x=-2
0, 0
-2, 0
-1, 0

x=1.5
-2, -2
0, 0
-1, -1

x=0.5
2, 2
2, 2
2, 2

x=-0.5
2, 2
2, 2
2, 2

odd

at x=2
0, -2
0, 0
0, -1

x=1
-2, 2
0, 2
-1, 0

x=0
2, 2
2, 2
0, 0

x=-1
2, 0
2, -2
0, 1

x=-2
0, 0
-2, 0
1, 0

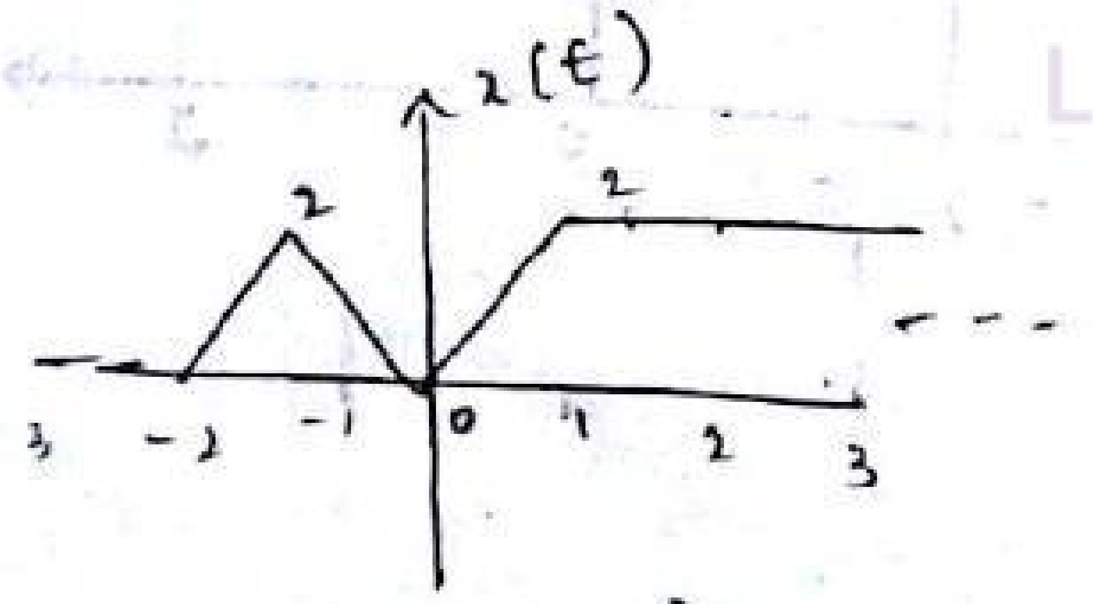
x=1.5
-2, -2
0, 0
-1, -1

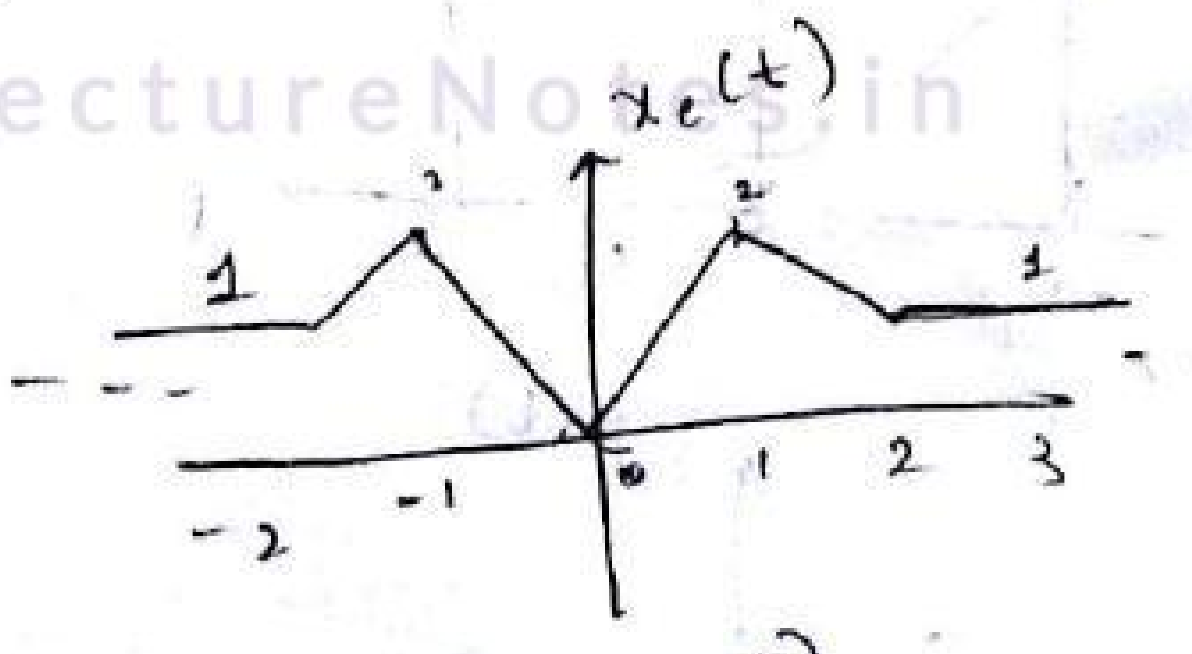
x=0.5
2, 2
2, 2
0, 0

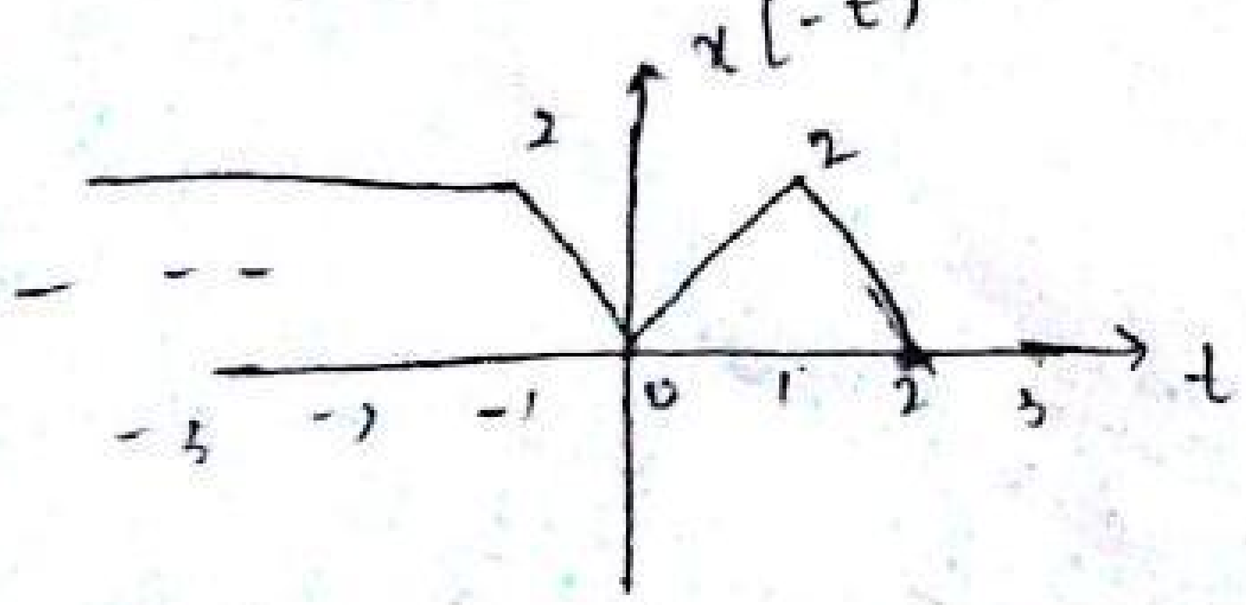
x=-0.5
2, 2
2, 2
0, 0

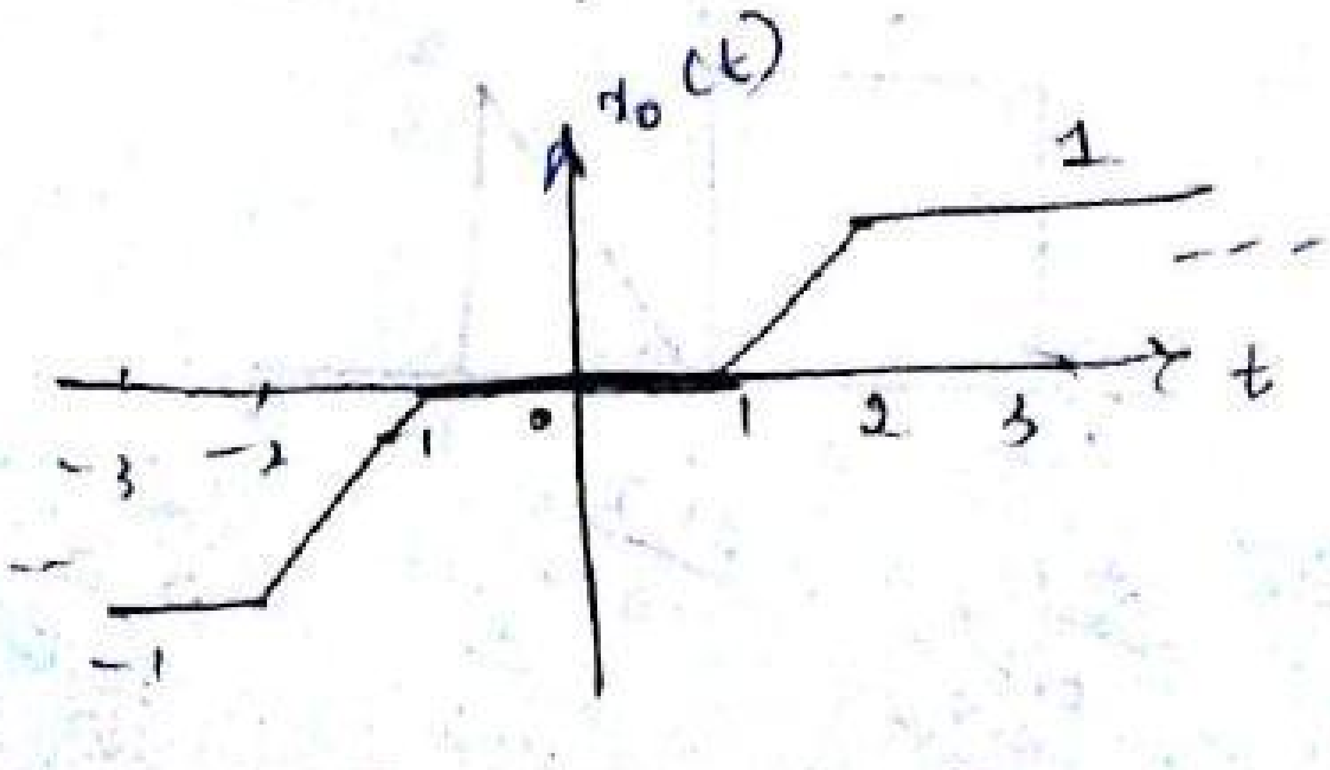
x=-1.5
0, 0
-2, -2
1, 1

(7)

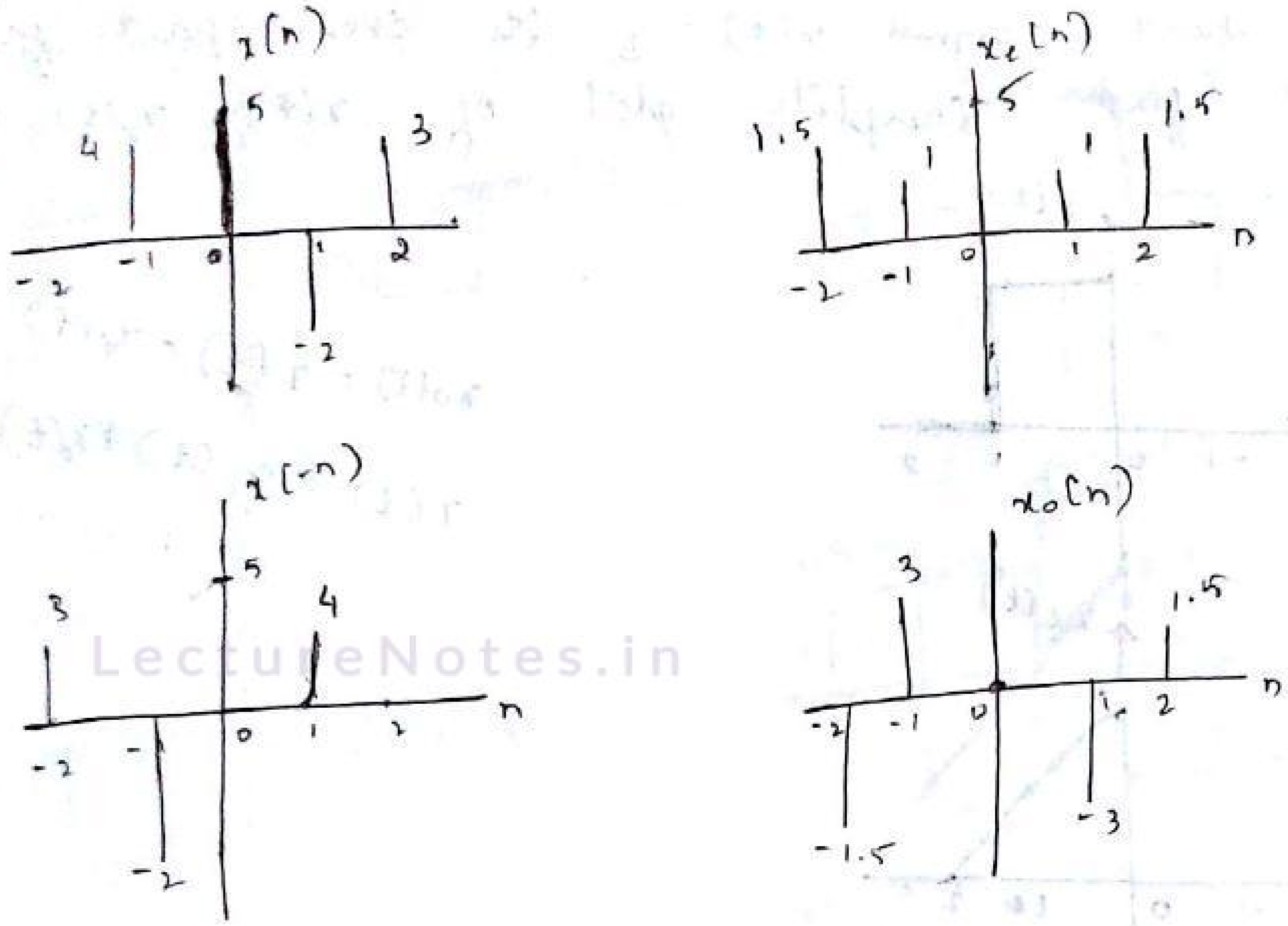








(8)



(9)

$$x(t) = \sin t + \cos t + \sin t \cos t$$

$$x(-t) = \sin(-t) + \cos(-t) + \sin(-t)\cos(-t)$$

$$x(-t) = -\sin t + \cos t - \sin t \cos t$$

$$x_e(t) = \cos t$$

$$x_o(t) = \sin t + \sin t \cos t$$

(10) $x(n) = n^2 \left(\frac{1}{2}\right)^n$

$$x(-n) = (-n)^2 \left(\frac{1}{2}\right)^{-n}$$

$$x(-n) = n^2 \left(\frac{1}{2}\right)^{-n}$$

$$x_e(n) = \frac{n^2}{2}\left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{-n}\right]$$

$$x_o(n) = \frac{n^2}{2}\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^{-n}\right]$$

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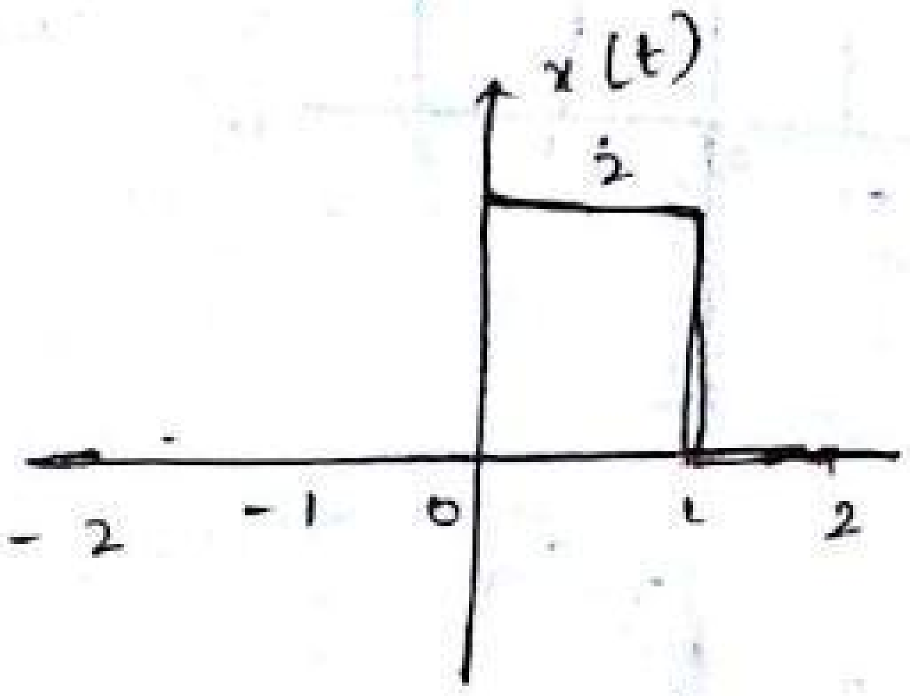
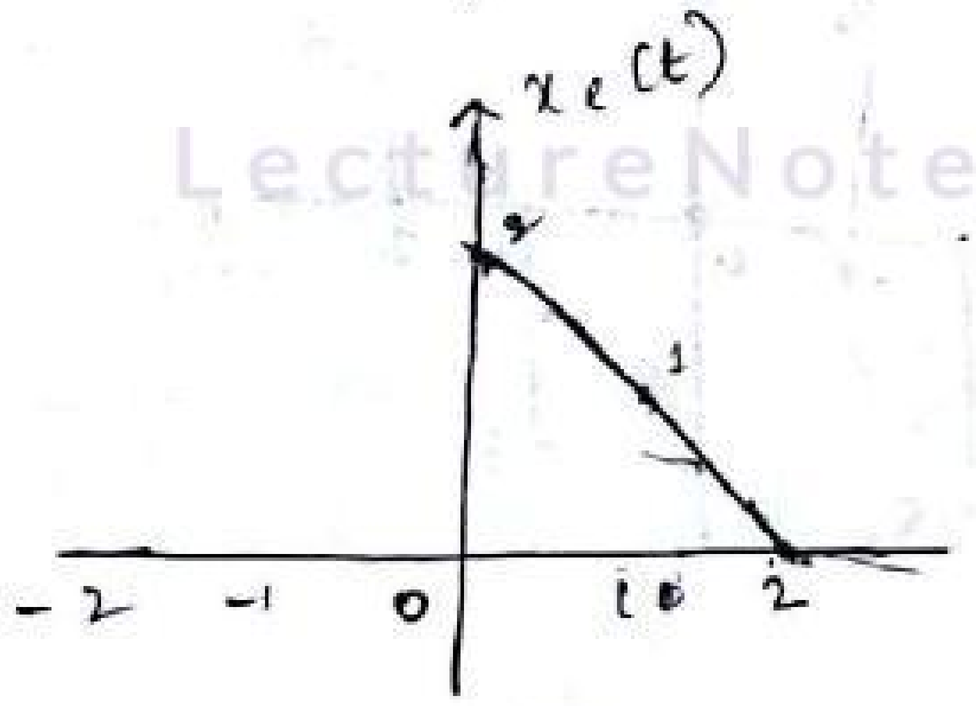
Q) Given that signal $x(t)$ & its even part for $t \geq 0$ find complete plot of $x(t)$, $x_e(t)$, $x_o(t)$

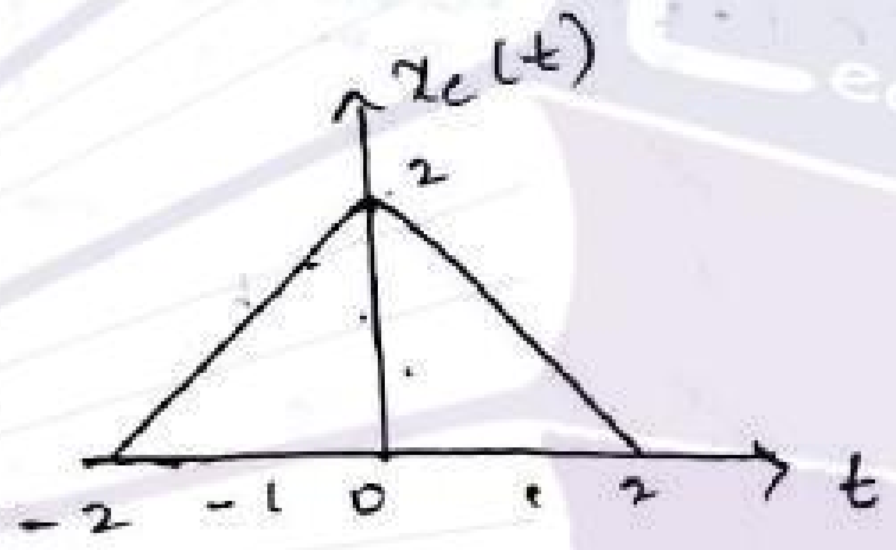


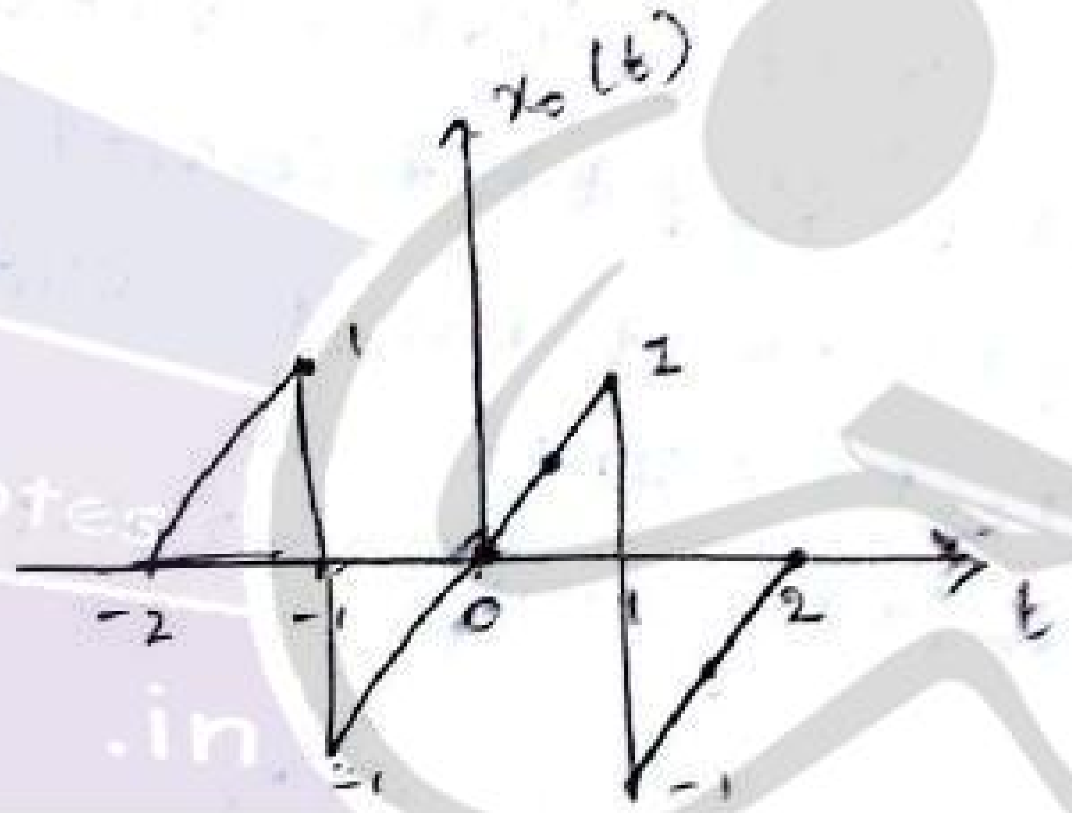
$x_o(t) = x(t) - x_e(t)$

$x(t) = x_e(t) + x_o(t)$



→ Even signal is symmetric so complete plot can be given as





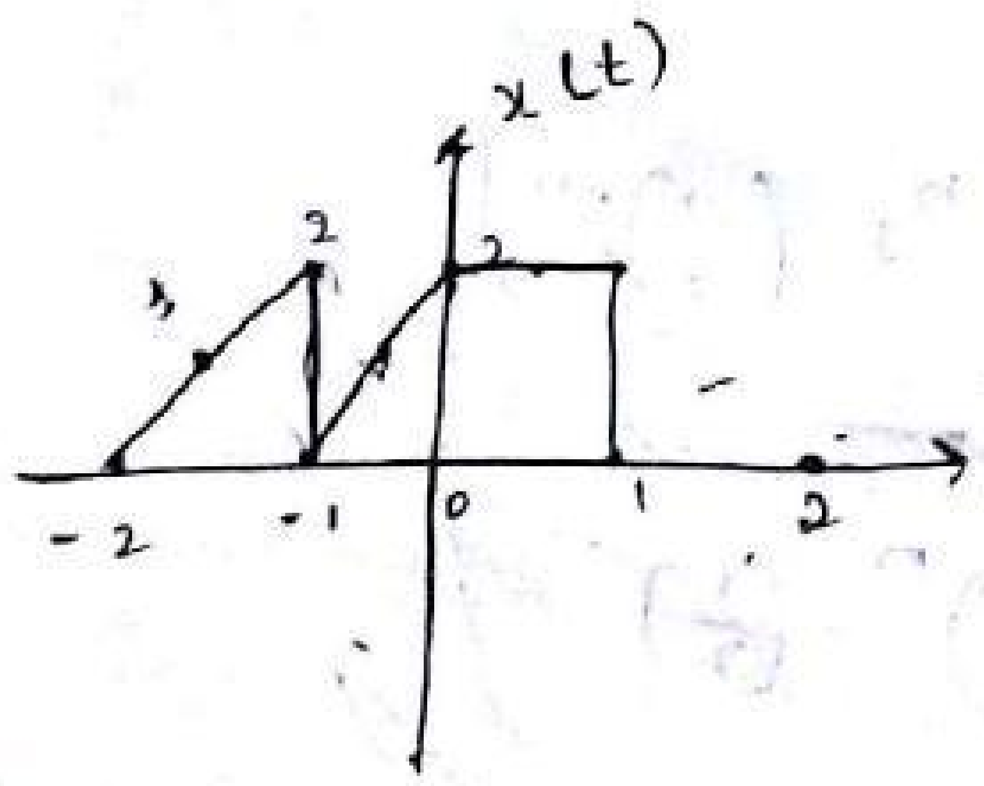
odd

at 0
2
2
0

at 1
0, 2
1, 1
-1, 1

at 1.5
0, 0
0.5, 0.5
-0.5, 0.5

at 0.5
0, 2
1.5, 1.5
0.5, 0.5



$x(t)$

at -1
-1, 1
-1, 1
0, 2

at 1
1, 1
-1, 1
0, 2

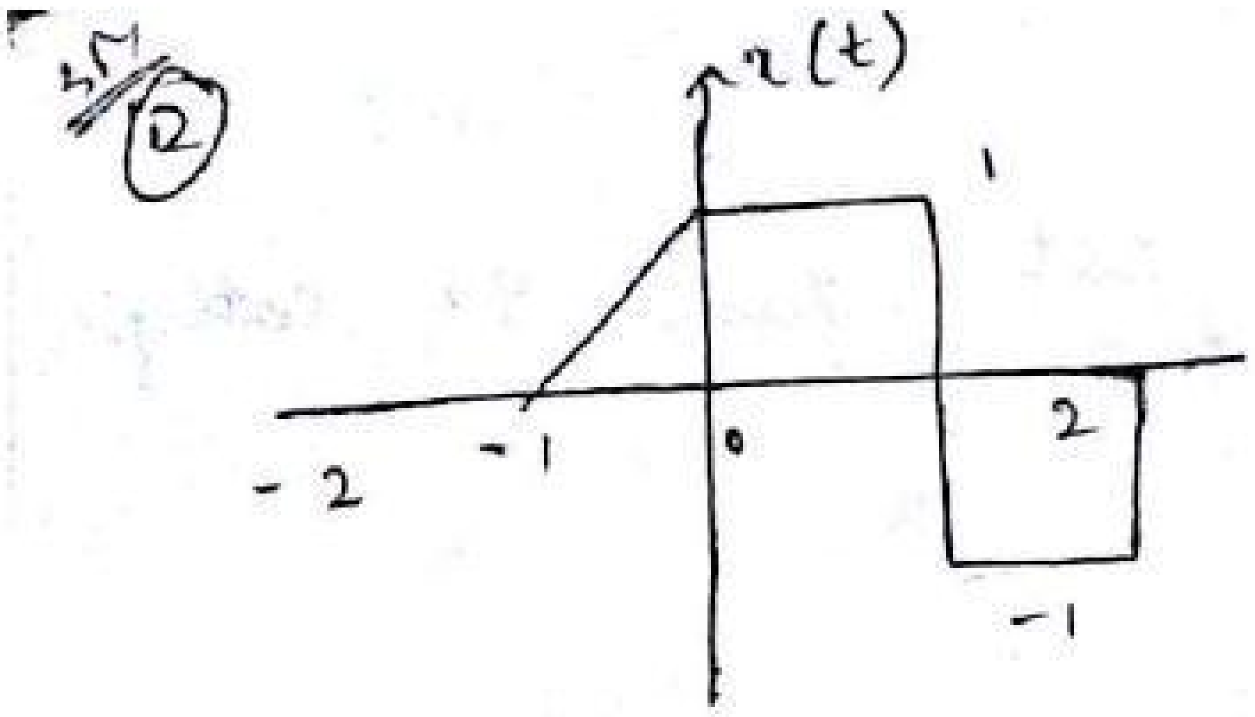
at 2
0, 0
0, 0

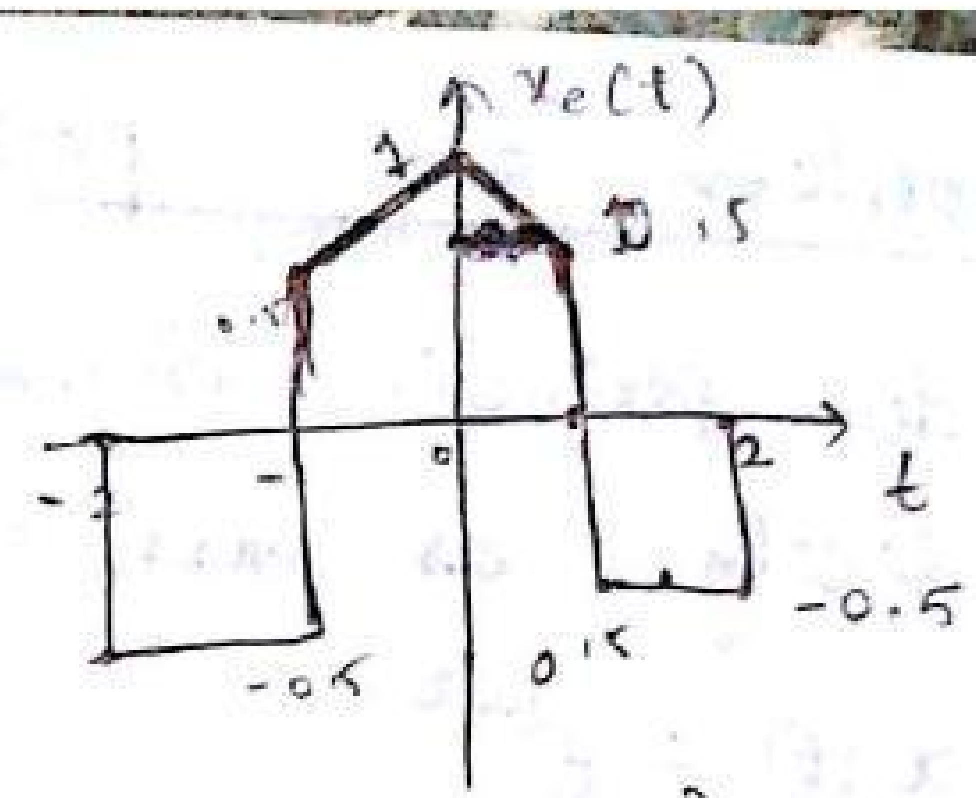
at -1.5
0.5, 0.5
0.5, 0.5
1, 1

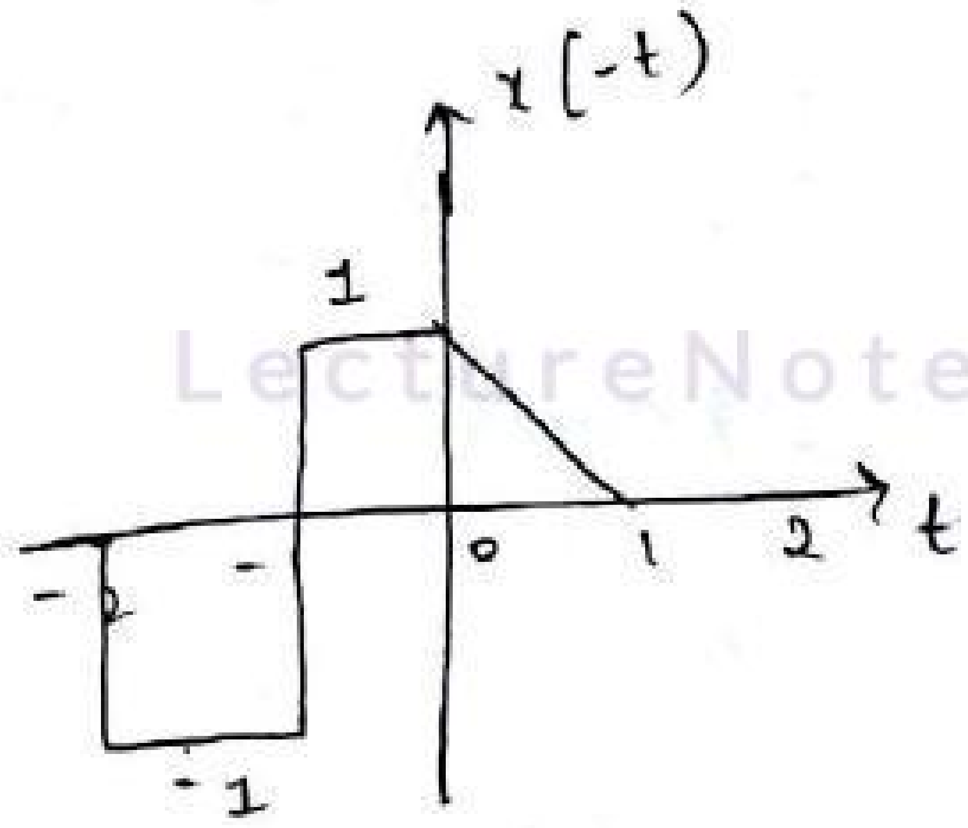
at -0.5
1.5, 1.5
-0.5, 0.5
1, 1

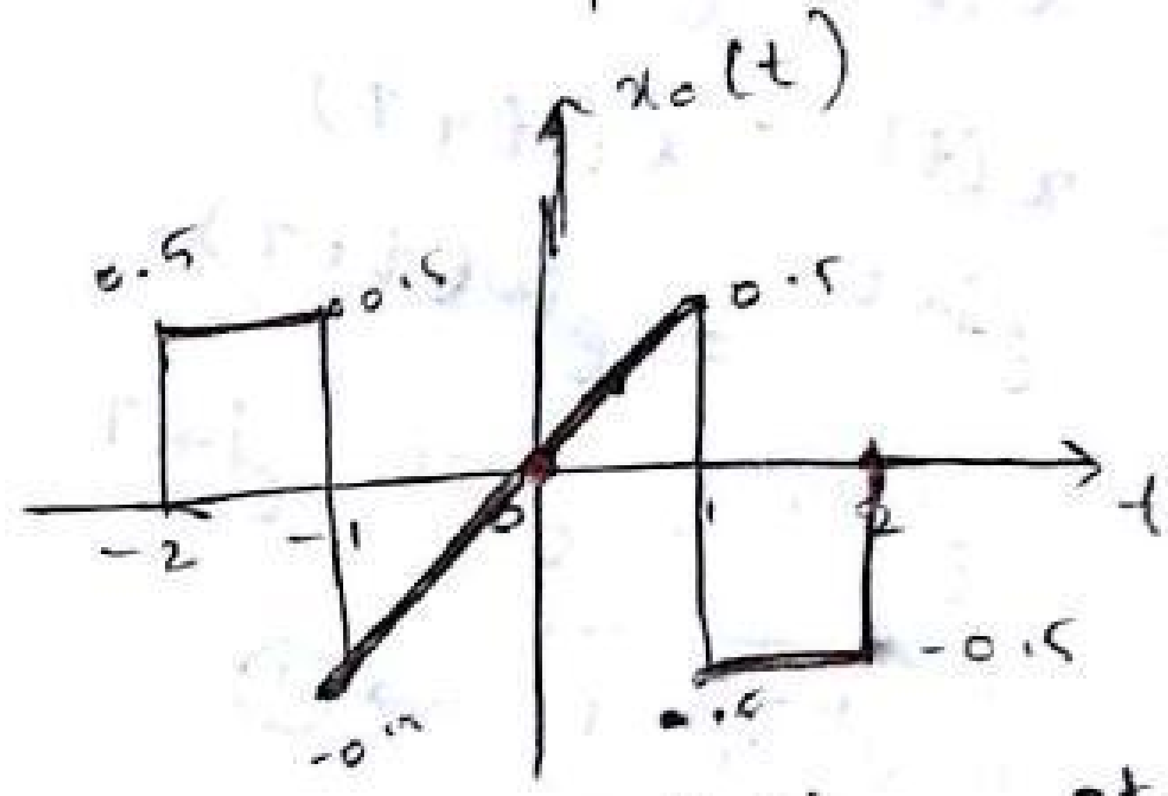
at 0.5
1.5, 1.5
0.5, 0.5
2, 2

3M (2)









even

at 2: 0, -1 ; 0, 0 ; 0, -0.5

at 1: -1, 1 ; 0, 0 ; -0.5, 0.5

at 0: 1, 1 ; 1, 1 ; 1, 1

at -1: 0, 0 ; 1, -1 ; 0.5, -0.5

at -2: 0, 0 ; -1, 0 ; -0.5, 0

at 1.5: -1, -1 ; 0, 0 ; -0.5, -0.5

at 0.5: 1, 1 ; 0.5, 0.5 ; 0.25, 0.25

at -0.5: 0.5, 0.5 ; 1, 1

odd

at 2: 0, -1 ; 0, 0 ; 0, -0.5

at 1: -1, 1 ; 0, 0 ; -0.5, 0.5

at 0: 1, 1 ; 1, 1 ; 0, 0

at -1: 0, 0 ; 1, -1 ; -0.5, 0.5

at -2: 0, 0 ; -1, 0 ; 0.5, 0

at 1.5: -1, -1 ; 0, 0 ; -0.5, -0.5

at 0.5: 1, 1 ; 0.5, 0.5 ; 0.25, 0.25

at -0.5: 0.5, 0.5 ; 1, 1 ; -0.25, -0.25

at -1.5: 0, 0 ; -1, -1 ; 0.5, 0.5

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Problems on periodicity

Consider periodic signal $x(t) = e^{j\omega t}$, then it satisfying periodicity as $x(t) = x(t+T)$

$$x(t) = e^{j\omega t}$$

$$x(t) = x(t+T)$$

$$e^{j\omega t} = e^{j\omega(t+T)}$$

$$e^{j\omega t} = e^{j\omega t} \cdot e^{j\omega T}$$

$$\boxed{e^{j\omega T} = 1} \rightarrow (1)$$

$$e^{j0} = \cos 0 + j\sin 0$$
$$= 1$$

$$e^{j2\pi} = \cos 2\pi + j\sin 2\pi$$
$$= 1$$

$$e^{j4\pi} = 1$$

$$e^{j6\pi} = 1$$

$$\vdots$$

$$\boxed{e^{jm2\pi} = 1} \rightarrow (2) \qquad m = 0, 1, 2, 3 - - -$$

equate eqn (1) & (2)

$$e^{j\omega T} = e^{jm2\pi}$$

$$\omega T = m2\pi$$

$$\boxed{T = \frac{m2\pi}{\omega}}$$ Time period

if $m = 1$

$$\boxed{T = \frac{2\pi}{\omega}}$$ fundamental Time period

$x(n) = e^{j\Omega n}$

$x(n) = x(n+N)$

$e^{j\Omega n} = e^{j\Omega(n+N)}$

$e^{j\Omega n} = e^{j\Omega n} \cdot e^{j\Omega N}$

$\boxed{e^{j\Omega N} = 1} \rightarrow (1)$

$e^{j0} = \cos 0 + j\sin 0$

$e^{j2\pi} = \cos 2\pi + j\sin 2\pi$

$= 1$

$e^{j4\pi} = 1$

$e^{j6\pi} = 1$

$\vdots$

$\boxed{e^{jm2\pi} = 1} \rightarrow (2) \quad m = 0, 1, 2, 3 - - - -$

equate eqn (1) & (2)

$e^{j\Omega N} = e^{jm2\pi}$

$\Omega N = m2\pi$

$\boxed{\frac{m}{N} = \frac{\Omega}{2\pi}}$

valid Time periods

$T = \frac{2\pi}{\omega}$

$\frac{m}{N} = \frac{\Omega}{2\pi}$

$T = 4, \ T = \pi, \ T = 2/3, \ T = \sqrt{2}$

$N = 4$ ✓ $\quad N = \pi$ ✗

$N = 2/5$ ✗ $\quad N = \sqrt{2}$ ✗

$m = 5$ ✓ $\quad m = \pi$ ✗ $\quad m = \sqrt{3}$ ✗

General periodic Signal

$\sin \omega t$

$\cos(\omega t + \theta)$

$e^{j\omega t}$

$\sin \Omega n$

$\cos \Omega n$

$e^{j\Omega n}$

(1) check for periodicity and find its Time period

(1) $x(t) = \sin\left(t + \pi/2\right)$

$\omega = 1$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{1}$

$\boxed{T = 2\pi}$

It is periodic at Time period 2π

(2) $x(t) = \cos\left(2\pi/_3\, t\right)$

$\omega = 2\pi/_3$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/_3}$

$\boxed{T = 3}$

It is periodic at Time period 3

(3) $x(n) = \sin \pi/_3\, n$

$\Omega = \pi/_3$

$\frac{m}{N} = \frac{\Omega}{2\pi} = \frac{\pi/_3}{2\pi} = 1/_6$

$\frac{m}{N} = \frac{1}{6}$

$m = 1$ ✓

$N = 6$ ✓

It is periodic at Time period 6

(4) $x(n) = \cos\left(\frac{3\pi}{5} n + 5\right)$

$\Omega = \frac{3\pi}{5}$

$$\frac{m}{N} = \frac{\Omega}{2\pi} = \frac{3\pi/5}{2\pi}$$

$$\frac{m}{N} = \frac{3}{10}$$

$m = 3$ ✓

$N = 10$ ✓

It is periodic at Time period 10

(5) $x(n) = \sin(5n)$

$\Omega = 5\Omega$

$$\frac{m}{N} = \frac{\Omega}{2\pi} = 5/2\pi$$

$$\frac{m}{N} = \frac{5}{2\pi}$$

$m = 5$ ✓

$N = 2\pi$ ×

It is Non - periodic

(6) $x(t) = \sin^2 t$

$$x(t) = \frac{1 - \cos 2t}{2}$$

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos 2t$$

$$\left\{ \begin{array}{l} \sin^2\theta = \frac{1-\cos 2\theta}{2} \\ \cos^2\theta = \frac{1+\cos 2\theta}{2} \end{array} \right.$$

$\omega = 2$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2}$$

$$\boxed{T = \pi}$$

It is periodic at Time period π

(7) $x(t) = \cos \pi/3\, t + \sin \pi/4\, t$

$\omega_1 = \pi/3$ $\quad \omega_2 = \pi/4$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi/3} \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi/4}$$

$$\boxed{T_1 = 6} \qquad \boxed{T_2 = 8}$$

Individually they are periodic for combination $x(t)$

step 1

Take ratio of Time periods

$\frac{T_1}{T_2} = \frac{6}{8} = 3/4$ It is rational number, so $x(t)$ is periodic

step 2

$\frac{T_1}{T_2} = \frac{3}{4}$

$4T_1 = 3T_2$

$4 \times 6 = 3 \times 8$

$\boxed{T = 24}$

$x(t)$ is periodic with Time period 24

NOTE

Rational

$5/3, 1/2, 2/9, 7/9, 9/3$

irrational

$\pi/2, 5/\sqrt{3}, 1/\pi$

(8) $x(n) = \sin \frac{\pi}{3} n + \cos \frac{\pi}{2} n$

$\boxed{\frac{m}{N} = \frac{\Omega}{2\pi}}$ $\Omega_1 = \pi/3$ $\Omega_2 = \pi/2$

$\frac{m}{N_1} = \frac{\pi/3}{2\pi}$ $\frac{m}{N_2} = \frac{\pi/2}{2\pi}$

$\boxed{\frac{m}{N_1} = \frac{1}{6}}$ $\boxed{\frac{m}{N_2} = 1/4}$

$\boxed{N_1 = 6}$ $\boxed{N_2 = 4}$

step-1

$\frac{N_1}{N_2} = \frac{6}{4} = \frac{3}{2}$

$\frac{3}{2}$ is rational number

step 2

$$\frac{N_1}{N_2} = \frac{3}{2}$$

$$2N_1 = 3N_2$$

$$2 \times 6 = 3 \times 4$$

$\boxed{N = 12}$ It is periodic at Time period 12

(9) $x(t) = \cos 2t + \sin 3t$

$\boxed{T = \frac{2\pi}{\omega}}$ $\omega_1 = 2$ $\omega_2 = 3$

$$T_1 = \frac{2\pi}{2} = \pi \qquad T_2 = \frac{2\pi}{3} = \frac{2\pi}{3}$$

$\boxed{T_1 = \pi}$ $\boxed{T_2 = 2\pi/3}$

step - 1

$$\frac{T_1}{T_2} = \frac{\pi}{2\pi/3} = \frac{3}{2}$$

$\frac{3}{2}$ is rational

step -2

$$\frac{T_1}{T_2} = \frac{3}{2}$$

$$2T_1 = 3T_2$$

$$2\pi = 3 \times \frac{2\pi}{3}$$

$\boxed{T = 2\pi}$

$x(t)$ is periodic with Time period 2π

(10) $x(t) = \sin t + \sin\sqrt{2}\,t$

$\boxed{T = \frac{2\pi}{\omega}}$ $\omega_1 = 1$ $\omega_2 = \sqrt{2}$

$$T_1 = \frac{2\pi}{1} = 2\pi \qquad T_2 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$$

$\boxed{T_1 = 2\pi}$ $\boxed{T_2 = \sqrt{2}\pi}$

step - 1

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\pi} = \frac{\sqrt{2}}{1}$$

It is irrational

~~step - 2~~ $\cancel{\frac{T_1}{T_2} = \frac{\sqrt{2}}{1} = T_1 = \sqrt{2}T_2 \Rightarrow \boxed{T = 2\pi}}$

$x(t)$ is ~~Time~~ Non-periodic ~~with Time 2π~~

(11) $x(n) = \sin \frac{\pi}{3} n \cdot \cos \frac{\pi}{5} n$

$$= \sin A \cos B$$

$$= \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{\pi}{3}+\frac{\pi}{5}\right)n + \sin\left(\frac{\pi}{3}-\frac{\pi}{5}\right)n\right]$$

$$= \frac{1}{2}\left[\sin \frac{8\pi}{15} n + \sin \frac{2\pi}{15} n\right]$$

$$= \frac{1}{2}\sin \frac{8\pi}{15} n + \frac{1}{2}\sin \frac{2\pi}{15} n$$

$$\Omega_1 = \frac{8\pi}{15} \qquad \Omega_2 = \frac{2\pi}{15}$$

$$\boxed{\frac{m}{N} = \frac{\Omega}{2\pi}} \qquad \frac{m}{N_1} = \frac{\frac{8\pi}{15}}{2\pi} = \frac{4}{15} \qquad \frac{m}{N_2} = \frac{\frac{2\pi}{15}}{2\pi} = \frac{1}{15}$$

$$\boxed{N_1 = 15} \qquad \boxed{N_2 = 15}$$

step 1

$$\frac{N_1}{N_2} = \frac{15}{15} = \frac{1}{1}$$

It is Rational

step 2

$$\frac{N_1}{N_2} = \frac{1}{1}$$

$$N_1 = N_2$$

$$\boxed{N = 15}$$

$x(t)$ is Time periodic with Time 15

(12) $x(n) = 5\sin(0.2\pi n)$

$$\Omega = 0.2\pi$$

$$\frac{m}{N} = \frac{\Omega}{2\pi} = \frac{0.2\cancel{\pi}}{2\cancel{\pi}} = \frac{0.2}{2} = \frac{1}{10} = \frac{1}{10}$$

$$m = 1 \qquad \boxed{N = 10}$$

$x(n)$ is ~~non~~-periodic with Time $\boxed{10}$

(13) $x(n) = (-1)^n$

$= \cos \pi n$

$\Omega = \pi$

$$\frac{m}{N} = \frac{\Omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$$

it is rational.

$\boxed{N = 2}$

$x(n)$ is periodic with period 2

(14) if $T_1 = 1.08$

$T_2 = 3.6$

$T_3 = 2.025$ then find

are Time period of $x_1(t)$, $x_2(t)$, $x_3(t)$

Time period of $x(t) = x_1(t) + x_2(t) + x_3(t)$

→ step 1

$$\frac{T_1}{T_2} = \frac{1.08}{3.6} = \frac{3}{10}$$

$$\frac{T_1}{T_3} = \frac{1.08}{2.025} = \frac{8}{15}$$

step 2

LCM of 10 & 15

$$\frac{a}{b} = \frac{10}{15} = \frac{2}{3}$$

$3a = 2b$

$3 \times 10 = 2 \times 15$

$\boxed{P = 30}$

step 3

$T = T_1 \times P$

$T = 1.08 \times 30$

$\boxed{T = 32.4}$

(15) if $T_1 = 2.16$

$T_2 = 7.2$

$T_3 = 4.5$

are Time period of $x_1(t)$, $x_2(t)$, $x_3(t)$ then find time period for $x(t) = x_1(t) + x_2(t) + x_3(t)$

Step 1

$$\frac{T_1}{T_2} = \frac{2.16}{7.2} = \frac{3}{10}$$

$$\frac{T_1}{T_3} = \frac{2.16}{4.5} = \frac{12}{25}$$

Step 2

LCM of 10, 25

$$\frac{a}{b} = \frac{10}{25} = \frac{2}{5}$$

$5a = 2b$

$5 \times 10 = 2 \times 25$

$50 = 50$

$\boxed{P = 50}$

Step -3

$T = T_1 \times P$

$= 2.16 \times 50$

$\boxed{T = 108}$

Energy and power Signal

Signal which as finite energy i.e., $0 < E < \infty$ is said to be energy signal its total energy can be given as

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

Signal which as finite power i.e, $0<P<\infty$ is Said to be power signal its average power can be written as $P_{avg} = \frac{1}{T}\int_0^T x^2(t)\,dt$

$$P_{avg} = \frac{1}{N}\sum_{n=0}^{N-1} x^2(n)$$

$\boxed{P \propto V^2}$

$P = V^2/R$

$\boxed{P \propto I^2}$

$P = I^2R$

$\boxed{\text{work done} \propto x^2(t)}$

NOTE

* All periodic signals are power signals.
* All Non-periodic signals are Energy signals
* If Result is zero or infinity then it is neither energy nor power

1) Calculate energy or power for the given signal

$$x(t) = \begin{cases} e^{-2t} & 0 \le t \le \infty \\ 0 & \text{otherwise} \end{cases}$$

Given problem signal is Non-periodic so, it is energy signal.

$$\therefore E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_0^{\infty} (e^{-2t})^2\,dt$$

$$\int_0^{\infty} e^{-4t}\,dt$$

$$\left[\frac{e^{-4t}}{-4}\right]_0^{\infty} : \frac{e^{-\infty} - e^{-0}}{-4}$$

$$\frac{0-1}{-4} = \boxed{\frac{1}{4} J}$$

② $x(t) = \cos\frac{2\pi}{3}t$

It is periodic, so it is power signal

$\omega = \frac{2\pi}{3}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/3} = 3$

$\boxed{T=3}$

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{3}\int_0^3 \cos^2\frac{2\pi}{3}t\,dt \qquad \cos^2\theta = \frac{1+\cos2\theta}{2}$$

$$= \frac{1}{3}\int_0^3 \frac{1+\cos\frac{4\pi}{3}t}{2}\,dt$$

$$= \frac{1}{6}\left[\int_0^3 1\,dt + \int_0^3 \cos\frac{4\pi}{3}t\,dt\right]$$

$$\cancel{\frac{1}{6}\left[3+\left(-\frac{9}{4}\right)\right]}$$

$$= \underline{\underline{\frac{1}{8}}} \qquad \frac{1}{6}\left(\int_0^3 t\,dt + \sin\frac{4\pi}{3}\Big|_0^3\right]$$

$$\frac{1}{6}\left[(3-0)+(0-0)\right)$$

$$= \frac{3}{6} = \boxed{\frac{1}{2}}$$

③ $x(t) = \begin{cases} t & 0 \le t \le \infty \\ 0 & \text{otherwise} \end{cases}$

Given signal is Non-periodic, so it is energy signal

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{0}^{\infty} t^2\,dt$$

$$= \frac{t^3}{3}\Big|_0^{\infty}$$

$$= \infty$$

It is neither energy nor power

4) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \le \infty \\ 0 & \text{otherwise} \end{cases}$

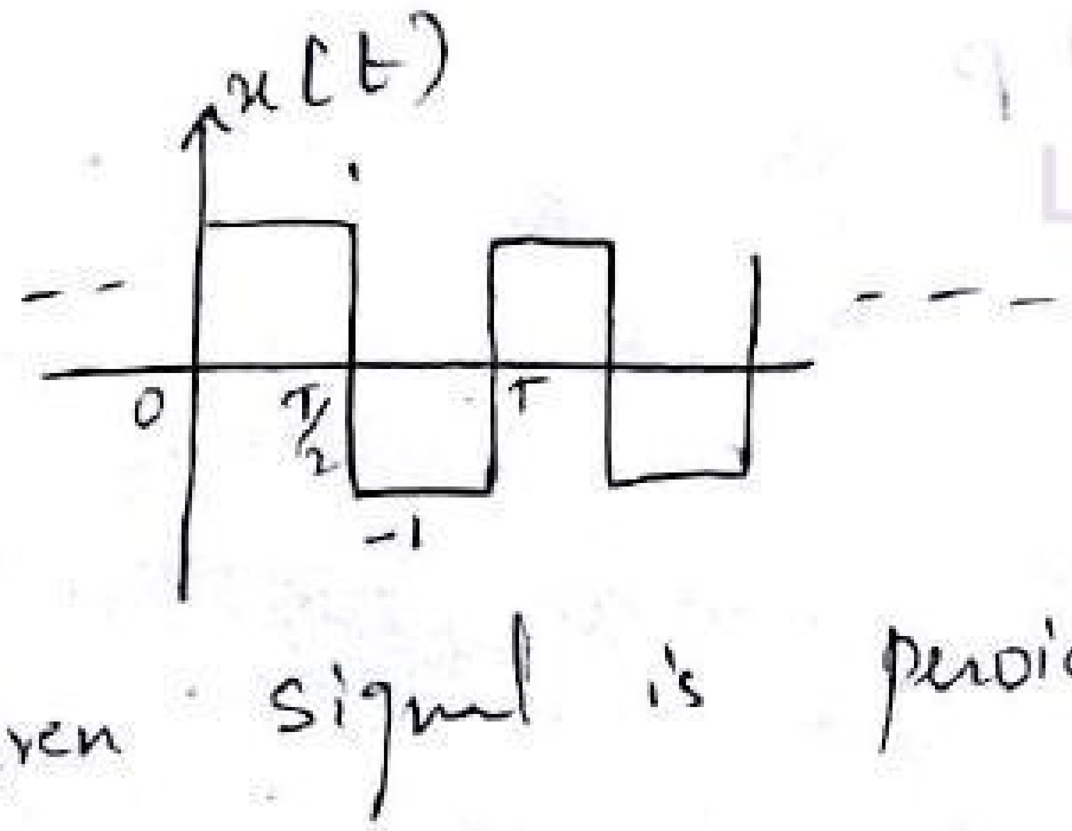
Given signal is Non-periodic, so it is energy signal.

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \frac{1}{1-\frac{1}{4}} = \underline{\underline{1.33\ J}}$$

5)



Given signal is periodic, so it is power signal

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/2} 1^2\,dt + \int_{T/2}^{T} (-1)^2\,dt\right]$$

$$= \frac{1}{T}\left[\frac{T}{2} + \left(T - \frac{T}{2}\right)\right.$$

$$\frac{1}{T}\left[\frac{T}{2} + \frac{T}{2}\right] \Rightarrow \frac{1}{T}\left[\frac{2T}{2}\right] = \underline{\underline{1}}$$

(6) $x(n) = \left(\frac{3}{2}\right)^n \quad 0 \le n \le \infty$

$\quad\quad 0 \quad$ otherwise

Given Signal is Non-periodic, so, it is energy Signal

$$E = \sum_{n=-\infty}^{\infty} x^2(t)\,dt$$

$$= \sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^{n^2} dt \Rightarrow \sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^{2^n} dt$$

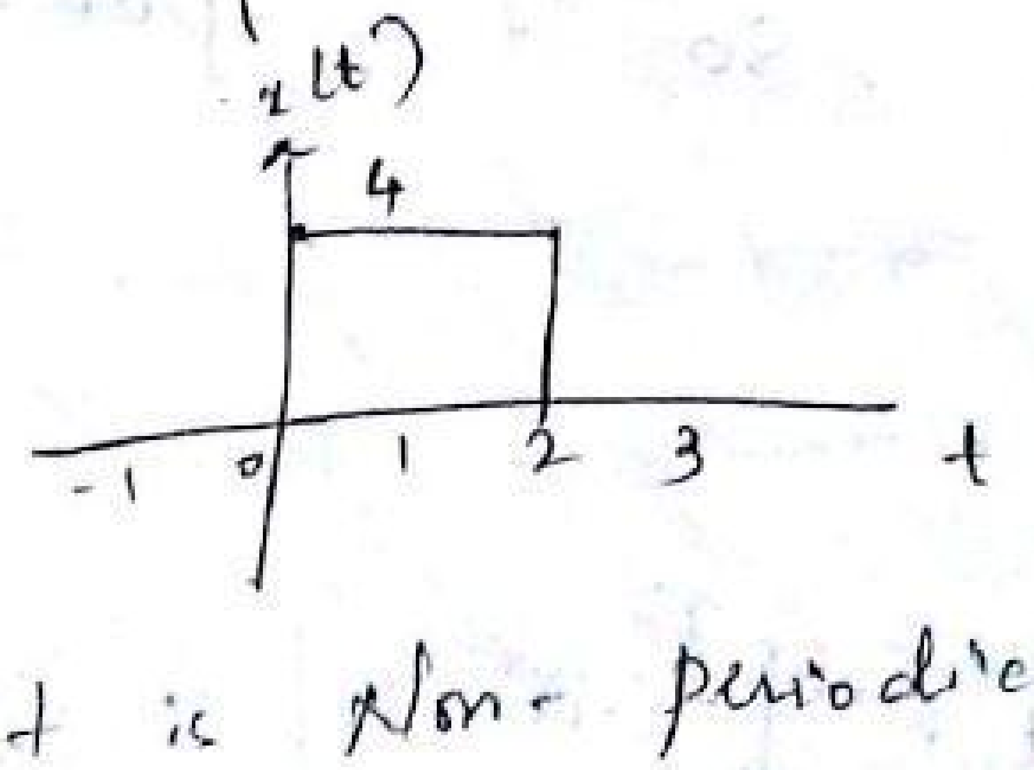
$$= \sum_{n=0}^{\infty} \left(\frac{9}{4}\right)^{n} dt$$

$$\sum_{n=0}^{\infty} (2.25)^n\,dt = \cancel{\frac{1}{1-\frac{9}{4}}} = \cancel{0}$$

$$= \underline{\underline{\infty}}$$

(7) sketch and find E and P

$$x(t) = \begin{cases} 4 & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$



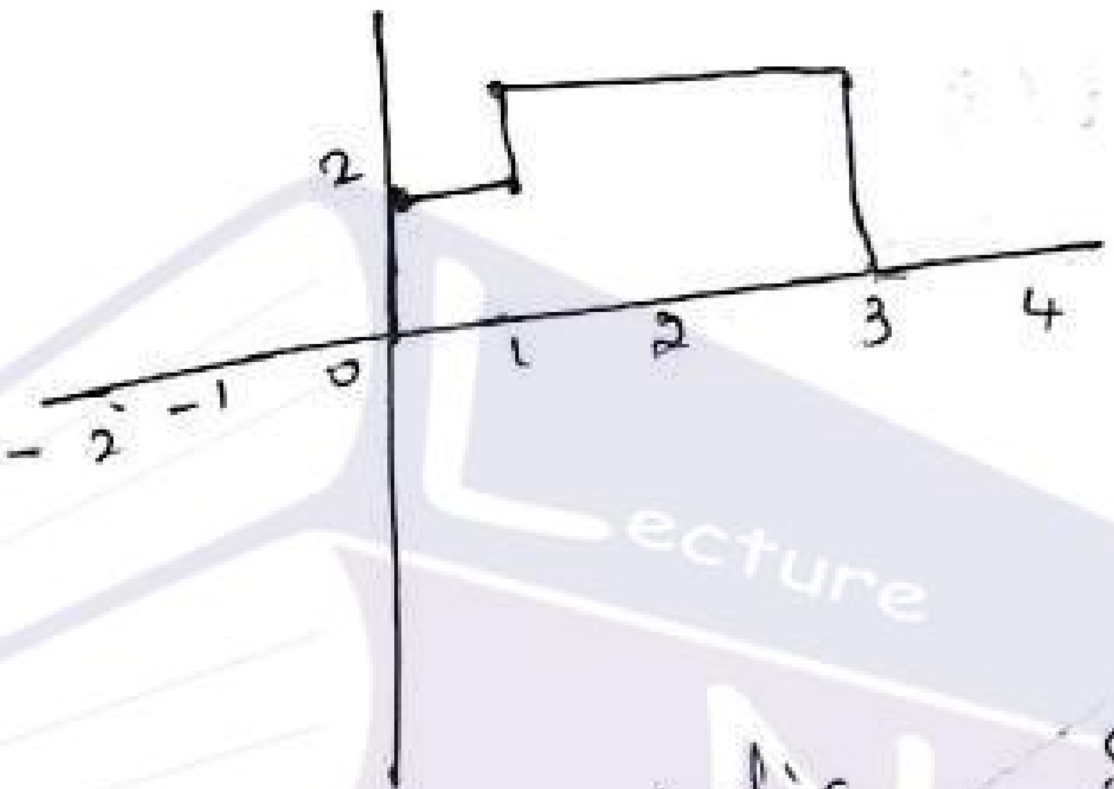
It is Non-periodic, So it is energy Signal

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^2 4^2\,dt$$

$$= 16t \Big|_0^2 dt$$

$$= \underline{\underline{32J}}$$

(8) $$x(t) = \begin{cases} 2 & 0 \le t \le 1 \\ 3 & 1 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$



It is Non-periodic, So it is energy signal

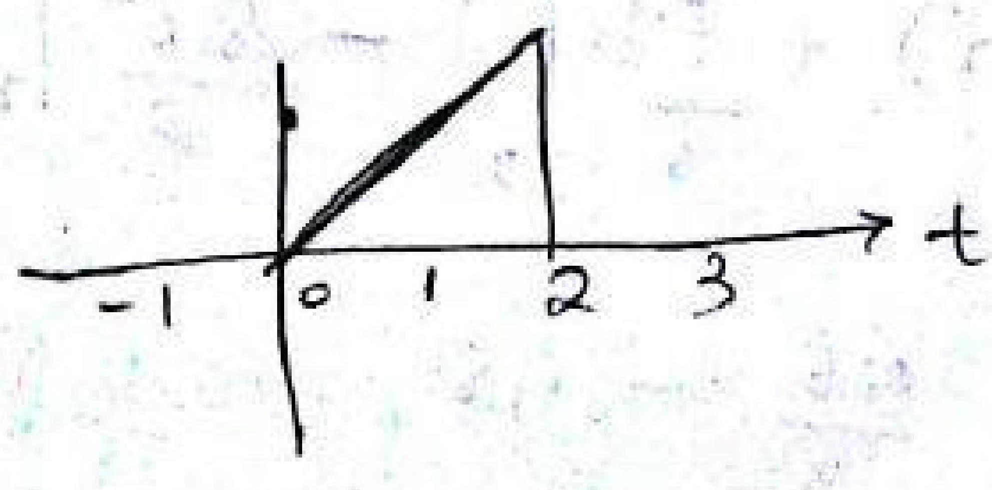
$$E = \int_{-\infty}^{\infty} x^2(t)\,dt =$$

$$= \int_0^1 2^2\,dt + \int_1^3 3^2\,dt$$

$$= 4 + (9 \times 3) - 9$$

$$4 + 18 = \underline{\underline{22J}}$$

(9) $$x(t) = \begin{cases} t & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$



$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

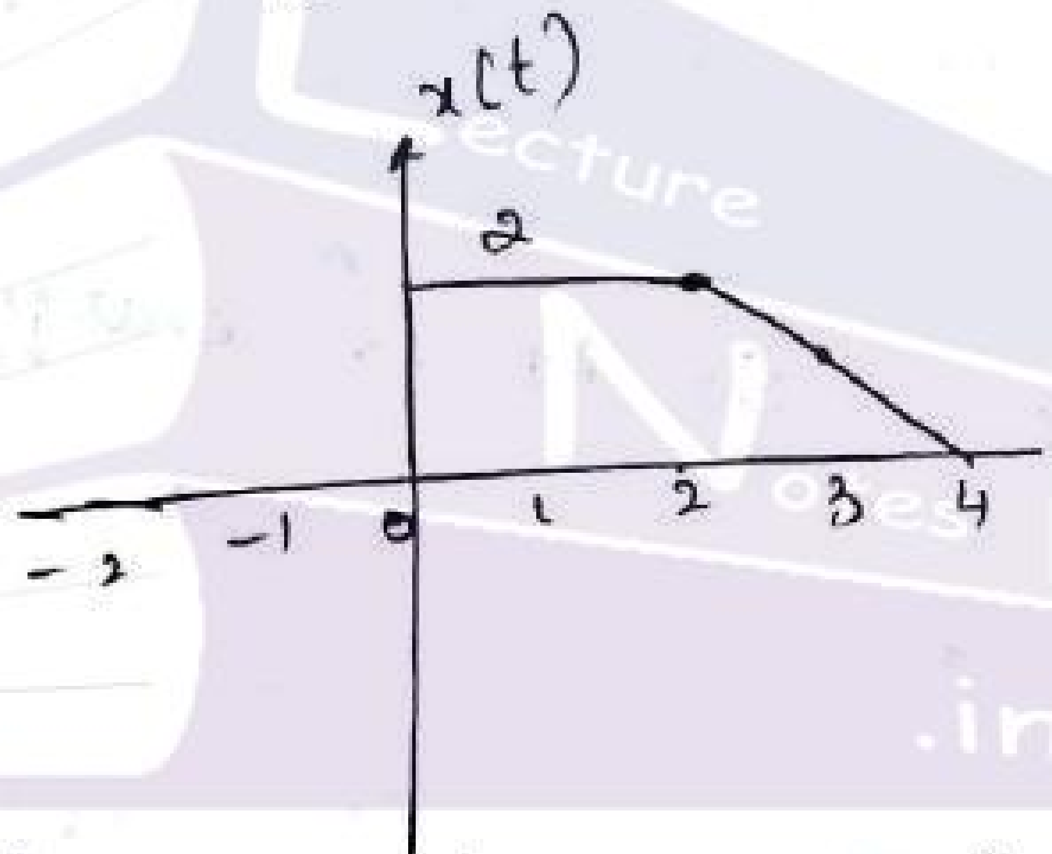
$$= \int_0^2 t^2\,dt$$

$$\frac{t^3}{3}\Big|_0^2$$

$$= \frac{8}{3} J$$

(10)

$$x(t) = \begin{cases} 2 & 0 \le t \le 2 \\ -t+4 & 2 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$



It is Non-periodic, so it is energy Signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_0^2 2^2\,dt + \int_2^4 (-t+4)^2\,dt$$

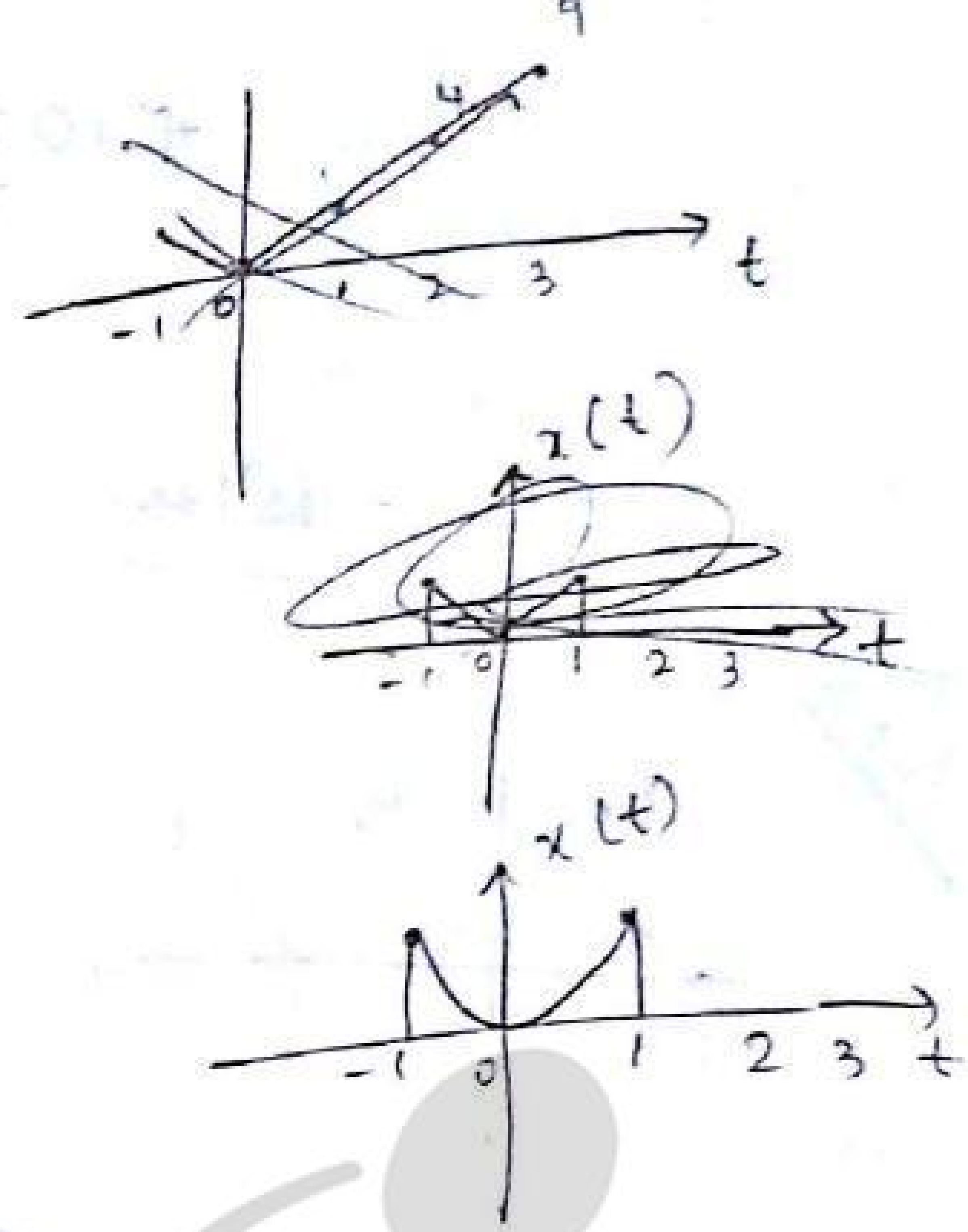
$$= 4(2) + \int_2^4 (4^2 + t^2 - 8t)\,dt$$

$$8 + 16(4-2) + \frac{t^3}{3}\Big|_2^4 - \frac{8t^2}{2}\Big|_2^4$$

$$8 + 32 + \frac{56}{3} - 48 = \frac{32}{3} J$$

(11) $x(t) = \begin{cases} t^2 & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

S-1 is Non-periodic, so it is energy signal



$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-1}^{1} t^4\,dt$$

$$= \left.\frac{t^5}{5}\right|_{-1}^{1}$$

$$= \frac{1}{5}[1+1]$$

$$= 2/5$$

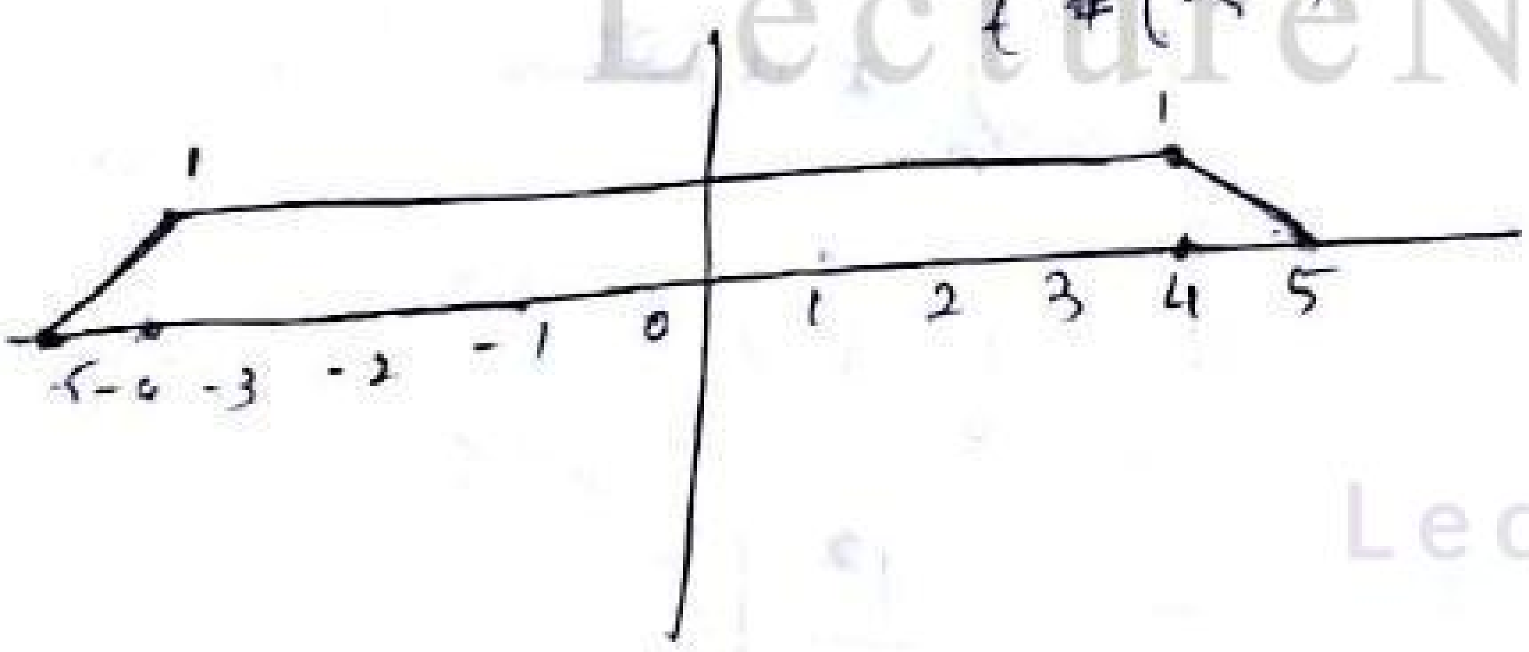
(12) $x(t) = \begin{cases} 5-t & 4 \le t \le 5 \\ 1 & -4 \le t \le 4 \\ t+5 & -5 \le t \le -4 \\ 0 & \text{otherwise.} \end{cases}$

$t-5$

$-t+5$

$t+5$

$t \neq (-5)$



$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{4}^{5} (5-t)^2\,dt + \int_{-4}^{4} 1^2\,dt + \int_{-5}^{-4} (t+5)^2$$

$$= \int_{4}^{5} (25+t^2-10t)\,dt + 8 + \int_{-5}^{4} t^2+25+10t$$

$= \cancel{25(5-4) + (25-16) - 10(5-4) + 8 +}$

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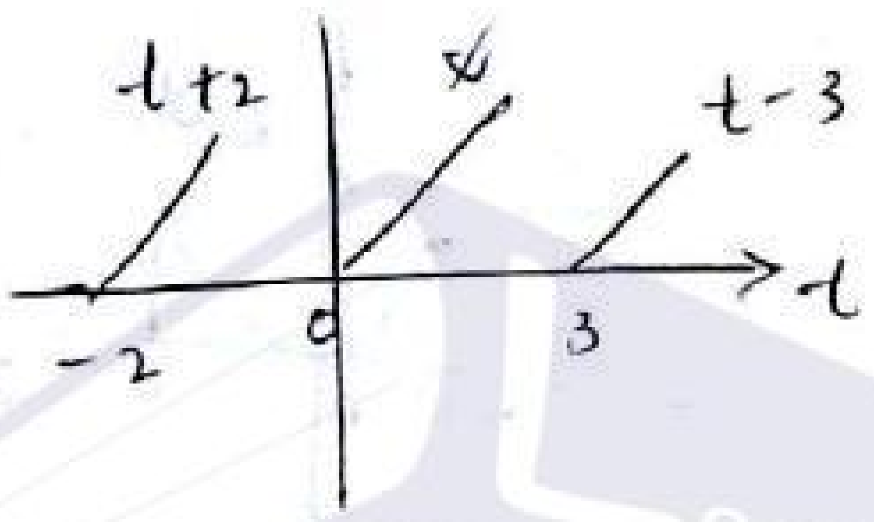
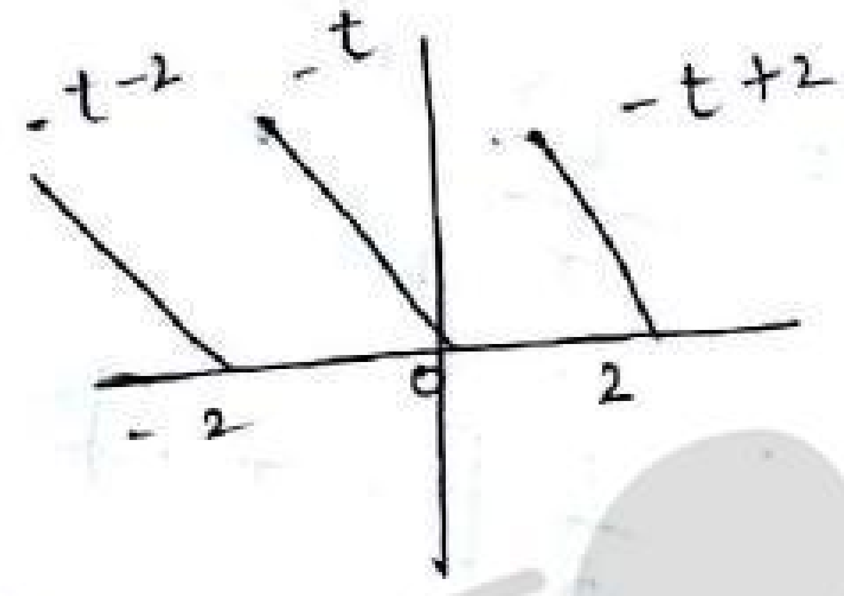
$= \cancel{25}\ 7$

$$25(5-4) + \frac{t^3}{3}\Big|_4^5 - 10\frac{t^2}{2}\Big|_4^5 + 8 + \frac{t^3}{3}\Big|_{-5}^{-4} + 25(4+5) + 10\frac{t^2}{2}\Big|_{-5}^{-4}$$

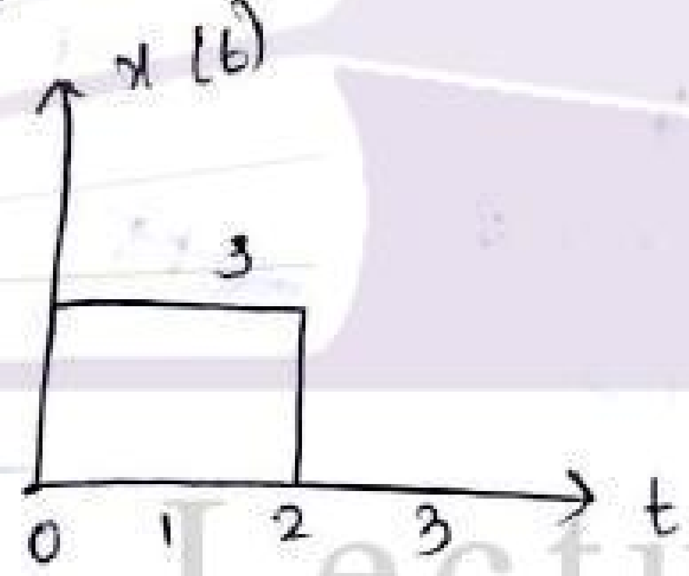
$= \cancel{281.33}\quad 8.66$

Note





(13) find E & P



$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_0^2 3^2\,dt$$

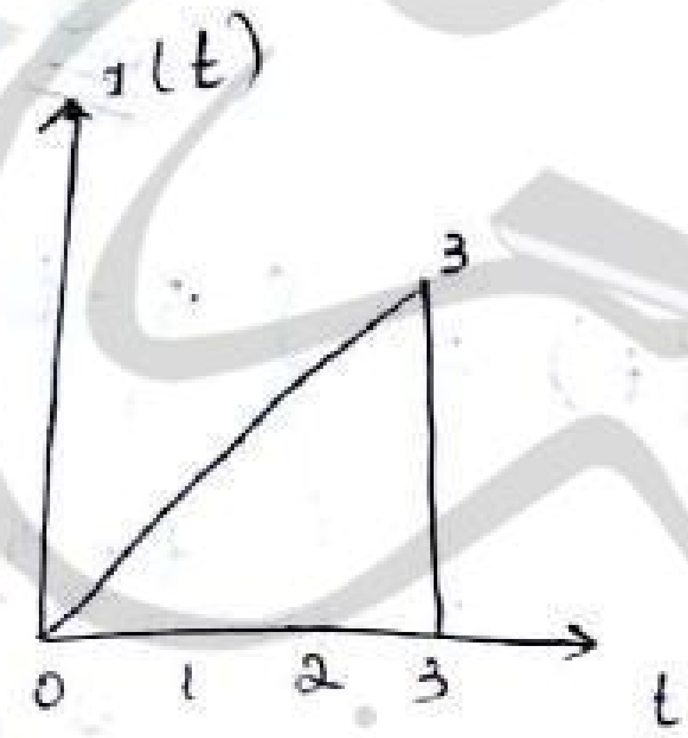
$$= 9\int_0^2 dt$$

$$= 9t\Big|_0^2$$

$$= 9(2)$$

$$= 18\,J$$

(14)



$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

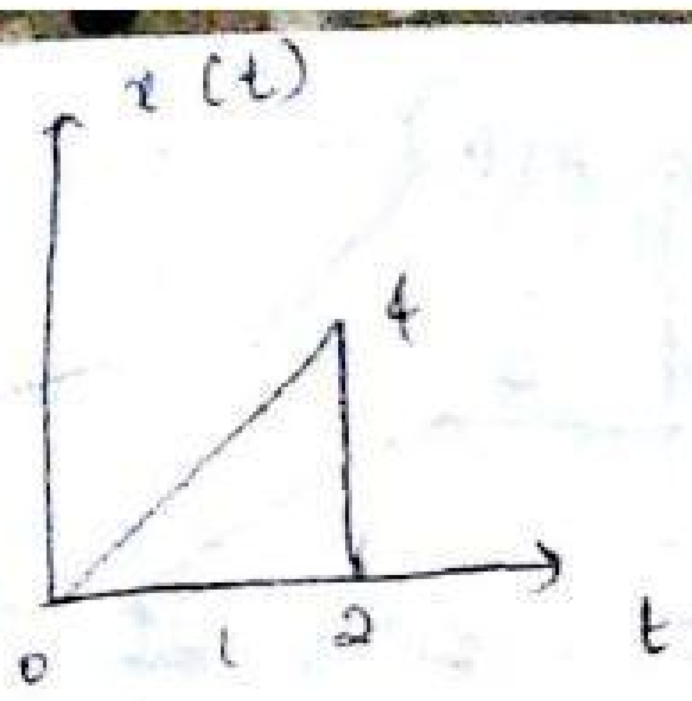
$$E = \int_0^3 t^2\,dt$$

$$= \frac{t^3}{2}\Big|_0^3$$

$$= \frac{1}{2}[9-0]$$

$$\boxed{= \frac{9}{2}} = 9\,J$$

(15)



$mx + c$

$mt + 0$ [since graph start from 3rd]

$M = \frac{4}{2} = 2 = \underline{\underline{2t}}$

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_0^2 (2t)^2 dt$$

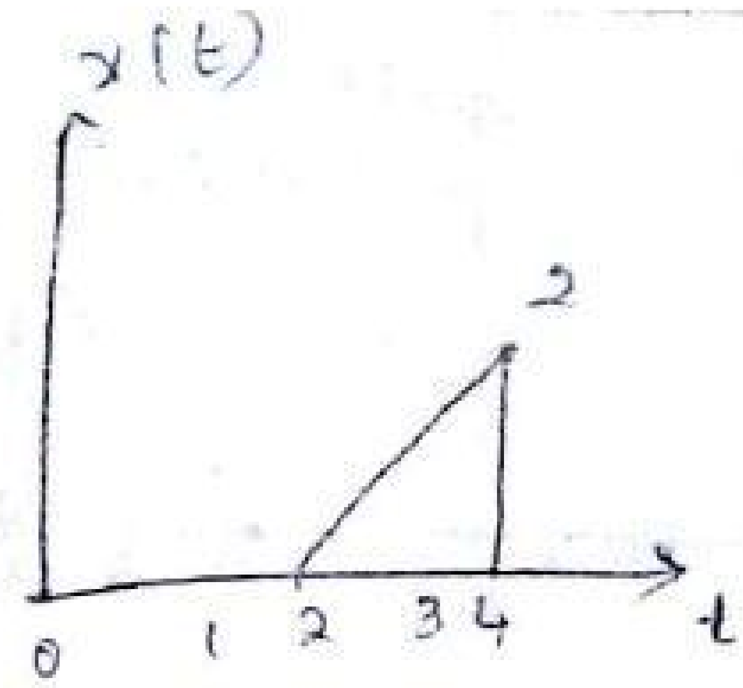
$$= \int_0^2 4t^2 dt$$

$$= 4 \frac{t^3}{3}\Big|_0^2$$

$$= \frac{4}{3}[8-0]$$

$$\boxed{= \frac{32}{3} J}$$

(16)



$mx + c$

$mt + c$

$m = \frac{2}{2} = 1$

$c = 2$ [it start from 2]

$1t - 2 = t - 2$

$$E = \int_{-\infty}^{\infty} (t-2)^2 dt$$

$$= \int_2^4 (t-2)^2 dt$$

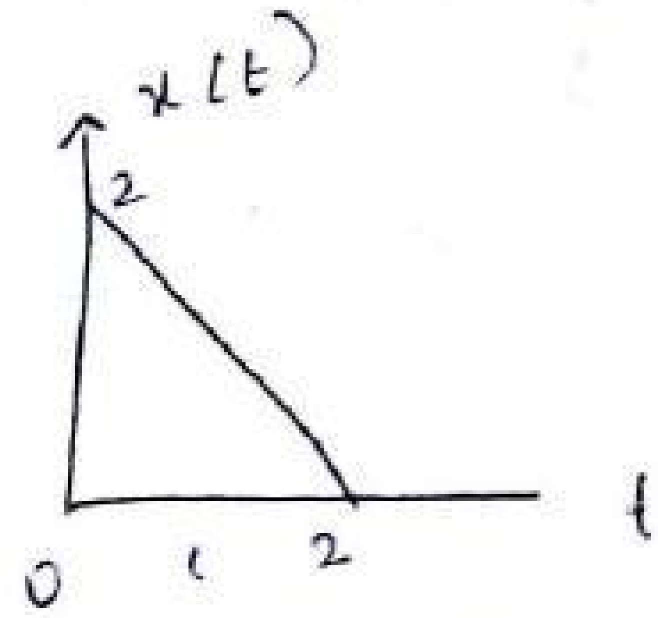
$$= \int_2^4 (t^2 + 4 - 4t)\, dt$$

$$= \frac{t^3}{3} + 4t - 4\frac{t^2}{2}\Big|_2^4$$

$$= \frac{1}{3}[56] + 4[2] - 2[12]$$

$$\boxed{= \frac{8}{3} J}$$

(17)



it belong to $(-t)$ group

$m(-t) + c$

$c = 2$. [+ve because opp direction]

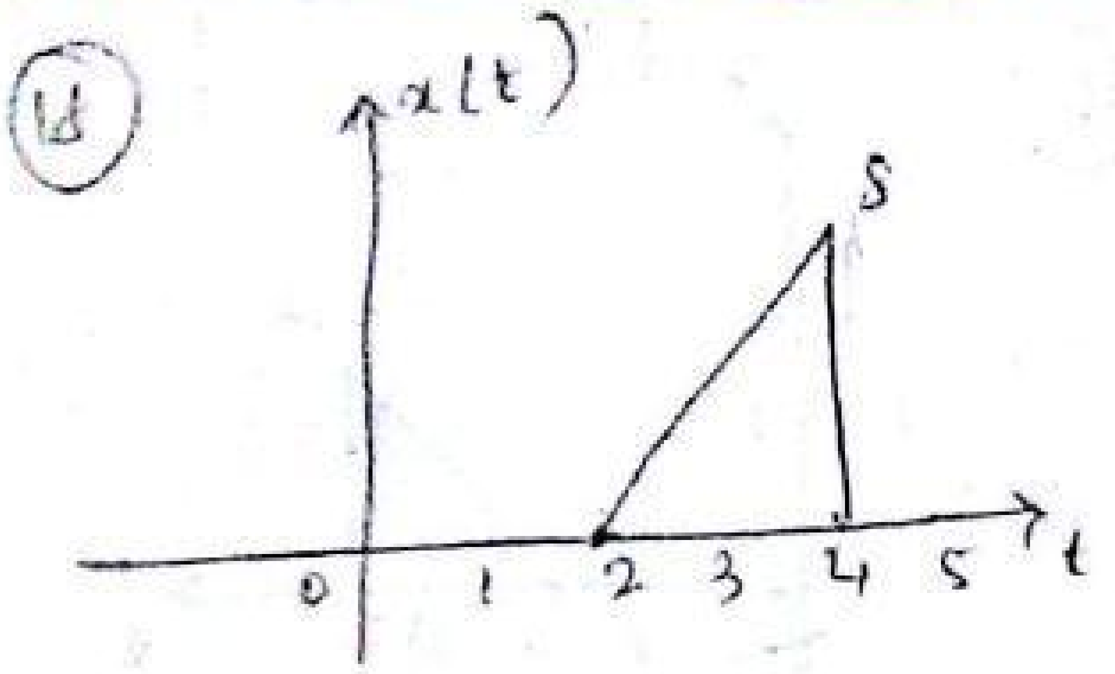
$m = \frac{2}{2} = 1$

$-t + 2$

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_0^2 (2-t)^2 dt$$

$$= \int_0^2 4 + t^2 - 4t \quad \boxed{8/3 \, J}$$

(18)



$mx + c$

$c = 2$

$mt - 2$

$m = \frac{8}{2} = 4$

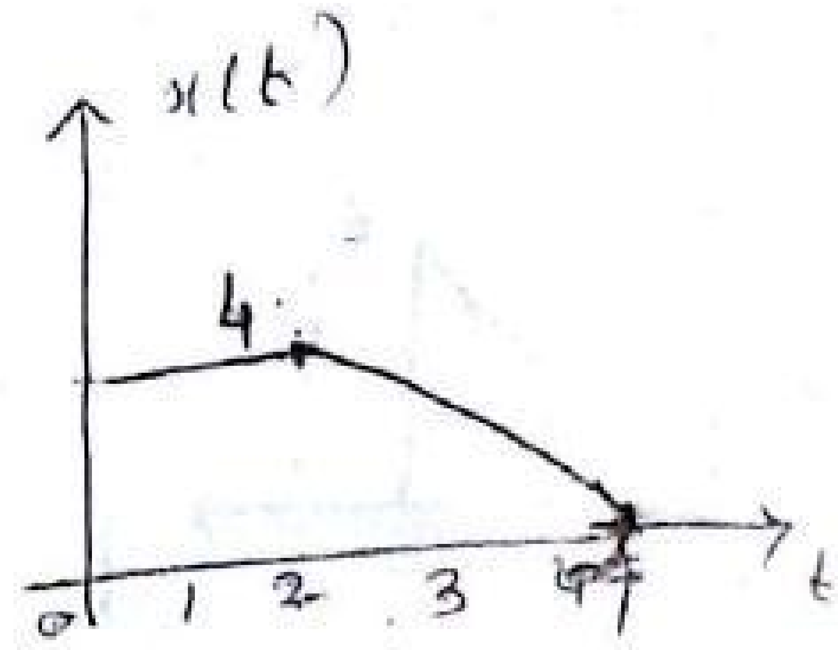
$4(t-2)$

$E = \int_{2}^{4} 4(t-2)^2 \, dt$

$= \cancel{210.66}\ J$

$\boxed{E = \frac{32}{3}\ J}$

(19)

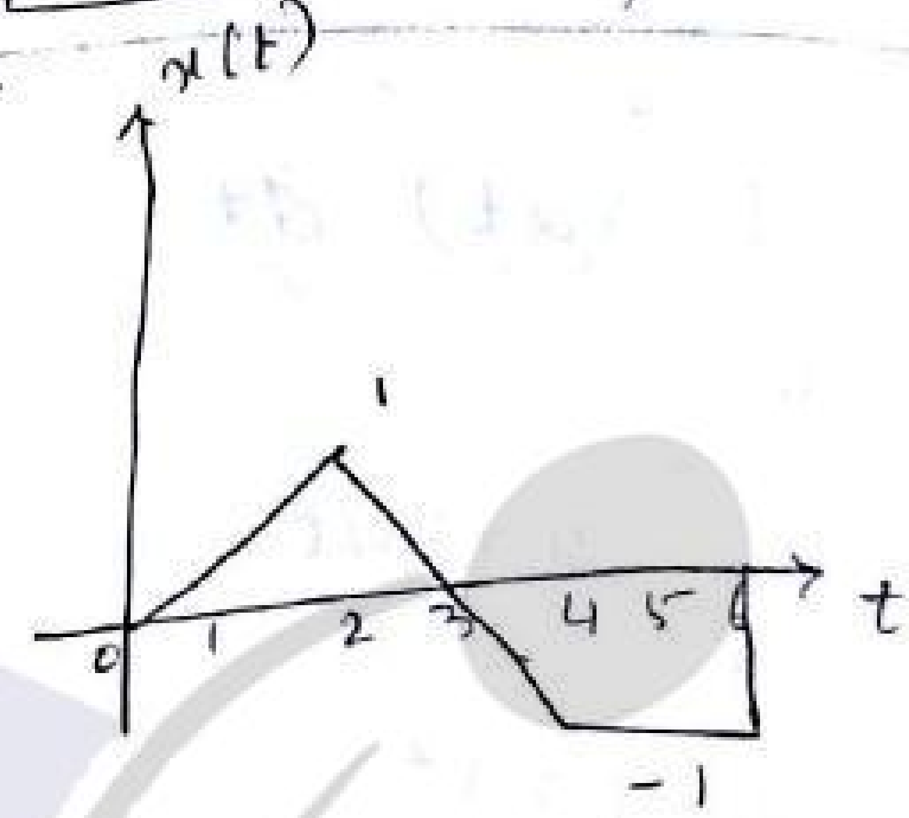


$\int_0^2 4^2 \, dt + \int_2^4 [2(4-t)]^2 \, dt$

$\boxed{= \frac{128}{3}\ J}$ $\quad \frac{128}{3}\ J$ //

$2t + 4$

(20)

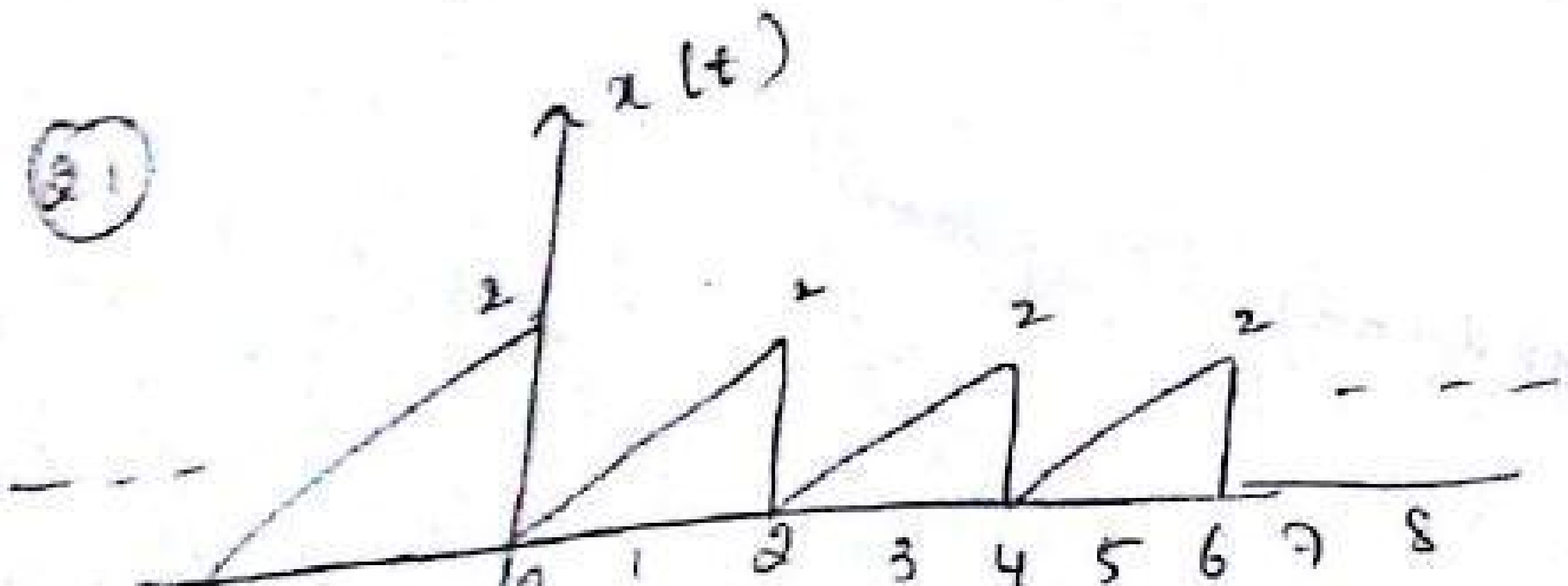


$\cancel{\int_0^2 \left(\frac{t}{2}\right)^2 dt + \int_2^3 (-t+3)^2 dt +}$

$\int_0^2 \left(\frac{t}{2}\right)^2 dt + \int_2^3 (-t+3)^2 dt + \int_3^4 (-t+3)^2 dt + \int_4^6 (-1)^2 dt$

$\boxed{= \frac{10}{3}\ J}$

(21)



It is a power signal

$$P = \frac{1}{T}\int_0^T x^2(t)\, dt \qquad \boxed{T = 2}$$

$$= \frac{1}{2}\left[\int_0^2 (t^2)\, dt\right]$$

$$= \frac{t^3}{6}\Big|_0^2$$

$$\frac{1}{6}[8-0]$$

$$= \frac{8}{6} = \boxed{\frac{4}{3}\, W}$$

Basic Operation on Signal

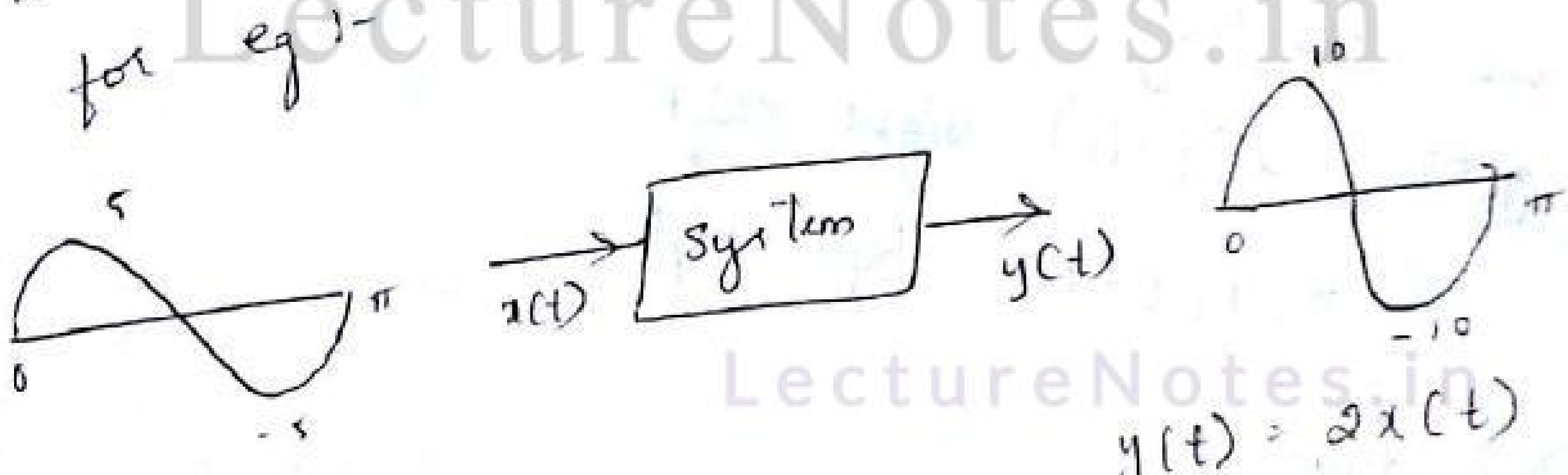
System can perform two basic operation on i/p signal

i) Amplitude operation

ii) Time operation.

i) Amplitude operation.

O/p signal is modified amplitude of i/p signal for eg:-

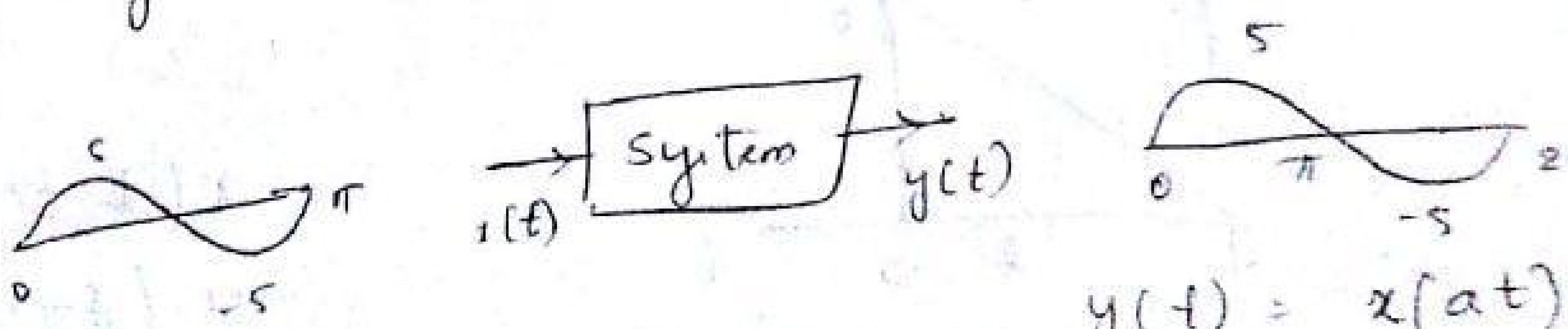


$y(t) = 2x(t)$

ii) Time operation

O/p signal is modified time of i/p signal

eg:



$y(t) = x(at)$

a → Multiplication factor

Amplitude operation

① amplitude scaling

$y(t) = a x(t)$

② Addition

$y(t) = x_1(t) + x_2(t)$

③ Multiplication

$y(t) = x_1(t) \cdot x_2(t)$

④ Integrator

$y(t) = \int x(t)$

⑤ Differentiation

$y(t) = \frac{d}{dt} x(t)$

Time operation

① time scaling

$y(t) = x(at)$

$a > 1$ compression

$a < 1$ expansion

② Time shifting

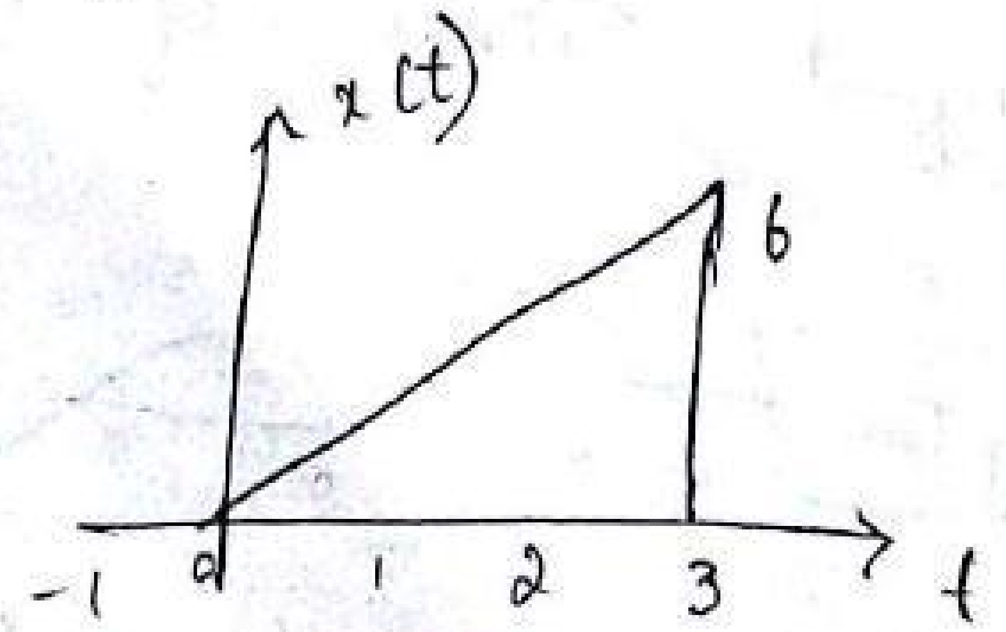
$y(t) = x(t - t_0)$ right shift

$= x(t + t_0)$ left shift

③ Reflection

$y(t) = x(-t)$

①



Sketch

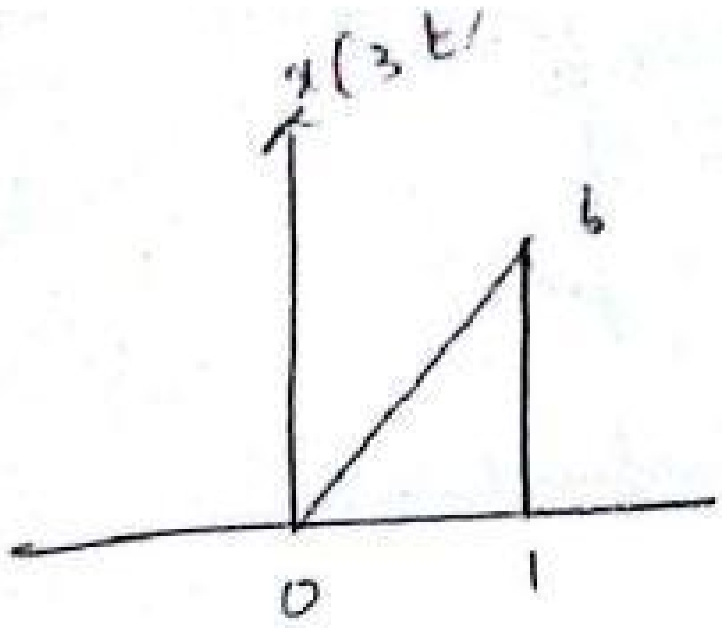
$x(3t)$

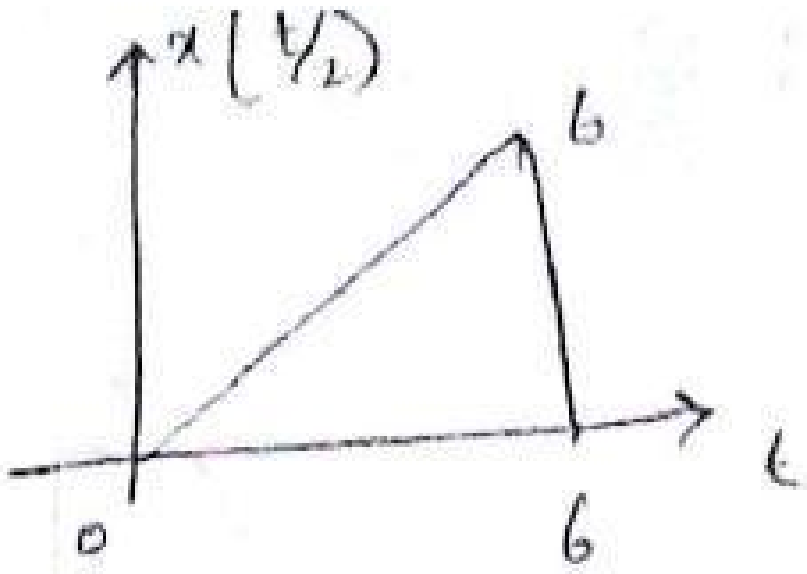
$x(t/2)$

$x(t-2)$

$x(t+1)$

$x(-t)$





$3t : 0, 3$

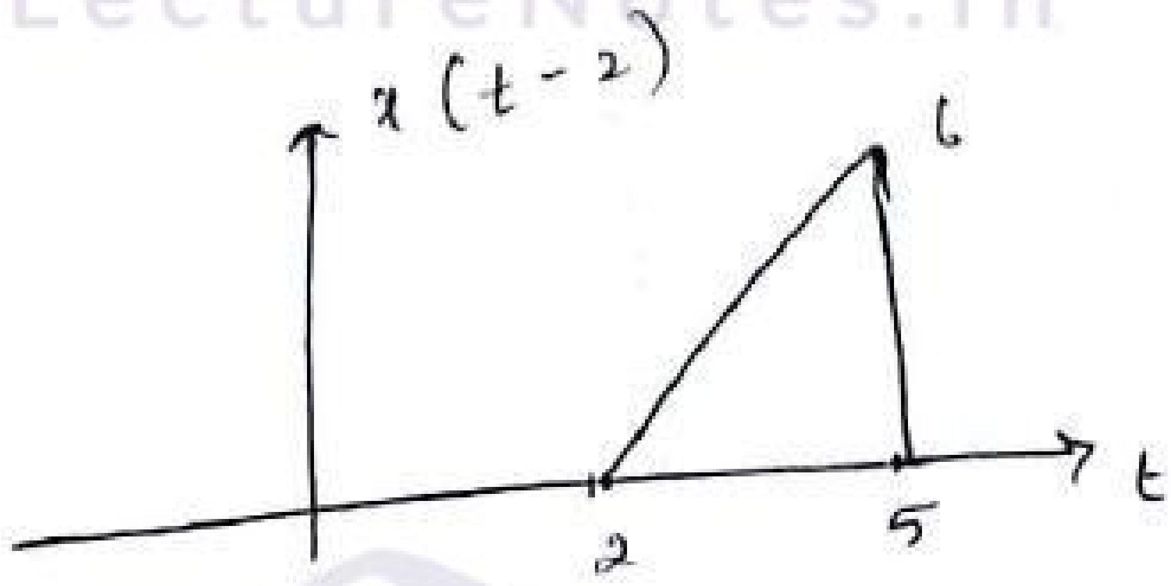
$\div 3$

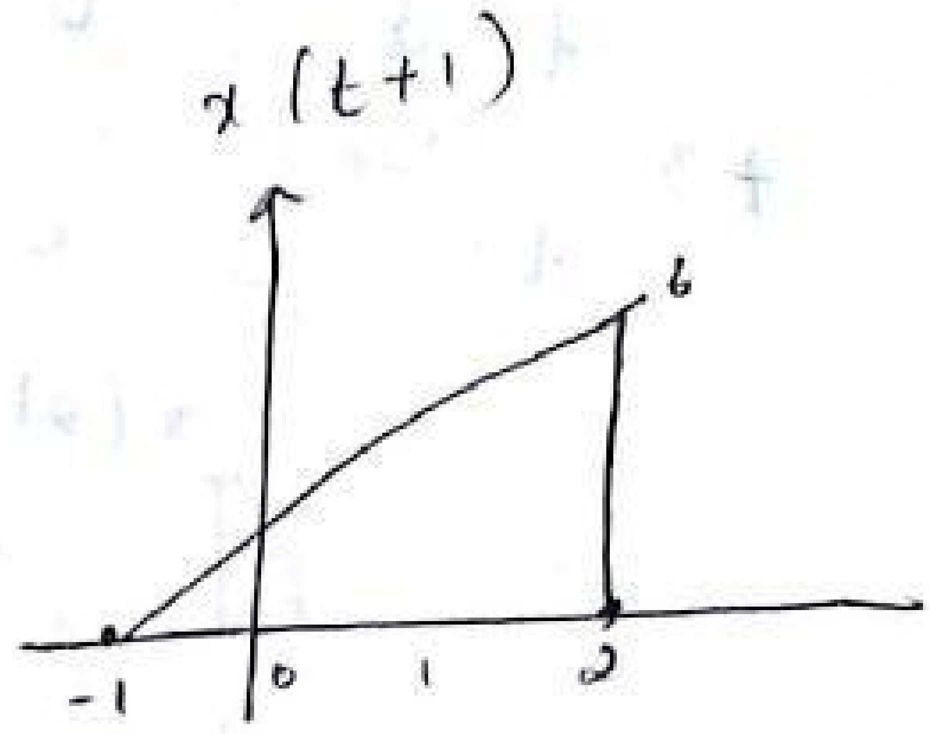
$t : 0, 1$

$t/2 : 0, 3$

$\times 2$

$t : 0, 6$





$t - 2 : 0, 3$

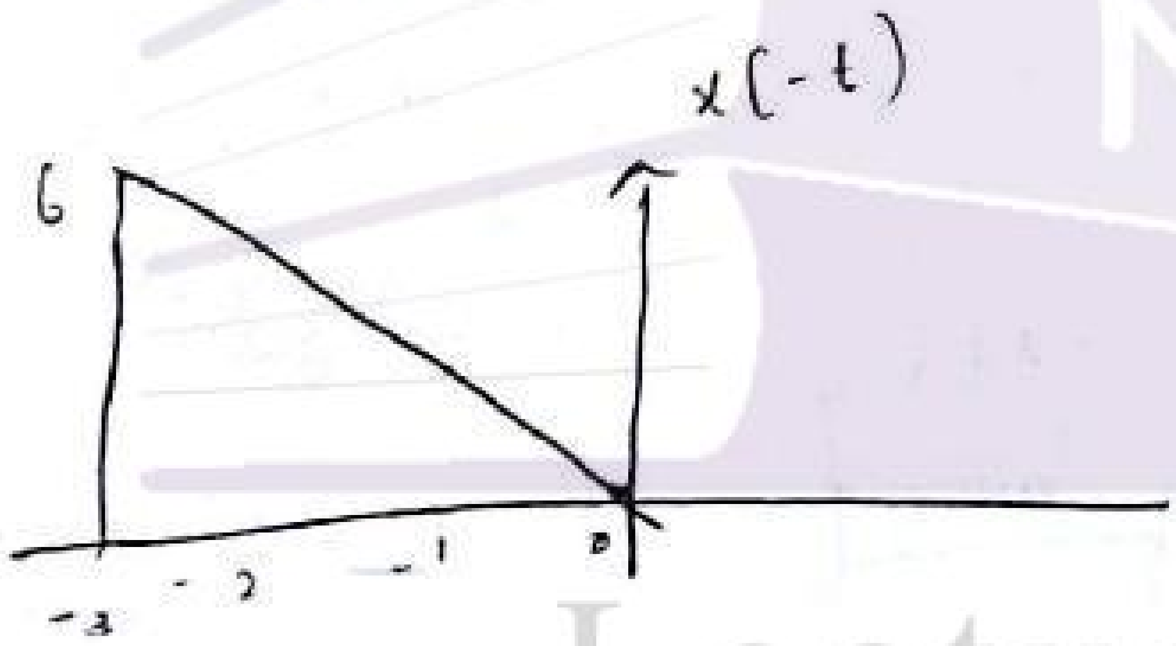
$\div 2$

$t : 2, 5$

$t + 1 : 0, 3$

-1

$t : -1, 2$

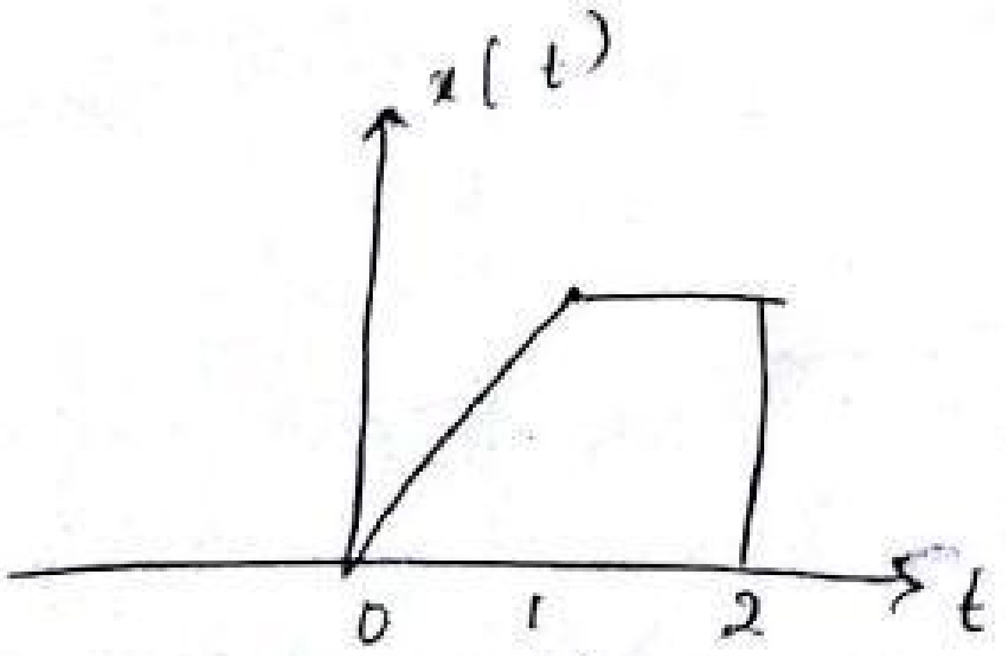


$-t : 0, 3$

$\times -1$

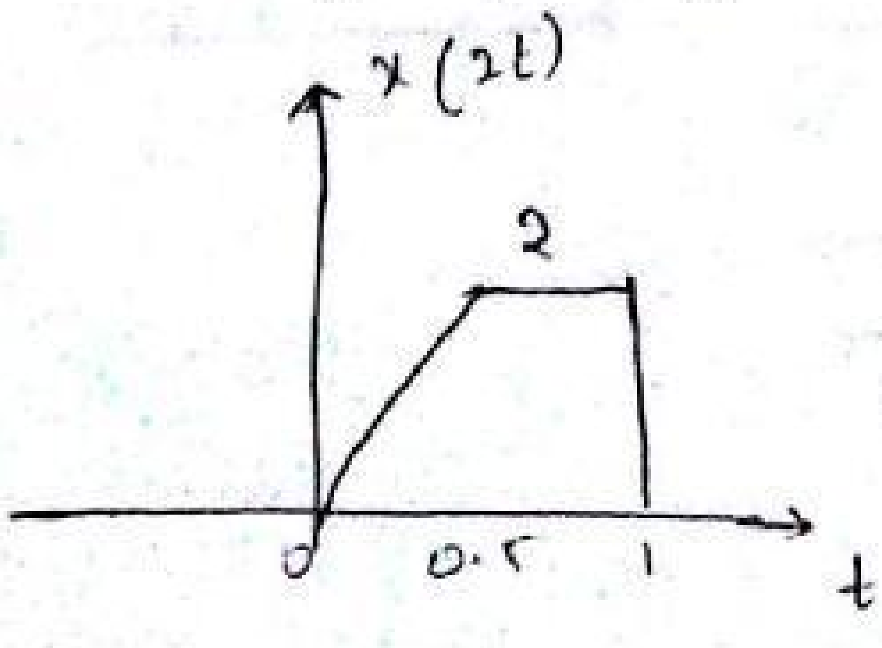
$t ; 0, -3$

(2)



sketch $x(2t)$

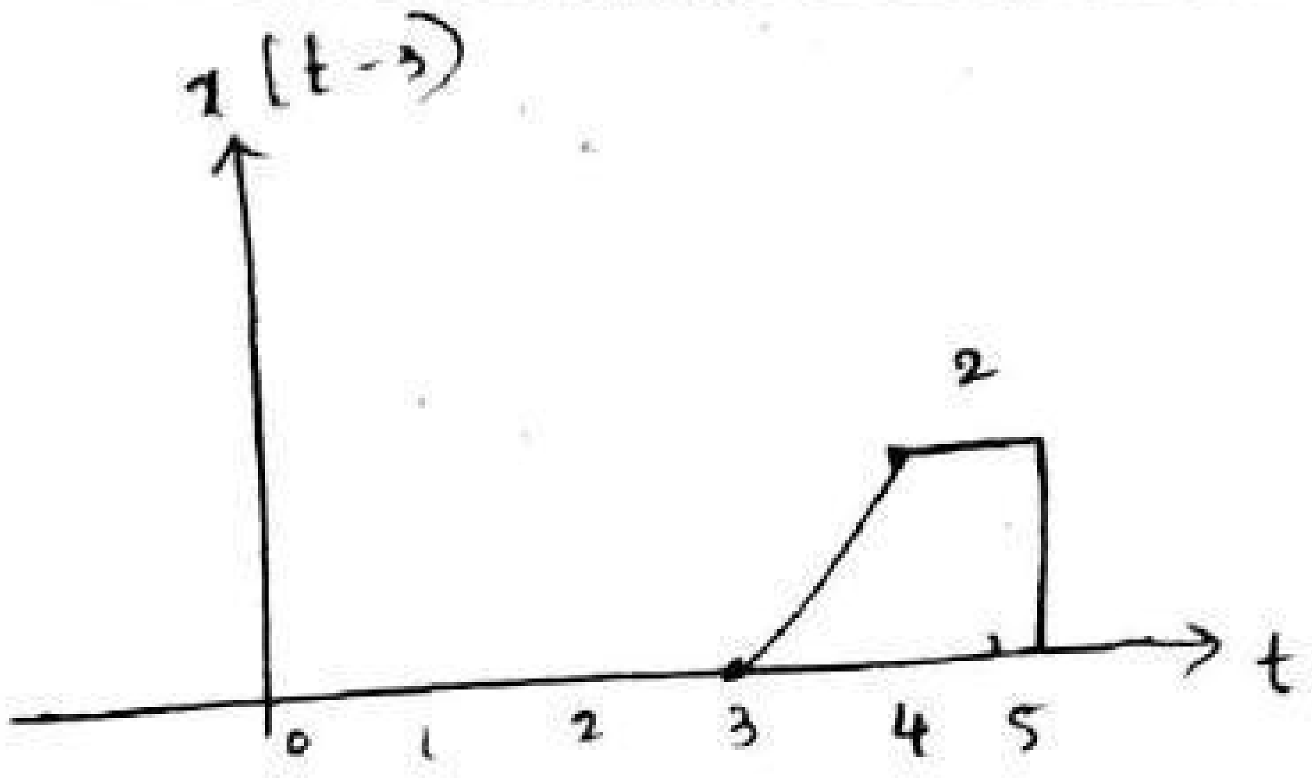
$x(t-3)$



$2t : 0, 1, 2$

$\div 2$

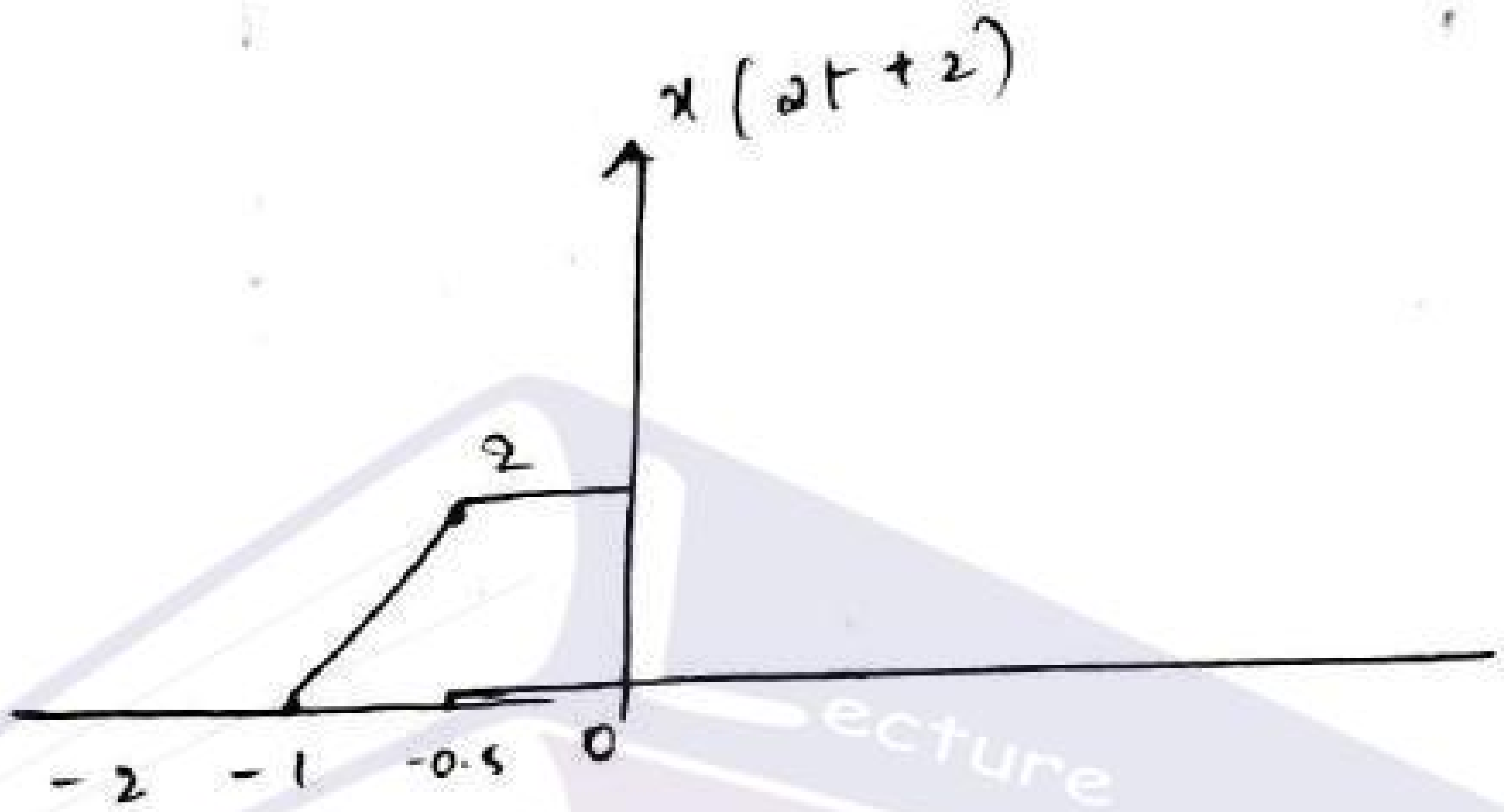
$t : 0, 0.5, 1$



$t-3$: 0, 1, 2

+3

t : 3, 4, 5



$2t+2$: 0, 1, 2

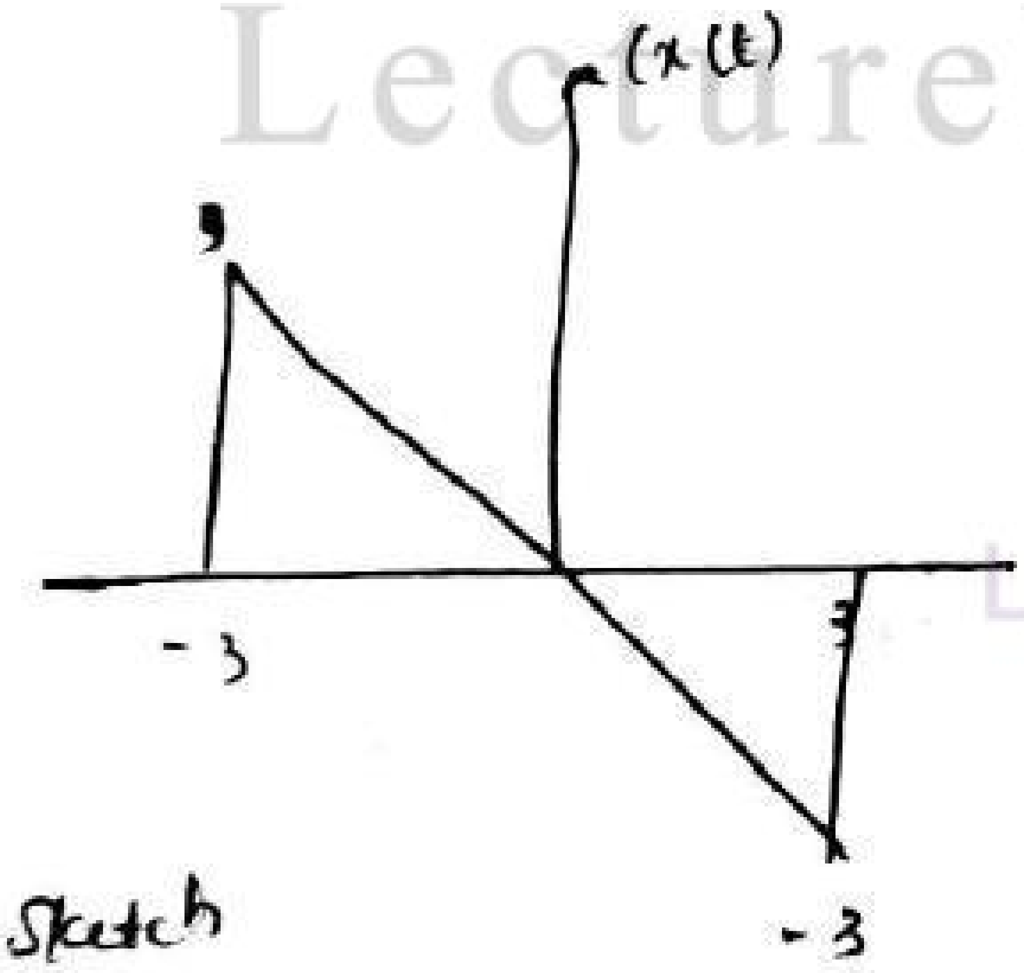
-2

$2t$: -2, -1, 0

$\div 2$

t : -1, -0.5, 0

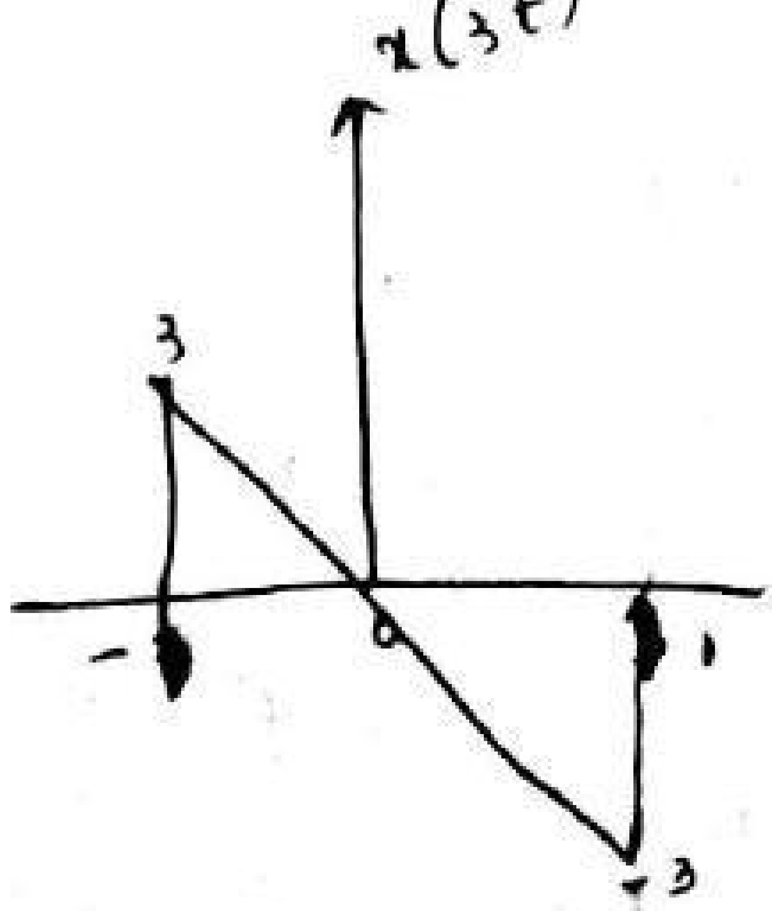
(3)



Sketch

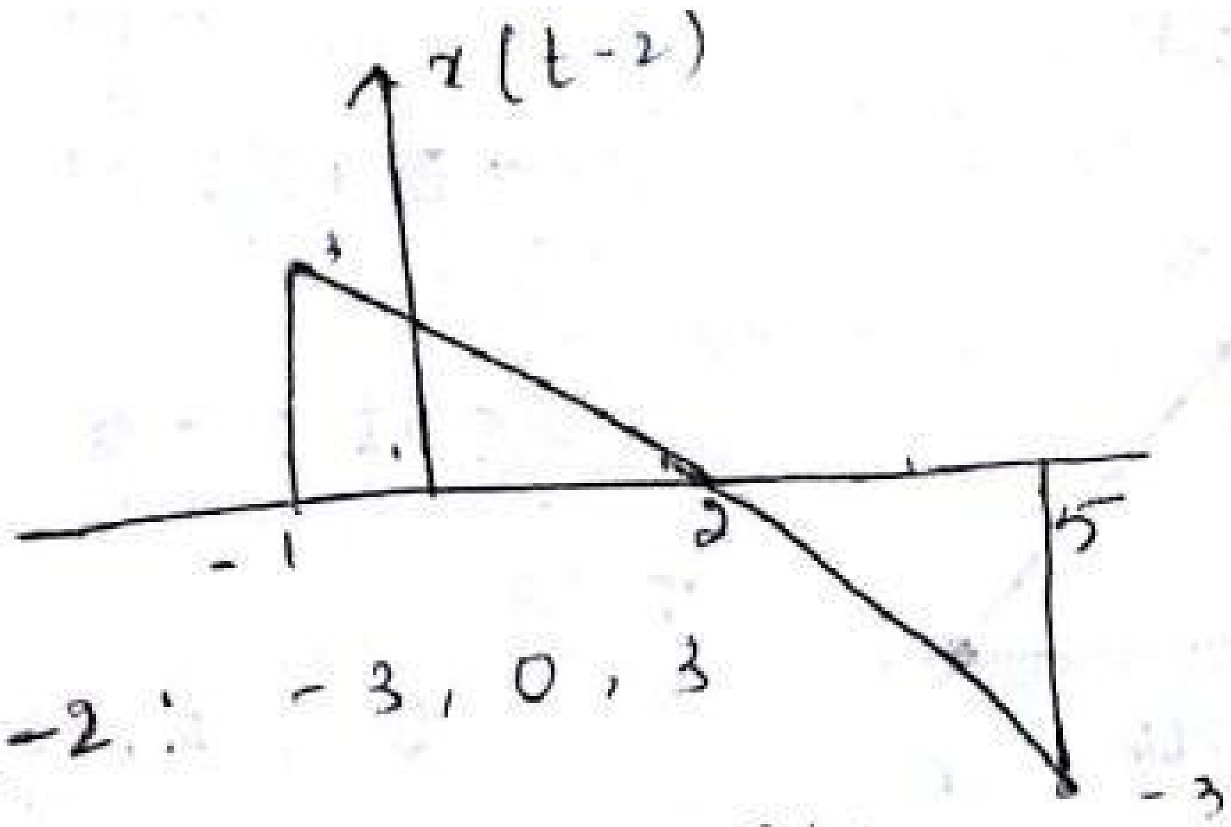
$x(3t)$

$x(t-2)$



$\div 3$ $3t$: -3, 0, 3

t = -1, 0, 1



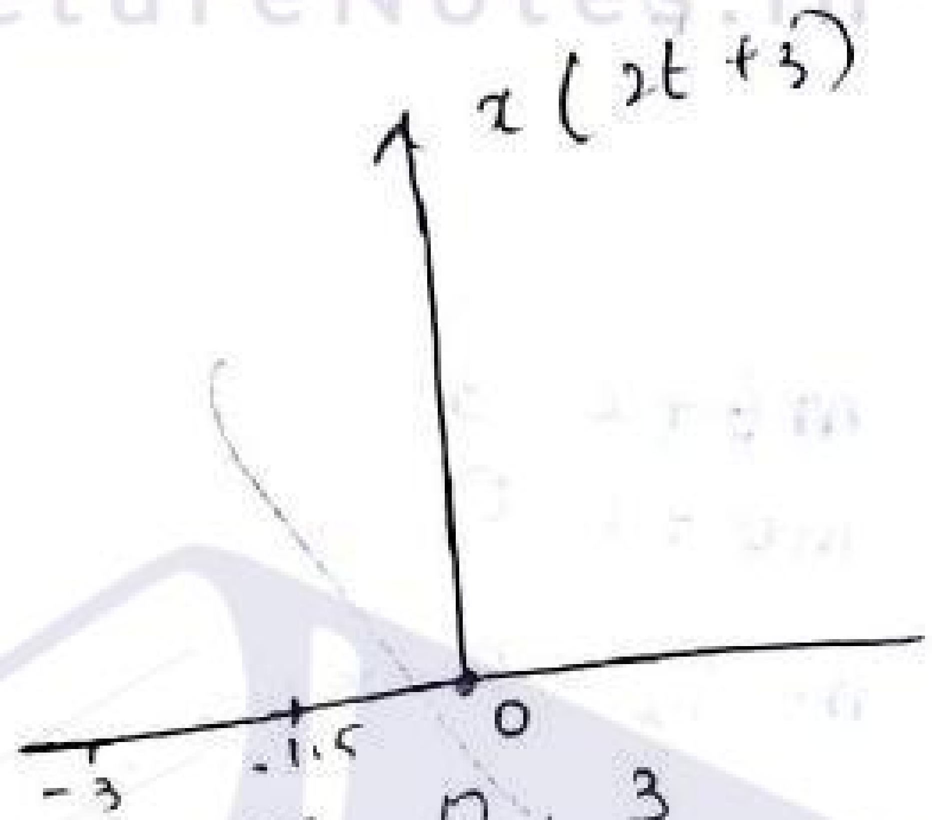
$t-2 : -3, 0, 3$

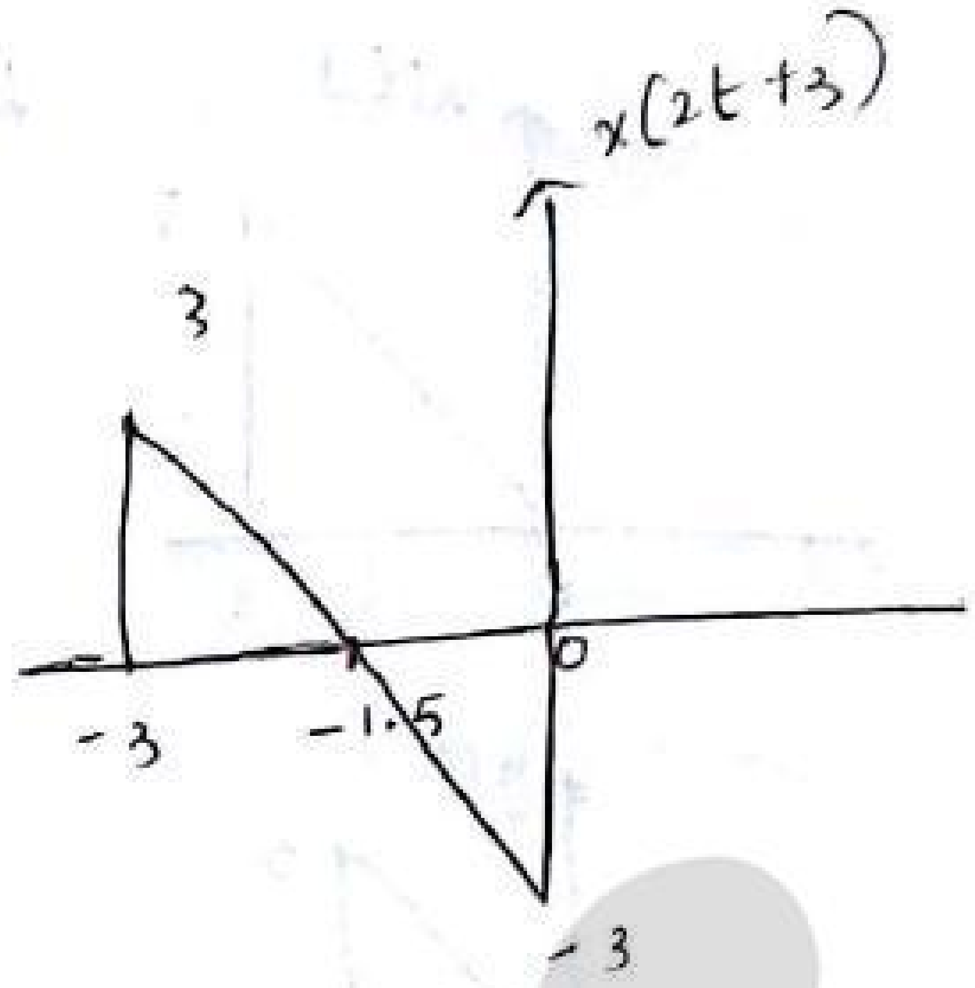
$+2$

$t = -3+2, 2, 3+2$

$t = -1, 2, 5$

(2)





$2t+3 : -3, 0, 3$

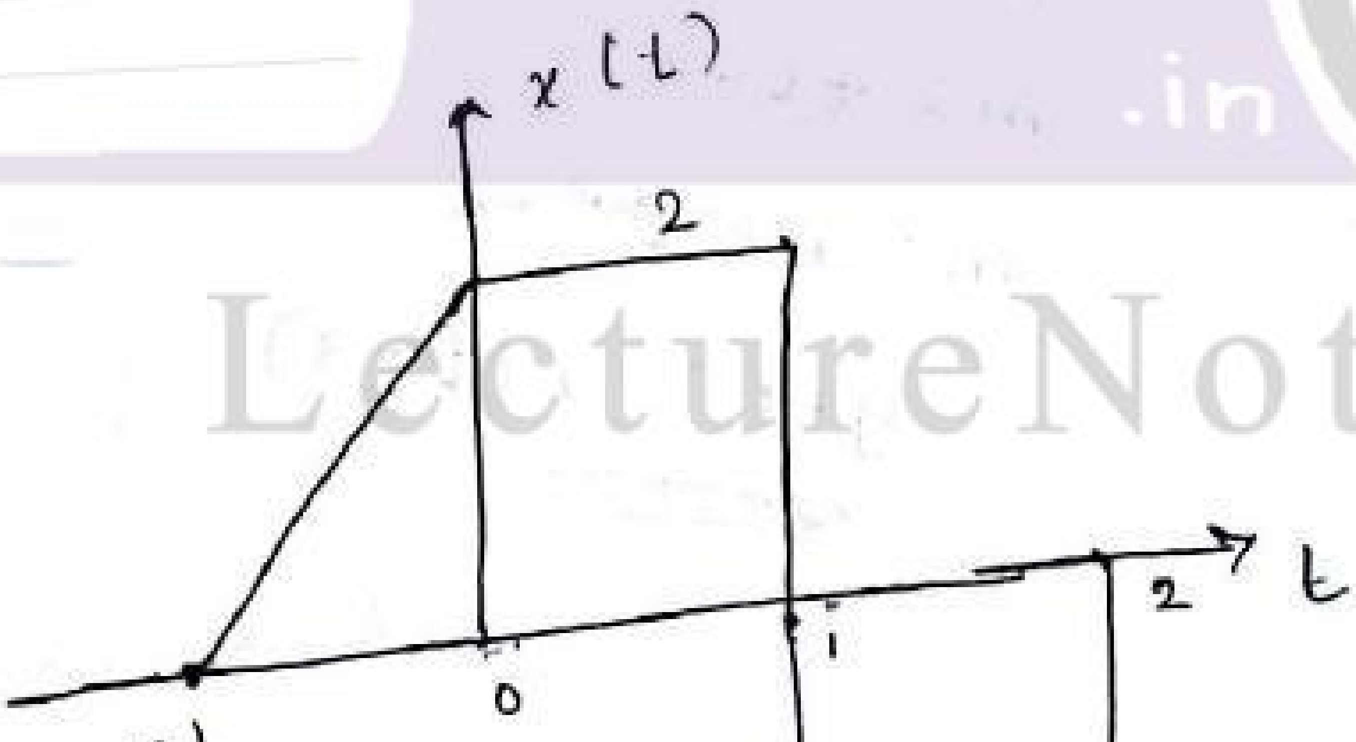
-3

$2t : -6, -3, 0$

$\div 2$

$t = -3, -1.5, 0$

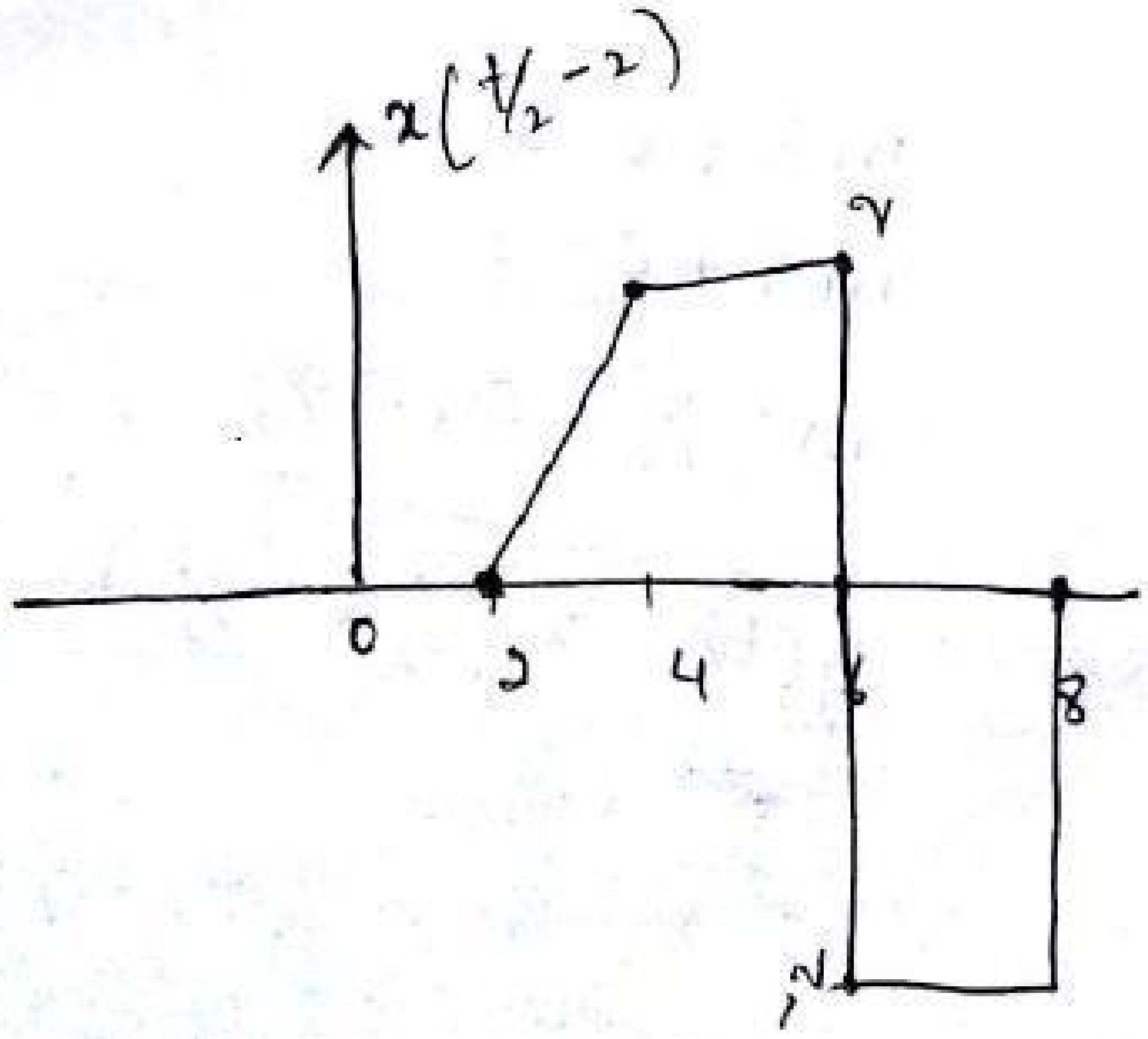
(4)



Sketch

$x(t/2 - 2)$

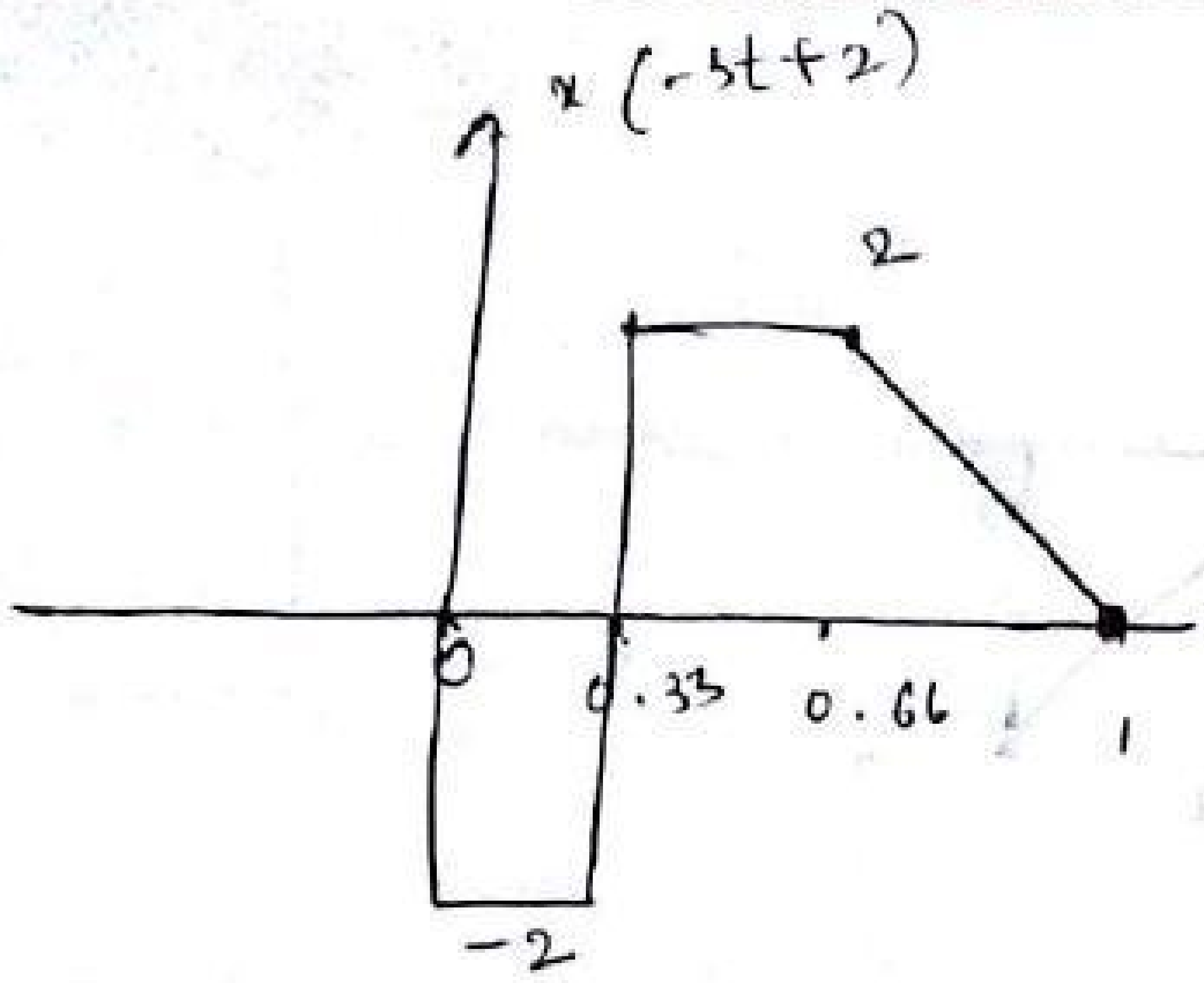
$x(-3t+2)$



$t/2 - 2 : -1, 0, 1, 2$

$+2$ $\quad t/2 : 1, 2, 3, 4$

$\times 2$ $\quad t : 2, 4, 6, 8$



$-3t+2 : -1, 0, 1, 2$

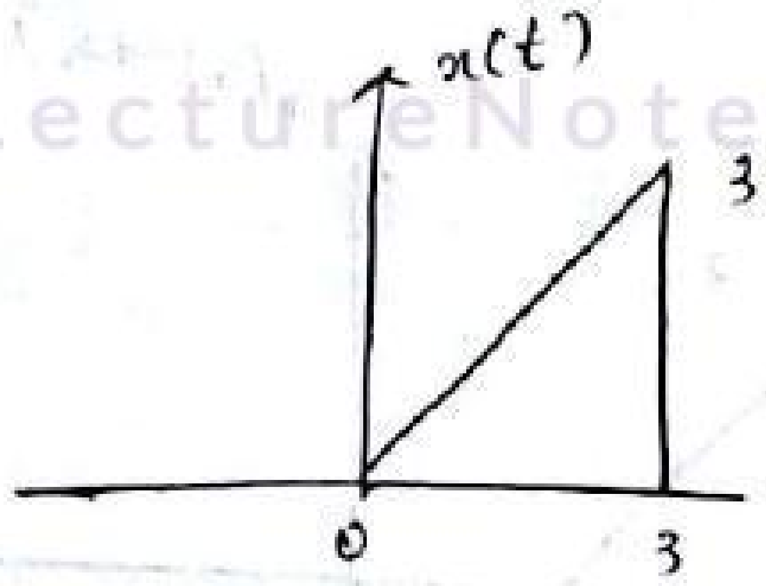
-2

$-3t : -3, -2, -1, 0$

$\div -3$

$t : 1, 0.66, 0.33, 0$

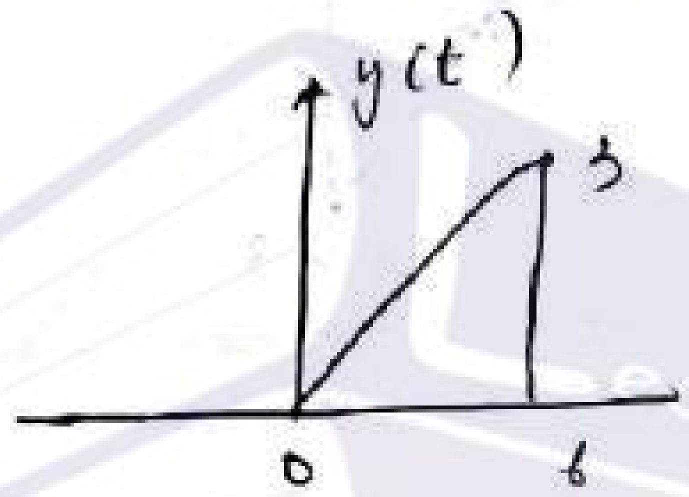
(5) Represent $y(t)$ interms of $x(t)$



$m6 + c = 3$

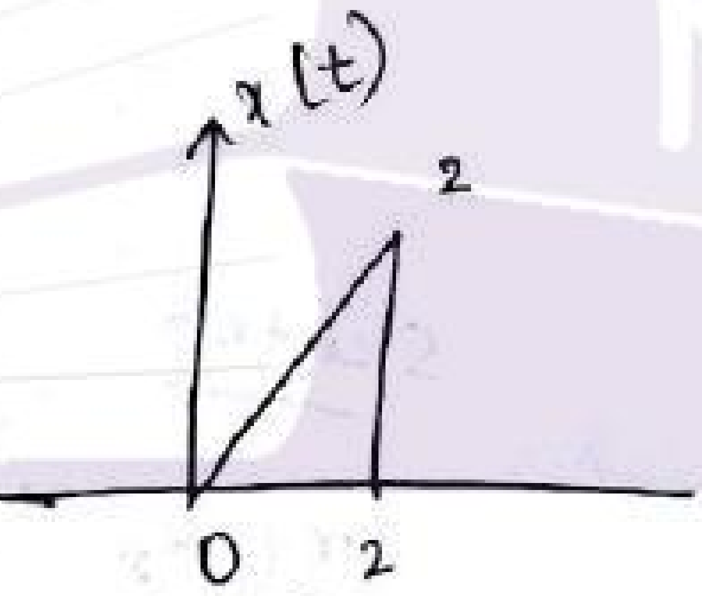
$m0 + c = 0$

$m = \frac{1}{2} \quad c = 0$



$y(t) = x(t/2)$

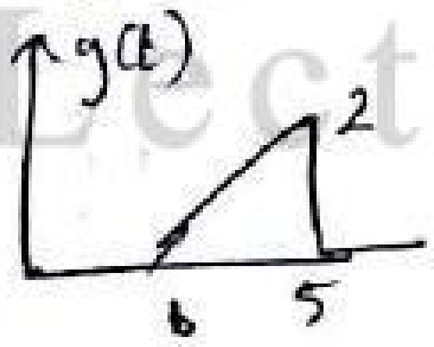
(6)



$m0 + c = 0$

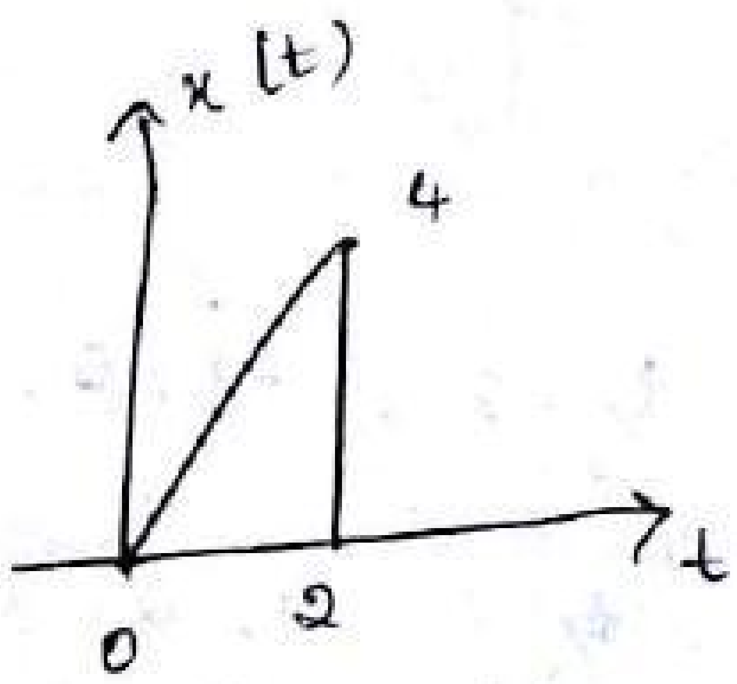
$m2 + c = 6$

$m = 1 \quad c = -3$



$y(t) = x(t-3)$

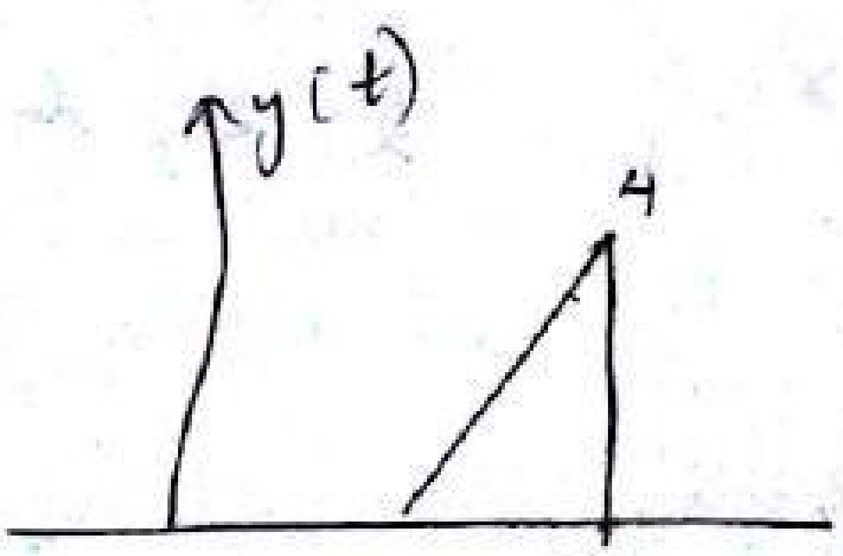
7)



$m3 + c = 0$

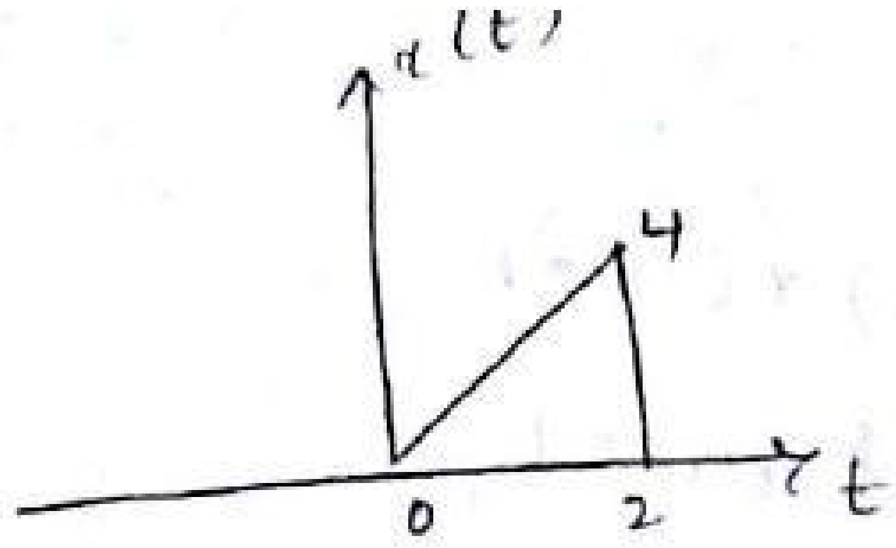
$m7 + c = 2$

$m = \frac{1}{2} \quad c = -\frac{3}{2}$



$y(t) = x\left(\frac{1}{2}t - \frac{3}{2}\right)$

(8)

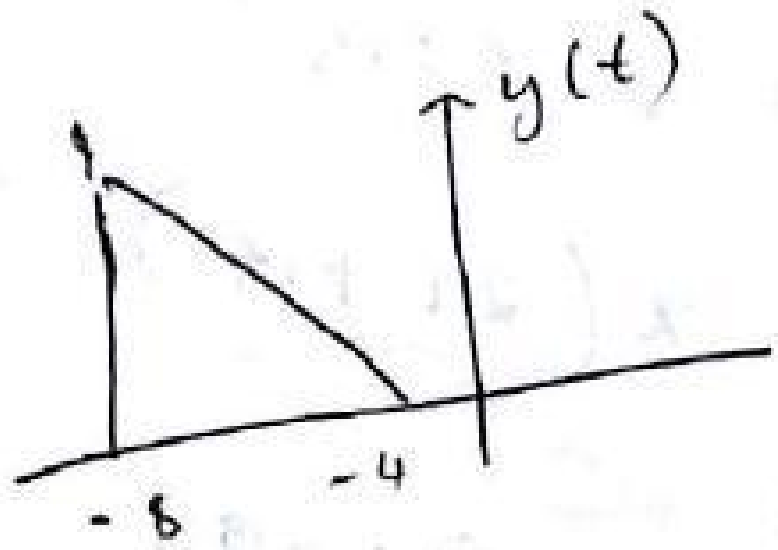


$Mt + C$

$m(-4) + C = 0$

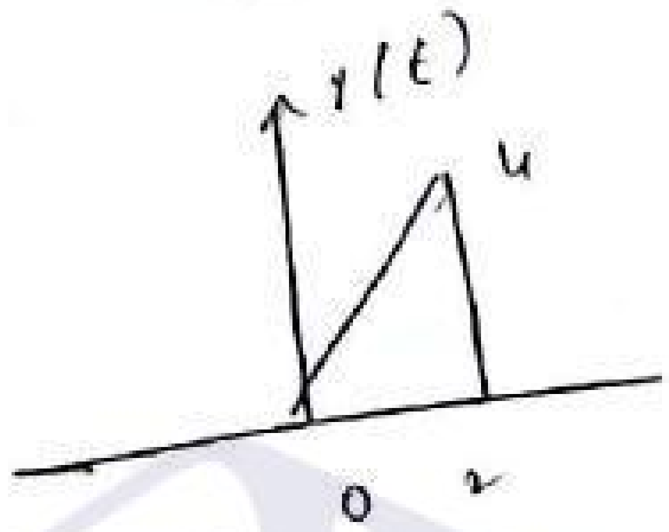
$M(-8) + C = 2$

$M = -\frac{1}{2}$, $C = -2$



$y(t) = x\left(-\frac{1}{2}t - 2\right)$

(9)

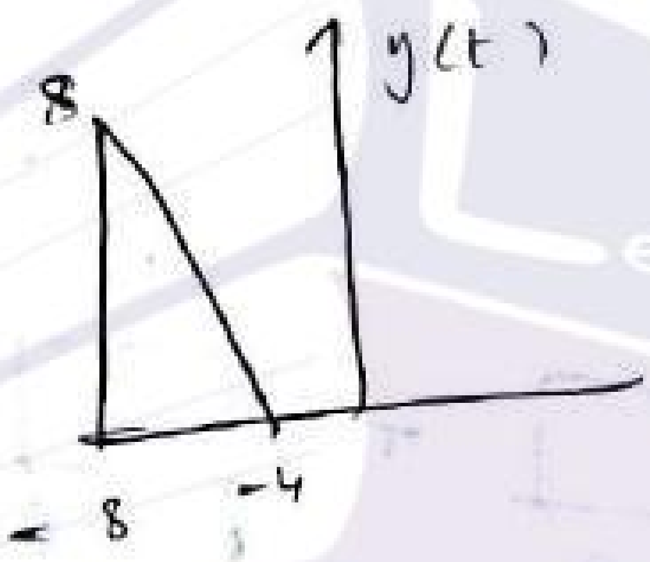


$m(-4) + C = 0$

$M(-8) + C = 2$

$M = -\frac{1}{2}$ $C = -2$

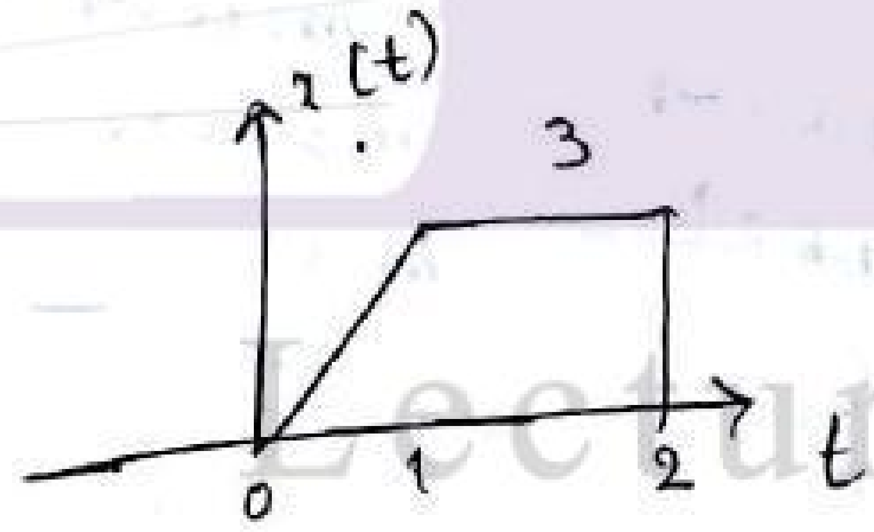
$y(t) \cdot x(t)$
$8 = 4$
$8 \times 4 \times \boxed{2}$

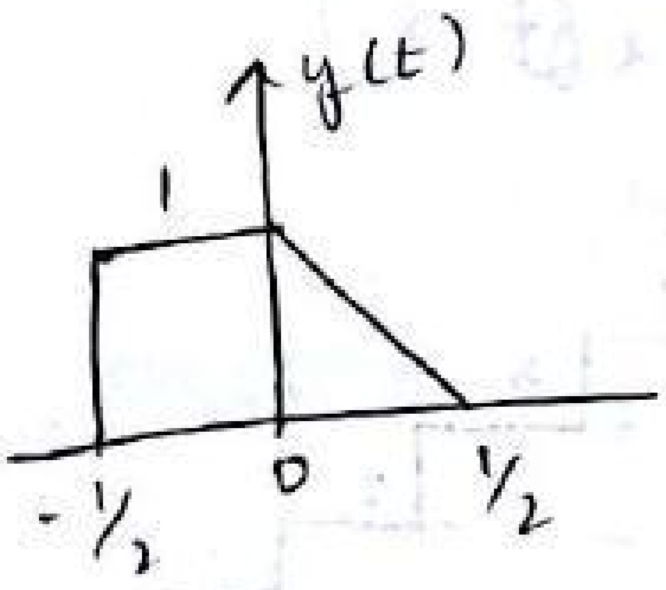


$y(t) = 2x\left(-\frac{1}{2}t - 2\right)$

$y(t)$ $x(t)$
$8 =$ 4×2

(10)





$m(-\frac{1}{2}) + C = 2$

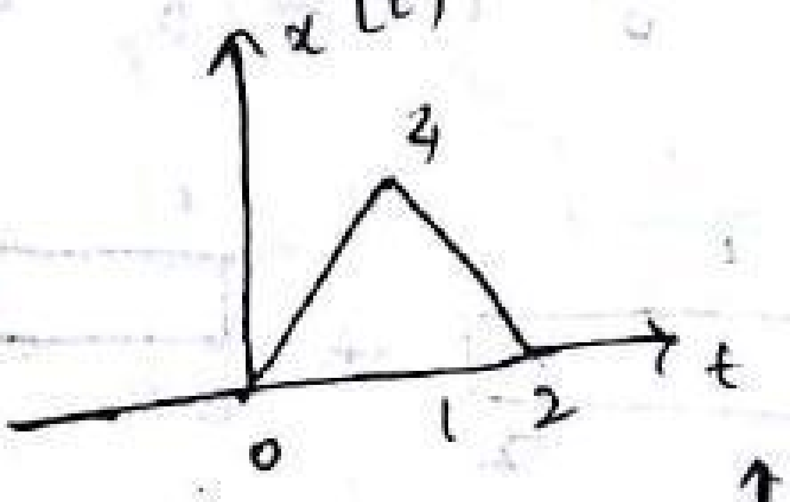
$m(\frac{1}{2}) + C = 0$

$M = -2$, $C = 1$

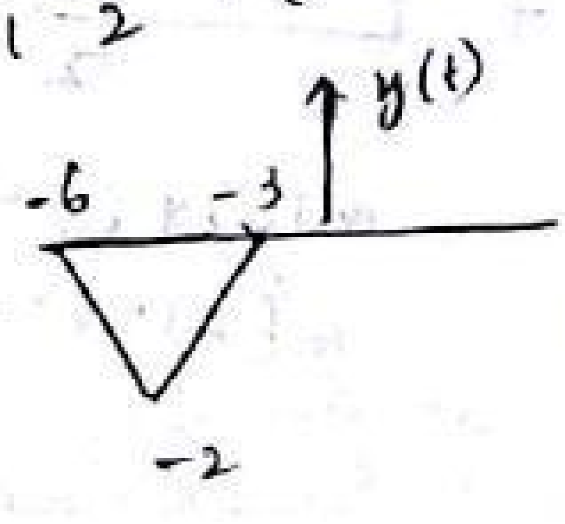
$y(t) = \frac{1}{3}x(-2t + 1)$

$y(t) = x(t)$
$1 = 3$
$1 = 3\boxed{\left(\frac{1}{3}\right)}$

(11)



$y(t) = x(t)$
$-2 = 4\left(\frac{1}{-2}\right)$



$M(-6) + C = 0$

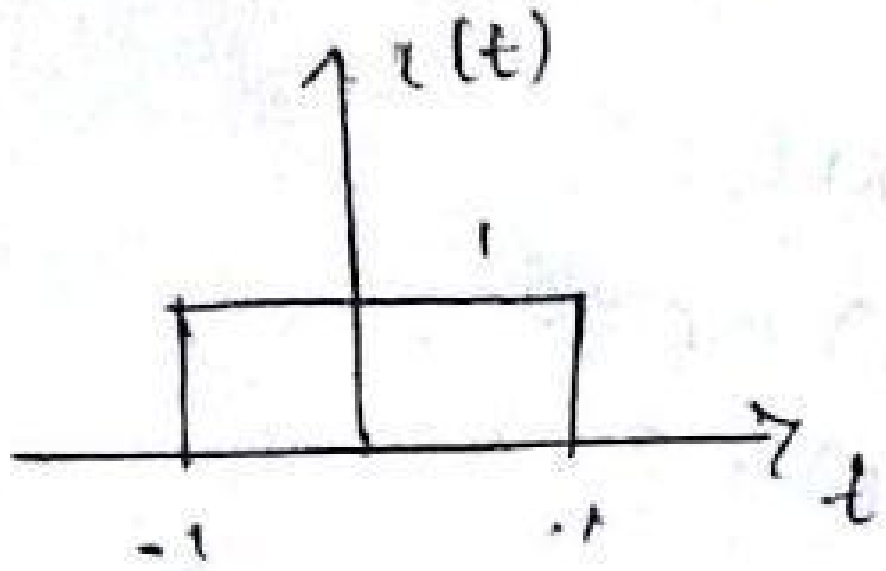
$M(-3) + C = 2$

$M = 2/3$

$C = 4$

$y(t) = -\frac{1}{2}x\left(\frac{2}{3}t + 4\right)$

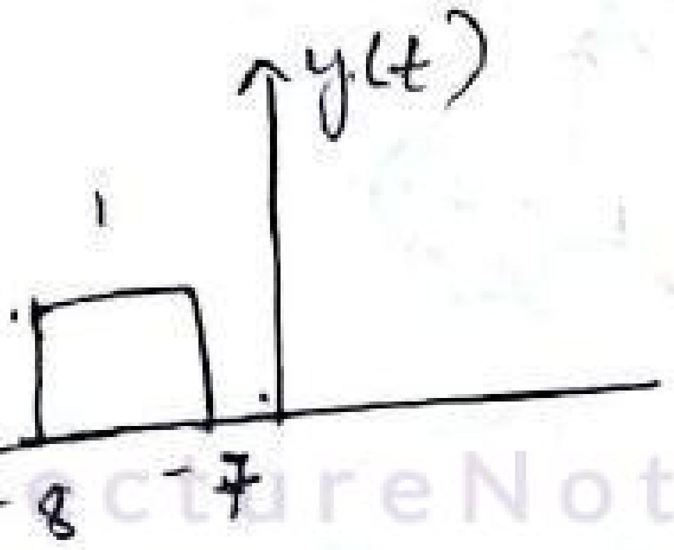
(12)



$m(-8)+c=-1$

$m(-7)+c=1$

$m=2, \quad c=15$



$y(t) = x(2t+15)$

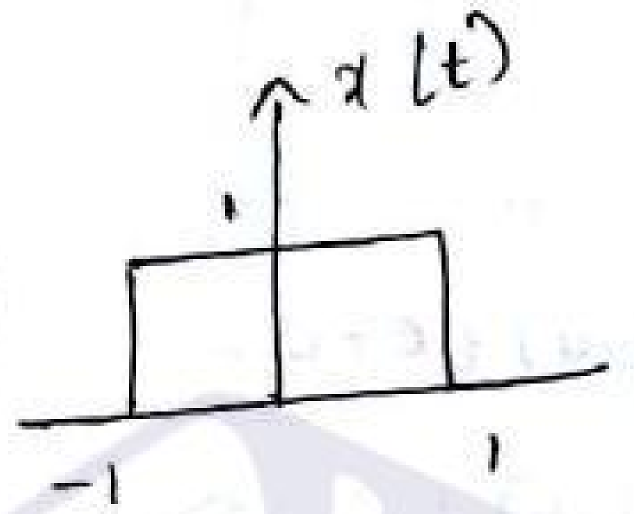
$m(-7)+c=1$

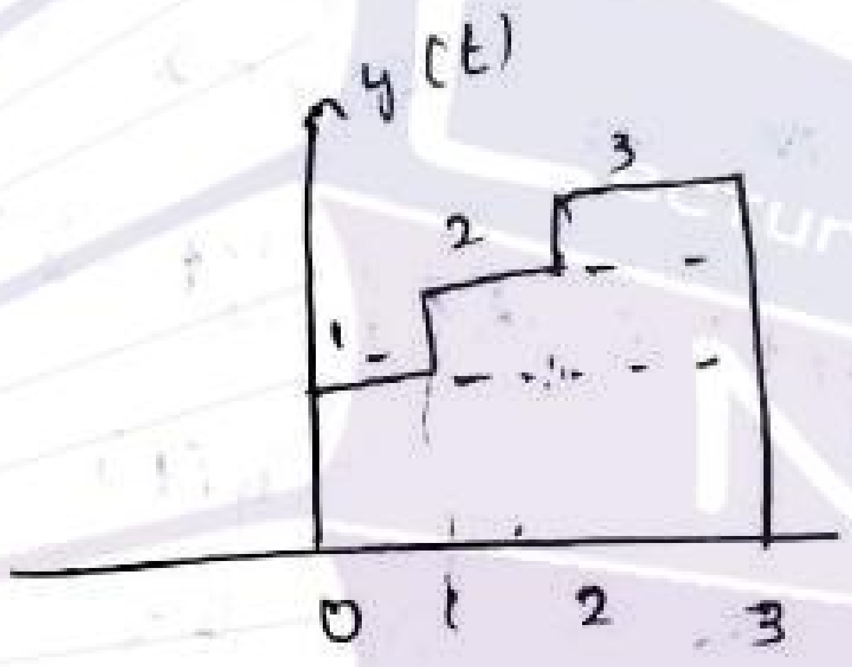
$m(-8)+c=-1$

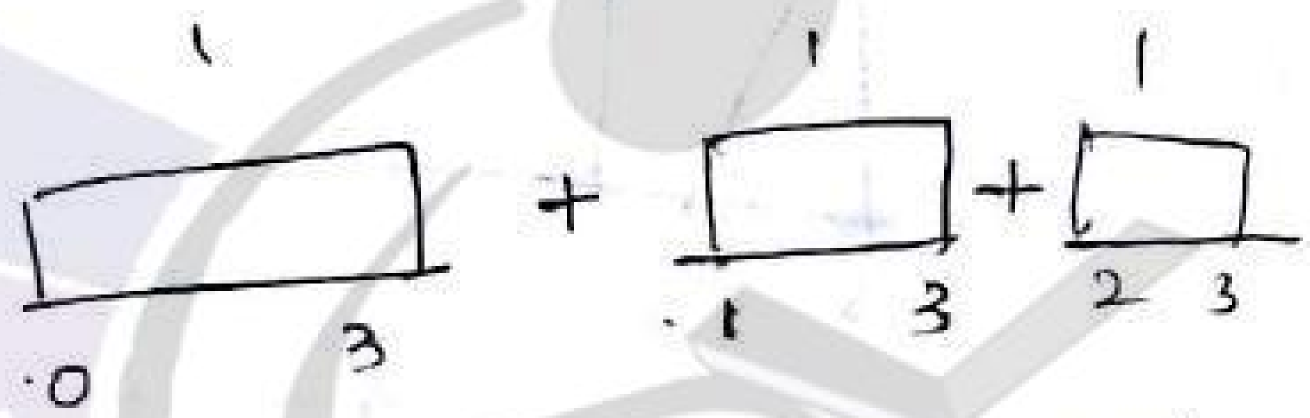
$\underline{\underline{x(2t+15)}}$

5 Marks

(13)







$m(0)+c=-1$

$m3+c=1$

$m=2/3$

$c=-1$

$m1+c=-1$

$m3+c=1$

$m=1$

$c=-2$

$m2+c=-1$

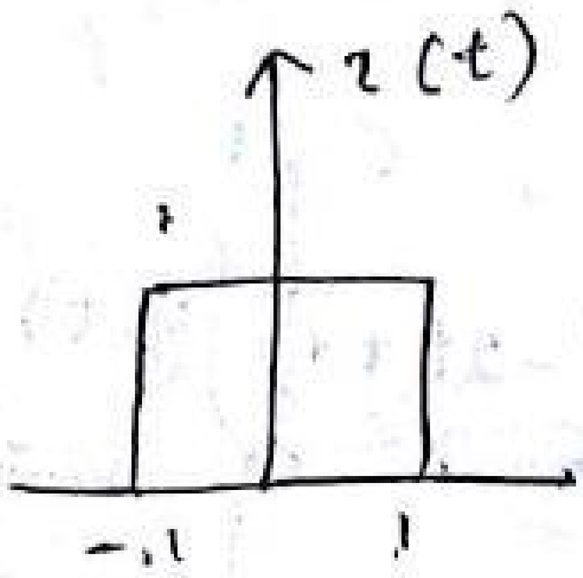
$m3+c=1$

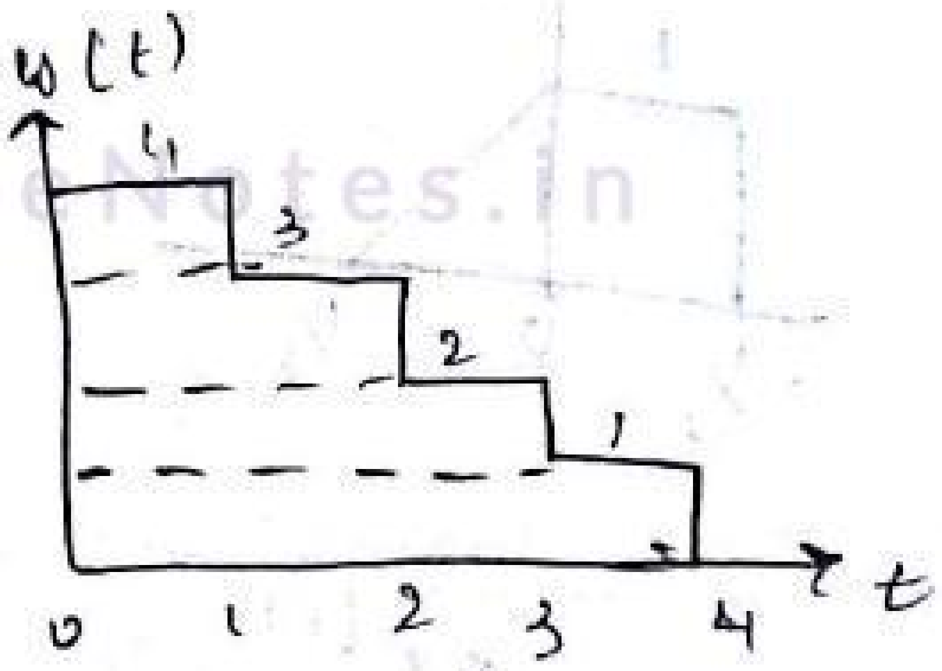
$m=2$

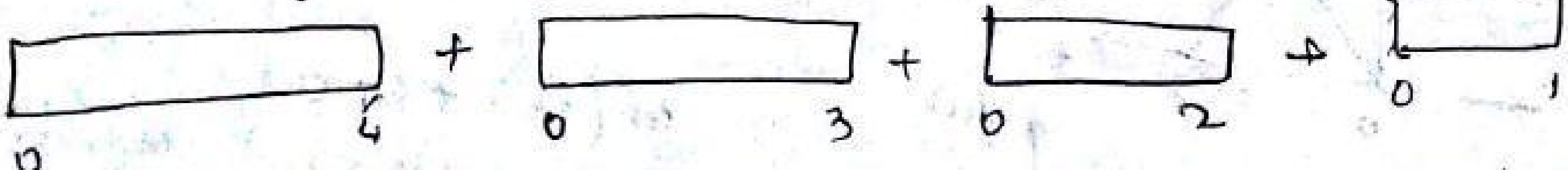
$c=-5$

$y(t) = x\left(\frac{2}{3}t-1\right)+x(t-2)+x(2t-5)$

(14)







$m(0)+c=-1$

$m(4)+c=1$

$m(0)+c=-1$

$m(3)+c=1$

$m(0)+c=-1$

$m(2)+c=1$

$m(0)+c=-1$

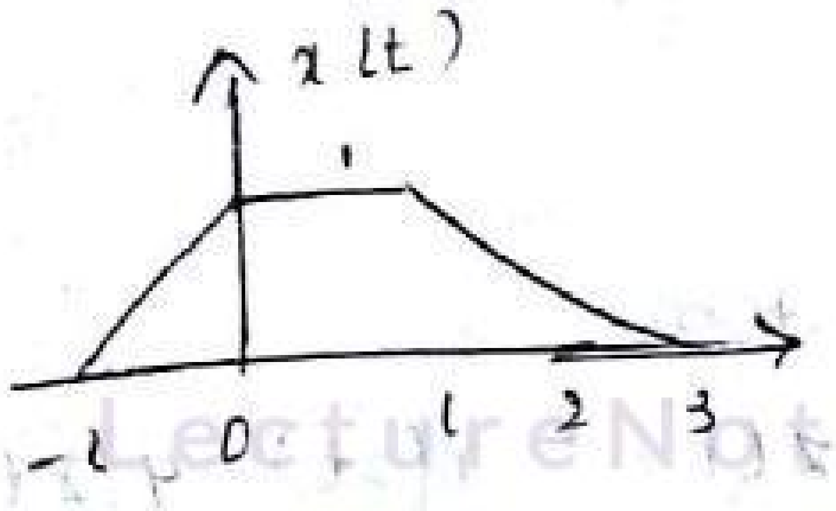
$m(1)+c=1$

$m = \frac{1}{2}$ $\quad$ $m = \frac{2}{3}$ $\quad$ $m = 1$ $\quad$ $m = 2$

$c = -1$ $\quad$ $c = -1$ $\quad$ $c = -1$ $\quad$ $c = -1$

$$y(t) = x\left(\tfrac{1}{2}t - 1\right) + x\left(\tfrac{2}{3}t - 1\right) + x(t-1) + x(2t-1)$$

(15)

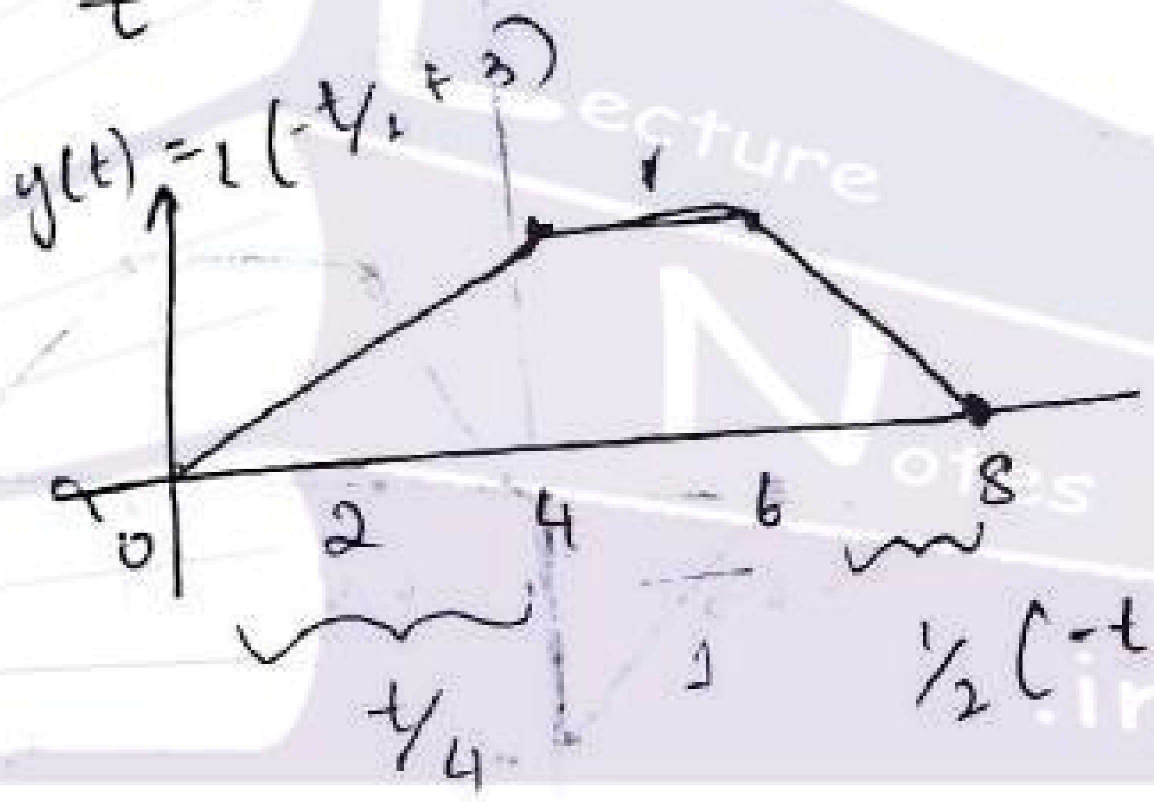


find energy of $x\left(-\frac{t}{2} + 3\right)$

$-\frac{t}{2} + 3 : -1, 0, 1, 3$

-3 $\quad$ $-\frac{t}{2} : -4, -3, -2, 0$

$\times -2$ $\quad$ $t = 8, 6, 4, 0$



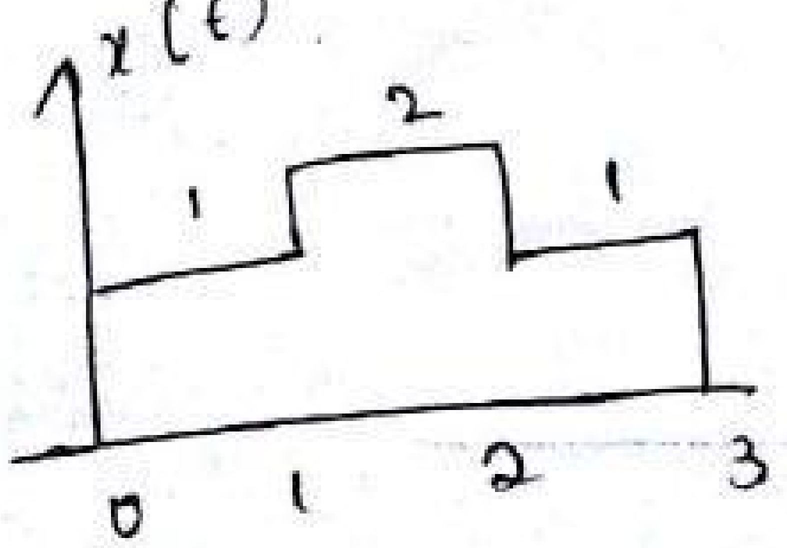
$\frac{1}{2}(-t+8) = -\frac{t}{2} + 4$

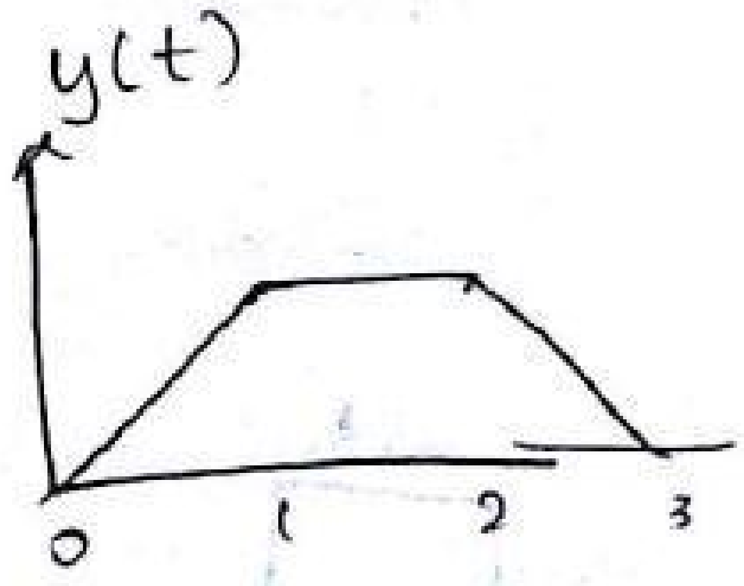
$$E = \int_{-\infty}^{\infty} y^2(t)\,dt$$

$$= \int_0^4 \left(\frac{t}{4}\right)^2 dt + \int_4^6 1^2\,dt + \int_6^8 \left(-\frac{t}{2} + 4\right)^2 dt$$

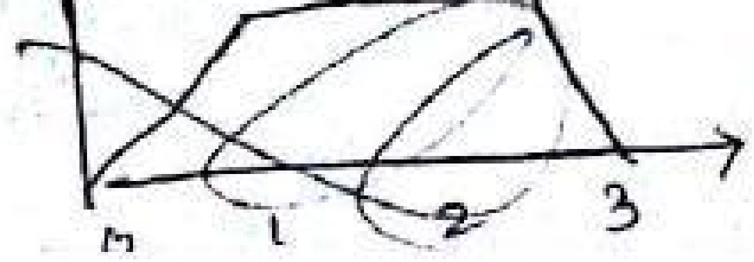
$\boxed{E = 4J}$

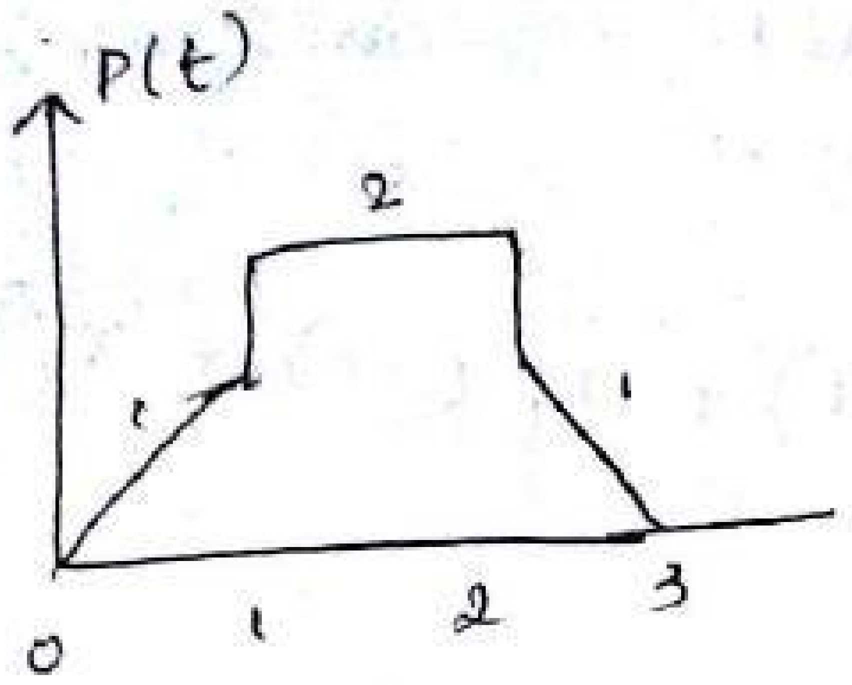
(16)





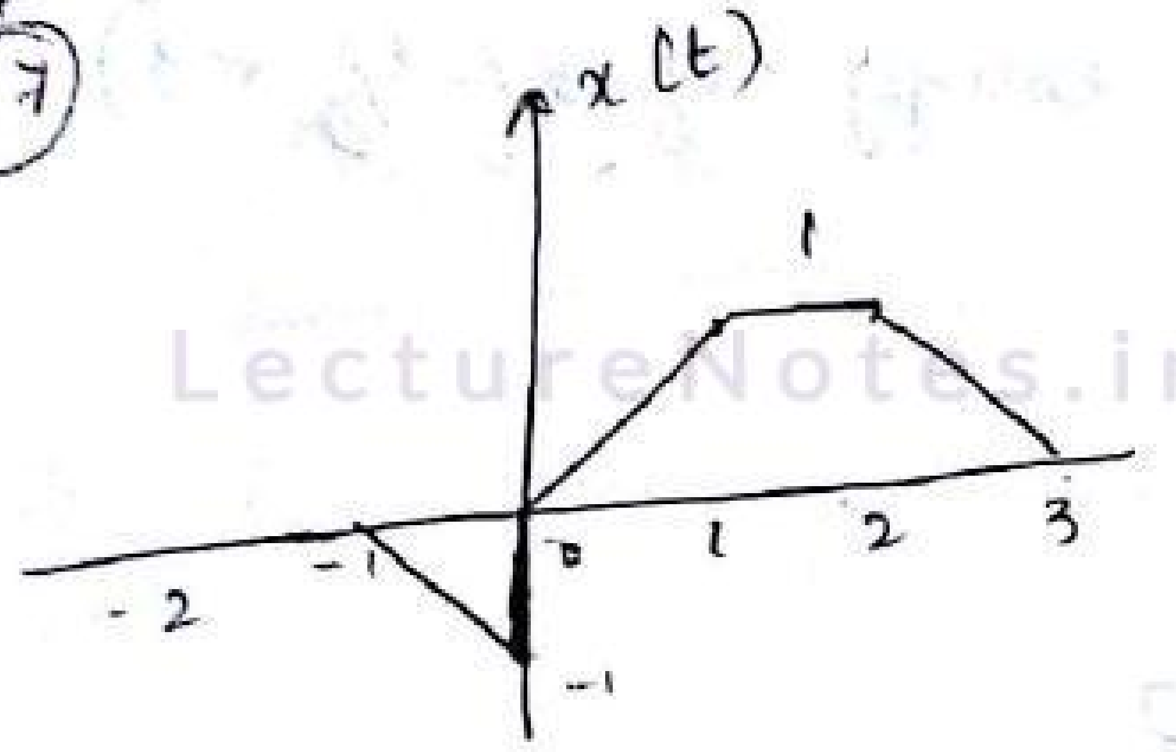
find $p(t) = x(t) \cdot y(t)$





5M

1.7



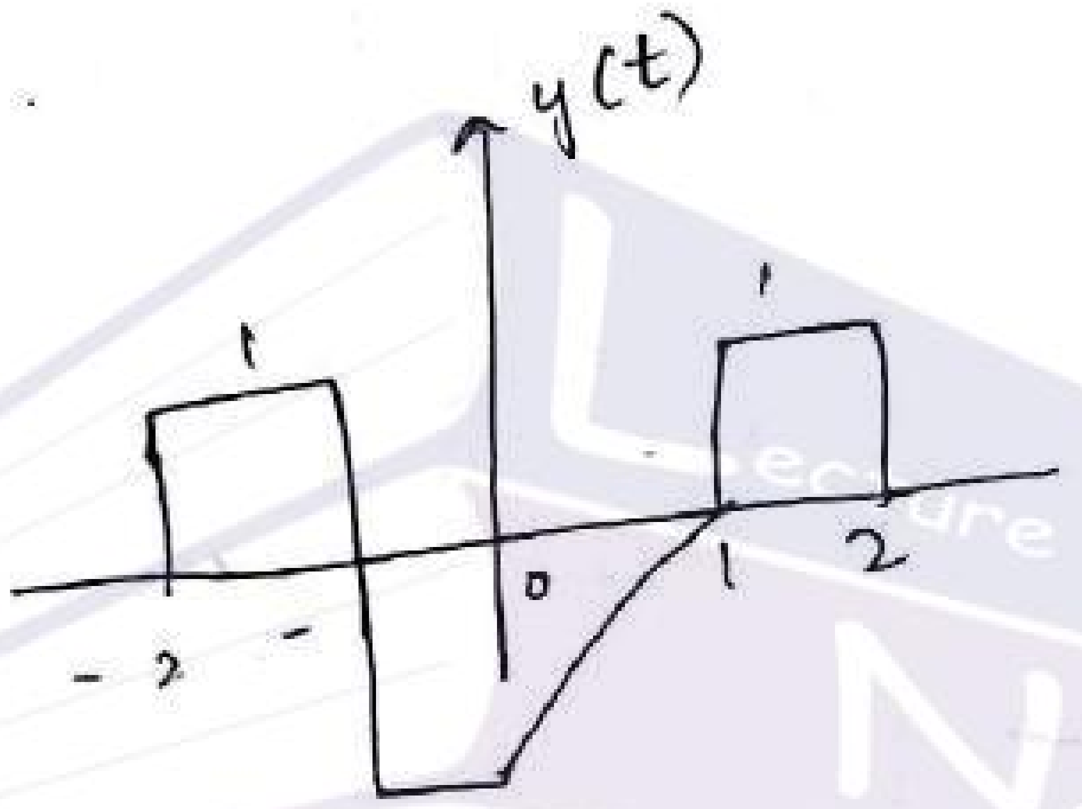
Sketch

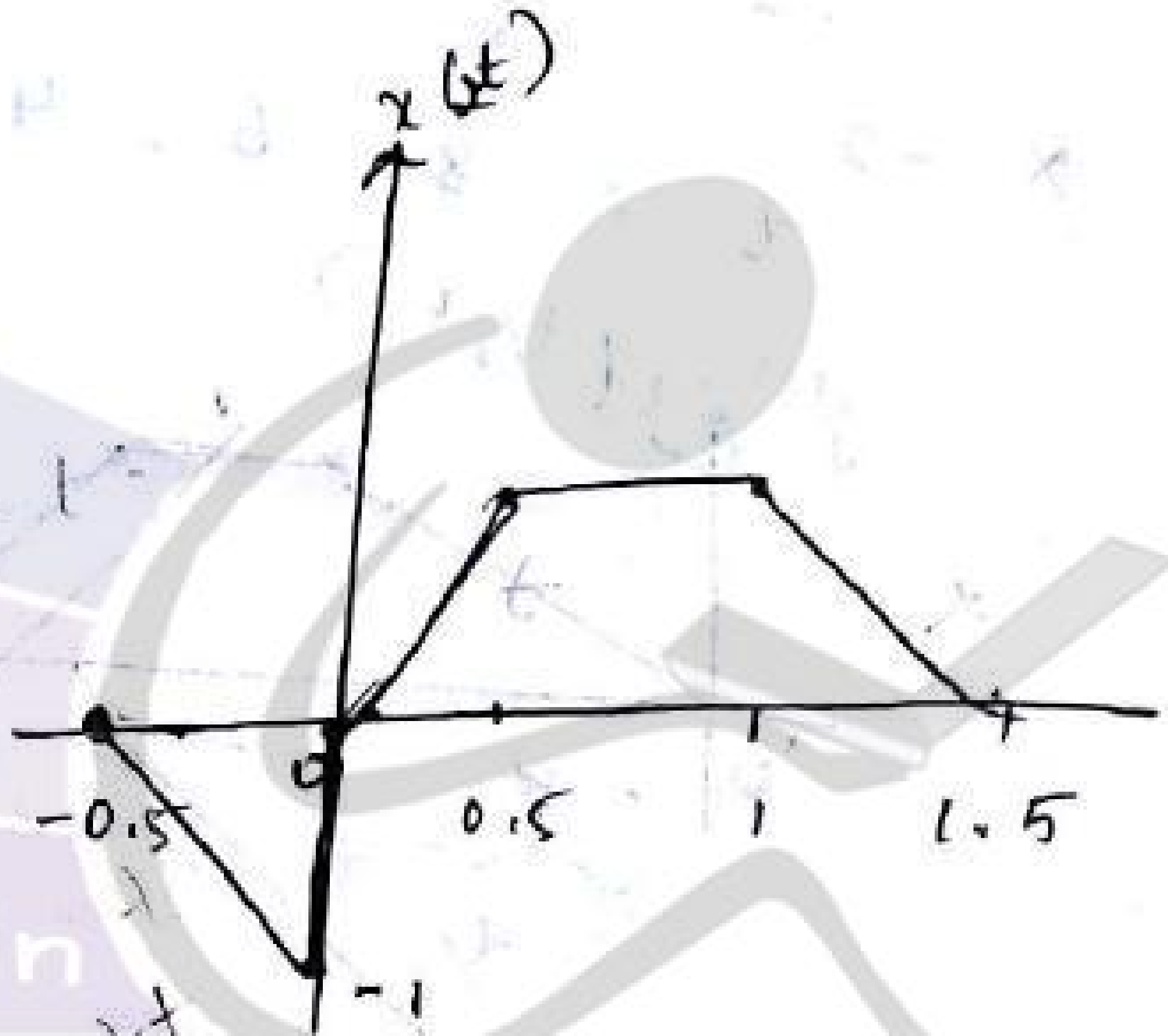
$z(t) = x(2t) \cdot y(2t+1)$

$2t : -1, 0, 1, 2, 3$

$\div 2$

$t : -0.5, 0, 0.5, 1, 1.5$





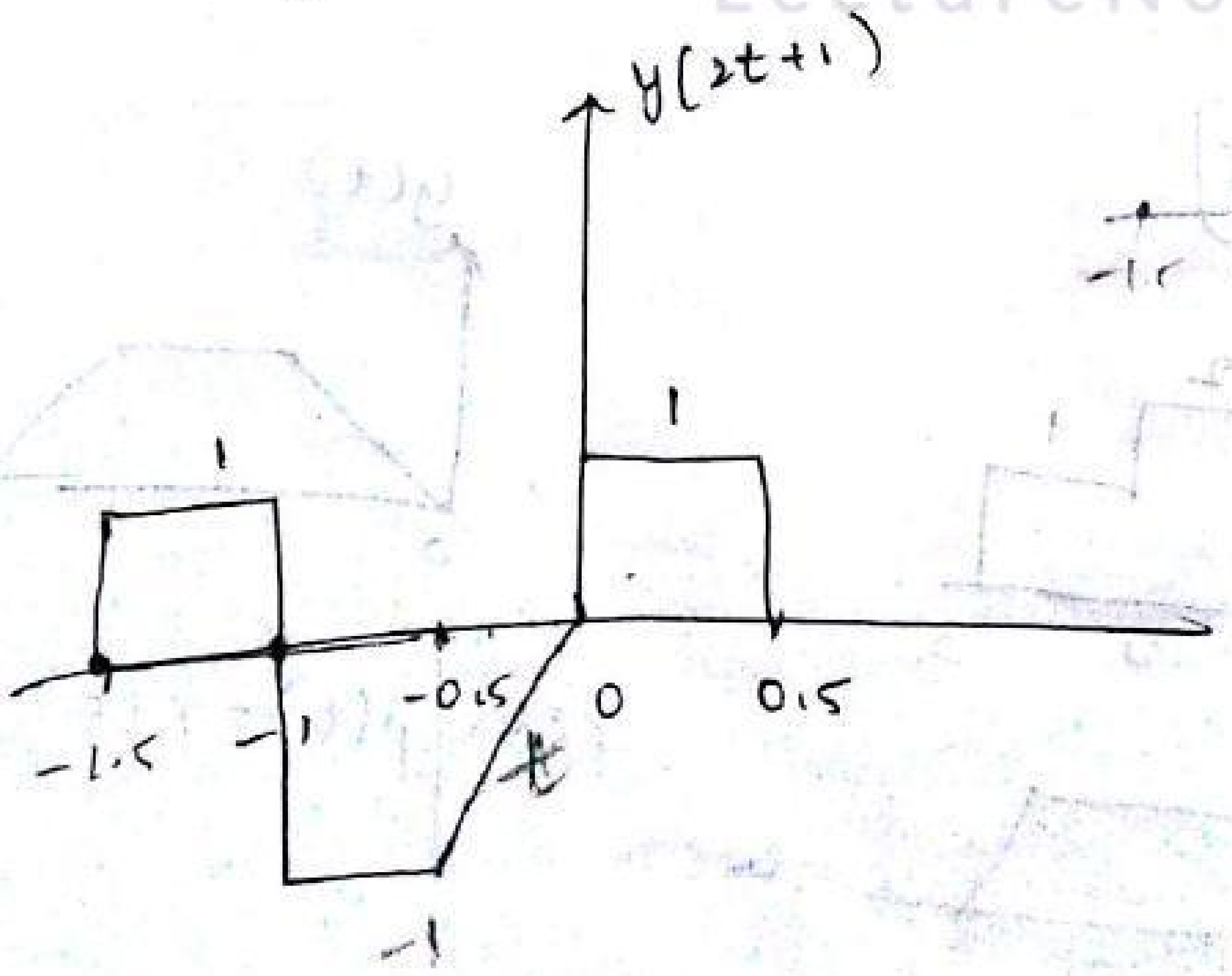
$2t+1 : -2, -1, 0, 1, 2$

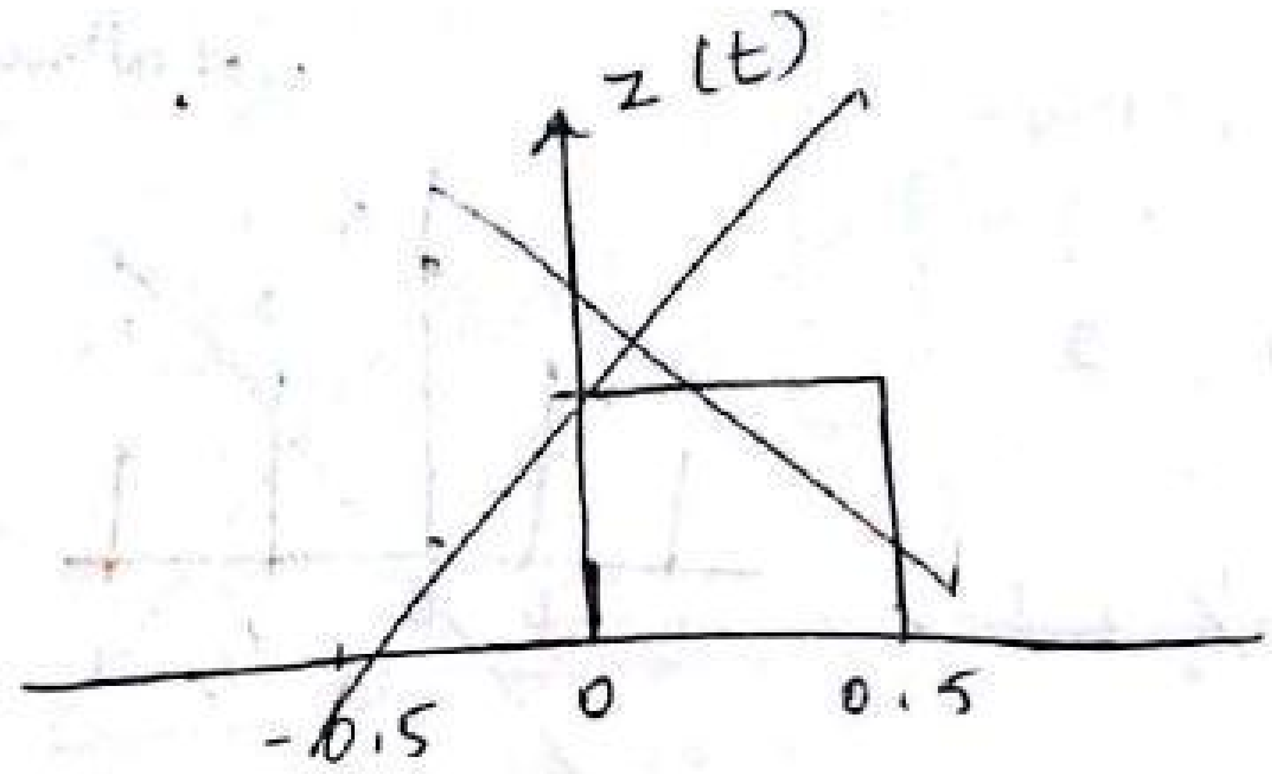
-1

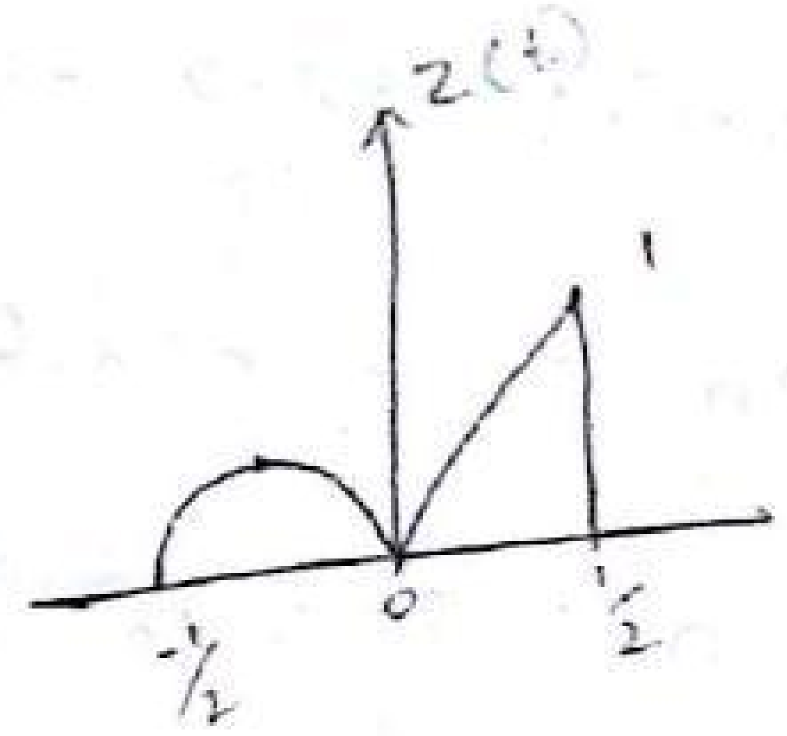
$2t : -3, -2, -1, 0, 1$

$\div 2$

$t = -1.5, -1, -0.5, 0, 0.5$

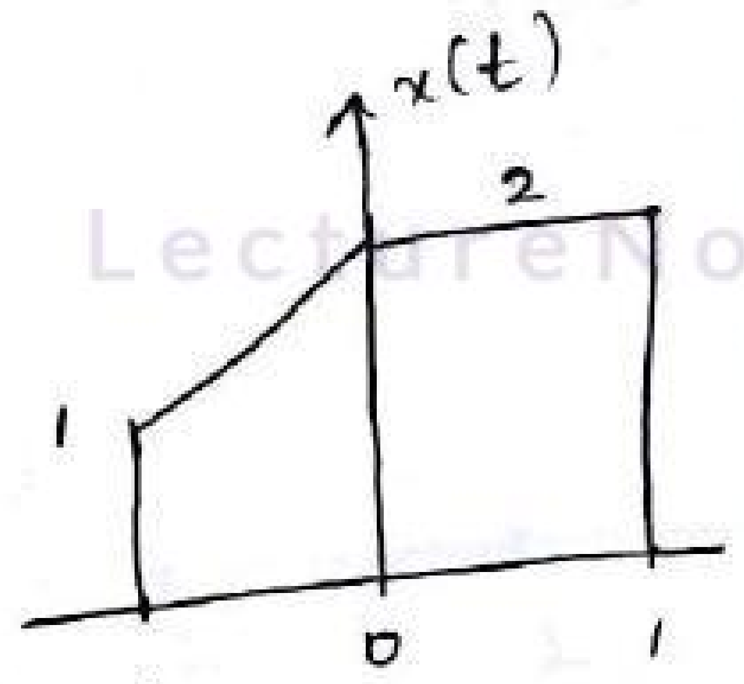






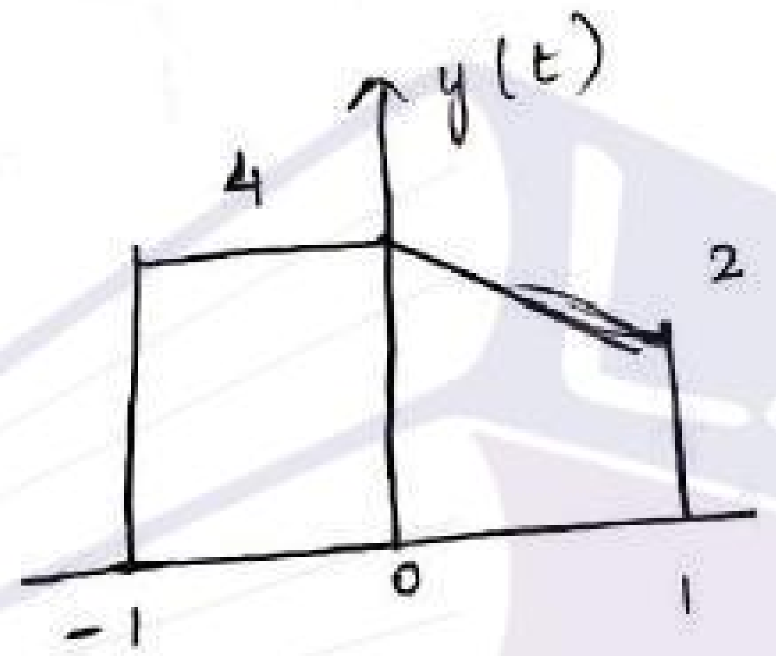
6M

(18)



Sketch

$$z(t) = x(1-t)\, y(t/2)$$



$1-t$: $-1, 0, 1$

-1

$-t$: $-2, -1, 0$

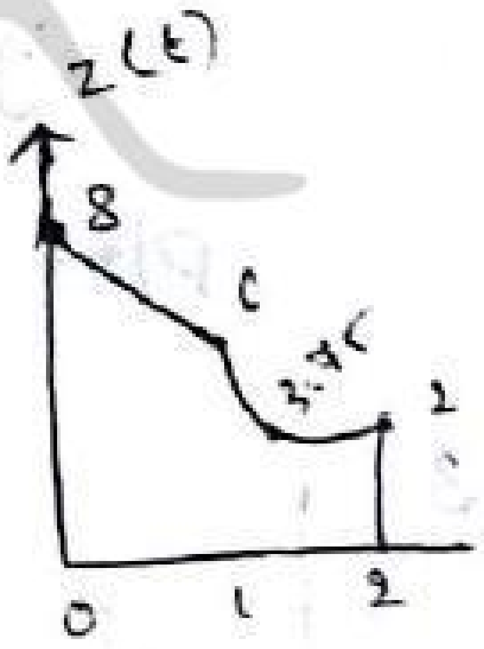
$\times -1$

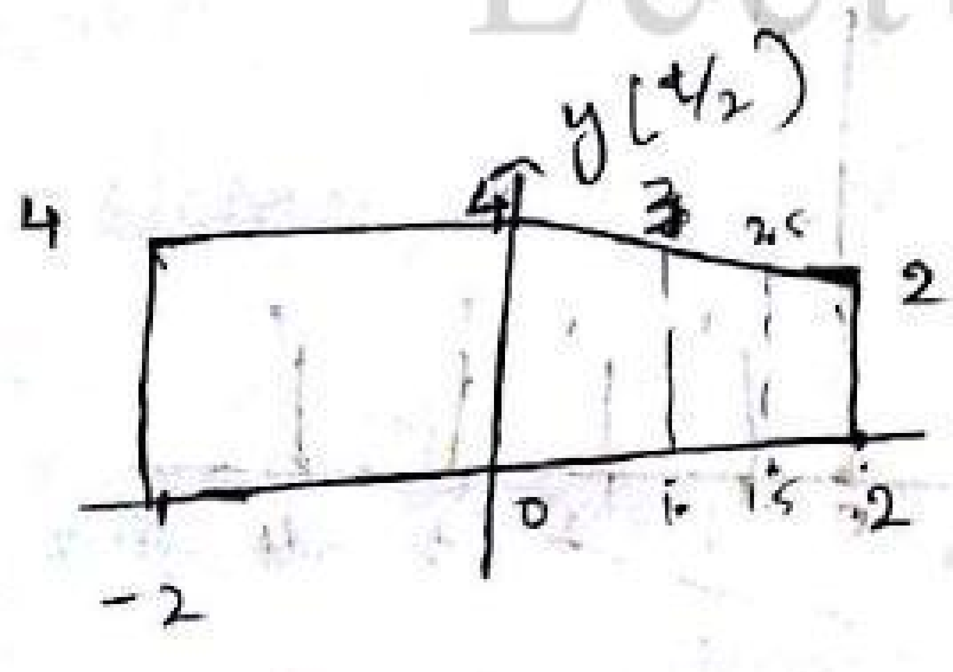
t : $2, 1, 0$

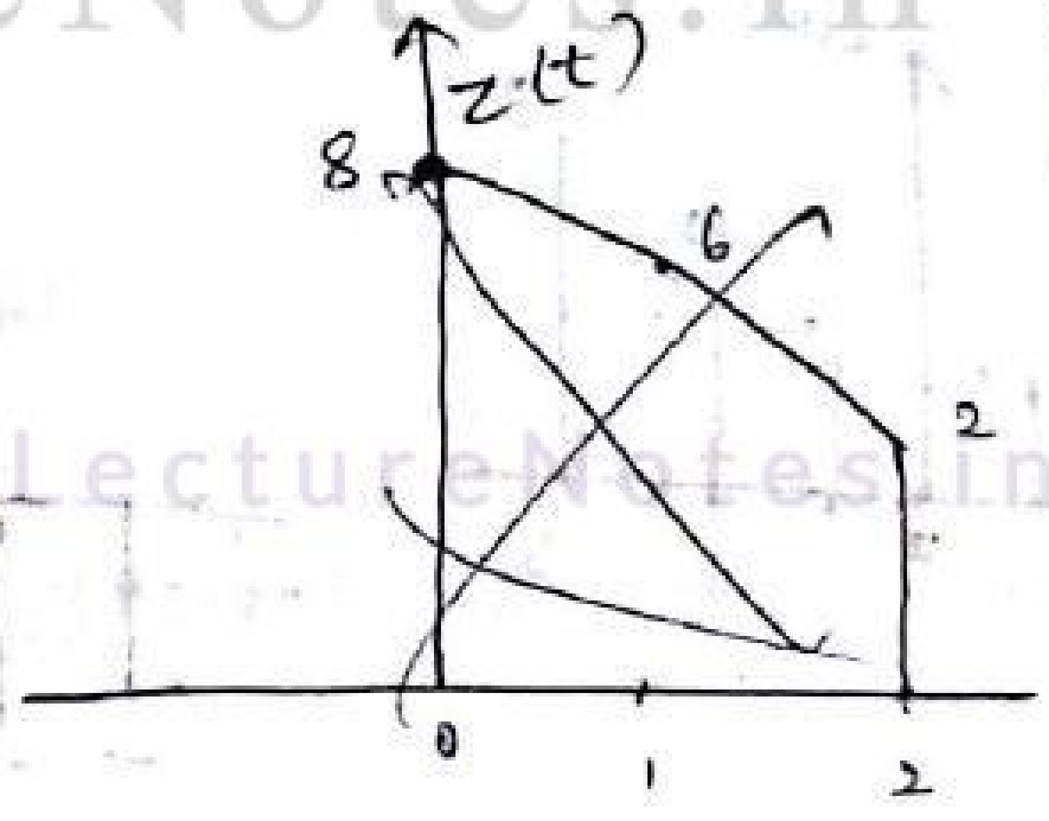
$t/2$: $-1, 0, 1$

$\times 2$

$t = -2, 0, 2$

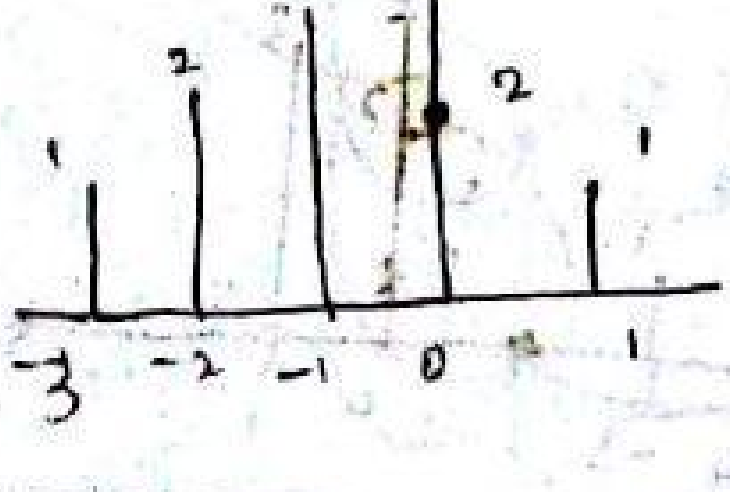






4M

(19)



find energy of $x(2n-1)$

not allowed

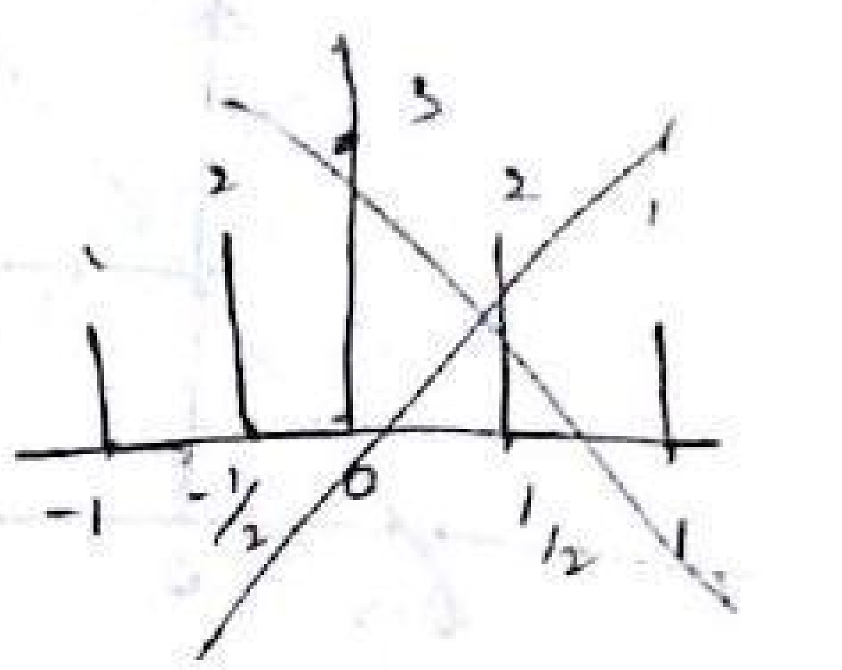
$(2n-1)$: $-3, -2, -1, 0, 1$

$+1$

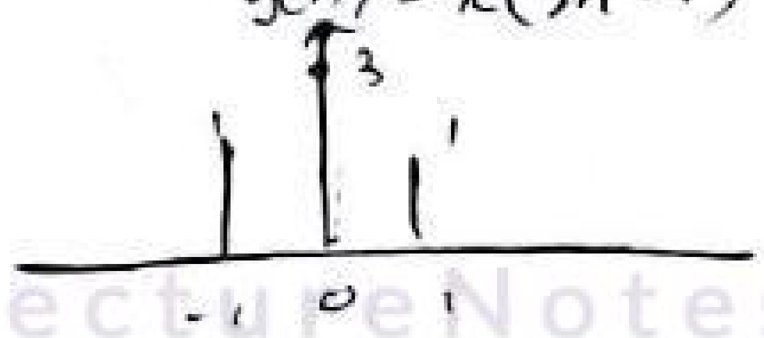
$2n$: $-2, -1, 0, 1, 2$

$\div 2$

n : $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1$



$y(n) = x(2n-1)$



$$E = \sum_{n=-\infty}^{\infty} y^2(n)$$

$$= \sum_{n=-1}^{1} y^2(n)$$

$$= y^2(-1) + y^2(0) + y^2(1)$$

$$= 1^2 + 3^2 + 1^2 = \underline{\underline{11\,J}}$$

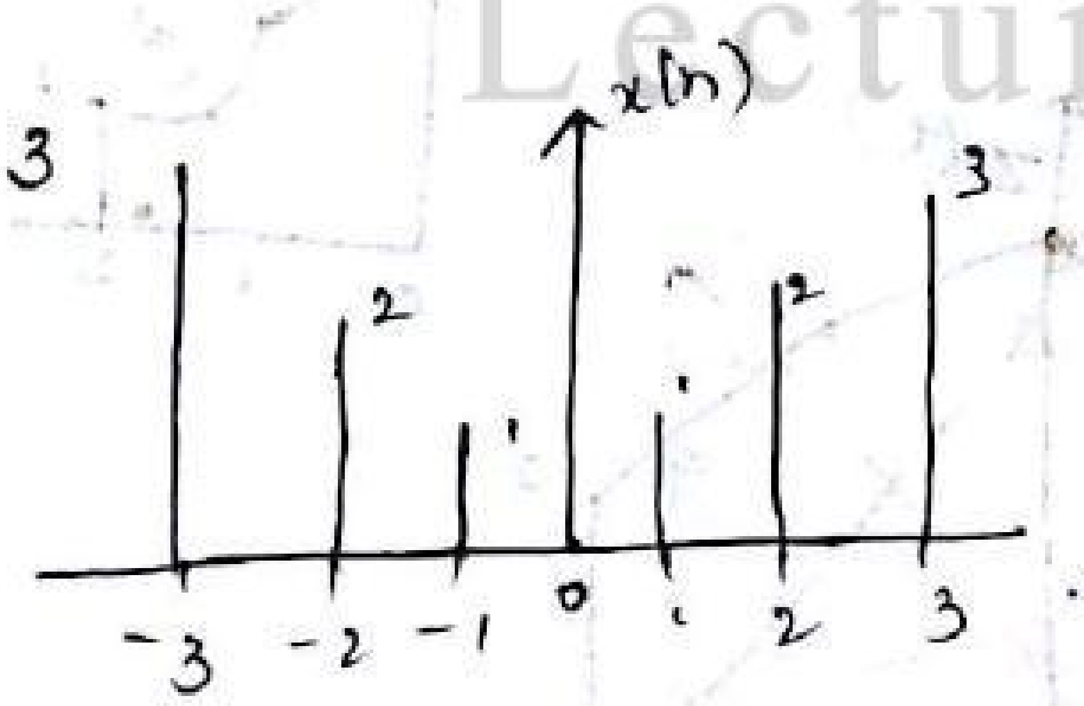
4M

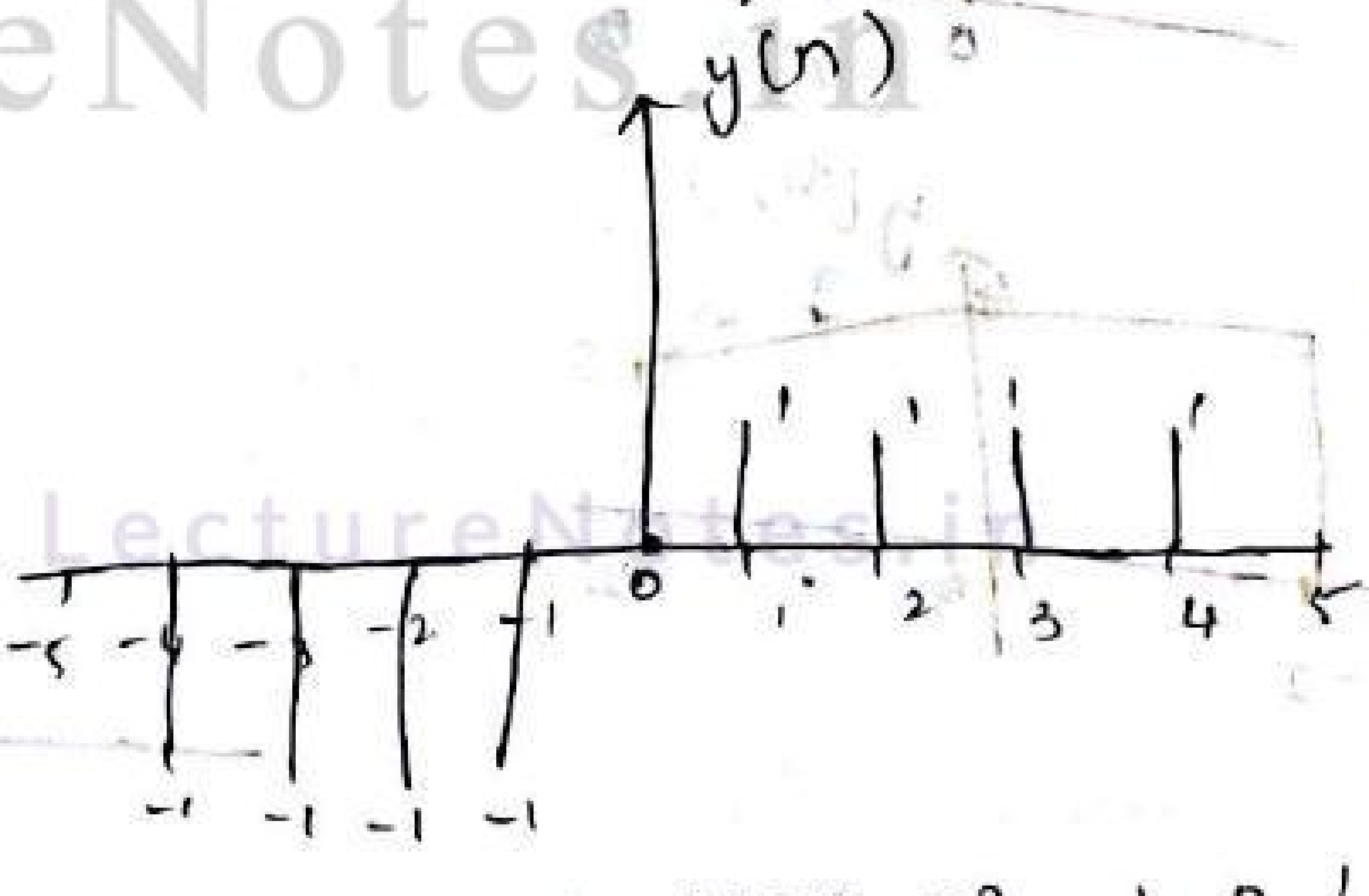
(20)

$x(n) = \{3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3\}$

$y(n) = \{-1, -1, -1, -1, \underset{\uparrow}{0}, 1, 1, 1, 1\}$

Plot $p(n) = x(n-2) + y(n+2)$





$n-2$; $-3, -2, -1, 0, 1, 2, 3$

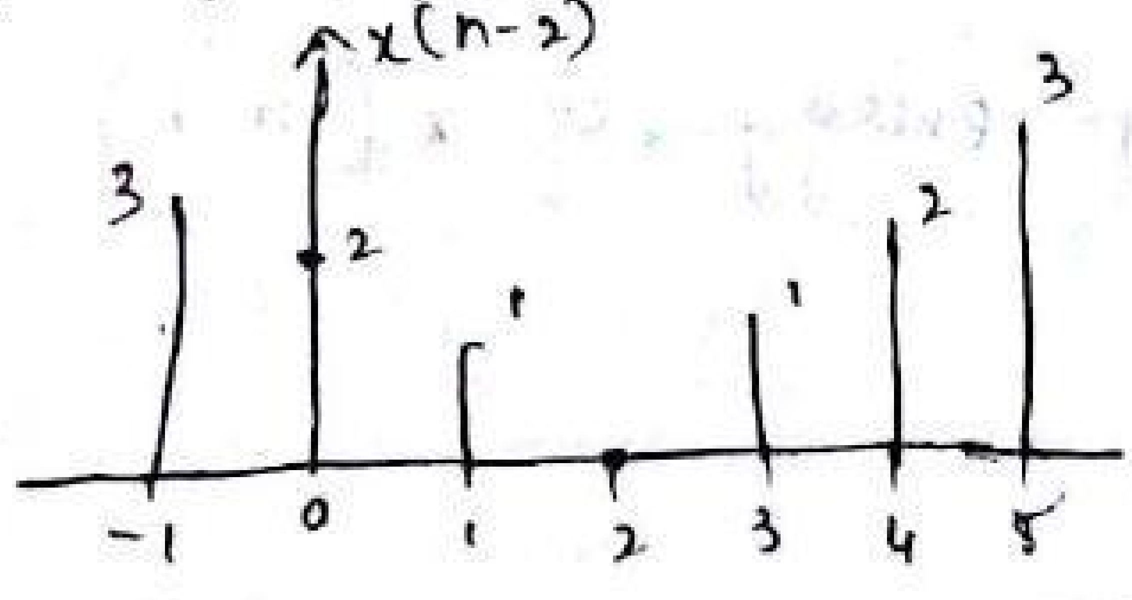
$+2$

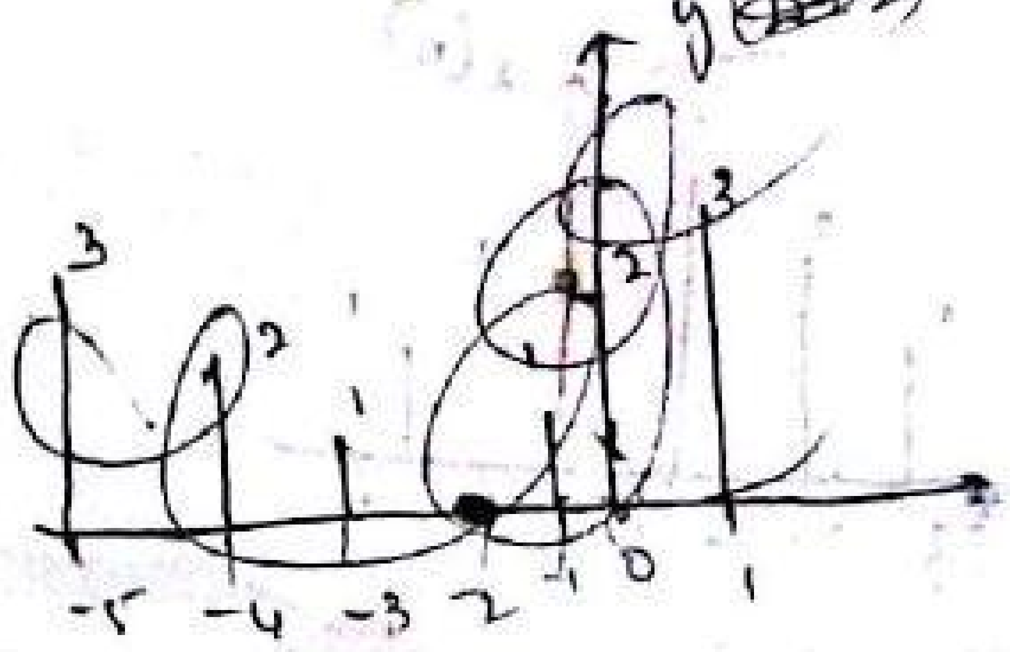
n : $-1, 0, 1, 2, 3, 4, 5$

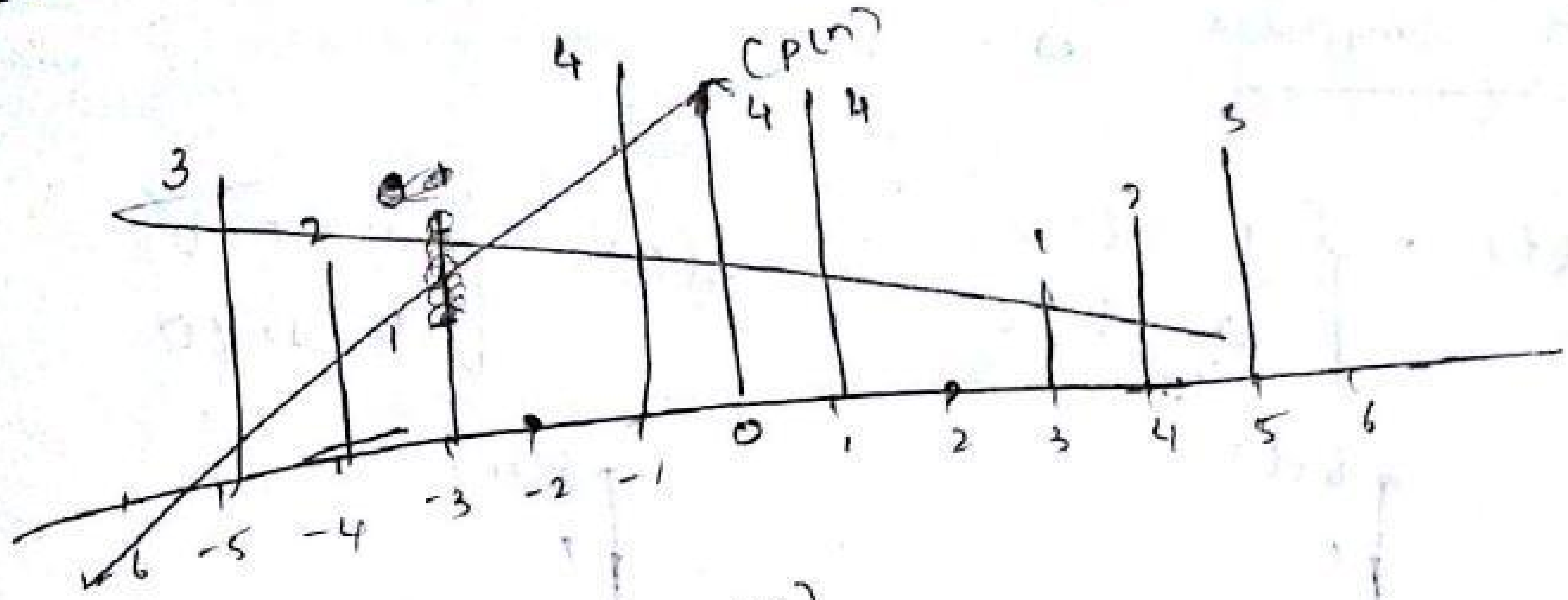
$n+2$; $-4, -3, -2, -1, 0, 1, 2, 3$

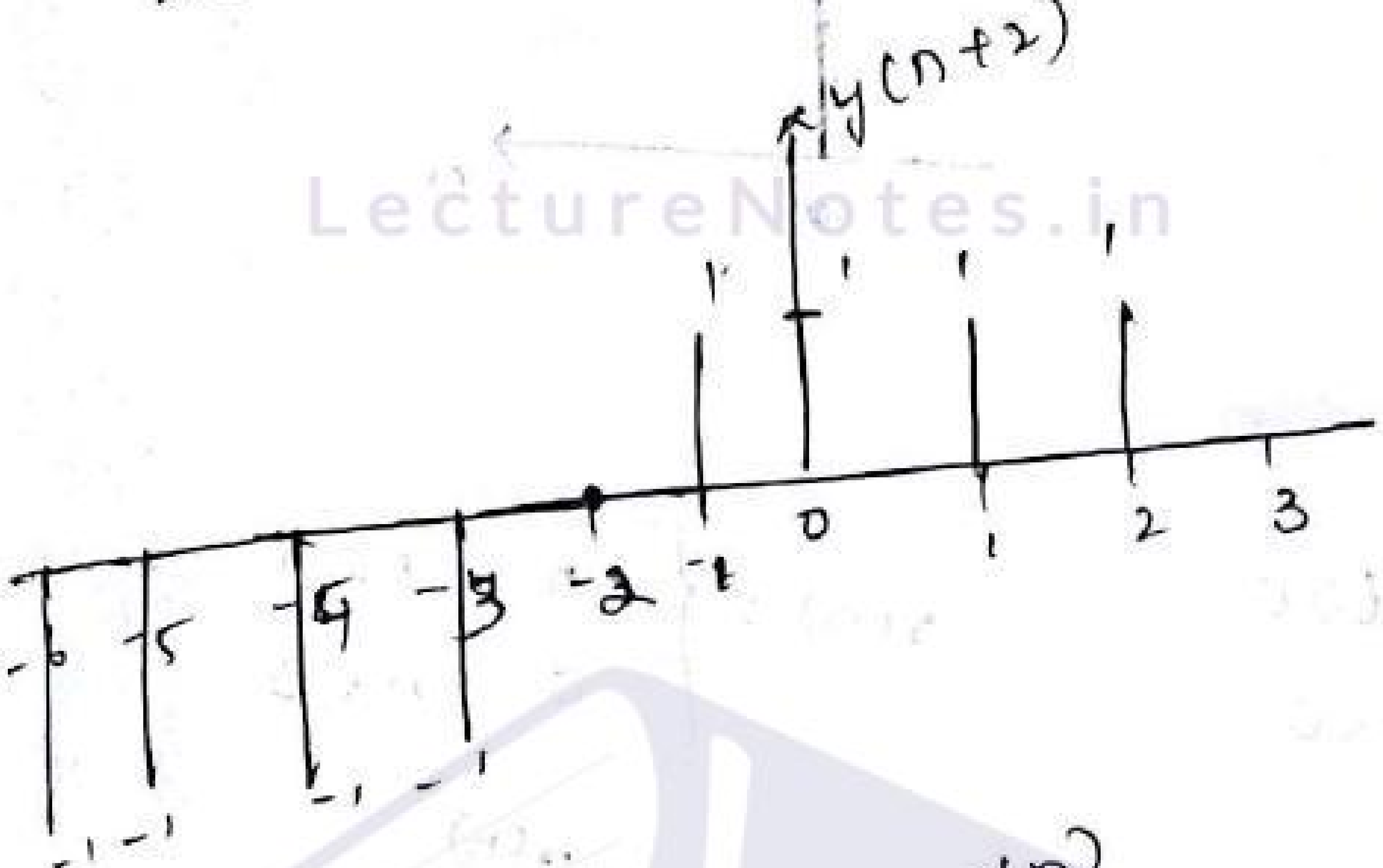
-2

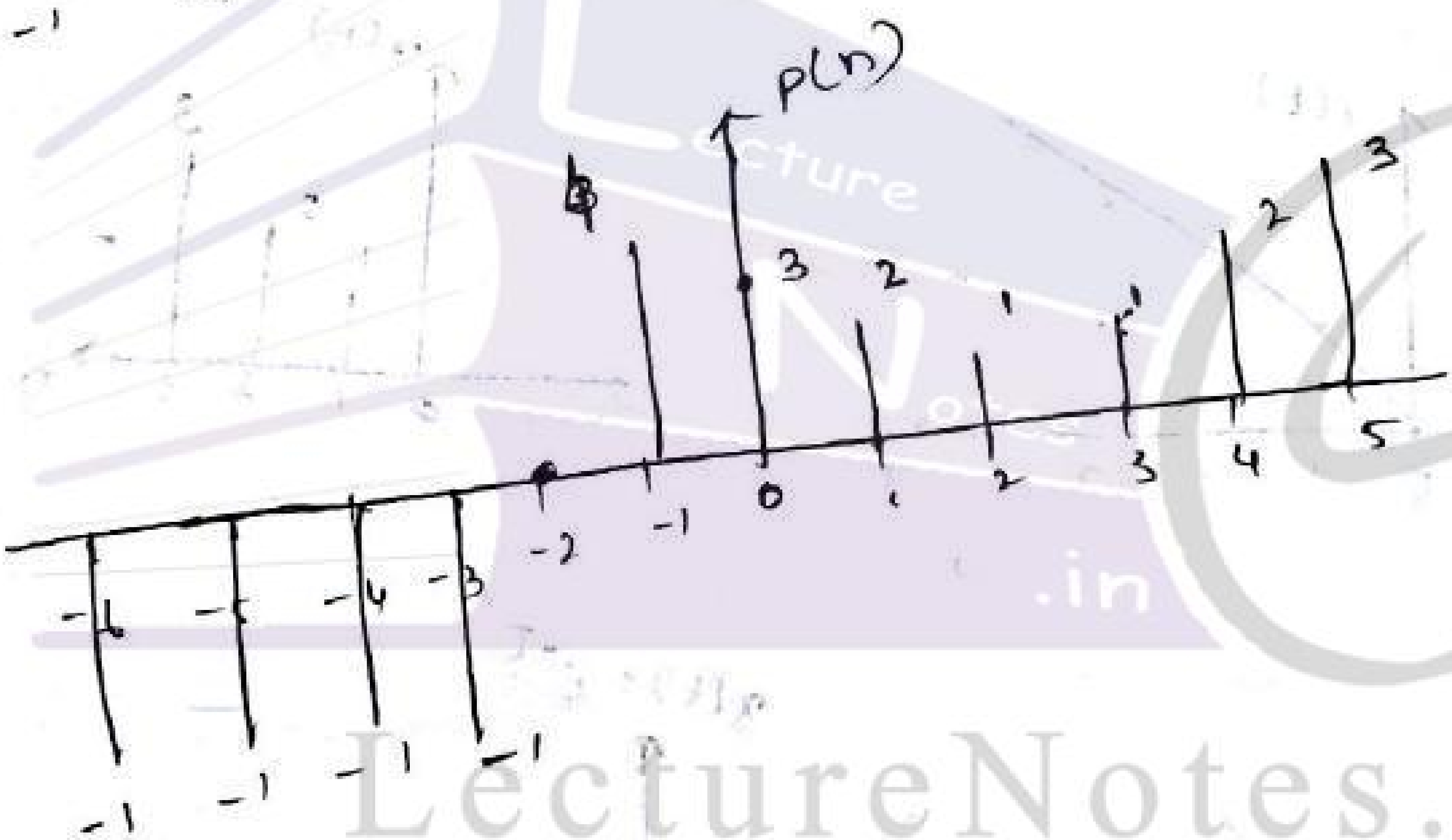
n : $-6, -5, -4, -3, -2, -1, 0, 1$









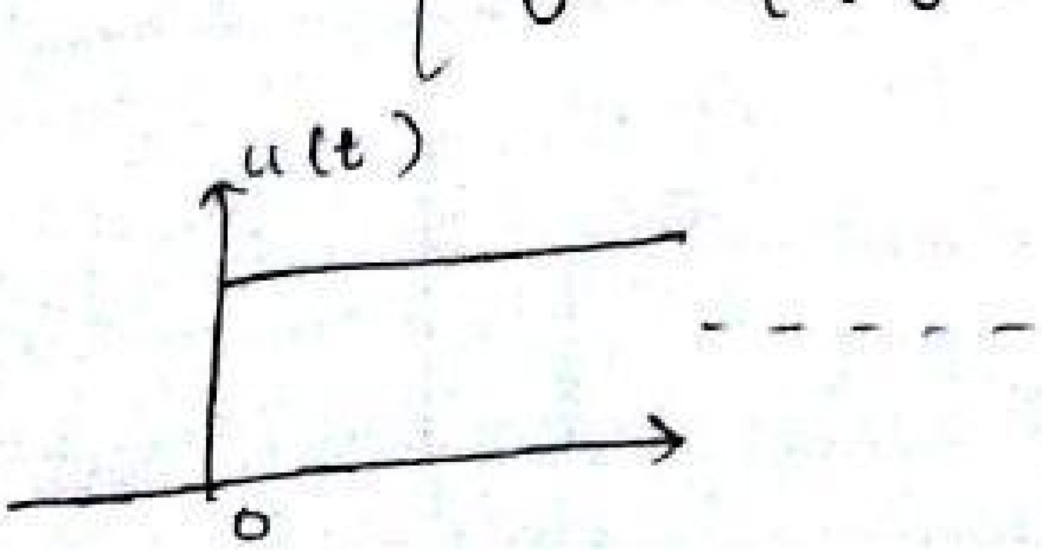


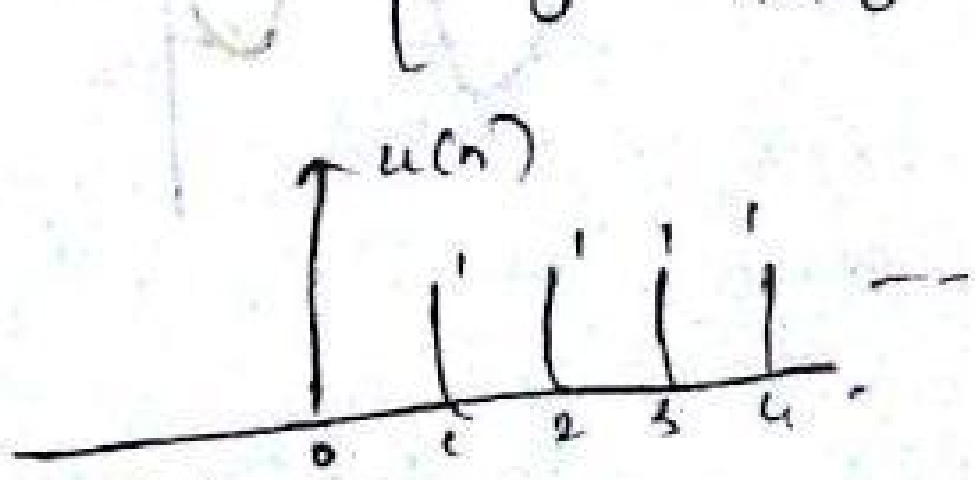
Elementary signal

They basic building blocks of any complex signals. They are the fundamental signals. Different type of elementary signals are

① unit step 'u'

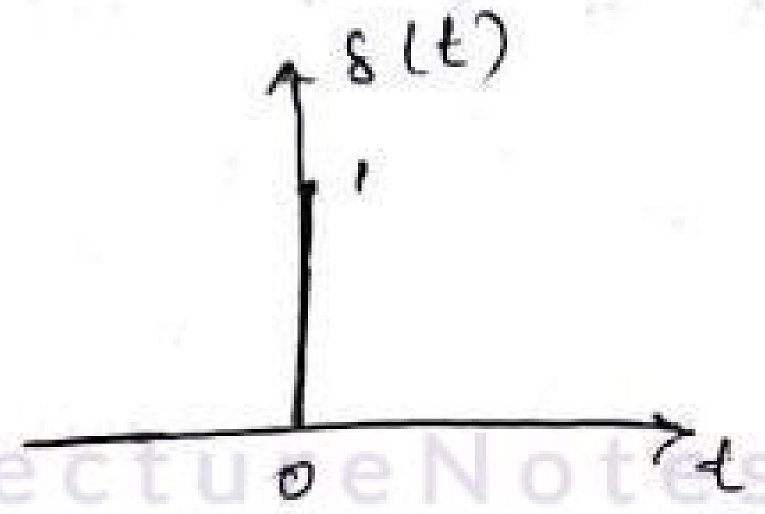
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

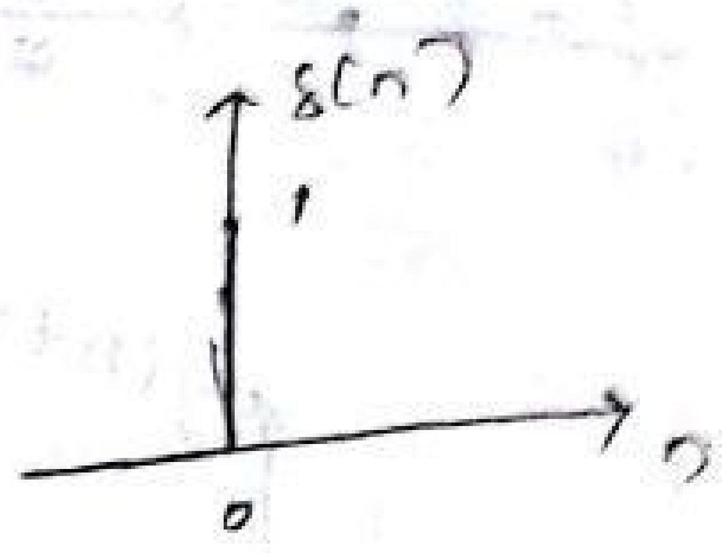




(2) unit Impulse 'δ'

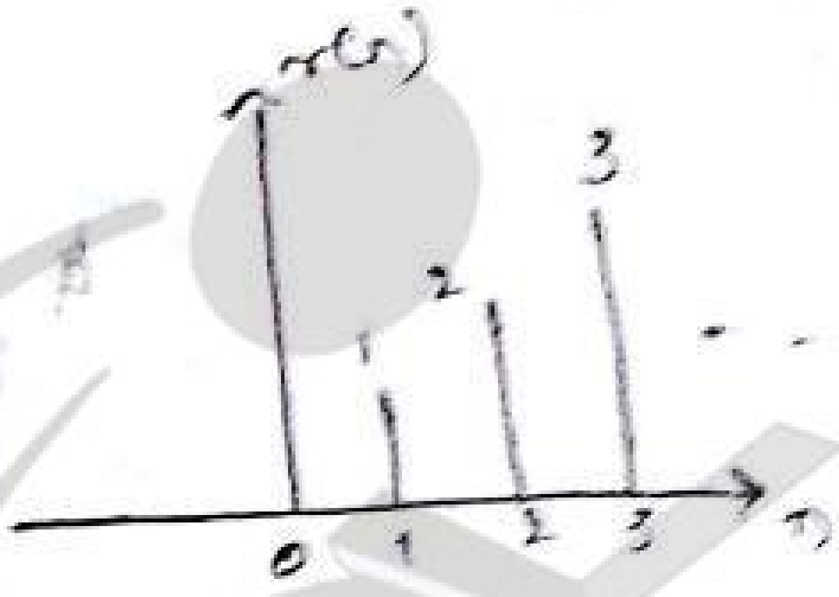
$$\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases} \qquad \delta(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$





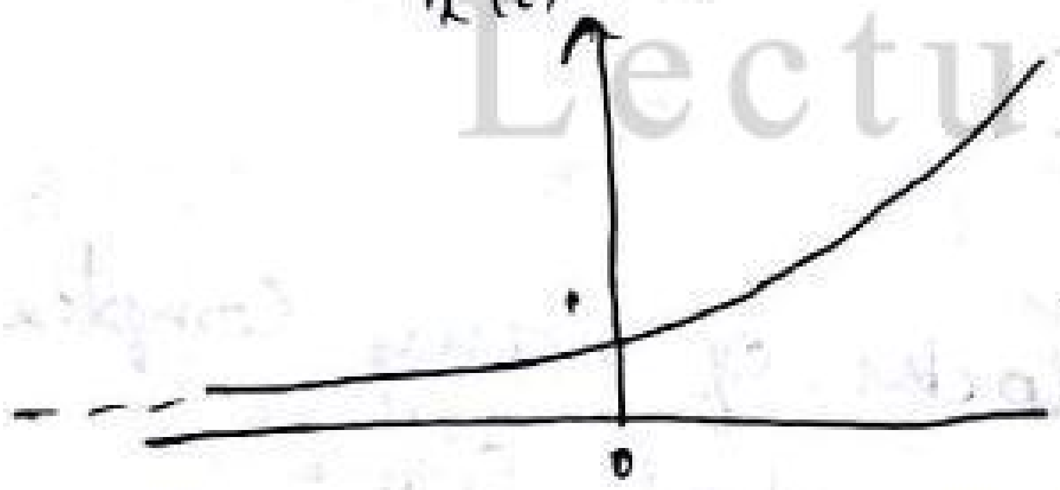
(3) unit ramp 'r'

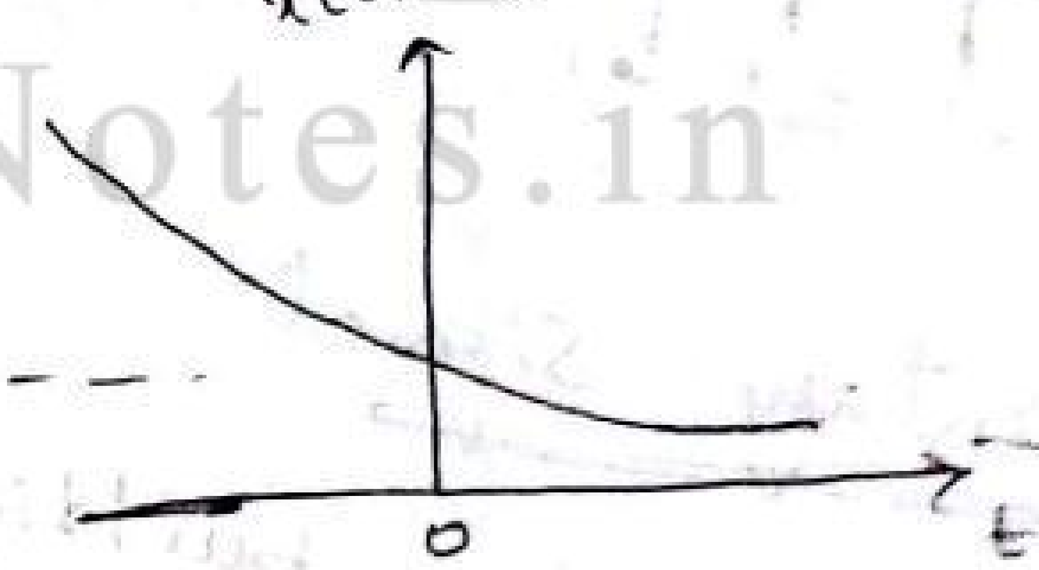
$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad r(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



(4) Exponential

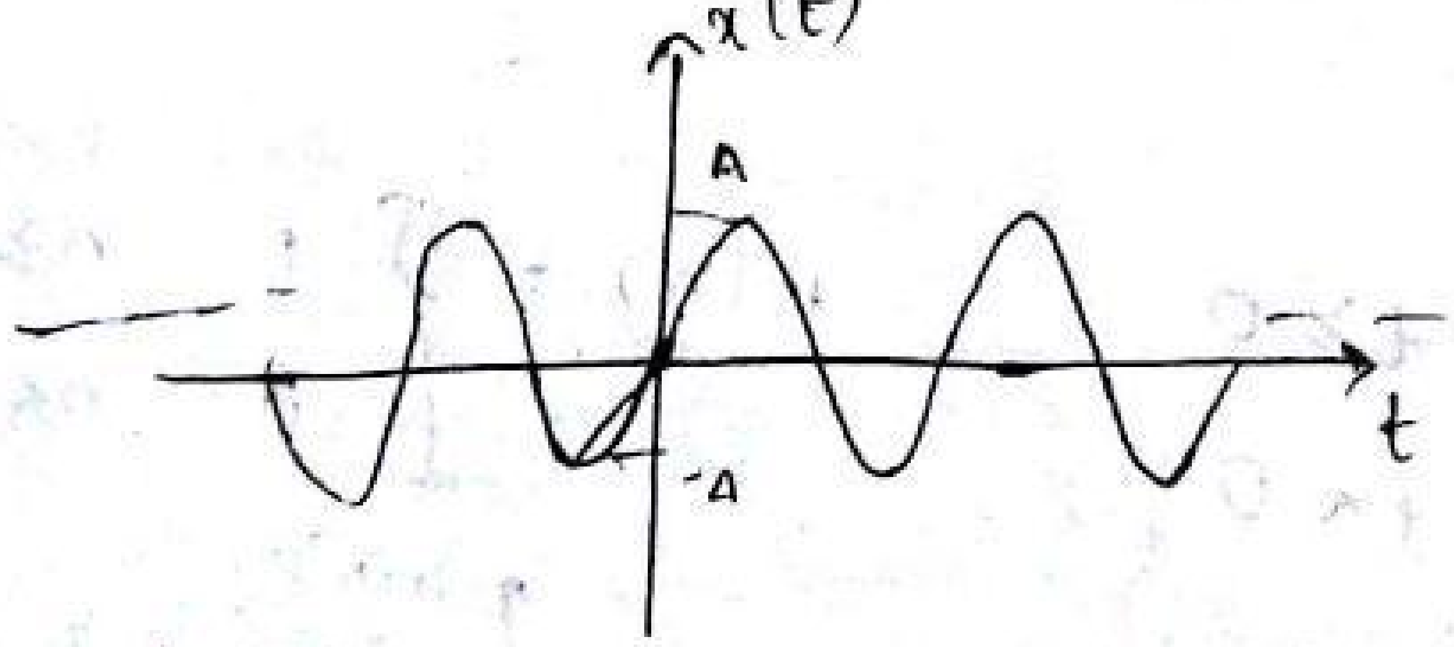
$x(t) = e^{t}$ $\qquad$ $x(t) = e^{-t}$





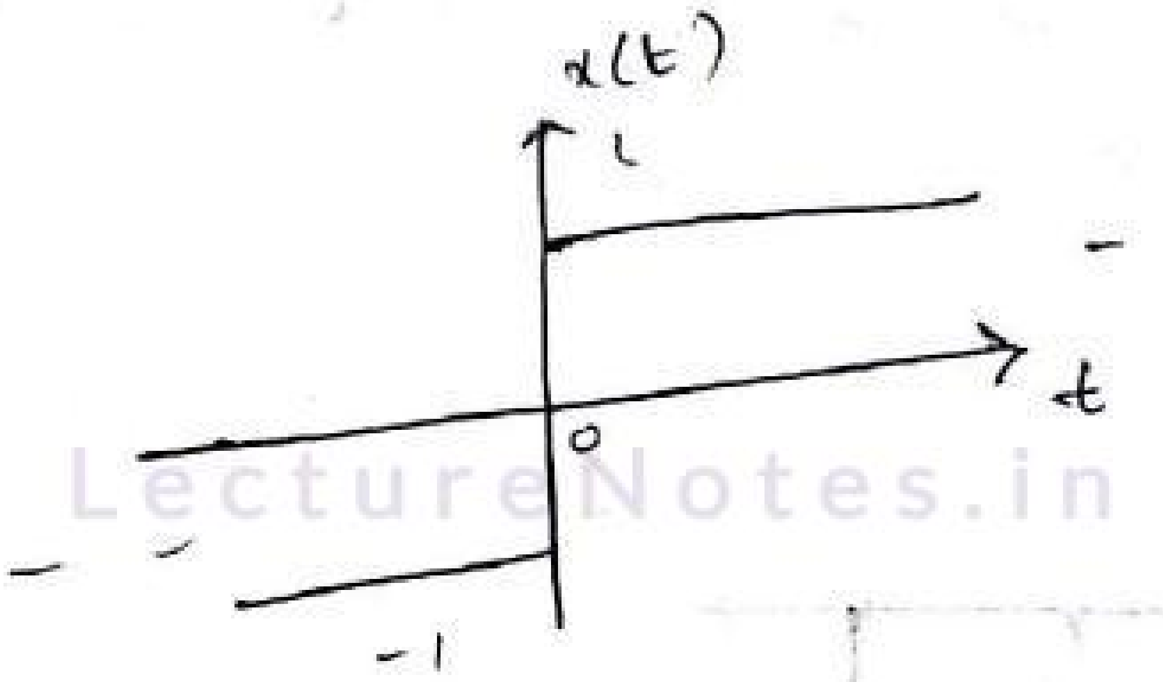
(5) Sinusoidal

$x(t) = A\sin\omega t$



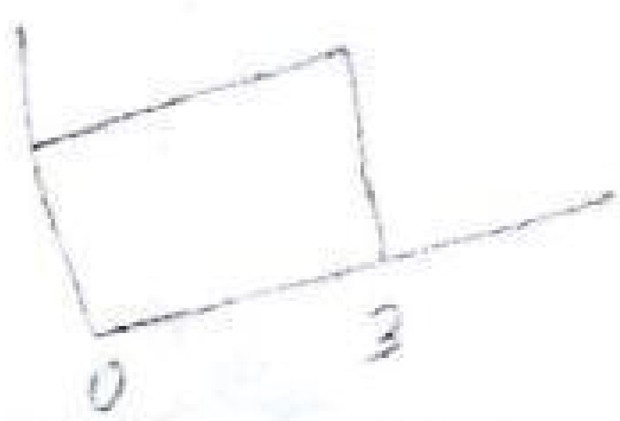
(6) Signum

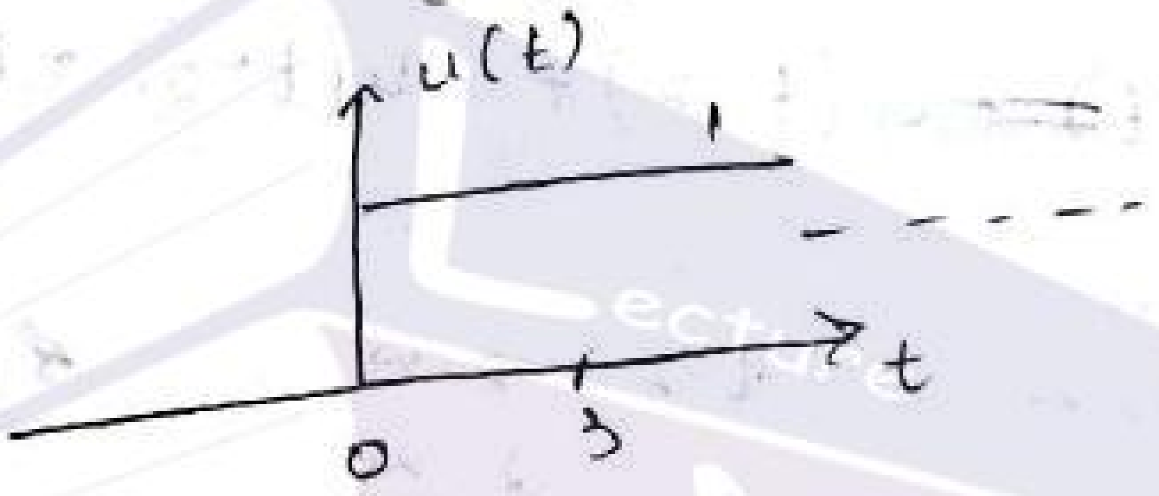
$$x(t) = \begin{cases} 1 & t \geq 0 \\ -1 & t \leq 0 \end{cases}$$



Sketch the following signal

(1) $x(t) = u(t) - u(t-3)$

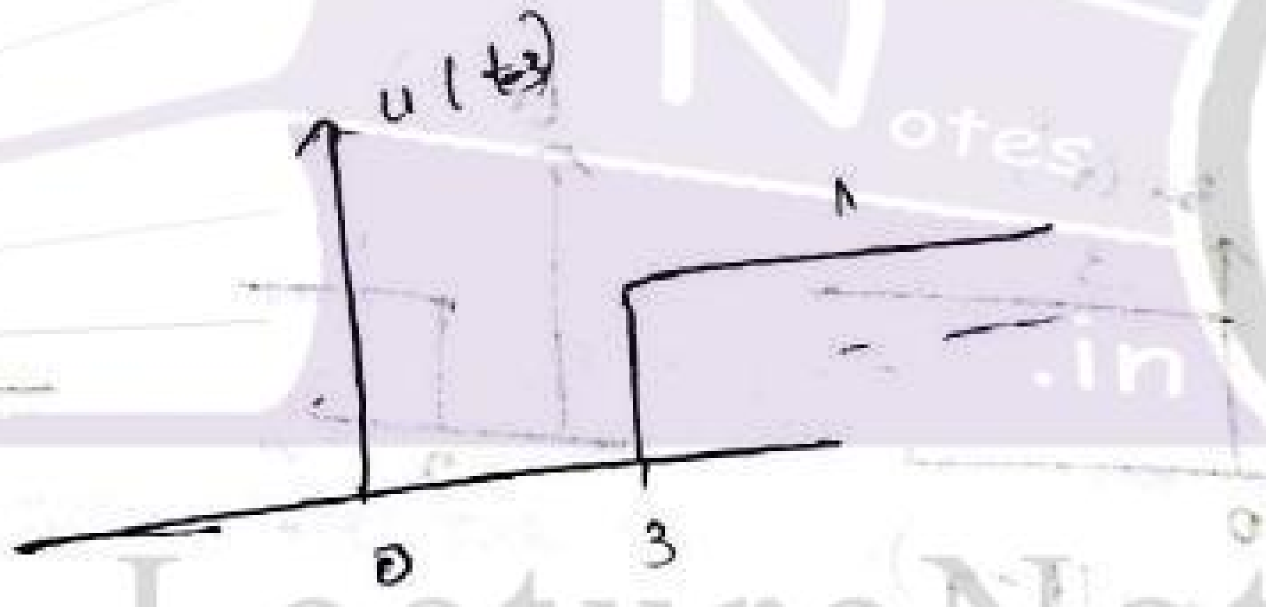


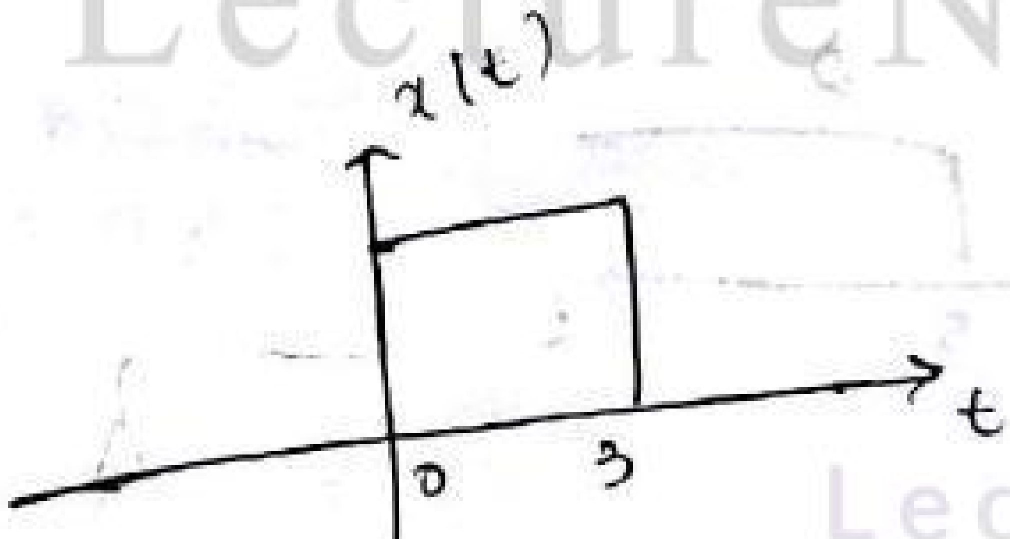


$t-3 : 0, \infty$

$+3$

$t : 3, \infty$

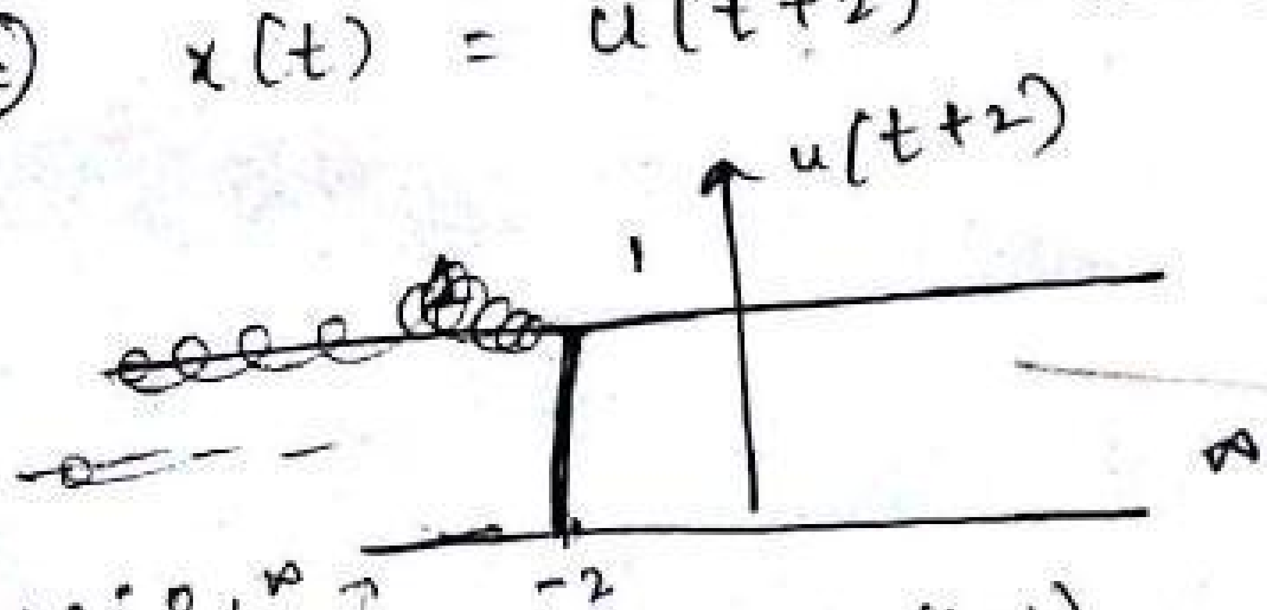


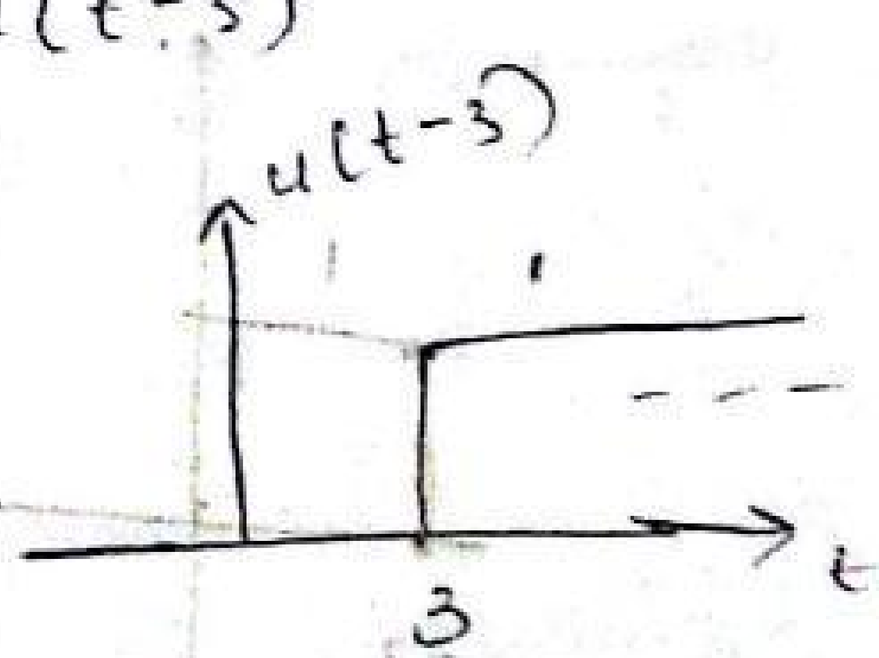


$t+2 : 0, \infty$

$t : -2, \infty$

(2) $x(t) = u(t+2) - 2u(t-1) + u(t-3)$



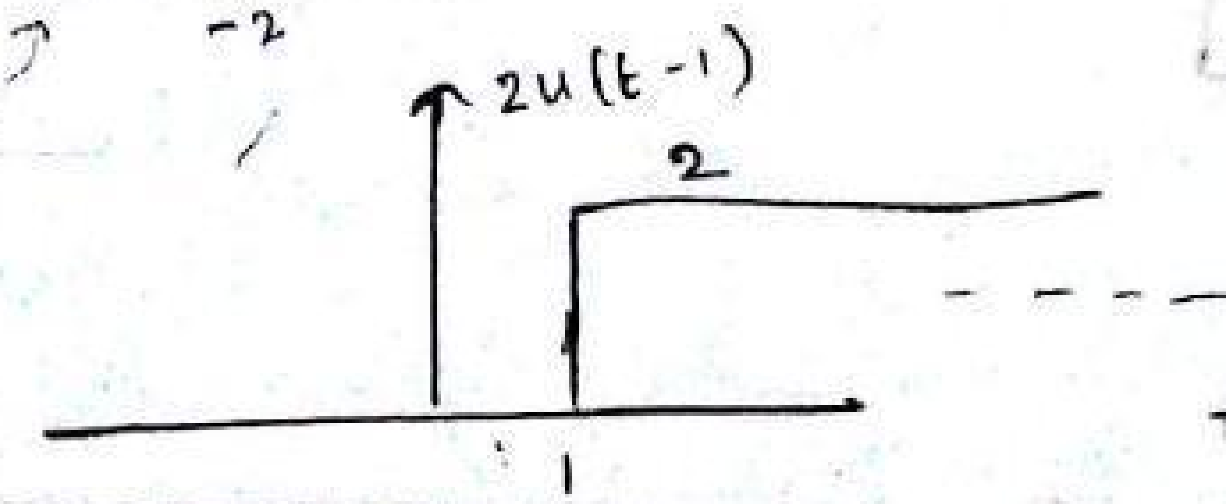


$t+2 ; 0, \infty$

$t : -2, \infty$

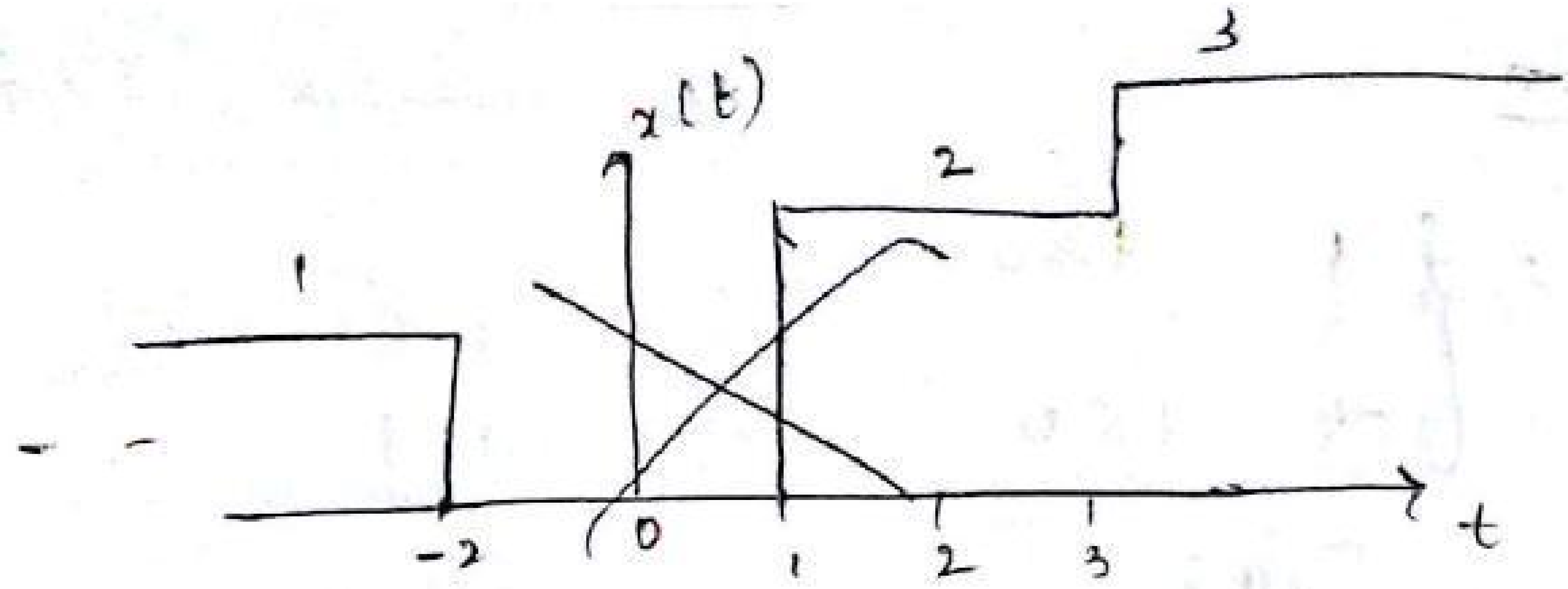
$t-3 ; 0, \infty$

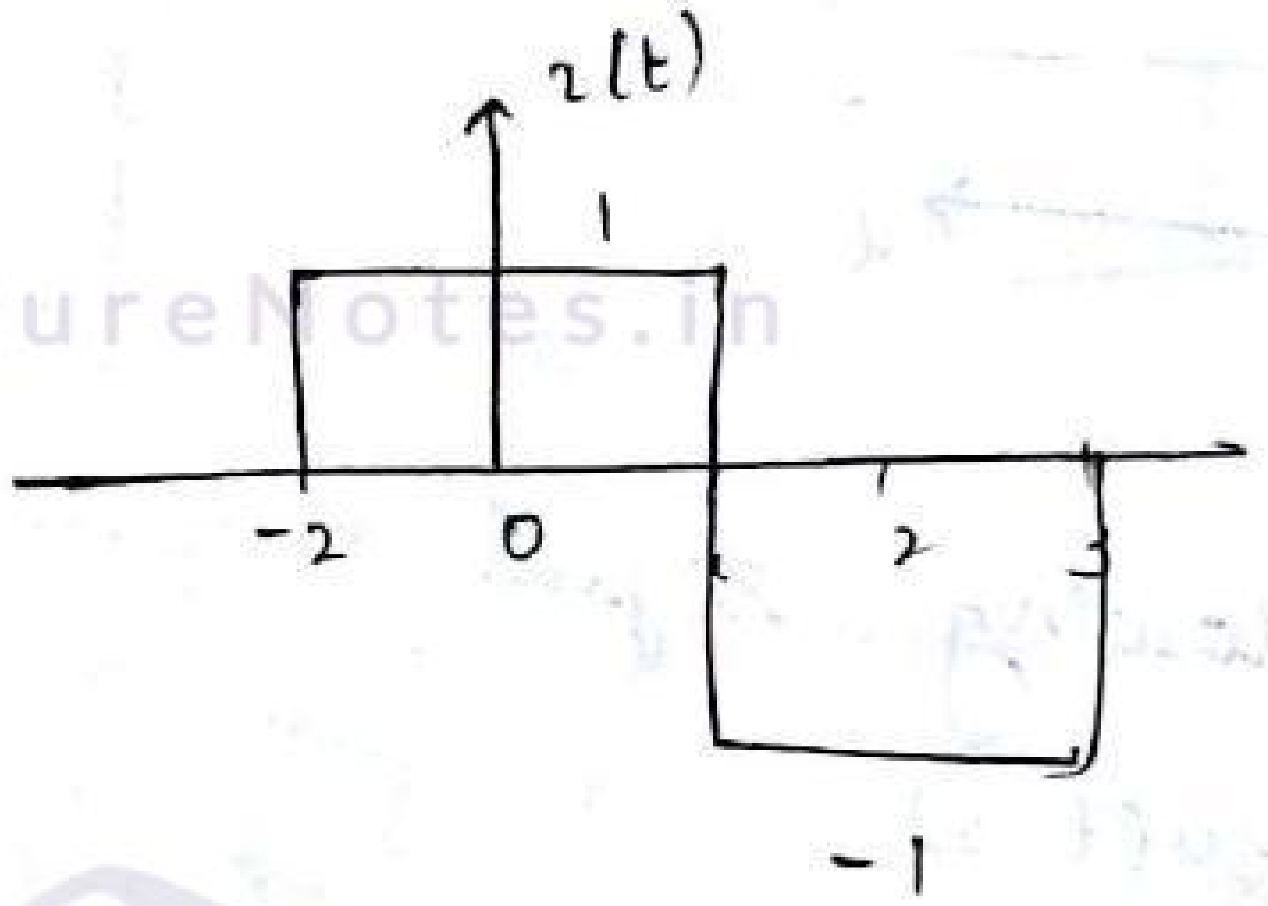
$t : 3, \infty$



$t-1 ; 0, \infty$

$t : 1, \infty$





2M

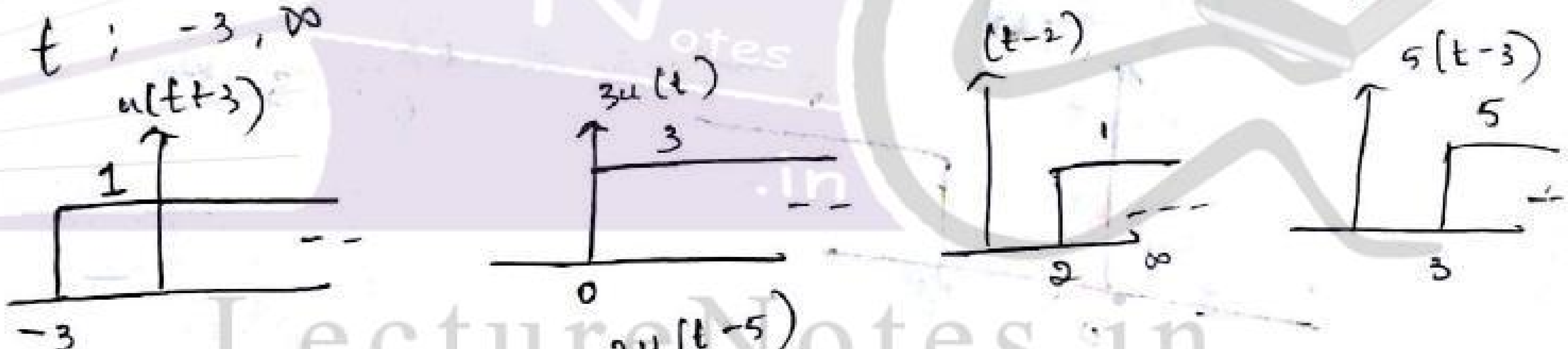
(3) $x(t) = u(t+3) - 3u(t) - u(t-2) + 5u(t-3) - 2u(t-5)$

$t+3 ;\ 0, \infty$
-3
$t ;\ -3, \infty$

$t ;\ 0, \infty$

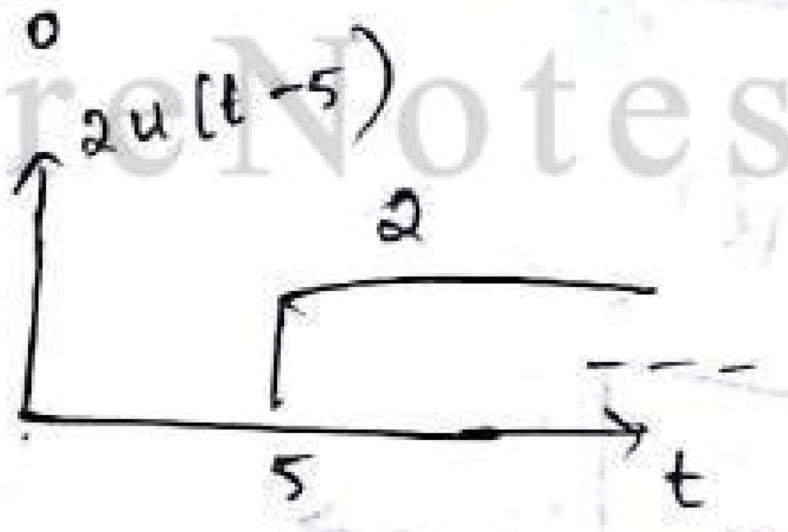
$t-2 ;\ 0, \infty$
$t ;\ 2, \infty$

$t-3 ;\ 0, \infty$
$t ;\ 3, \infty$

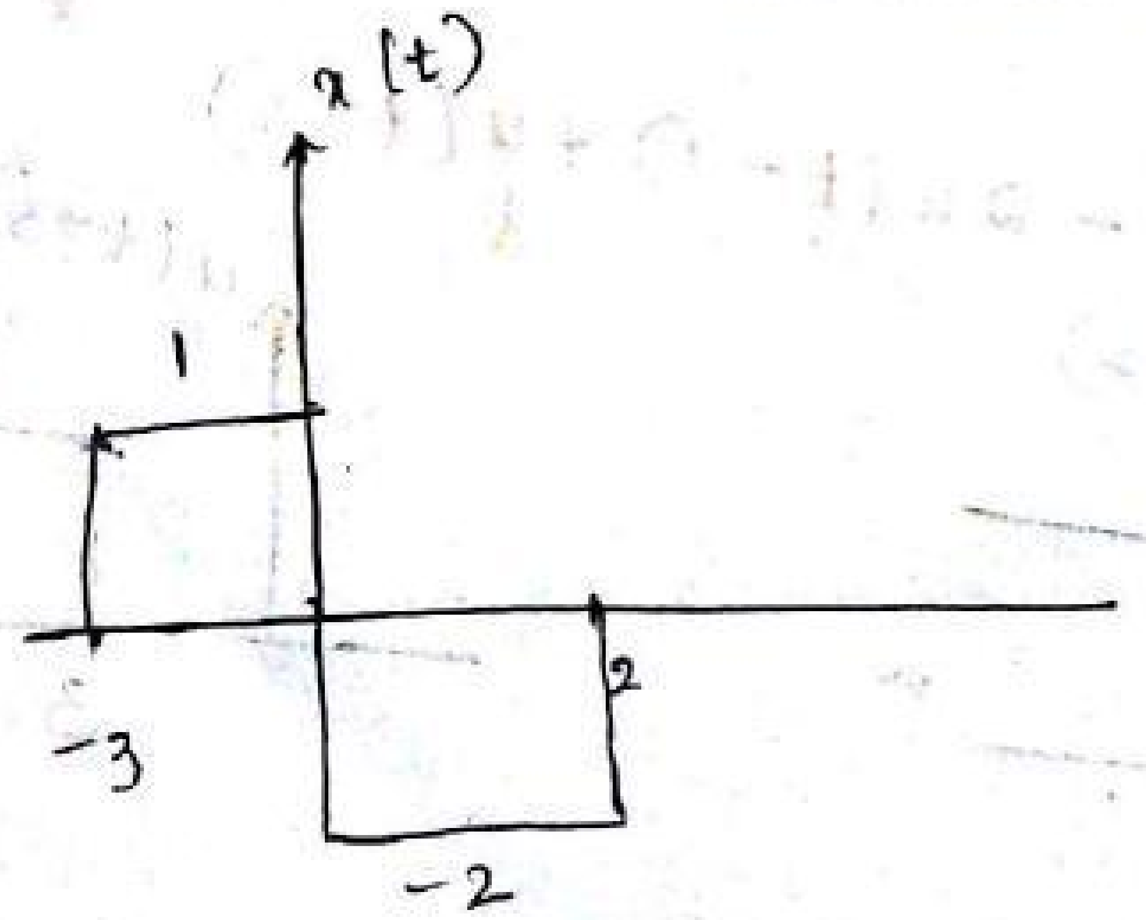


~~t-5~~

$t-5 ;\ 0, \infty$
$5, \infty$



$-\infty$ to -3
-3 to 0
∞ to 0 . 2

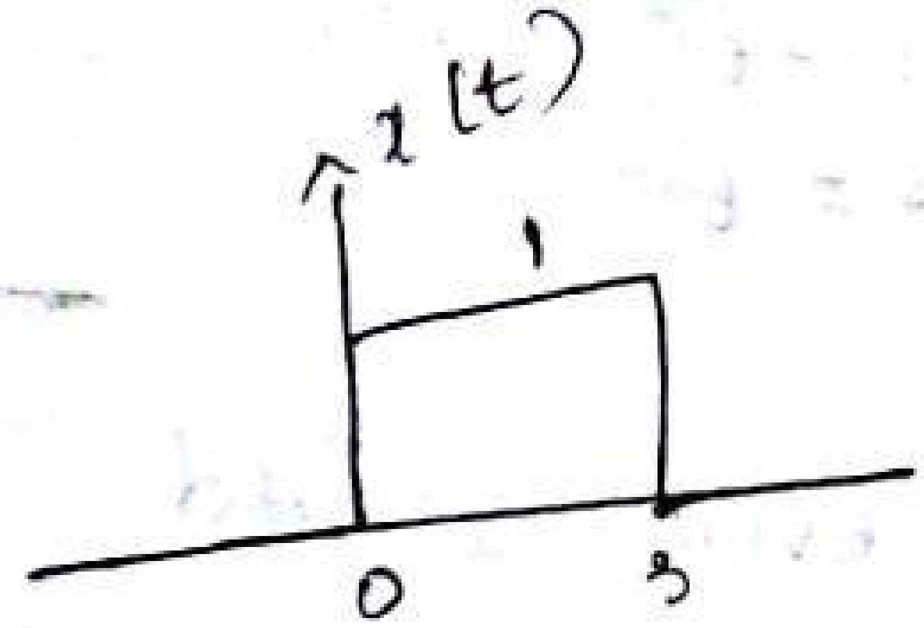


other method

① $x(t) = u(t) - u(t-3)$

0 3

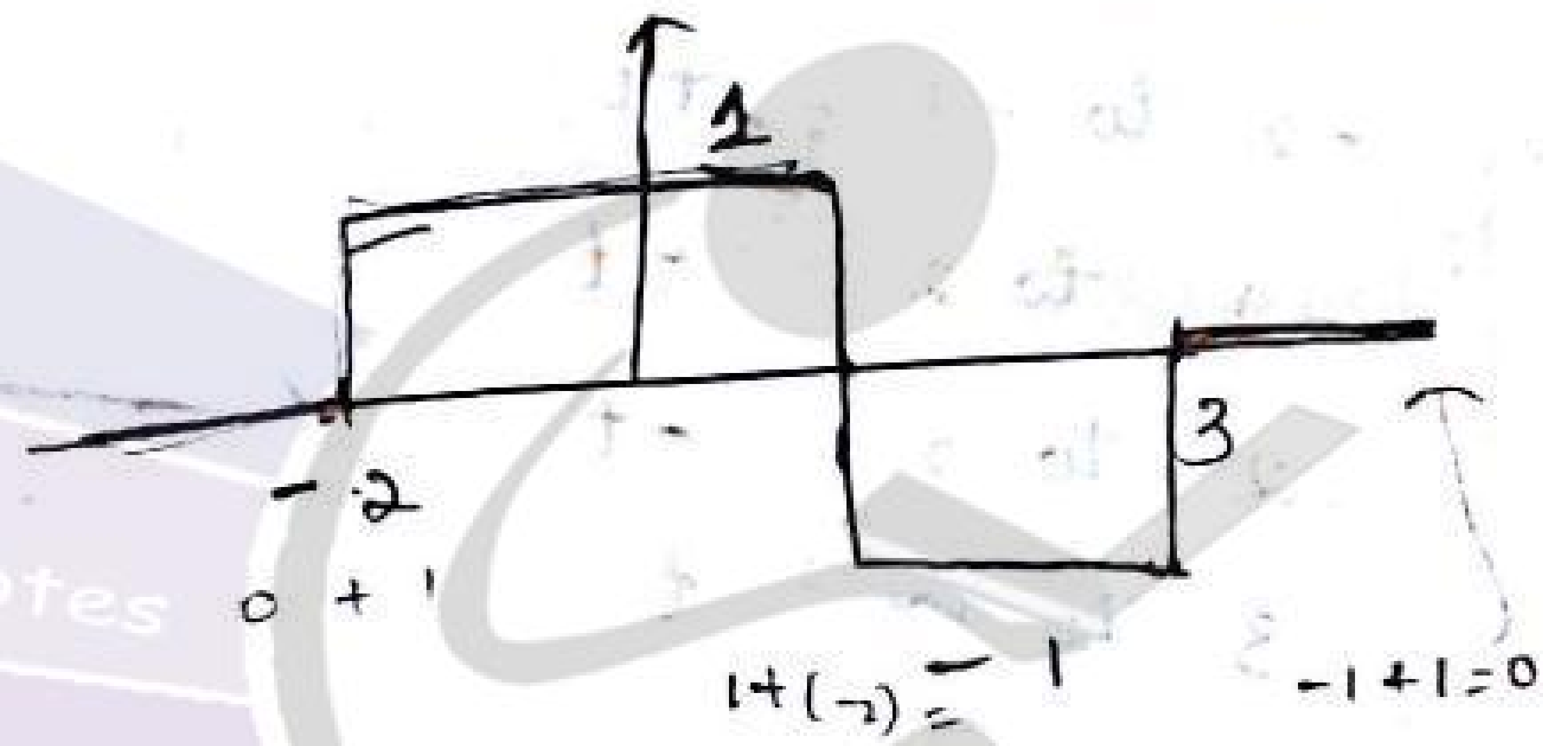
time	Amplitude
$-\infty$ to 0	0
0 to 3°	1
3 to ∞	-1



② $x(t) = u(t+2) - 2u(t-1) + u(t-3)$

-2 1 3

time	Amplitude
$-\infty$ to -2	0
-2 to 1	1
1 to 3	-2
3 to ∞	1

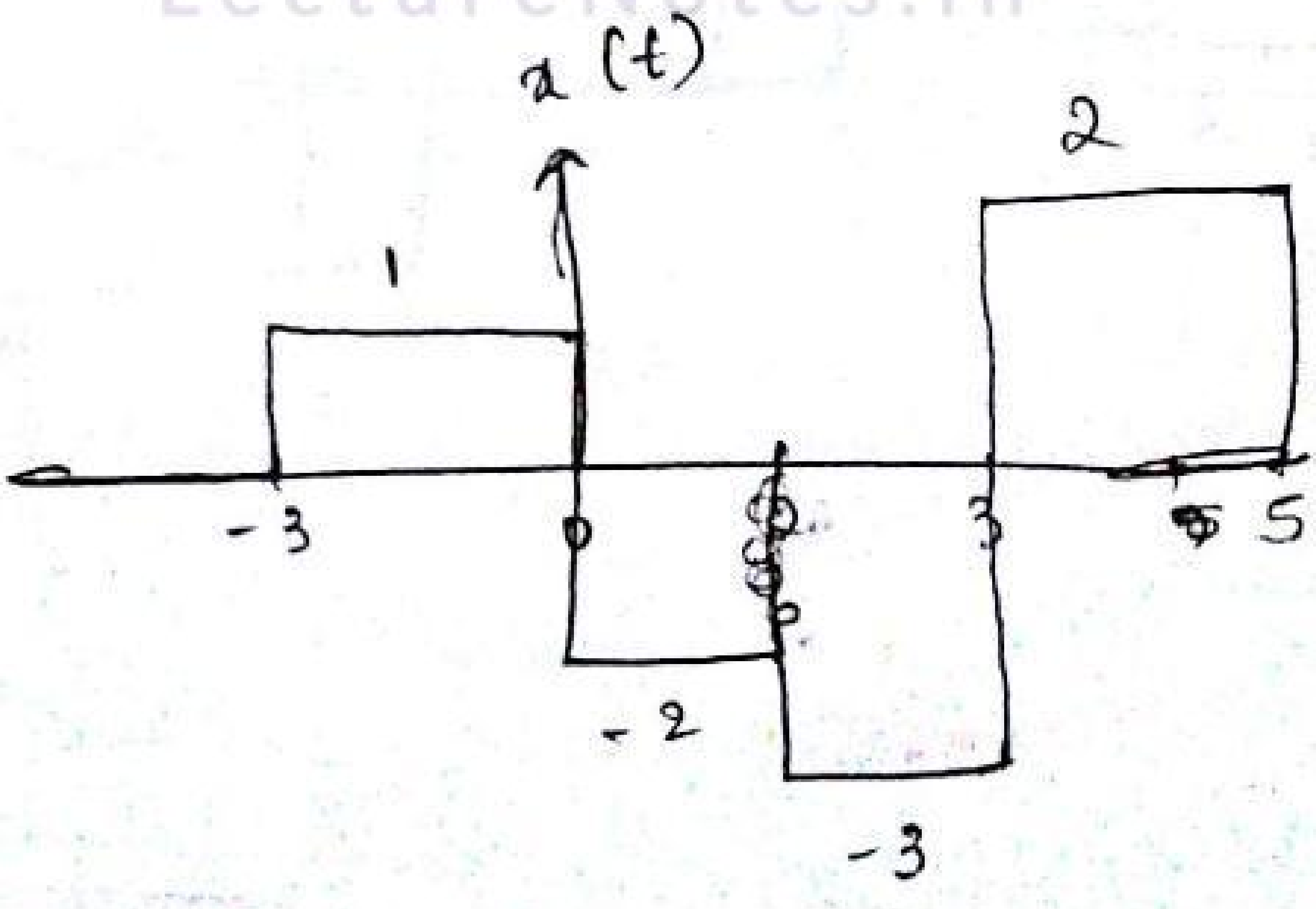


2M

③ $x(t) = u(t+3) - 3u(t) - u(t-2) + 5(t-3) - 2u(t-5)$

-3 0 2 3 5

time	Amp
$-\infty$ to -3	= 0
-3 to 0	= 1
0 to 2	= -3
2 to 3	= -1
3 to 5	= 5
5 to ∞	= -2



(4) $x(t) = r(t+1) - r(t) + r(t-2)$

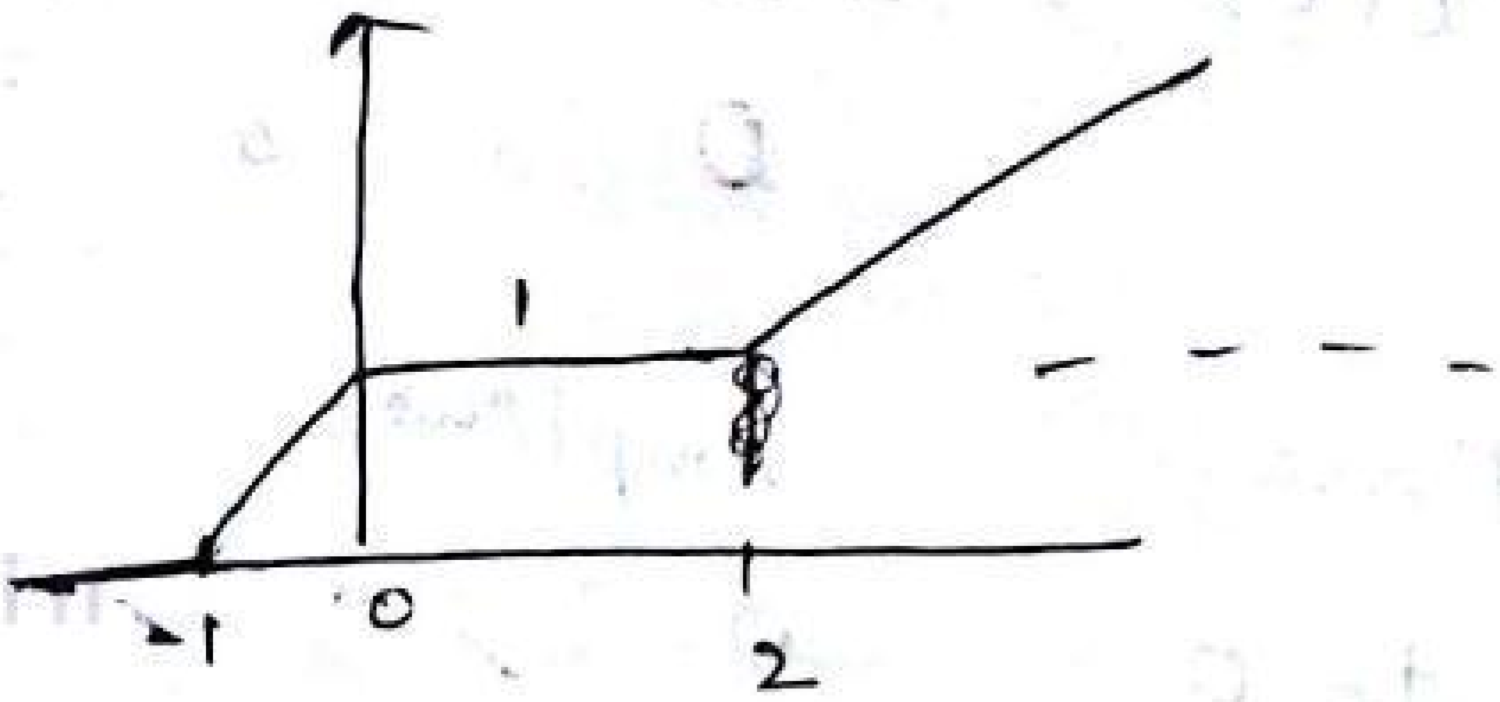
$-1, \quad 0 \quad 2$

$-\infty$ to $-1 = 0$

-1 to $0 = t$

0 t $2 = -t$

2 to $\infty = t$



find even & odd

(5) $x(t) = r(t+2) - r(t+1) - r(t-2) + r(t-3)$

$-2 \quad -1 \quad 2 \quad 3$

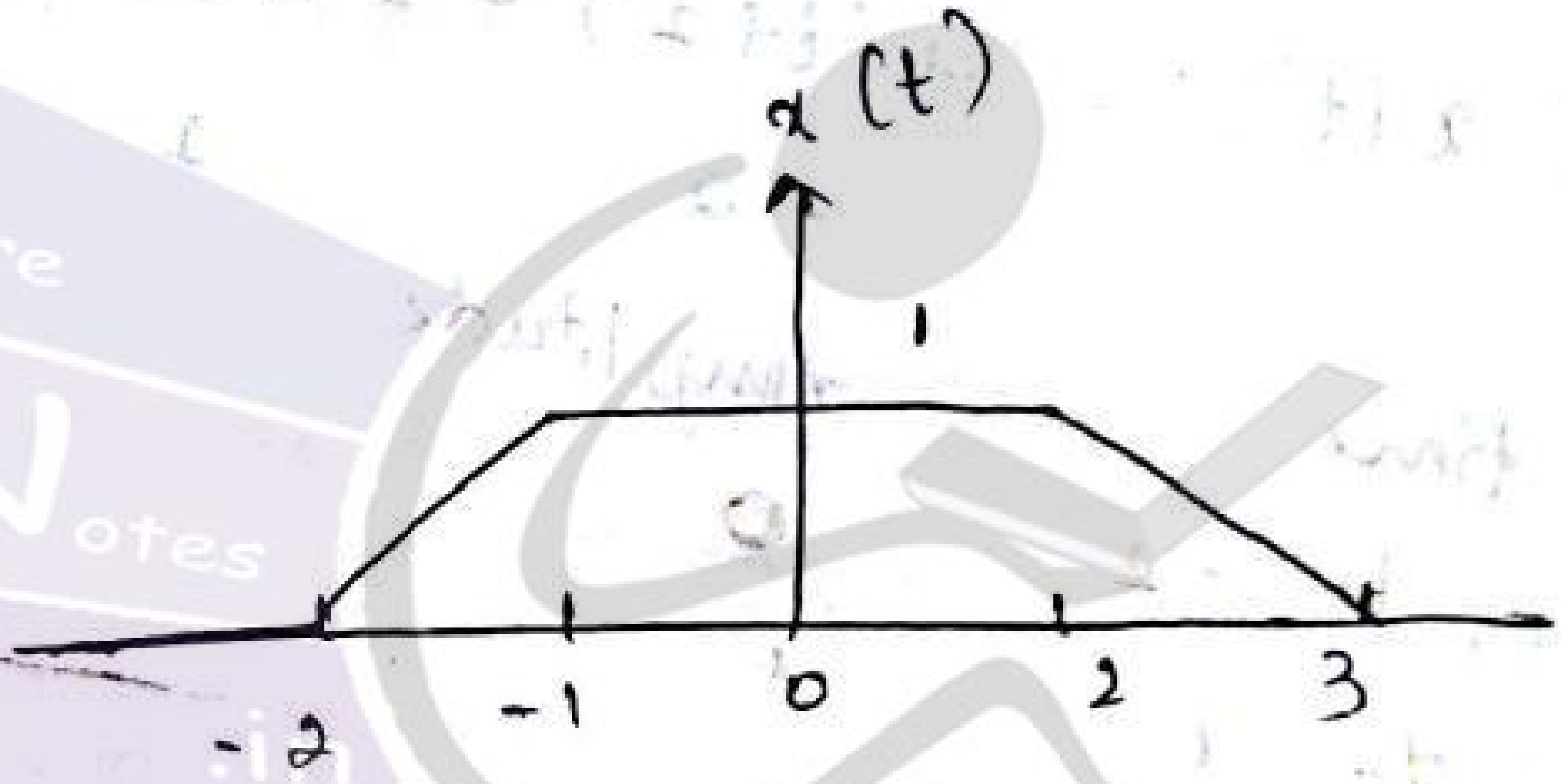
$-\infty$ to $-2 = 0$

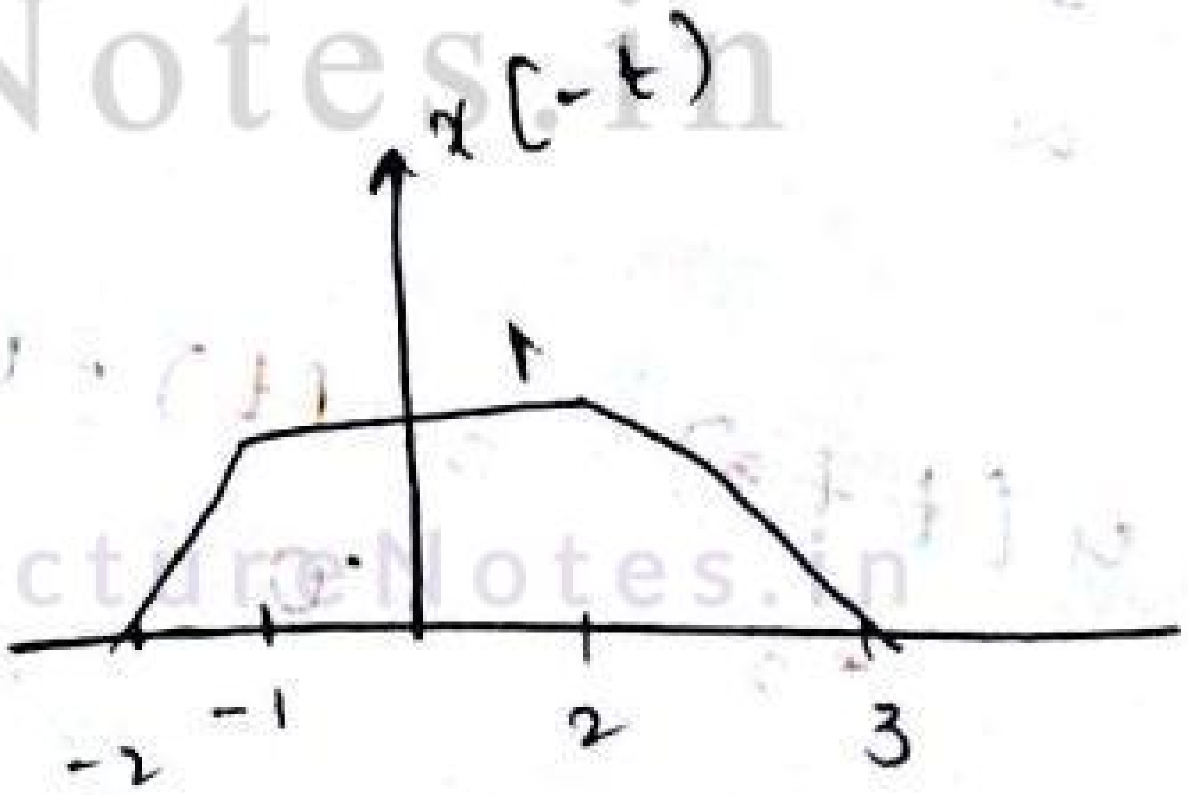
-2 to $-1 = +t$

-1 to $2 = -t$

2 to $3 = -t$

3 to $\infty = t$





4M find even & odd

6) $x(n) = u(n+2) - 3u(n-1) + 2u(n-5)$

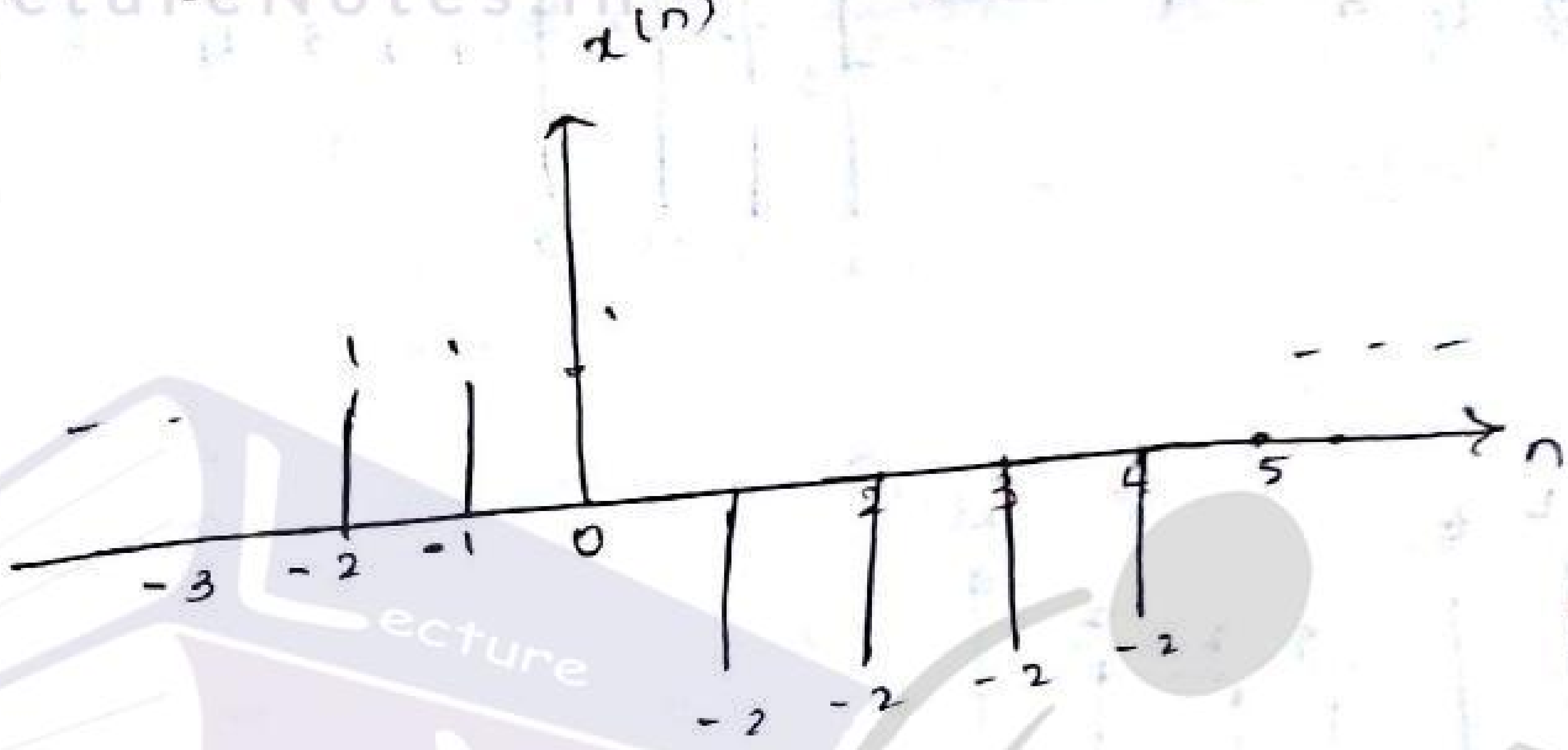
$-2 \qquad 1 \qquad 5$

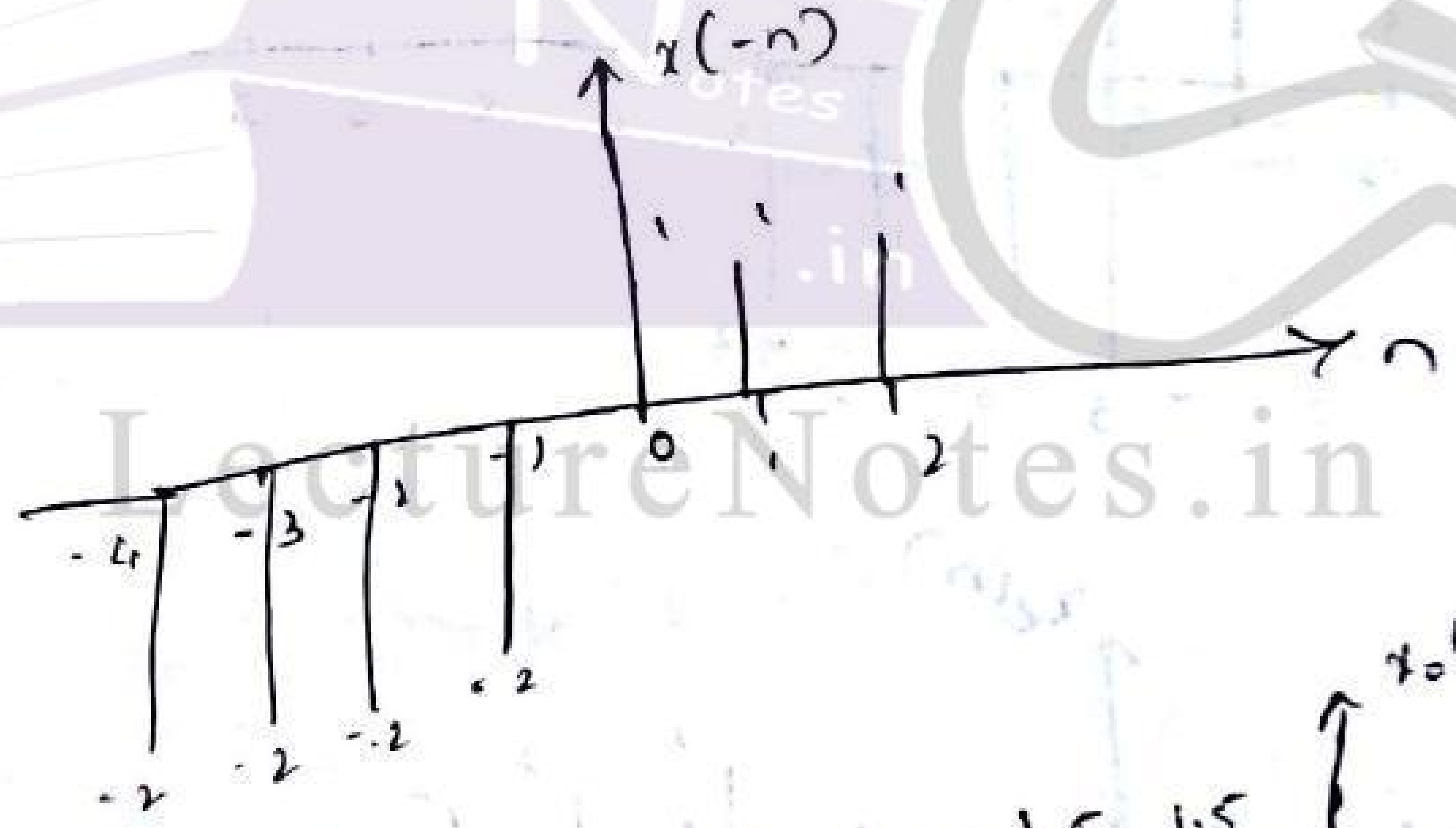
$-\infty$ to $-2 = 0$

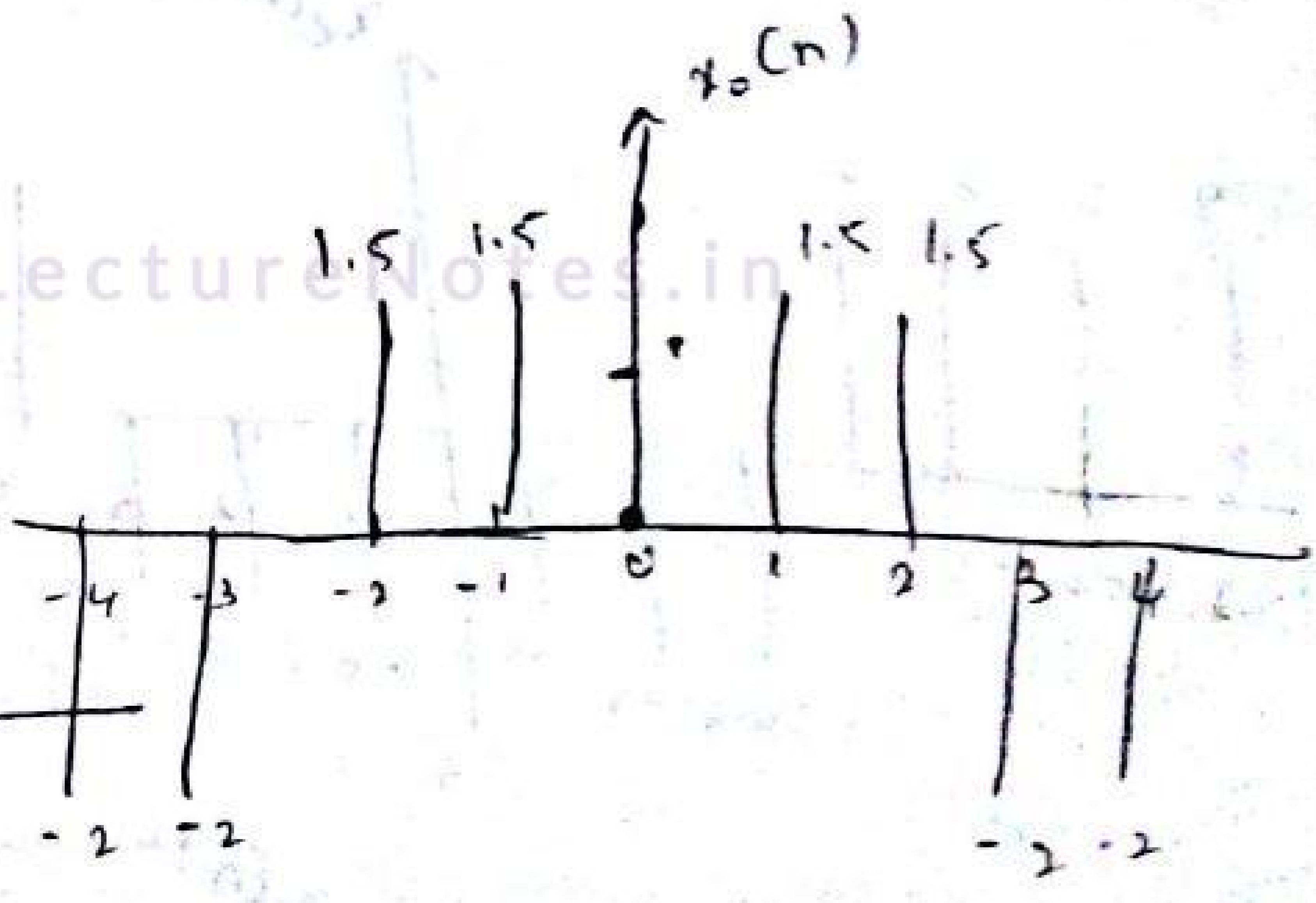
-2 to $1 = 1$

1 to $5 = -3$

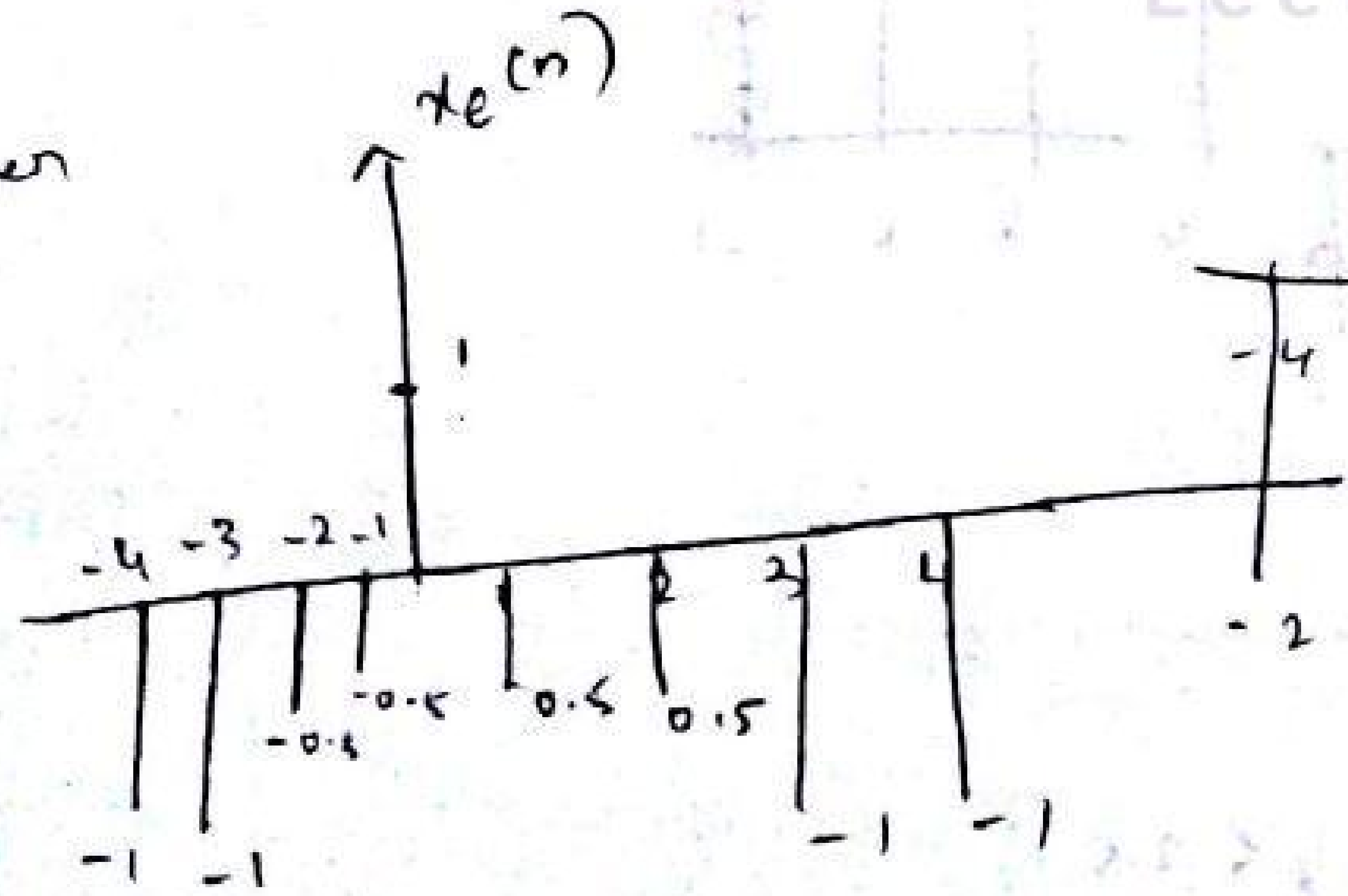
5 to $\infty = 2$







even



7) $x(n) = -3u(n+3) + 5u(n-1) + 2u(n-4) - 4u(n-7)$

$-3 \qquad 1 \qquad 4 \qquad 7$

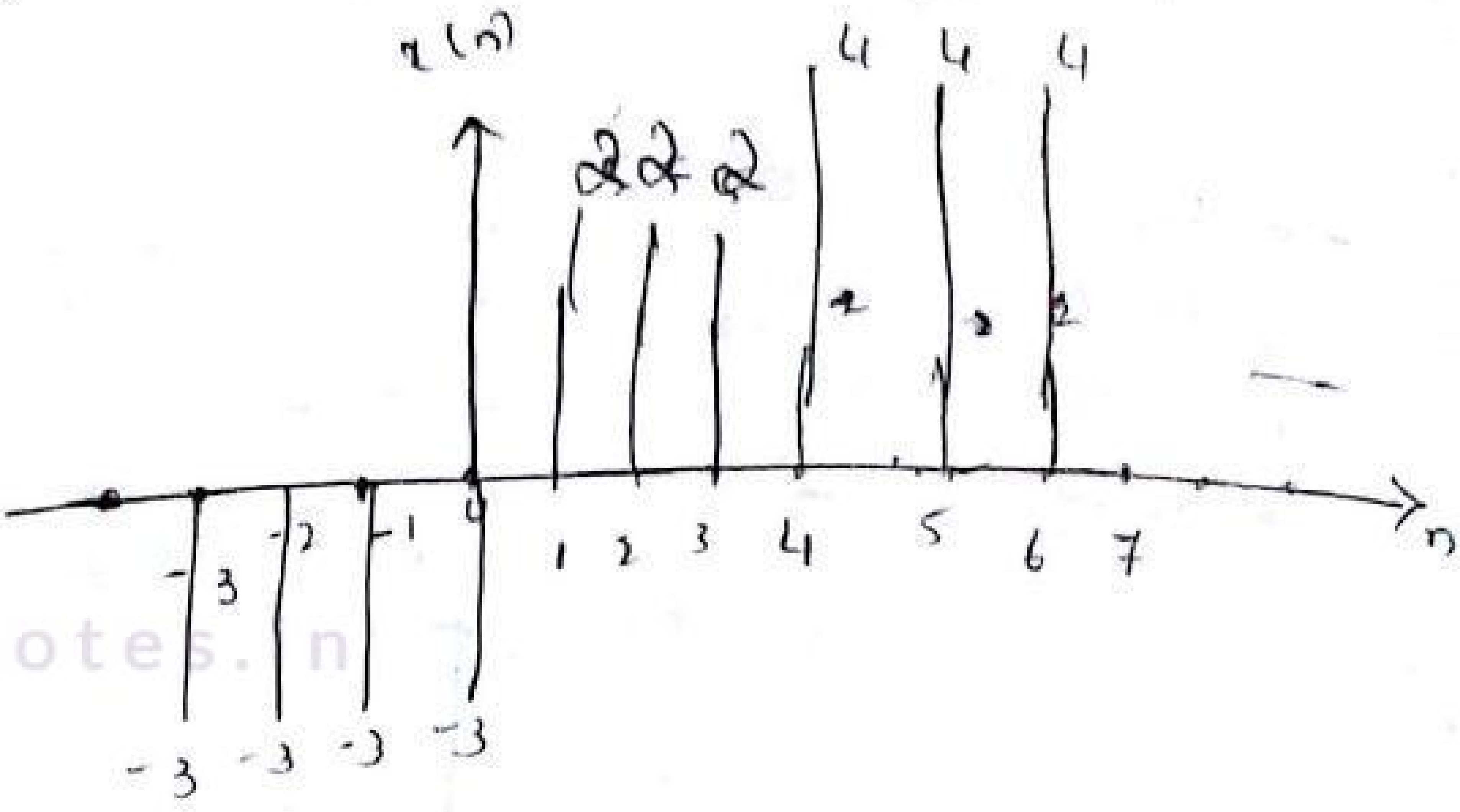
$-\infty$ to $-3 = 0$

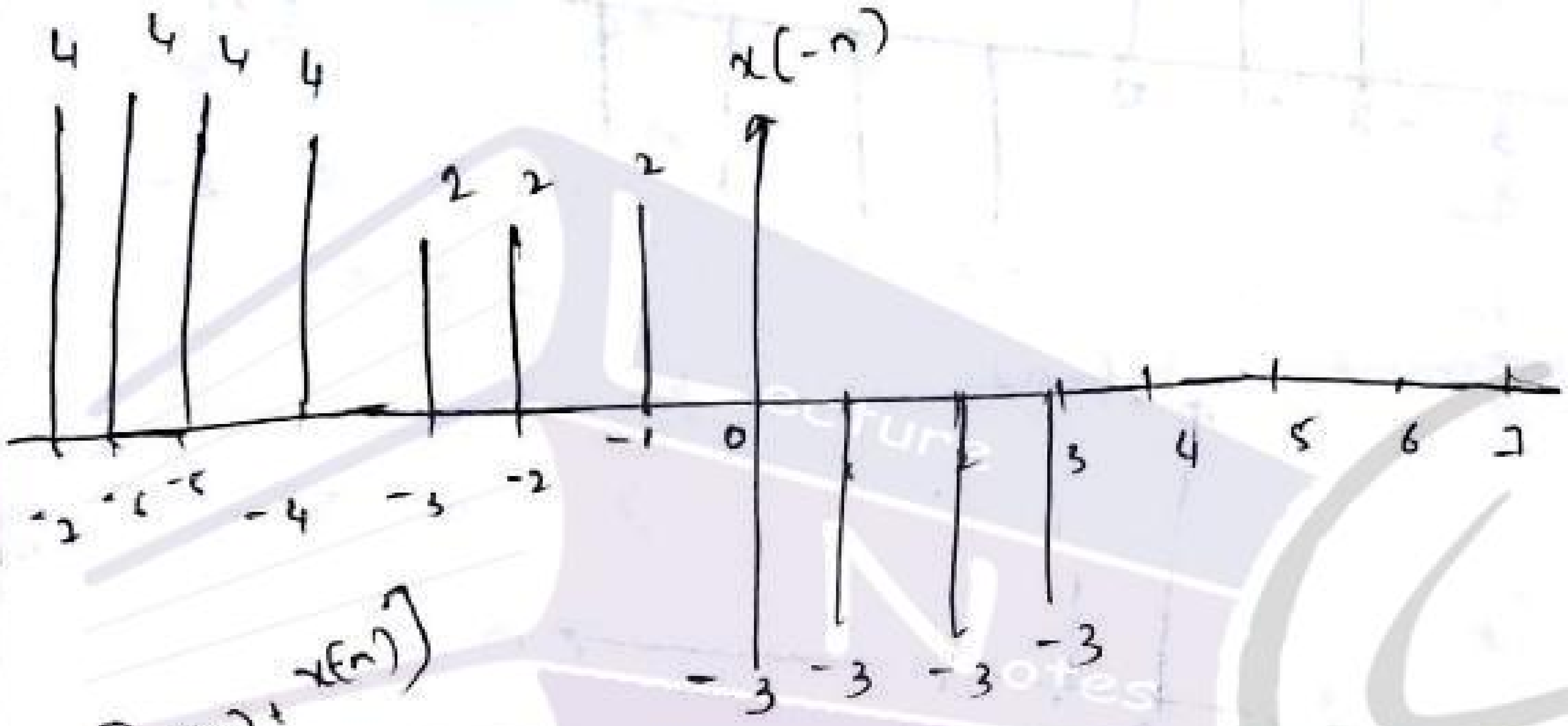
-3 to $1 = -3$

1 to $4 = 5$

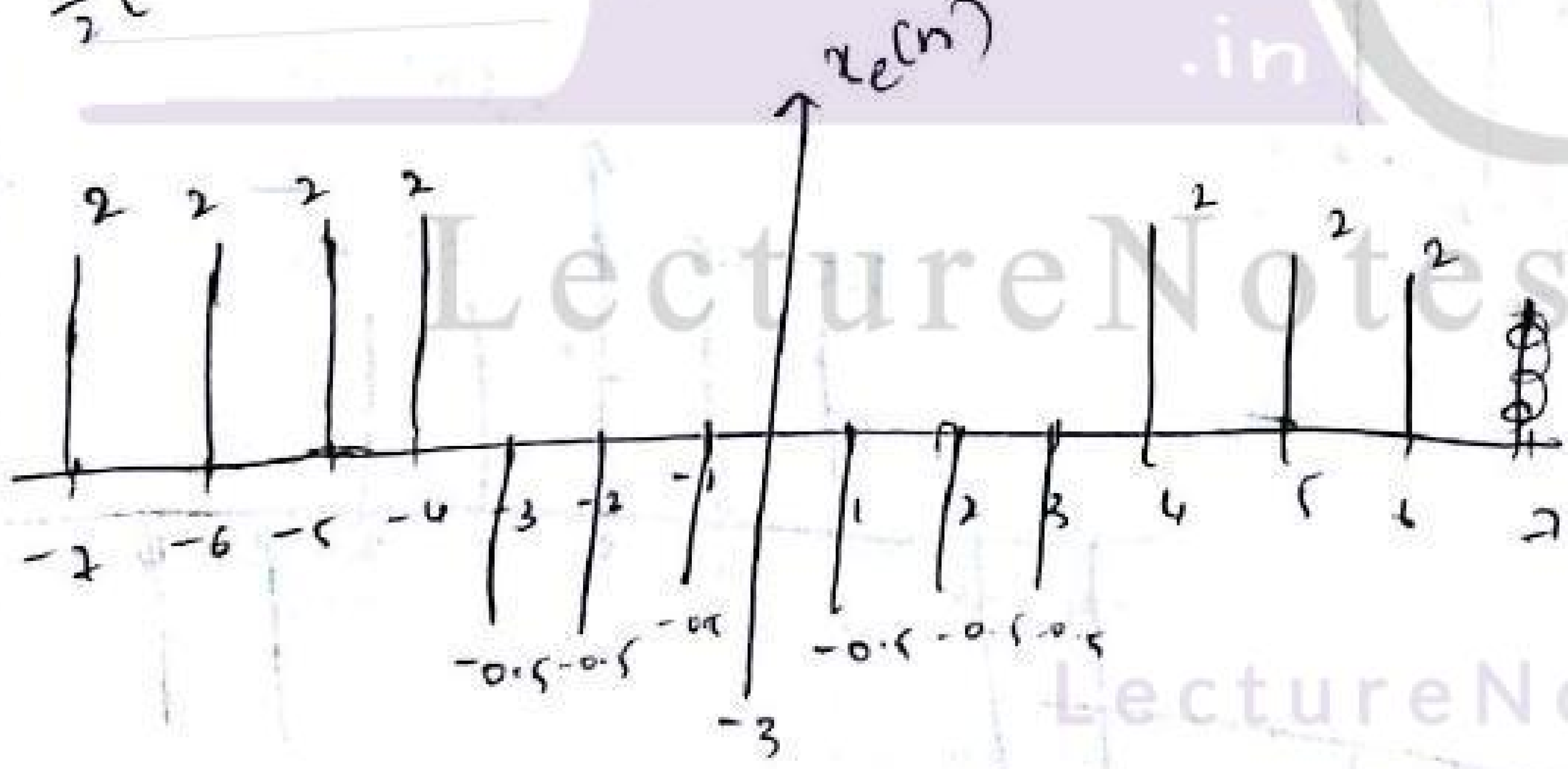
4 to $7 = 2$

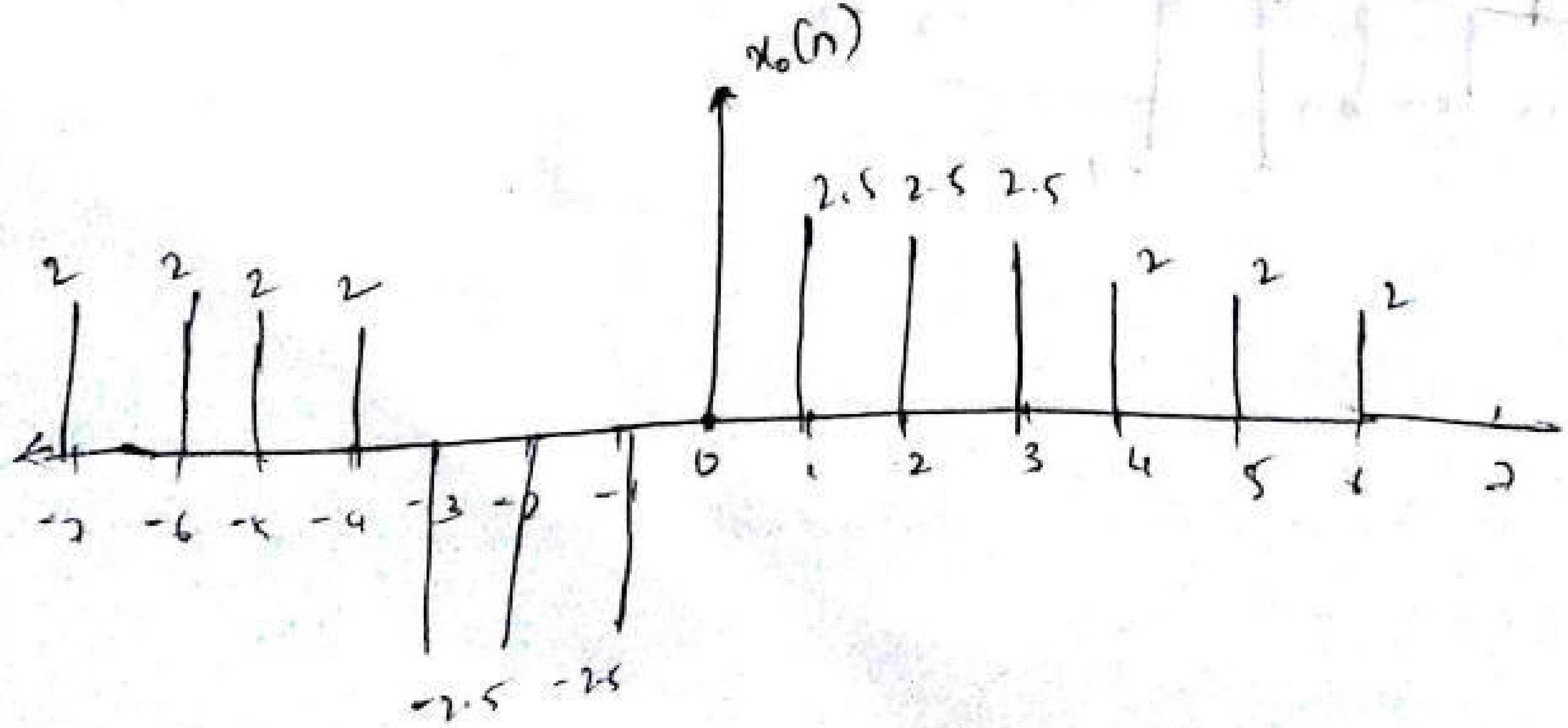
7 to ∞ -4





$\frac{1}{2}[x(n) + x(-n)]$

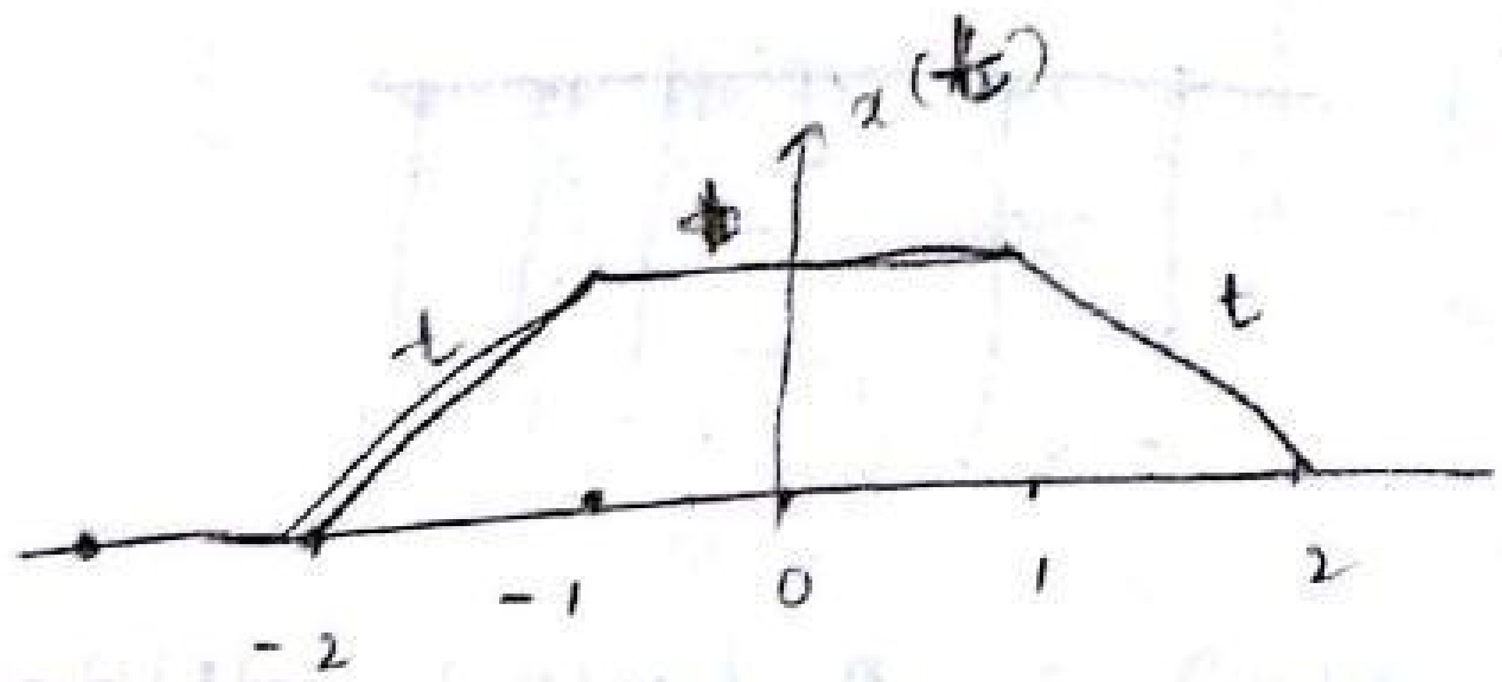


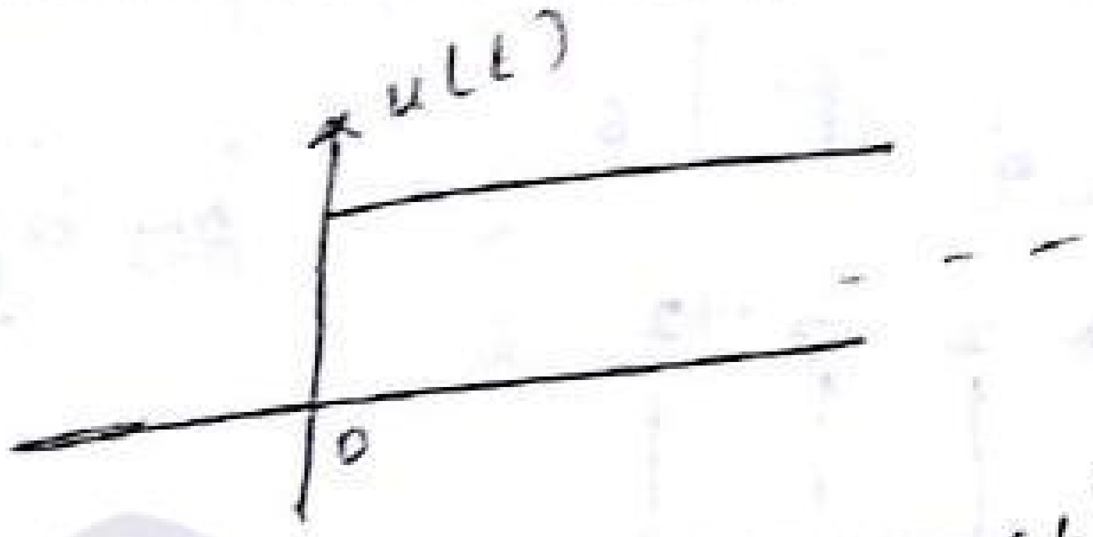


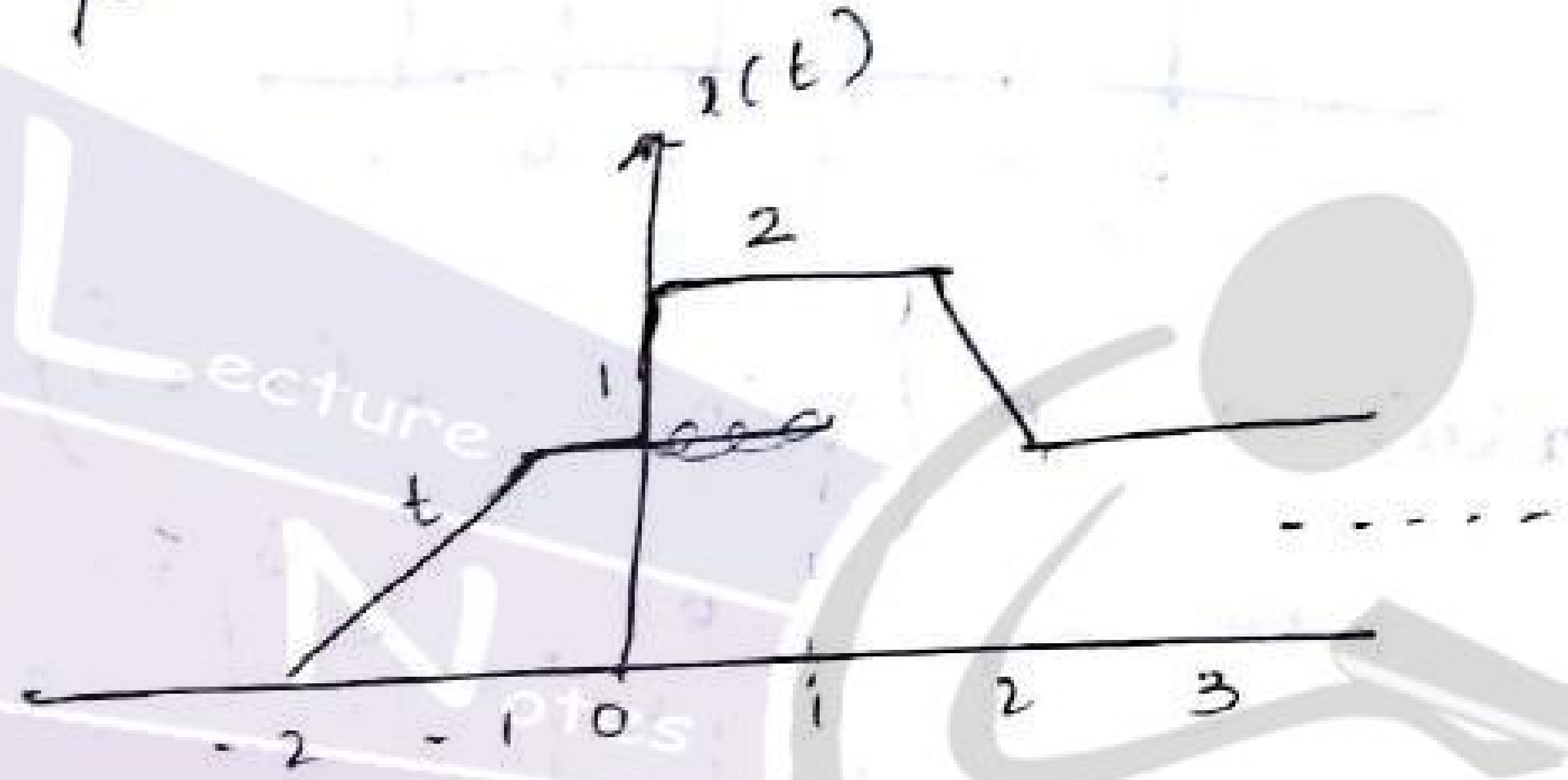
⑧ $x(t) = [u(t)] + [r(t+2) - r(t+1) - r(t-1) + r(t-2)]$

$-2 \qquad -1 \qquad 1 \qquad 2$

$-\infty$ to -2 : 0
-2 to -1 : t
-1 to 1 : $-t$
1 to 2 : $-t$
2 to ∞ : t

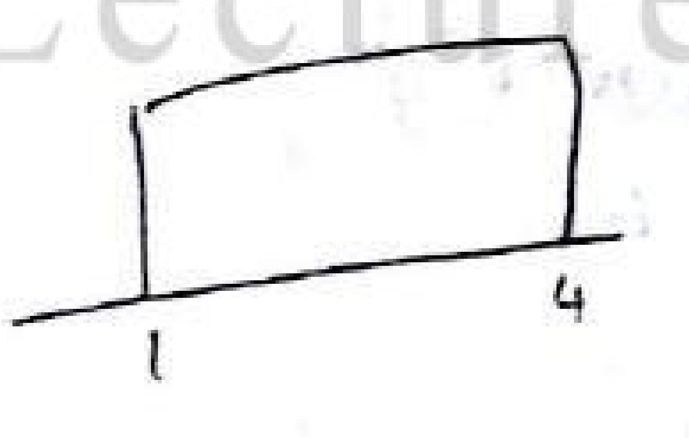






⑨ $x(t) = u(t-1) - u(t-4)$

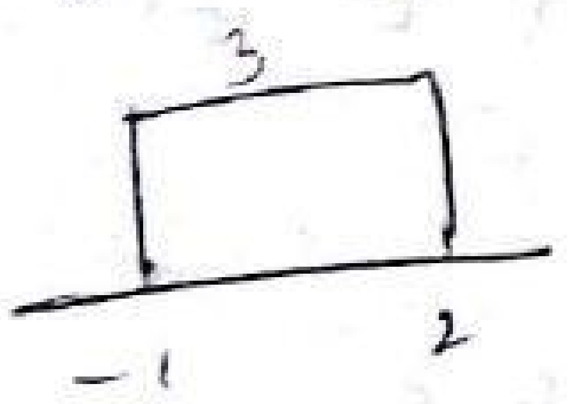
$1 \qquad 4$



only 2 - signal it should be ↑ and amplitude should be same

⑩ $x(t) = 3[u(t+1) - u(t-2)]$

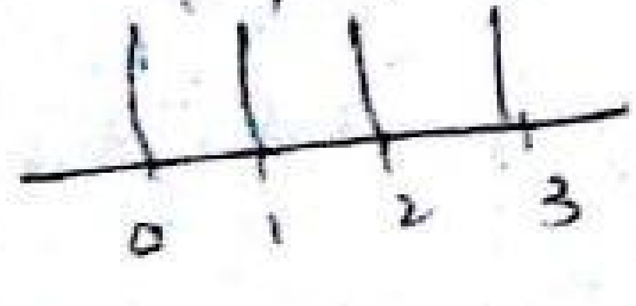
$-1 \qquad 2$



⑪ $x(n) = u(n) - u(n-4)$

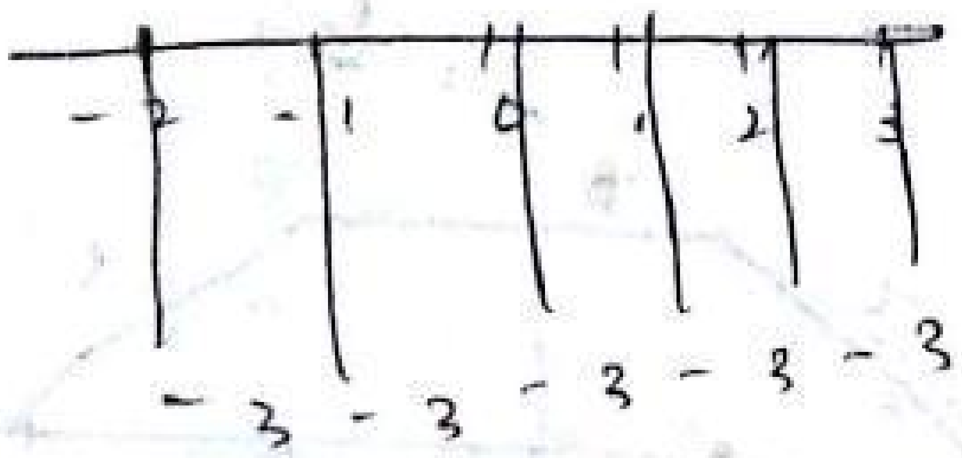
$0 \qquad 4$

one no. before



(12) $x(n) = -3[u(n+2) - u(n-4)]$

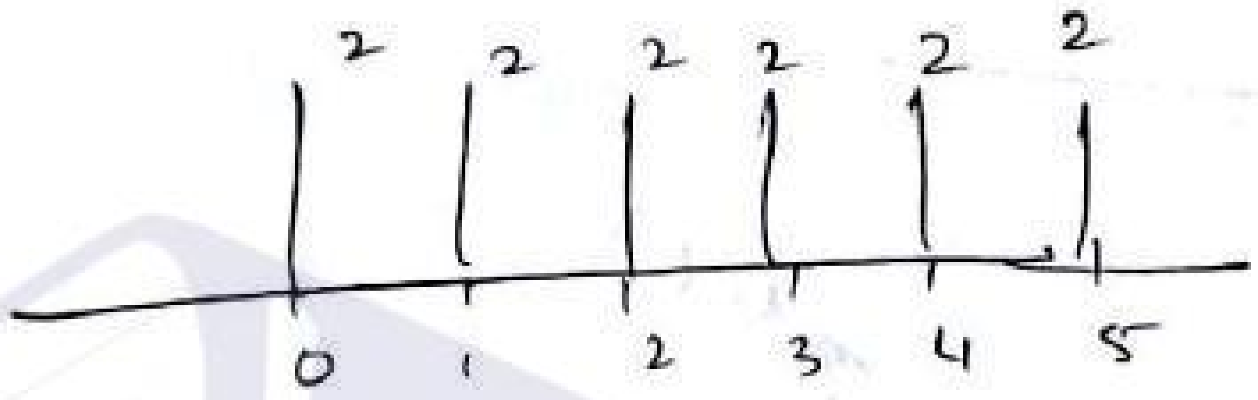
$-2 \qquad 4$



(13) $x(n) = 2[u(n) - u(n-6)]$

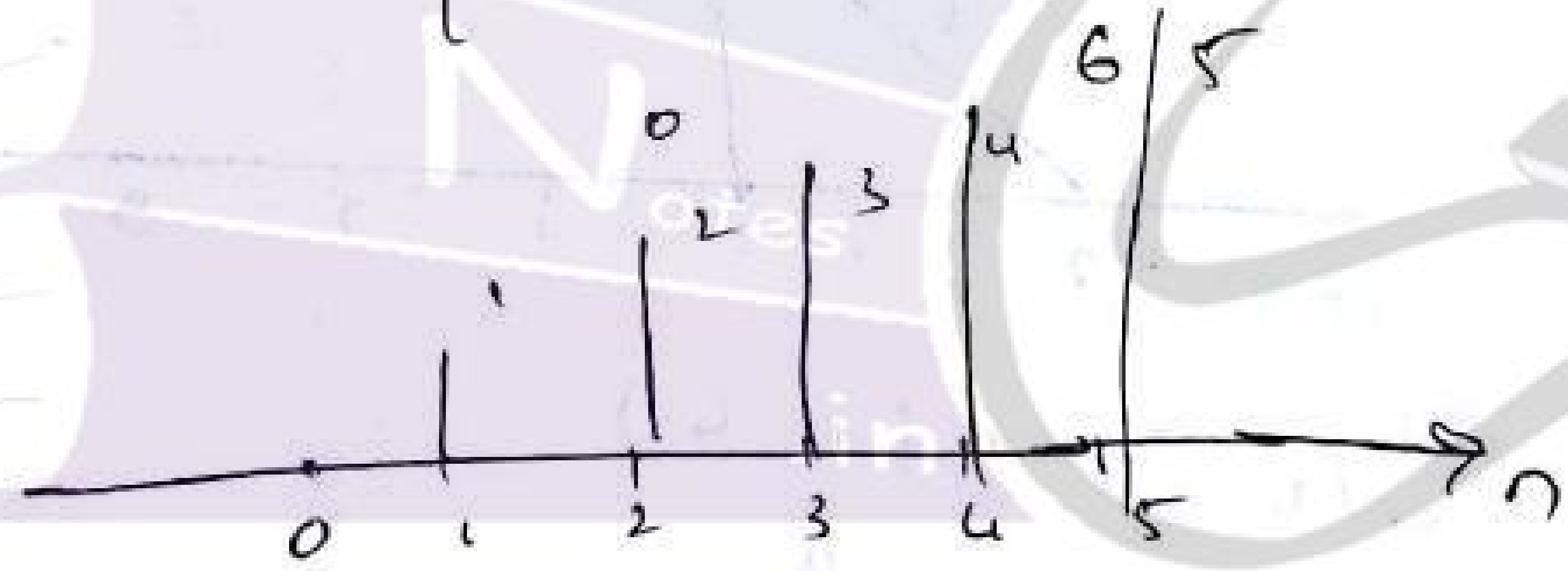
$0 \qquad 6$

Done before



(14) $x(n) = n[u(n) - u(n-6)]$

$0 \qquad 6$



4M

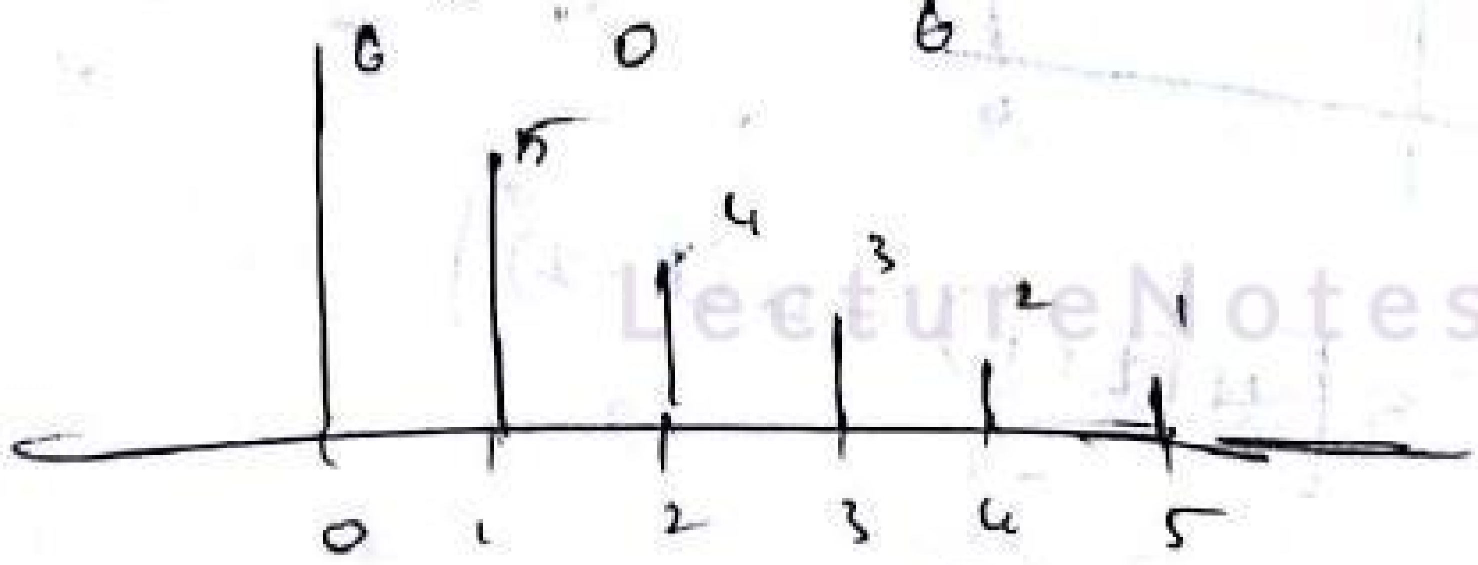
(15) $x(n) = [6-n][u(n) - u(n-6)]$

$0 \qquad 6$

sketch

a) $x(2n-3)$

b) $x(4-n)$



(a)

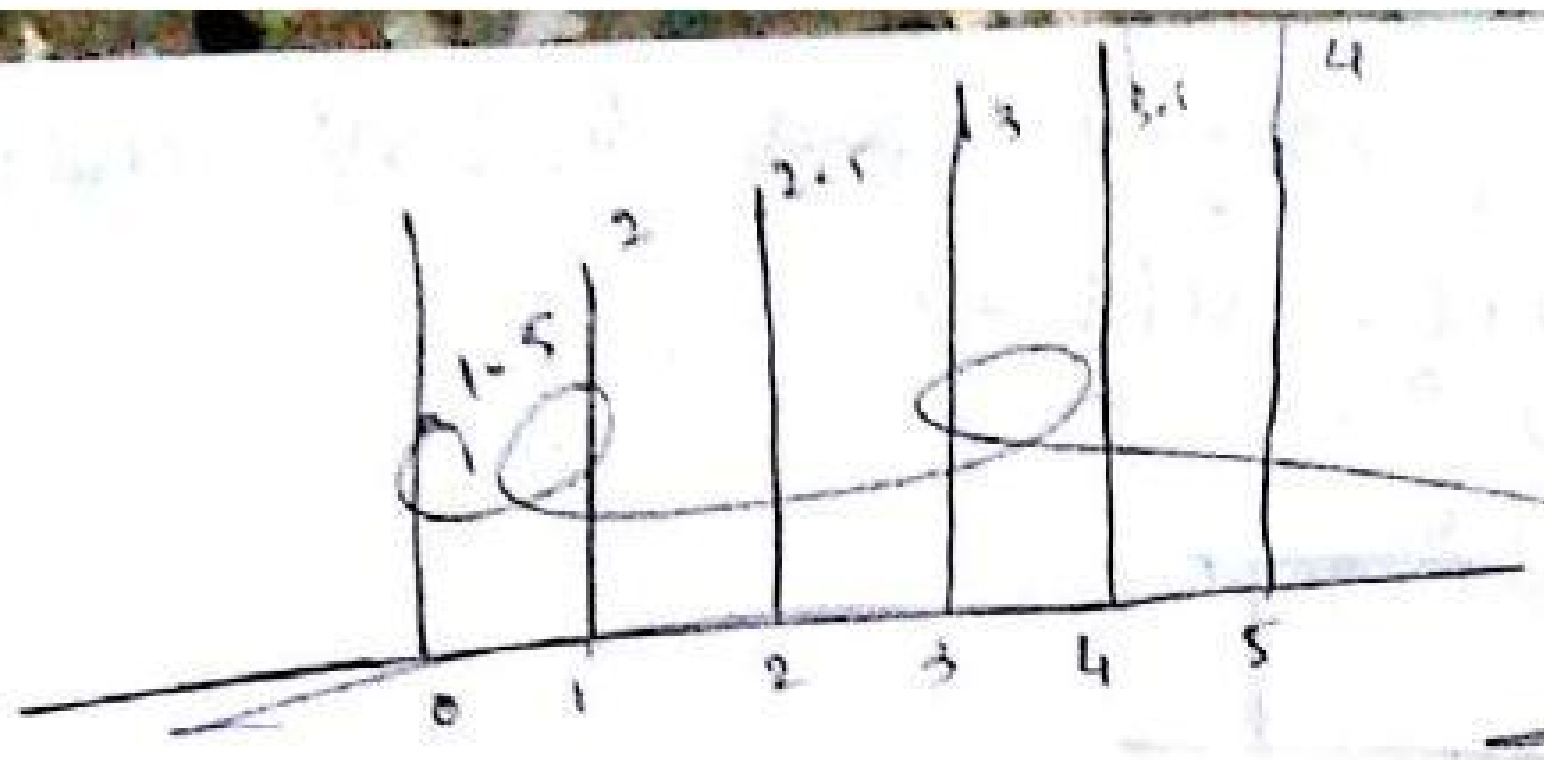
$2n-3$; 0, 1, 2, 3, 4, 5

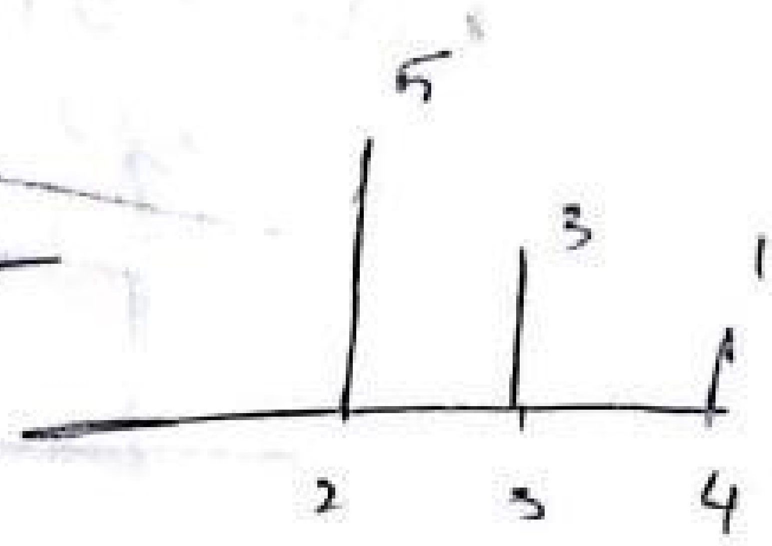
$+3$

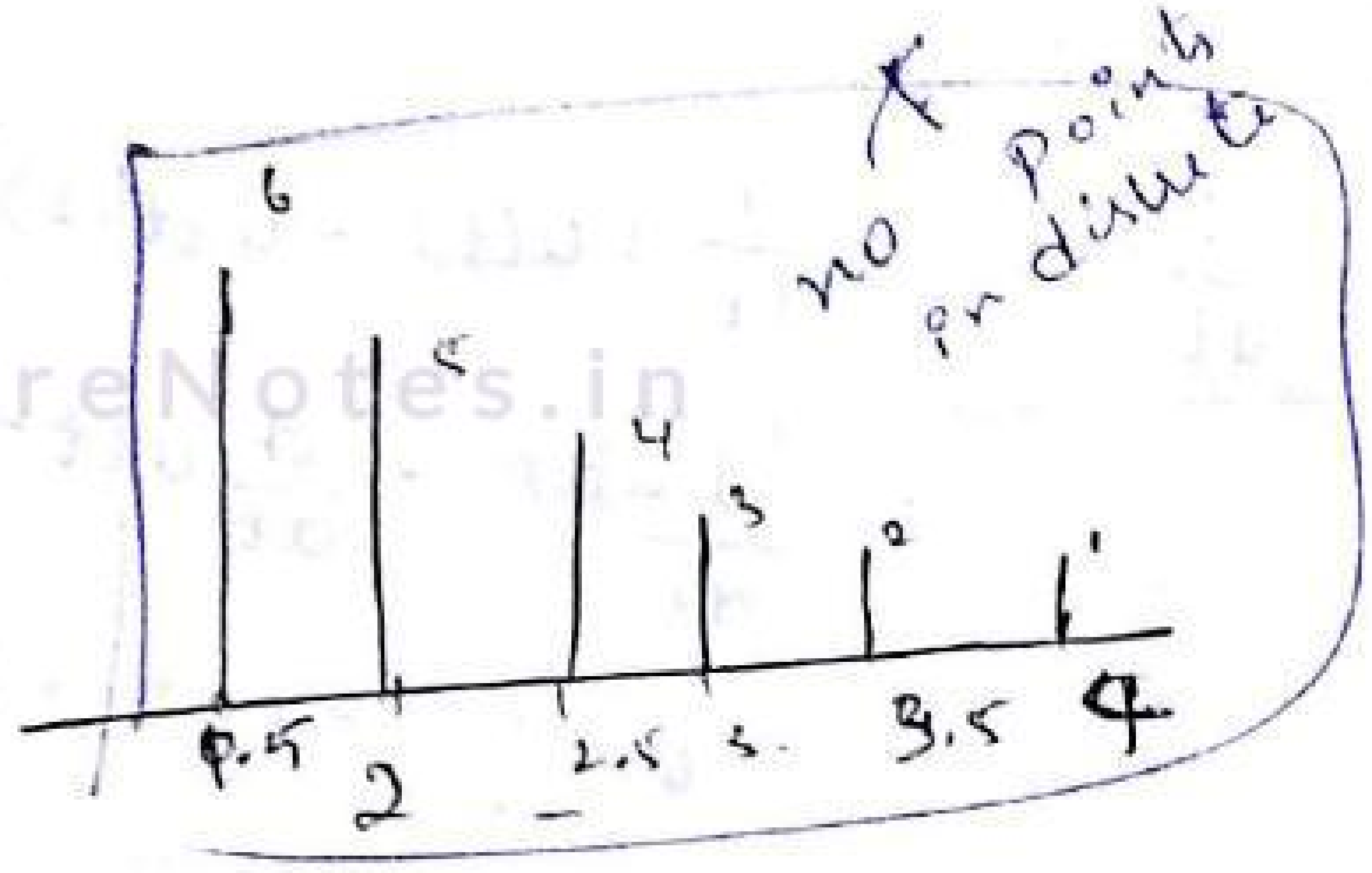
$2n$; 3, 4, 5, 6, 7, 8

$\div 2$

n ; 1.5, 2, 2.5, 3, 3.5, 4







(b) $4-n$

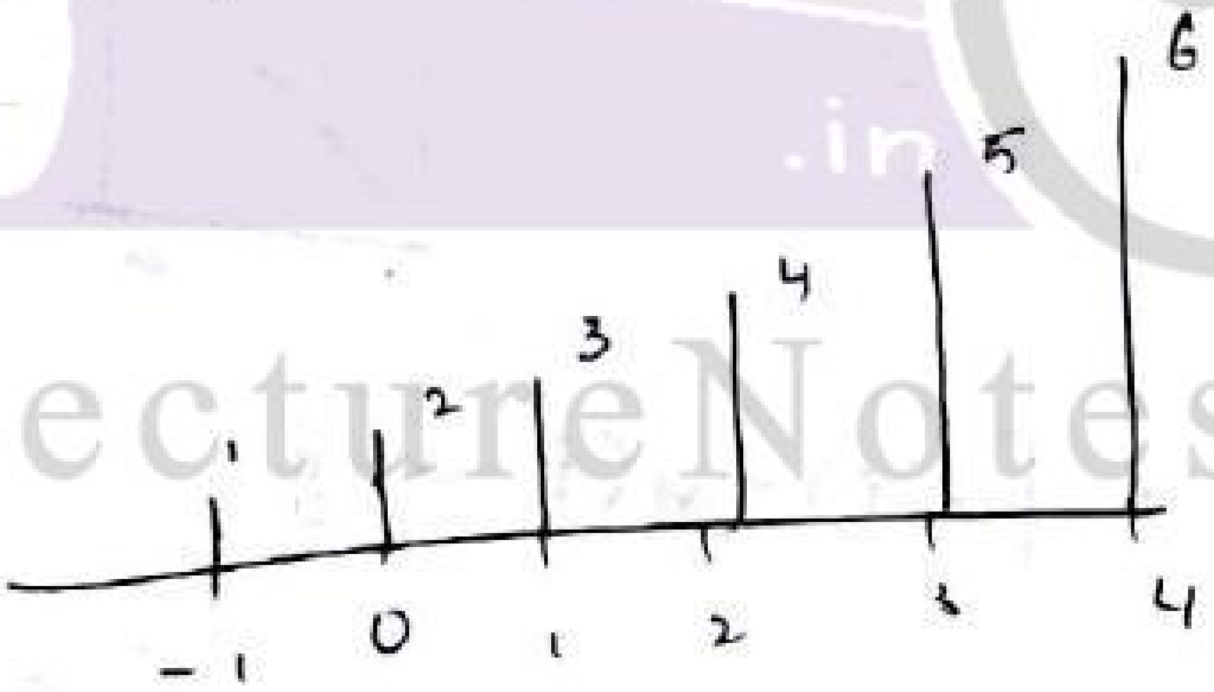
$4-n$; $0, 1, 2, 3, 4, 5$

-4

$-n$; $-4, -3, -2, -1, 0, 1$

$\times -1$

$n = 4, 3, 2, 1, 0, -1$



Derivative of Signal

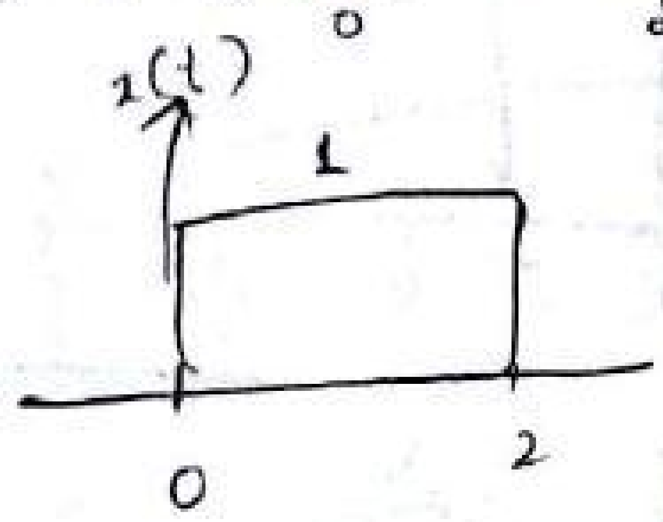
$$\frac{d}{dt} r(t) = u(t)$$

$$\frac{d}{dt} u(t) = \delta(t)$$

$$\frac{d}{dt} \delta(t) = 0$$

(16) Sketch the signal and its 1st derivative

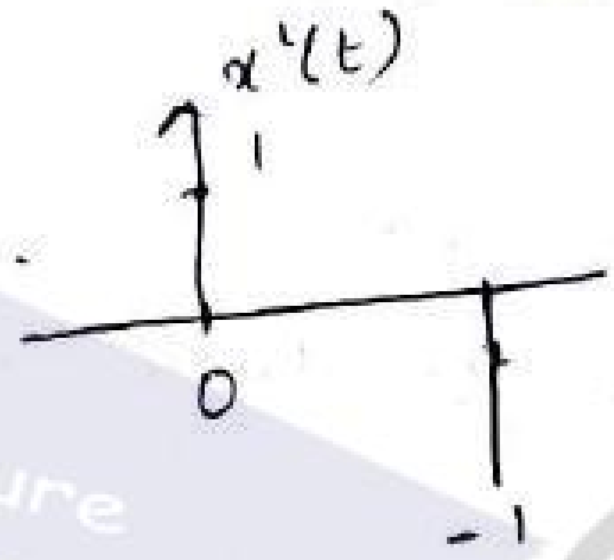
$x(t) = u(t) - u(t-2)$



$x'(t) = \frac{d}{dt} x(t) = \frac{d}{dt}\left[u(t) - u(t-2)\right]$

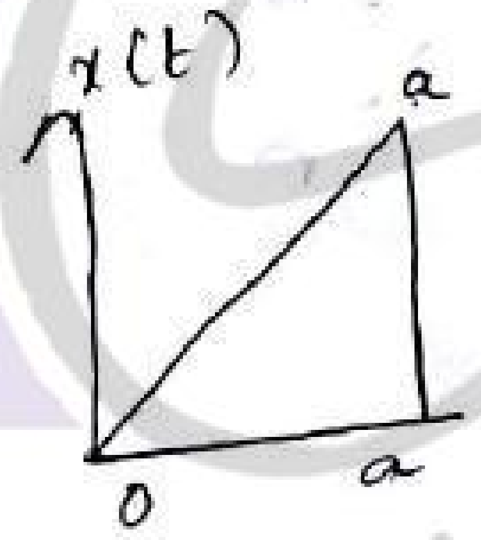
$= \frac{d\,u(t)}{dt} - \frac{d}{dt} u(t-2)$

$\delta(t) - \delta(t-2)$



(17) $x(t) = t\left[u(t) - u(t-a)\right]$

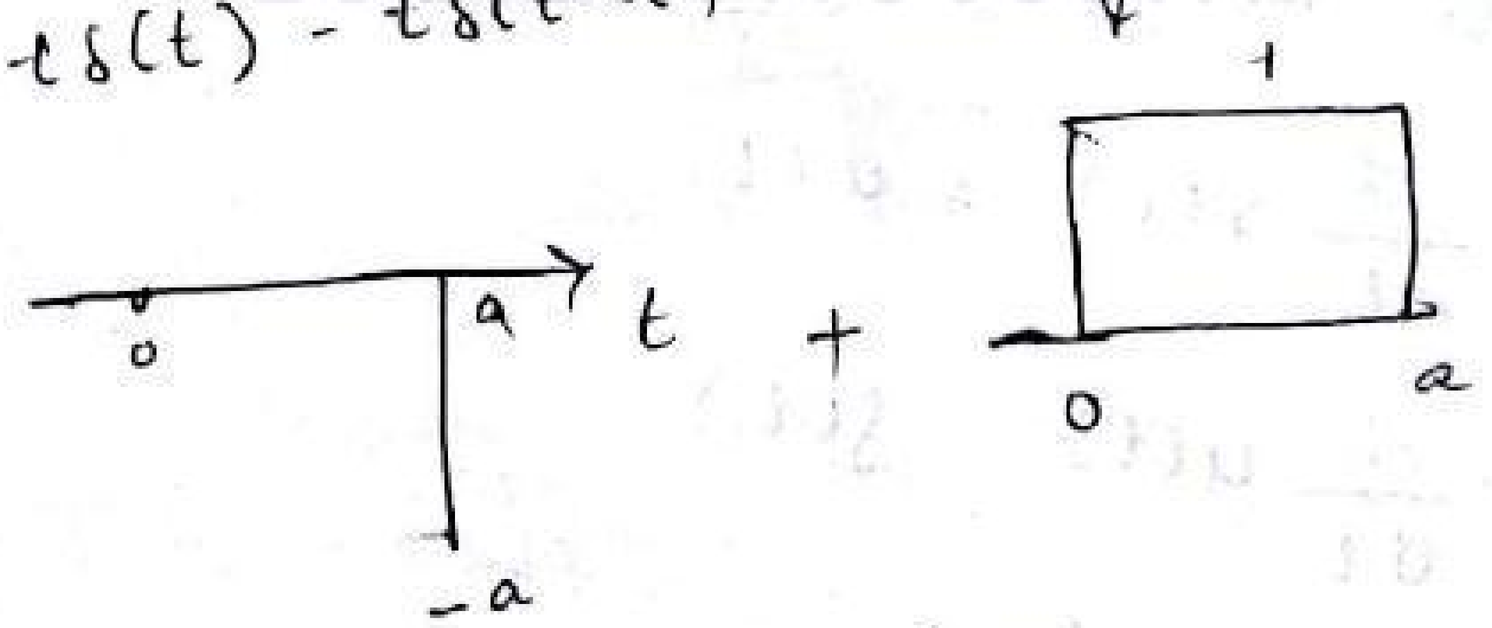
sketch
$x(t)$ & $x'(t)$



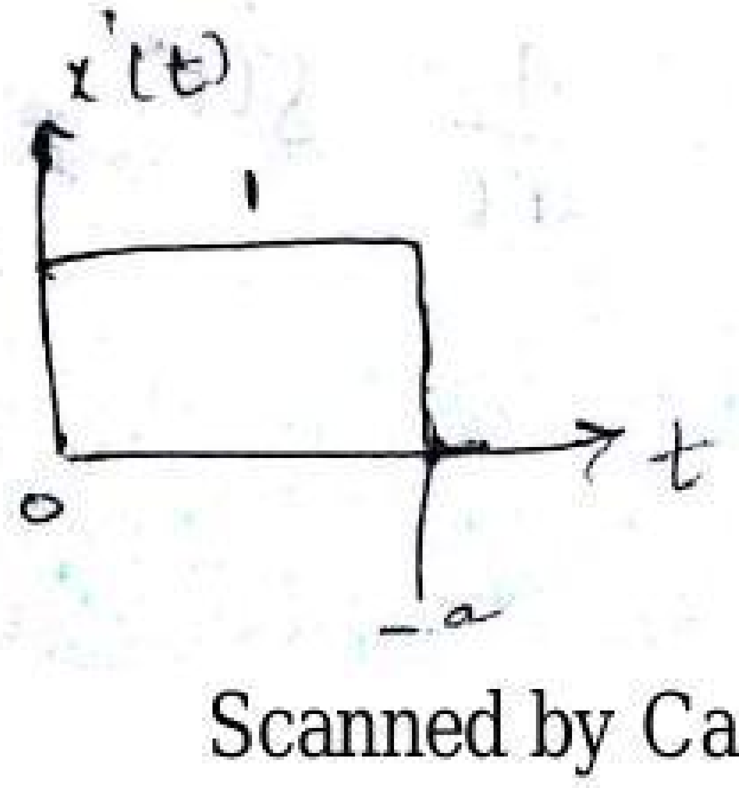
$x'(t) = \frac{d}{dt}\left[t\left[u(t) - u(t-a)\right]\right]$

$x'(t) = t\left[\delta(t) - \delta(t-a)\right] + \left[u(t) - u(t-a)\right]$

$t\delta(t) - t\delta(t-a)$



$\Downarrow$

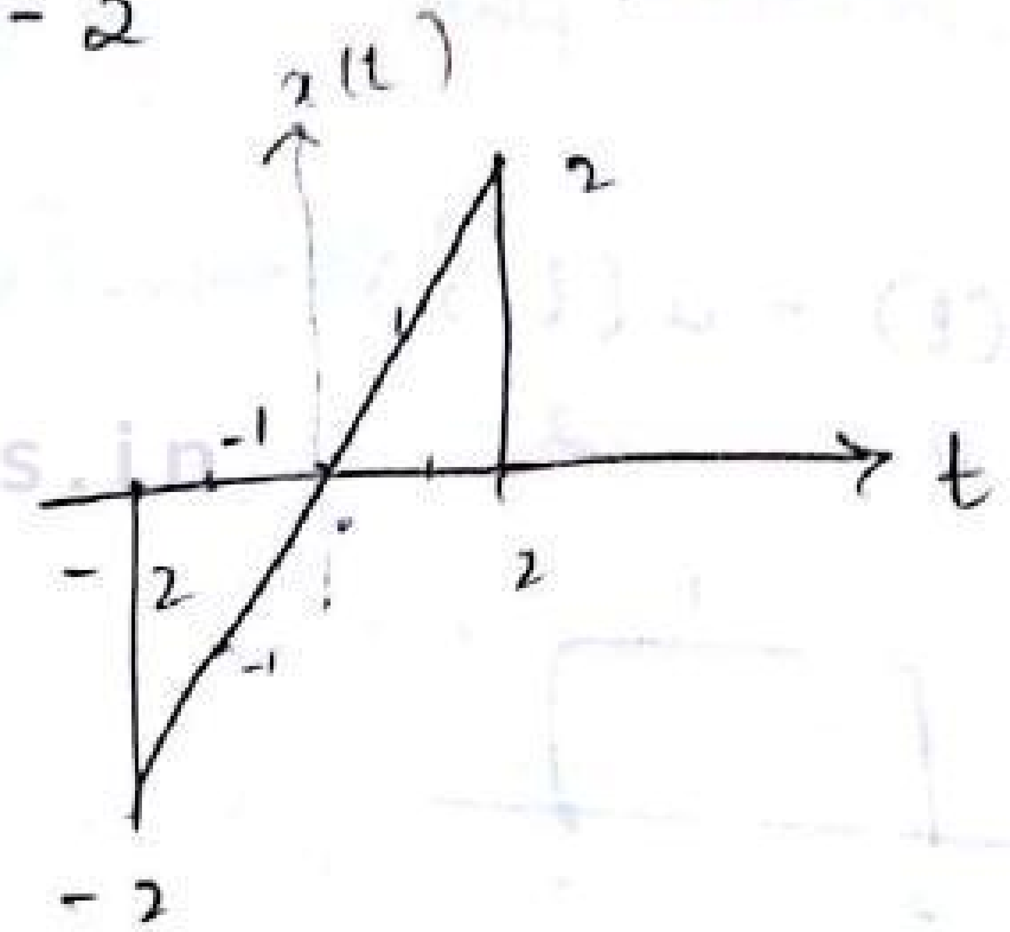


(18) $x(t) = t\,[u(t+2) - u(t-2)]$

sketch $x(t)$ & $x'(t)$

$$t\,u(t+2) - t\,u(t-2)$$

-2 $\qquad$ $+2$



$$x'(t) = \frac{d}{dt}\Big[\underset{u}{t}\,\underset{v}{u(t+2)} - t\,u(t-2)\Big]$$

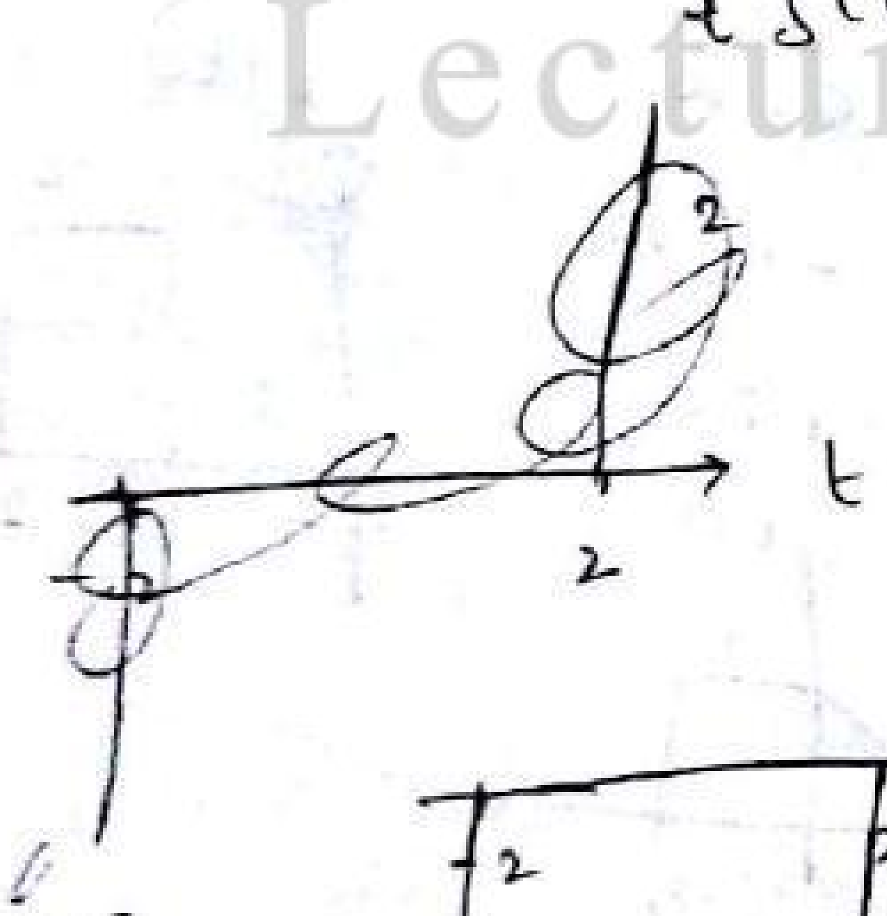
$$\cancel{t\,[\delta(t+2) - \delta(t-2)] + u(t+2)} \cdot$$

$$t\,\delta(t+2) + u(t+2) - u(t-2) - t\,\delta(t-2)$$

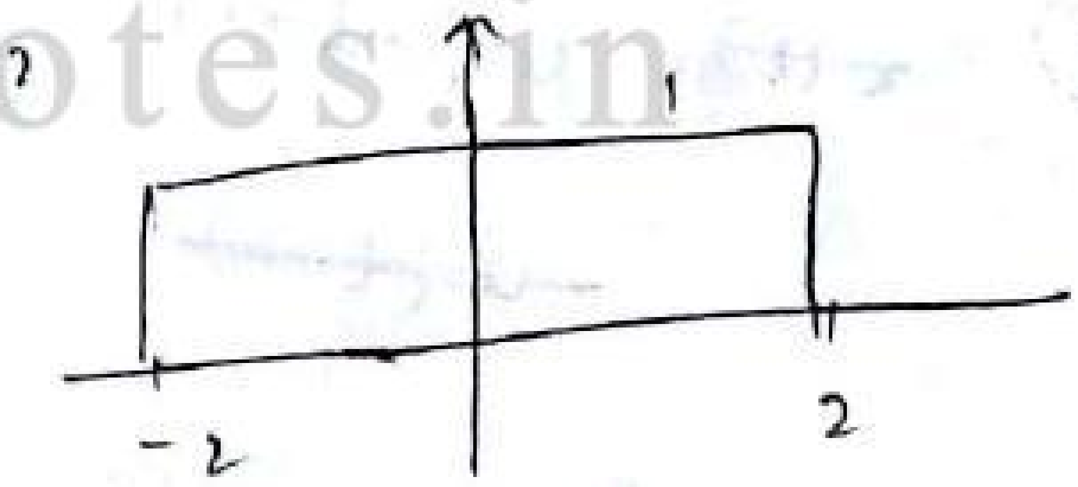
$$t\,[\delta(t+2) - \delta(t-2)] + u(t+2) - u(t-2)$$

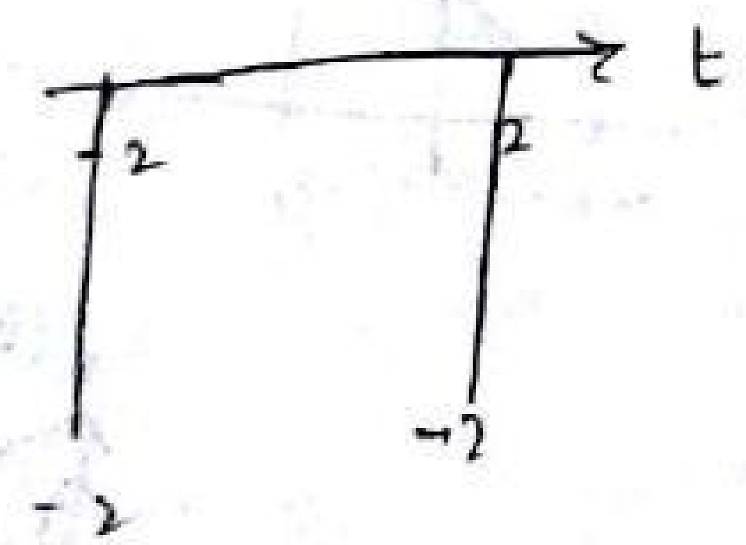
$$t\,\delta(t+2) - t\,\delta(t-2)$$

-2 $\qquad$ 2

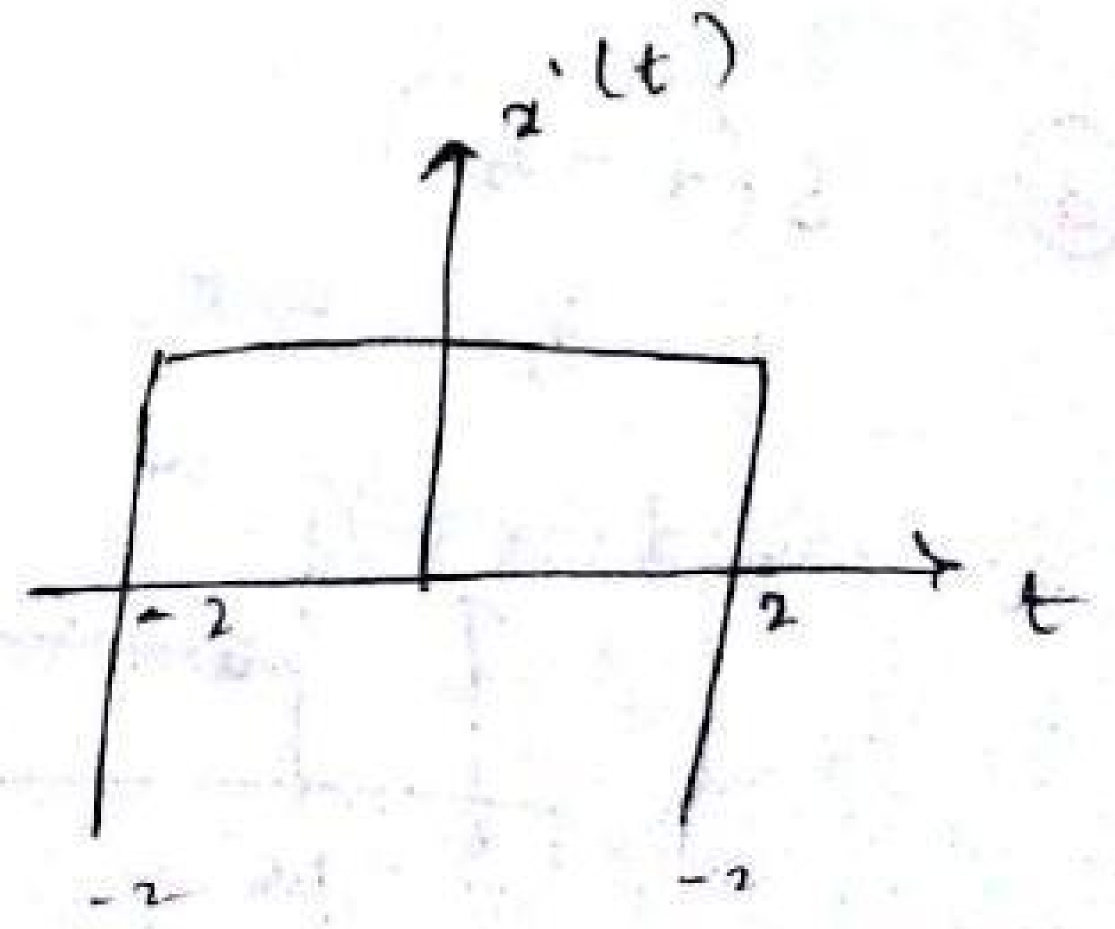


+

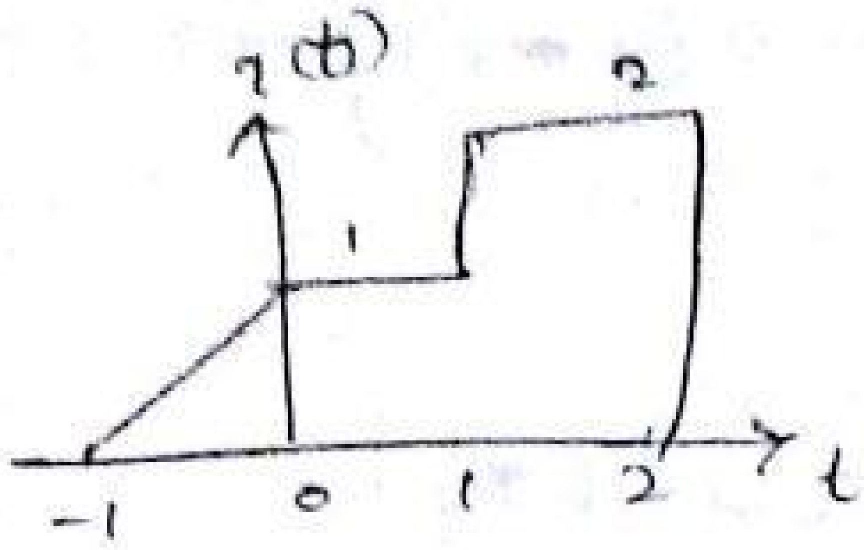




$\Rightarrow$



(19)



Sketch

(a) $x(t)\,[u(t)-u(t-2)]$

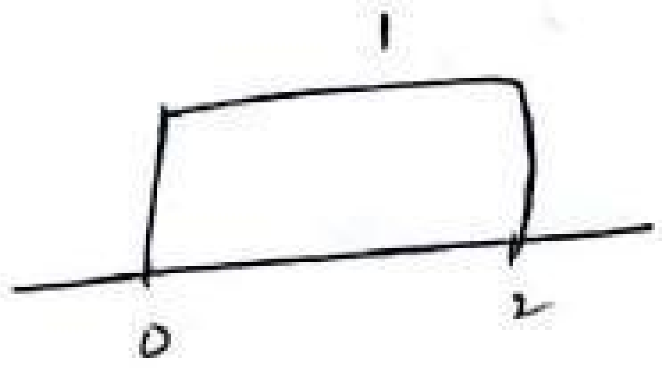
(b) $x(t)\cdot[u(t+1)-u(t)]$

(c) $x(t)\cdot u(-t+1)$

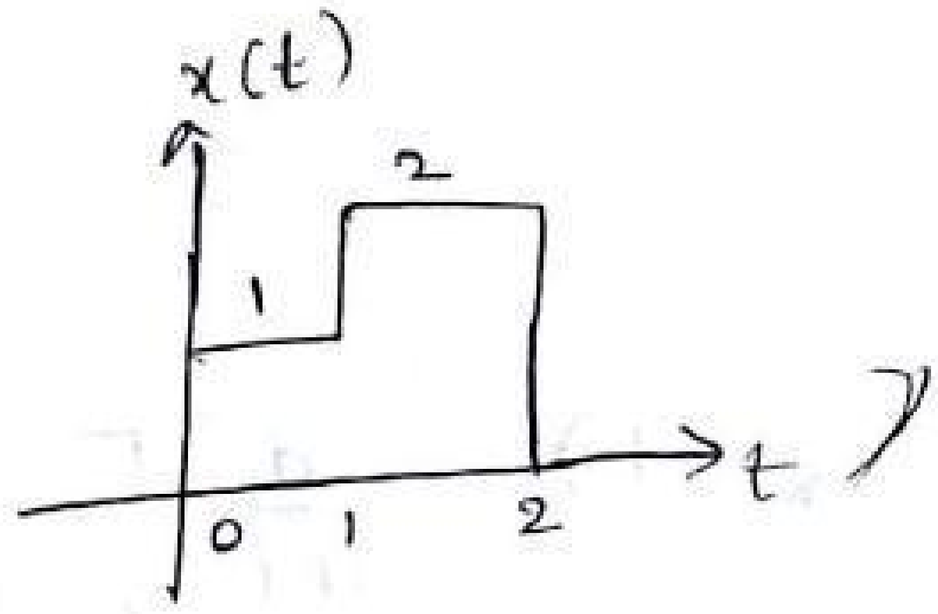
(d) $x(t)\cdot\delta(t-3/2)$

(a) $u(t)-u(t-2)$

0 2

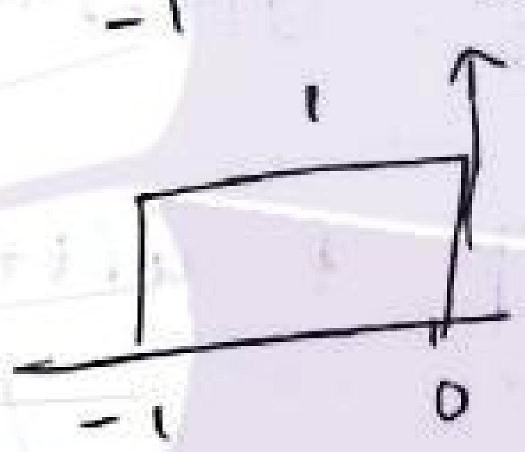


$x(t).\ u(t)-u(t-2)$

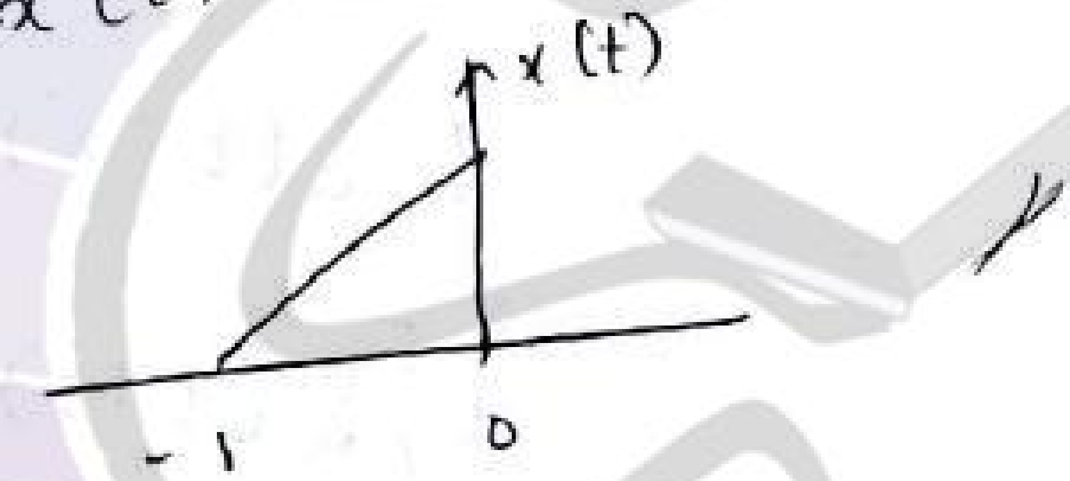


(b) $u(t+1)-u(t)$

-1 0



$x(t).\ u(t+1)-u(t)$



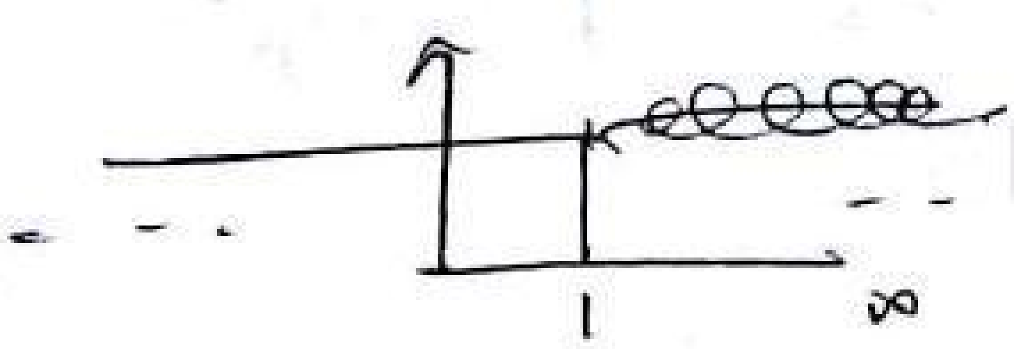
(c) ~~x(t)~~ $u(-t+1)$

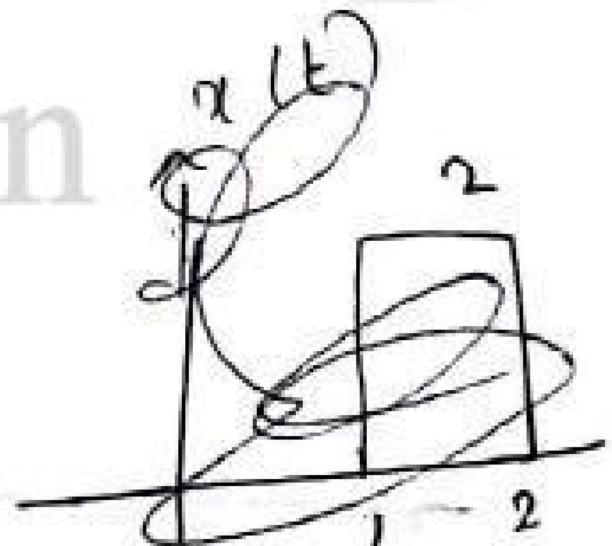
~~u(t+1)~~

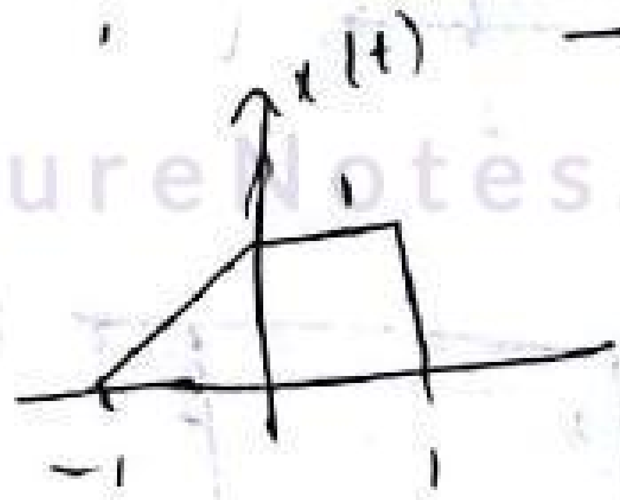
$-t+1$; 0, ∞

$-t$; -1, ∞

t ; 1, ∞



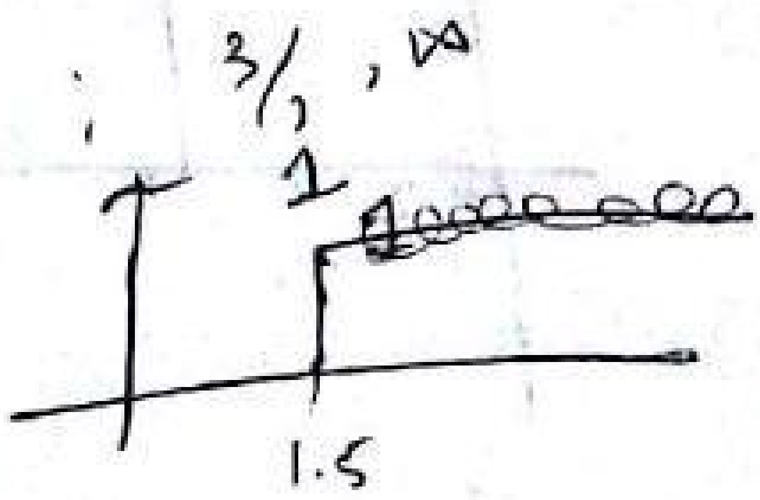


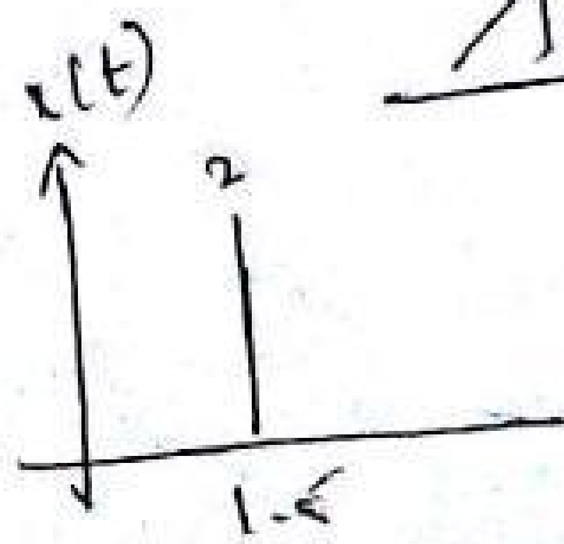


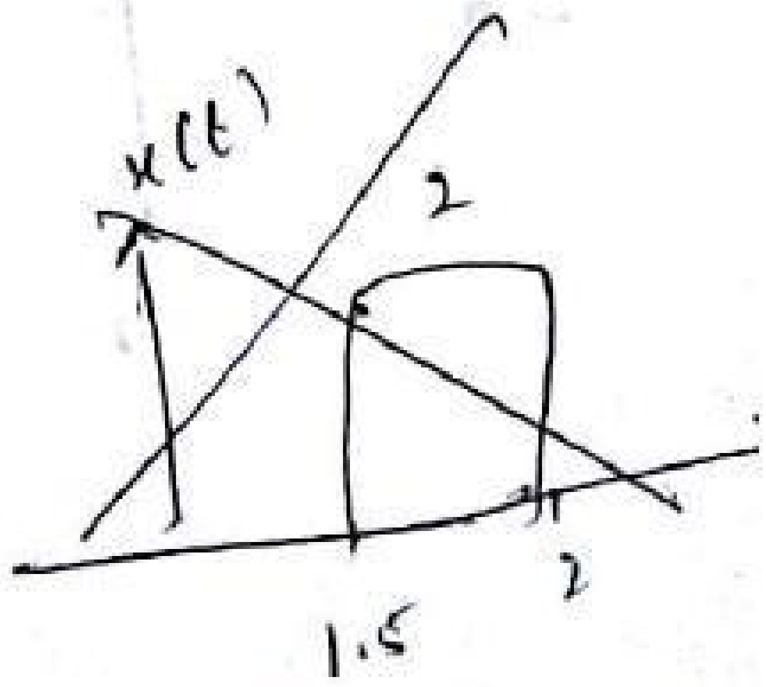
(d) $\delta(t-3/2)$

$t-3/2$; 0, ∞

t ; 3/2, ∞







Properties of System

① **Linearity** :- System is said to be linear for any change in it in i/p amplitude gives same change in o/p Amplitude.

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

$$x(t) \longrightarrow y(t)$$
$$a\, x_1(t) \rightarrow a y_1(t)$$
$$b\, x_2(t) \rightarrow b y_2(t)$$

Superposition theorem

$$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$$

② **Time invarient**

System is said to be Time invarient if shift in i/p time gives same shift in o/p time.

$$x(t) \longrightarrow y(t)$$
$$x(t-a) \longrightarrow y(t-a)$$

③ **Memory**

System is said to be memory if o/p depends on past or futhure value of i/p

④ **Causal** [practically possible]

System is said to be causal if o/p depends on present or past value of the i/p

System is said to be Non-causal if o/p depends on futhure value of the i/p.

$$x(t) \rightarrow \text{present}$$
$$x(t+1) \rightarrow \text{futhure}$$
$$x(t-1) \rightarrow \text{past}$$

$$\left.\begin{array}{l} x(-t) \\ x(t/2) \\ x(3t) \end{array}\right\} ?$$

$y(t) = x(t+3)$ NC, M

$y(t) = x(t-1) + x(t)$ C, M

$y(t) = x(t)$ C, ML

$y(t) = x(t) + x(t+2)$ NC, M

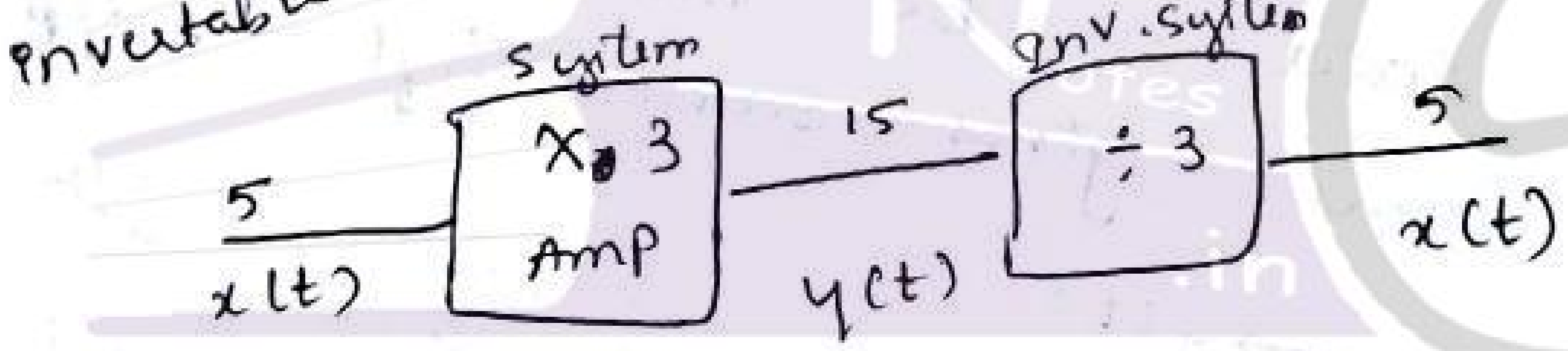
⑤ Stability [Working]

System is said to be stable if bounded i/p [finite i/p] gives bounded o/p

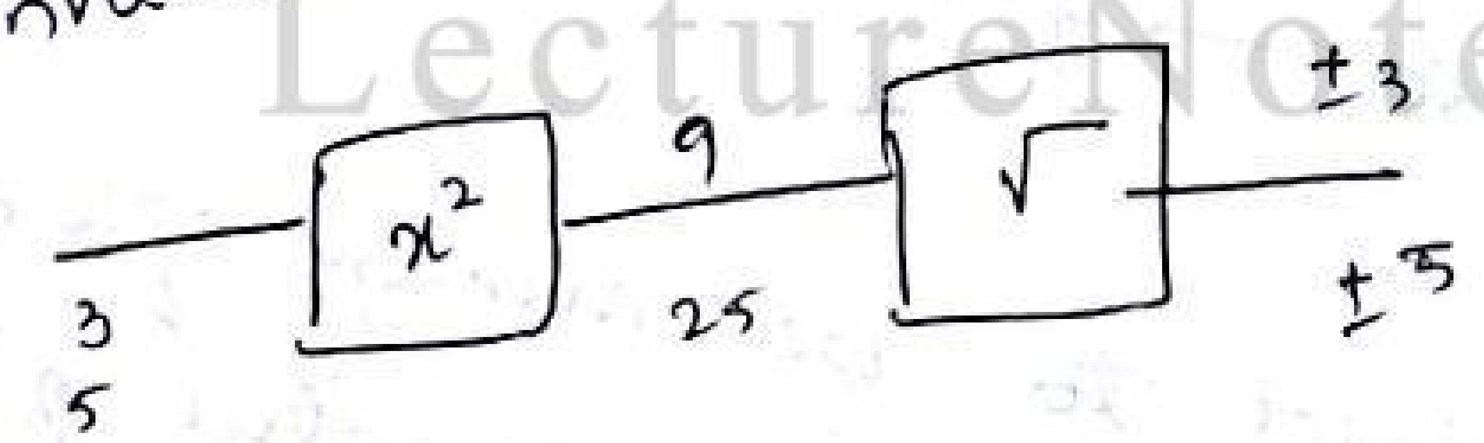
⑥ Invertable

System is said to be invertable if we can Recover the i/p from the o/p

invertable



Non-invertable



⑦ static and dynamic

If system character varies with respect to amplitude and time then such systems are dynamic system

If system character remains constant then such system are called static system.

Check for linearity

1) $y(t) = \cos[x(t)]$

$x(t) \rightarrow y(t)$

$x(t) \rightarrow \cos(x(t))$

$ax_1(t) \rightarrow \cos[ax_1(t)]$

$bx_2(t) \rightarrow \cos[bx_2(t)]$

add the o/p

$\rightarrow \cos[ax_1(t)] + \cos[bx_2(t)] \rightarrow (1)$

add the i/p

$ax_1(t) + bx_2(t)$

$x_3(t) \rightarrow \cos(x_3(t))$

$\rightarrow \cos(ax_1(t) + bx_2(t)) \rightarrow (2)$

$(1) \neq (2)$

Non-linear.

2) $y(t) = e^{x(t)}$

$x(t) \rightarrow y(t)$

$x(t) \rightarrow e^{x(t)}$

$ax_1(t) \rightarrow e^{ax_1(t)}$

$bx_2(t) \rightarrow e^{bx_2(t)}$

add the o/p

$e^{ax_1(t)} + e^{bx_2(t)} \rightarrow (1)$

$\left\{ e^{a+b} = e^a \cdot e^b \right.$

add the i/p

$ax_1(t) + bx_2(t) \rightarrow$

$x_3(t) \rightarrow e^{x_3(t)}$

$\rightarrow e^{ax_1(t) + bx_2(t)} \rightarrow (2)$

$(1) \neq (2)$

Non-linear

③ $y(n) = \log[x(n)]$

$x(n) \rightarrow y(n)$

$x(n) = \log[x(n)]$

$ax_1(n) = a\log[ax(n)]$

$bx_2(n) = b\log[bx(n)]$

add o/p

$a\log[ax(n)] + b\log[bx(n)] \rightarrow$ ①

add i/p

$\underbrace{ax_1(n) + bx_2(n)}_{x_3(t)} \rightarrow$

$x_3(t) \rightarrow \log[ax_1(n) + bx_2(n)] \rightarrow$ ②

① $\neq$ ②

Non - linear

④ $y(n) = x(-n)$

$x(n) \rightarrow y(n)$

$x(n) = x(-n)$

$ax_1(n) = ax_1(-n)$

$bx_2(n) = bx_2(-n)$

add o/p

$ax_1(-n) + bx_2(-n) \rightarrow$ ①

$\underbrace{ax_1(n) + bx_2(n)}_{x_3(t)} =$

$x_3(t) \rightarrow x(-n)$

$\rightarrow ax_1(-n) + bx_2(-n) \rightarrow$ ②

① = ②

linear

5) check for Time invarient

$y(t) = \cos[x(t)]$

$x(t) \to y(t)$

$x(t) \to \cos[x(t)]$

delay the i/p

$x(t-t_0) \to \cos[x(t-t_0)] \to ①$

change $t \to t-t_0$

$x(t-t_0) \to \cos[x(t-t_0)] \to ②$

① = ②

Time in-varient

6)

$y(t) = e^{x(t)}$

$x(t) \to y(t)$

$x(t) \to e^{x(t)}$

delay the i/p

$x(t-t_0) \to e^{x(t-t_0)} \to ①$

change $t \to t-t_0$

$x(t-t_0) \to e^{x(t-t_0)} \to ②$

① = ②

Time in-varient

7)

$y(t) = \log[x(t)]$

$x(t) = y(t)$

$x(t) = \log[x(t)]$

delay i/p

$x(t-t_0) = \log[x(t-t_0)] \to ①$

change t to $t-t_0$

$x(t-t_0) = \log[x(t-t_0)] \to ②$

① = ②

Time in-varient

8)

$y(n) = x(-n)$

$x(n) = y(n)$

$x(n) = x(-n)$

(12) $y(n) = x(-n)$

$x(n) \rightarrow x(-n)$

n	$-n$	
-3	3	past
-2	2	
-1	1	
0	0	present
1	-1	future
2	-2	
3	-3	

a) Memory except $n=0$
b) Non causal $n>0$
c) stable
d) invertable
e) dynamic

(13) $y(t) = 2x(t) + 5$

* $x(t) \rightarrow y(t)$

$x(t) \rightarrow 2x(t) + 5$

$ax_1(t) \rightarrow 2ax_1(t) + 5$

$bx_2(t) \rightarrow 2bx_2(t) + 5$

add O/p

$2a_x(t) + 2bx_2(t) + 10 \rightarrow (1)$

add i/p

$\underbrace{ax_1(t) + bx_2(t)}$

$x_3(t) \rightarrow y_3(t) \quad 2x_3(t) + 5$

$x_3(t) \rightarrow 2[ax_1(t) + bx_2(t)] + 5$

$x_3(t) \rightarrow 2a_x(t) + 2bx_2(t) + 5 \rightarrow (2)$

$(1) \neq (2)$

Non - linear

* $x(t) \rightarrow y(t)$

$x(t) \rightarrow 2x(t) + 5$

Delay the i/p

$x(t-t_0) \rightarrow 2x(t-t_0) + 5 \rightarrow (1)$

change t to $(t - t_0)$

$x(t-t_0) \rightarrow 2x(t-t_0)+5 \rightarrow (2)$

$(1) = (2)$

Time invarient.

* It is Memory less [only present i/p]
* It is Causal [There is no future i/p]
* stable [finite i/p & finite o/p]
* It is invertable
* It is dynamic

(14) $y(n) = x(n) + u(n+1)$

$x(n) \rightarrow y(n)$

$x(n) \rightarrow x(n) + u(n+1)$

* $a x_1(n) \rightarrow a x_1(n) + u(n+1)$

$b x_2(n) \rightarrow b x_2(n) + u(n+1)$

add o/p

$a x_1(n) + b x_2(n) + 2u(n+1) \rightarrow (1)$

add i/p

$\underbrace{a x(n) + b x(n)}_{x_3(n)}$

$x_3(n) \rightarrow x_3(n) + u(n+1)$

$x_3(n) \rightarrow a x_1(n) + b x_2(n) + u(n+1) \rightarrow (2)$

$(1) \neq (2)$

Non - linear

* dynamic
* Delay time

$x(n) \rightarrow x(n) + u(n+1)$

$x(n-n_0) \rightarrow x(n-n_0) + u(n+1) \rightarrow (1)$

Scanned by CamScanner

Change n to n-no

$x(n-n_0) \to x(n-n_0) + u(n^{n_0}+1) \to (2)$

$(1) \neq (2)$

Time variant.

+ It is Memoryless
+ It is Causal
+ It is Stable
+ invertable.

(15) $y(n) = x(n-1) + x(n+2)$

$x(n) \to y(n)$

$x(n) \to x(n-1) + x(n+2)$

+ $a x_1(n) \to a x_1(n-1) + a x_1(n+2)$

$b x_2(n) \to b x_2(n-1) + b x_2(n+2)$

add o/p

$a x_1(n-1) + a x_1(n+2) + b x_2(n-1) + b x_2(n+2) \to (1)$

add i/p

$\underbrace{a x_1(n) + b x_2(n)}_{x_3(n)} \Rightarrow x_3(n-1) + x_3(n+2)$

$\to a x_1(n-1) + b x_2(n-1) +$

$a x_1(n+2) + b x_2(n+2) \to (2)$

$(1) = (2)$

linear

+ Delay time

$x(n-n_0) \to x(n-n_0-1) + x(n-n_0+2) \to (1)$

change n to n-no

$x(n-n_0) \to x(n-n_0-1) + x(n-n_0+2) \to (2)$

$(1) = (2)$

Time invariant

* Static
* Memory
* Non-causal
* Stable
* Non invertable [2 i/p]

(16) $y(t) = x(t/2)$

$x(t) \to y(t)$

$x(t) \to x(t/2)$

* $ax_1(t) \to ax_1(t/2)$

$bx_2(t) \to bx_2(t/2)$

add o/p

$ax_1(t/2) + bx_2(t/2) \to (1)$

add i/p

$\underbrace{ax_1(t) + bx_2(t)}$

$x_3(t) \to x_3(t/2)$

$x_3(t) = ax_1(t/2) + bx_2(t/2) \to (2)$

$(1) = (2)$

linear.

* Delay time.

$x(t-t_0) = \cancel{x\left(\frac{t-t_0}{2}\right)} \to (1)$

$x\left(\frac{t}{2} - t_0\right)$

Change t to $t - t_0$

$x(t-t_0) = x\left(\frac{t-t_0}{2}\right) \to (2)$

$(1) \neq (2)$

Time variant

* dynamic
* Memory except $t=0$
* Non causal for $t \geq 1$
* stable
* invertable

(17) $y(n) = x^2(n-2) = [x(n-2)]^2$

$x(n) \rightarrow y(n)$

$x(n) \rightarrow [x(n-2)]^2$

* $a x_1(n) \rightarrow [a x_1(n-2)]^2$

$b x_2(n) \rightarrow [b x_2(n-2)]^2$

add o/p

$[a x_1(n-2)]^2 + [b x_2(n-2)]^2 \rightarrow (1)$

add i/p

$\underbrace{a x_1(n) + b x_2(n)}$

$x_3(n) \rightarrow [x_3(n-2)]^2$

$\rightarrow [a x_1(n-2) + b x_2(n-2)]^2$

$x_3(n) \rightarrow [a x_1(n-2)]^2 + [b x_2(n-2)]^2 + 2a x_1(n-2) b x_2(n-2) \rightarrow (2)$

$(1) \neq (2)$

~~Non~~ Non-linear

* Delay time

$x(n-n_0) \rightarrow [x(n-n_0-2)]^2 \rightarrow (1)$

Change n to $n-n_0$

$x(n-n_0) \rightarrow [x(n-n_0-2)]^2 \rightarrow (2)$

$(1) = (2)$

time invariant

* ~~Static~~ Dynamic
* memory
* Causal
* stable
* Non-invertable

18) $y(t) = \frac{d}{dt}\left[e^{-t}x(t)\right]$

$x(t) \rightarrow y(t)$

$x(t) \rightarrow \frac{d}{dt}\left[e^{-t}x(t)\right]$

$ax_1(t) \rightarrow \frac{d}{dt}\left[e^{-t}ax_1(t)\right]$

$bx_2(t) \rightarrow \frac{d}{dt}\left[e^{-t}bx_2(t)\right]$

add o/p

$$\frac{d}{dt}\left[e^{-t}ax_1(t)\right] + \frac{d}{dt}\left[e^{-t}bx_2(t)\right] \rightarrow ①$$

add i/p

$$\underbrace{ax_1(t) + bx_2(t)}_{x_3(t)} \rightarrow \frac{d}{dt}\left[e^{-t}x_3(t)\right]$$

$$\rightarrow \frac{d}{dt}\left[e^{-t}\left[ax_1(t) + bx_2(t)\right]\right]$$

$$= \frac{d}{dt}\left[e^{-t}ax_1(t) + e^{-t}bx_2(t)\right]$$

$$= \frac{d}{dt}\left[e^{-t}ax_1(t)\right] + \frac{d}{dt}\left[e^{-t}bx_2(t)\right] \rightarrow ②$$

① = ②

Linear

Delay time

$$x(t-t_0) = \frac{d}{dt}\left[e^{-t}x(t-t_0)\right] \rightarrow ①$$

Change $t = t_0 - t_0$

$$x(t-t_0) = \frac{d}{d(t)}\left[e^{-(t-t_0)}x(t-t_0)\right] \rightarrow ②$$

① ≠ ②

Time variant

Module - 2

Convolution / Time domain Representation of $\frac{1}{m}$

Convolution :- It is a Mathematical operator which is sum of product of two sequence with one sequence as time shifted quantity.

Consider sequences $p(n)$ & $q(n)$ then the Convolution $r(n) = p(n) * q(n)$

Convolution Sum

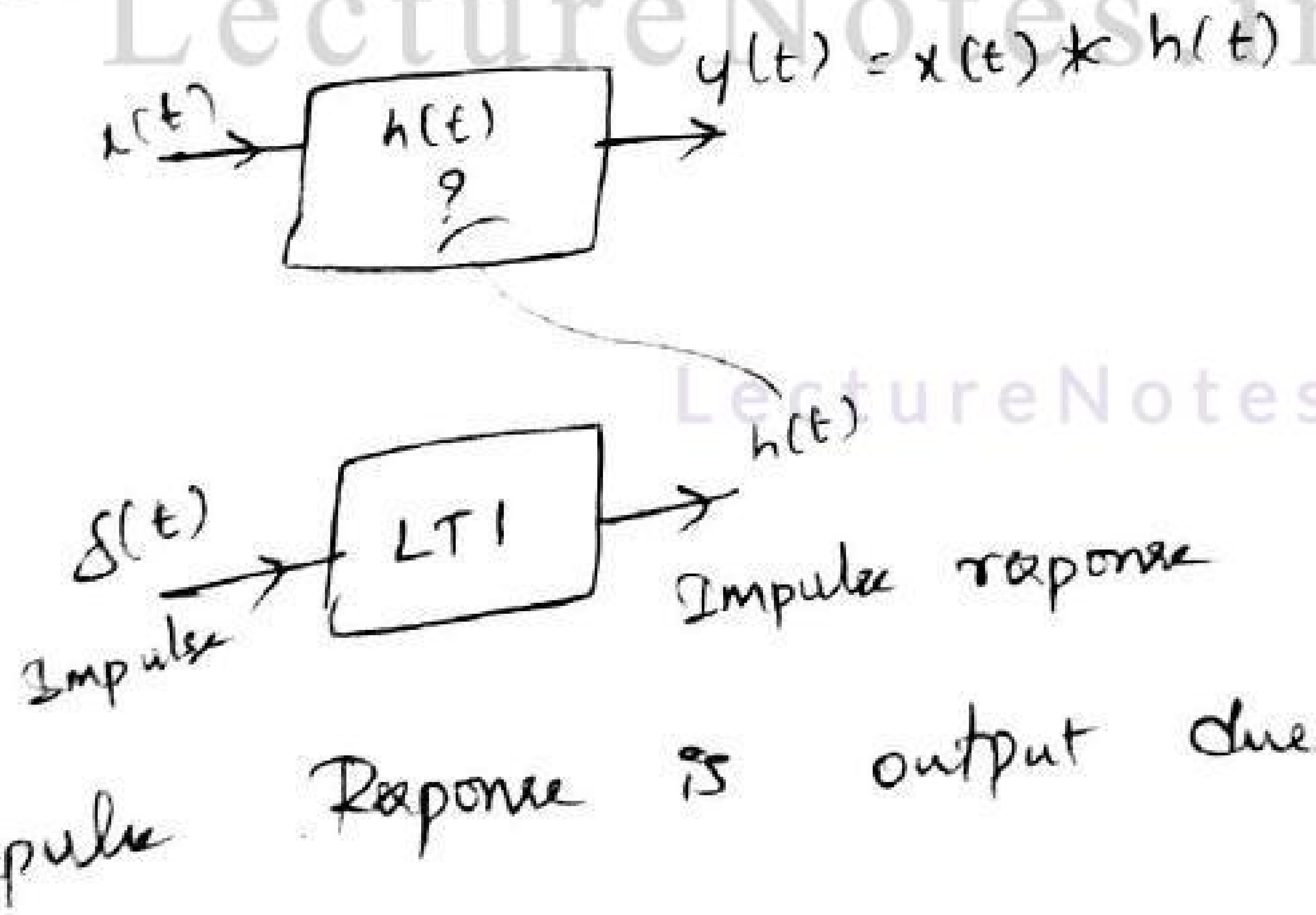
$$\boxed{r(n) = \sum_{k=-\infty}^{\infty} \underset{\text{Dummy}}{p(k)} \cdot q(\underset{\text{Main}}{n} - k)}$$

Illy $r(t) = p(t) * q(t)$

Convolution integral

$$\boxed{r(t) = \int_{\tau=-\infty}^{\infty} \underset{\text{Dummy}}{p(\tau)} \cdot q(\underset{\text{main}}{t} - \tau)\, d\tau}$$

Impulse Response Representation



Impulse Response is output due to impulse i/p.

- Impulse Response Completely Characterizes the behaviour of LTI System convolution sum

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

convolution integral

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

1) $x(n) = \{\underset{\uparrow}{2}, 3, 1\}$

$h(n) = \{\underset{\uparrow}{1}, 2\}$

find $y(n) = x(n) * h(n)$

Sol^n

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(n) = \sum_{k=0}^{2} x(k)\, h(n-k)$$

$\boxed{n=0}$

$$y(0) = \sum_{k=0}^{2} x(k)\, h(-k)$$

$$y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2)$$

$$= 2\times1 + 3\times0 + 1\times0$$

$$= 2 + 0 + 0$$

$$\boxed{y(0) = 2}$$

$\boxed{n=1}$

$$y(1) = \sum_{k=0}^{2} x(k)\, h(1-k)$$

$$= x(0)h(1-0) + x(1)h(1-1) + x(2)h(1-2)$$

$$= x(0)h(1) + x(1)h(0) + x(2)h(-1)$$

$$= 2\times2 + 3\times1 + 1\times0$$

$$= 4 + 3 + 0$$

$$\boxed{y(1) = 7}$$

$\boxed{n=2}$

$$y(2) = \sum_{k=0}^{2} x(k)\, h(2-k)$$

$$= x(0)h(2-0) + x(1)h(2-1) + x(2)h(2-2)$$

$$= x(0)h(2) + x(1)h(1) + x(2)h(0)$$

$$= 2\times 0 + 3\times 2 + 1\times 1$$

$$6+1$$

$$\boxed{y(2) = 7}$$

Short cut

	2	3	1
1	2	3	1
2	4	6	2

$\boxed{2 \;\; 7 \;\; 7 \;\; 2}$

$\boxed{n=3}$

$$y(3) = \sum_{k=0}^{2} x(k)\, h(3-k)$$

$$= x(0)h(3-0) + x(1)h(3-1) + x(2)h(3-2)$$

$$= x(0)h(3) + x(1)h(2) + x(2)h(1)$$

$$= 2\times 0 + 3\times 0 + 1\times 2$$

$$\boxed{y(3) = 2}$$

$$y(n) = \{\underset{\uparrow}{2}, 7, 7, 2\}$$

NOTE:-

i) Summation limit is x - limit.

ii) Total length of $y(n)$ is $L = M+N-1$
where M is length of $x(n)$,
N is length of $h(n)$

iii) $y(n)$ varies from total values in negative side to total values in positive side in both $x(n)$ & $h(n)$

② $x(n) = \{2, \underset{\uparrow}{1}, 3, -2\}$

$h(n) = \{5, \underset{\uparrow}{2}, 1\}$

$y(n) = x(n) * h(n)$

$L = M + N - 1$

$4 + 3 - 1 = 6$

	2	1	3	-2
5	10	5	15	-10
→ 2	4	2	6	-4
1	2	1	3	-2

$10, 9, \underset{\uparrow}{19}, -3, -1, -2$

Convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$n=0 \quad y(0) = \sum x(k) h(-k)$

$n=1 \quad y(1) = \sum x(k)(-k+1)$

$n=2 \quad y(2) = \sum x(k)\, h(-k+2)$

$n=3 \quad y(3) = \sum x(k)\, h(-k+3)$

NOTE [Convolution sum]

i) Given Signals are $x(n)$ & $h(n)$

ii) Modify $x(n) \to x(k)$

iii) Modify $h(n) \to h(k) \to h(-k) \to h(-k+n)$

iv) $x(k)$ is fixed and $h(-k+n)$ is shifting Right side by varying n-value from $-\infty$ to ∞

v) when Both Signal is having Common area Multiply & Add to get final Result $y(n)$

NOTE [Convolution Integral]

1) Given signals are $x(t)$ and $h(t)$

2) Modify $x(t) \rightarrow x(\tau)$

3) Modify $h(t) \rightarrow h(\tau) \rightarrow h(-\tau) \rightarrow h(-\tau+t)$

4) $x(\tau)$ is fixed and $h(-\tau+t)$ is shifting Right side by variying 't' value from $-\infty$ to ∞

5) when both signal is having Common area Multiply & integrate to get final Result $y(t)$

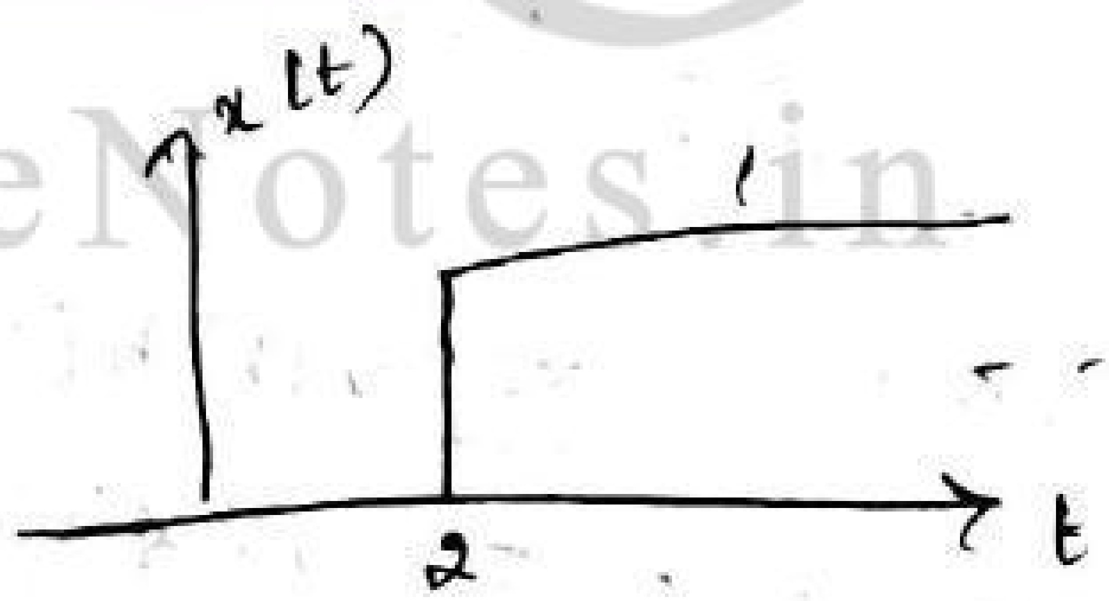
③ $x(t) = u(t-2)$

$h(t) = u(t+1)$

find $y(t) = x(t) * h(t)$

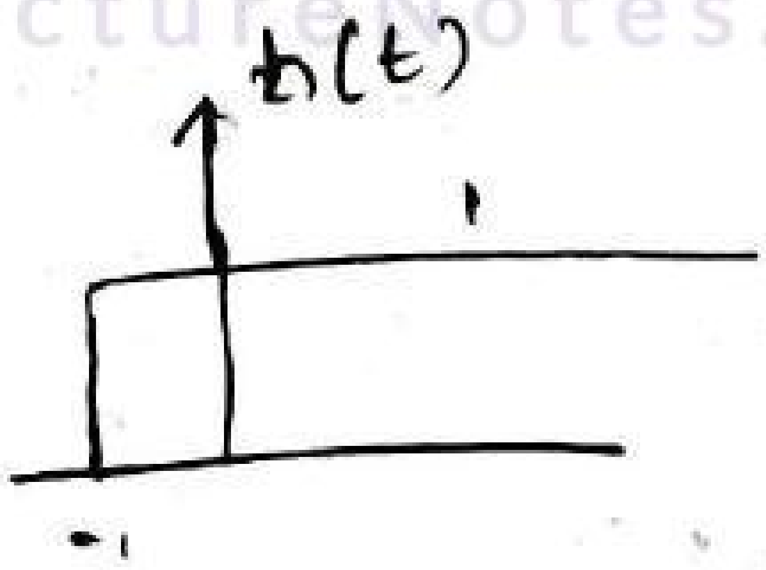
$t-2 ; 0, \infty$

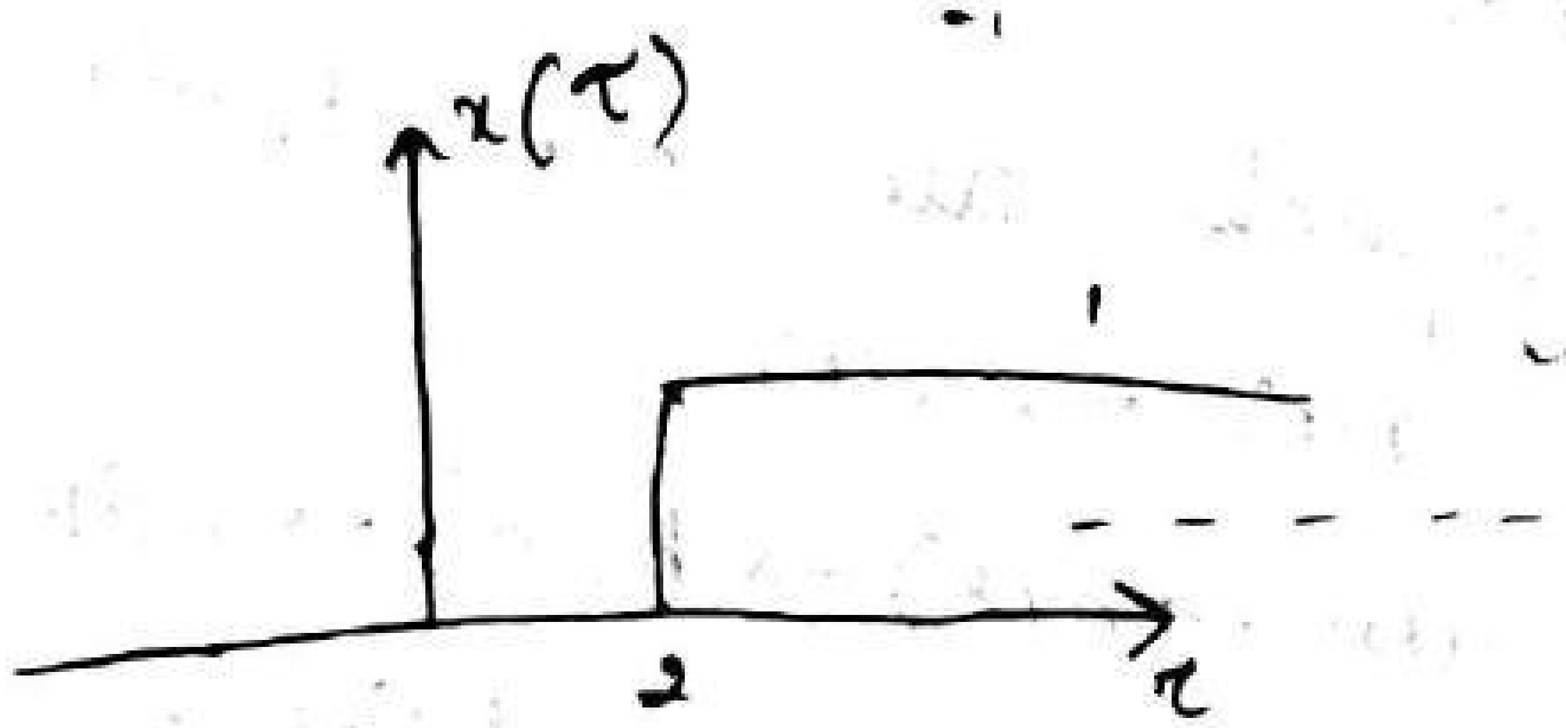
$t ; 2, \infty$

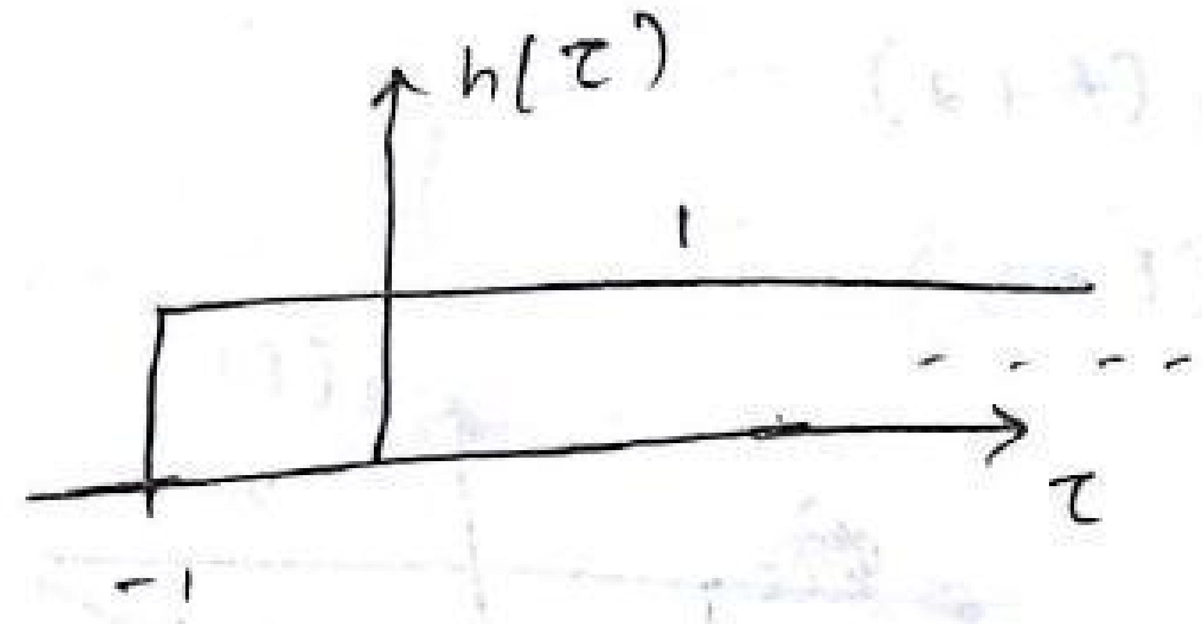


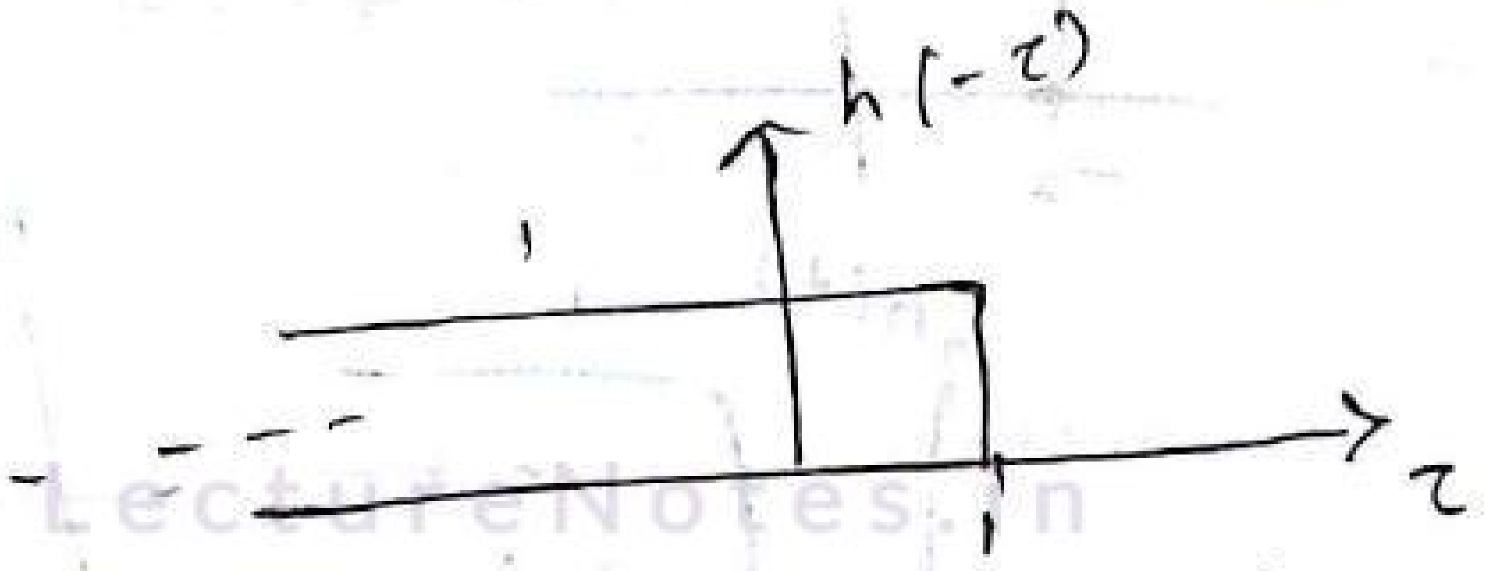
$t+1 ; 0, \infty$

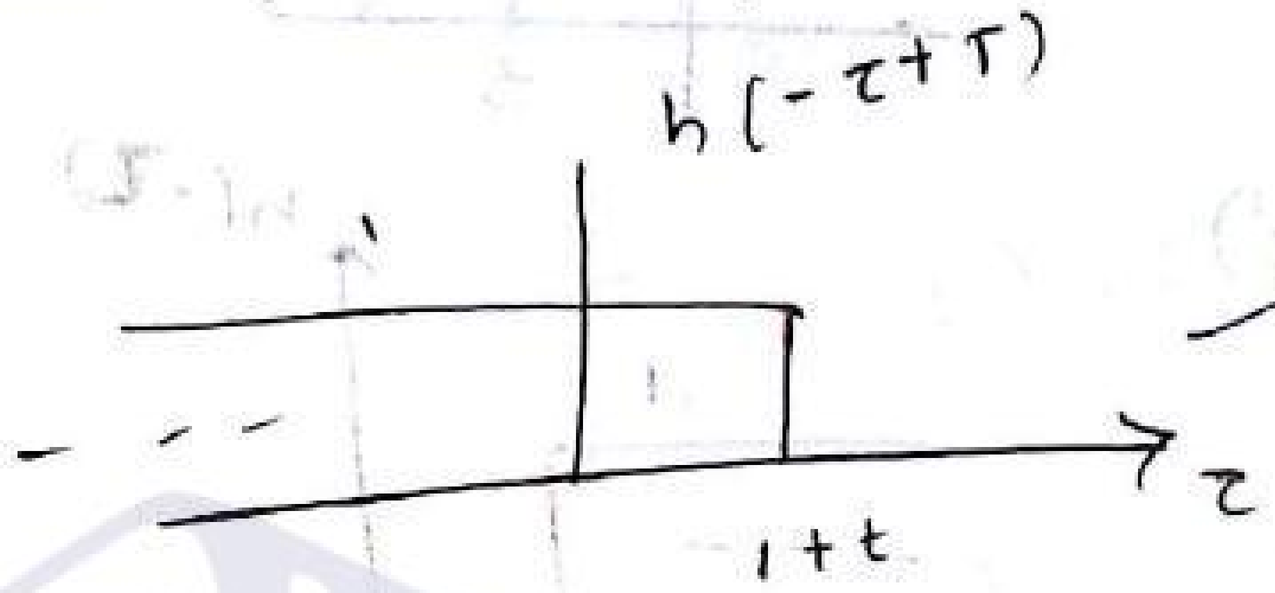
$t ; -1, \infty$

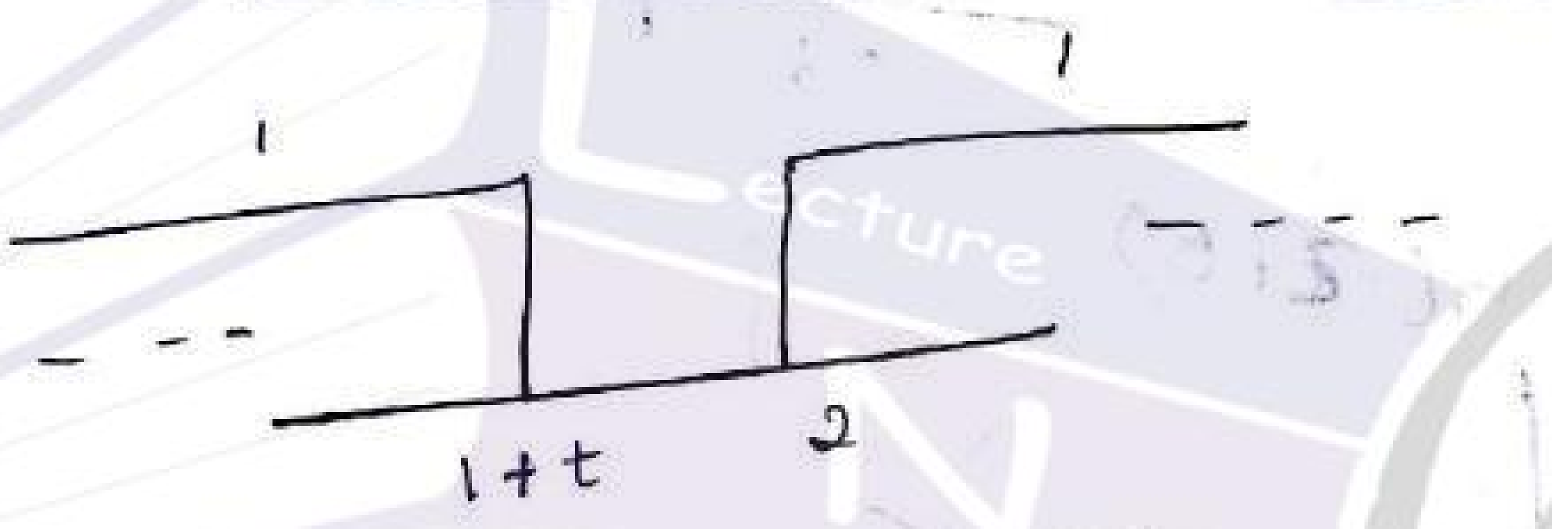




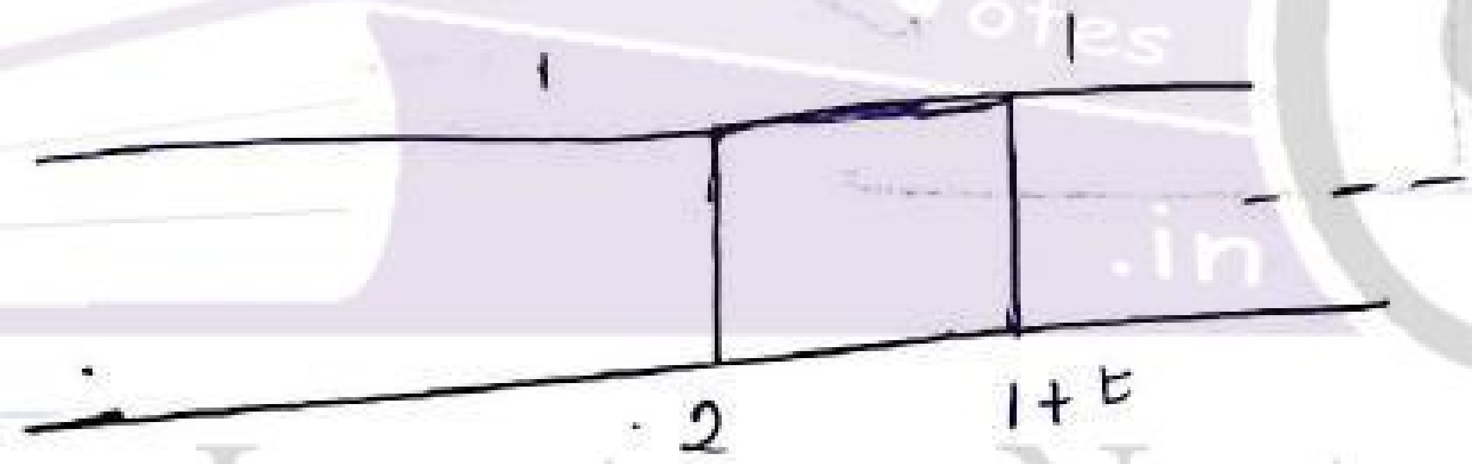








$= 0 \quad -\infty \le t < 1$



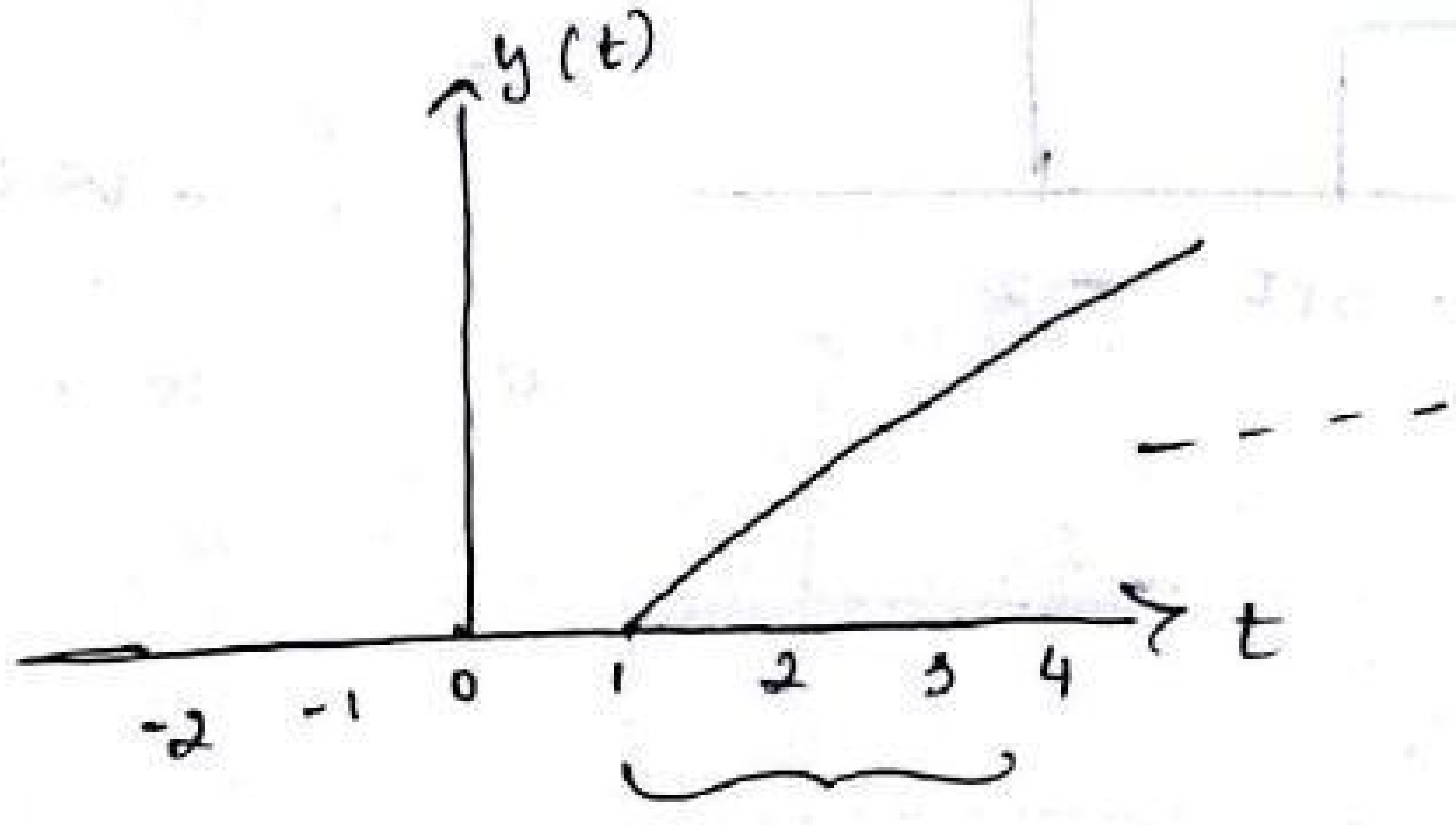
$$\int_{2}^{1+t} 1.1 \, d\tau \qquad 1 \le t \le \infty$$

$$= [\tau]_2^{1+t}$$

$$= 1 + t - 2$$

$$= t - 1$$

$$y(t) = \begin{cases} 0 & -\infty \le t < 1 \\ t-1 & 1 \le t \le \infty \end{cases}$$

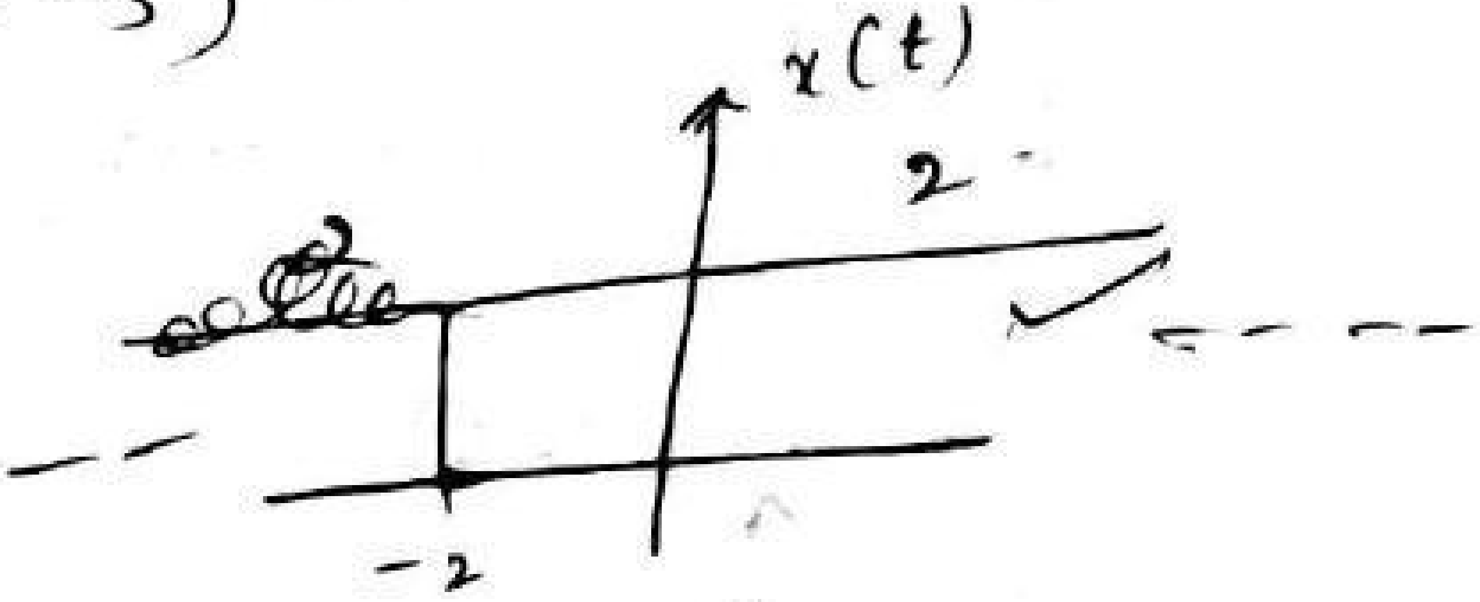


(4) $x(t) = 2u(t+2)$

$h(t) = u(t-3)$

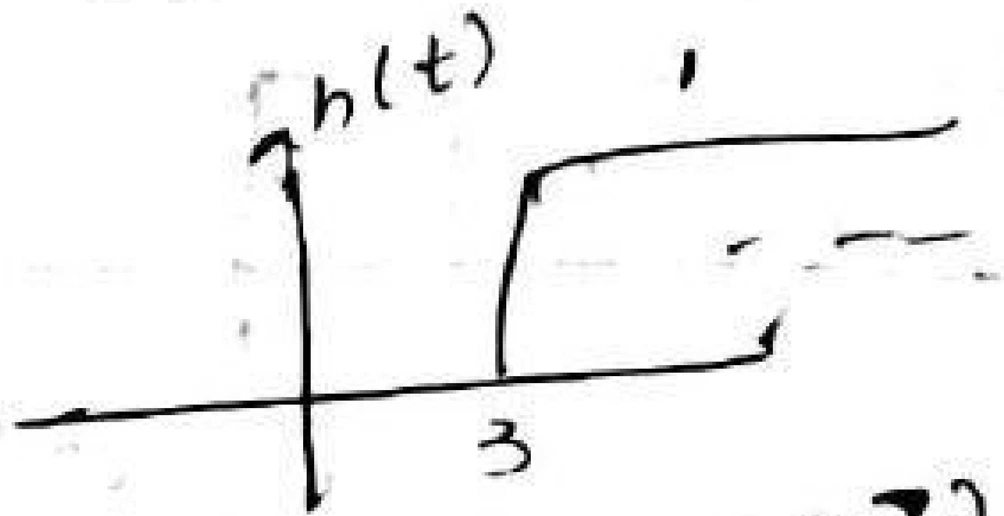
$t+2 : 0, \infty$

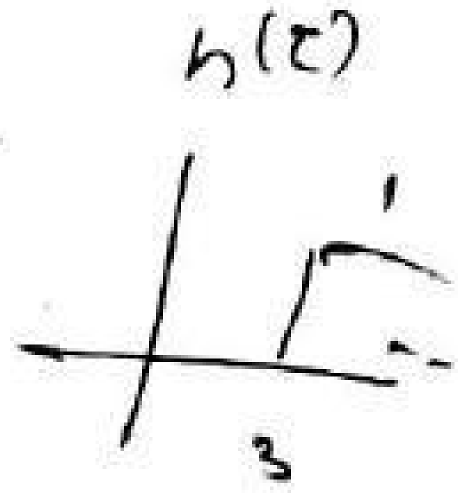
$t : -2, \infty$

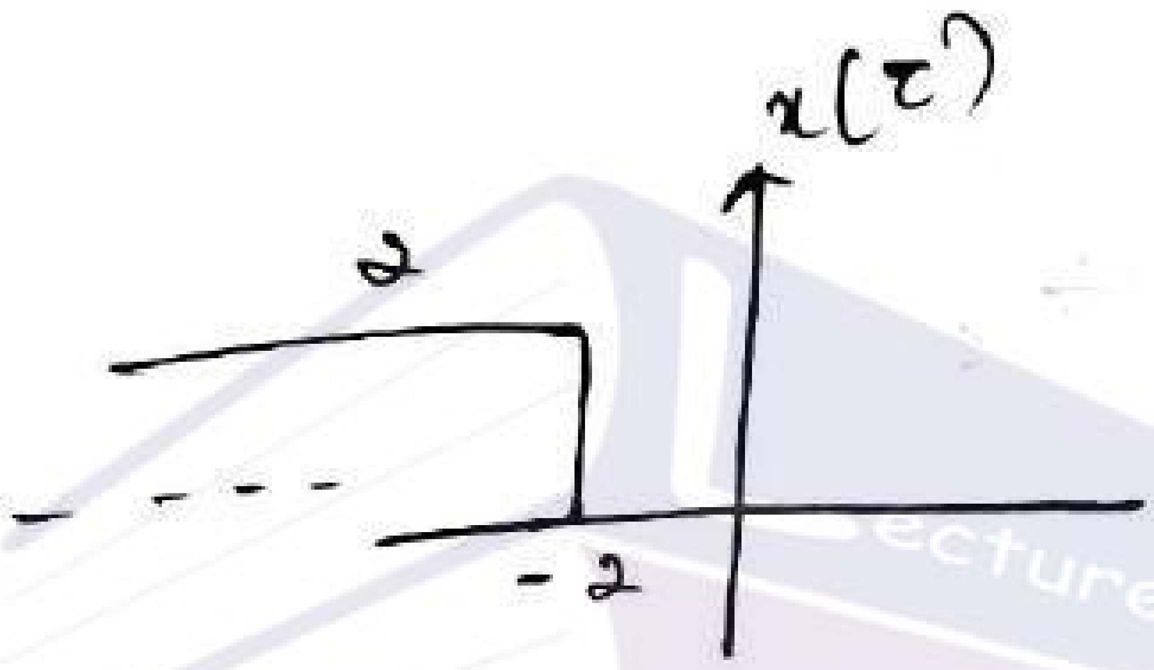


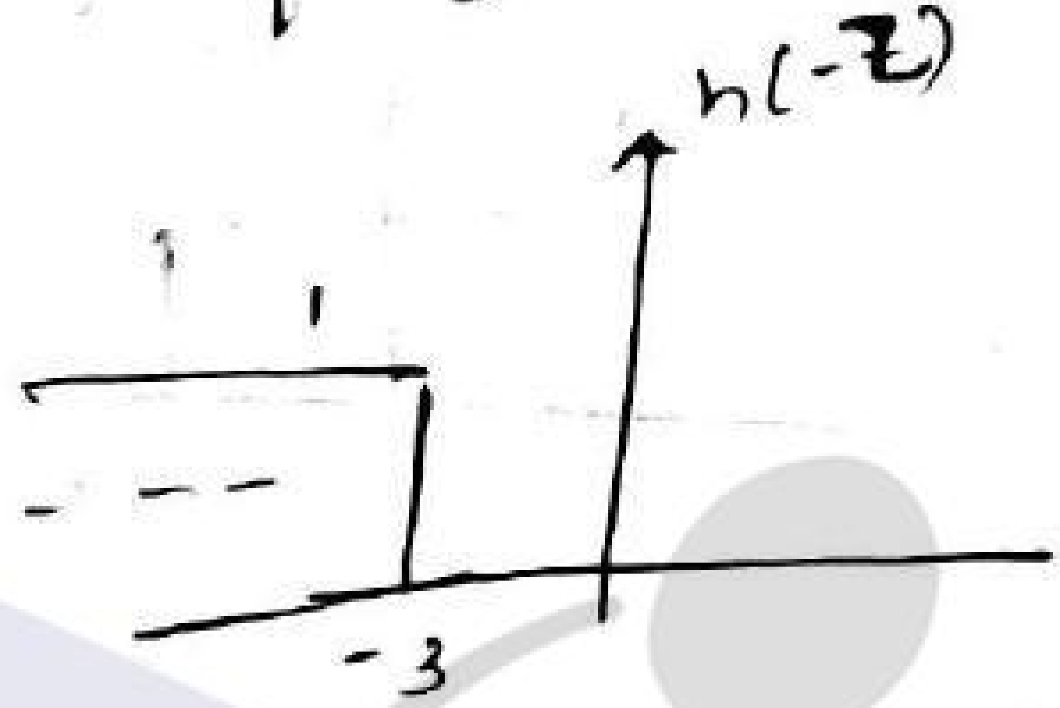
$t-3 ; 0, \infty$

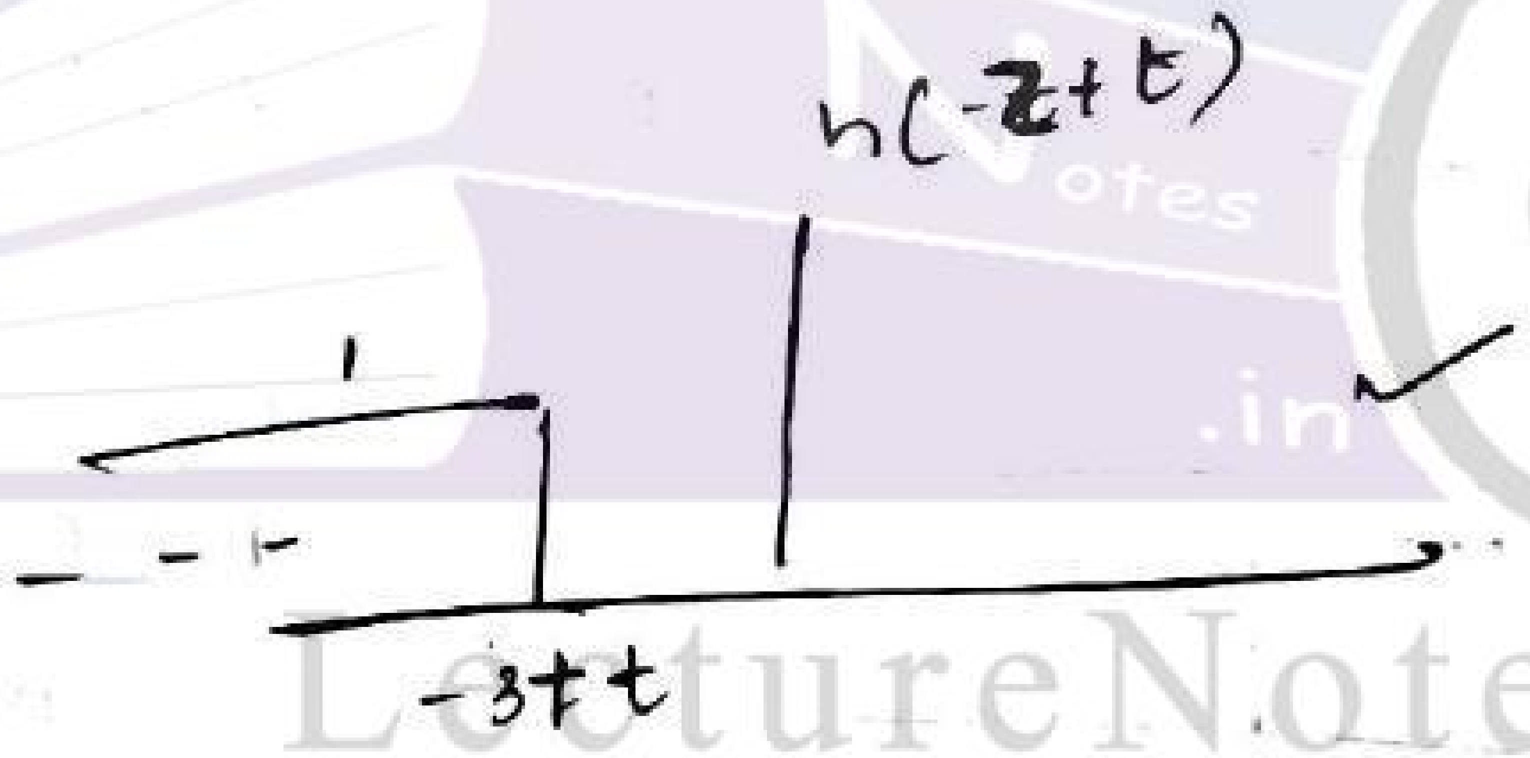
$t ; 3, \infty$

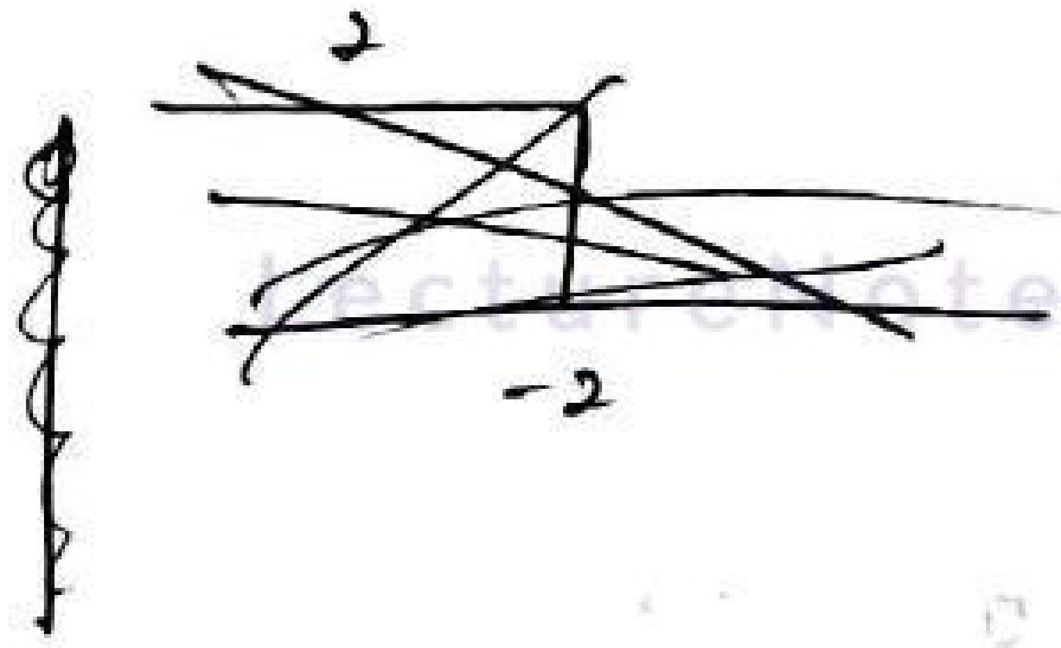


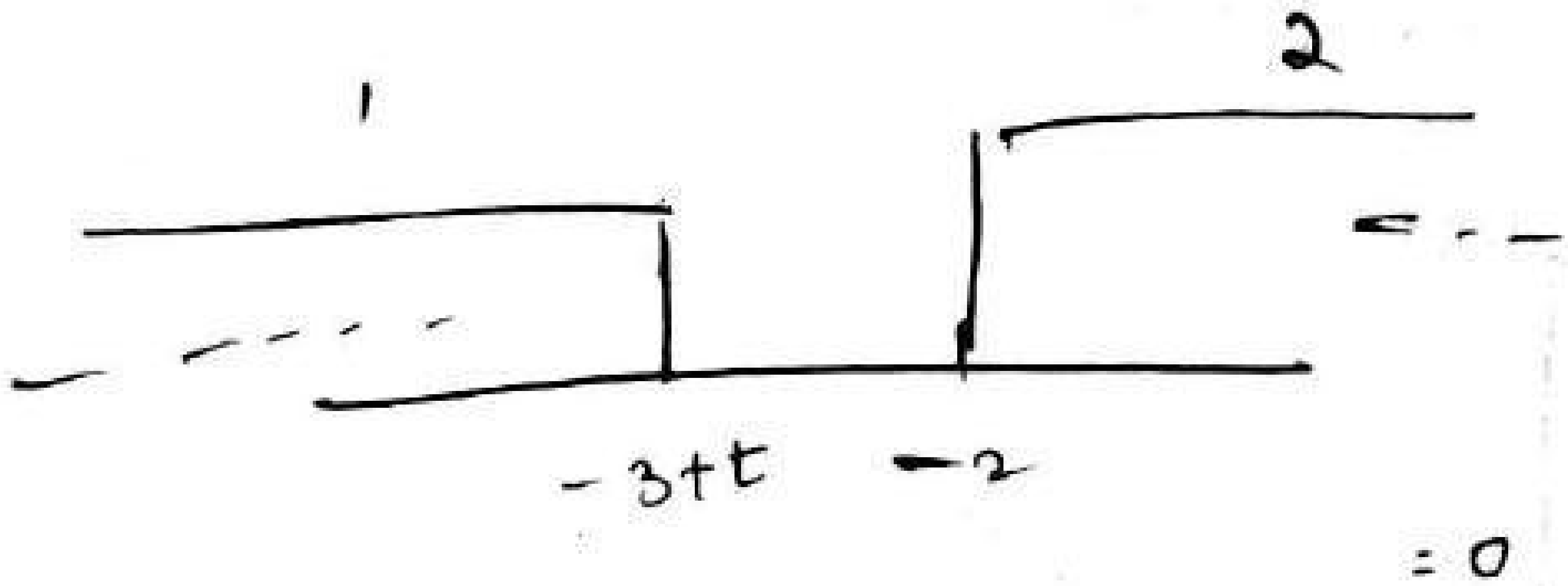






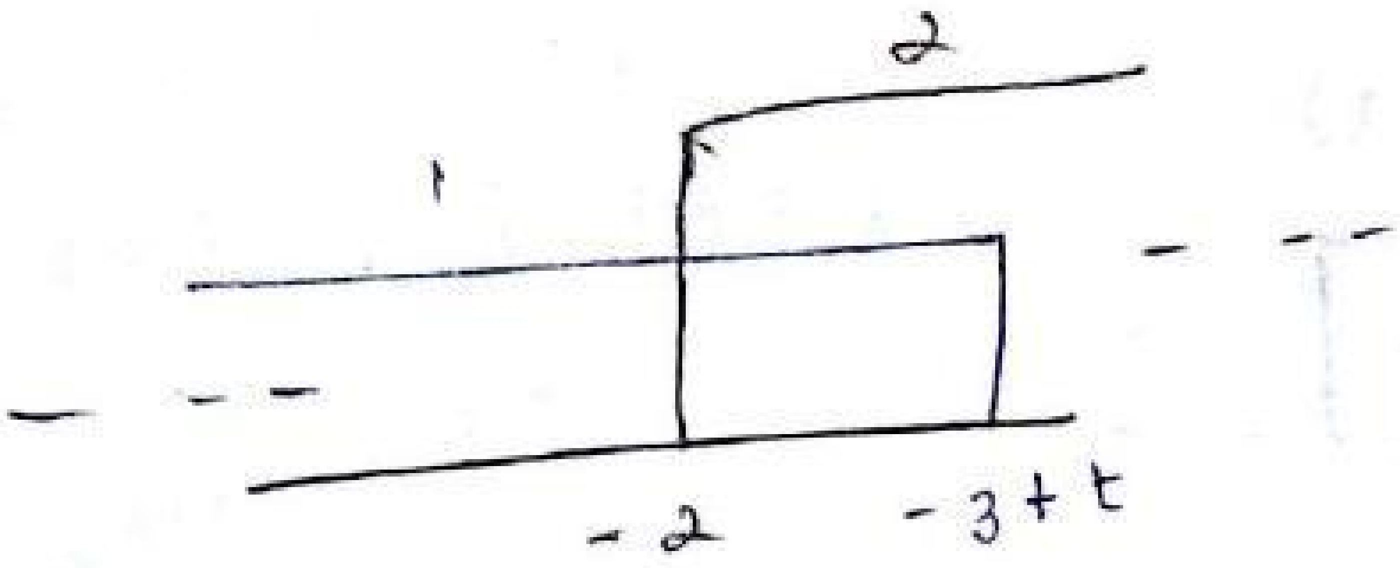






$-\infty < t < 1$

$= 0$



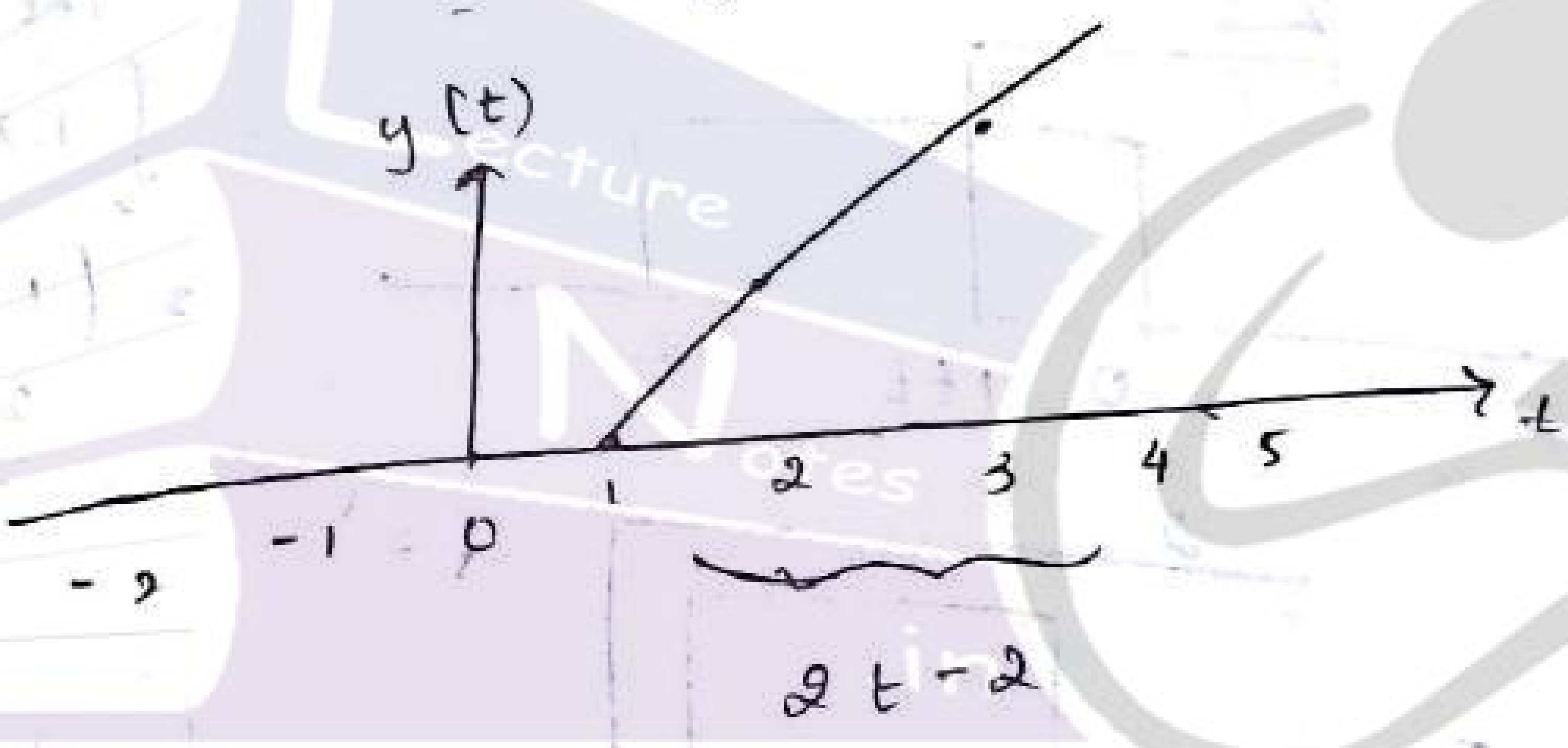
$1 \le t \le \infty$

$$\int_{t=-2}^{-3+t} 2.1\, d\tau$$

$$= 2\left[\tau\right]_{-2}^{-3+t}$$

$$= 2\left[(-3+t)-(-2)\right]$$

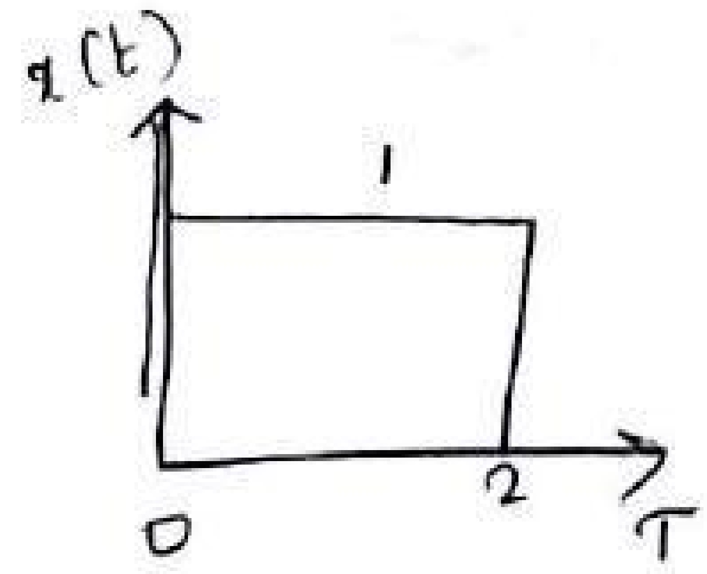
$$= 2(t-1) = 2t-2$$



(5) $x(t) = u(t) - u(t-2)$

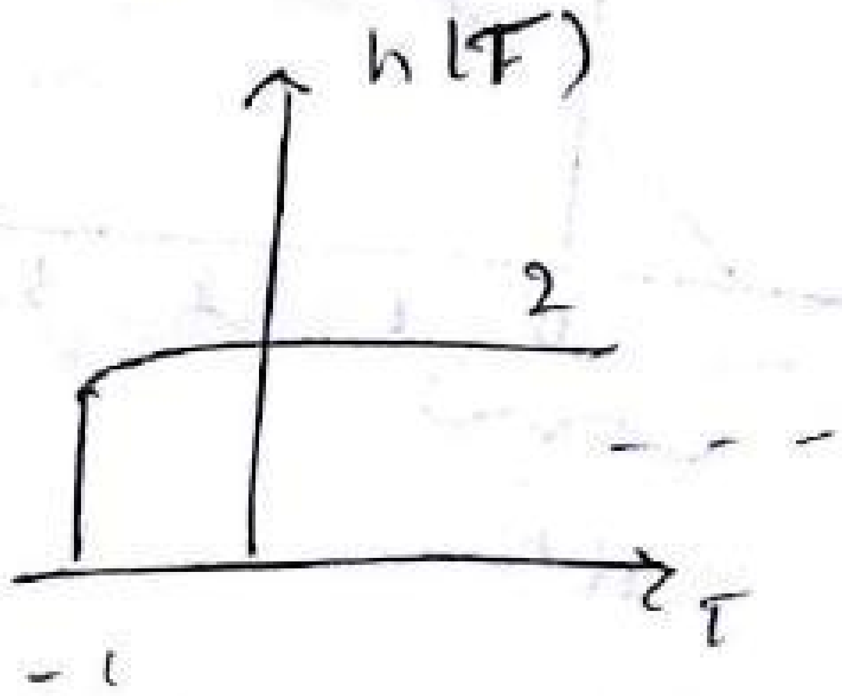
$h(t) = 2u(t+1)$

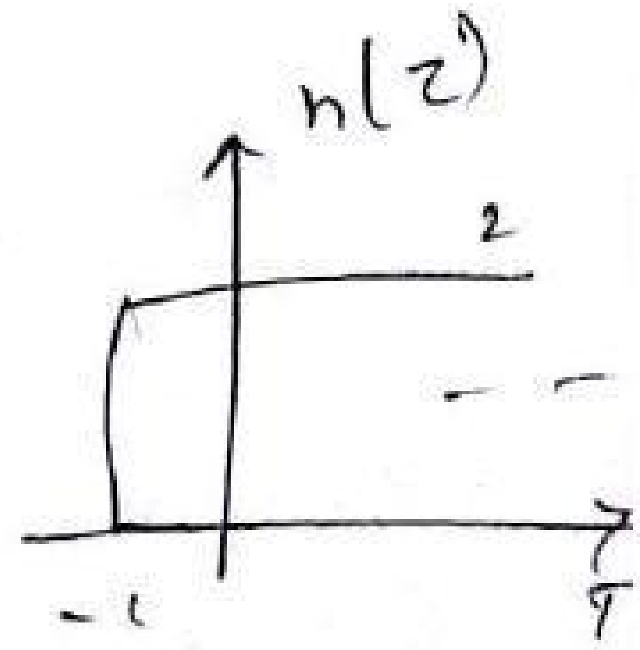
find $y(t) = x(t) * h(t)$



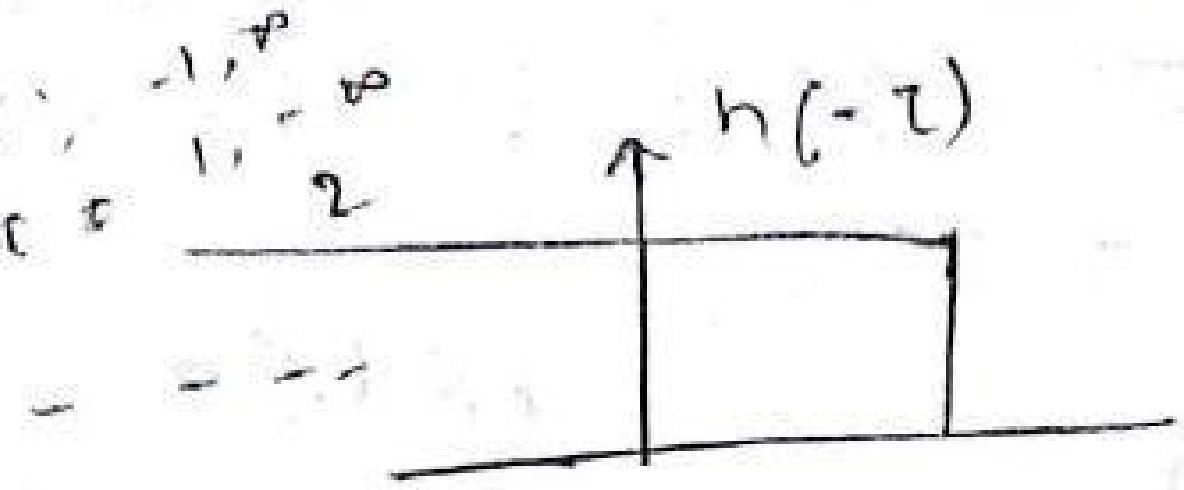
$t+1 : 0, \infty$

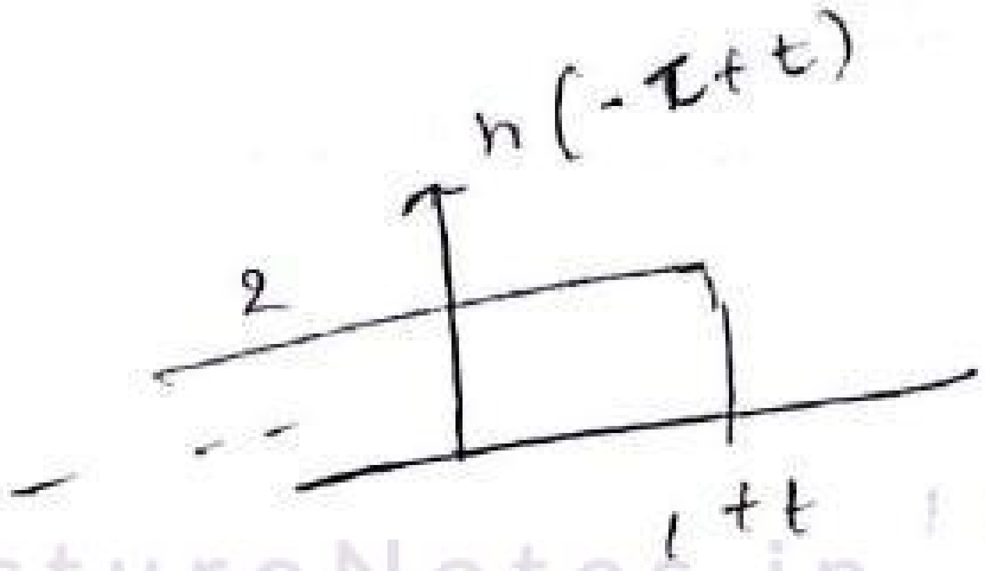
$t : -1, \infty$

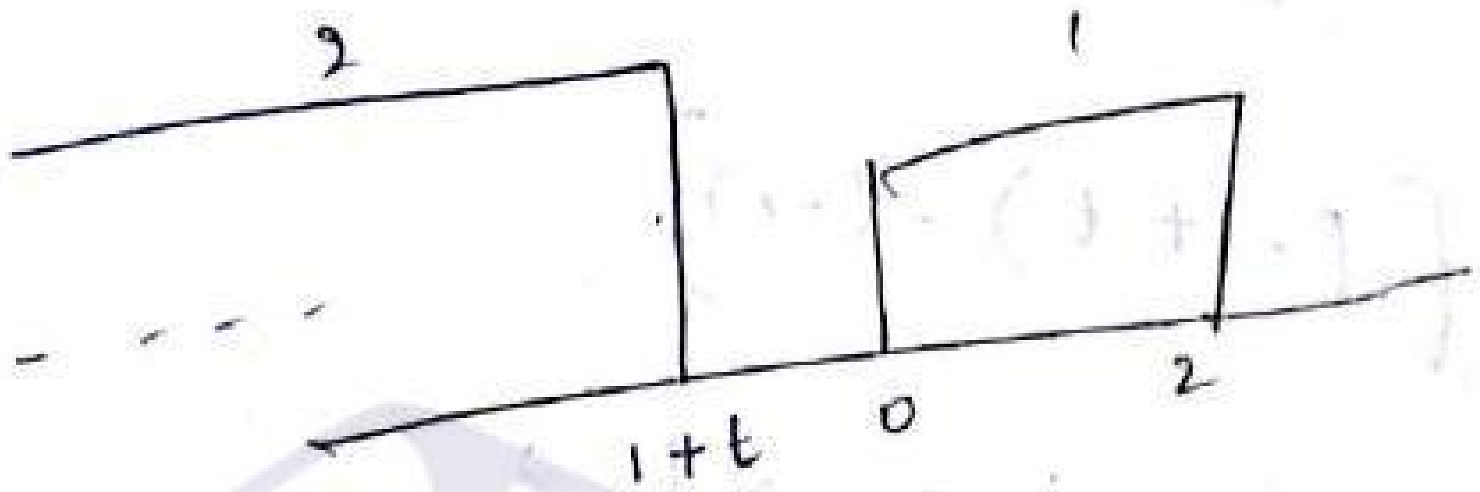




τ : $-1, \infty$
τ : $1, -\infty$

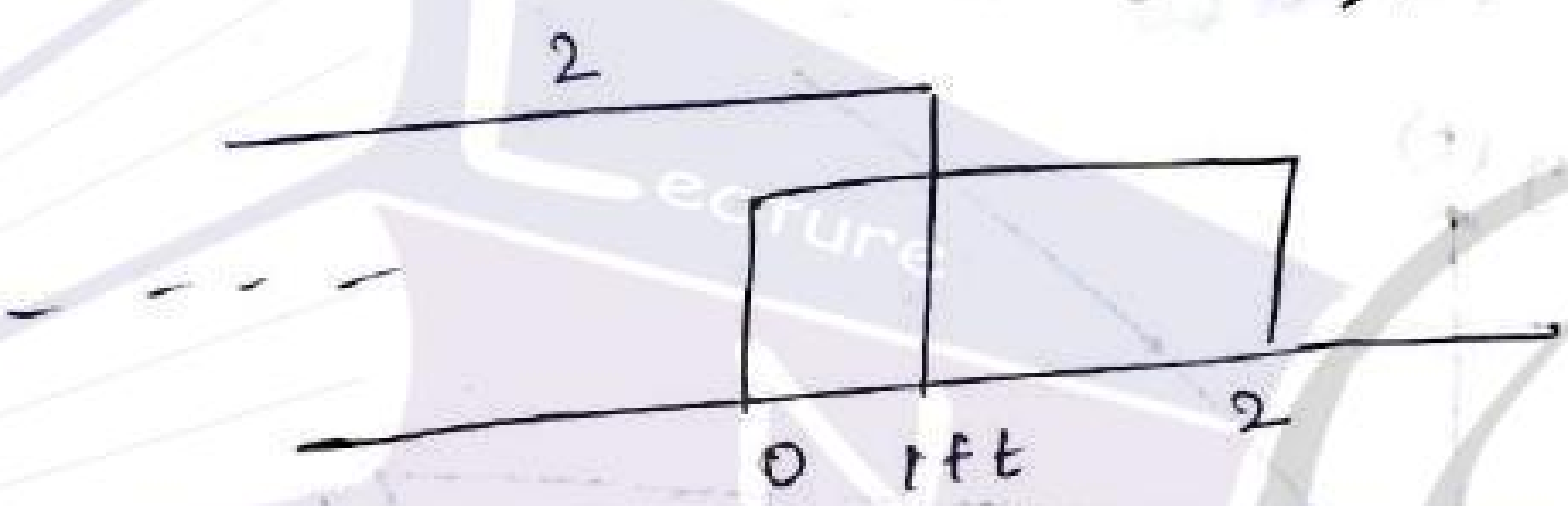






$-\infty \le t \le -1$

$-1 \le t \le 1$

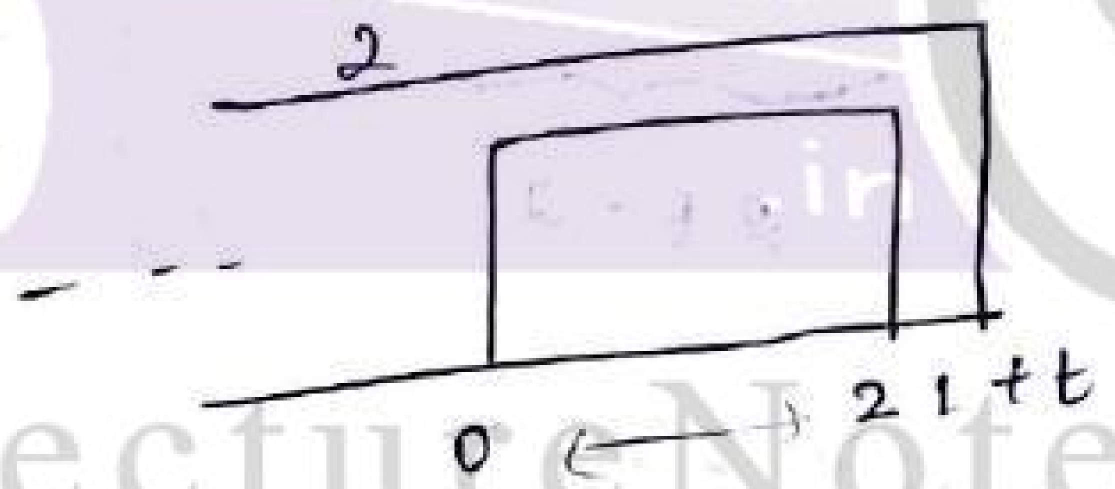


$$= \int_0^{1+t} 1.2\, d\tau$$

$$2\left[1+t-0\right]$$

$$= \underline{\underline{2t+2}}$$

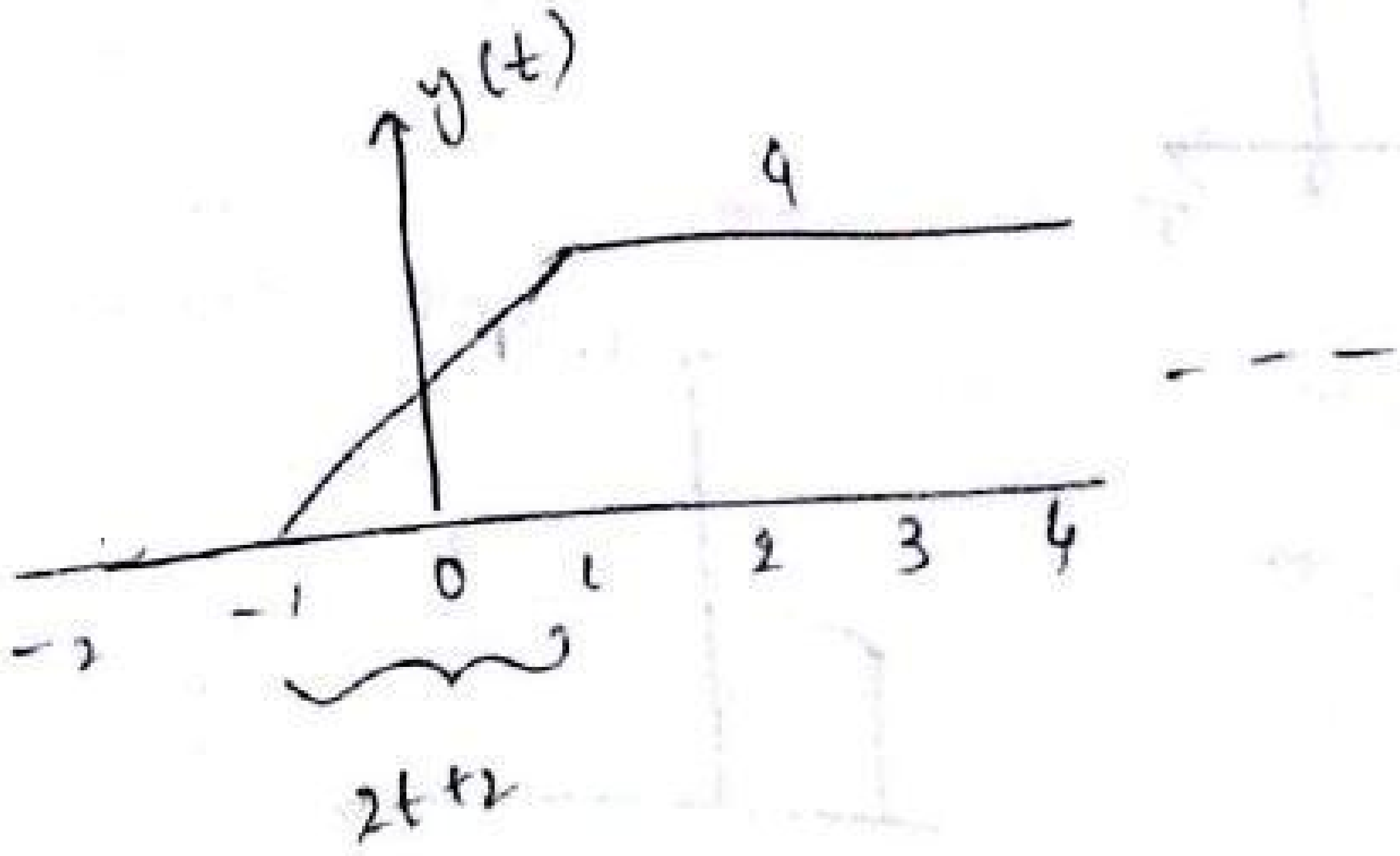
$1 \le t \le \infty$



$$= \int_0^2 1.2\, d\tau$$

$$2(2-0) = 4 //$$

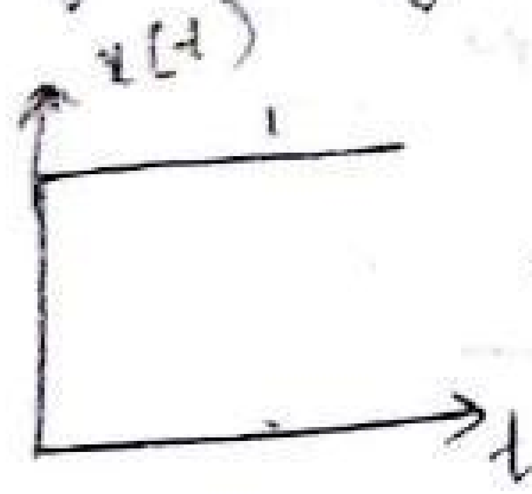
$$y(t) = \begin{cases} 0 & -\infty \le t \le -1 \\ 2t+2 & -1 \le t \le 1 \\ 4 & 1 \le t \le \infty \end{cases}$$

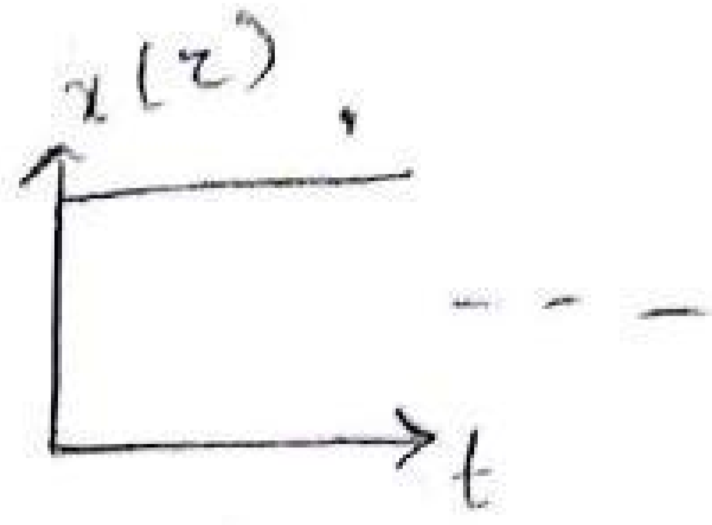


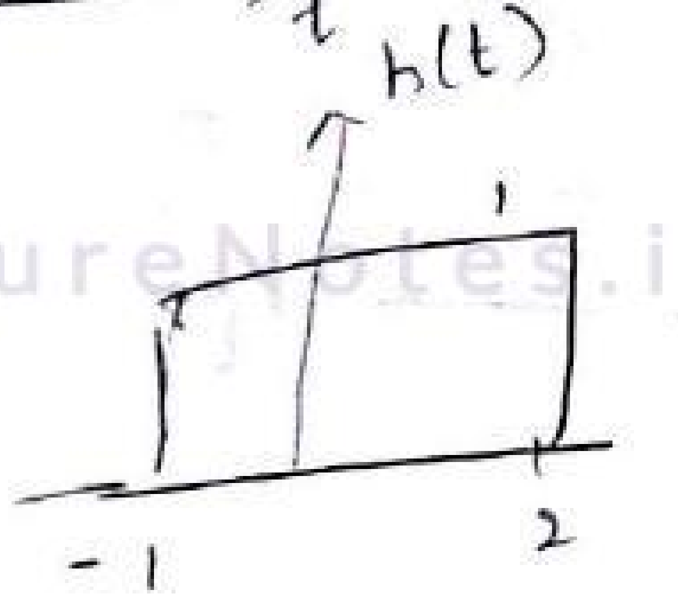
(5) $x(t) = u(t)$

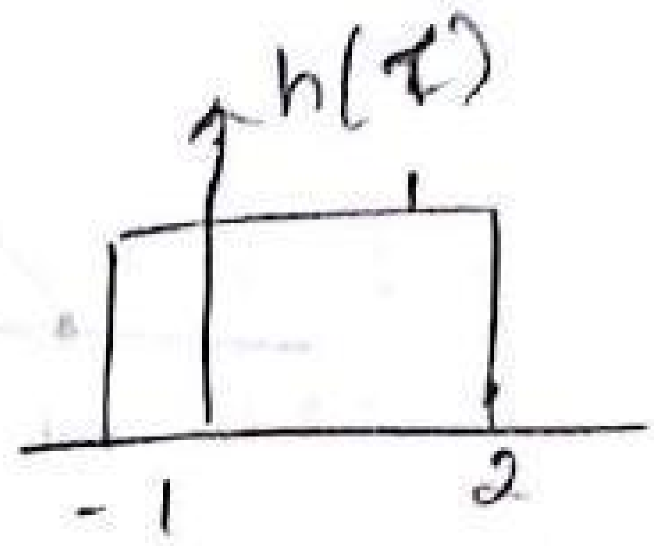
$h(t) = u(t+1) - u(t-2)$

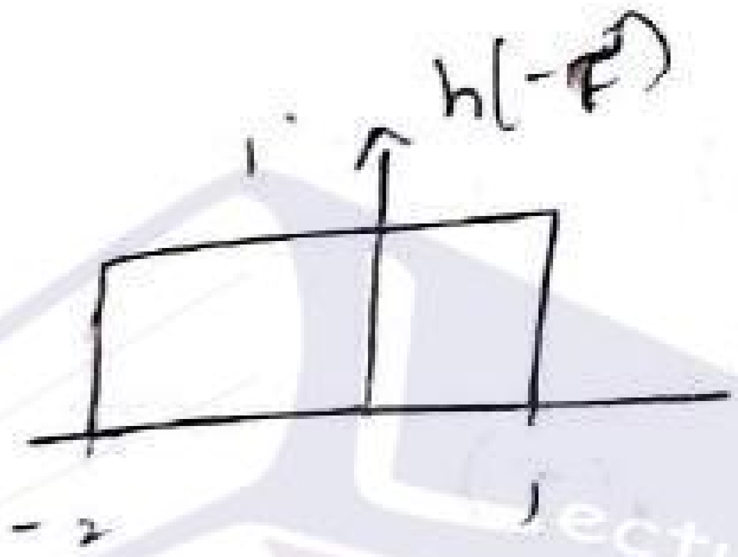
find $y(t) = x(t) * h(t)$

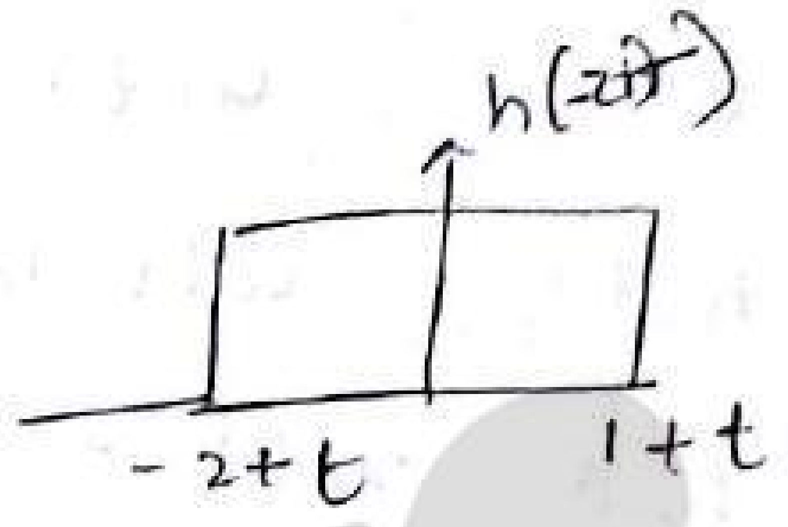


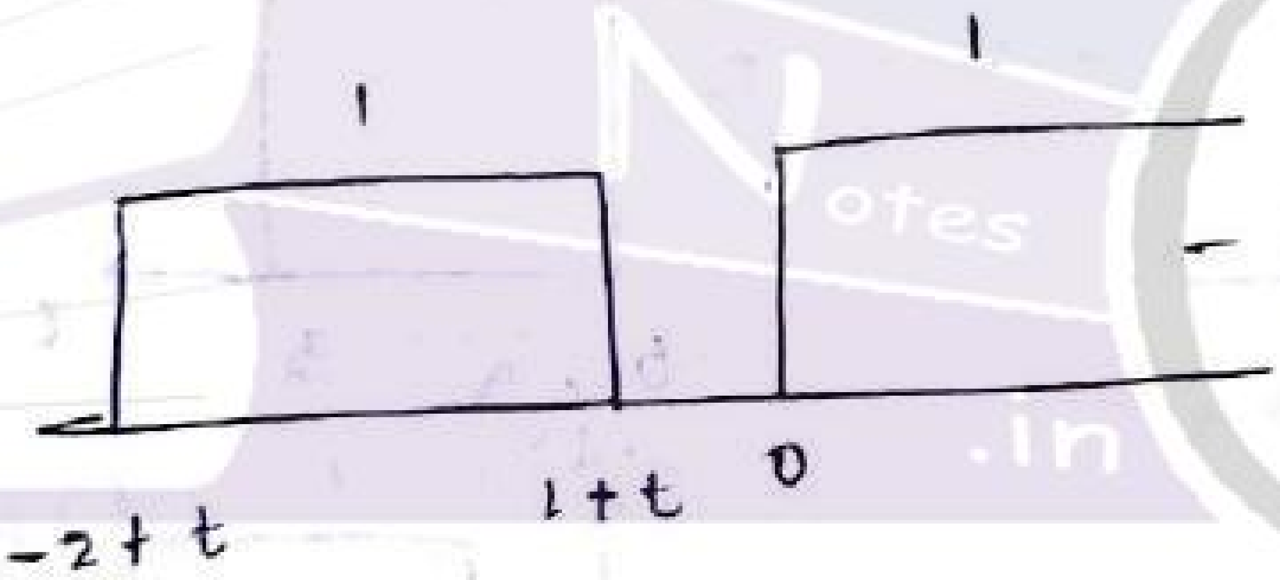






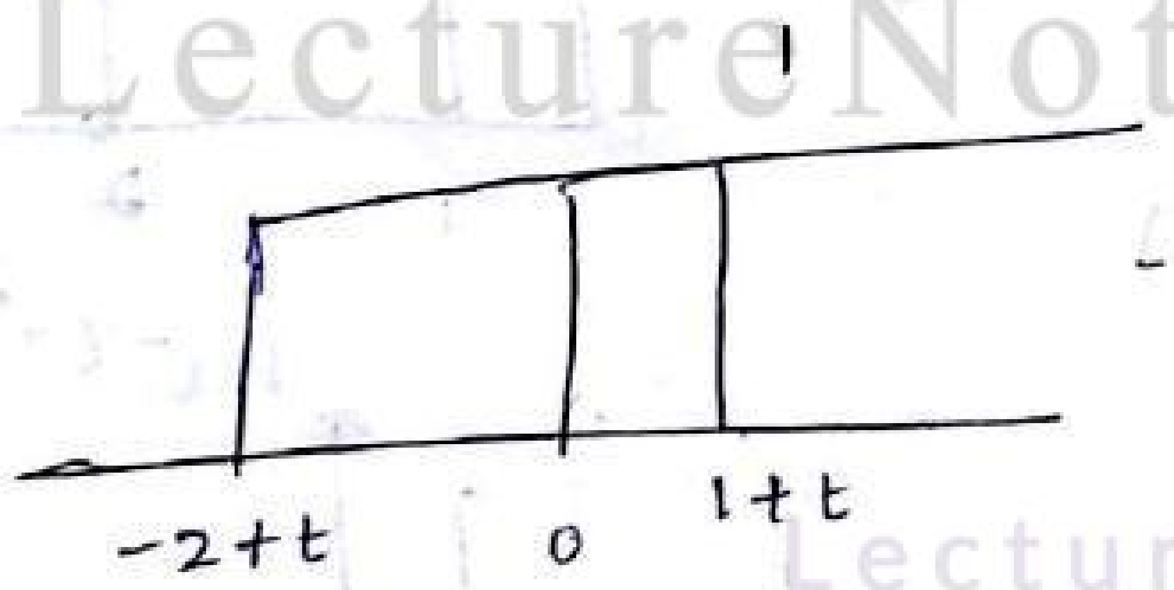






$-\infty < t < -1$

$-1 \le t \le 2$



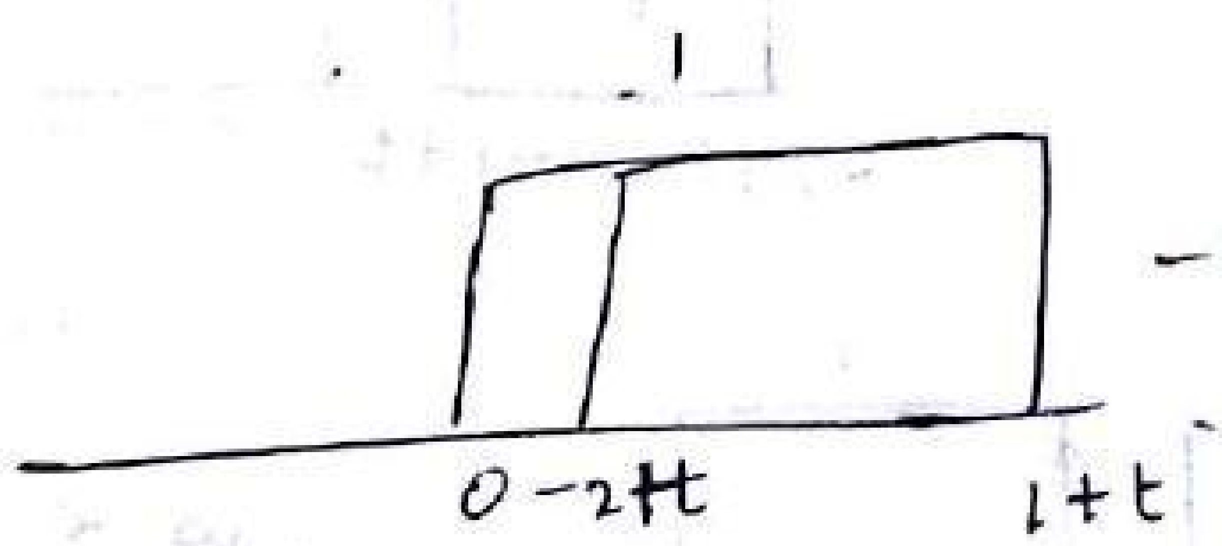
$$\int_0^{1+t} 1 \cdot 1 \, d\tau$$

$1+t-0$

$= \underline{\underline{1+t}}$

$-2+t = 0$

$t = 2$



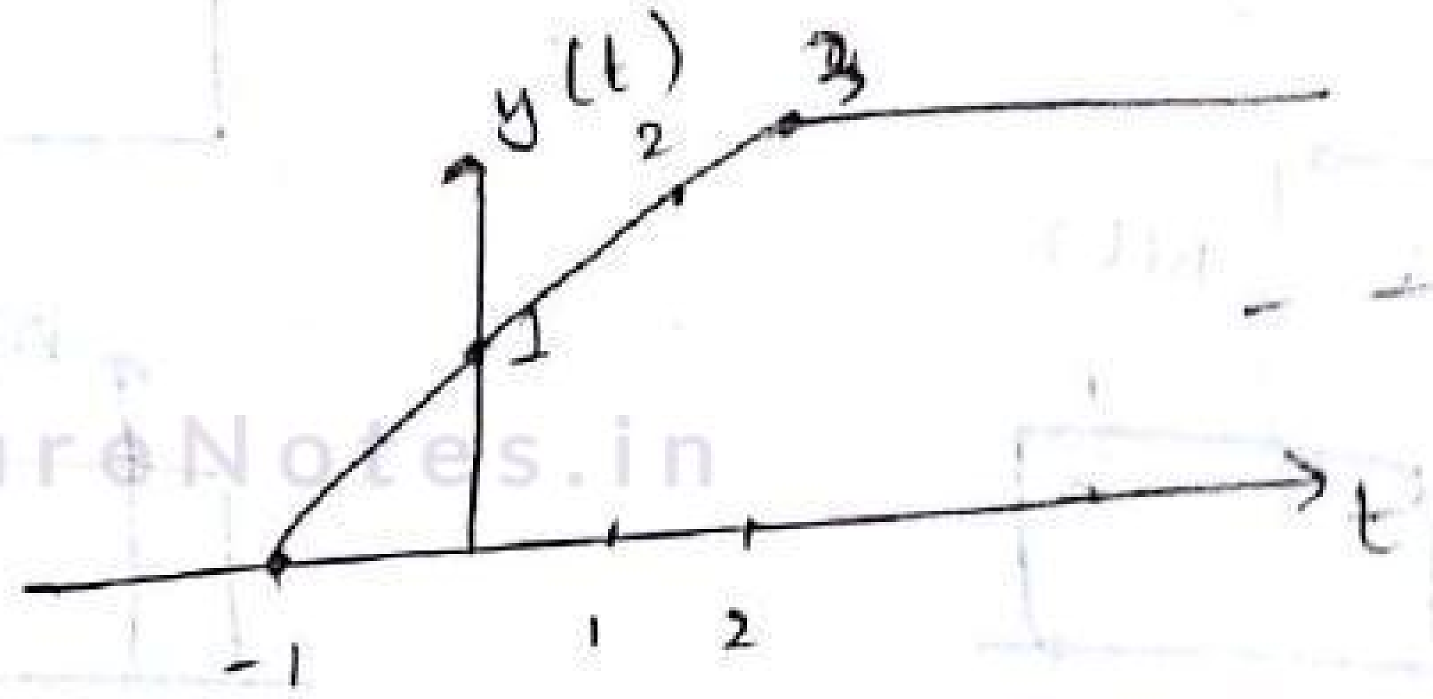
$2 < t \le \infty$

$$\int_{-2+t}^{1+t} 1 \cdot 1 \, d\tau$$

$1+t-(-2+t)$

$1+\cancel{t}+2-\cancel{t} = 3$

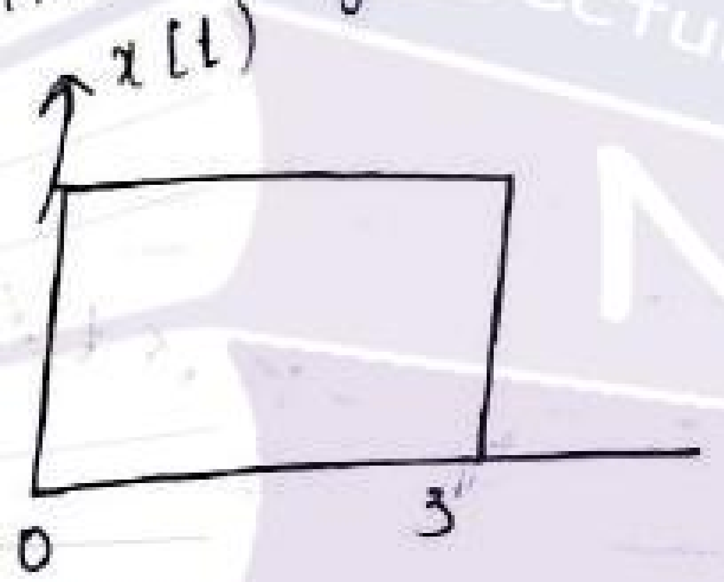
$$y(t) = \begin{cases} 0 & -\infty \le t \le -1 \\ 1+t & -1 \le t \le 2 \\ 3 & 2 \le t \le \infty \end{cases}$$

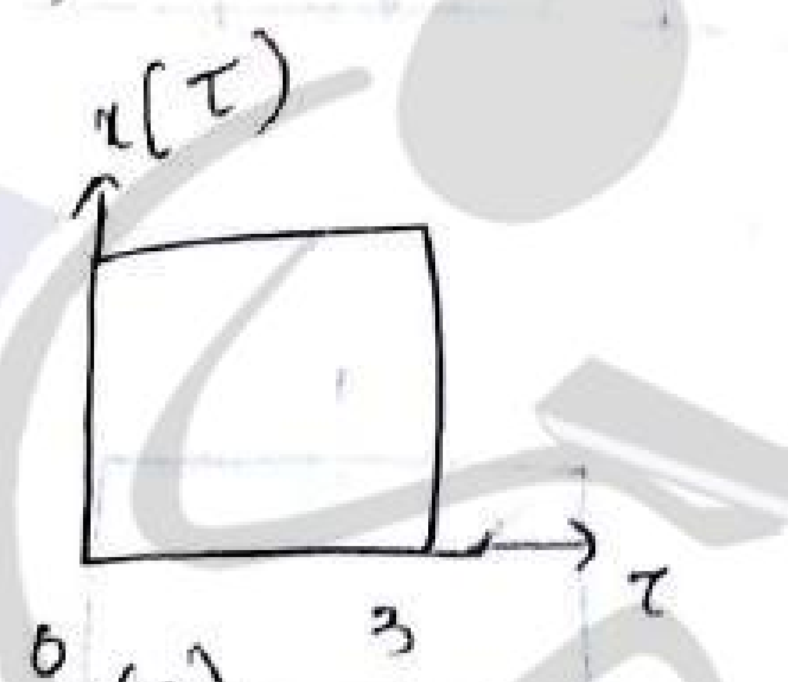


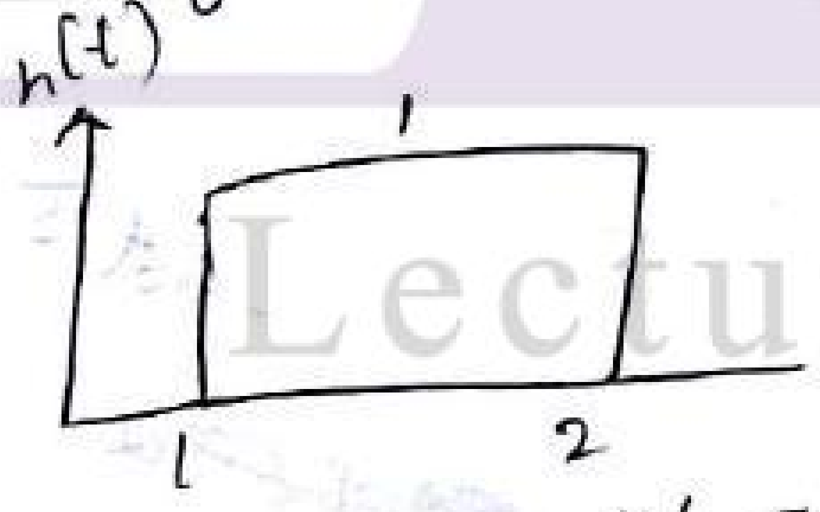
(2) $x(t) = u(t) - u(t-3)$

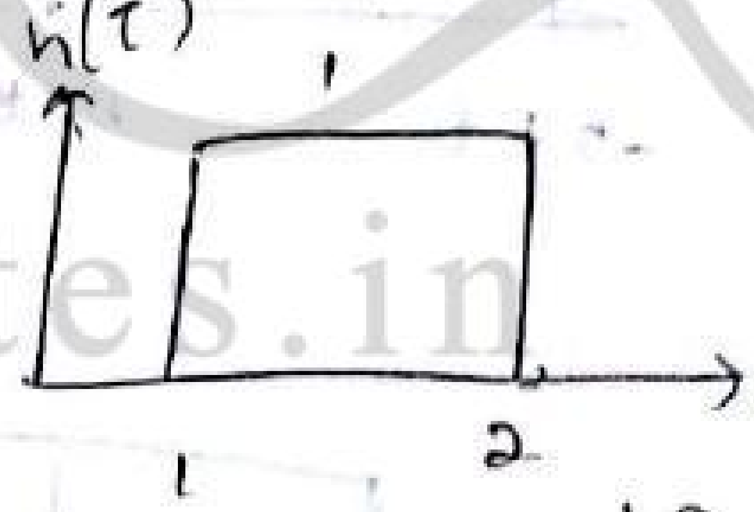
$h(t) = u(t-1) - u(t-2)$

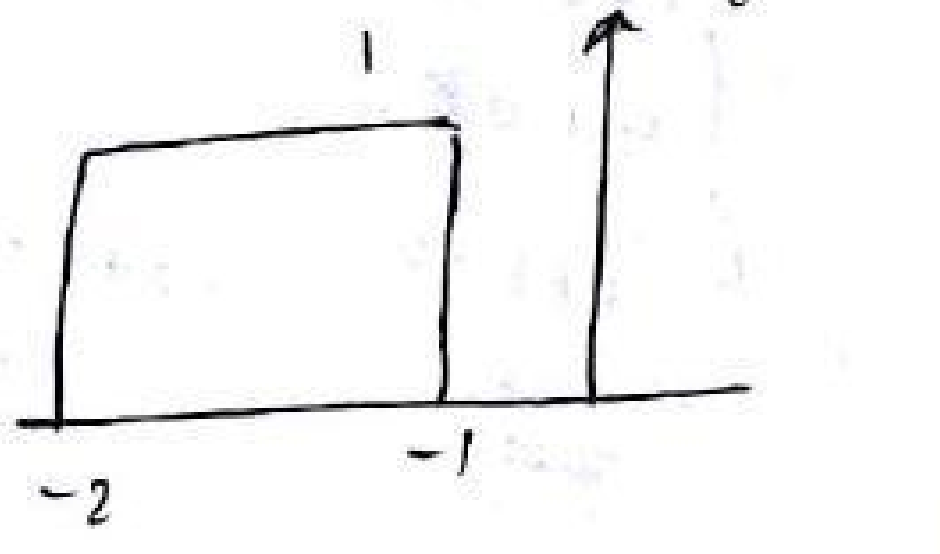
find $y(t) = x(t) * h(t)$

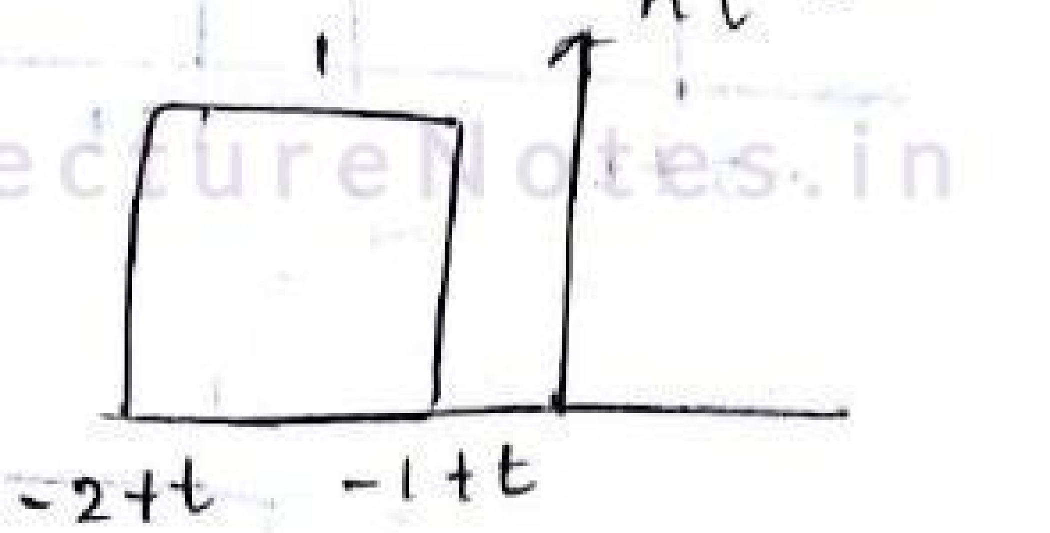


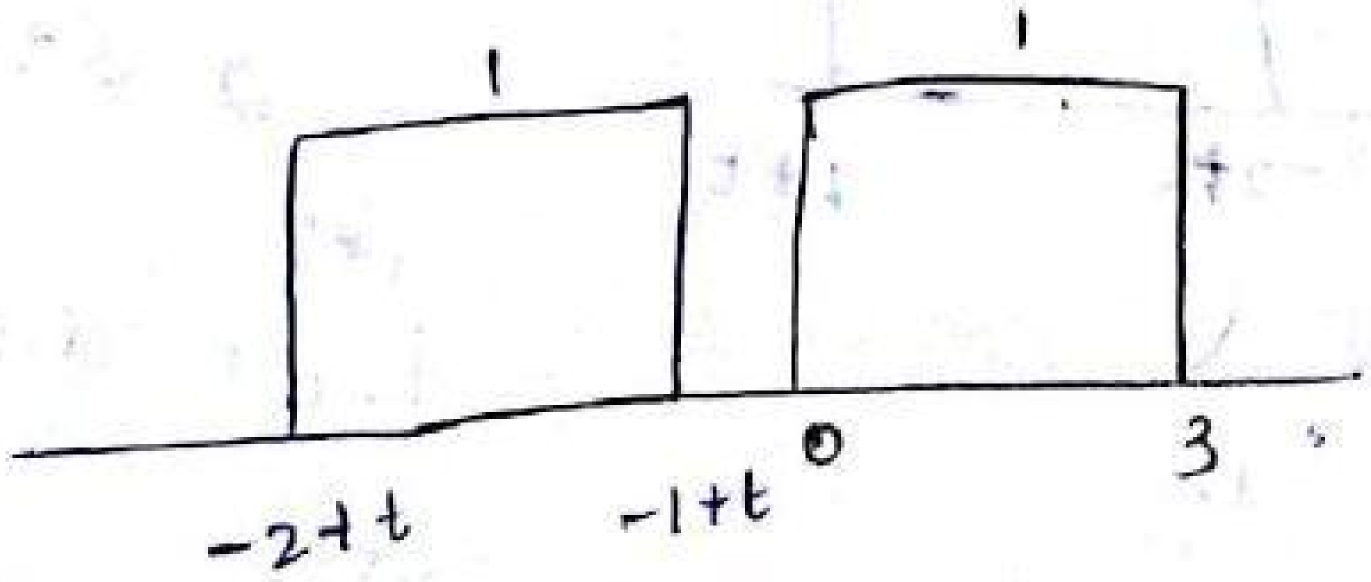






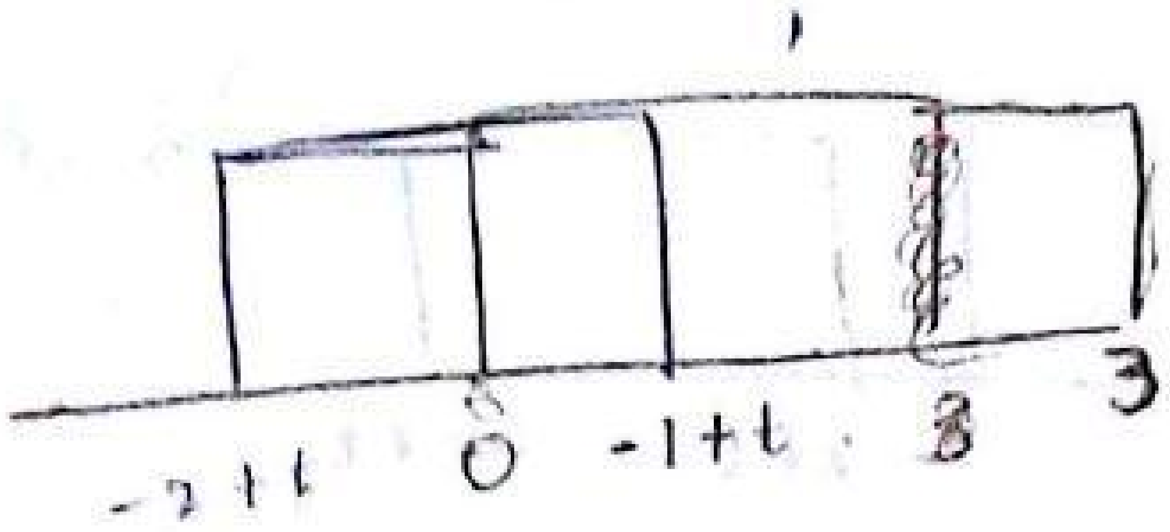






$-\infty \le t \le 1$

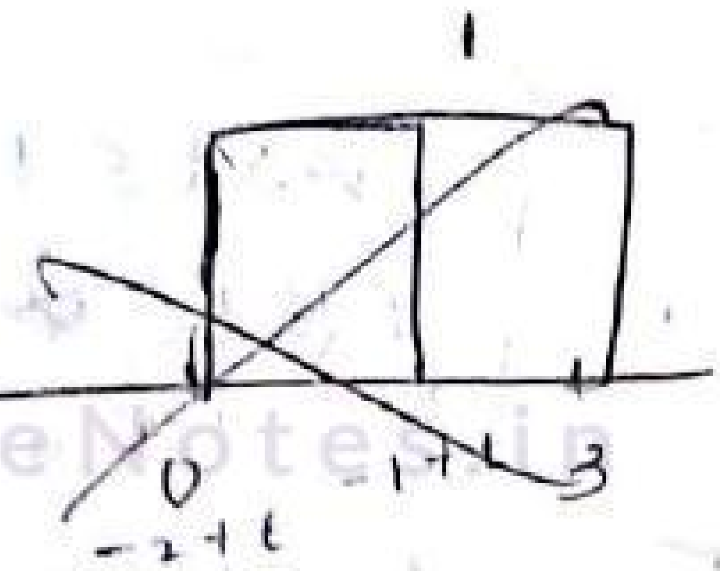
$= 0$ //



$1 \le t \le 2$

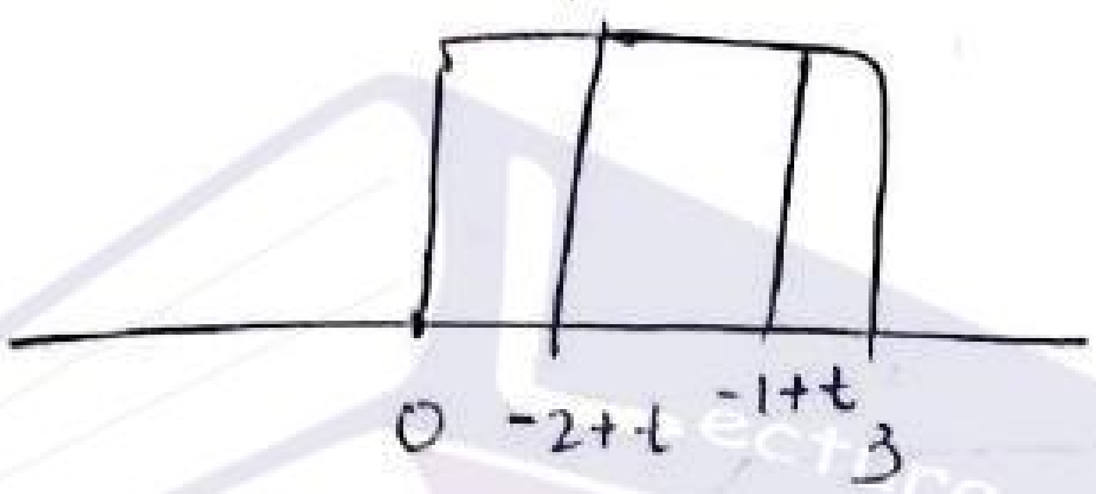
$$\int_{0}^{-1+t} 1.1 \, d\tau$$

$$= \underline{\underline{-1+t}}$$



$2 \le t \le 4$

$$\int_{-2+t}^{-1+t} 1.1 \, d\tau$$



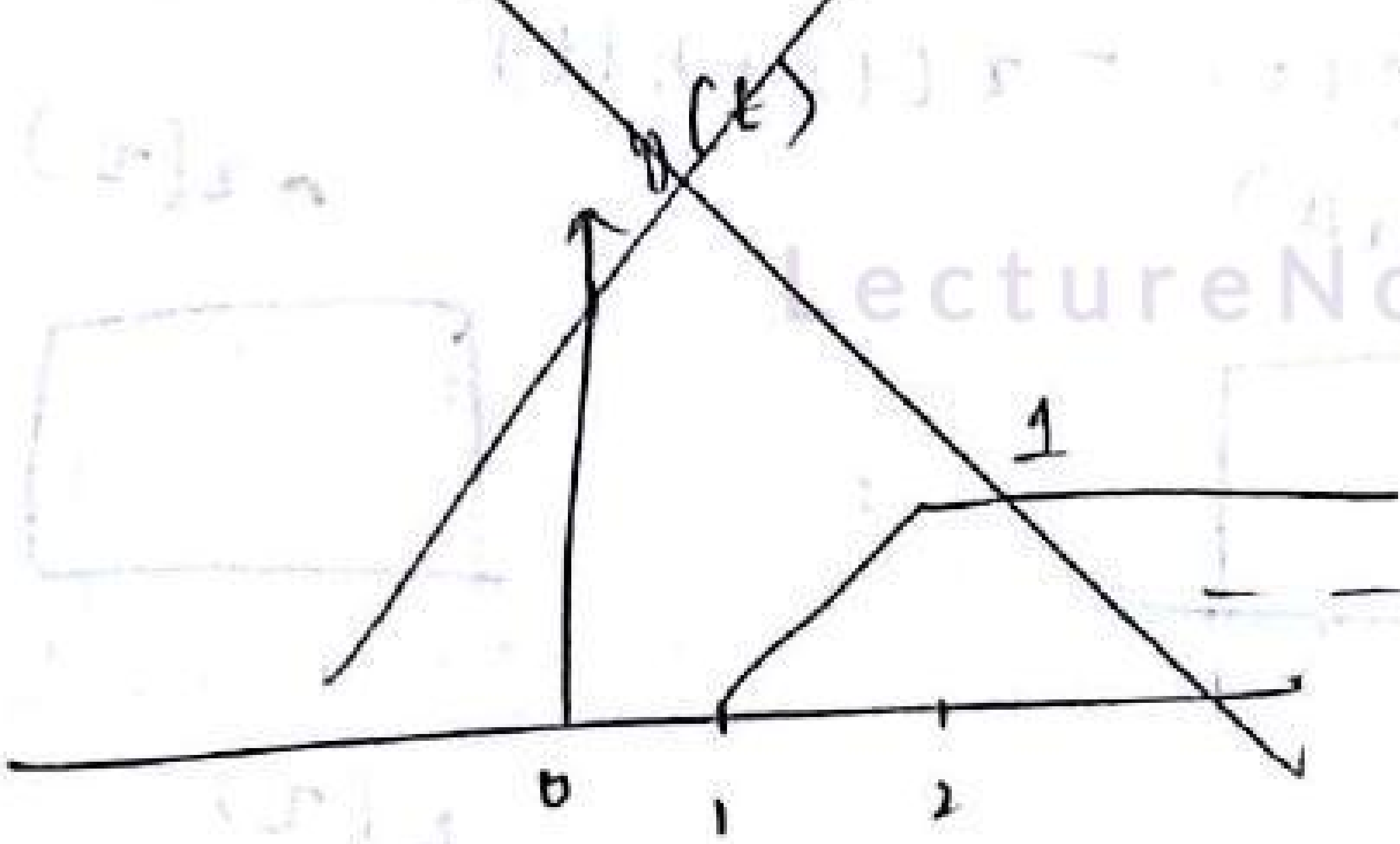
$$-1+t - (-2+t)$$

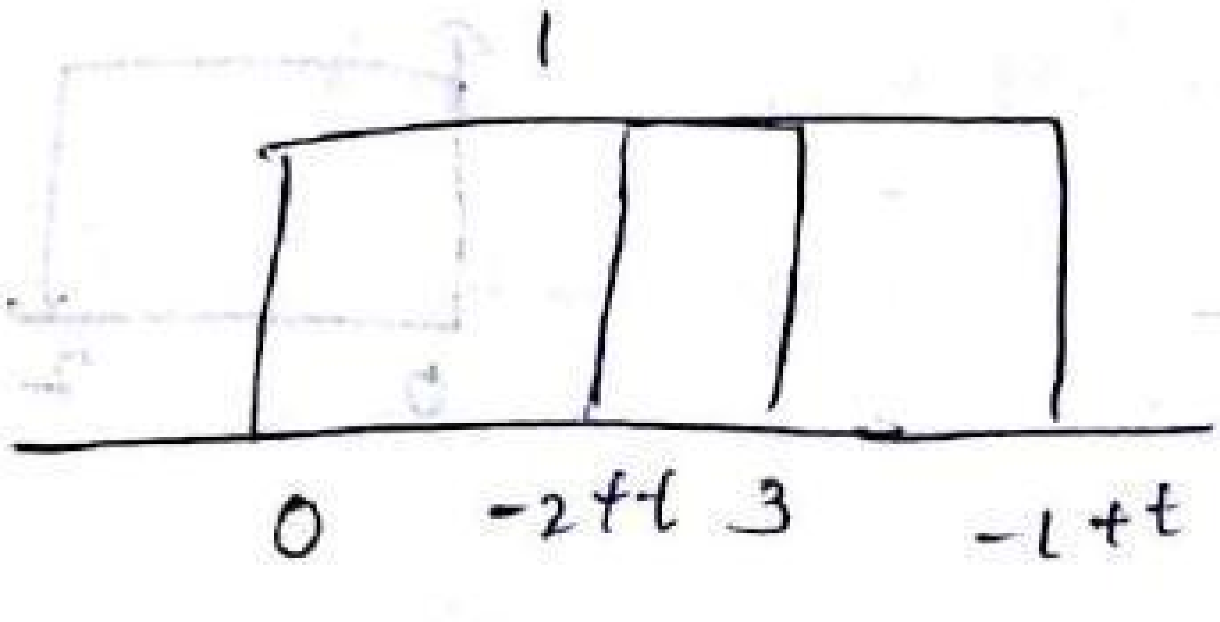
$$-1+t+2-t$$

$$= \underline{\underline{1}}$$

$2 \le t \le 4$ =

$$y(t) = \begin{cases} 0 & -\infty \le t \le 1 \\ -1+t & 1 \le t \le 2 \\ 1 & 2 \le t \le 4 \end{cases}$$





$4 \le t \le 5$

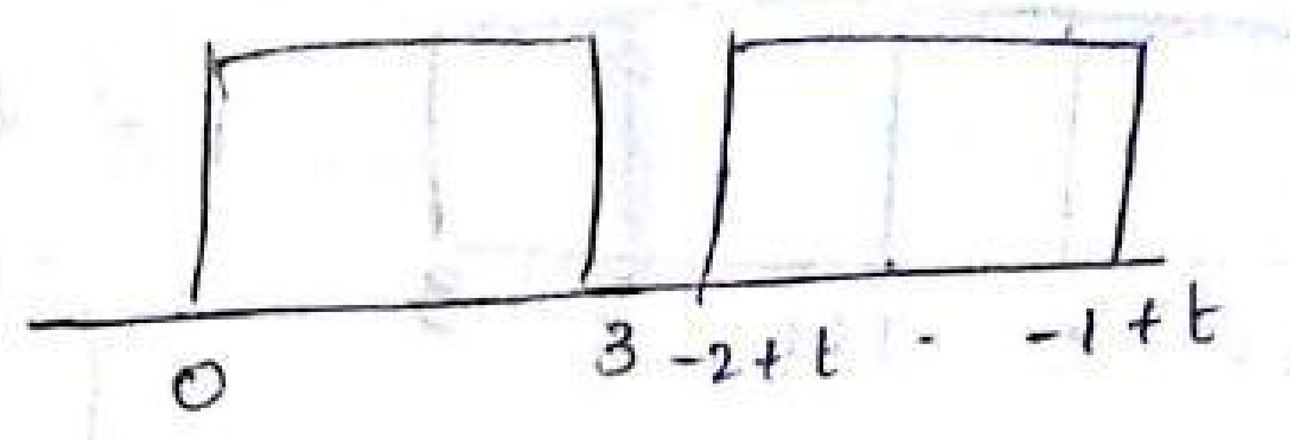
$$\int_{-2+t}^{3} 1.1 \, d\tau$$

$$3+2-t$$

$$-t+5$$

$$= \cancel{-1+t+3}$$

$$= \cancel{4} \; t-4$$

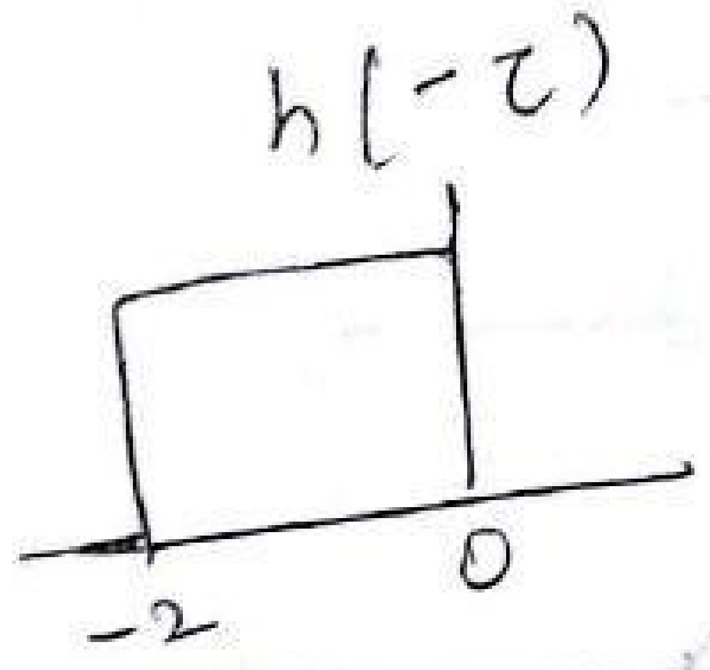


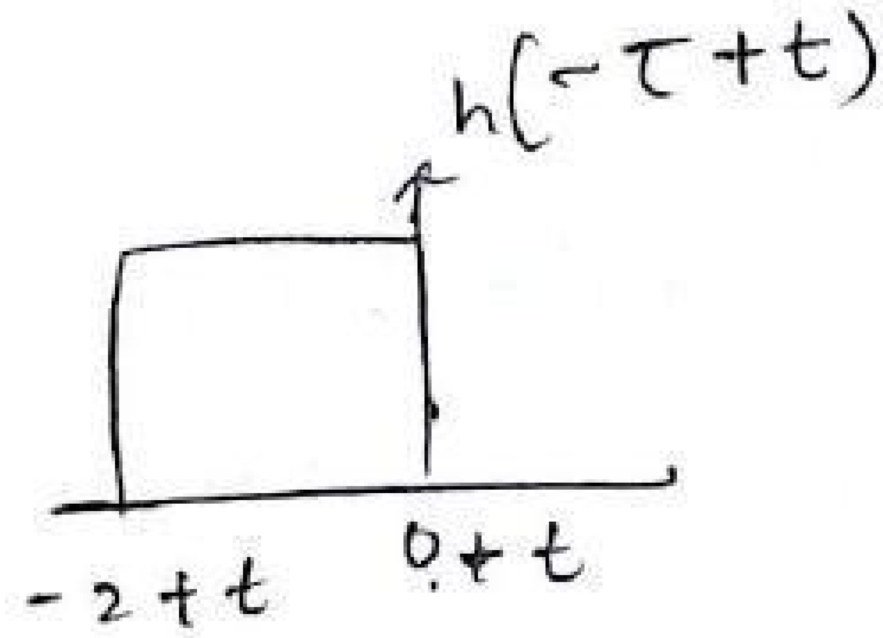
$5 \leq t \leq \infty$

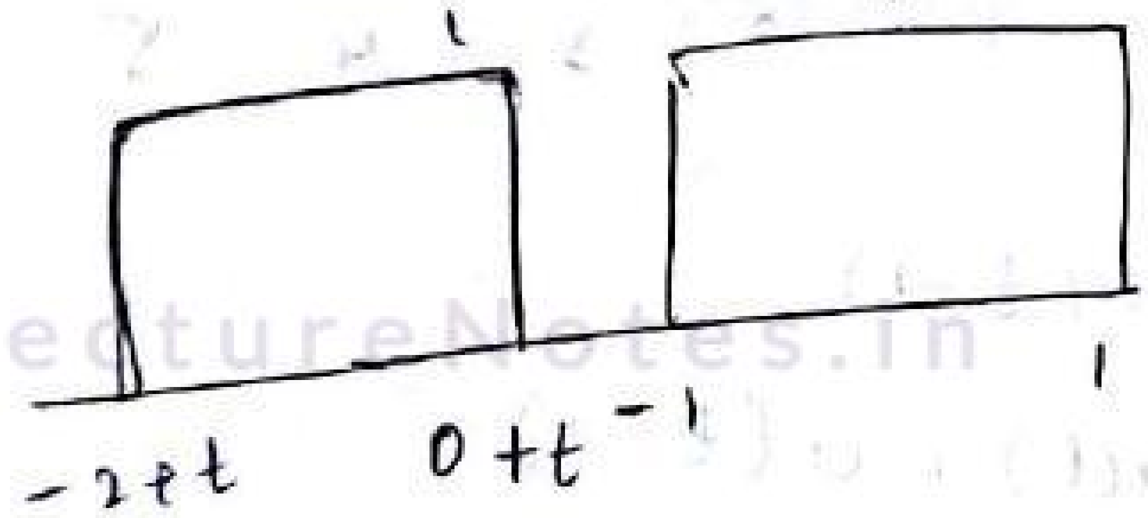
$$y(t) = \begin{cases} 0 & -\infty \leq t \leq 1 \\ -1+t & 1 \leq t \leq 2 \\ 1 & 2 \leq t \leq 4 \\ -t+5 & 4 \leq t \leq 5 \\ 0 & 5 \leq t \leq \infty \end{cases}$$

8) $x(t) = u(t+1) - u(t-1)$
$h(t) = u(t) - u(t-2)$

find $y(t) = x(t) * h(t)$

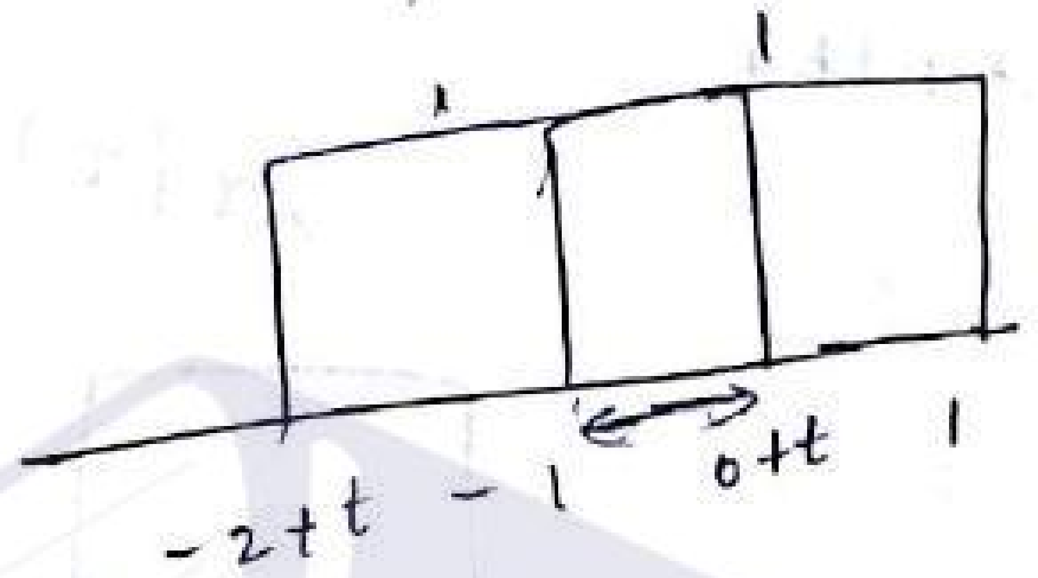






$-\infty \le t \le -1$

$= \underline{\underline{0}}$



$-1 \le t \le 1$

$$\int_{-1}^{t} 1.1 \, d\tau$$

$t - (-1)$

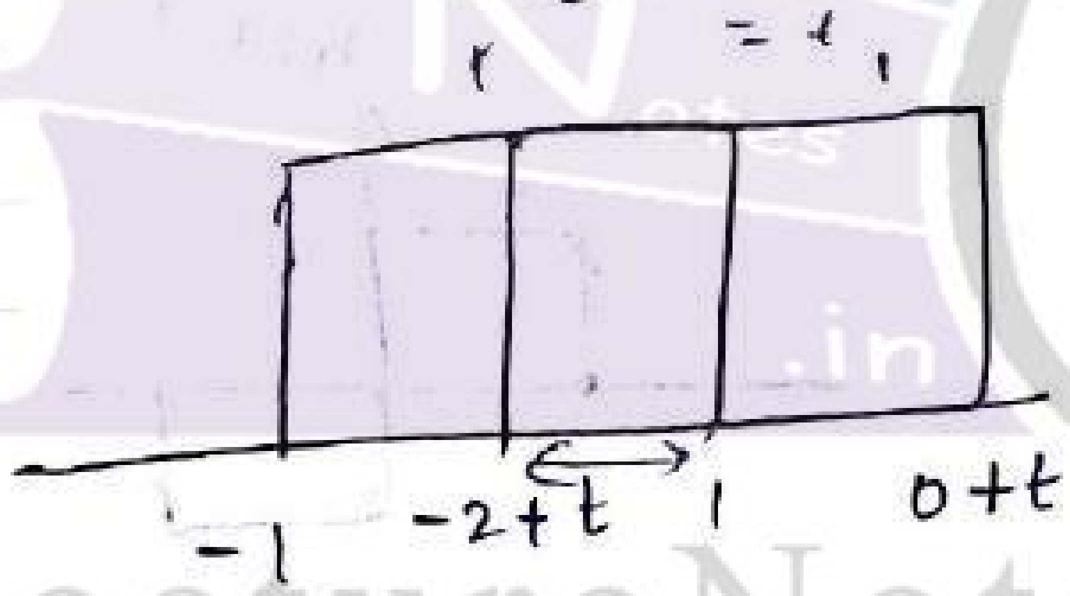
$= \underline{\underline{t+1}}$

$-2+t = -1$

$t = -1+2$

$= 1$

-2+t 0+t



$1 \le t \le 3$

$$\int_{-2+t}^{1} 1.1 \, d\tau$$

$1 - (-2+t)$

$1 + 2 - t$

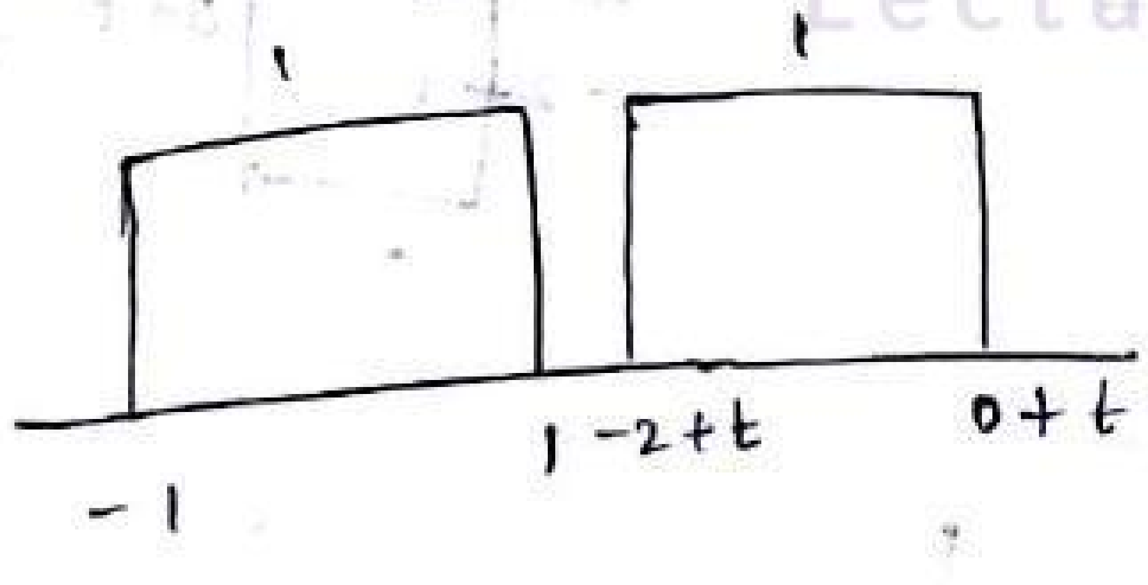
$= \underline{\underline{3 - t}}$

$-2+t = -1$

$t = 1$

$-2+t = 1$

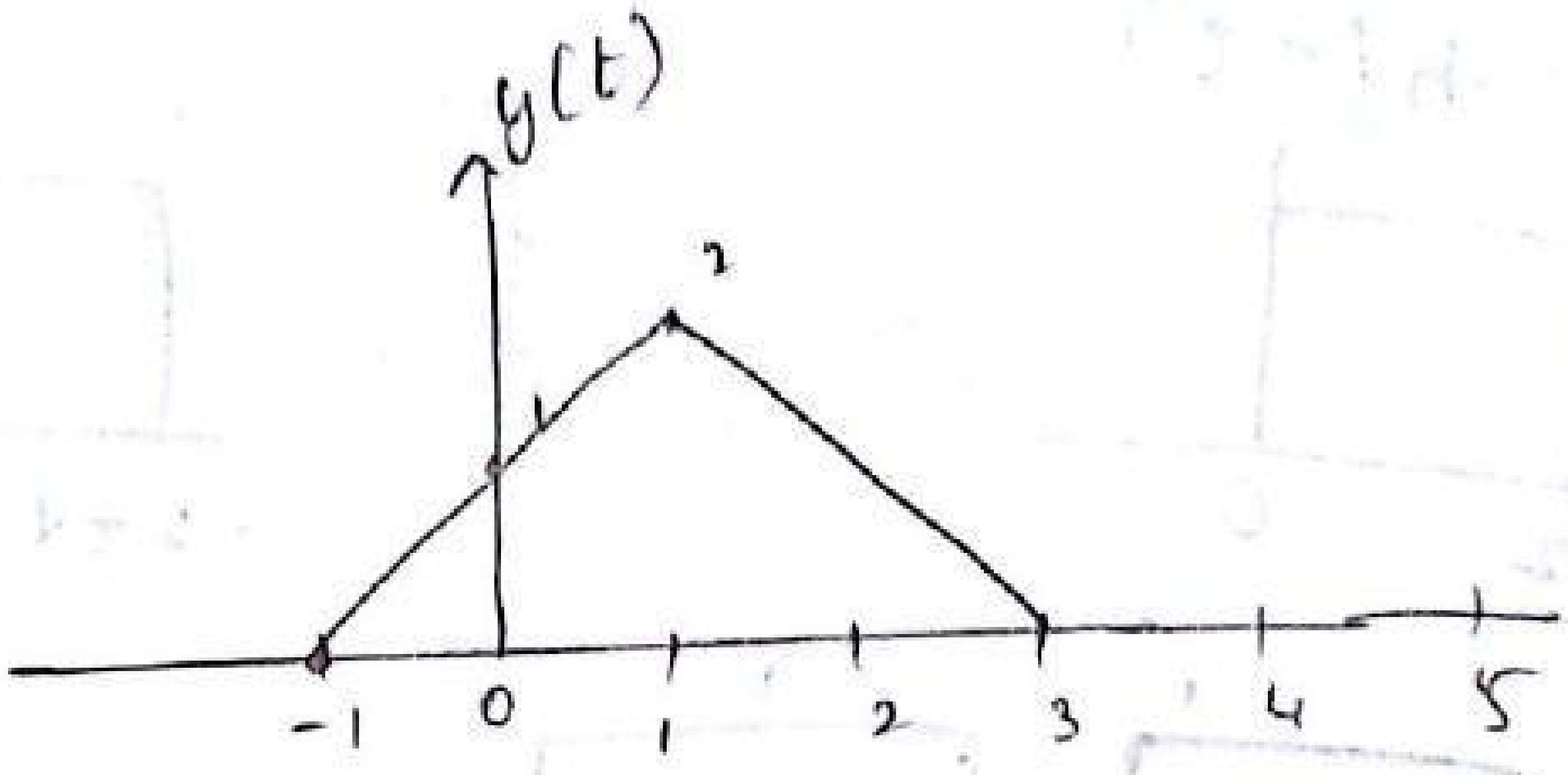
$t = 3$



$3 \le t \le \infty$

$= 0$

$$y(t) = \begin{cases} 0 & -\infty \le t \le -1 \\ t+1 & -1 \le t \le 1 \\ 3-t & 1 \le t \le 3 \\ 0 & 3 \le t \le \infty \end{cases}$$



(9) $x(t) = u(t+1) - u(t-1)$

$h(t) = u(t+2) - 2u(t) + u(t-2)$;

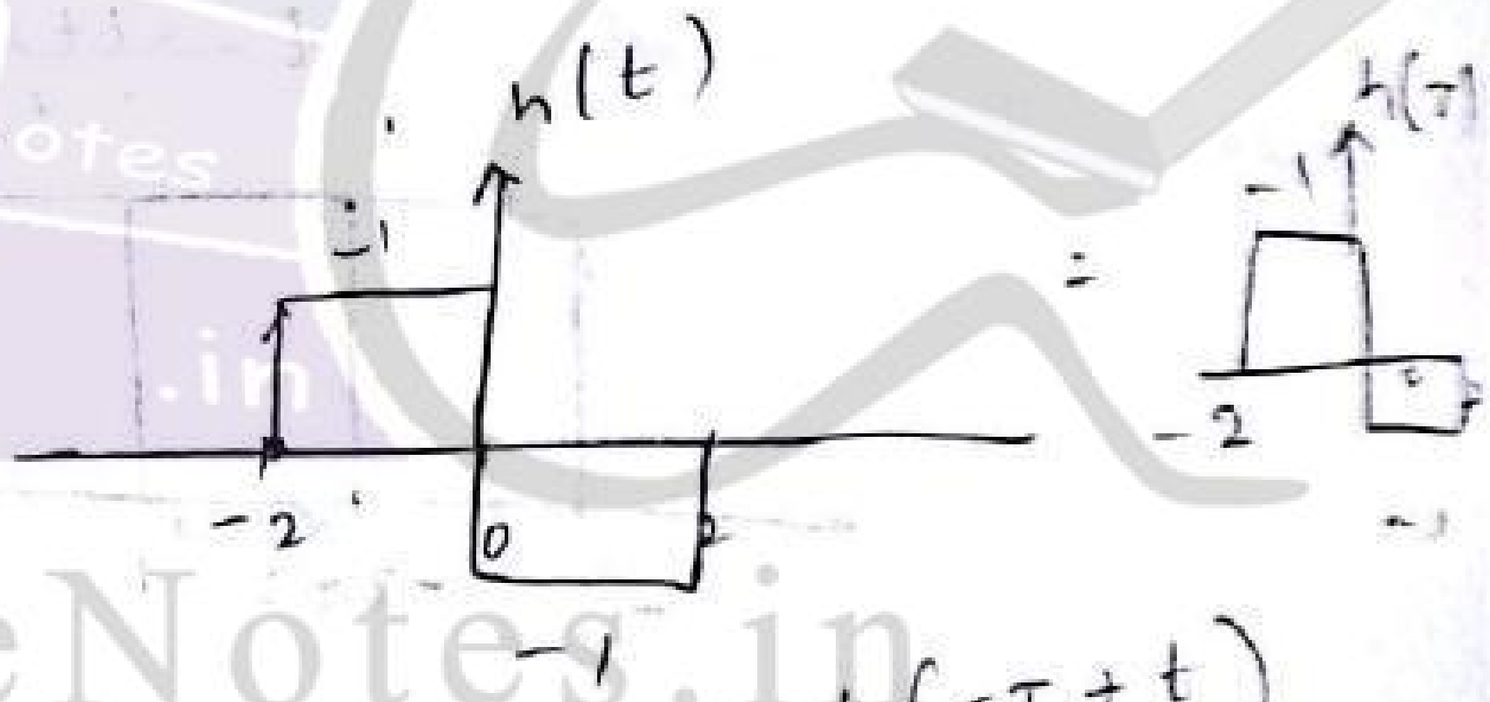
find $y(t) = x(t) * h(t)$

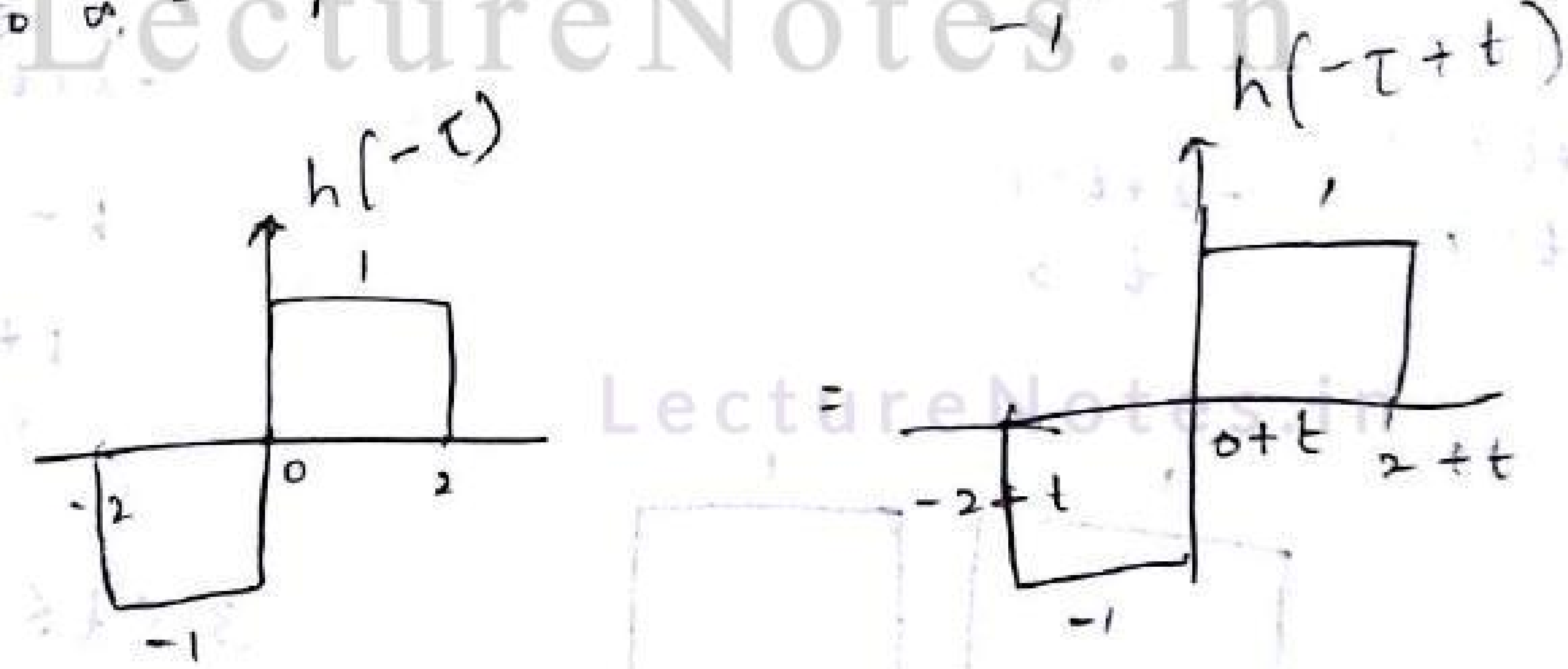
$-\infty$ to -2 : 0

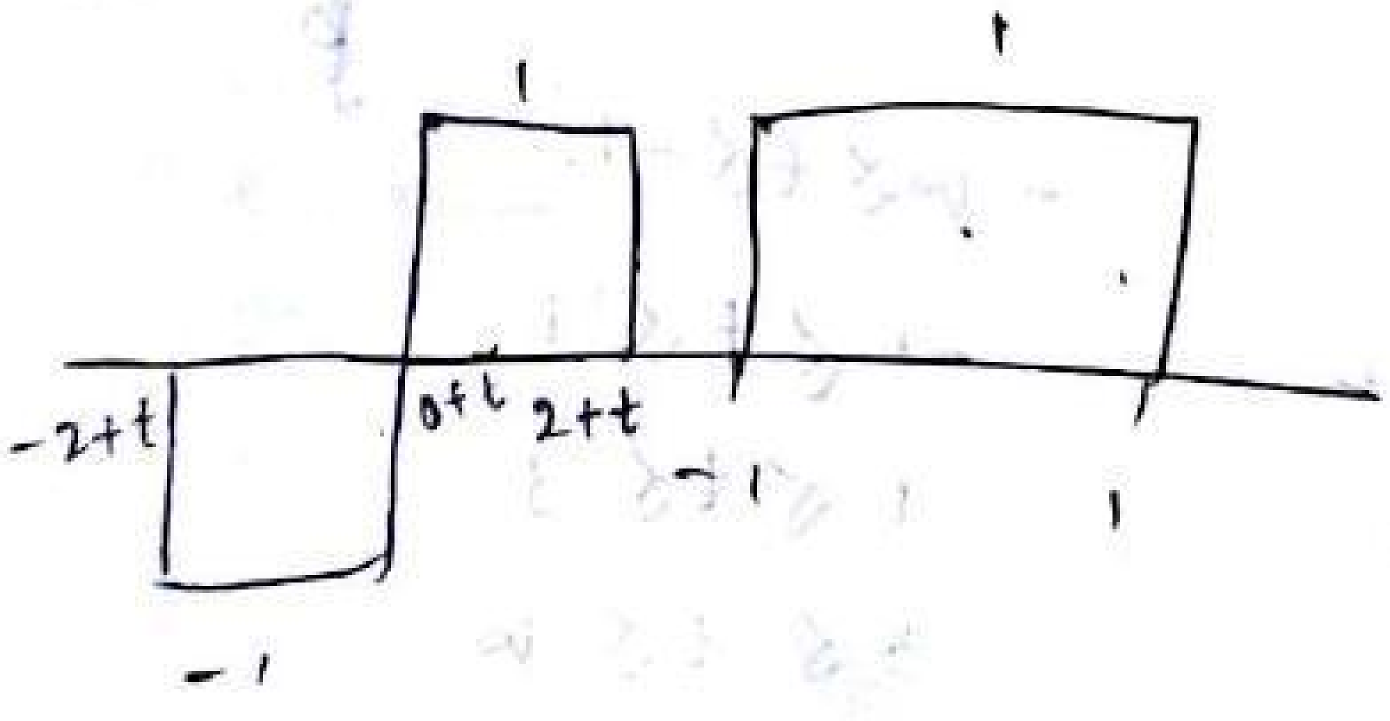
-2 to 0 : 1

0 to 2 = -2

2 to ∞ : 1

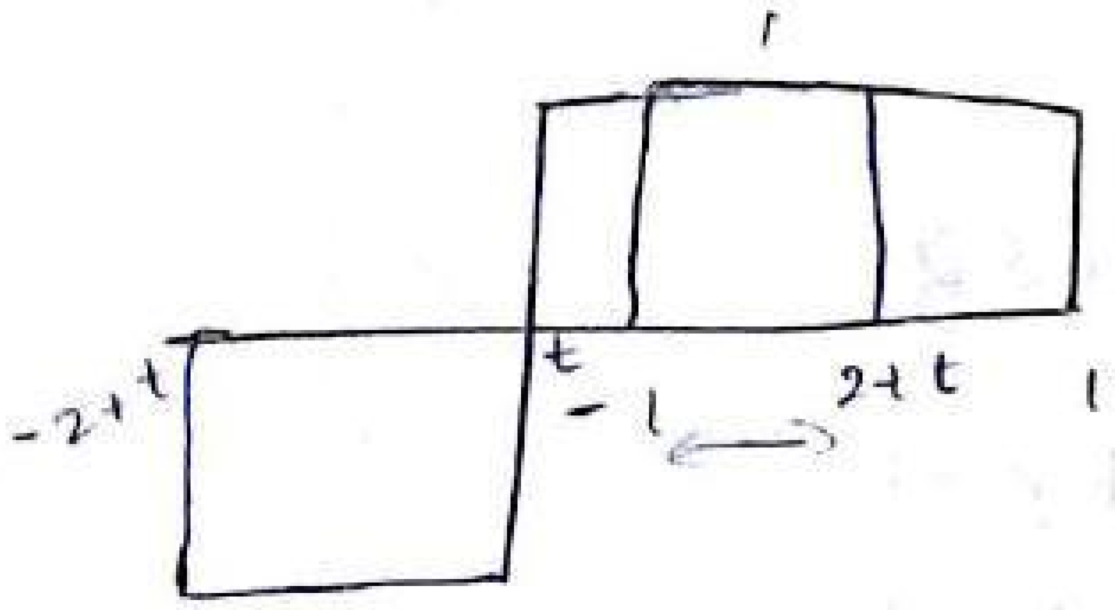






$-\infty \le t \le -3$

$\underline{\underline{0}}$

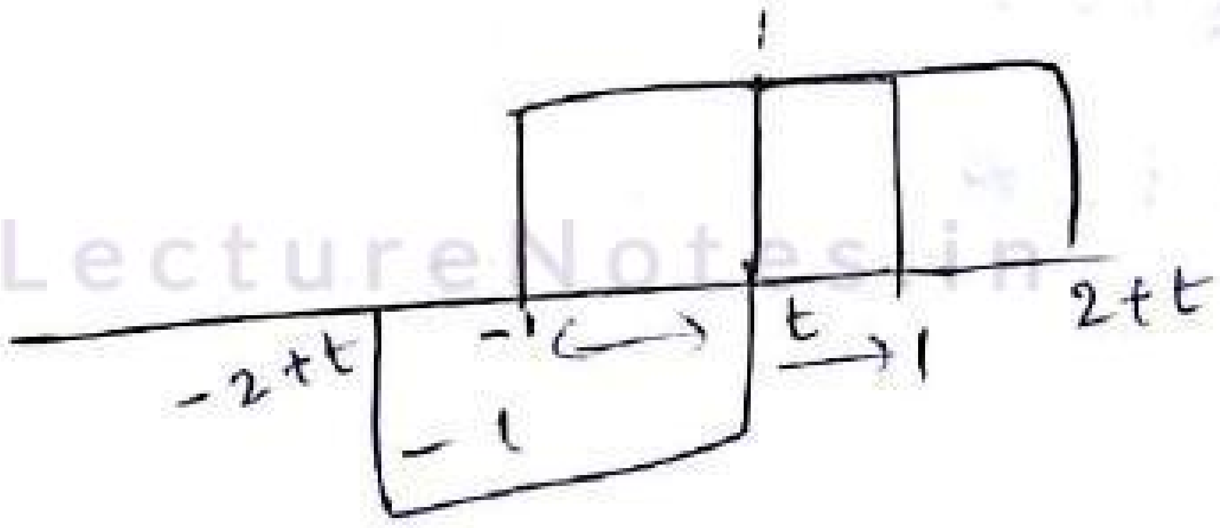


$-3 \le t \le -1$

$\int_{-1}^{2+t} 1.1\, d\tau$

$2+t+1$

$= \underline{3+t}$

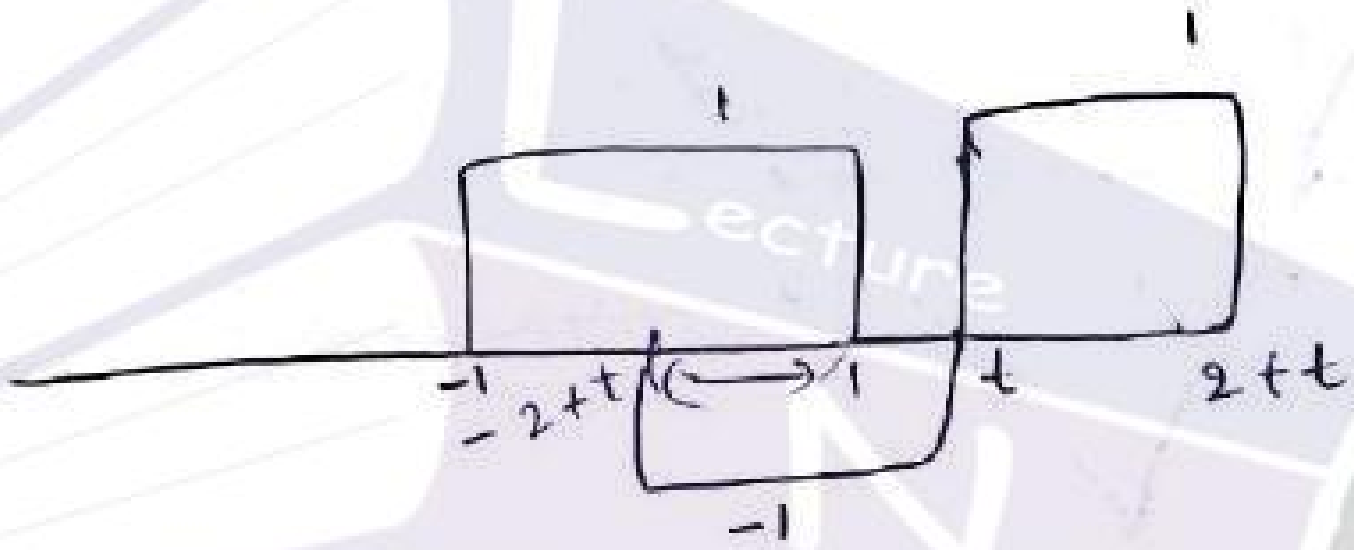


$-1 \le t \le 1$

$\int_{t}^{1} 1.1\, d\tau + \int_{-1}^{t} 1.(-1) d\tau$

$= \underline{1-t+-t-1}$

$\underline{-2t}$



$1 \le t \le 3$

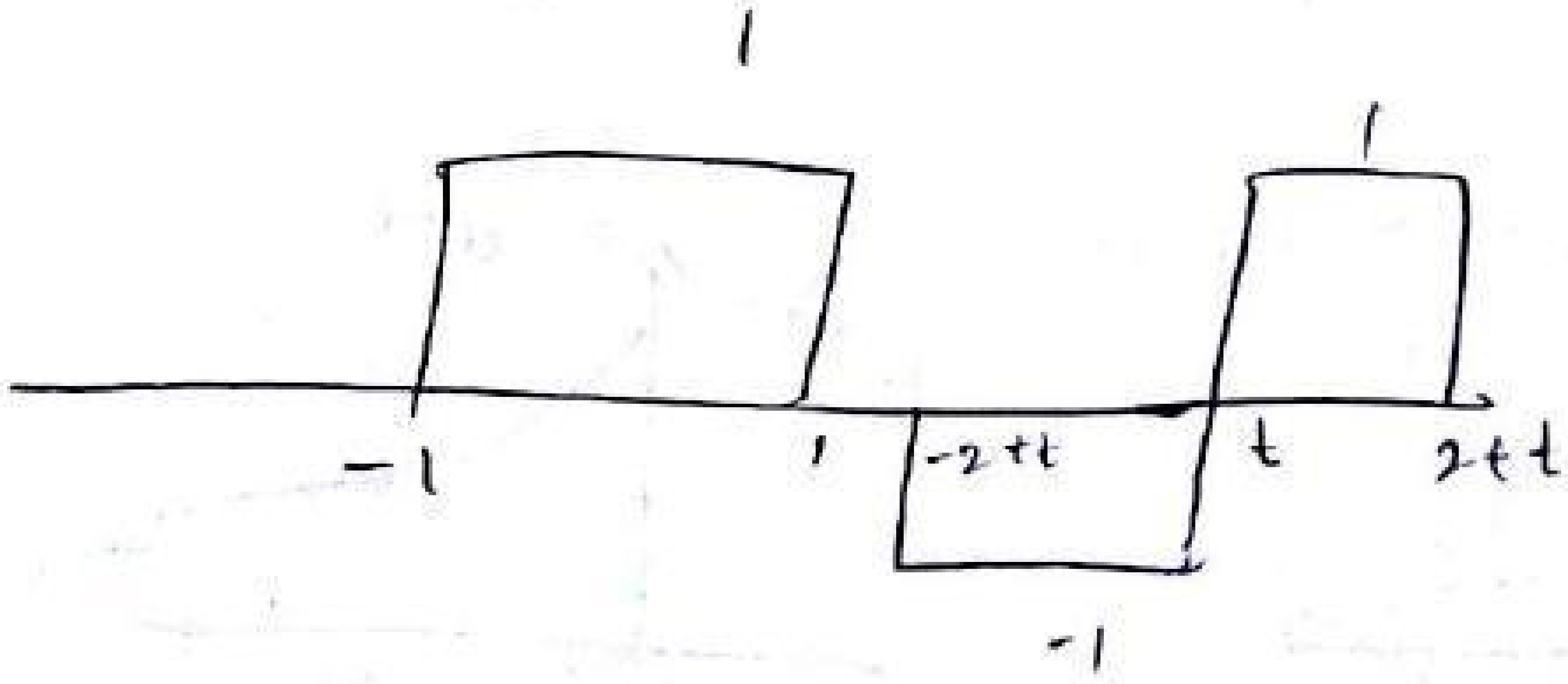
$\int_{-2+t}^{1} 1.(-1)\, d\tau$

$-\left[1-(-2+t)\right]$

$-\left[1+2-t\right]$

$-3+t$

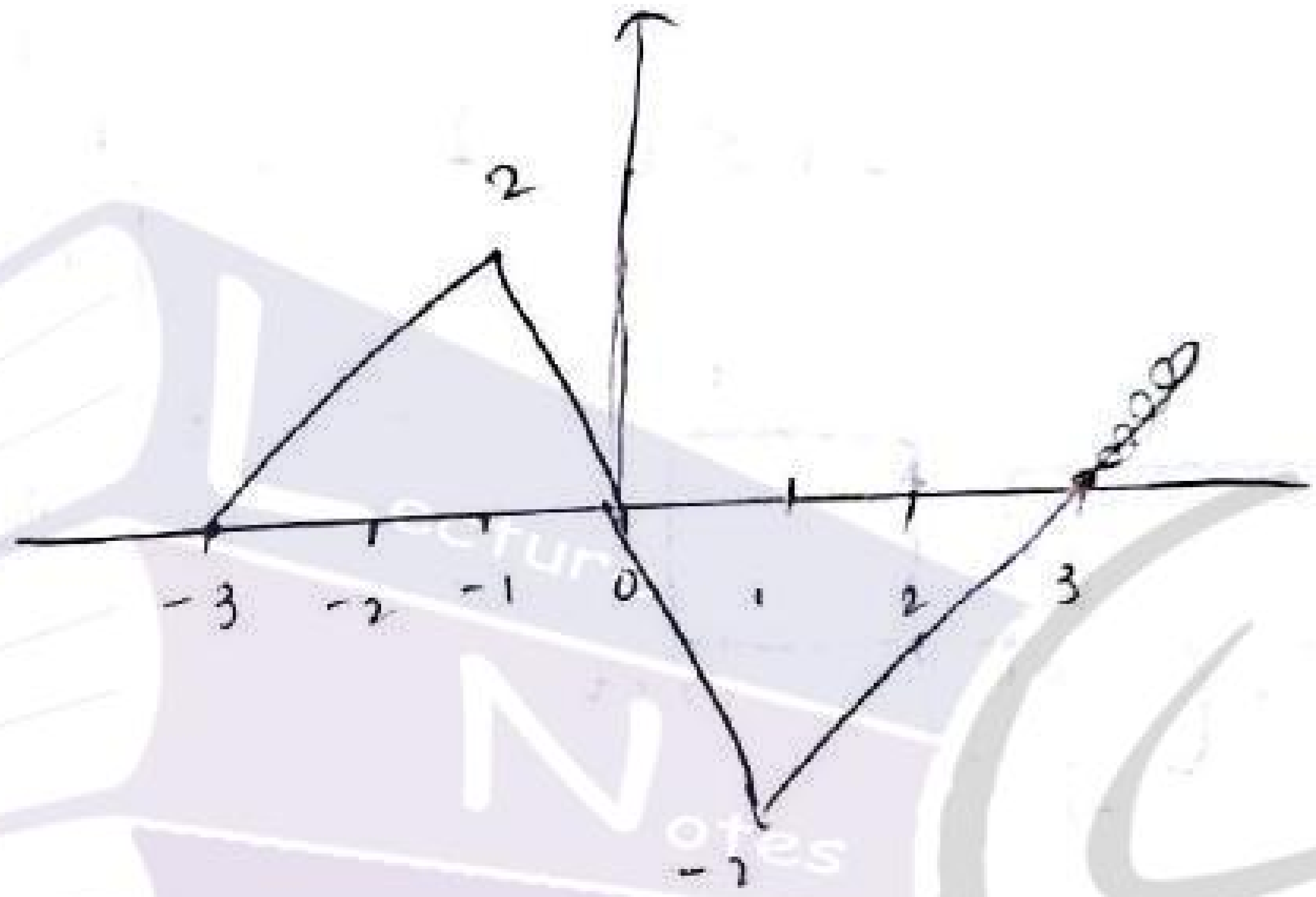
$= \underline{t-3}$



$3 \le t \le \infty$

$=0$

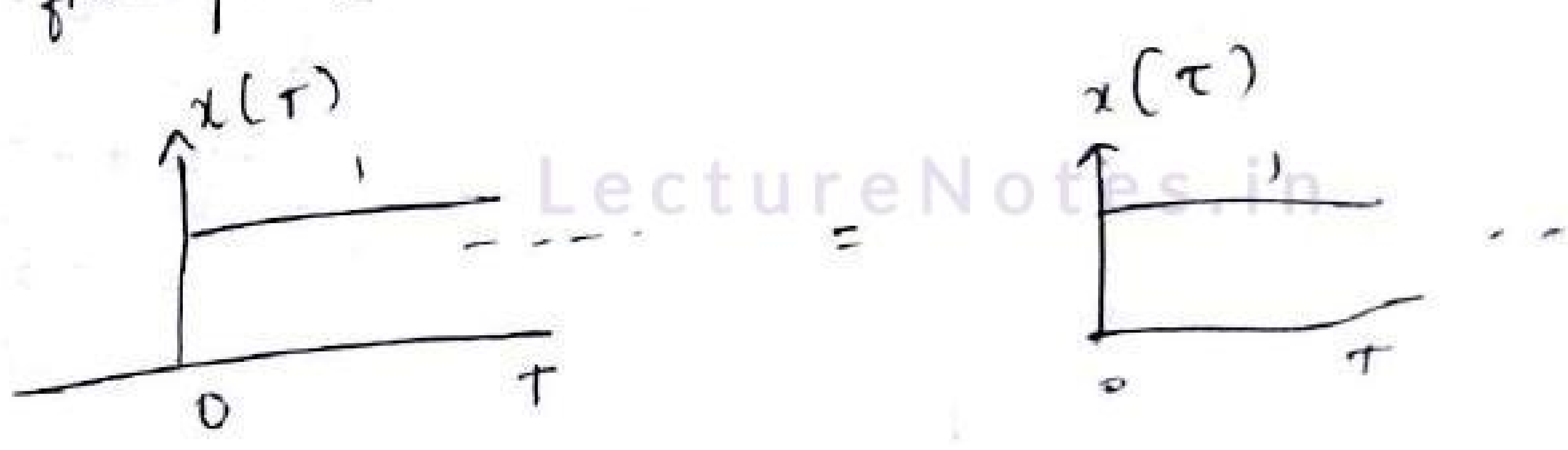
$$y(t) = \begin{cases} 0 & -\infty \leq t \leq -3 \\ 3+t & -3 \leq t \leq -1 \\ -2t & -1 \leq t \leq 1 \\ t-3 & 1 \leq t \leq 3 \\ 0 & 3 \leq t \leq \infty \end{cases}$$

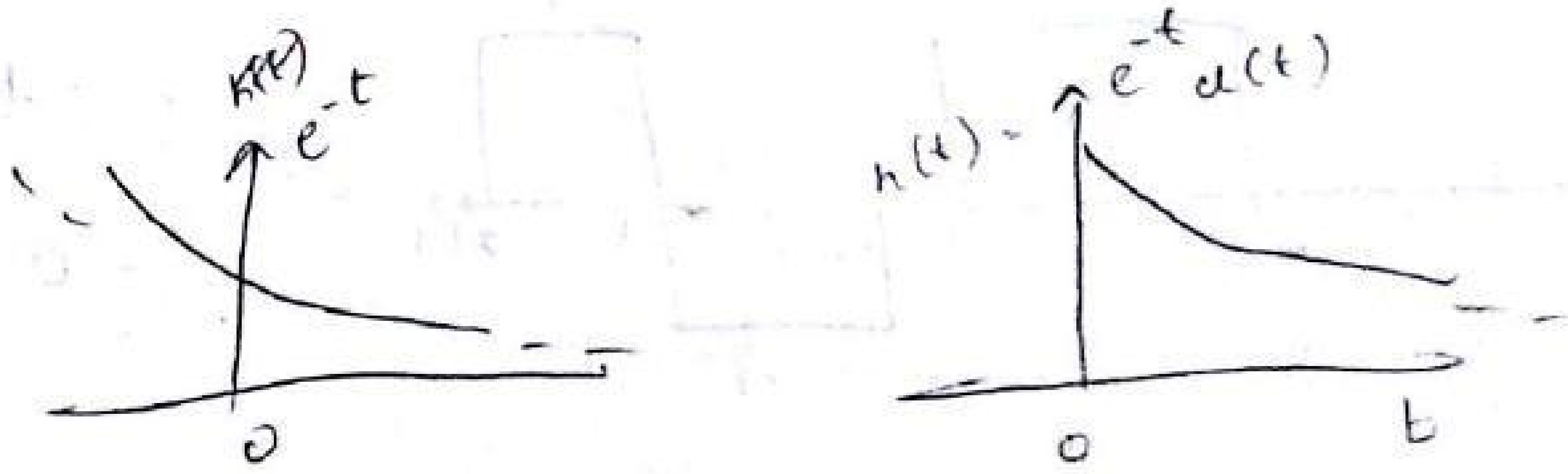


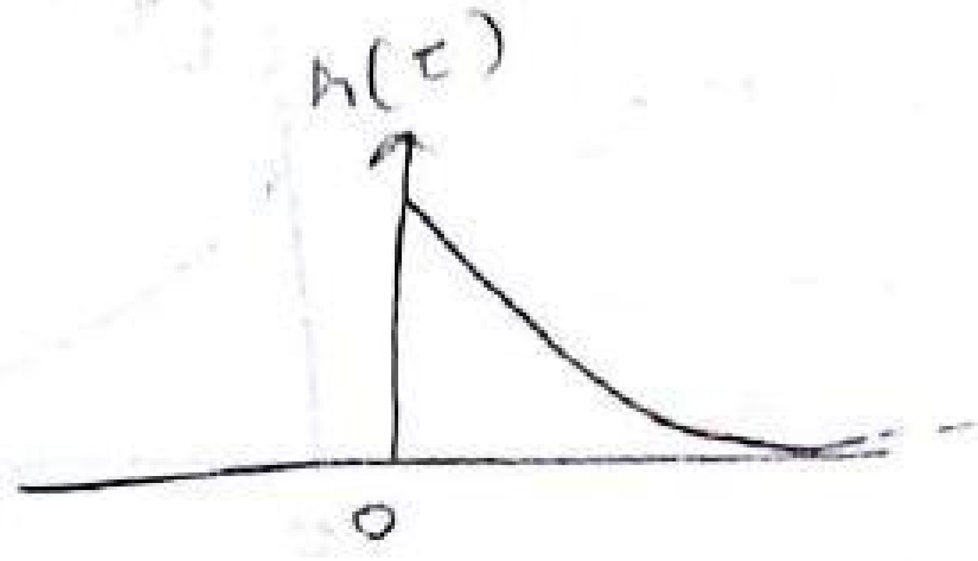
(10) $x(t) = u(t)$

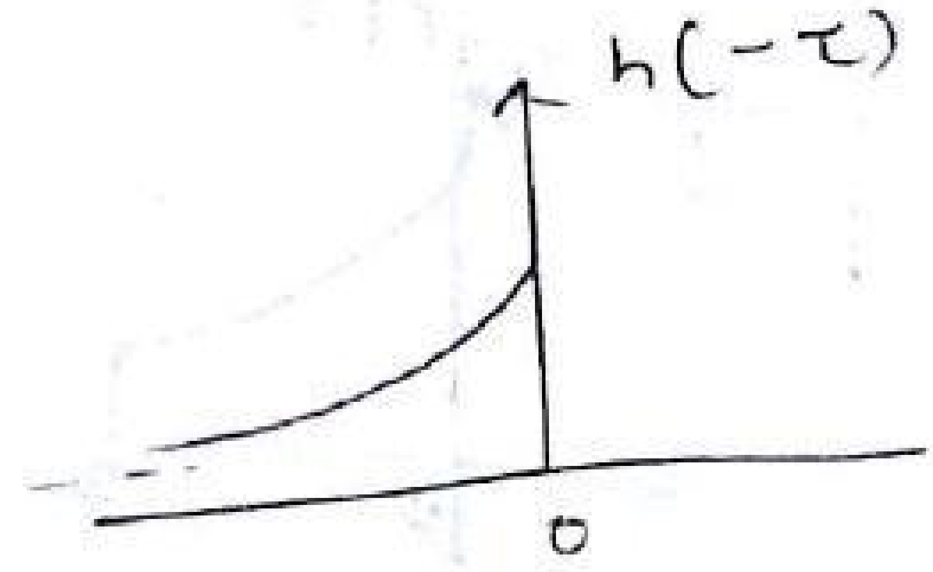
$h(t) = e^{-t}.u(t)$

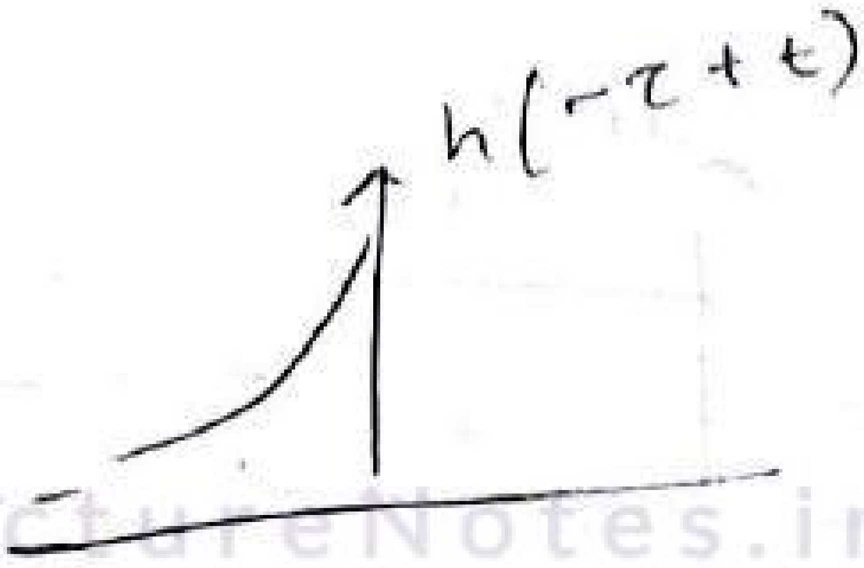
find $y(t) = x(t) * h(t)$

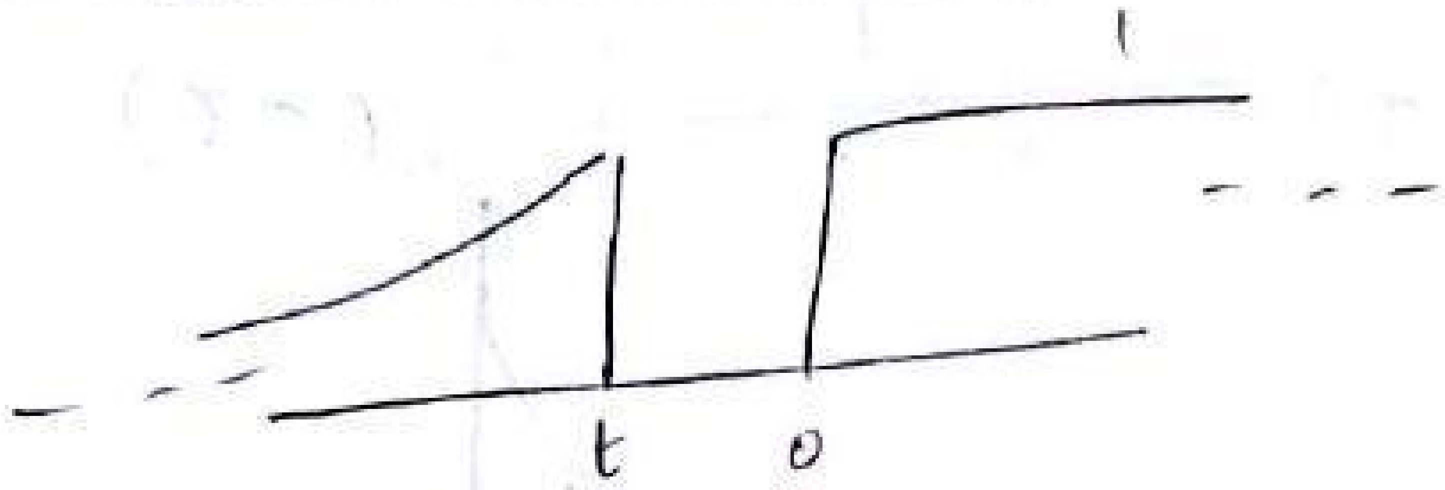






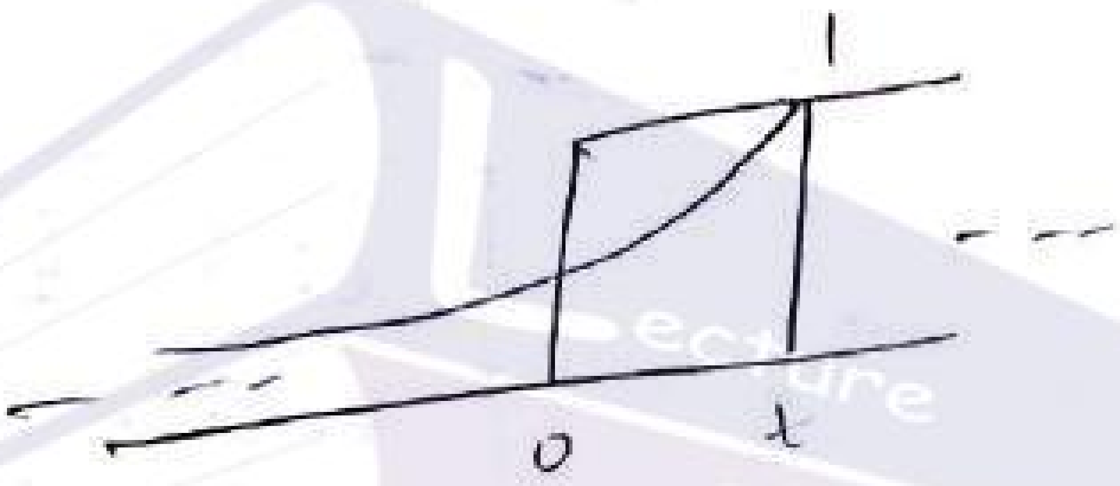






$-\infty < t < 0$

$= 0$



$0 \le t \le \infty$

$$\int_0^t 1\, e^{\tau - t}\, d\tau$$

$$= e^{-t} \int_0^t e^{\tau}\, d\tau$$

$$= e^{-t} \left[e^{\tau} \right]_0^t$$

$$= e^{-t} \left[e^{t} - e^{0} \right]$$

$$\underline{1 - e^{-t}}$$

$x(t) = 1$

$x(\tau) = 1$

$h(t) = e^{-t}$

$h(\tau) = e^{-\tau}$

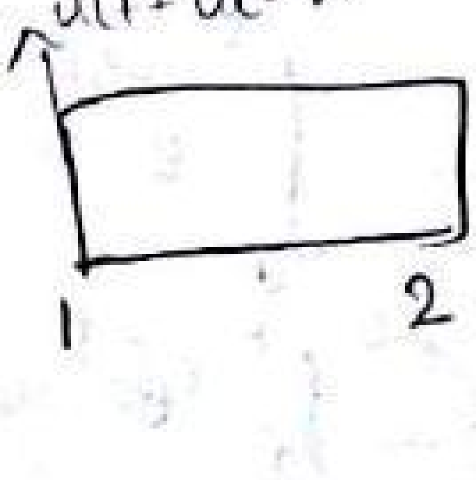
$h(-\tau) = e^{-(-\tau)}$

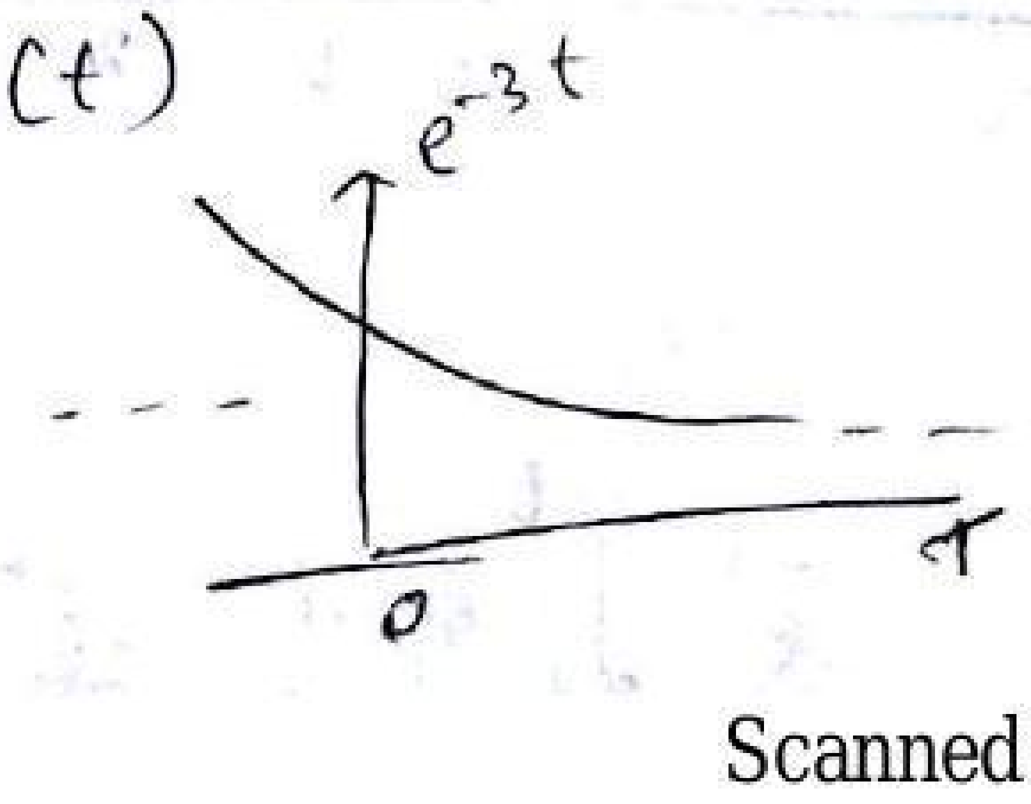
$h(-\tau + t) = e^{-(-\tau + t)}$

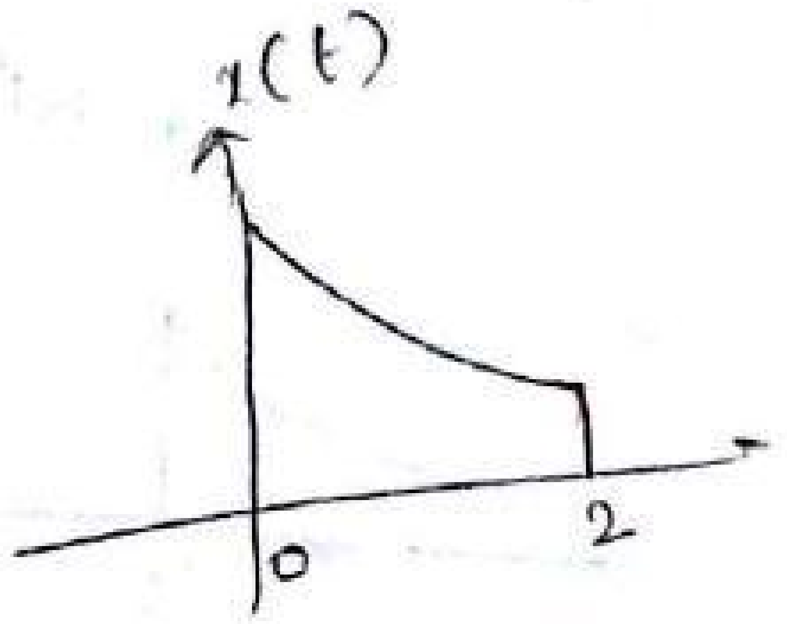
$= \underline{e^{\tau - t}}$

(ii) $x(t) = e^{-3t}\left[u(t) - u(t-2)\right]$

$h(t) = e^{t}\, u(t)$







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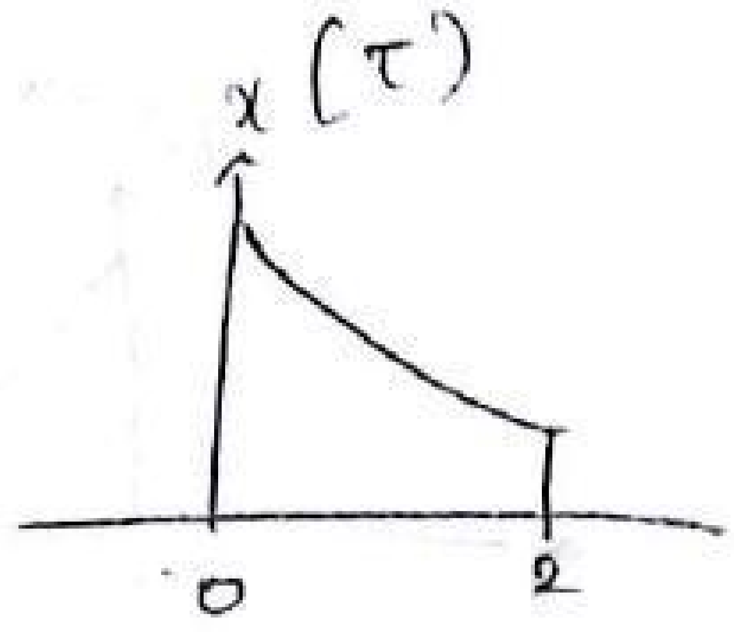
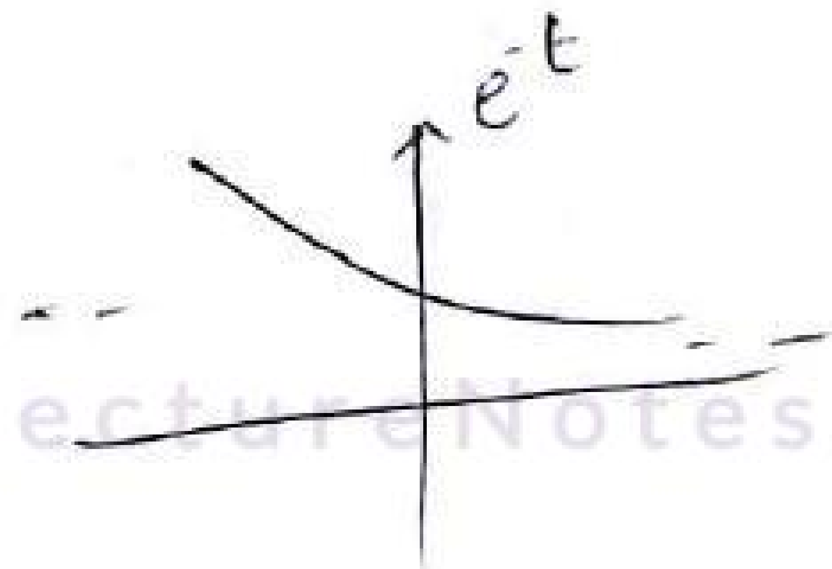
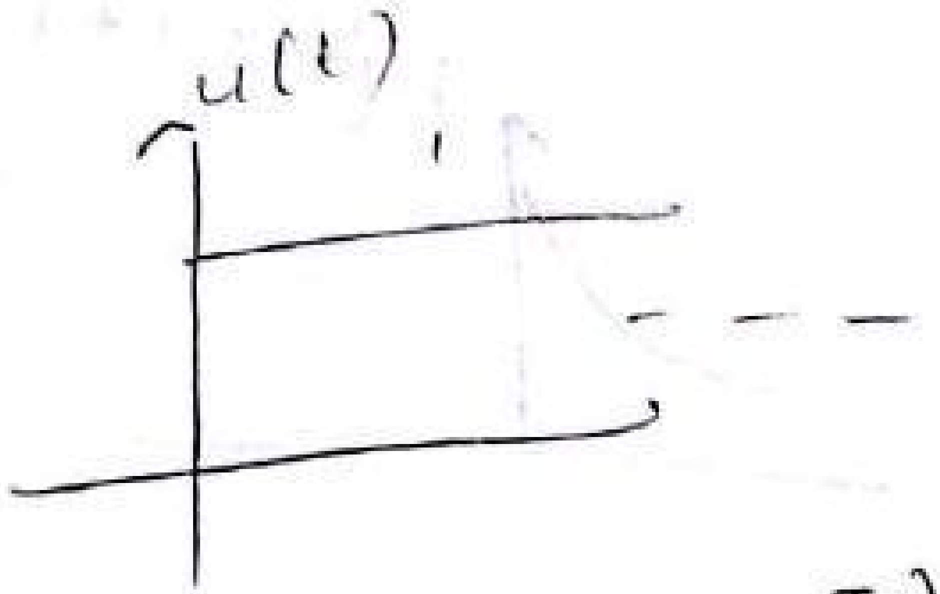
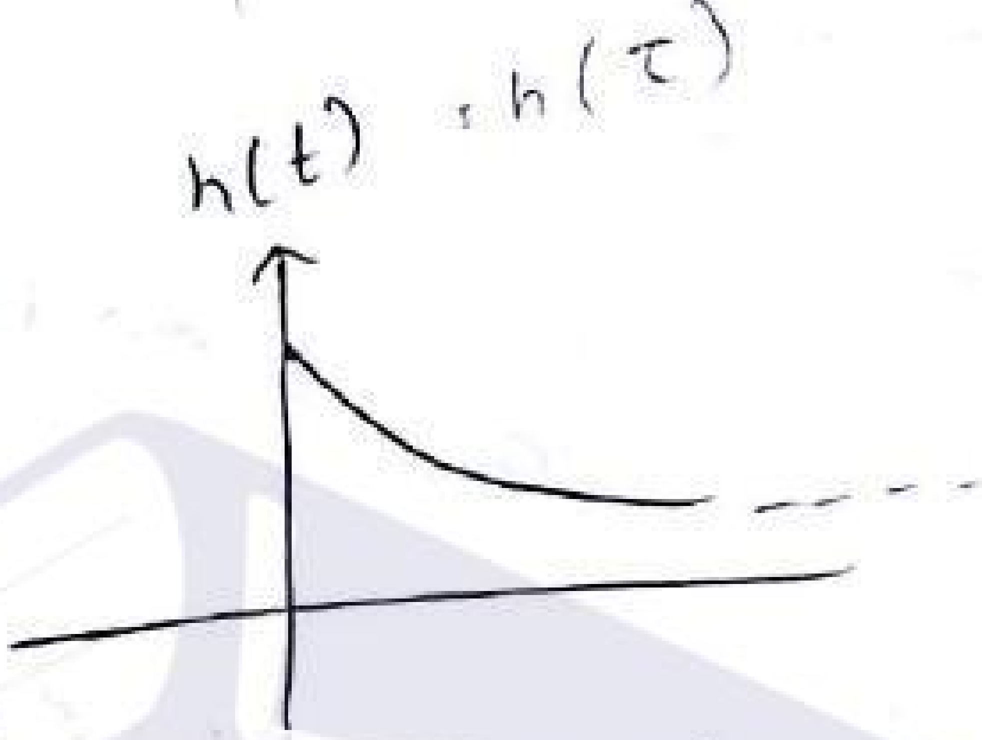
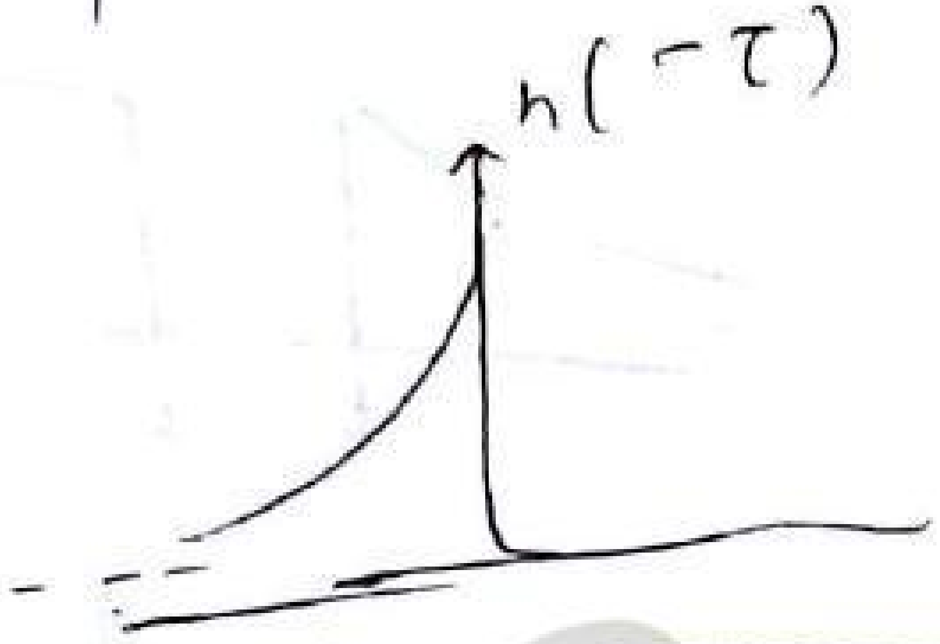
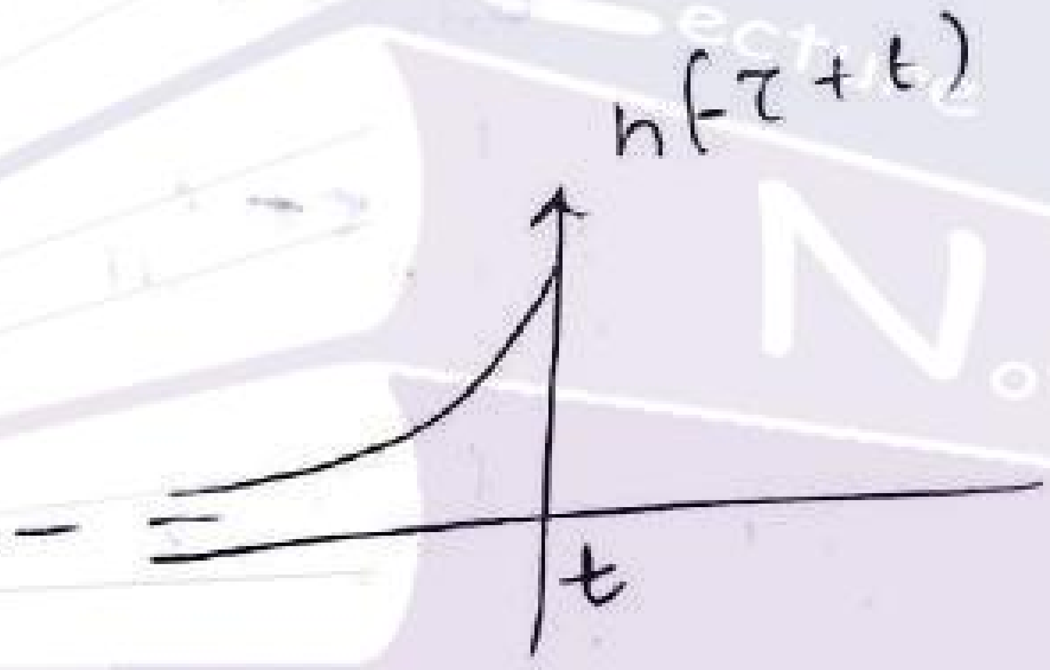












$x(t) = e^{-3t}$

$x(\tau) = e^{-3\tau}$

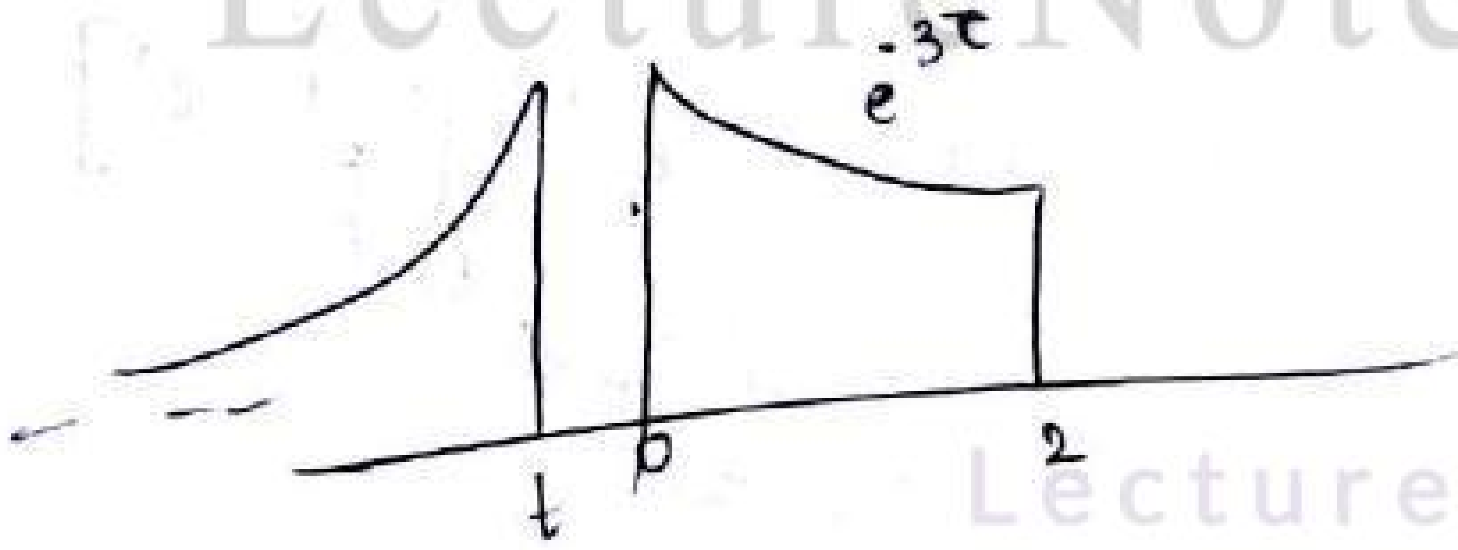
$h(t) = e^{-t}$

$h(\tau) = e^{-\tau}$

$h(-\tau) = e^{-(-\tau)}$

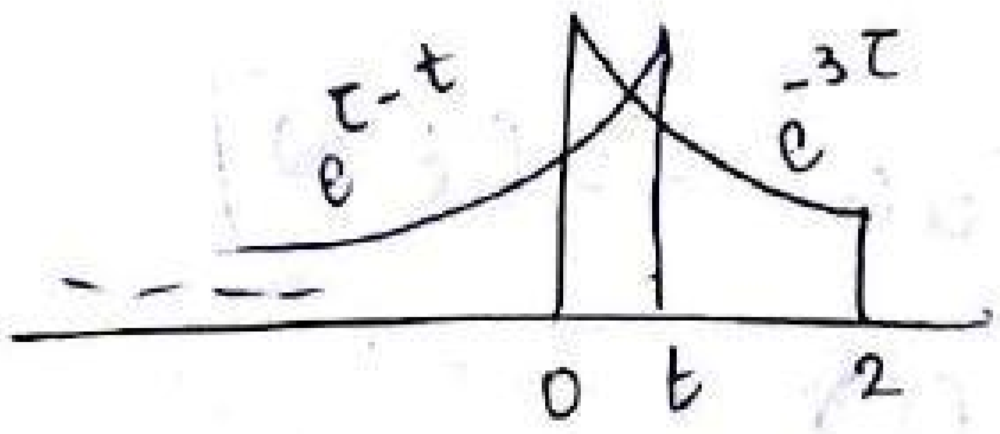
$h(-\tau+t) = e^{-(-\tau+t)}$

$= e^{\tau-t}$



$-\infty \le t \le 0$

$= 0$



$0 \le t \le 2$

$$\int_0^t e^{-3\tau} \cdot e^{\tau-t} \cdot d\tau$$

$$\int_0^t e^{-3\tau} \cdot e^{\tau} \cdot e^{-t} \, d\tau$$

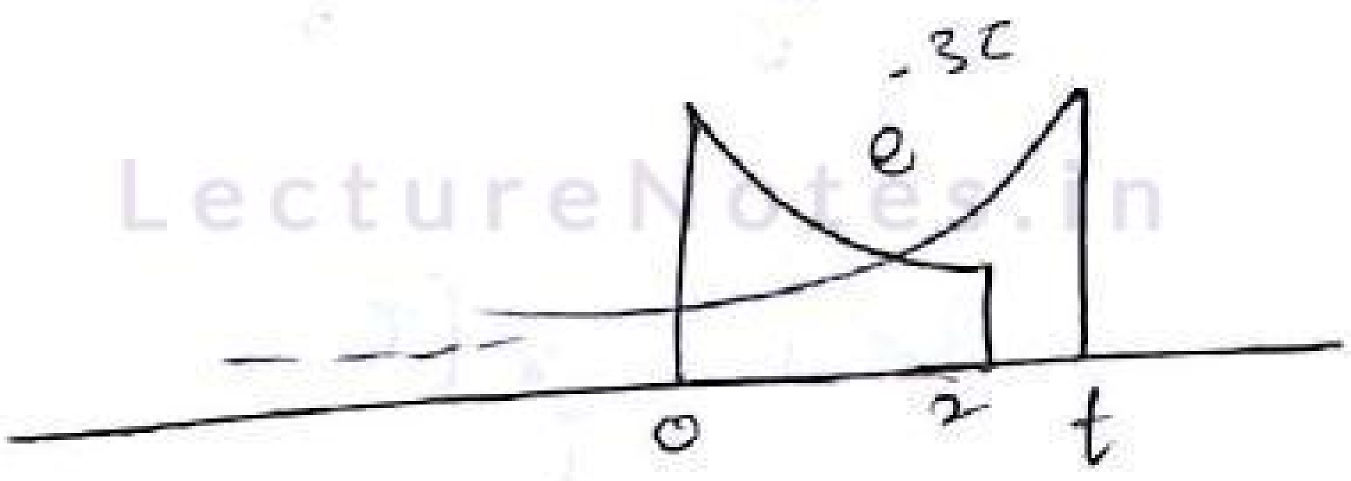
$$e^{-t} \int_0^t e^{-2\tau} d\tau = \frac{-e^{-t}}{2}\left[e^{-2t} - 1\right] \cancel{e^{-2\tau} \int_0^t e^{-t} dt} \quad \frac{-e^{-3t} + 1}{2}$$

$$\frac{-e^{-3t}+e^{-t}}{2}$$

$$= \cancel{e^{-2\tau}\frac{e^{-t}}{-1}\Big|_0^t}$$

$$\cancel{-e^{-2\tau}\left[e^{-t}-1\right]}$$

$$\cancel{= -e^{-2\tau-t}+e^{-2\tau}}$$



$2 \le t \le \infty$

$$\int_0^2 e^{-3\tau}\cdot e^{\tau-t}\,d\tau$$

$$= e^{-t}\int_0^2 e^{-2\tau}\,d\tau$$

$$= \frac{-e^{-t}}{2}\left[e^{-4}-e^{0}\right] = \frac{-e^{-t}}{2}\left[e^{-4}-1\right] \quad \cancel{-e^{-2\tau}\left[e^{-2}-e^{0}\right]}$$

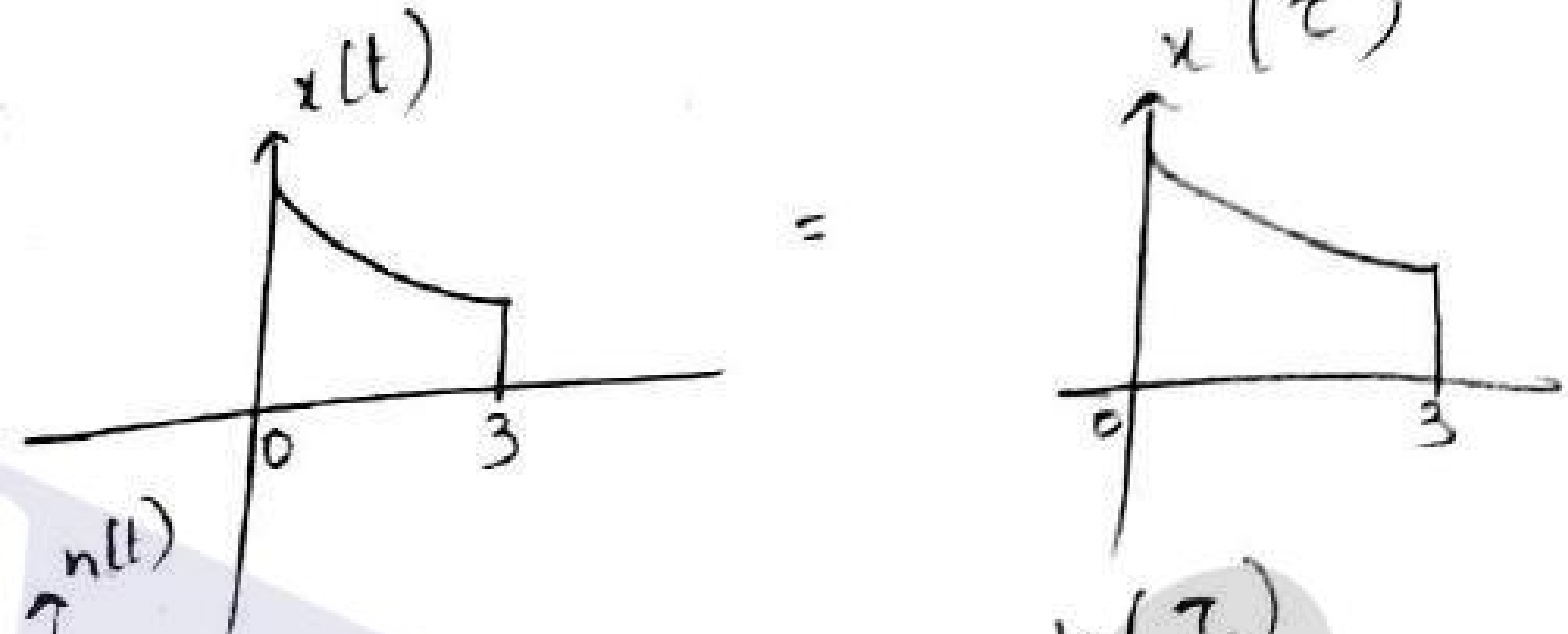
$$\cancel{-e^{-2\tau-2}+e^{-2\tau}}$$

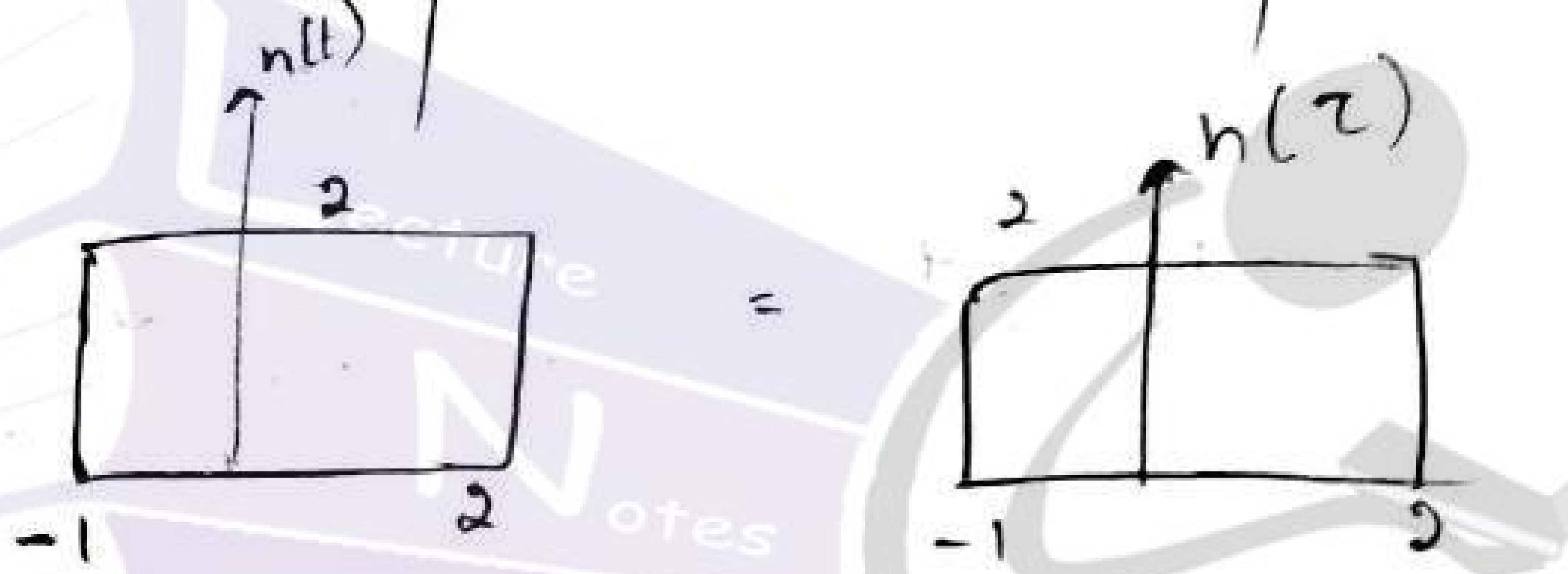
$$\cancel{y(t) = \begin{cases} 0 & -\infty \le t \le 0 \\ -e^{-2\tau}\left[e^{-t}-1\right] & 0 \le t \le 2 \\ -e^{-2\tau}\left[e^{-2}-1\right] & 2 \le t \le \infty \end{cases}}$$

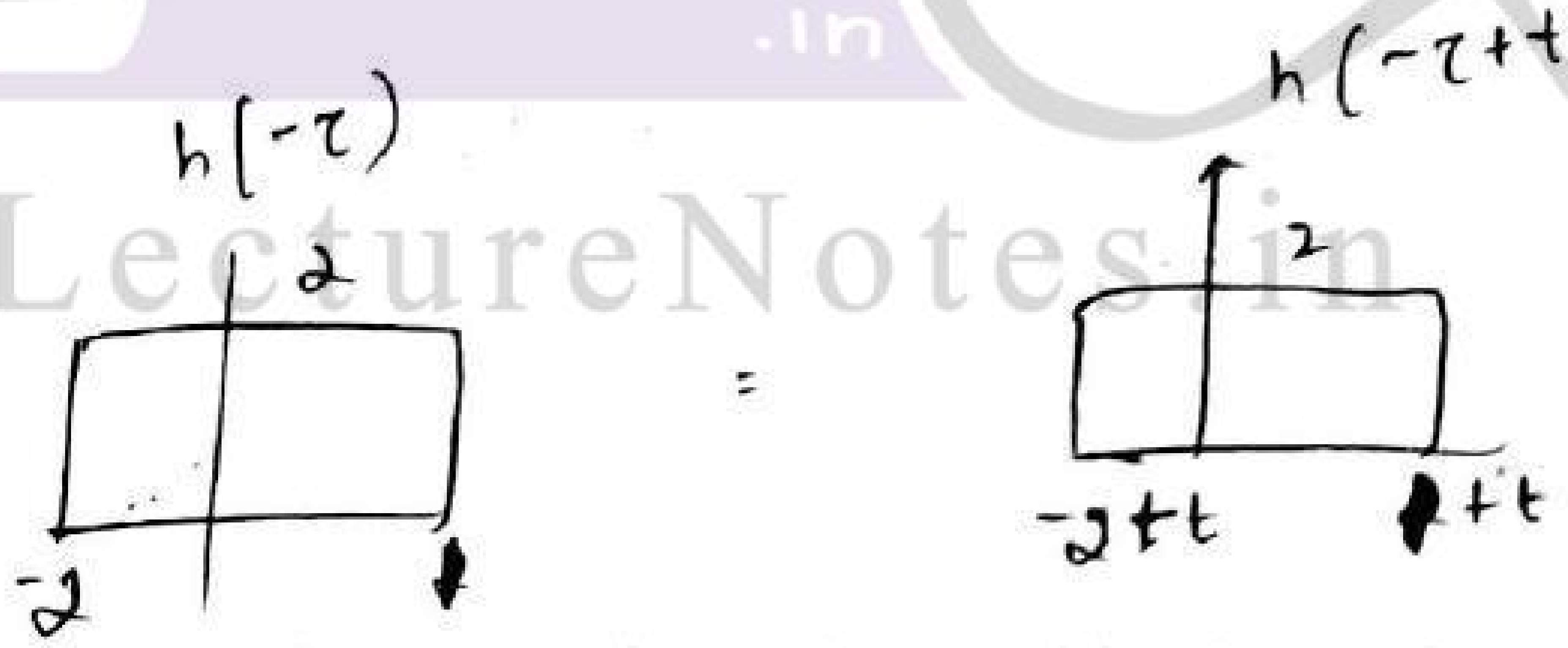
$$\boxed{y(t) = \begin{cases} 0 & -\infty \le t \le 0 \\ \dfrac{-e^{-3t}+e^{-t}}{2} & 0 \le t \le 2 \\ \dfrac{-e^{-t}}{2}\left[e^{-4}-1\right] & 2 \le t \le \infty \end{cases}}$$

(12) $x(t) = e^{-2t}\left[u(t) - u(t-3)\right]$

$h(t) = 2\left[u(t+1) - u(t-2)\right]$

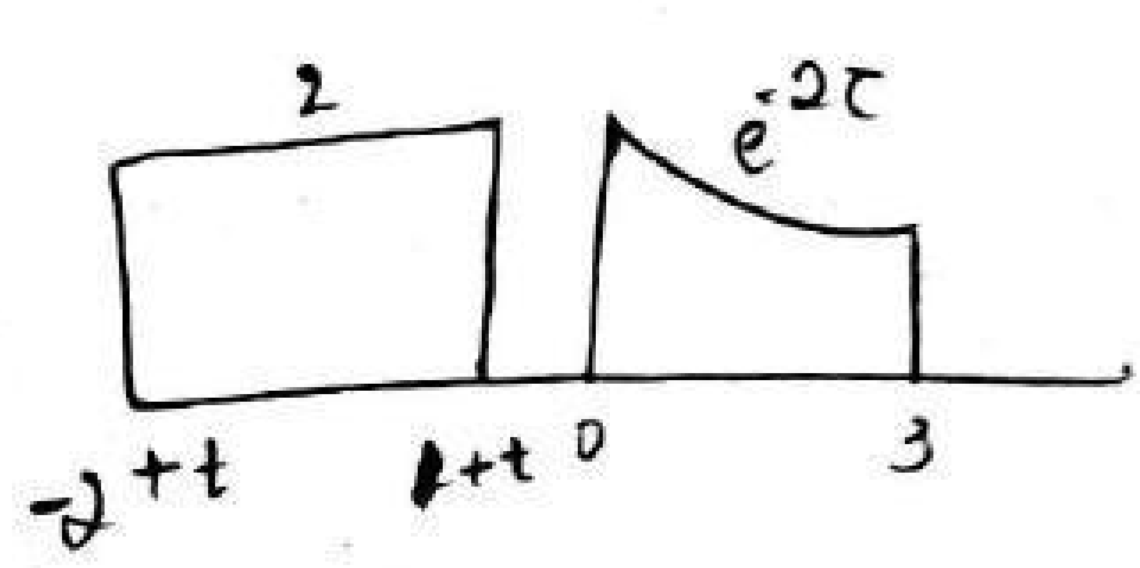






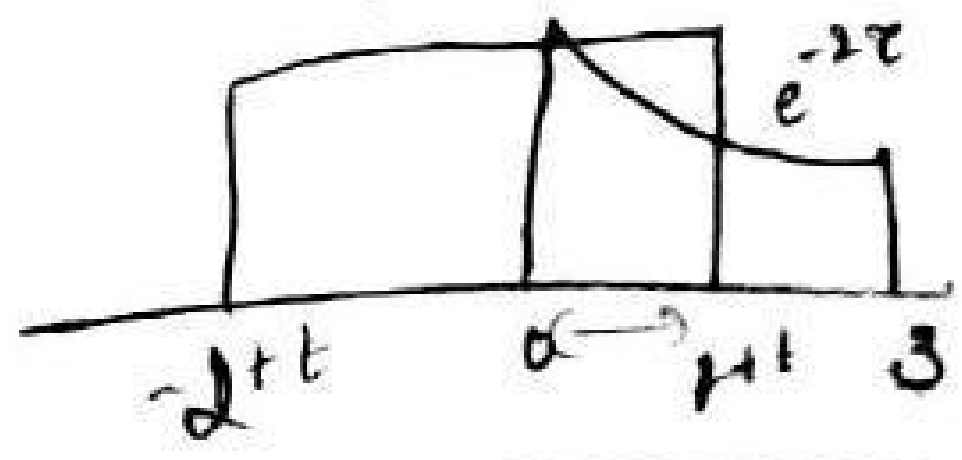
$x(t) = e^{-2t}$

$x(\tau) = e^{-2\tau}$



$-\infty \leq t \leq -2$

$\boxed{=0}$



$-2 \leq t \leq 2$

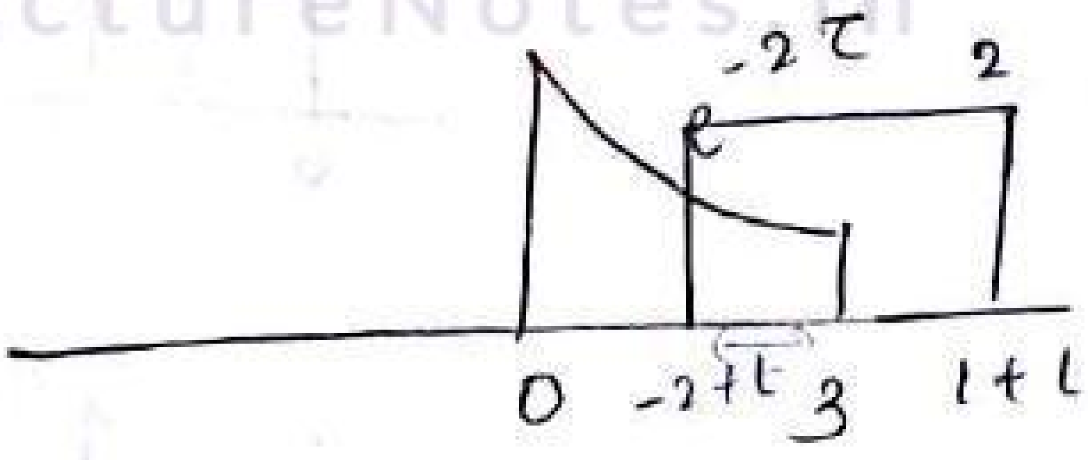
$\int_0^{1+t} 2 \cdot e^{-2\tau}\, d\tau$

$$2\left.\frac{e^{-2\tau}}{-2}\right|_0^{1+t}$$

$$-\left[e^{-2[1+t]}-e^{0}\right]$$

$$-e^{-2-2t}+1$$

$$=\boxed{1-e^{-2(1+t)}}$$



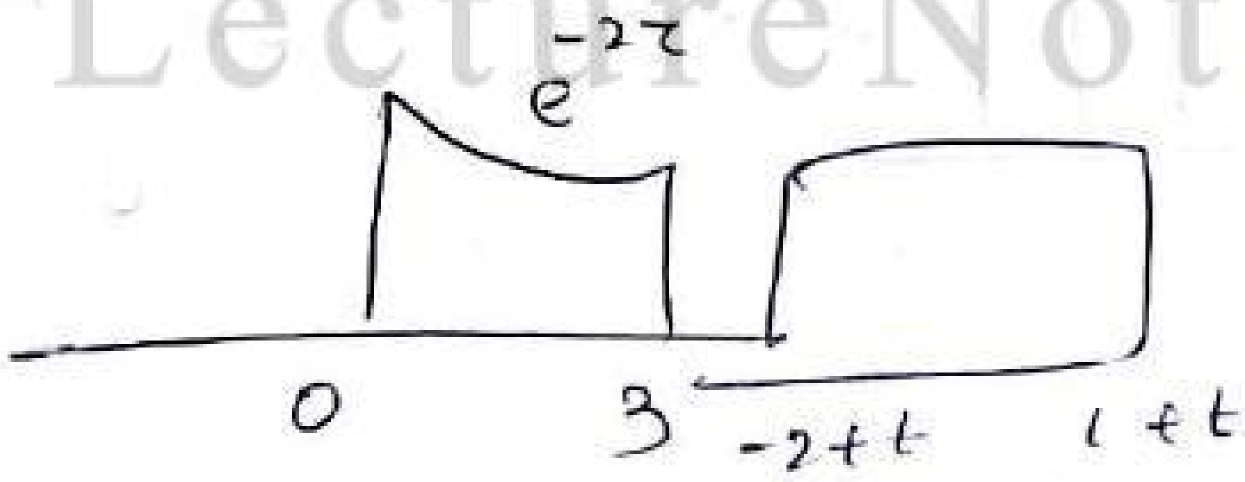
$$2\leq t\leq 5$$

$$\int_{-2+t}^{3} e^{-2\tau}\cdot 2$$

$$\cancel{\frac{2}{-2}\left[e^{-2[3+2-1}\right.}$$

$$-1\left[e^{-2(3)}-e^{-2(-2+t)}\right]$$

$$\boxed{=-e^{-6}+e^{4-2t}}$$



$$5\leq t\leq\infty$$

$$\boxed{=0}$$

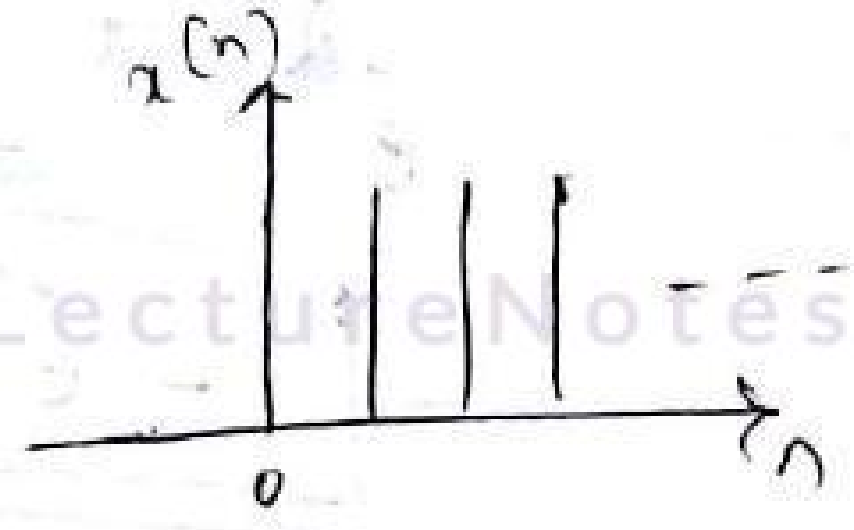
$$\boxed{y(t)=\begin{cases}0 & -\infty\leq t\leq 2\\ 1-e^{-2(1+t)} & -2\leq t\leq 2\\ e^{4-2t}-e^{-6} & 2\leq t\leq 5\\ 0 & 5\leq t\leq\infty\end{cases}}$$

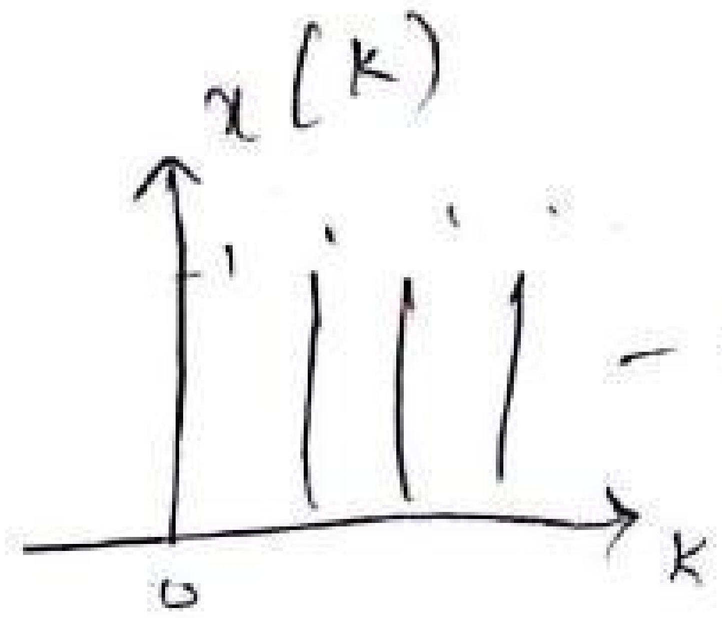
CONVOLUTION SUM

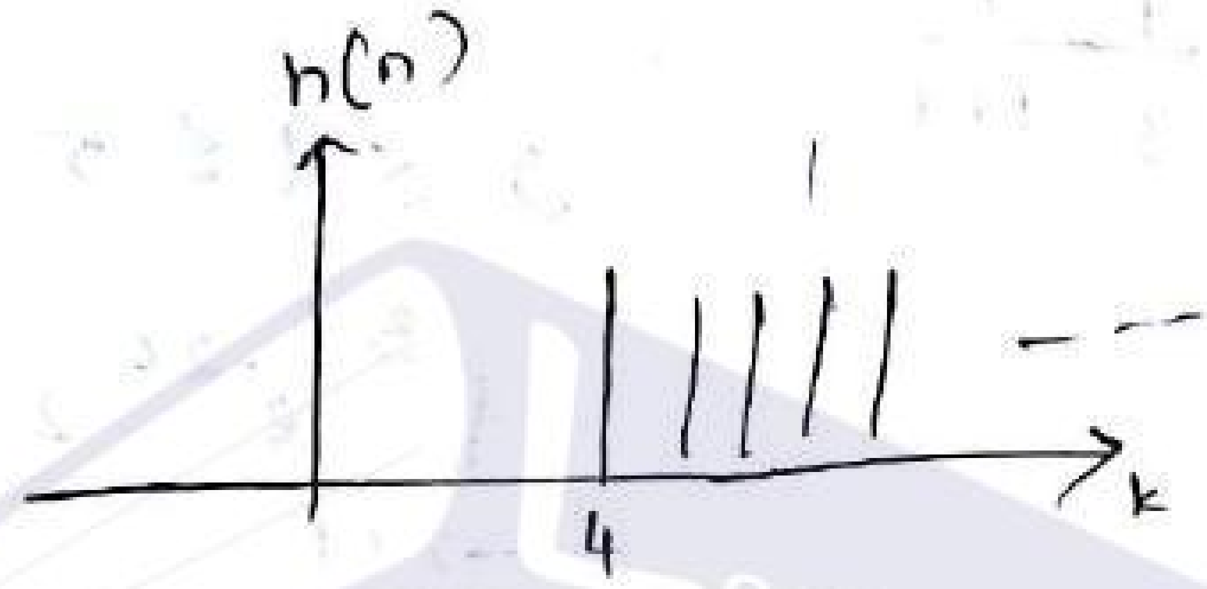
(1) $x(n) = u(n)$

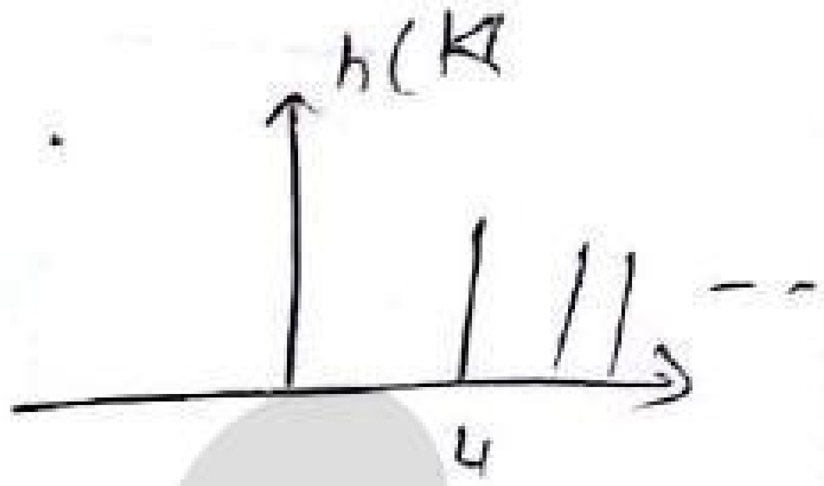
$h(n) = u(n-4)$

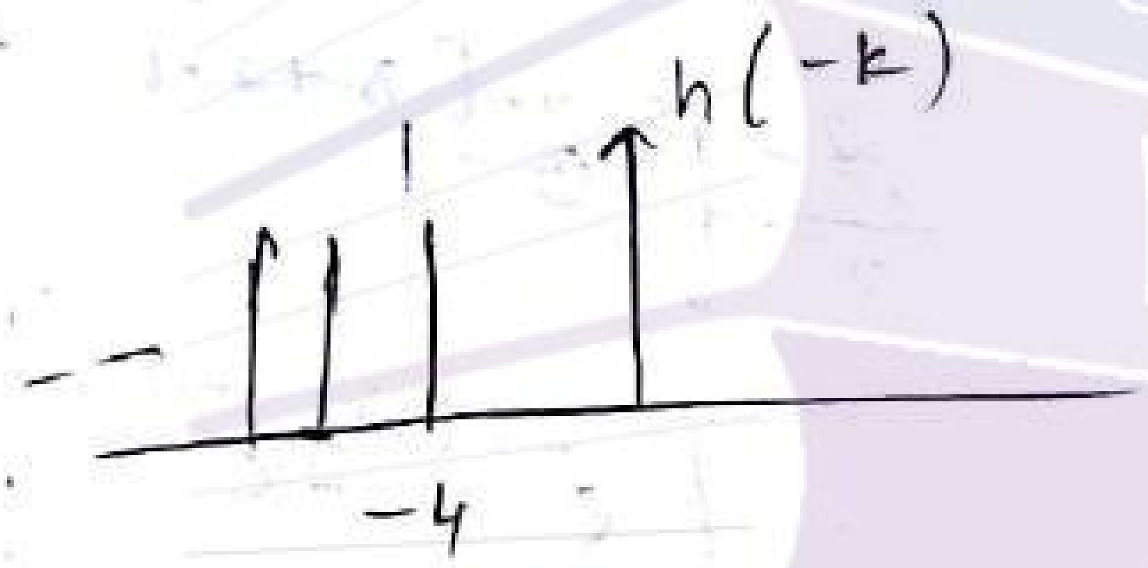
find $y(n)$

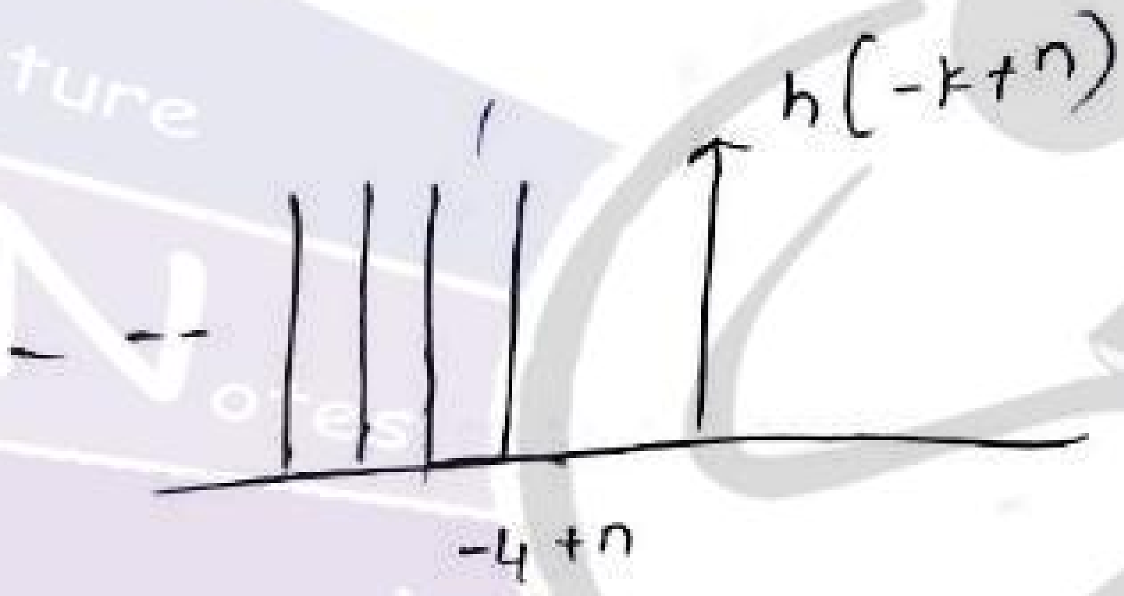


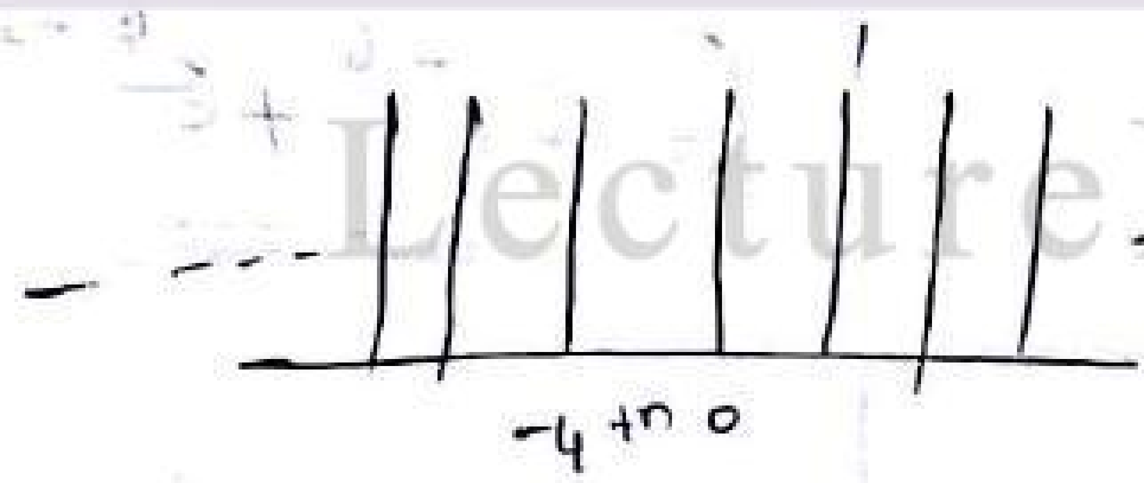






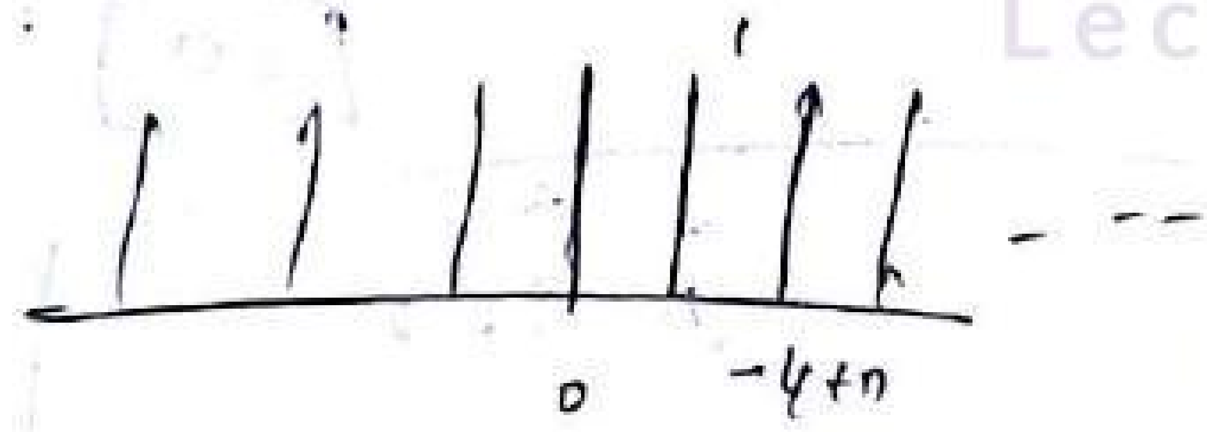






$-\infty \le n < 4$

$= 0$



$4 \le n \le \infty$

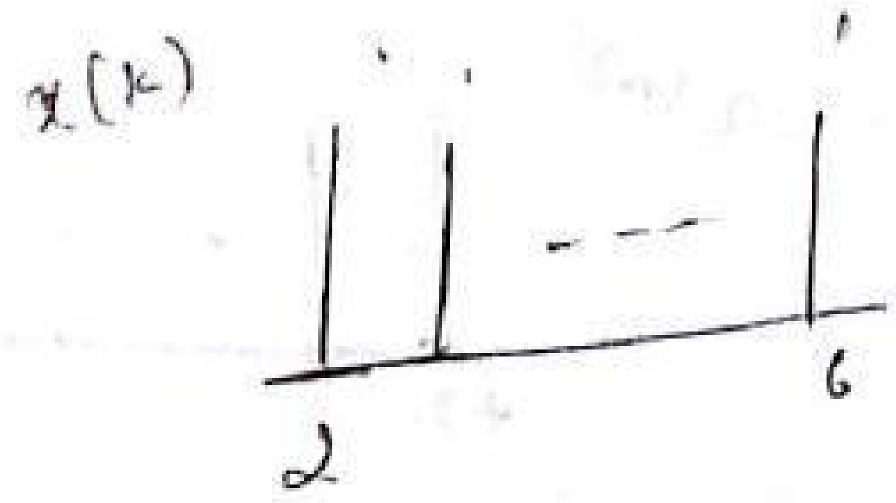
$$\sum_{k=0}^{-4+n} 1 \cdot 1 = \sum_{k=0}^{n-4} 1 = n-4+1$$

$$= \underline{\underline{n-3}}$$

$$\boxed{y(n) = \begin{cases} 0 & -\infty \le n < 4 \\ n-3 & 4 \le n \le \infty \end{cases}}$$

(2) $x(n) = u(n-2) - u(n-7)$

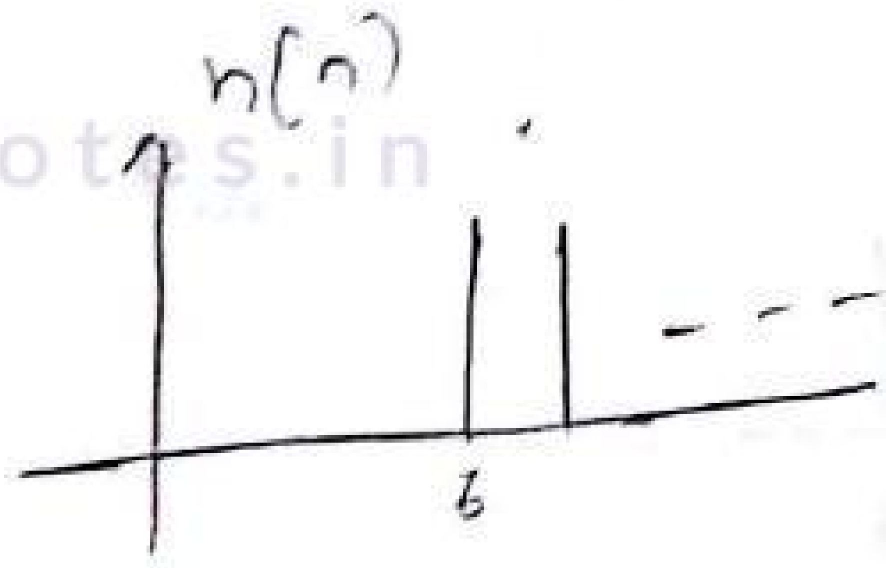
$h(n) = u(n-6)$

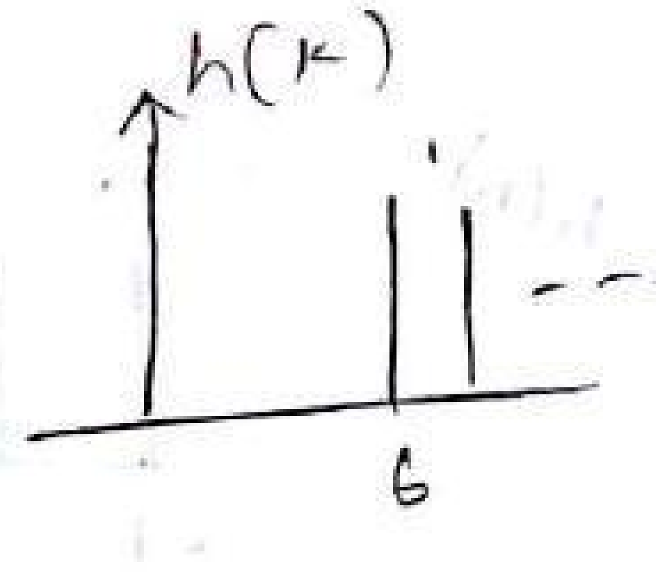


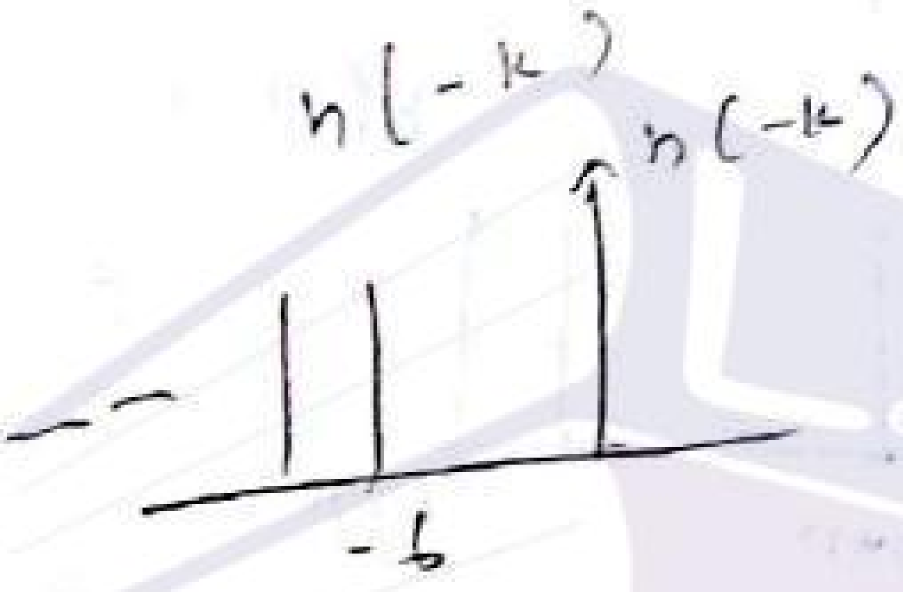
h(n)

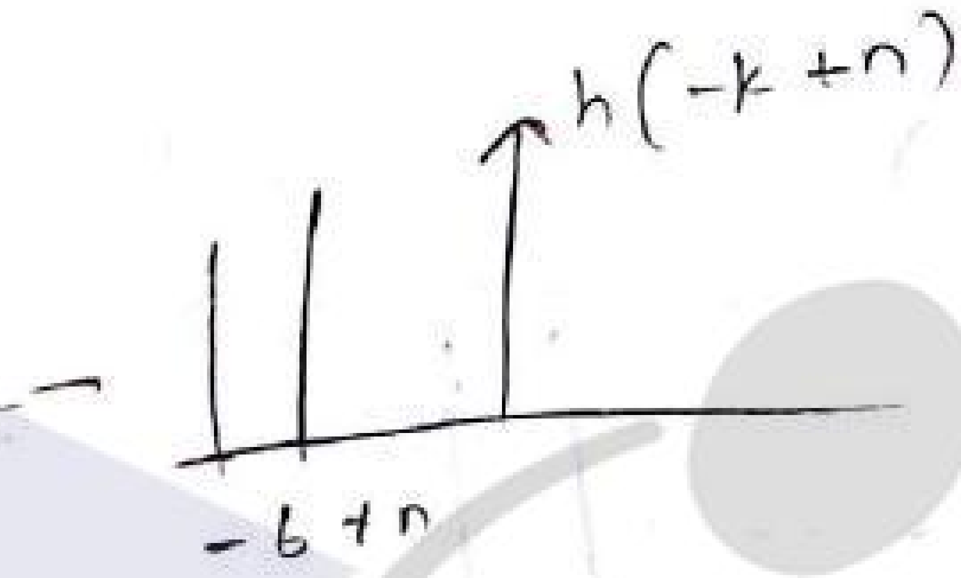
$n-6 \geq 0, \infty$

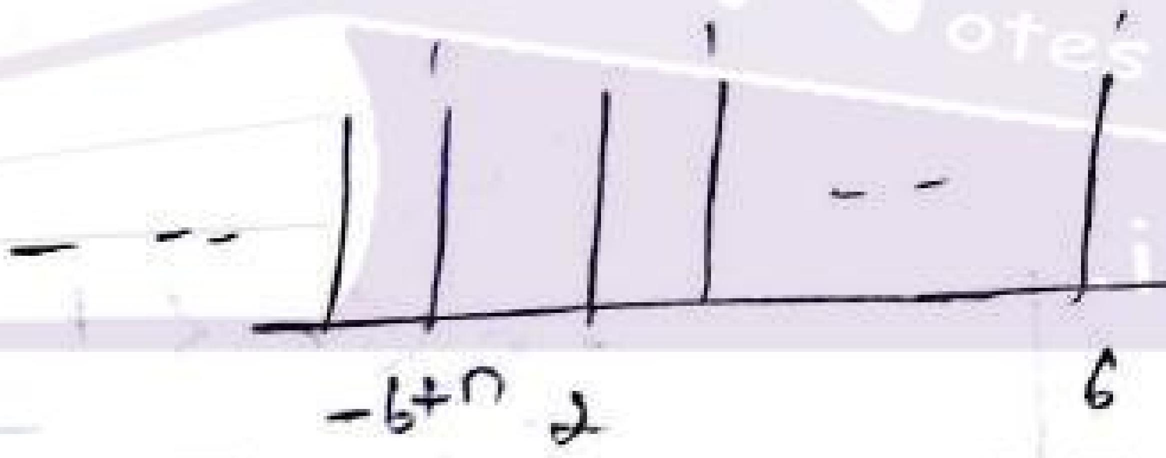
$n \geq 6, \infty$





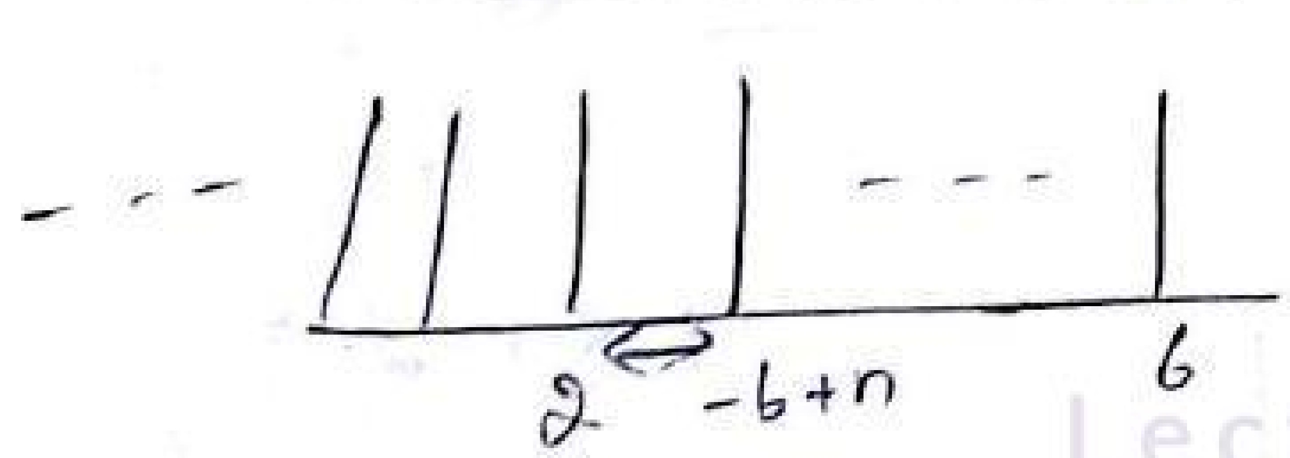






$-\infty < n \leq 7$

$= 0$



$7 \leq n < 12$

$k-2 = l$

$k=2 \;\; l=0$

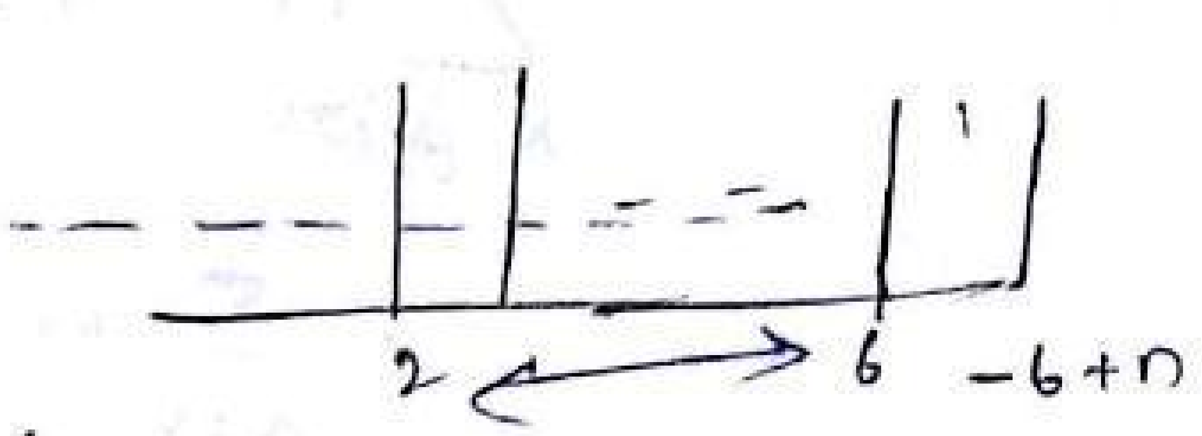
$k=n-6 \;\; l=n-8$

$$\sum_{k=2}^{-6+n} 1\cdot 1 = \sum_{l=0}^{n-8} 1\cdot 1$$

$k = 2 \Rightarrow 0$

$= n-8+1$

$\boxed{n-7}$



$12 \leq n \leq \infty$

$$\sum_{k=2}^{6} 1\cdot 1 = \cancel{6+1}$$

$= \cancel{7}$

$k=2 \;\; l=0$

$k=6 \;\; l=4$

$\sum_{l=0}^{4}$

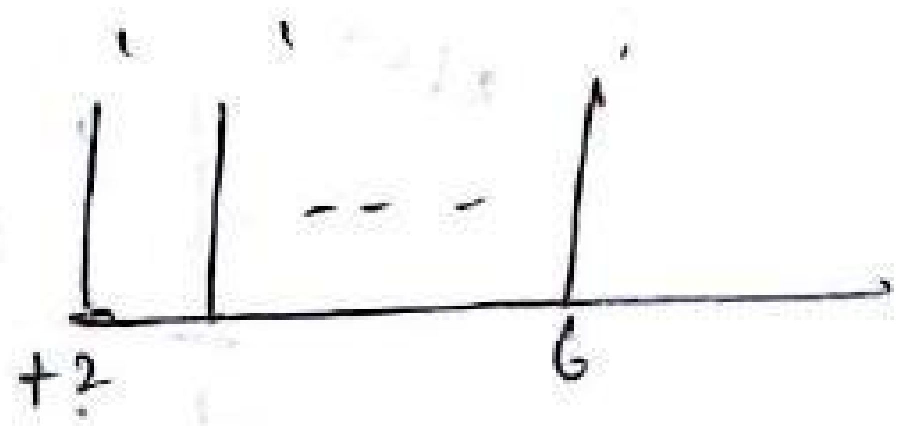
$4+1$

$\boxed{5}$

$$\boxed{y(n) = \begin{cases} 0 & -\infty < n \leq 7 \\ n-7 & 7 \leq n < 12 \\ 5 & 12 \leq n \leq \infty \end{cases}}$$

③ $x(n) = u(n-2) - u(n-7)$

$h(n) = u(n+1) - u(n-4)$



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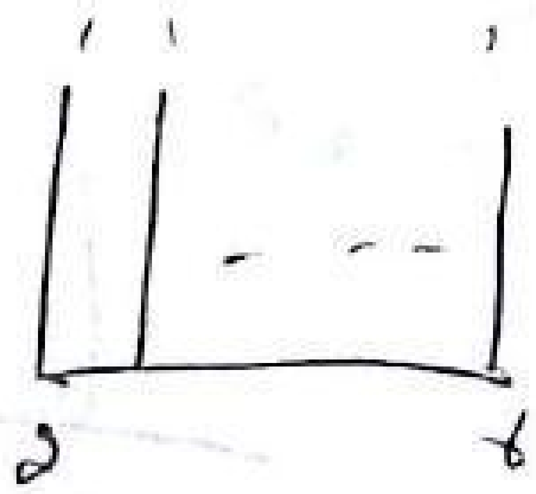

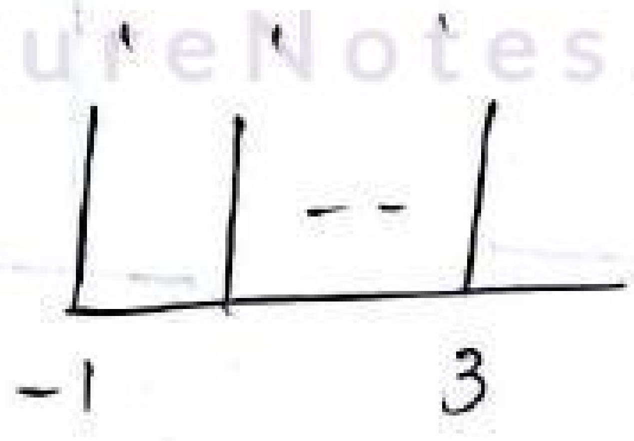

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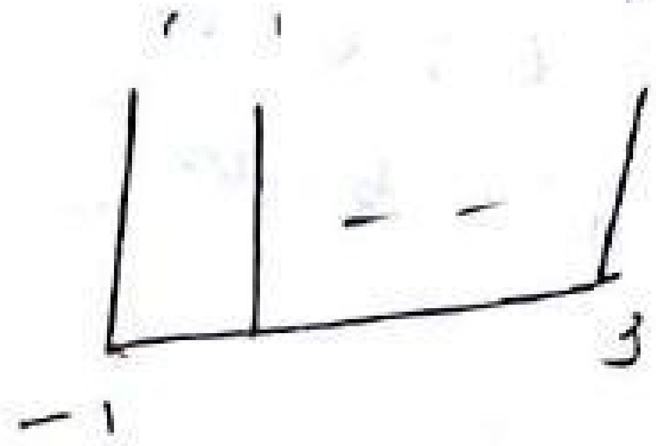

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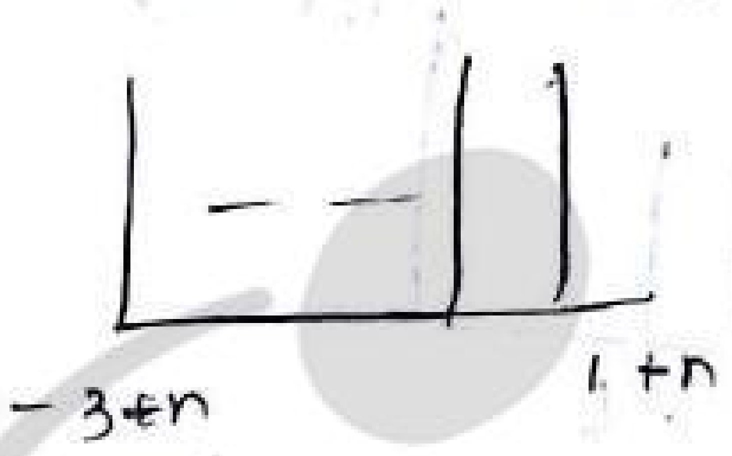
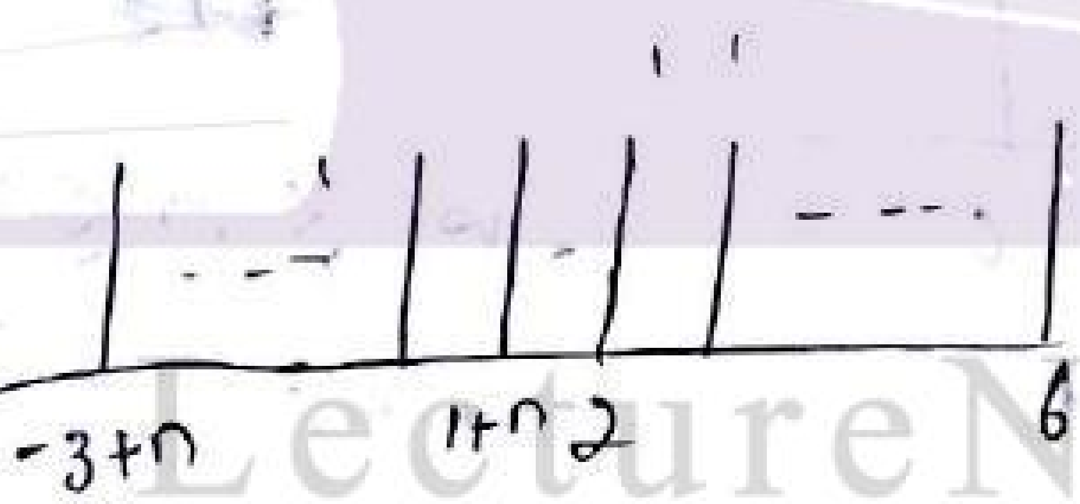
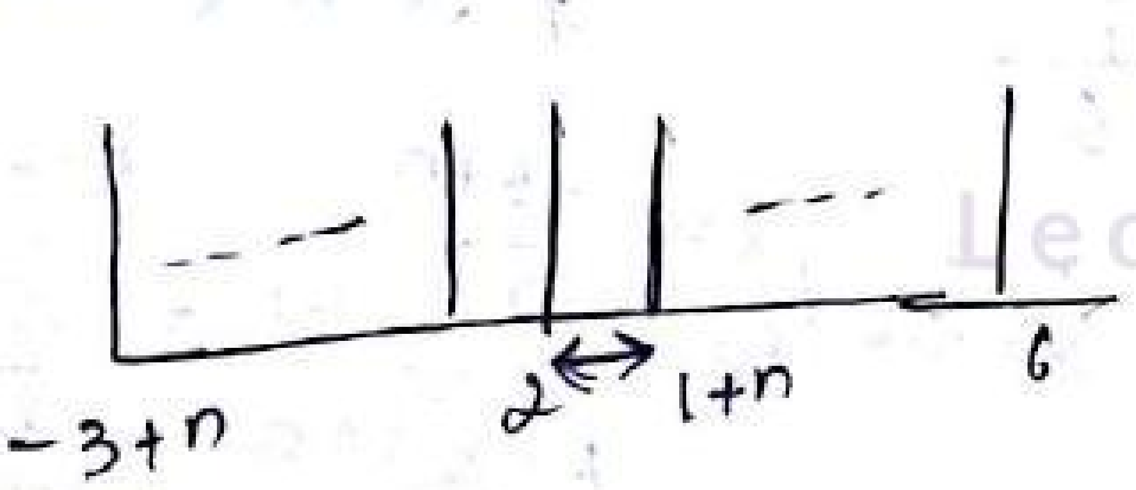




$-\infty < n < 1$

$\boxed{=0}$



$1 \leq n < 5$

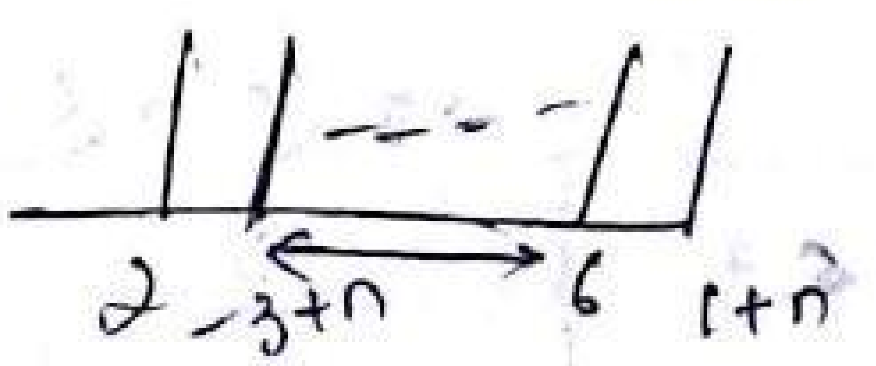
$\sum_{k=\boxed{2}\Rightarrow 0}^{1+n} 1.1$

$k=2 \quad l=$

$k=1+n \quad l=$

$\sum_{l=0}^{n-1} 1.1 = n-1+$

$\boxed{=n}$



$5 \leq n \leq 9$

$\sum_{k=-3+n}^{6} 1.1$

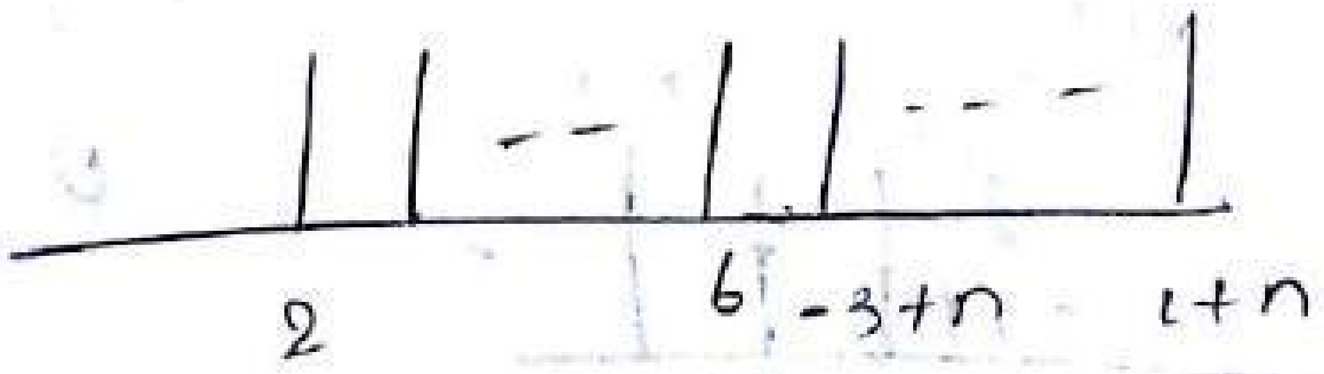
$k = -3+n \quad l$

$k = 6 \quad l =$

$6 - (-3+n)$

$$\sum_{l=0}^{9-n} 1.1 \qquad 9-n+1$$

$$\boxed{= 10-n}$$



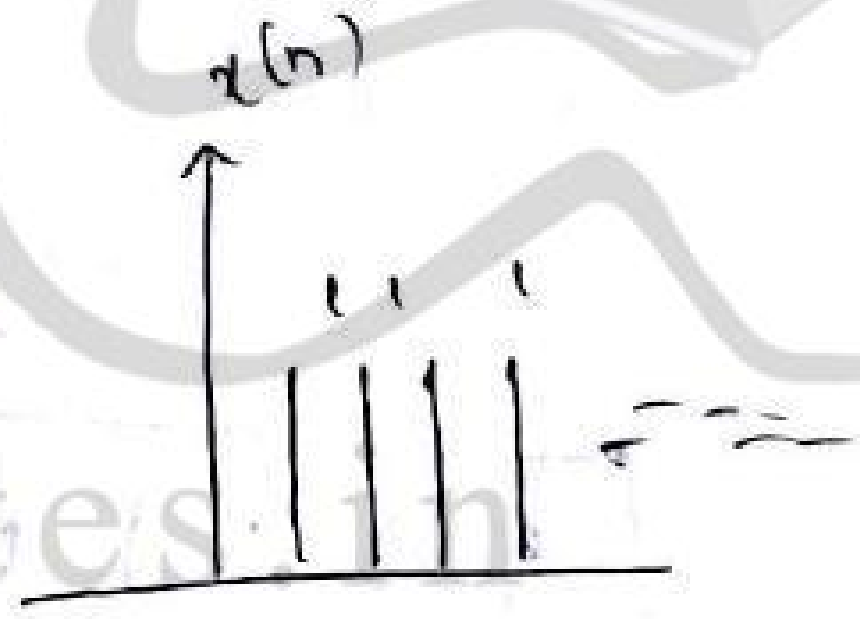
$9 \leq n \leq \infty$

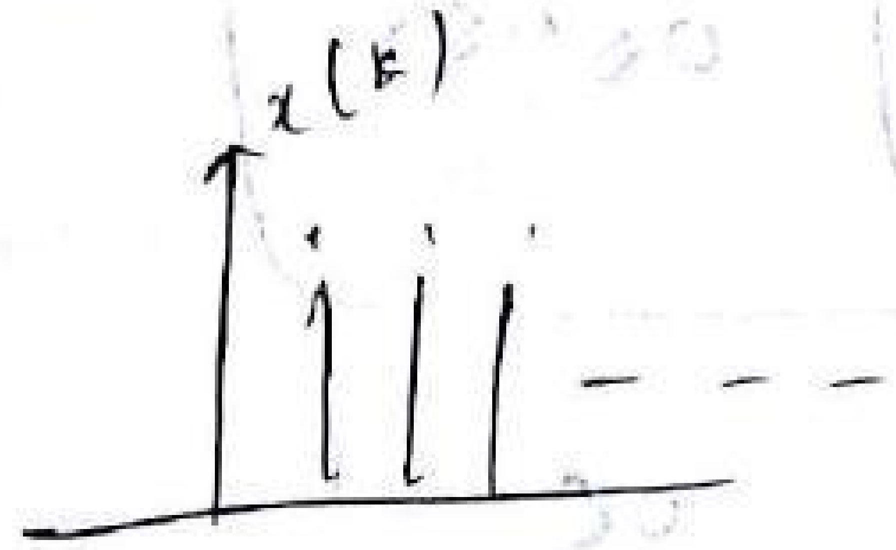
$= 0$

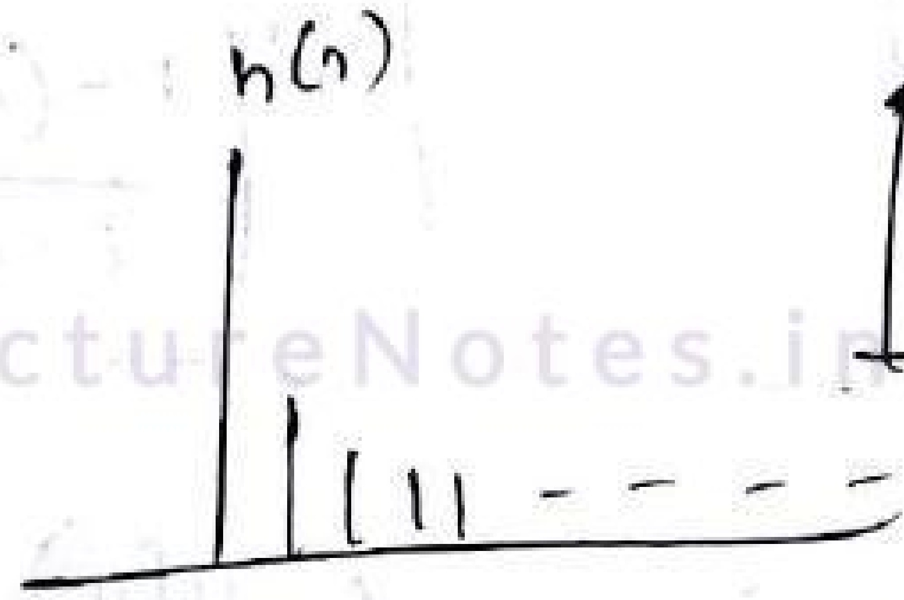
$$\boxed{y(n) = \begin{cases} 0 & -\infty \leq n < 1 \\ n & 1 \leq n < 5 \\ 10-n & 5 \leq n < 9 \\ 0 & 9 \leq n \leq \infty \end{cases}}$$

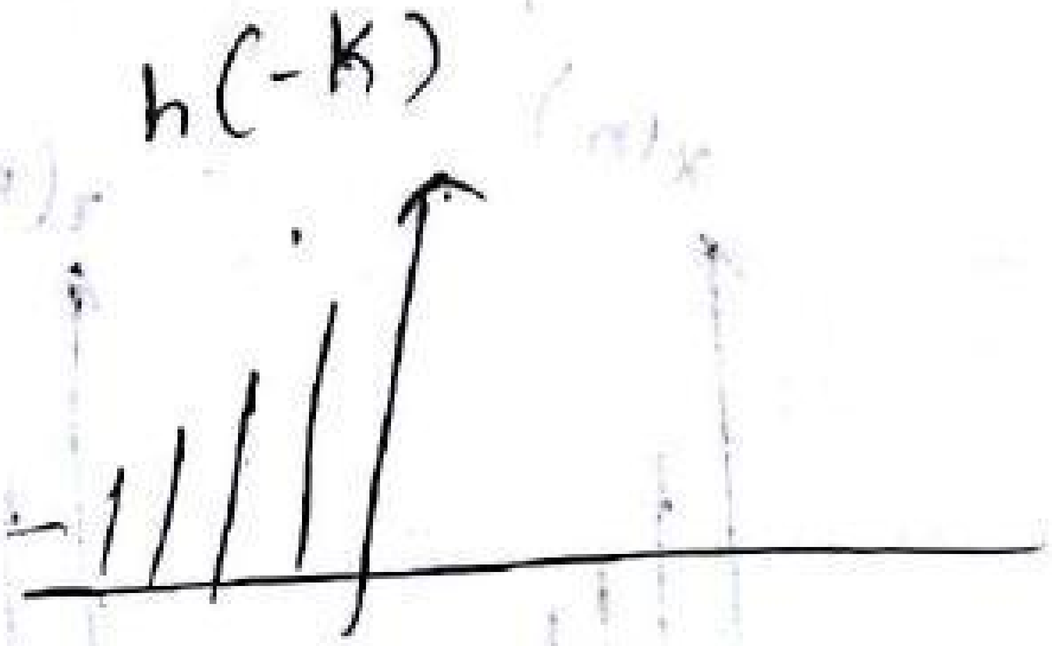
(4) $x(n) = u(n)$ $\qquad 0 < \alpha < 1$

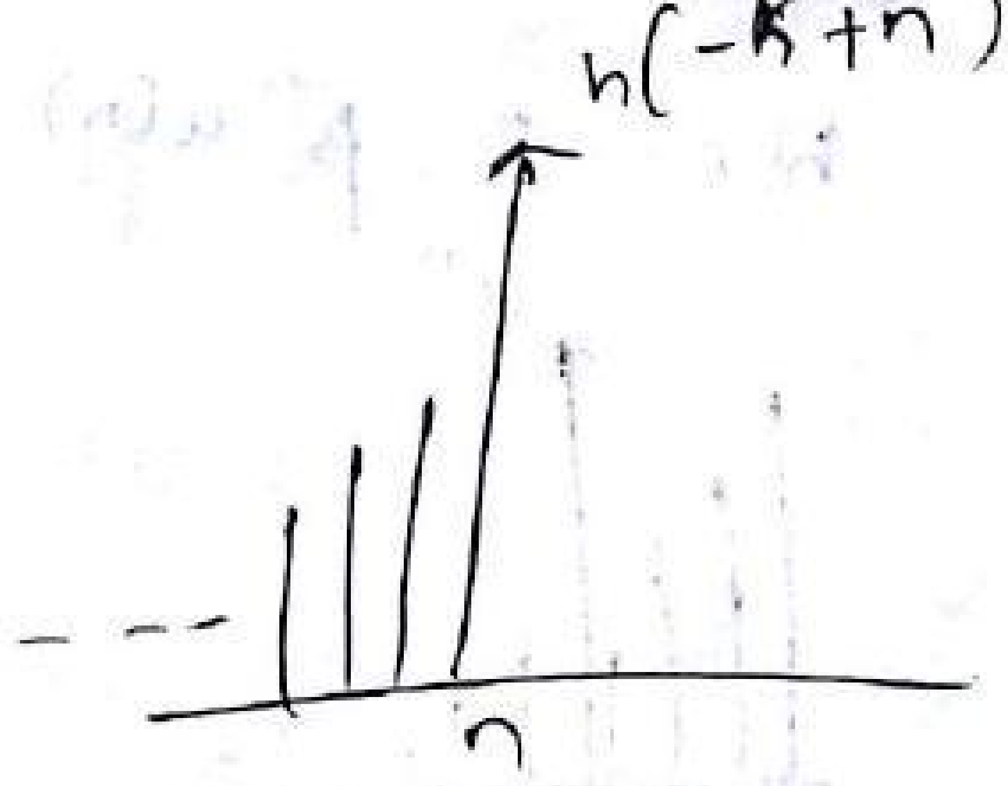
$h(n) = \alpha^n u(n)$

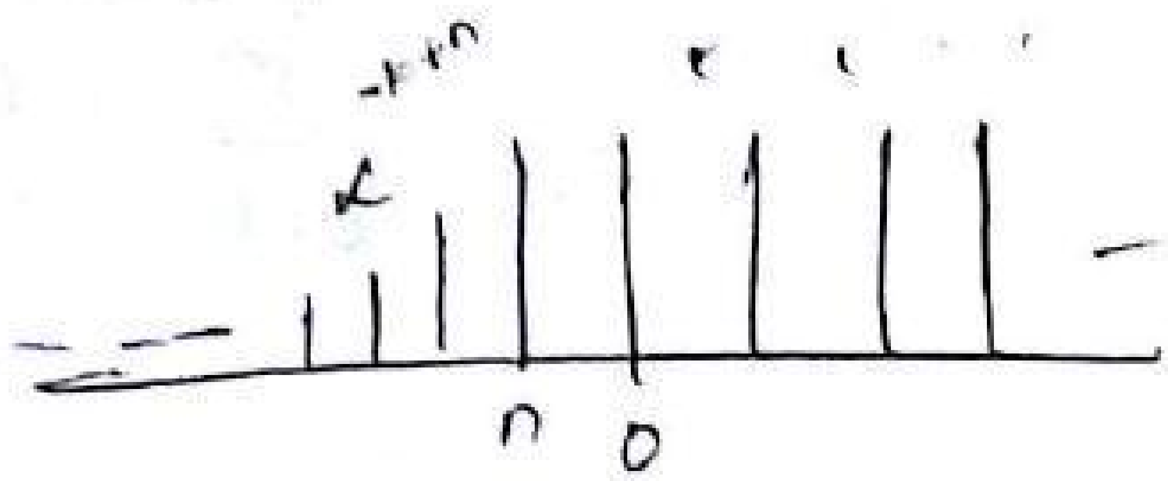












$-\infty \le n < 0$

$0 \le n \le \infty$

$x(n) = 1$

$x(k) = 1$

$h(n) = \alpha^n$

$h(k) = \alpha^k$

$h(-k) = \alpha^{-k}$

$h(-k+n) = \alpha^{-k+n}$

$$\sum_{k=0}^{n} 1 \cdot \alpha^{-k+n}$$

$$\sum_{k=0}^{n} \alpha^{n} \alpha^{-k}$$

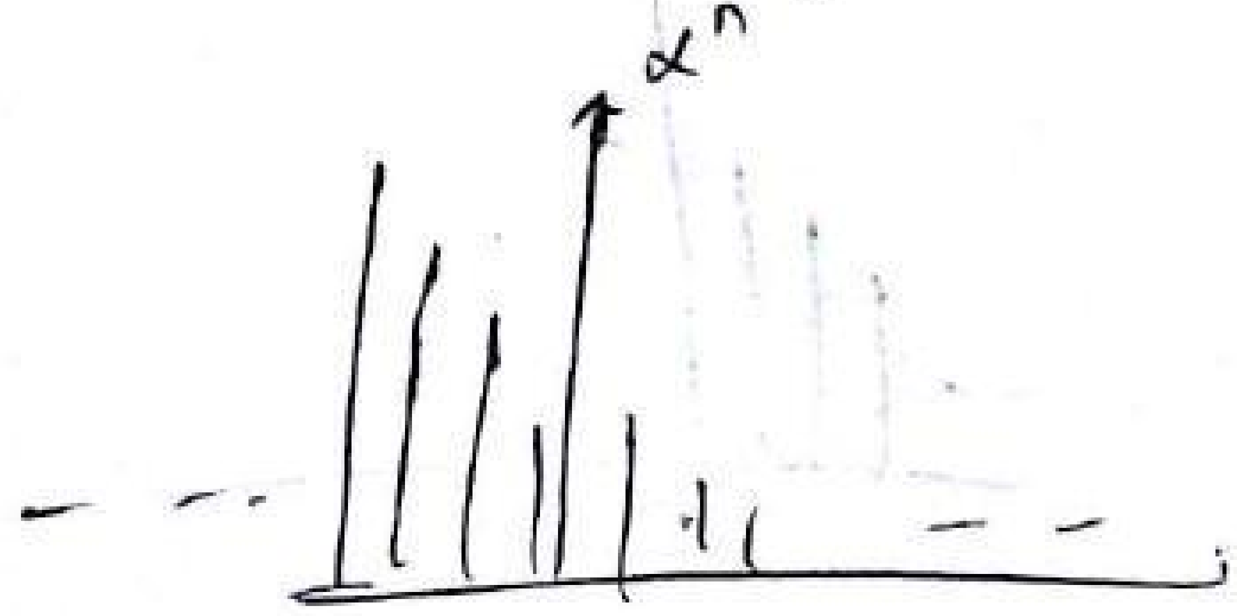
$$\alpha^{n} \sum_{k=0}^{n} (\alpha^{-1})^{k}$$

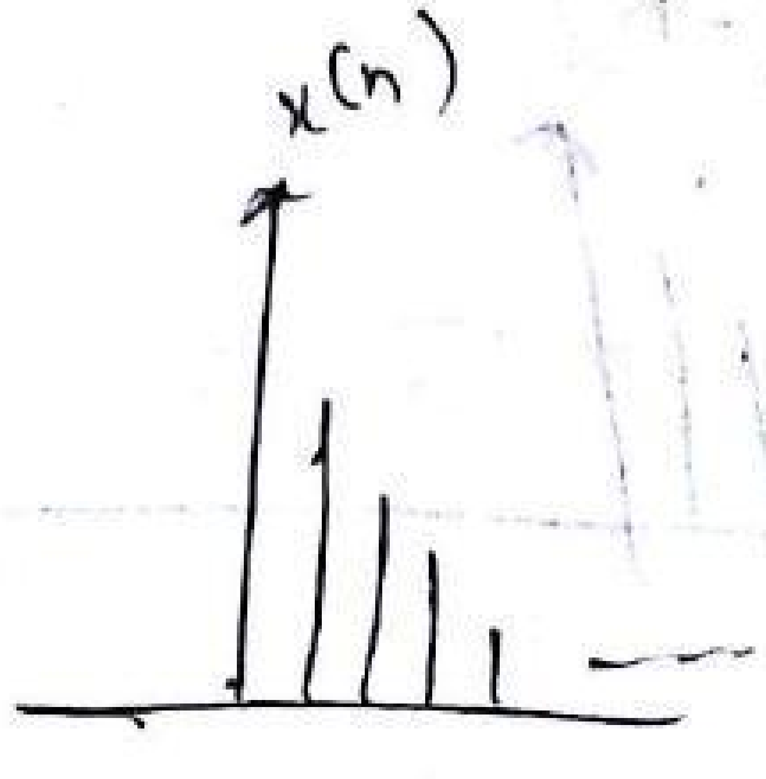
$$= \alpha^{n} \left[\frac{1 - (\alpha^{-1})^{n+1}}{1 - \alpha^{-1}} \right]$$

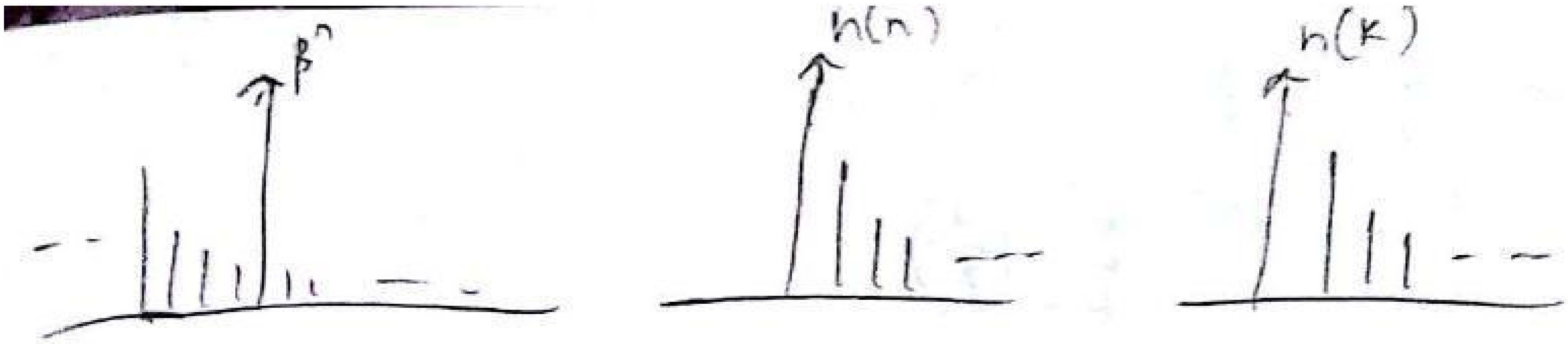
$$\boxed{y(n) = \begin{cases} 0 & -\infty \le n < 0 \\ \alpha^{n} \left[\dfrac{1 - (\alpha^{-1})^{n+1}}{1 - \alpha^{-1}} \right] & 0 \le n \le 8 \end{cases}}$$

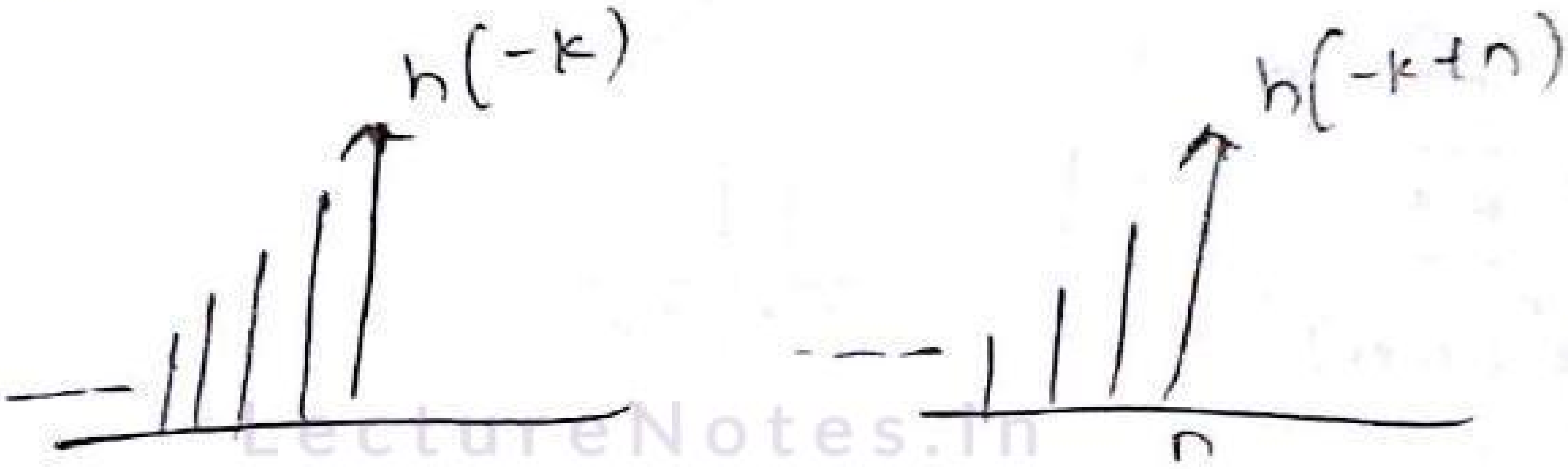
(5) $x(n) = \alpha^{n} u(n)$ $\quad 0 < \alpha < 1$

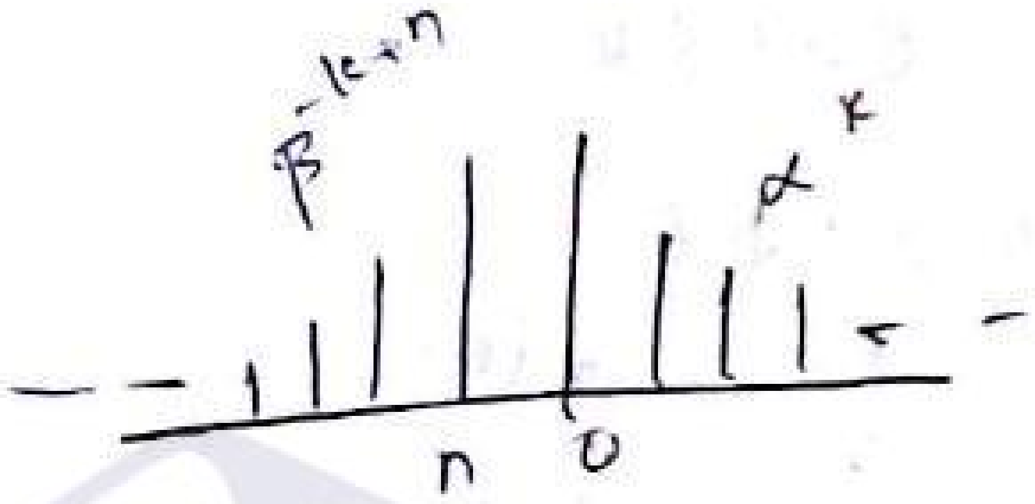
$h(n) = \beta^{n} u(n)$ $\quad 0 < \beta < 1$





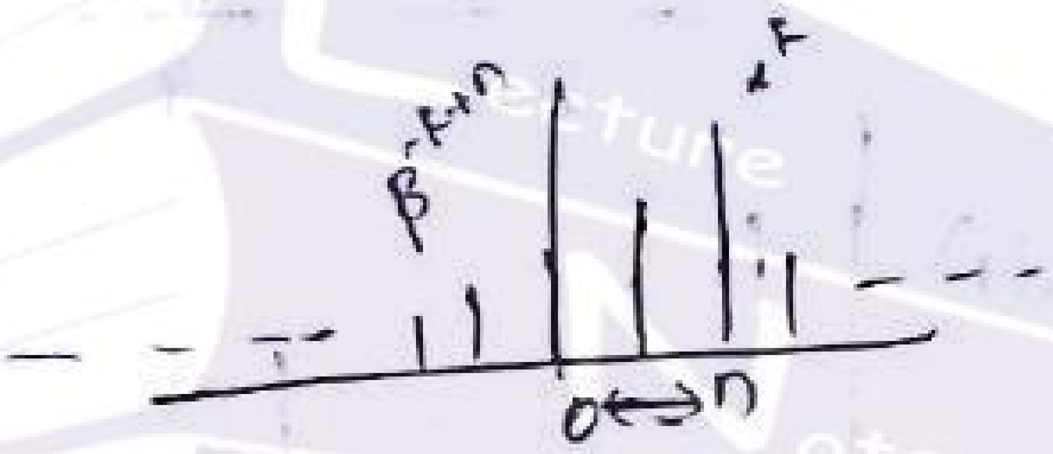






$-\infty \le n < 0$

$= \underline{\underline{0}}$



$0 \le n < \infty$

$x(n) = \alpha^n$

$x(k) = \alpha^k$

$h(n) = \beta^n$

$h(k) = \beta^k$

$h(-k) = \beta^{-k}$

$h(-k+n) = \underline{\underline{\beta^{-k+n}}}$

$$\sum_{k=0}^{n} \alpha^k \cdot \beta^{-k+n}$$

$$\sum_{k=0}^{n} \alpha^k \beta^{-k} \cdot \beta^n$$

$$\beta^n \sum_{k=0}^{n} \alpha^k \beta^{-k}$$

$$\beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k \to (1)$$

$$\boxed{= \beta^n \left[\frac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}} \right] \text{ if } \beta \neq \alpha.}$$

$$\boxed{y(n) = \begin{cases} 0 & -\infty \le n < 0 \\ \beta^n \left[\dfrac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}} \right] \text{ if } \beta \neq \alpha & 0 \le n < \infty \\ \beta^n (n+1) \text{ if } \beta = \alpha & \end{cases}}$$

→ if $\alpha = \beta$

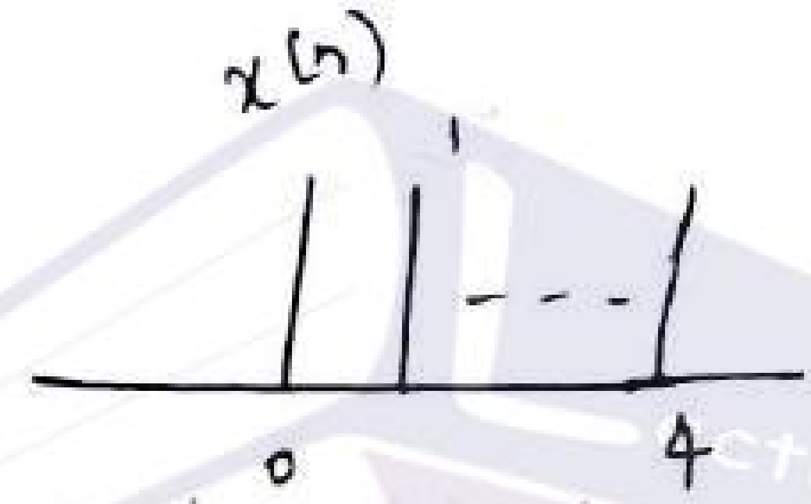
$$① \Rightarrow \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^n$$

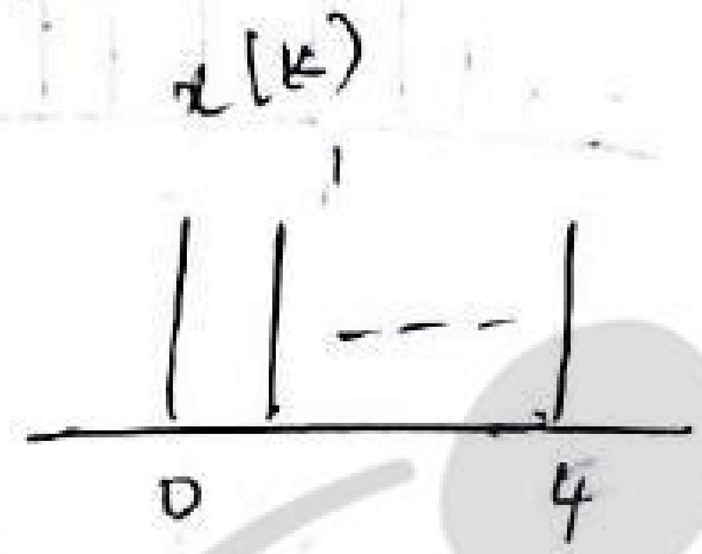
$$\beta^n \sum_{k=0}^{n} 1^n$$

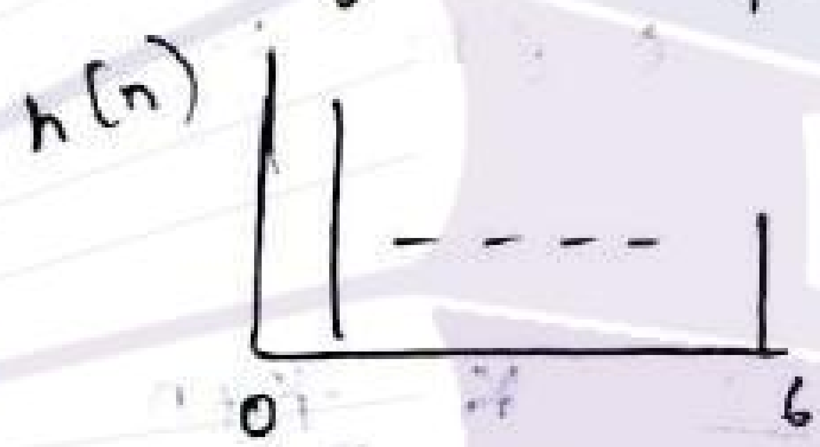
$$\boxed{= \beta^n (n+1)}$$

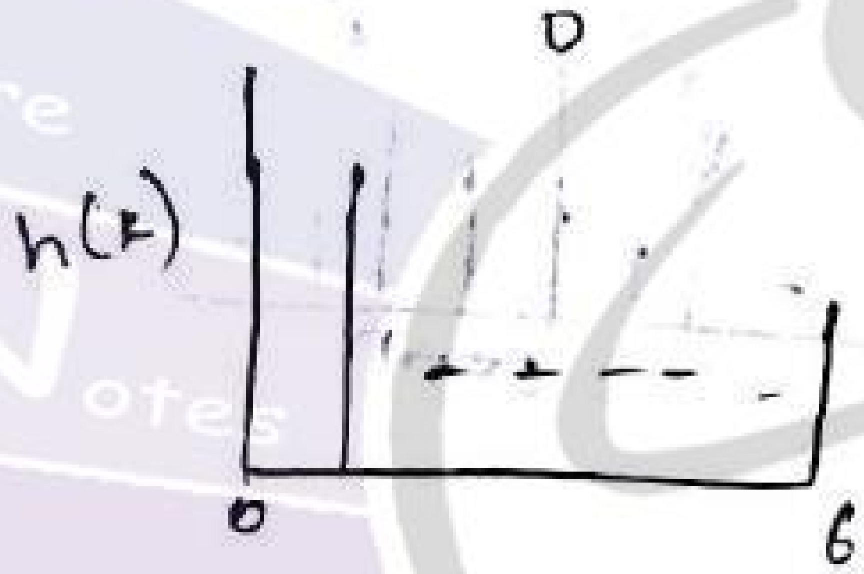
⑥ $x(n) = 1 \qquad 0 \leq n \leq 4$

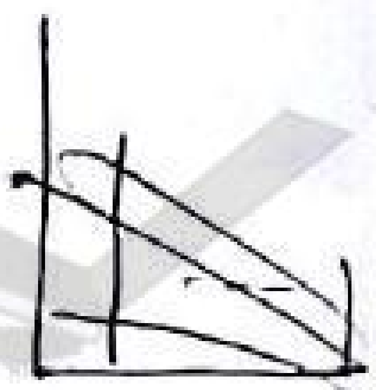
$h(n) = \alpha^n \qquad 0 \leq n \leq 6$

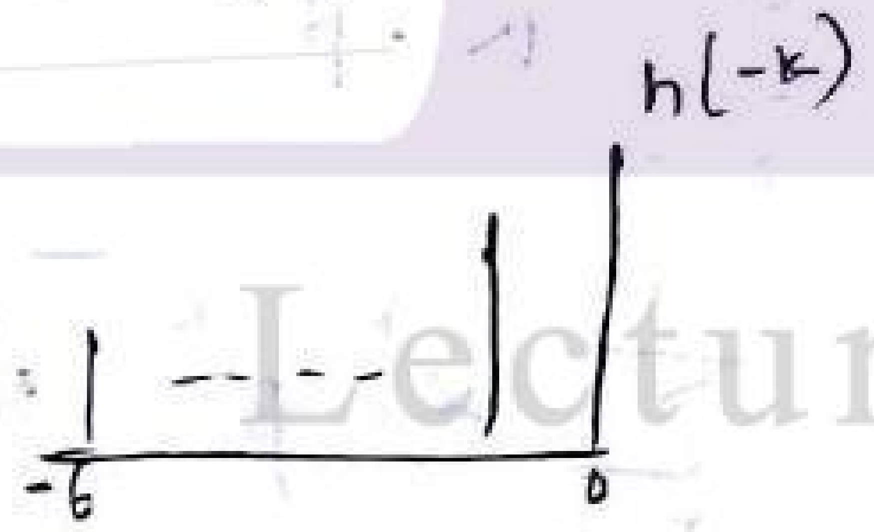


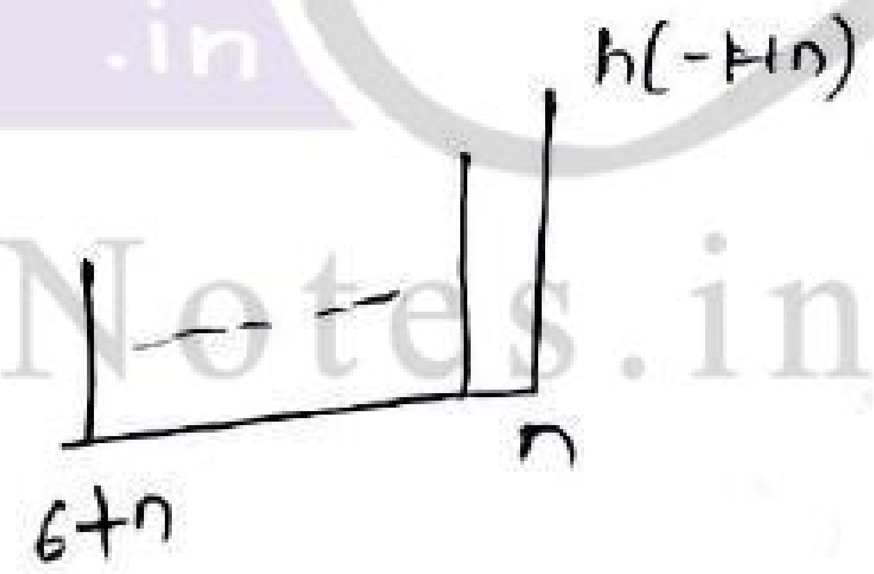












$x(n) = 1$

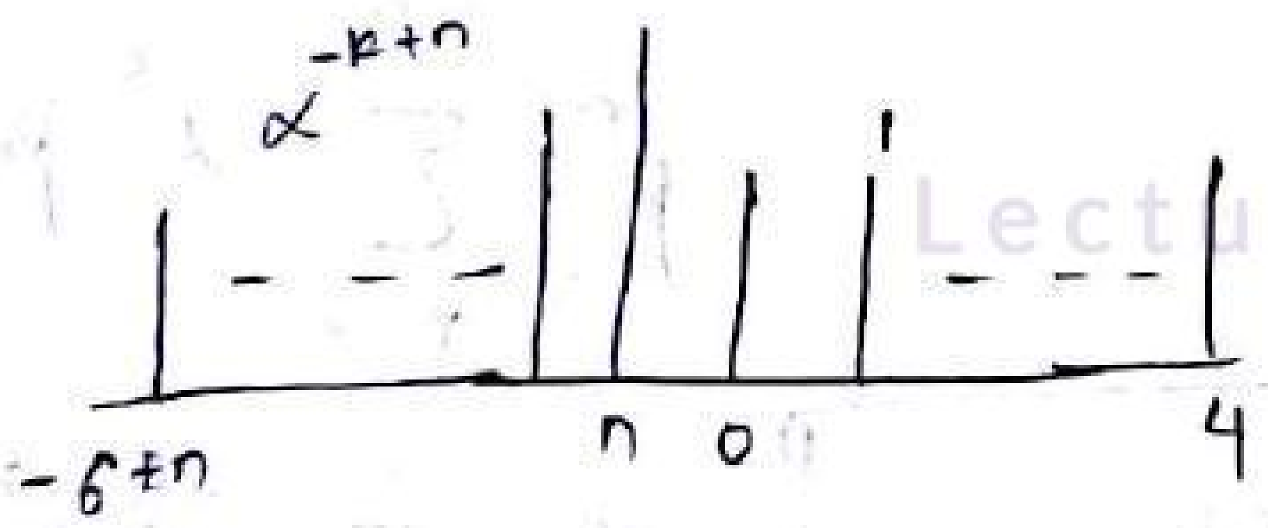
$x(k) = 1$

$h(n) = \alpha^n$

$h(k) = \alpha^k$

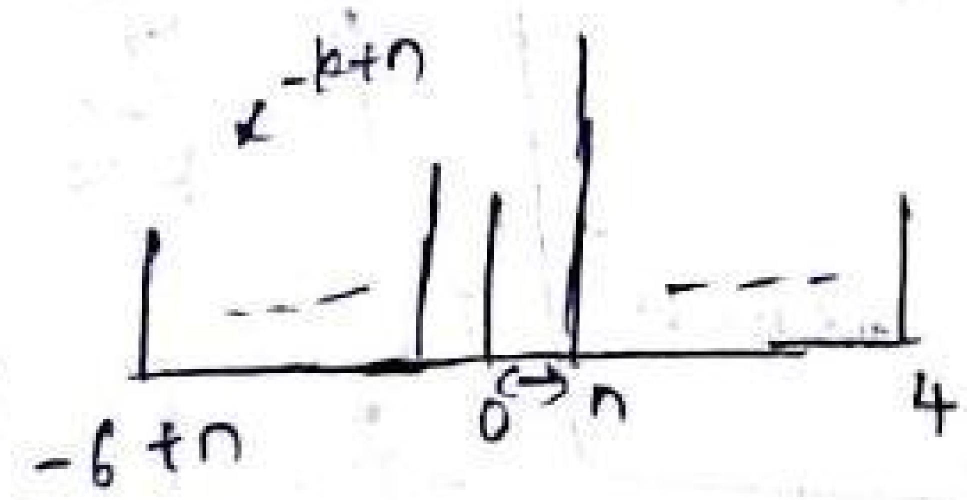
$h(-k) = \alpha^{-k}$

$h(-k+n) = \alpha^{-k+n}$



$-\infty \leq n < 0$

$\boxed{= 0}$

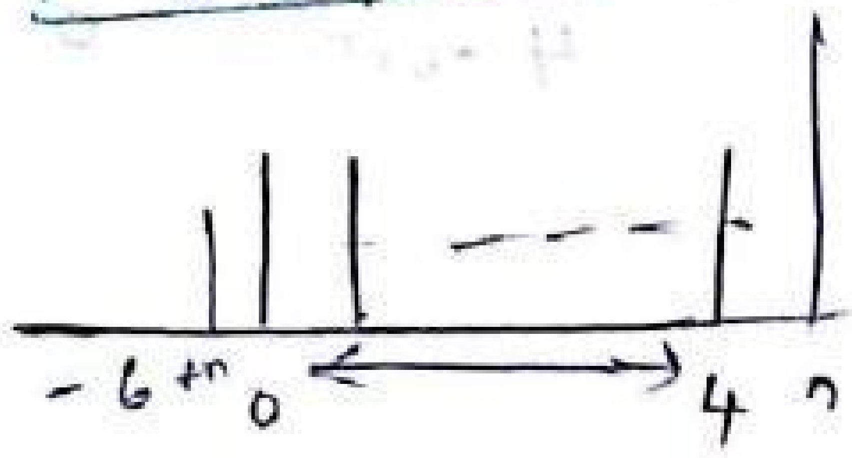


$0 \leq n < 4$

$$\sum_{k=0}^{n} \alpha^{-k+n}$$

$$\alpha^n \sum_{k=0}^{n} \alpha^{-k} = \alpha^n \sum_{k=0}^{n} \alpha^{-k}$$

$$\boxed{= \alpha^n \left[\frac{1-(\alpha^{-1})^{n+1}}{1-\alpha^{-1}}\right]}$$



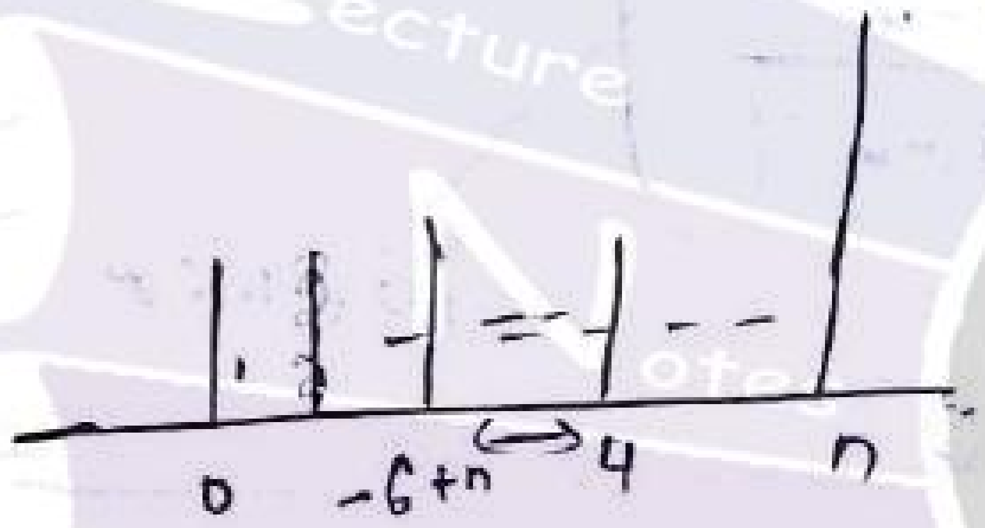
$4 < n < 6$

$$\sum_{k=0}^{4} 1.\alpha^{-k+n}$$

$$\alpha^n \sum_{k=0}^{4} (\alpha^{-1})^k$$

$$\alpha^n \left[\frac{1-(\alpha^{-1})^{4+1}}{1-\alpha^{-1}}\right]$$

$$\boxed{\alpha^n \left[\frac{1-\alpha^{-5}}{1-\alpha^{-1}}\right]}$$



$+6 \le n < 10$

$$\sum_{k=\boxed{-6+n}}^{4} 1.\alpha^{-k+n} \Rightarrow 0$$

$k = -6+n \Rightarrow k+6-n = l$

$k = l-6+n$

$k = -6+n \quad l = 0$

$k = 4 \quad l = 10-n$

$$\sum_{l=0}^{10-n} \alpha^{-(l-6+n)+n}$$

$$\sum_{l=0}^{10-n} \alpha^{-l+6-\cancel{n}+\cancel{n}}$$

$$\sum_{n=0}^{10-n} \alpha^{-l+6}$$

$$\alpha^6 \sum_{n=0}^{10-n} (\alpha^{-1})^l$$

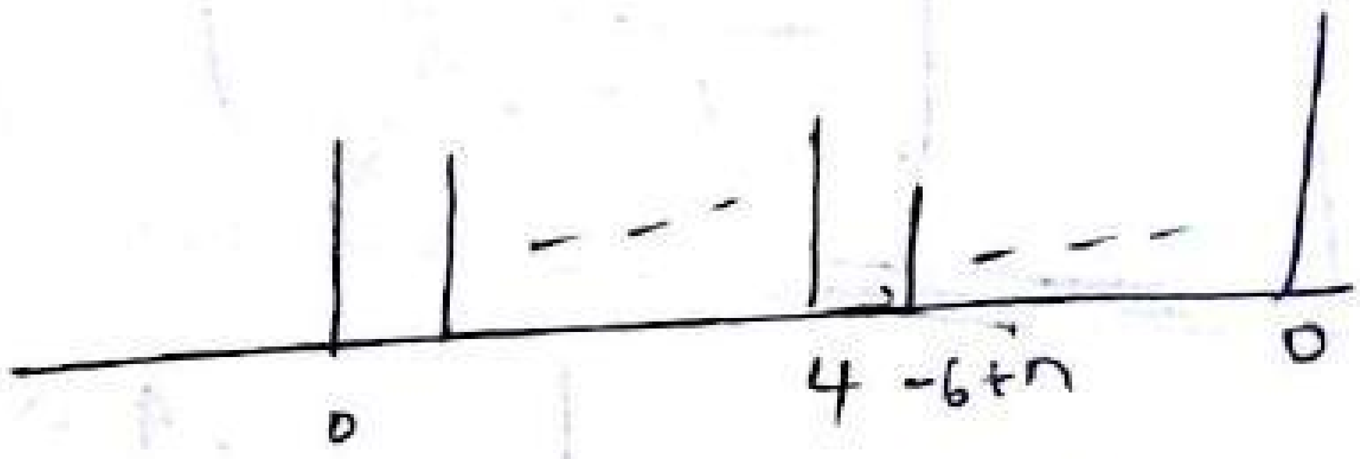
$$\alpha^6 \left[\frac{1-(\alpha^{-1})^{10-n+1}}{1-\alpha^{-1}}\right]$$

$$\boxed{= \alpha^6 \left[\frac{1-(\alpha^{-1})^{11-n}}{1-\alpha^{-1}}\right]}$$

$$\cancel{\alpha^n \sum_{l=0}^{10-n} (\alpha^{-1})^n}$$

$$\cancel{\alpha^n \left[\frac{1-(\alpha^{-1})^{10-n}}{1-\alpha^{-1}}\right]}$$

$$\cancel{\alpha^n \left[\frac{1-\alpha^{-10+n}}{1-\alpha^{-1}}\right]}$$



$10 \le n \le \infty$

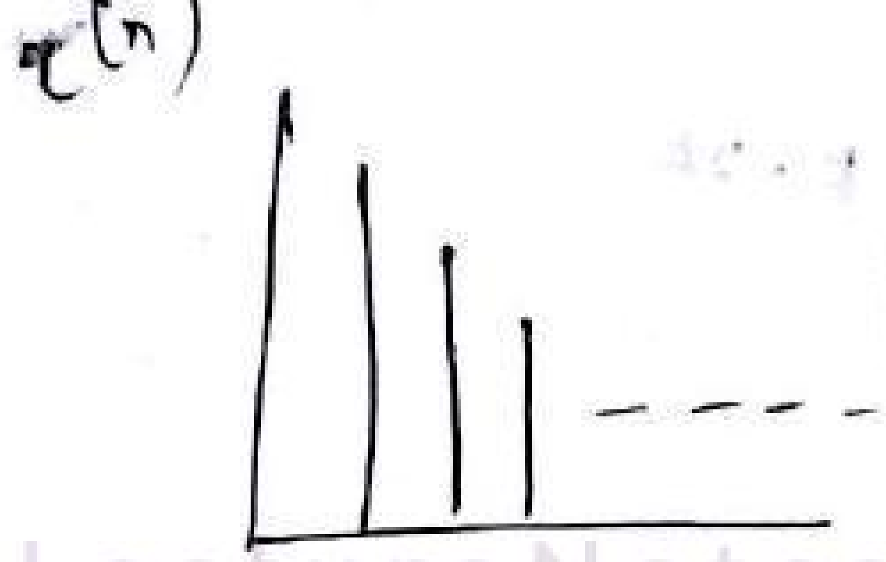
$\boxed{=0}$

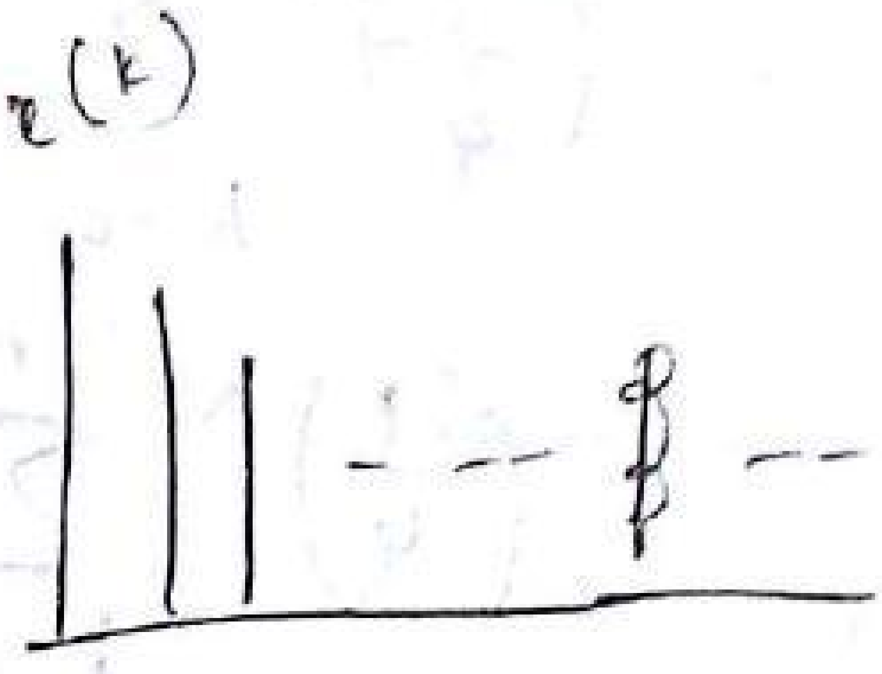
$$y(n) = \begin{cases} 0 & -\infty \le n < 0 \\ \alpha^{n}\left[\dfrac{1-(\alpha^{-1})^{n+1}}{1-\alpha^{-1}}\right] & 0 \le n < 4 \\ \alpha^{6}\left[\dfrac{1-(\alpha^{-1})^{1+n}}{1-\alpha^{-1}}\right] & 4 \le n < 6 \\ \alpha^{n}\left[\dfrac{1-\alpha^{-10+n}}{1-\alpha^{-1}}\right] & 6 \le n < 10 \\ 0 & 10 \le n \le \infty \end{cases}$$

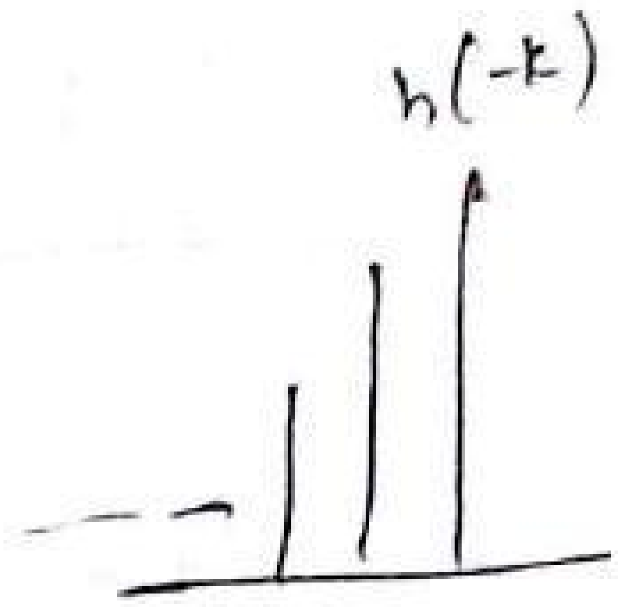
$$y(n) = \begin{cases} 0 & -\infty \le n < 0 \\ \alpha^{n}\left[\dfrac{1-(\alpha^{-1})^{n+1}}{1-\alpha^{-1}}\right] & 0 \le n < 4 \\ \alpha^{n}\left[\dfrac{1-(\alpha^{-1})^{5}}{1-\alpha^{-1}}\right] & 4 \le n < 6 \\ \alpha^{6}\left[\dfrac{1-(\alpha^{-1})^{11-n}}{1-\alpha^{-1}}\right] & 6 \le n < 10 \\ 0 & 10 \le n \le \infty \end{cases}$$

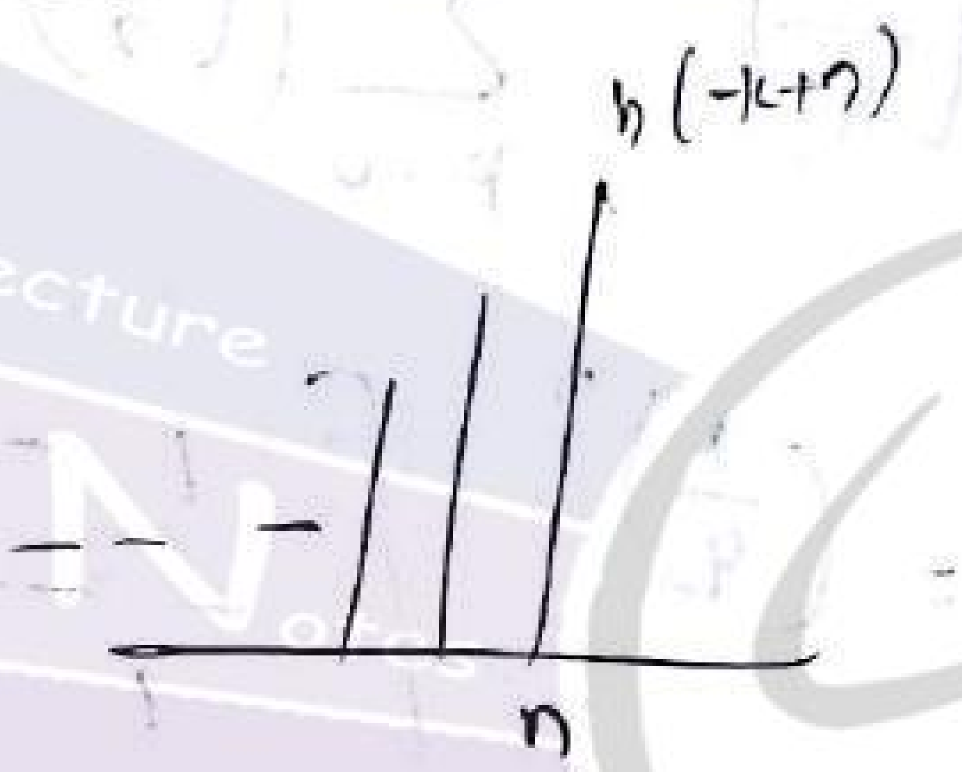
7) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$h(n) = \left(\frac{1}{4}\right)^n u(n)$









$x(n) = \frac{1}{2}^n$

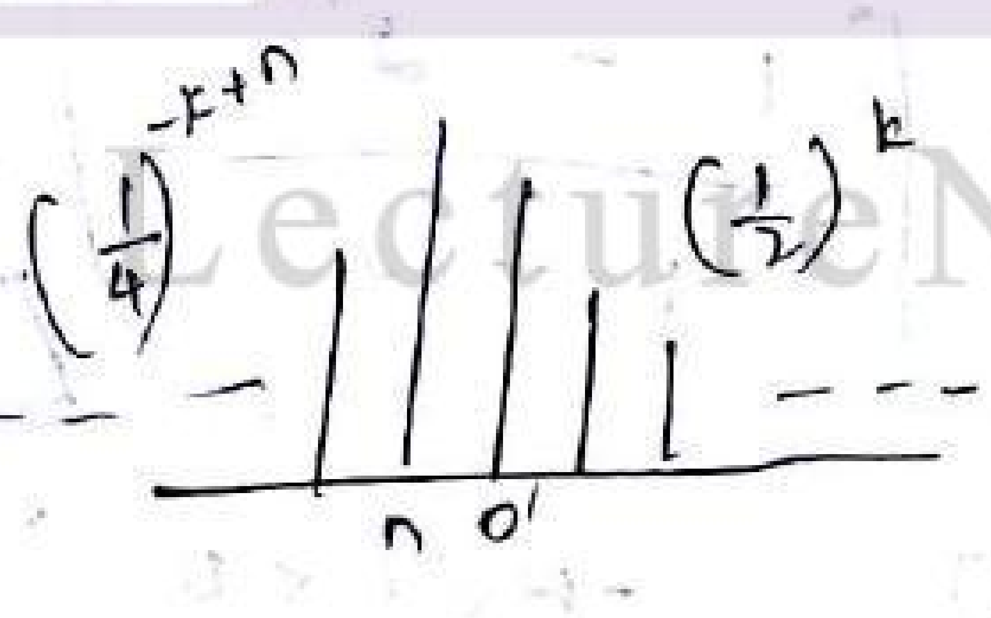
$x(k) = \frac{1}{2}^k$

$h(n) = \left(\frac{1}{4}\right)^n$

$h(k) = \left(\frac{1}{4}\right)^k$

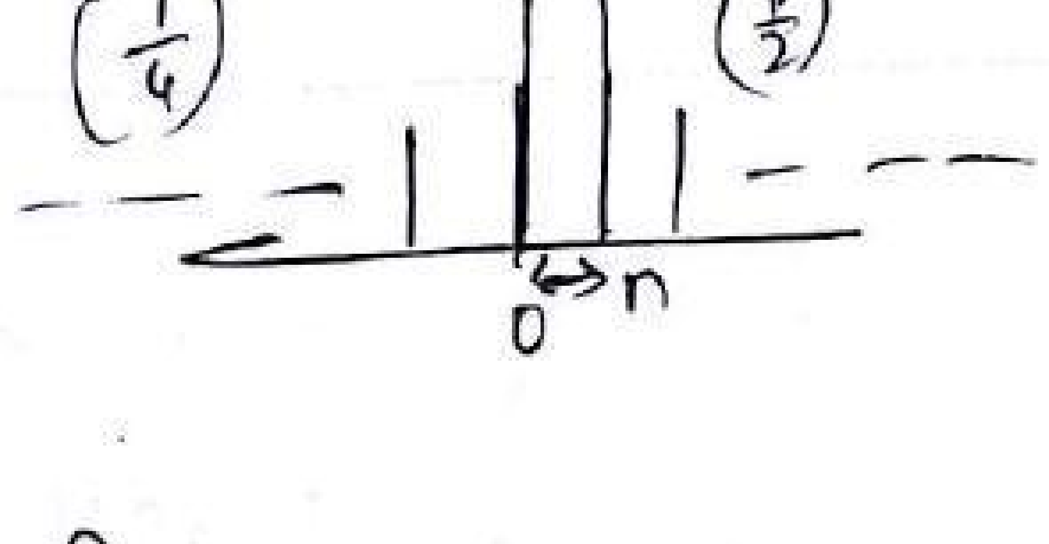
$h(-k) = \left(\frac{1}{4}\right)^{-k}$

$h(-k+n)\ \left(\frac{1}{4}\right)^{-k+n}$



$-\infty \leq n < 0$

$= \boxed{0}$



$0 \leq n \leq \infty$

$$\sum_{k=0}^{n} \left(\frac{1}{4}\right)^{-k+n} \left(\frac{1}{2}\right)^k.$$

$$\left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \cancel{\ } \left(\frac{1}{2^2}\right)^{-k} \left(\frac{1}{2}\right)^k = \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^{-k} \left(\frac{1}{2}\right)^k$$

Scanned by CamScanner

8)

$$\left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-2k} \left(\frac{1}{2}\right)^{k}$$

$$\left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k-2k}$$

$$\left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k}$$

$$\left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\left(\frac{1}{2}\right)^{-1}\right)^{k}$$

$$= \left(\frac{1}{4}\right)^n \left[\frac{1 - \left[\left(\frac{1}{2}\right)^{-1}\right]^{n+1}}{1 - \left(\frac{1}{2}\right)^{-1}}\right]$$

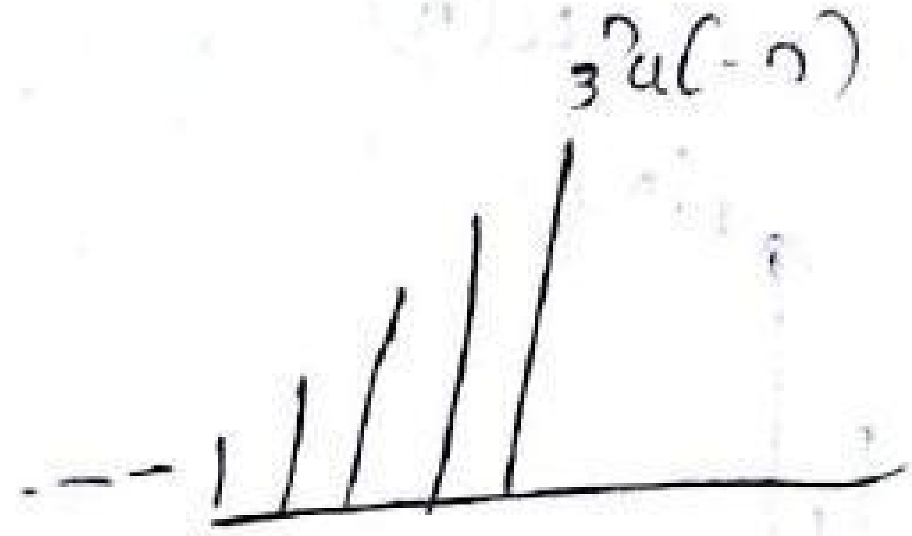
$$= \left(\frac{1}{4}\right)^n \left[\frac{1 - 2^{n+1}}{1 - 2}\right] \Rightarrow \left(\frac{1}{4}\right)^n \left[\frac{1-2}{-1}\right.$$

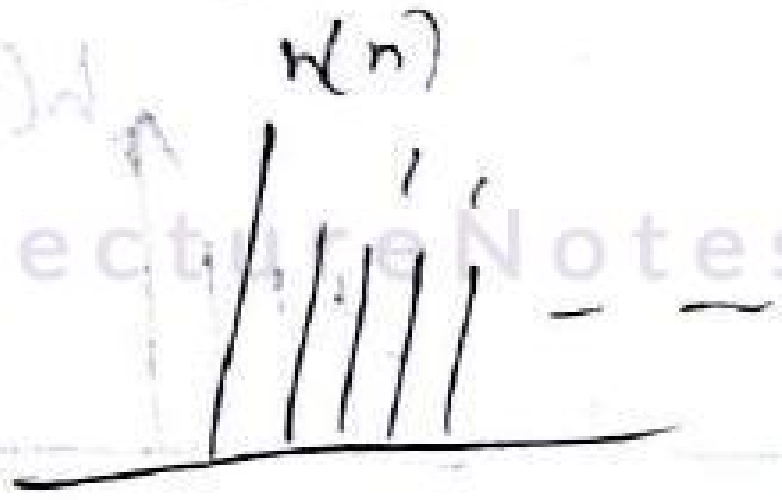
$$\boxed{= \left(\frac{1}{4}\right)^n \left[2^{n+1} - 1\right.}$$

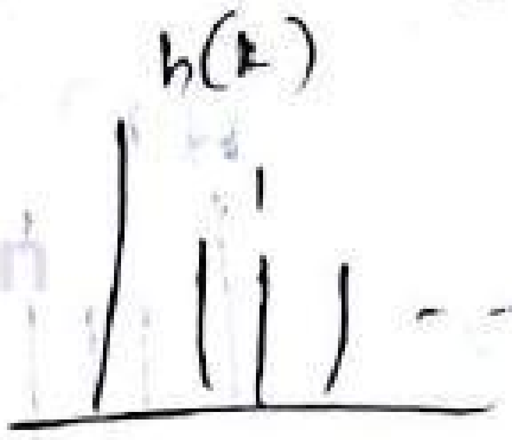
$$\boxed{y(n) = \begin{cases} 0 & -\infty < n < 0 \\ \left(\frac{1}{4}\right)^n \left[\frac{1 - 2^{n+1}}{1 - 2}\right] & 0 \le n \le \infty \end{cases}}$$

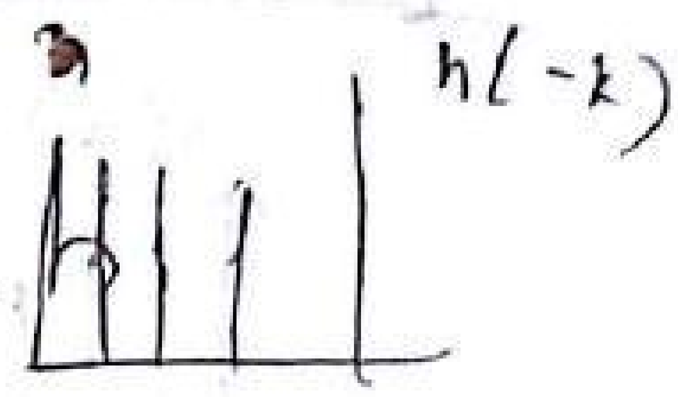
8) $x(n) = 3^n u(-n)$

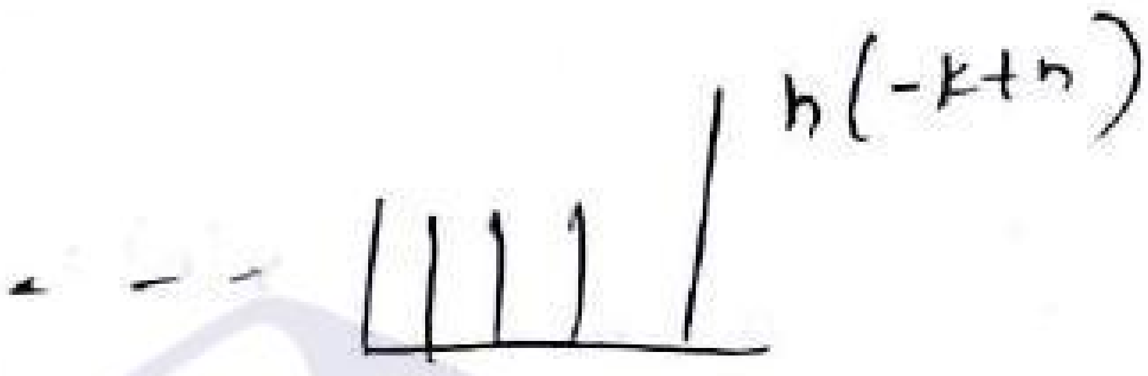
$h(n) = u(n)$

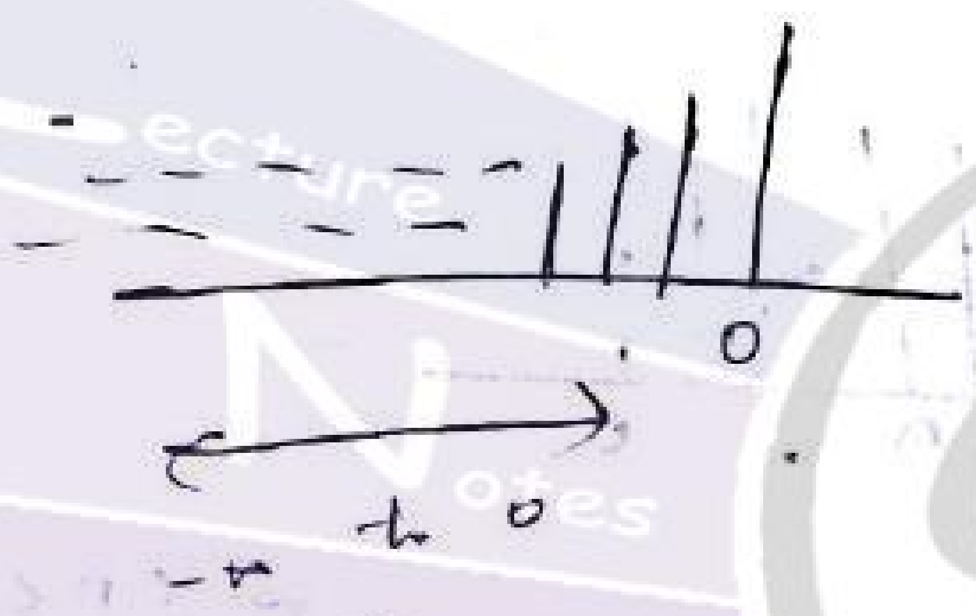












$-\infty < n < 0$

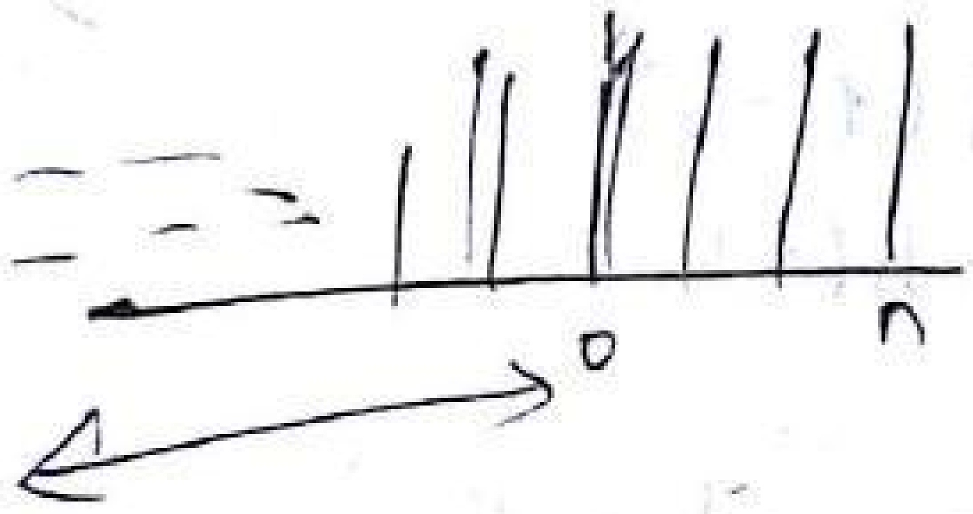
$$\sum_{k=-\infty}^{n} 3^k$$

$k \to -k$

$$\sum_{k=\infty}^{-n} 3^{-k}$$

$$= \sum_{k=-n}^{\infty} \left(\frac{1}{3}\right)^k$$

$$\boxed{= \frac{\left(\frac{1}{3}\right)^{-n}}{1-\frac{1}{3}}}$$



$0 \le n \le \infty$

$$\sum_{k=-\infty}^{0} 3^{k}$$

$k \to -k$

$$= \sum_{k=\infty}^{0} 3^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{k} = \frac{1}{1-\frac{1}{3}}$$

$$\boxed{y(n) = \begin{cases} \dfrac{\left(\frac{1}{3}\right)^{n}}{1-\frac{1}{3}} & -\infty \le n < 0 \\[2ex] \dfrac{1}{1-\frac{1}{3}} & 0 \le n \le \infty \end{cases}}$$

Properties of Convolution

ⓐ $u(t) * \delta(t) = u(t)$

ⓑ $u(t) * \delta(t-2) = u(t-2)$

ⓒ $u(t+2) * \delta(t-1) = u(t-1+2)$
$= u(t+1)$

ⓓ $3u(t-3) * 2\delta(t+1) = 6u(t+1-3)$
$= 6u(t-2)$

ⓔ $t\,u(t) * \delta(t-1) = (t-1)\,u(t-1)$

ⓕ $(2t-1)u(t+3) * 2\delta(t-2) = 2\left[2(t-2)-1\right]u(t-2+3)$
$= 2(2t-4-1)\,u(t+1)$
$= 2(2t-5)\,u(t+1)$

ⓖ $4\delta(t) * 3\delta(t-2) = 12\,\delta(t-2)$

① $x(t) = 3u(t) - 3u(t-4)$

$h(t) = 2\delta(t-1) + \delta(t-3)$

find $y(t) = x(t) * h(t)$

$$y(t) = [3u(t) - 3u(t-4)] * [2\delta(t-1) + \delta(t-3)]$$

$$= 6u(t-1) - 6u(t-1-4) + 3u(t-3) - 3u(t-3-4)$$

$$= 6u(t-1) - 6u(t-5) + 3u(t-3) - 3u(t-7)$$

1 5 3 7

Re-arrange

$$= 6u(t-1) + 3u(t-3) - 6u(t-5) - 3u(t-7)$$

1 3 5 7.

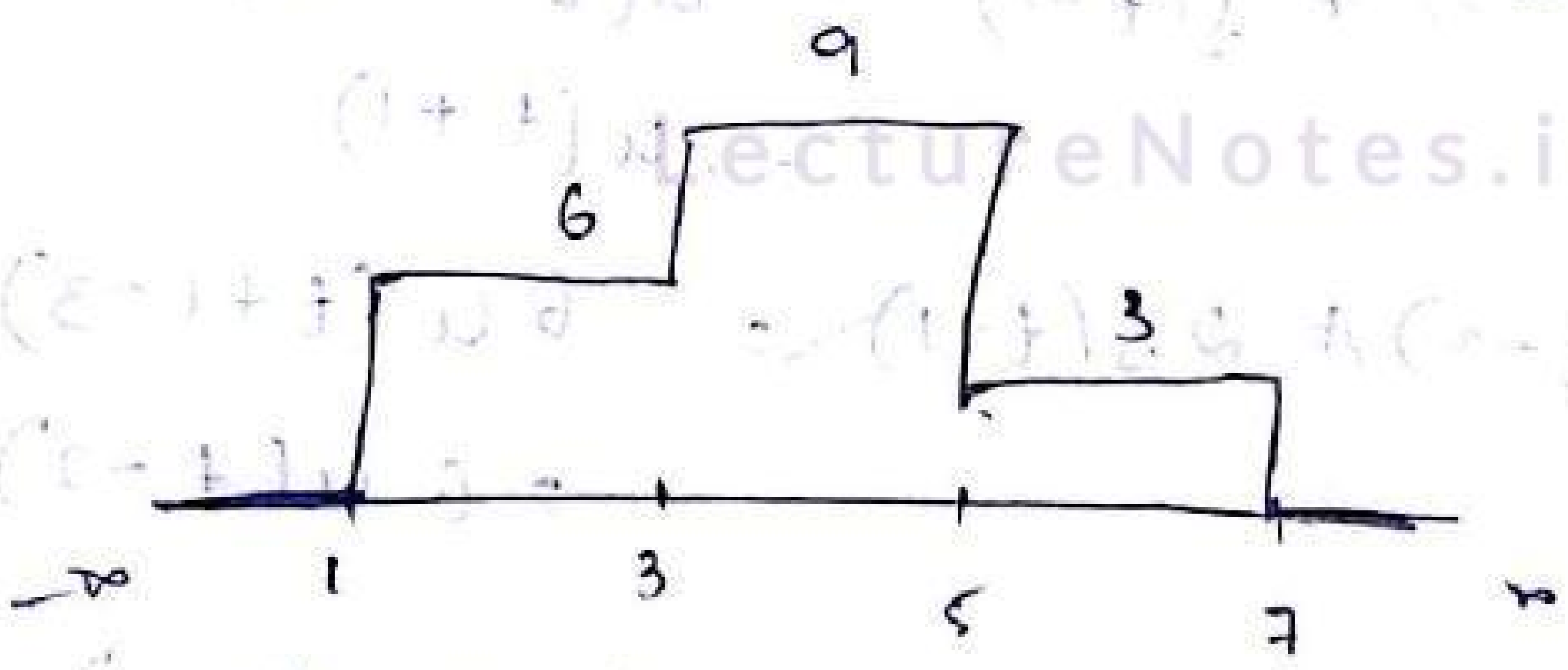
$-\infty$ to 1 : 0

1 to 3 : 6

3 to 5 : 3

5 to 7 : −6

7 to ∞ : −3



② $x(t) = 2[u(t+1) - u(t-1)]$

$h(t) = \delta(t) - 2\delta(t-1) + 8\delta(t-2)$

find $y(t) = x(t) * h(t)$

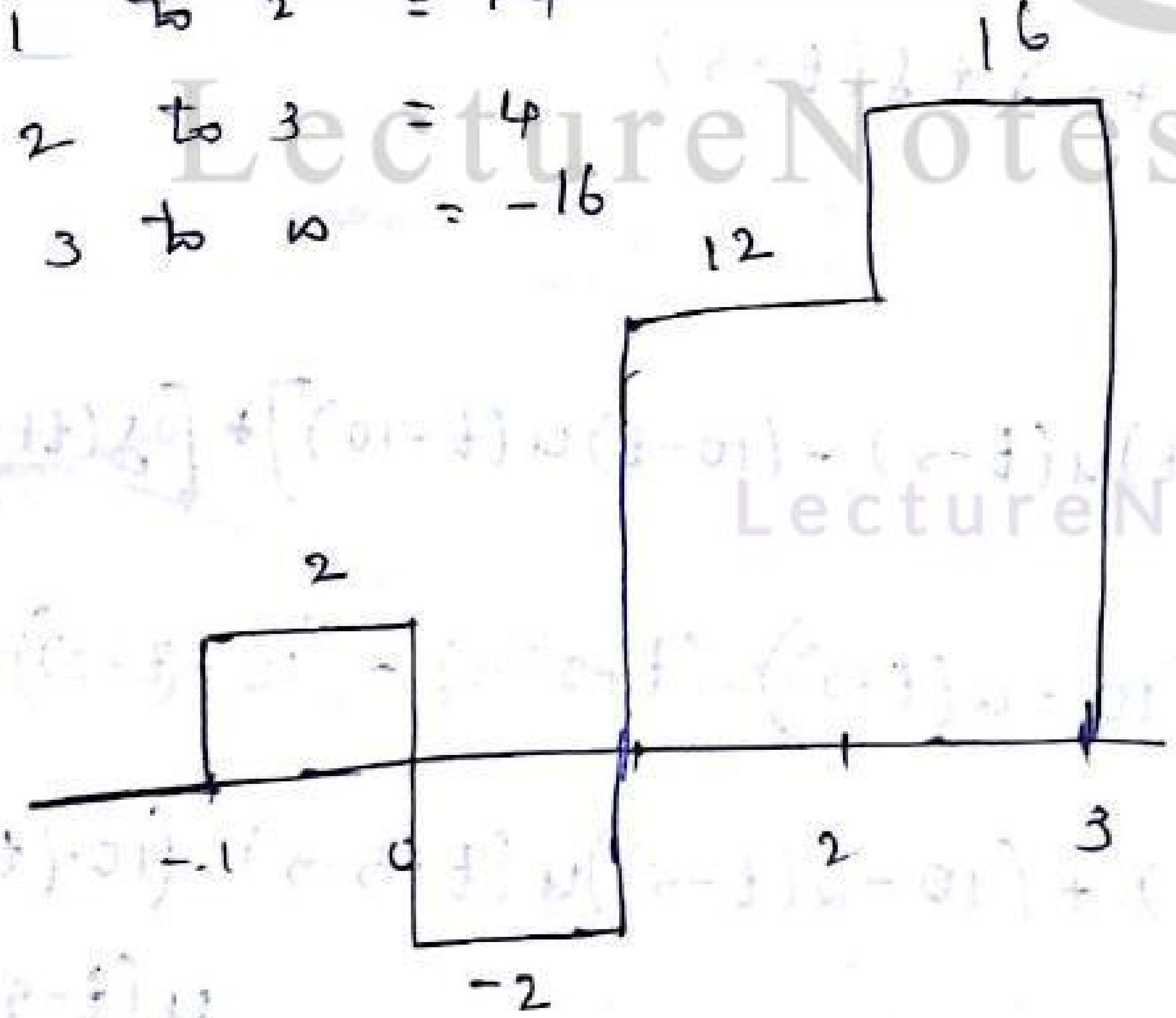
$$y(t) = \left[2u(t+1) - 2u(t-1)\right] * \left[\delta(t) - 2\delta(t-1) + 8\,\delta(t-2)\right]$$

$$= 2u(t+1) - 2u(t-1) - 4u(t-1+1) + 4u(t-1-1) + 16u(t-2+1) - 16u(t-2-1)$$

$$= \underset{-1}{2u(t+1)} - \underset{1}{2u(t-1)} - \underset{0}{4u(t)} + \underset{2}{4u(t-2)} + \underset{1}{16u(t-1)} - \underset{3}{16u(t-3)}$$

$$= \underset{-1}{2u(t+1)} - \underset{0}{4u(t)} + \underset{1}{14u(t-1)} + \underset{2}{4u(t-2)} - \underset{3}{16u(t-3)}$$

$-\infty$ to -1 = 0
-1 to 0 = 2
0 to 1 = -4
1 to 2 = 14
2 to 3 = 4
3 to ∞ = -16



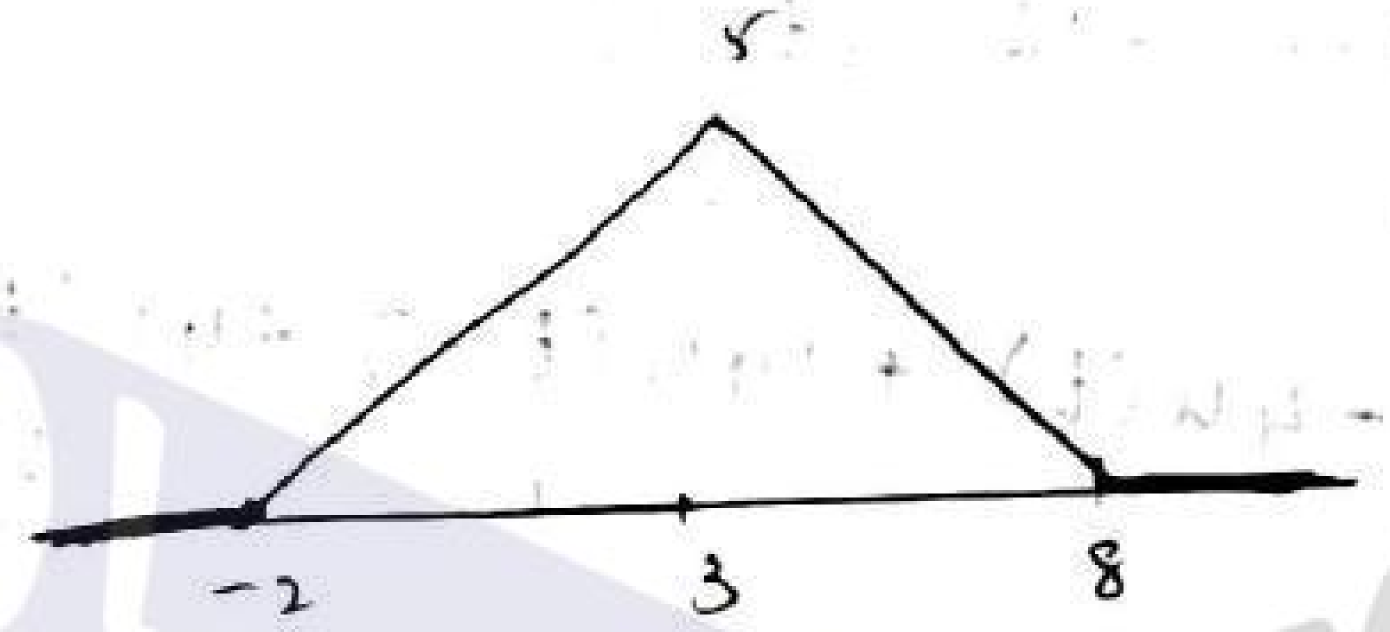
(3) $x(t) = \underset{-2}{(t+2)u(t+2)} + \underset{3}{(6-2(t))u(t-3)} - \underset{8}{(8-t)u(t-8)}$

$-\infty$ to $-2 = 0$

-2 to $3 = t+2$

3 to $\infty = 6-2t$

8 to $\infty = -(8-t)$



$\frac{t+2}{\frac{0}{t+2}}$ $\frac{\begin{matrix}6-2t\\ t+2\end{matrix}}{8-t}$ $\frac{\begin{matrix}-(8-t)\\ 8-t\end{matrix}}{0}$

(4) $x(t) = tu(t) + (10-2t)u(t-5) - (10-t)u(t-10)$

$h(t) = \delta(t+2) + \delta(t-5)$

find $y(t)$

$= [tu(t) + (10-2t)u(t-5) - (10-t)u(t-10)] * [\delta(t+2) + \delta(t-5)]$

$= [(t+2)u(t+2) + (10-2(t+2))u(t+2-5) - (10-(t+2))u(t+2-10)$

$+ (t-5)u(t-5) + (10-2(t-5))u(t-5-5) - (10-(t-5))$
$u(t-5-10)]$

$= [(t+2)u(t+2) + (10-2t-4)u(t-3) - (10-t-2)u(t-8)$

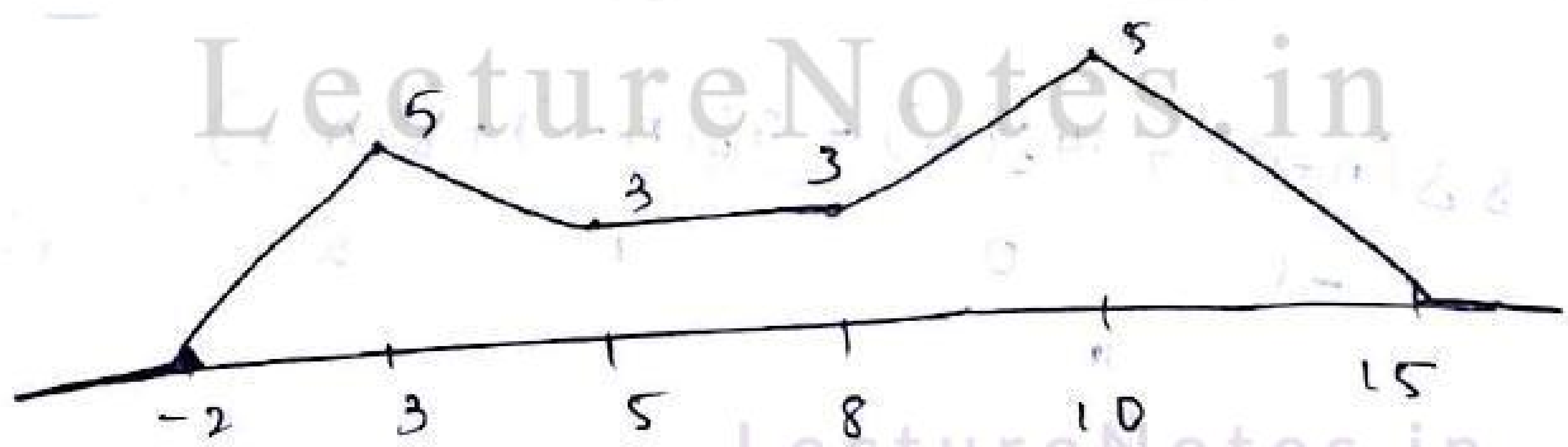
$+ (t-5)u(t-5) + (10-2t+10)u(t-10) - (10-t+5)$
$u(t-15)]$

$$= \Big[(t+2)\underset{-2}{u(t+2)} + (6-2t)\underset{3}{u(t-3)} - (8-t)\underset{8}{u(t-8)}$$
$$+ (t-5)\underset{5}{u(t-5)} + (20-2t)\underset{10}{u(t-10)} - (15-t)\underset{15}{u(t-15)}$$

$$= \Big[(t+2)\underset{-2}{u(t+2)} + (6-2t)\underset{3}{u(t-3)} + (t-5)\underset{5}{u(t-5)}$$
$$- (8-t)\underset{8}{u(t-8)} + (20-2t)\underset{10}{u(t-10)} - (15-t)\underset{15}{u(t-15)}$$

$-\infty$ to -2	$=$	0	$t+2$
-2 to 3	:	$t+2$	$8-t$
3 to 5	:	$(6-2t)$	3
5 to 8	:	$(t-5)$	$-5+t$
8 to 10	:	$-(8-t)$	$15-t$
10 to 15	:	$(20-2t)$	0
15 to ∞	=	$-(15-t)$	

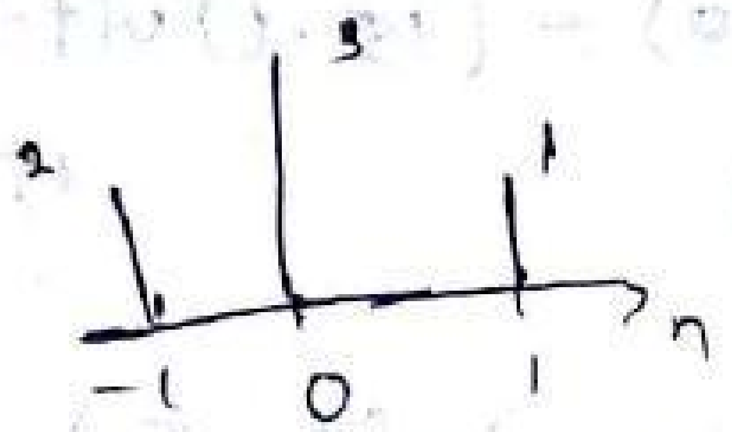
$t+2$
$\underline{6-2t}$
$8-t$
$\underline{t-5}$
3
$\underline{-(8+t)}$
$-5+t$
$\underline{20-2t}$
$15-t$
$-(15-t)$



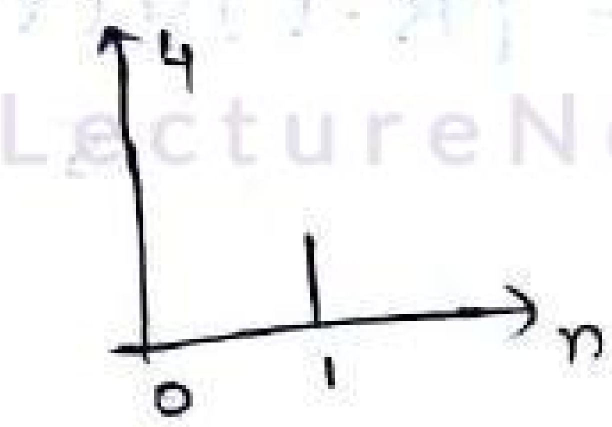
(5) $x(n) = \{2, \underset{\uparrow}{3}, 1\}$

$h(n) = \{\underset{\uparrow}{4}, 1\}$

find $y(n) = x(n) * h(n)$ using property (Analytical Method).



$$x(n) = 2\delta(n+1) + 3\delta(n) + \delta(n-1)$$



$$h(n) = 4\delta(n) + \delta(n-1)$$

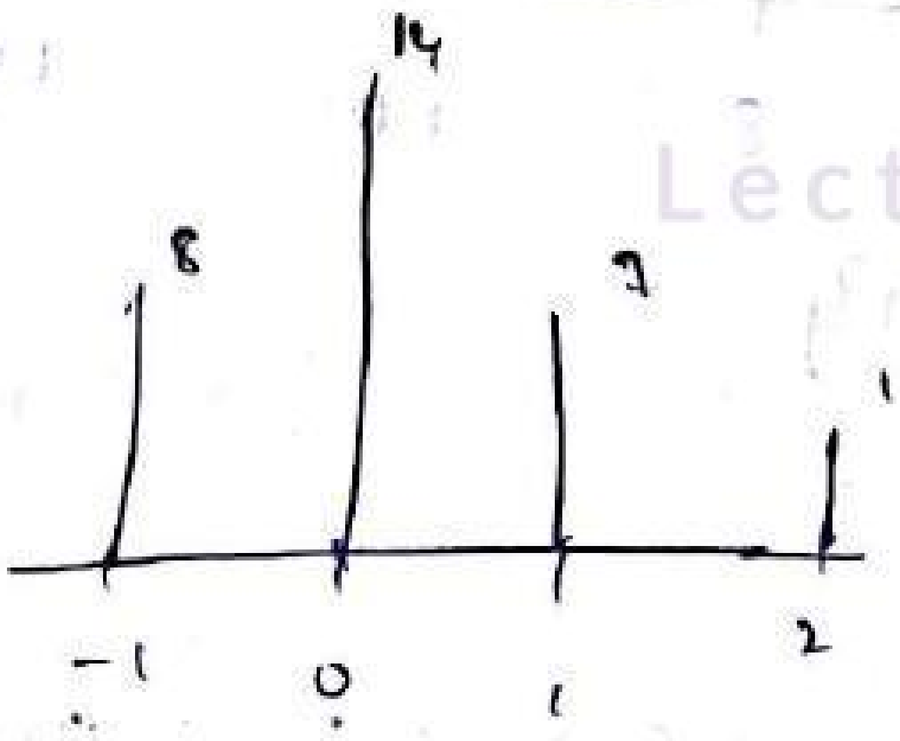
$$y(n) = x(n) * h(n)$$

$$= \left[2\delta(n+1) + 3\delta(n) + \delta(n-1)\right] * \left[4\delta(n) + \delta(n-1)\right]$$

$$= 8\delta(n+1) + 12\delta(n) + 4\delta(n-1) + 2\delta(n-1+1) + 3\delta(n-1) + \delta(n-1-1)$$

$$= 8\delta(n+1) + 12\delta(n) + 4\delta(n-1) + 2\delta(n) + 3\delta(n-1) + \delta(n-2)$$

$$= \underset{-1}{8\delta(n+1)} + \underset{0}{14\delta(n)} + \underset{1}{7\delta(n-1)} + \underset{2}{\delta(n-2)}$$

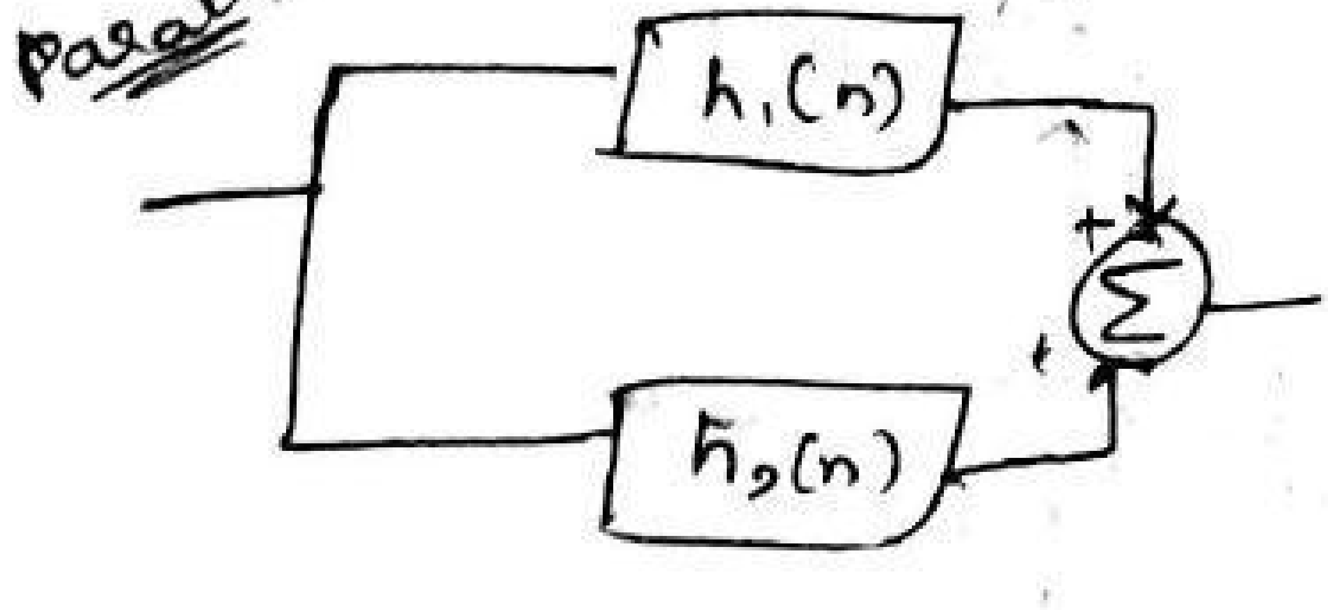


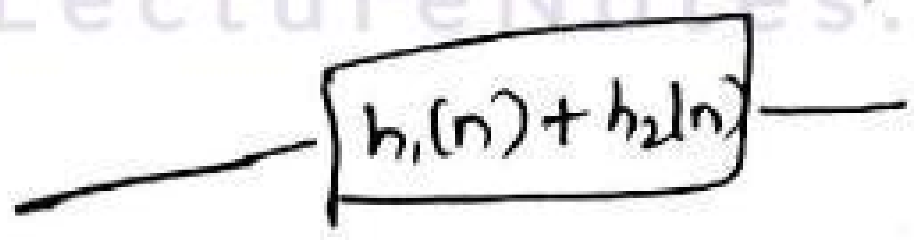
$$y(n) = \{8, \underset{\uparrow}{14}, 7, 1\}$$

Module - 3

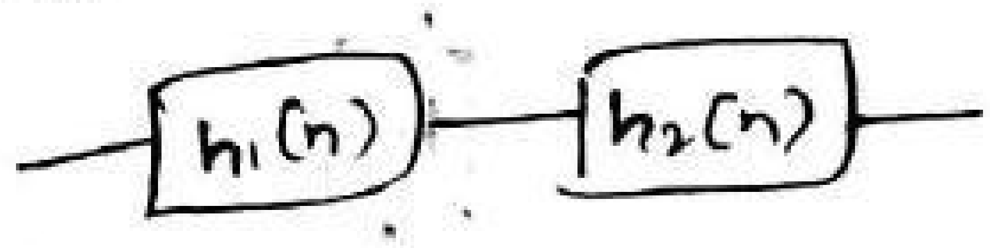
System interconnection

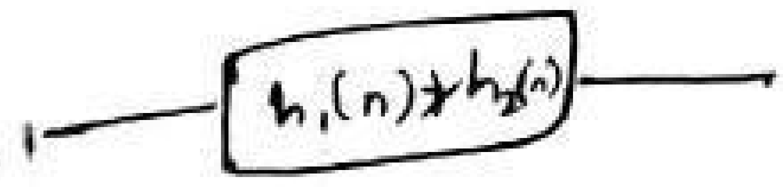
Parallel



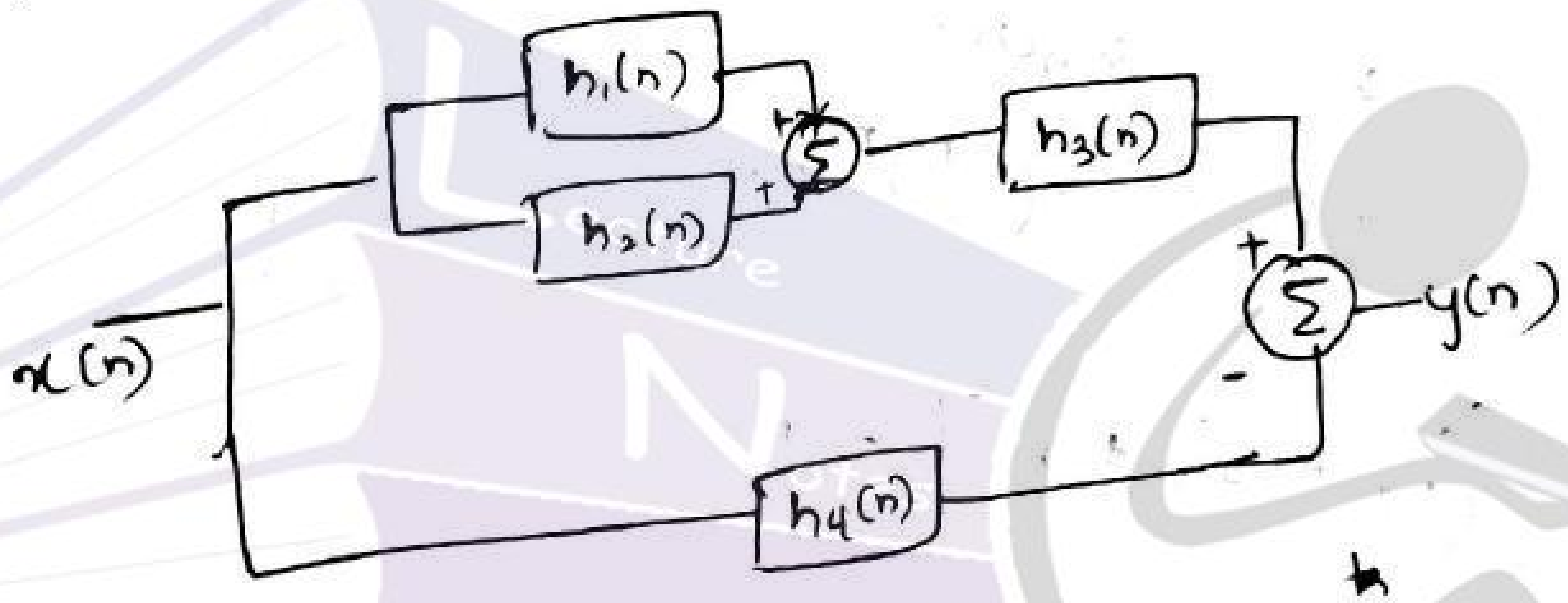


Series





① Find overall impulse response.



$h_1 = u(n)$

$h_2 = u(n+2) - u(n)$

$h_3 = \delta(n-2)$

$h_4 = a^n u(n)$

$= \left[[h_1(n) + h_2(n)] * h_3 \right] - h_4$

$= \left(\cancel{u(n)} + u(n+2) - \cancel{u(n)} \right) * \delta(n-2) \Big] - a^n u(n)$

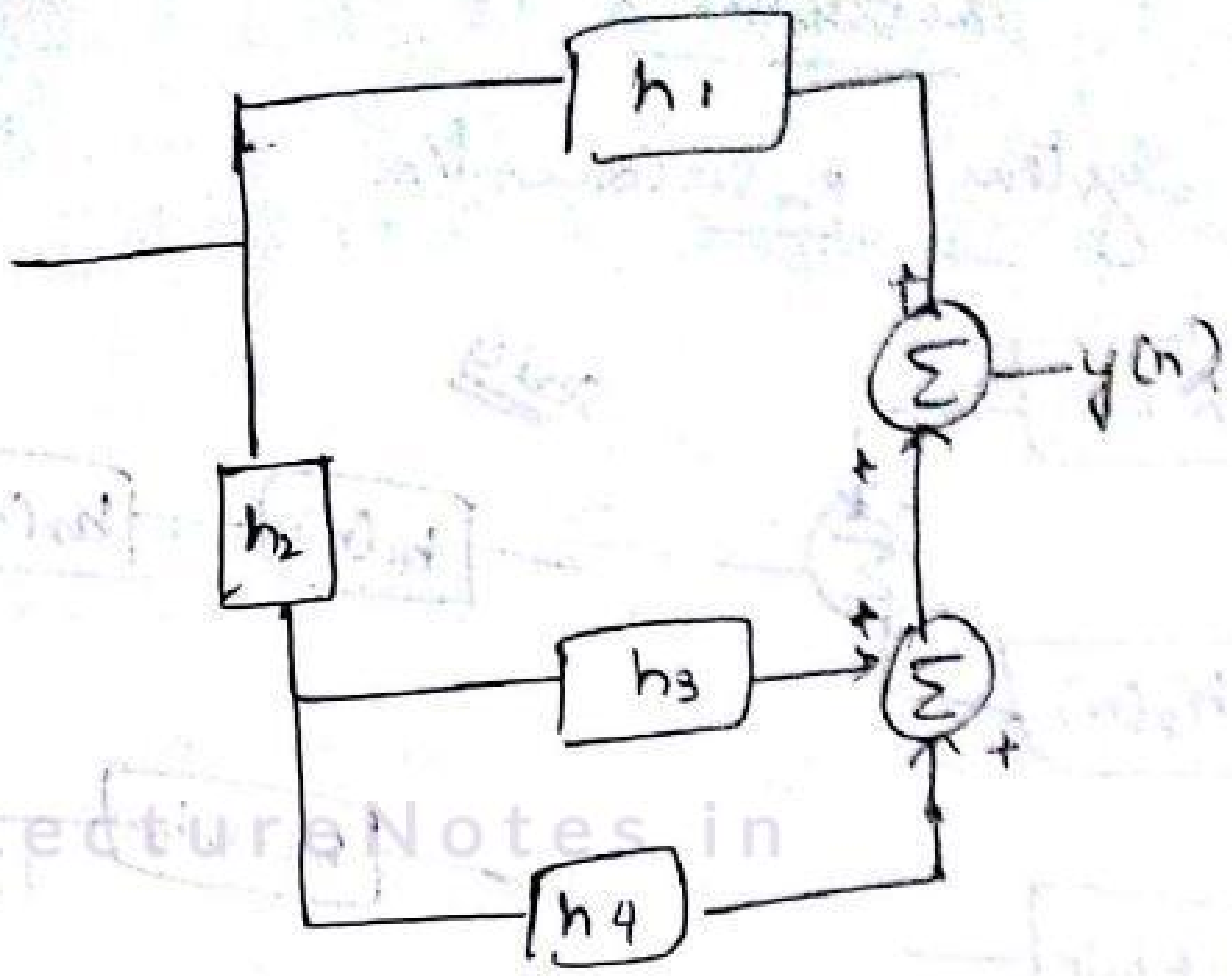
$= \left[u(n+2) * \delta(n-2) \right] - a^n u(n)$

$= u(n-2+2) - a^n u(n)$

$= u(n) - a^n u(n)$

$= u(n) \left[1 - a^n \right]$ //

2)

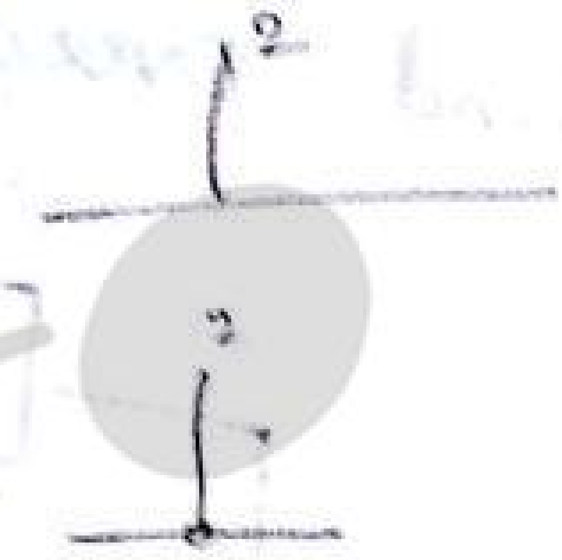


$$h_1 = \delta(n) + \tfrac{1}{2}\delta(n-1)$$

$$h_2 = \tfrac{1}{2}\delta(n) - \tfrac{1}{4}\delta(n-1)$$

$$h_3 = 2\delta(n)$$

$$h_4 = -2\left(\tfrac{1}{2}\right)^n u(n)$$



$$\rightarrow \quad \left[(h_3 + h_4) * h_2\right] + h_1$$

$$= \left[\left(2\delta(n) - 2\left(\tfrac{1}{2}\right)^n u(n)\right) * \left(\tfrac{1}{2}\delta(n) - \tfrac{1}{4}\delta(n-1)\right)\right] + \delta(n) + \tfrac{1}{2}\delta(n-1)$$

$$= \left[2\delta(n) * \left(\tfrac{1}{2}\delta(n) - \tfrac{1}{4}\delta(n-1)\right)\right] \cancel{+ \delta(n) + \tfrac{1}{2}\delta(n-1)}$$

$$- \left[2\left(\tfrac{1}{2}\right)^n u(n) * \left(\tfrac{1}{2}\delta(n) - \tfrac{1}{4}\delta(n-1)\right)\right] + \delta(n) + \tfrac{1}{2}\delta(n-1)$$

$$= \delta(n) \cancel{- \tfrac{1}{2}\delta(n-1)} - \left(\tfrac{1}{2}\right)^n u(n) + \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{n-1} u(n-1) + \delta(n) \cancel{+ \tfrac{1}{2}\delta(n-1)}$$

$$= \delta(n) - \left(\tfrac{1}{2}\right)^n u(n) + \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{n-1} u(n-1) + \delta(n)$$

$$= 2\delta(n) - \left(\tfrac{1}{2}\right)^n u(n) + \left(\tfrac{1}{2}\right)^n u(n-1)$$

$$= 2\delta(n) - \left[u(n) - u(n-1)\right]\left(\tfrac{1}{2}\right)^n$$

$$= 2\delta(n) - \left(\tfrac{1}{2}\right)^n \delta(n)$$

$$= \delta(n)\left[2 - \left(\tfrac{1}{2}\right)^n\right]$$

= at discrete $n = 0$

$$= \delta(n)\left[2 - 1\right]$$

$$= \underline{\underline{\delta(n)}}$$

<u>Memory</u>

System is said to be memory if impulse response $h(n)$ exist for $n \neq 0$

<u>Causal</u>

S/m is said to be causal if impulse response $h(n)$ does not exist for $n < 0$

<u>Stability</u>

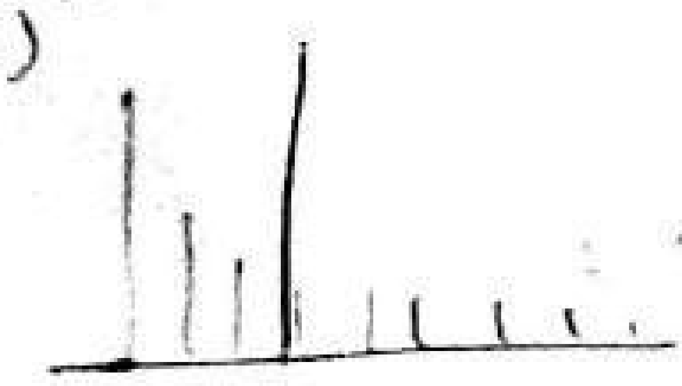
S/m is said to be stable if sum of impulse response is finite.

i.e., $$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

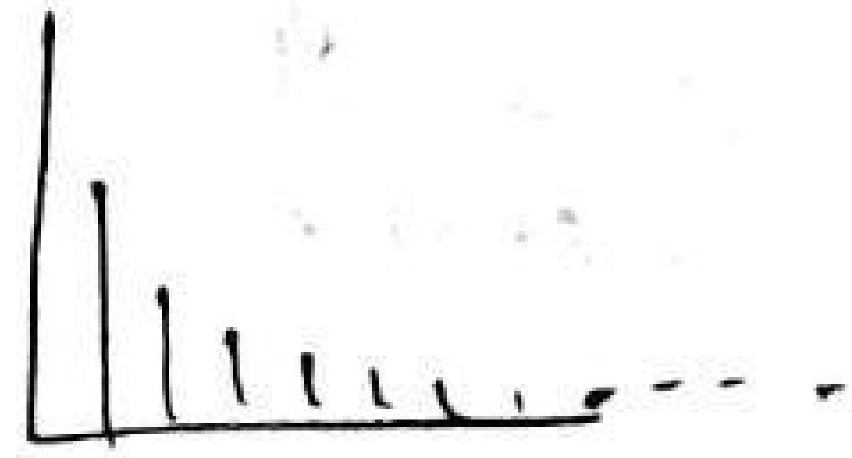
*** ① Check for memory, causal & stability.

i) $h(n) = (0.5)^n u(n)$

$(0.5)^n$



$(0.5)^n u(n)$



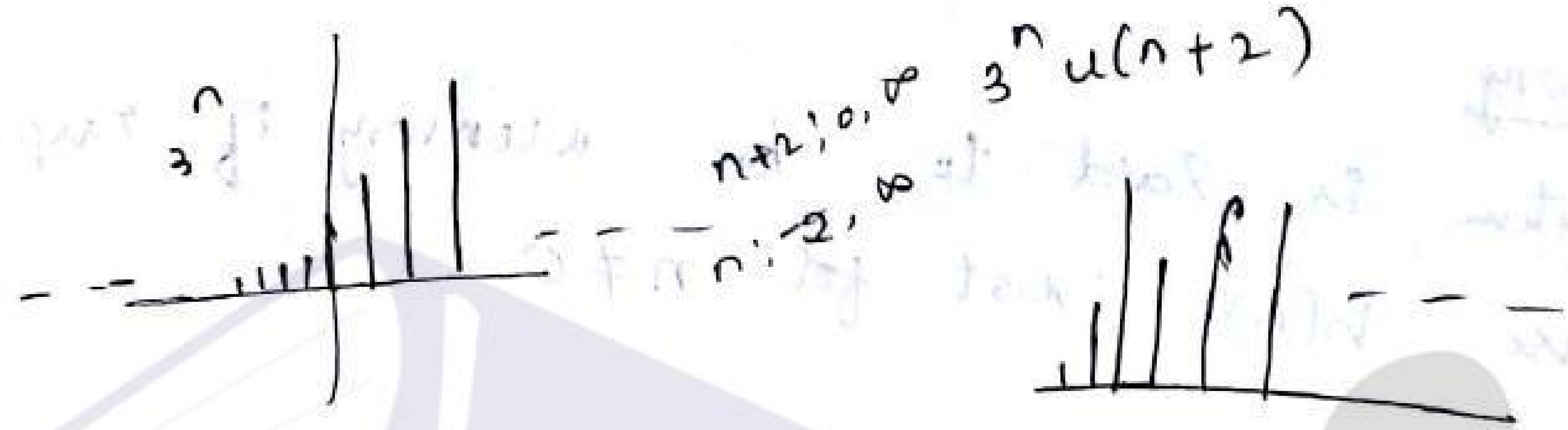
Memory
Causal
Stable.

(b) It is Memory because $h(n)$ exist for $n \neq 0$

It is Causal because $h(n)$ doesnot exist for $n<0$

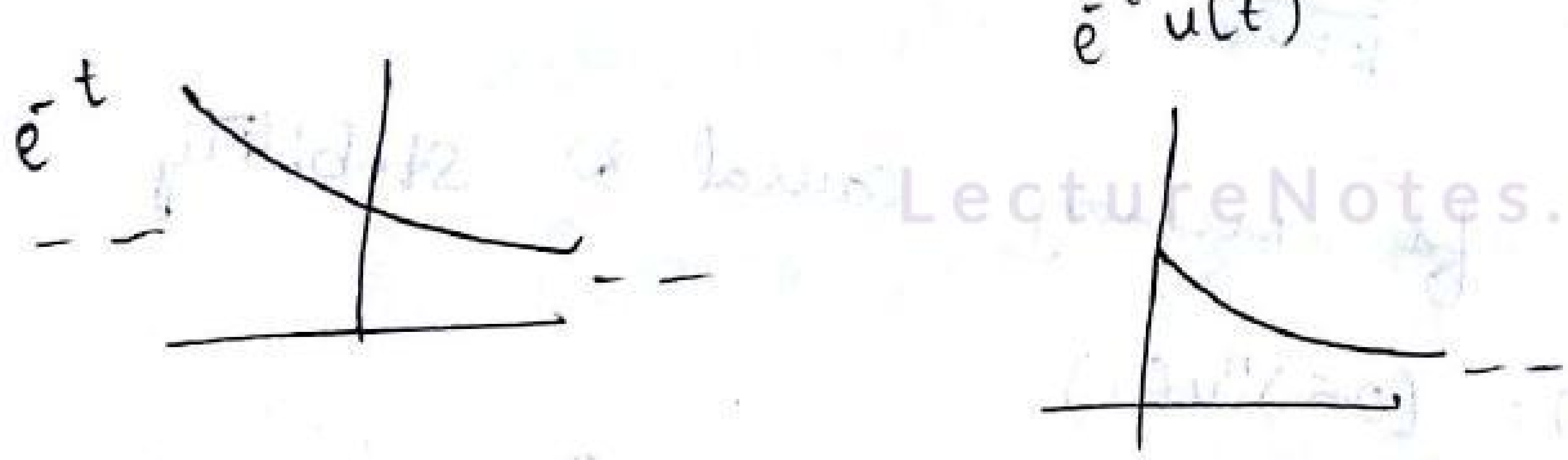
It is stable because impulse response is converging & Result is finite.

(2) $h(n) = 3^n u(n+2)$

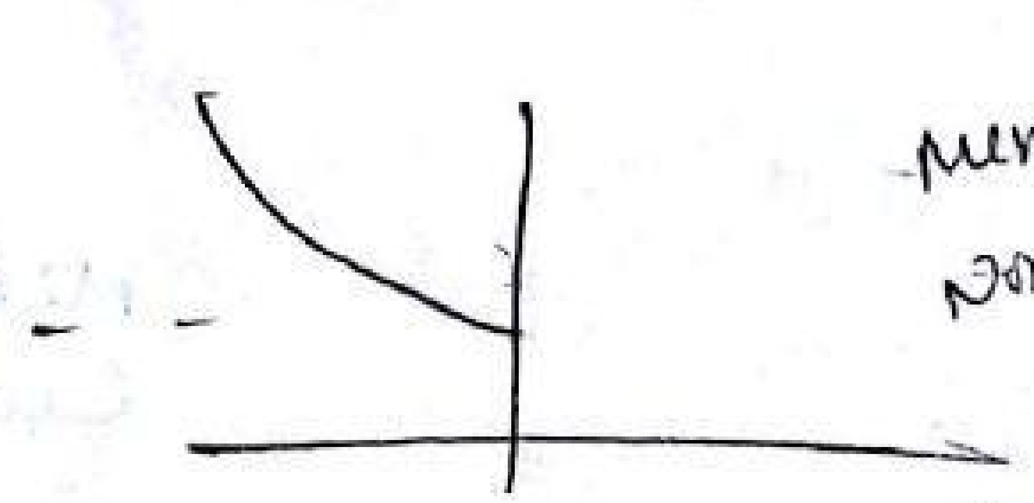


It is Memory because $h(n)$ exist for $n \neq 0$
It is non Causal " $h(n)$ exist for $n<0$
It is unstable " impulse response is diverging & Result is infinite

(3) $h(t) = e^{-t} u(t)$



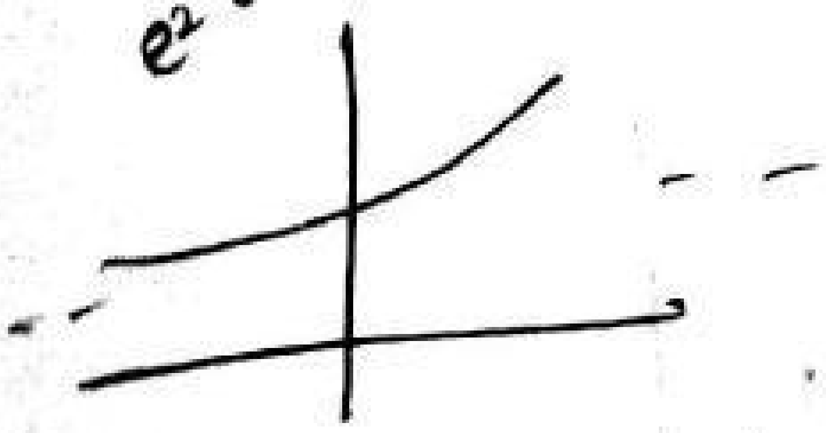
* Memory
* Causal
* Stable



Memory
Non Causal
unstable

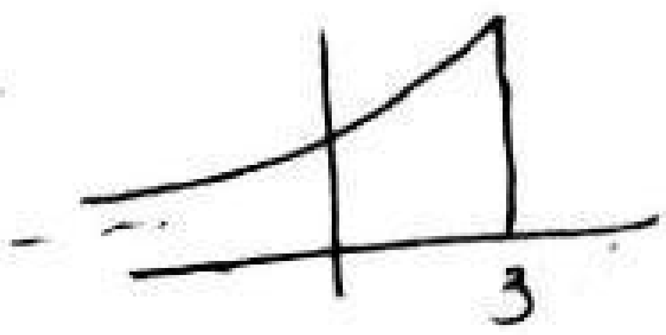
4) $h(t) = e^{2t} u(-t+3)$

e^{2t}



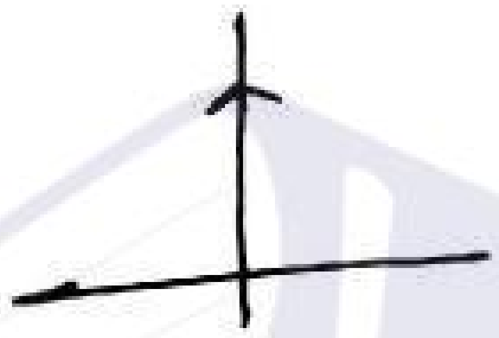
$-t+3 ; 0, \infty$
$-t ; -3, \infty$
$t ; 3, -\infty$

$e^{2t} u(-t+3)$



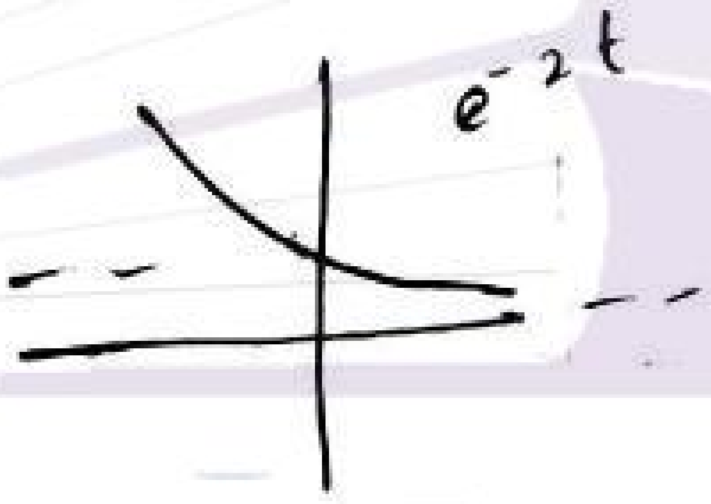
Memory
Non-Causal
Stable

5) $h(n) = \delta(n)$

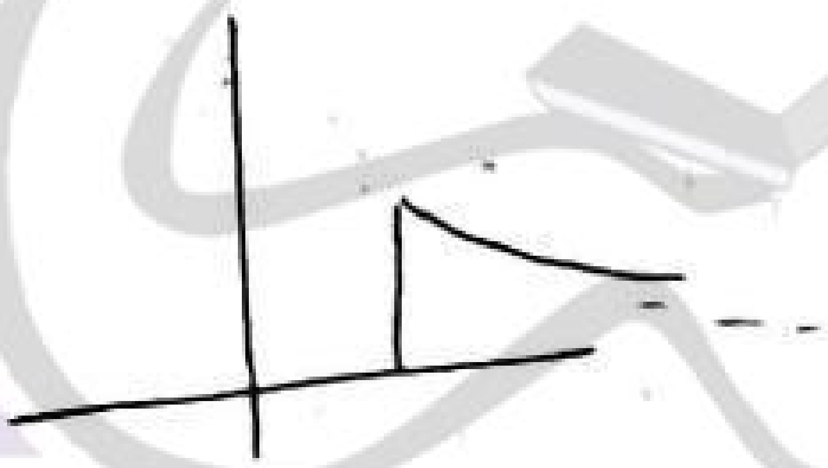


Memory less
Causal
Stable

6) $h(t) = e^{-2t} u(t-3)$



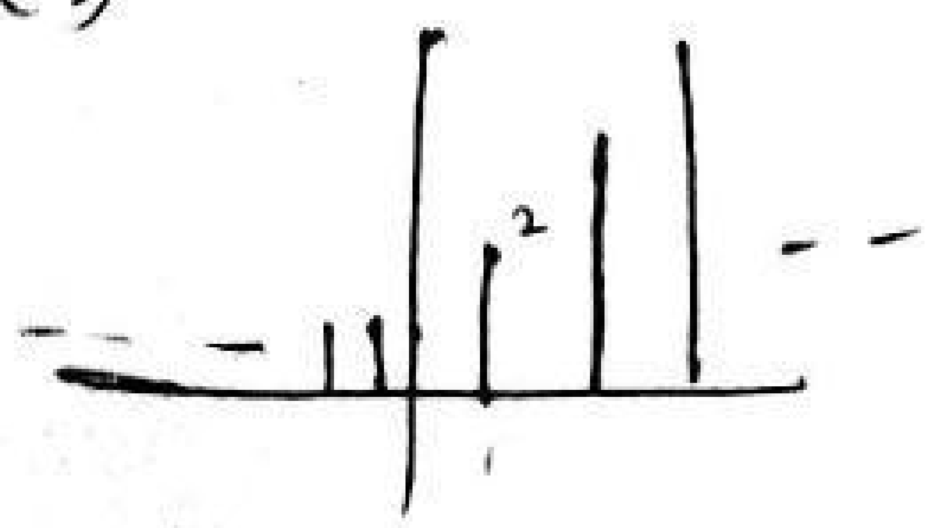
$t-3 ; 0, \infty$
$t ; 3, \infty$



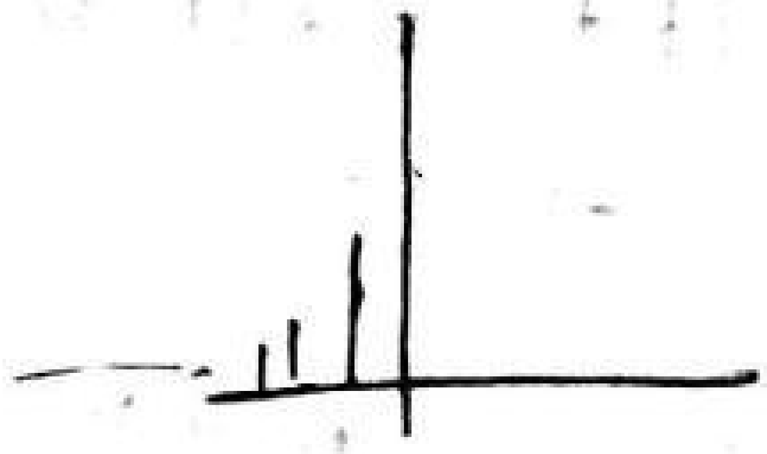
Memory
Causal
Stable

7) $h(n) = \left(\frac{1}{2}\right)^{-n} u(-n)$

$\left(\frac{1}{2}\right)^{-n}$



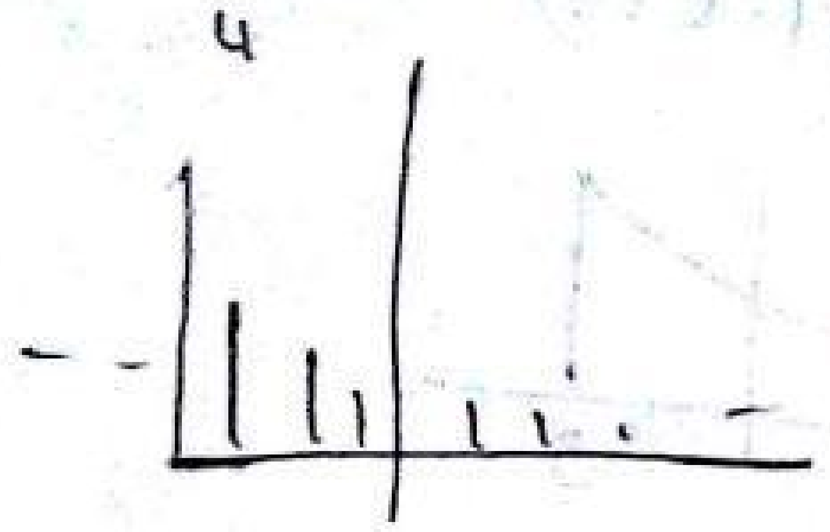
$\left(\frac{1}{2}\right)^{-n} u(-n)$



Memory
Non-Causal
stable

8) $h(n) = 4^{-n} u(-n+2)$

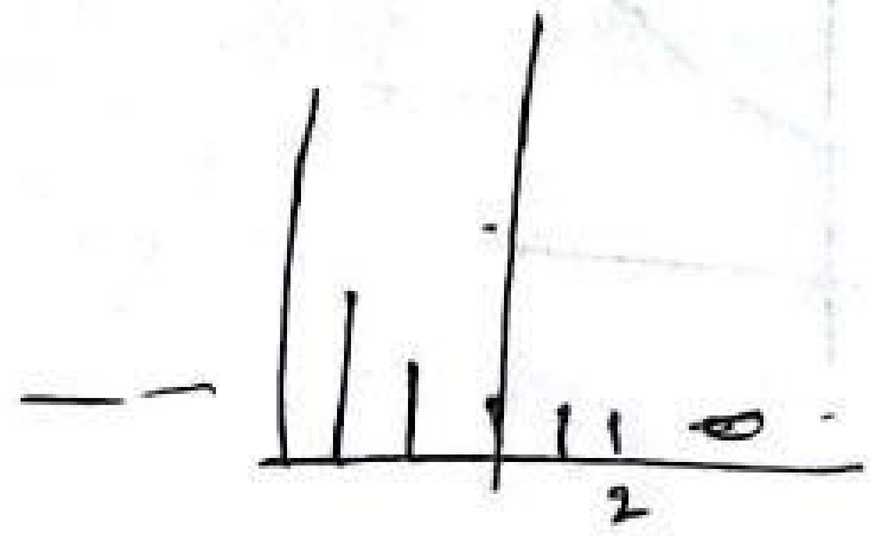
4^{-n}



$-n+2 ; 0, \infty$

$-n ; -2, \infty$

$n ; 2, -\infty$



Memory
Non-Causal
unstable

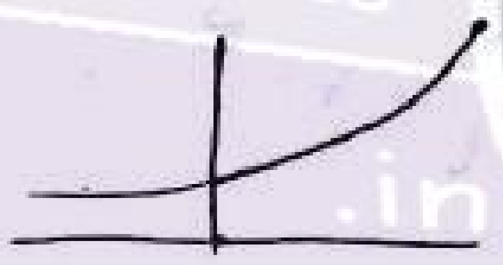
(9) $h(t) = t u(t)$

$t u(t)$

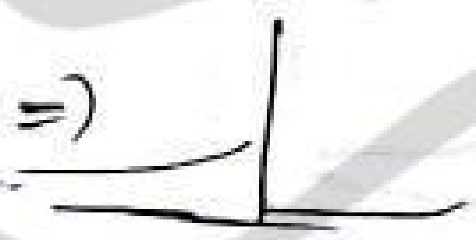
Memory
Causal
unstable.

(10) $h(t) = e^{-|t|}$

e^{t} $\{-\infty, 0\}$

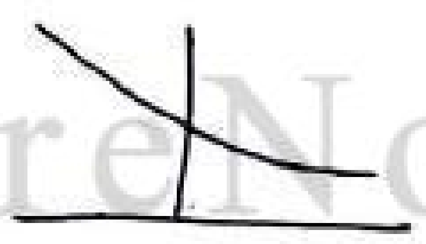


$\Rightarrow$



M
NC
S

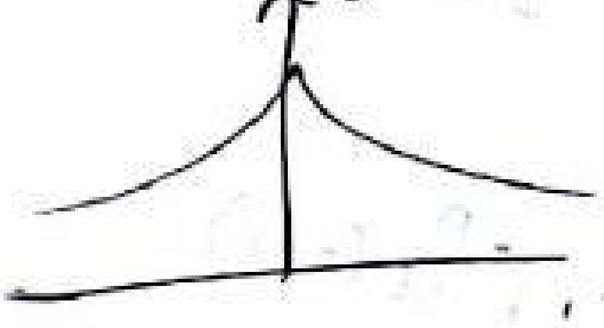
e^{-t} $\{0, \infty\}$



$\Rightarrow$

M
C
S

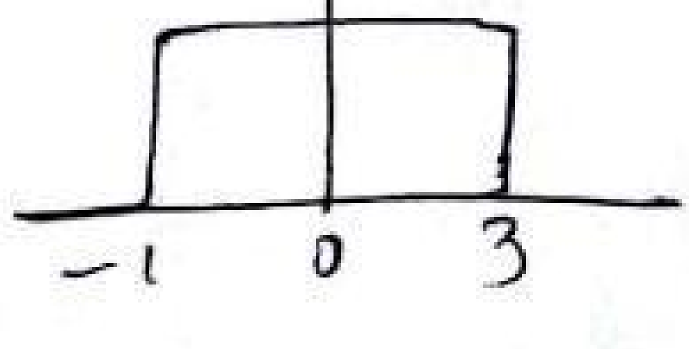
$e^{-|t|}$



Memory
Non Causal
Stable

(11) $h(t) = u(t+1) - u(t-3)$

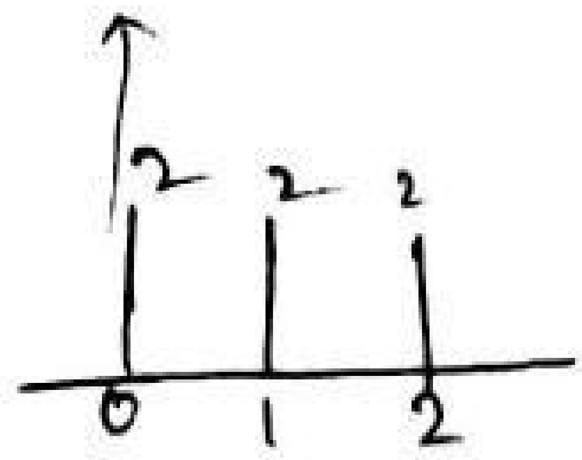
-1 3



Memory
Non causal
Stable

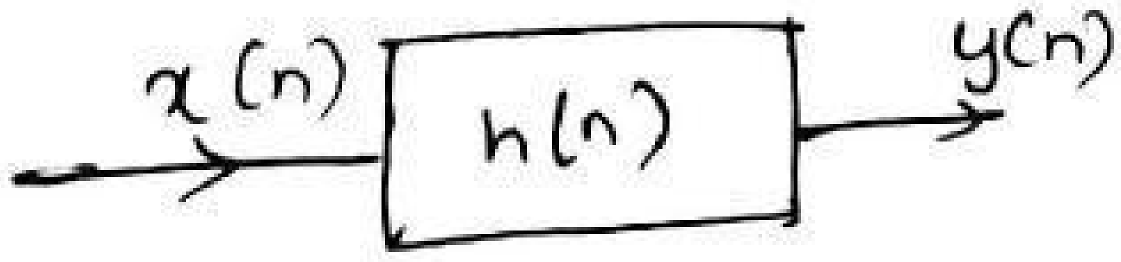
(12) $h(n) = 2[u(n) - u(n-3)]$

0 · 2



Memory
causal
Stable.

Step Response in terms of impulse Response



$y(n) = x(n) * h(n)$

$s(n) = u(n) * h(n)$

$s(n) = h(n) * u(n)$

$s(n) = h(n) * u(n)$

$$s(n) = \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$$

$n - k \geq 0$

$n \geq k$

$\boxed{k \leq n}$

$$\boxed{s(n) = \sum_{k=-\infty}^{n} h(k)}$$

ly

$$\boxed{s(t) = \int_{\tau=-\infty}^{t} h(\tau)\, d\tau}$$

Find step response for the given impulse Response

(1) $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

$$= \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^{k} u(k)$$

$k \geq 0$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k}$$

$$s(n) = \frac{1 - \left(\frac{1}{2}\right)^{k+1}}{1 - \frac{1}{2}}$$

② $h(n) = \left(\frac{1}{2}\right)^{n} u(n-3)$

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

$$= \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^{k} u(k-3)$$

$k-3 \geq 0$

$\boxed{k \geq 3}$

$$= \sum_{k=3}^{n} \left(\frac{1}{2}\right)^{k}$$

$k-3 = m$ | $k = 3$ | $m = 0$

$k = m+3$ | $k = n$ | $m = n-3$

$$= \sum_{m=0}^{n-3} \left(\frac{1}{2}\right)^{m+3}$$

$$= \sum_{m=0}^{n-3} \left(\frac{1}{2}\right)^{m} \left(\frac{1}{2}\right)^{3}$$

$$= \frac{1}{8} \sum_{m=0}^{n-3} \left(\frac{1}{2}\right)^{m}$$

$$s(n) = \frac{1}{8} \left[\frac{1 - \left(\frac{1}{2}\right)^{n-2}}{1 - \frac{1}{2}} \right]$$

③ $h(t) = t^2 u(t)$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$= \int_{-\infty}^{t} \tau^2 u(\tau)\,d\tau$$

$t \geq 0$

$$= \int_{0}^{t} \tau^2\,d\tau$$

$$= \left[\frac{\tau^3}{3}\right|_0^t$$

$$s(t) = \frac{t^3}{3}$$

④ $h(t) = e^{-at} u(t-2)$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$= \int_{-\infty}^{t} e^{-a\tau} u(\tau-2)\,d\tau$$

$$= \int_{2}^{t} e^{-a\tau}\,d\tau$$

$$\left.\frac{e^{-a\tau}}{-a}\right|_2^t$$

$$s(t) = \frac{-1}{a}\left[e^{-at} - e^{-2a}\right]$$

⑤ $h(t) = u(t+1) - u(t-1)$

$-1 \quad 1$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,dt = \int_{-1}^{1} u(\tau+1) - u(\tau-1)$$

$$= \int_{-1}^{1} 1\,d\tau = t\Big|_{-1}^{1} \Rightarrow [1-(-1)]$$

$$\boxed{s(t) = 2}$$

6) $h(t) = e^{-3t}u(t)$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$= \int_{-\infty}^{t} e^{-3t}u(t)\,d\tau$$

$$= \int_{0}^{t} e^{-3\tau}\,d\tau$$

$$\frac{e^{-3\tau}}{-3}\Big|_{0}^{t}$$

$$-\frac{1}{3}\left[e^{-3t} - 1\right]$$

$$s(t) = \frac{1}{3} - \frac{e^{-3t}}{3}$$

(7) $h(t) = e^{-2|t|}$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$= \int_{-\infty}^{0} e^{+2\tau}\,d\tau + \int_{0}^{\infty} e^{-2\tau}\,d\tau$$

$$= \frac{e^{2\tau}}{2}\Big|_{-\infty}^{0} + \frac{e^{-2\tau}}{-2}\Big|_{0}^{\infty}$$

$$\frac{1}{2}\left[1 - 0\right] + \frac{1}{-2}\left[0 - 1\right]$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$\boxed{s(t) = 1}$$

Fourier Series (CTFS / FS)

continuous and periodic

$$x(t) \xrightarrow{F.S} X(k)$$

F.S

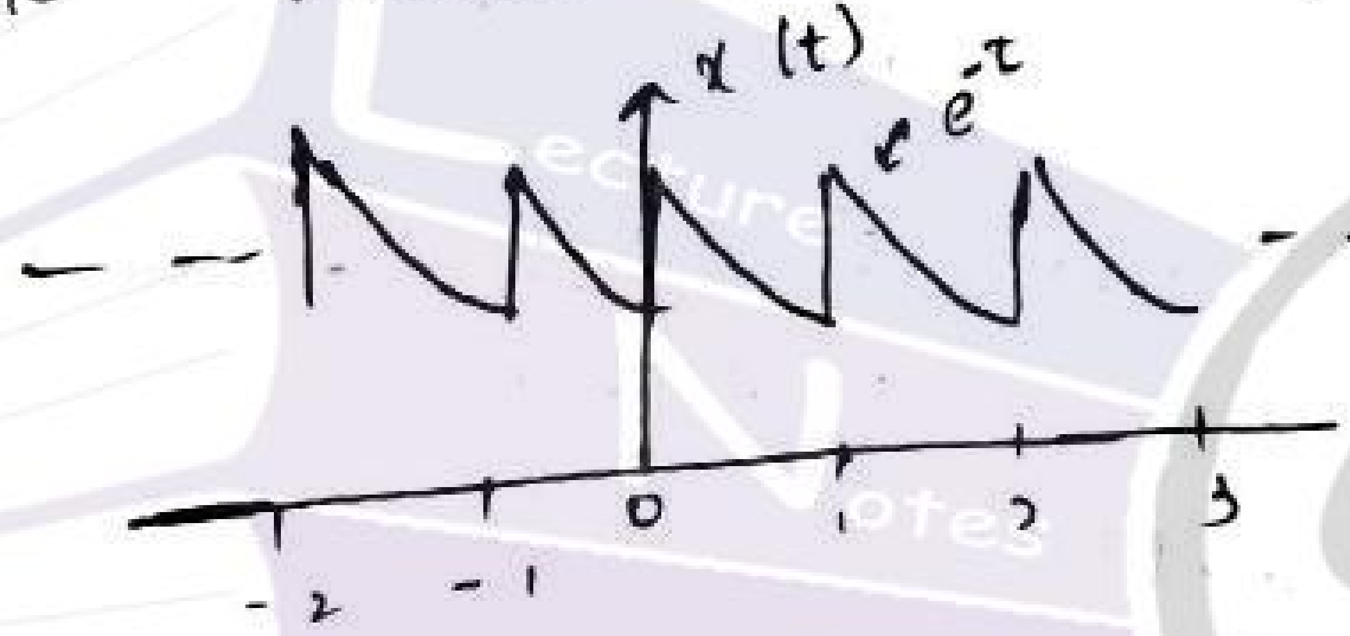
$$X(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

IFS

$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{-jk\omega_0 t}$$

$\omega_0 = \frac{2\pi}{T}$

① Find Fourier Series Co-efficient $X(k)$



$$X(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt.$$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$

$T = 1$

$$= \frac{1}{1}\int_0^1 e^{-t}\, e^{-jk\omega_0 t}\, dt$$

$$= \int_0^1 e^{-(1+jk\omega_0)t}\, dt$$

$$= \left.\frac{e^{-(1+jk\omega_0)t}}{-(1+jk\omega_0)}\right|_0^1$$

$$= \frac{e^{-(1+jk\omega_0)} - 1}{-(1+jk\omega_0)}$$

$$= \frac{1 - e^{-(1+j\omega_0)}}{(1+jk\omega_0)}$$

For Magnitude

$$\frac{1 - e^{-1} e^{-jk2\pi}}{1 + jk2\pi}$$

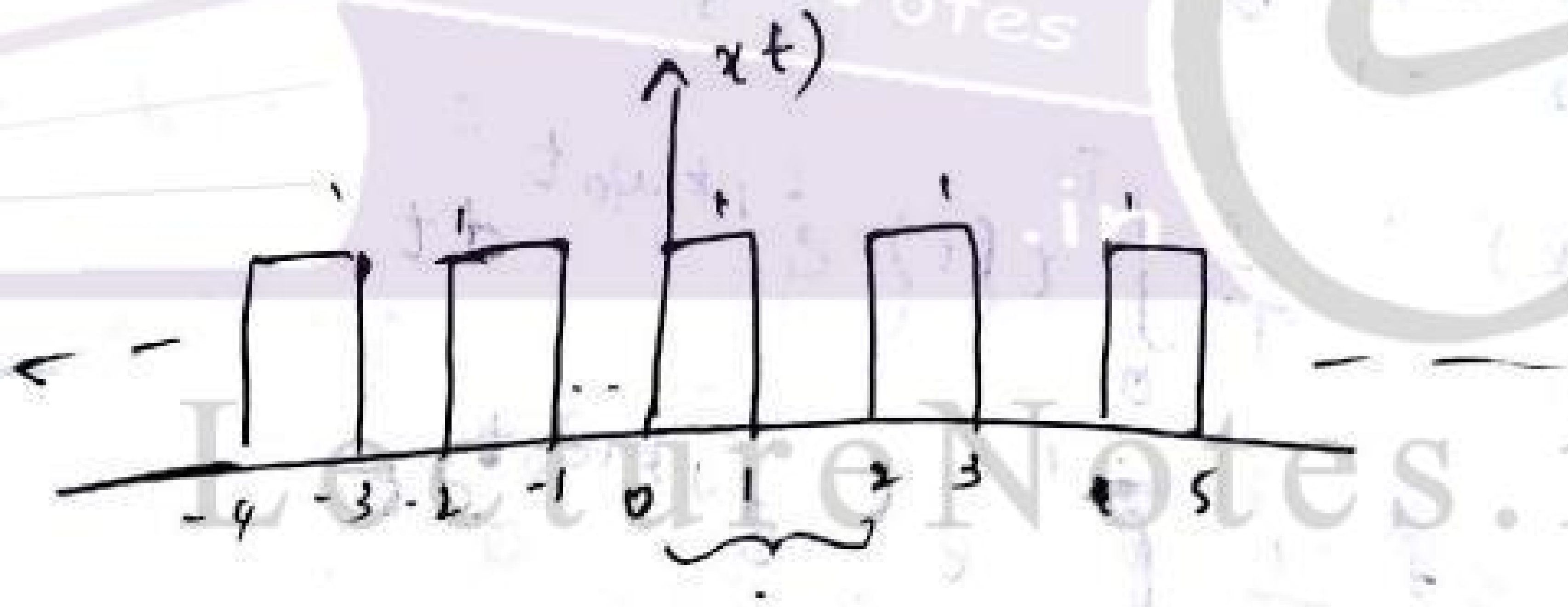
$$\frac{1 - e^{-1}(\cos k2\pi - j\sin k2\pi)}{1 - jk2\pi}$$

$$\frac{1 - e^{-1}\cos k2\pi + je^{-1}\sin k2\pi}{1 - jk2\pi}$$

$$|X(k\omega_0)| = \frac{\sqrt{(1 - e^{-1}\cos 2\pi k)^2 + (e^{-1}\sin k2\pi)^2}}{\sqrt{1^2 + (k2\pi)^2}}$$

$$\angle X(k) = \tan^{-1}\left(\frac{e^{-1}\sin k2\pi}{1 - e^{-1}\cos k2\pi}\right) - \tan^{-1}\left(\frac{-k2\pi}{1}\right)$$

(2)



$T = 2$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$$x(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

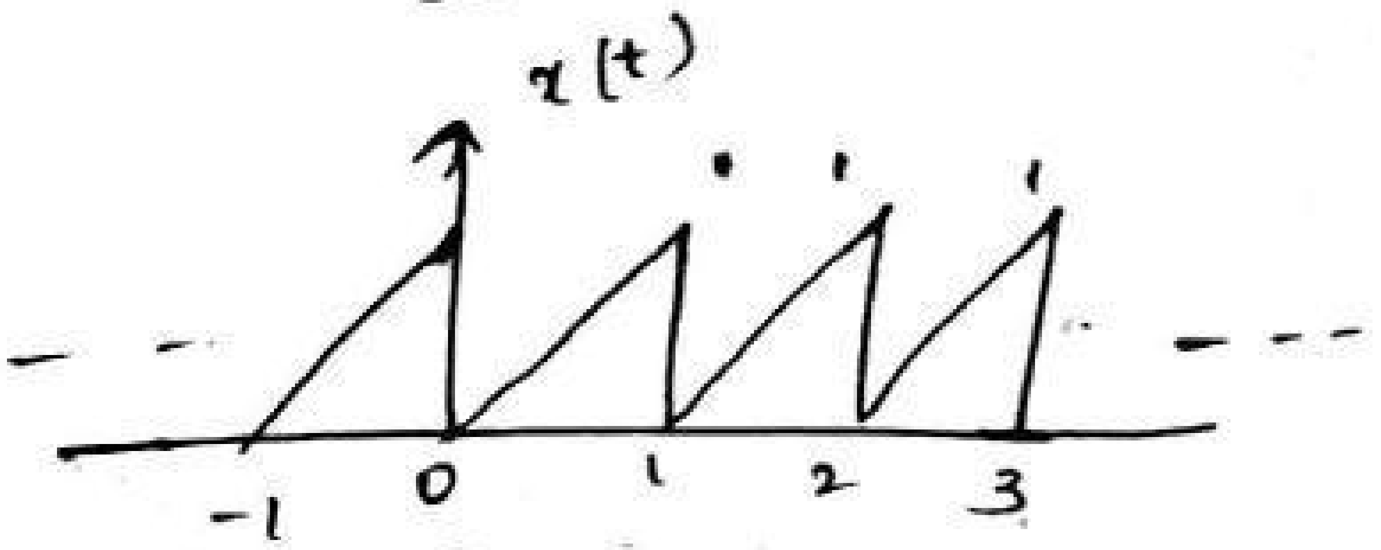
$$= \frac{1}{2}\int_0^2 e^{-jk\pi t}\, dt$$

$$\frac{1}{2}\left.\frac{e^{-jk\pi t}}{-jk\pi}\right|_0^2$$

$$-\frac{1}{2jk\pi}\left[e^{-jk\pi 2} - 1\right]$$

$$x(k) = \frac{1}{2}\left[\frac{1 - e^{-jk2\pi}}{jk\pi}\right]$$

(3)



$T = 1$

$\omega_0 = \frac{2\pi}{1} = 2\pi$

$$x(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{1}\int_0^1 t\, e^{-jk2\pi t}\, dt$$

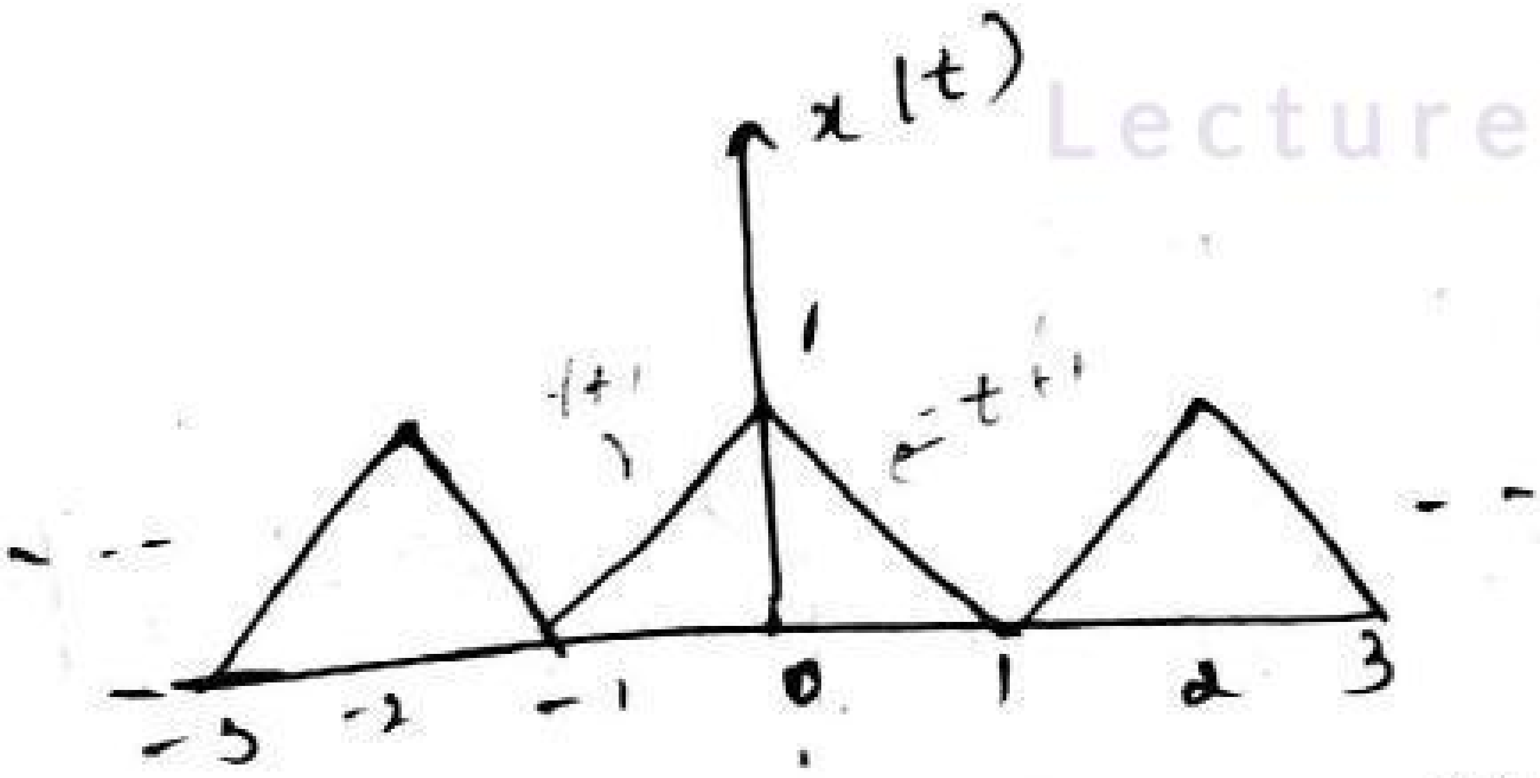
$$= t\,\frac{e^{-jk2\pi t}}{-jk2\pi} - \frac{e^{-jk2\pi t}}{(jk2\pi)^2}\Bigg|_0^1$$

$$= \left[\frac{-e^{-jk2\pi} + 0}{-jk2\pi}\right] + \frac{1}{(k2\pi)^2}\left[e^{-jk2\pi} - 1\right]$$

$$= \frac{-e^{-jk2\pi}}{+jk2\pi} + \frac{e^{-jk2\pi}}{k^2 4\pi^2} - \frac{1}{k^2 4\pi^2}$$

H.W

(4)



$T = 2$

$\omega_0 = \frac{2\pi}{2} = \pi$

$$x(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2}\left[\int_{-1}^{0} t+1\; e^{-jk2\pi t}\, dt + \int_0^1 -t+1\; e^{-jk2\pi t}\, dt\right]$$

$$= \frac{1}{2}\left\{ \left[(t+1) \frac{e^{-jk\pi t}}{-jk\pi} - \frac{e^{-jk\pi t}}{(-jk\pi)^2} \right]_{-1}^{0} + \left[(t+1) \frac{e^{-jk\pi t}}{-jk\pi} - (-1)\frac{e^{-jk\pi t}}{(-jk\pi)^2} \right]_{0}^{1} \right.$$

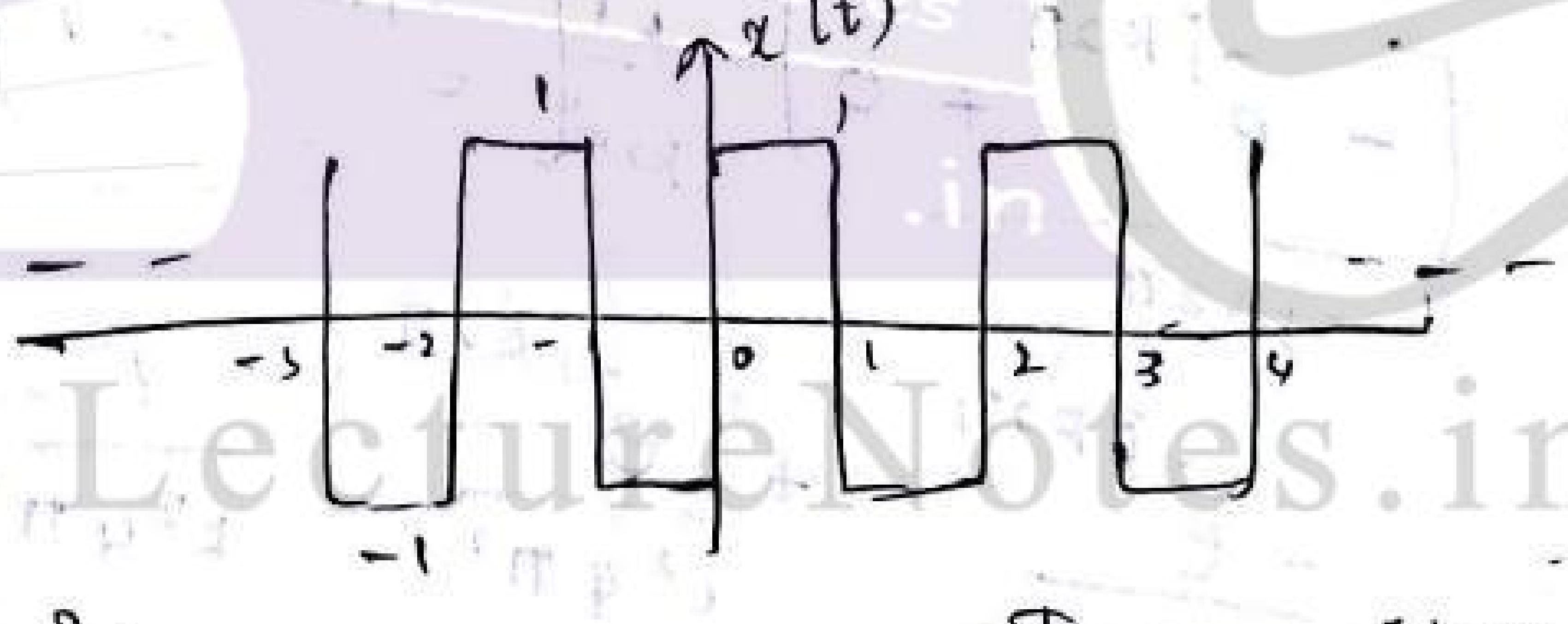
$$= \frac{1}{2}\left[e^{0}\left(\frac{1}{-jk\pi} + \frac{1}{k^2\pi^2}\right) - \left[0 + \frac{e^{jk\pi}}{k^2\pi^2}\right] + \left[0 - \frac{e^{-jk\pi}}{k^2\pi^2}\right] - \left[\frac{1}{-jk\pi} - \frac{1}{k^2\pi^2}\right] \right]$$

$$= \frac{1}{2}\left[\cancel{\frac{1}{-jk\pi}} + \frac{1}{k^2\pi^2} - \frac{e^{jk\pi}}{k^2\pi^2} - \frac{e^{-jk\pi}}{k^2\pi^2} + \cancel{\frac{1}{jk\pi}} + \frac{1}{k^2\pi^2} \right]$$

$$= \frac{1}{k^2\pi^2} - \frac{1}{2}\frac{e^{jk\pi}}{k^2\pi^2} - \frac{e^{-jk\pi}}{2k^2\pi^2} \quad //$$

A.W

5



$T = 2.$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \qquad X(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$

$$= \frac{1}{2}\int_0^2 x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2}\left[\int_0^1 1\, e^{-jk\omega_0 t}\, dt + \int_1^2 -1\, e^{-jk\omega_0 t}\, dt \right]$$

$$= \frac{1}{2}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Big|_0^1 - \frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Big|_1^2 \right]$$

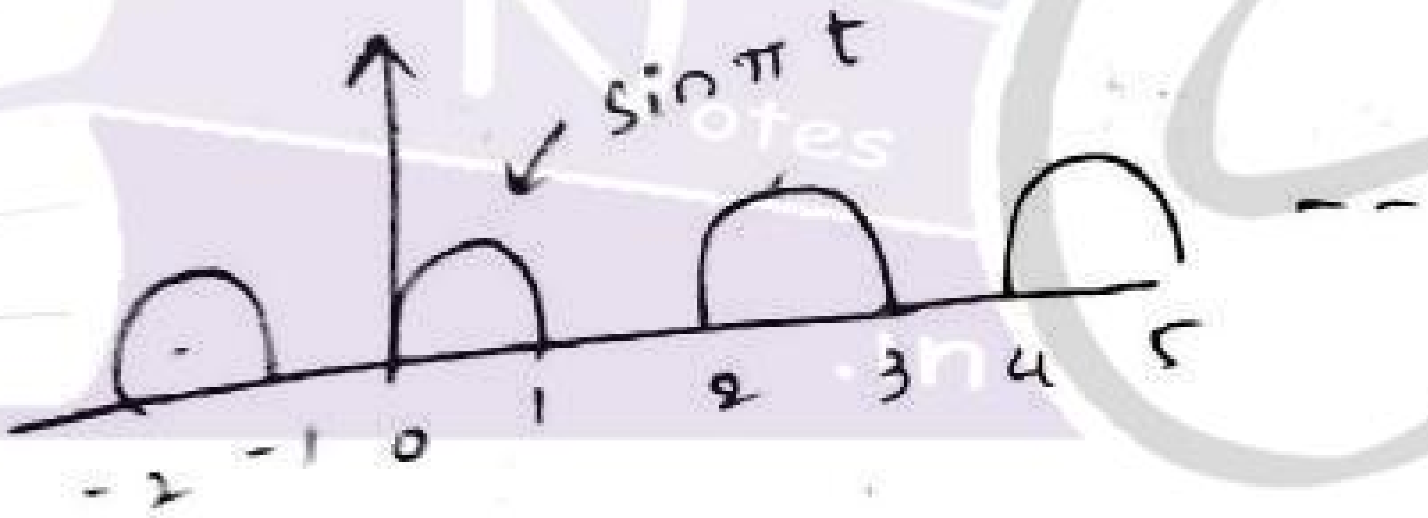
$$= \frac{1}{2}\left[\frac{1}{-jk\omega_0}\left[e^{-jk\omega_0} - 1\right] + \frac{1}{jk\omega_0}\left[e^{-2jk\omega_0} - e^{-jk\omega_0}\right]\right]$$

$$= \frac{1}{2}\left[\frac{e^{-jk\omega_0}}{-jk\omega_0} + \frac{1}{jk\omega_0} + \frac{e^{-2jk\omega_0}}{jk\omega_0} - \frac{e^{-jk\omega_0}}{jk\omega_0}\right]$$

$$= \frac{e^{-jk\omega_0}}{-jk\omega_0} + \frac{1}{2jk\omega_0} + \frac{e^{-2jk\omega_0}}{2jk\omega_0}.$$

H.W

6



$T = 2 \quad \omega = \pi \quad X(k) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$

$$= \frac{1}{2}\int_0^2 \sin\pi t\; e^{-jk\pi t}\, dt$$

$\cancel{\sin\pi t\left[\cos k\pi t - j\sin k\pi t\right.}$

$\sin\pi t\cos k\pi t - j\sin\pi t\sin k\pi t$

$$= \frac{1}{2}\int_0^2 \left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right) e^{-jk\pi t}\, dt$$

$$= \frac{1}{4j}\int_0 e^{(1-k)j\pi t} - e^{-(1+k)j\pi t}\, dT$$

$$\frac{1}{4\pi}\left[\frac{e^{(1-k)j\pi t}}{(1-k)j\pi}\Big|_0^1 - \frac{e^{-(1+k)j\pi t}}{-(1+k)j\pi}\Big|_0^1\right]$$

$$= \frac{1}{4\pi}\left\{\left[\frac{e^{(1-k)j\pi}}{(1-k)j\pi} - \frac{1}{(1-k)j\pi}\right] + \right.$$

$$\left.\left[\frac{e^{-(1+k)j\pi}}{(1+k)j\pi} - \frac{1}{-(1+k)j\pi}\right)\right.$$

$$= \frac{1}{4\pi}\left[\frac{e^{(1-k)j\pi}}{(1-k)j\pi} - \frac{1}{(1-k)j\pi} + \frac{e^{-(1+k)j\pi}}{(1+k)j\pi} - \frac{1}{(1+k)j\pi}\right.$$

⑦ $x(t) = \sin 2\pi t$ find fourier series Co-efficient

$x(k)$

$$\left\{ x(t) = \sum_{x=-\infty}^{\infty} x(k)\, e^{jk\omega_0 t} \right.$$

$\rightarrow \quad \omega_0 = 2\pi \qquad = \frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}$

$$\frac{1}{2j}\, e^{j2\pi t(1)} - \frac{1}{2j}\, e^{j(-1)2\pi t}$$

$x(k) = \frac{1}{2j} \qquad k = 1$

$\quad = \frac{-1}{2j} \qquad k = -1$

⑧ $x(t) = 2\cos\frac{2\pi}{3}t$

$\omega_0 = \frac{2\pi}{3}$

$$= \cancel{2}\left[\frac{e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t}}{\cancel{2}}\right]$$

$$x(t) = 1\, e^{j\overset{k}{1}\overset{\omega_0}{\frac{2\pi}{3}} t} + 1\, e^{j\overset{k}{(-1)}\overset{\omega_0}{\frac{2\pi}{3}} t}$$

$X(k) = 1 \quad k = 1$

$1 \quad k = -1$

(9) $x(t) = \sin 2\pi t + \cos 3\pi t$

$\omega_1 = 2\pi \qquad \omega_2 = 3\pi$

$T = \frac{2\pi}{\omega}$

$T_1 = \frac{2\pi}{2\pi} = 1 \qquad T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$

$T = \frac{2\pi}{\omega}$

$\frac{T_1}{T_2}$

$\frac{T_1}{T_2} = \frac{1}{2/3} = \frac{3}{2}$

$T \quad \omega = \frac{2\pi}{T}$

$2T_1 = 3T_2$

$2 \times 1 = 3 \times \frac{2}{3}$

$\boxed{T = 2}$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$\boxed{\omega_0 = \pi}$

$$x(k) = \frac{e^{j2\pi t} - e^{-j2\pi t}}{2j} + \frac{e^{j3\pi t} + e^{-j3\pi t}}{2}$$

$$= \frac{1}{2j} e^{j\overset{k}{(2)}\overset{\omega_0}{\pi} t} - \frac{1}{2j} e^{j\overset{k}{(-2)}\overset{\omega_0}{\pi} t} + \frac{1}{2} e^{j\overset{k}{(3)}\overset{\omega_0}{\pi} t} + \frac{1}{2} e^{j\overset{k}{(-3)}\overset{\omega_0}{\pi} t}$$

$x(k) = \frac{1}{2j} \quad k=2$

$-\frac{1}{2j} \quad k=2$

$\frac{1}{2} \quad k=3,-3$

(10) $x(t) = \cos(4t) + \sin(8t)$

$\omega_1 = 4$ $\quad \omega_2 = 8$

$T_2 = \frac{2\pi}{\omega_2} = \frac{\cancel{2}\pi}{\cancel{8}4} = \frac{\pi}{4}$

$T_1 = \frac{2\pi}{\omega} = \frac{\cancel{2}\pi}{\cancel{4}2} = \pi/2$

$\frac{T_1}{T_2} = \frac{\pi/2}{\pi/4} = \frac{\cancel{\pi}}{\cancel{2}} \times \frac{4}{\cancel{\pi}} = 2$

$T = \frac{2\pi}{\omega}$

$\frac{T_1}{T_2} = \frac{2}{1} \quad T_1 = 2T_2$

$\boxed{T = \frac{\pi}{2}}$

$\frac{2\pi}{T} = \frac{2\pi}{\pi/2} = 4$

$\boxed{\omega = 4}$

$x(t) = \frac{e^{j4t} + e^{-j4t}}{2} + \frac{e^{j8t} - e^{-j8t}}{2j}$

$x(t) = \frac{1}{2} e^{j4t} + \frac{1}{2} e^{-j4t} + \frac{e^{j8t}}{2j} - \frac{1}{2j} e^{-j8t}$

$e^{jk\omega_0 t}$

$x(k) = \frac{1}{2} \quad k=1$

$\frac{1}{2} \quad k=-1$

$\frac{1}{2j} \quad k=2$

$-\frac{1}{2j} \quad k=-2$

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Discrete time fourier series (DTFS)

Discrete & periodic $x(n) \xrightarrow{DTFS} X(k)$

DTFS

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jk\Omega_0 n}$$

IDTFS

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{jk\Omega_0 n}$$

① Find DTFS co-eff $X(k)$

i) $x(n) = \cos \frac{\pi}{3} n$

$\Omega_0 = \frac{\pi}{3}$

$$= \frac{e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}}{2}$$

$$x(n) = \frac{1}{2} e^{j(1)\frac{\pi}{3}n} + \frac{1}{2} e^{j(-1)\frac{\pi}{3}n}$$

$X(k) = \frac{1}{2} \quad k = 1, -1$

② $x(n) = \sin 3\pi n$

$\Omega_0 = 3\pi$

$$= \frac{e^{j3\pi n} - e^{-j3\pi n}}{2j}$$

$$x(n) = \frac{1}{2j} e^{j(3\pi)n(1)} - \frac{1}{2j} e^{j3\pi(-1)n}$$

$X(k) = \frac{1}{2j} \quad k = 1$

$= -\frac{1}{2j} \quad k = -1$

(3) $x(n) = \cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)$

$= \cancel{\cos\frac{2\pi}{3}n + \cos\frac{\pi}{3}}$

$\Omega_0 = \frac{2\pi}{3}$

$= \dfrac{e^{j\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)} + e^{-j\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)}}{2}$

$= \frac{1}{2} e^{j\frac{2\pi}{3}n} e^{j\pi/3} + \frac{1}{2} e^{-j\frac{2\pi}{3}n} e^{-j\pi/3}$

$= \frac{1}{2} e^{j\pi/3} e^{j(1)\frac{2\pi}{3}n} + \frac{1}{2} e^{-j\pi/3} e^{j(-1)\frac{2\pi}{3}n}$

$x(k) = \frac{1}{2} e^{j\pi/3} \quad k = 1$

$= \frac{1}{2} e^{-j\pi/3} \quad k = -1$

(4) $x(n) = \sin\frac{2\pi}{3}n + \cos\frac{\pi}{3}n$

$\Omega_1 = \frac{2\pi}{3} \qquad \Omega_2 = \frac{\pi}{3}$

$\frac{m}{N} = \frac{\Omega}{2\pi}$

$\frac{m}{N_1} = \frac{2\pi}{3 \cdot 2\pi} = \frac{1}{3}$

$\frac{m}{N_2} = \frac{\pi}{3 \cdot 2\pi} = \frac{1}{6}$

$N_1 = 3 \qquad N_2 = 6$

$\frac{N_1}{N_2} = \frac{3}{6} = \frac{1}{2} \qquad 2N_1 = N_2$

$2 \times 3 = 6$

$$\boxed{N=6}$$

$$\Omega = \frac{2\pi}{N} = \frac{2\pi}{6_3} = \frac{\pi}{3}$$

$$\boxed{\Omega = \frac{\pi}{3}}$$

$$= \frac{e^{j\frac{2\pi}{3}n} - e^{-j\frac{2\pi}{3}n}}{2j} + \frac{e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}}{2}$$

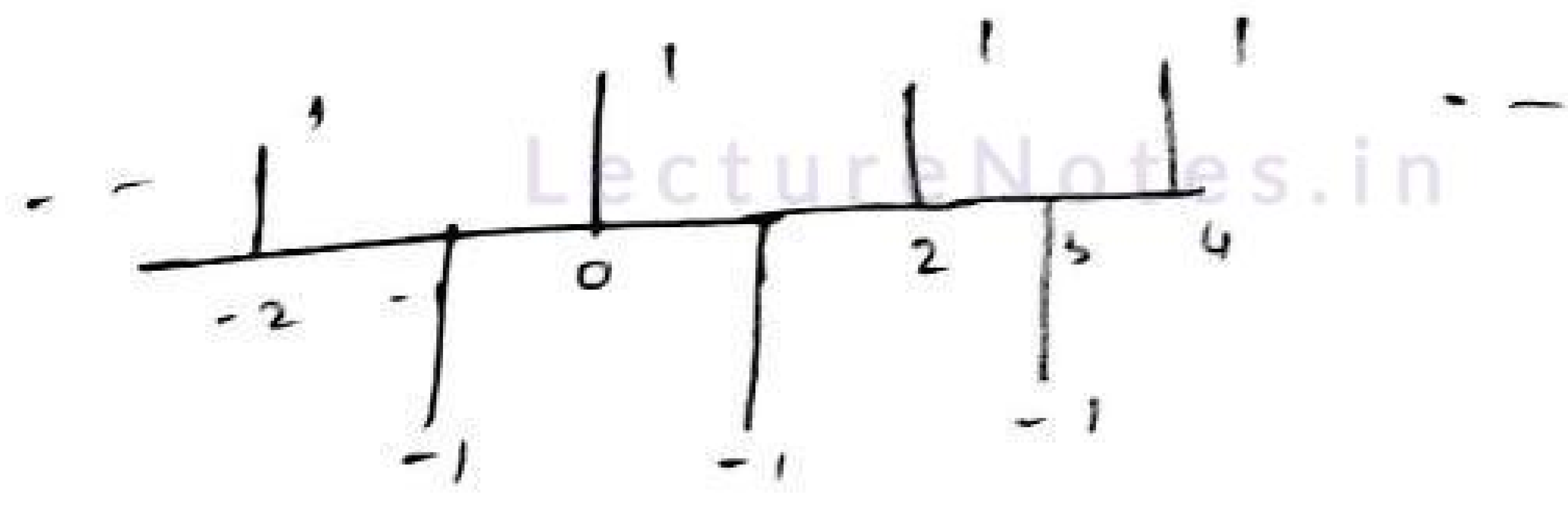
$$= \frac{1}{2j} e^{j(2)\frac{\pi}{3}n} - \frac{1}{2j} e^{j(-2)\frac{\pi}{3}n} + \frac{1}{2} e^{j(1)\frac{\pi}{3}n} + \frac{1}{2} e^{j(-1)\frac{\pi}{3}}$$

$$x(k) = \frac{1}{2j} \quad k=2$$

$$= \frac{1}{-2j} \quad k=-2$$

$$\frac{1}{2} \quad k=1,-1$$

(5) $x(n) = (-1)^n$



$N=2$ $\qquad \Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{2} = \pi$

$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(k)\, e^{-jk\Omega_0 n}$$

$$= \frac{1}{2}\sum_{n=0}^{1} x(n)\, e^{-jk\pi n}$$

$$n=0 \qquad\qquad n=1$$

$$= \frac{1}{2}\left[x(0)\,e^{-jk\pi 0} + x(1)\,e^{-jk\pi(1)}\right]$$

$$= \frac{1}{2}\left[1 \times 1 + (-1)\,e^{-jk\pi}\right]$$

$$x(k) = \frac{1}{2}\left[1 - e^{-jk\pi}\right]$$

Module-4

12/5/17

CTFT & DTFT

DTFT → Discrete Non-periodic Signal and it converts to spectrum [frequency domain

DTFT $$\boxed{X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}}$$

IDTFT $$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, e^{j\Omega n}\, d\Omega$$

① $x(n) = a^n u(n)$ find DTFT & Magnitude & phase

→ $$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n)\, e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} a^n\, e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} (a e^{-j\Omega})^n$$

$$= \frac{(a e^{-j\Omega})^0}{1 - a e^{-j\Omega}} = \frac{1}{1 - a e^{-j\Omega}}$$

$$\boxed{X(e^{j\Omega}) = \frac{1}{1 - a e^{-j\Omega}}}$$

$$= \frac{1}{1 - a(\cos\Omega - j\sin\Omega)}$$

$$= \frac{1}{(1 - a\cos\Omega) + j(a\sin\Omega)}$$

$$|X(e^{j\Omega})| = \frac{1}{\sqrt{(1-a\cos\Omega)^2 + (a\sin\Omega)^2}}$$ → Magnitude

$$\angle X(e^{j\Omega}) = 0 - \tan^{-1}\left(\frac{a\sin\Omega}{1-a\cos\Omega}\right)$$

$$\angle X(e^{j\Omega}) = -\tan^{-1}\left(\frac{a\sin\Omega}{1-a\cos\Omega}\right)$$ → phase

(2) $x(n) = 2^n u(n-3)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u(n-3)\, e^{-j\Omega n}$$

$n-3 = 0, \infty$

$n = 3, \infty$

$$= \sum_{n=3}^{\infty} 2^n e^{-j\Omega n}$$

$$= \sum_{n=3}^{\infty} (2e^{-j\Omega})^n$$

$$X(e^{j\Omega}) = \frac{(2e^{-j\Omega})^3}{1 - 2e^{-j\Omega}}$$

$$= \frac{8\,e^{-3j\Omega}}{1 - 2(\cos\Omega - j\sin\Omega)}$$

$$= \frac{8\,e^{-3j\Omega}}{1 - 2\cos\Omega + 2j\sin\Omega}$$

$$= \frac{8\,\overbrace{[\cos 3\Omega - j\sin 3\Omega]}^{e^{-3j\Omega}}}{1 - 2\cos\Omega + 2j\sin\Omega}$$

$$X(e^{j\Omega}) = \frac{\overbrace{8\cos 3\Omega - 8j\sin 3\Omega}^{8\,e^{-3j\Omega}}}{(1 - 2\cos\Omega) + 2j\sin\Omega}$$

$$|X(e^{j\Omega})| = \frac{\sqrt{(8\cos 3\Omega)^2 - (8\sin 3\Omega)^2}\ \ 8}{\sqrt{(1 - 2\cos\Omega)^2 + (2\sin\Omega)^2}}$$

$$\angle X(e^{j\Omega}) = \overbrace{\tan^{-1}\left(\frac{8\sin 3\Omega}{8\cos 3\Omega}\right)}^{-3\Omega} - \tan^{-1}\left(\frac{2\sin\Omega}{1 - 2\cos\Omega}\right)$$

3. $x(n) = \left(\frac{1}{3}\right)^{n+1} u(n+1)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right) u(n+1)\, e^{-j\Omega n}$$

$n+1 ; 0, \infty$

$n ; -1, \infty$

$$= \frac{1}{3} \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n+1)\, e^{-j\Omega n}$$

$$= \frac{1}{3} \sum_{n=-1}^{\infty} \left(\frac{1}{3}\right)^n e^{-j\Omega n}$$

$$= \frac{1}{3} \sum_{n=-1}^{\infty} \left(\frac{1}{3} e^{-j\Omega}\right)^n$$

$$= \frac{1}{3} \left[\frac{\left(\frac{1}{3} e^{-j\Omega}\right)^{-1}}{1 - \frac{1}{3} e^{-j\Omega}} \right]$$

$$X(e^{j\Omega}) = \frac{\frac{1}{9} e^{j\Omega}}{1 - \frac{1}{3} e^{-j\Omega}}$$

$$= \frac{\frac{1}{9} e^{j\Omega}}{1 - \frac{1}{3}\left[\cos\Omega - j\sin\Omega\right]}$$

$$= \frac{\frac{1}{9} e^{j\Omega}}{1 - \frac{1}{3}\cos\Omega + \frac{1}{3} j \sin\Omega}$$

$$|X(e^{j\Omega})| = \frac{\frac{1}{7}}{\sqrt{\left(1 - \frac{1}{3}\cos\Omega\right)^2 + \left(\frac{1}{3}\sin\Omega\right)^2}} \rightarrow \text{Mag}$$

$$\angle X(e^{j\Omega}) = \Omega - \tan^{-1}\left(\frac{\frac{1}{3}\sin\Omega}{1 - \frac{1}{3}\cos\Omega}\right) \rightarrow \text{phase}$$

(4) $x(n) = 3^n u(-n)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} 3^n u(-n)\, e^{-j\Omega n}$$

$-n : 0, \infty$

$n : 0, -\infty$

$$= \sum_{n=0}^{-\infty} 3^n e^{-j\Omega n}$$

$n \to -n$

$$= \sum_{n=0}^{\infty} 3^{-n} e^{j\Omega n}$$

$$\sum_{n=0}^{\infty} \left(3^{-1} e^{j\Omega}\right)^n$$

$$X(e^{j\Omega}) = \frac{1}{1 - \frac{1}{3} e^{j\Omega}}$$

$$\cancel{|X(e^{j\Omega})|} = \frac{1}{1 - \frac{1}{3}\left[\cos\Omega + j\sin\Omega\right]}$$

$$|X(e^{j\Omega})| = \frac{1}{\sqrt{\left(1 - \frac{1}{3}\cos\Omega\right)^2 + \left(\frac{1}{3}\sin\Omega\right)^2}}$$

$$\angle X(e^{j\Omega}) = 0 - \tan^{-1}\left(\frac{\frac{1}{3}\sin\Omega}{1 - \frac{1}{3}\cos\Omega}\right)$$

(5) $x(n) = a^{|n|}$

$|n| \to n$ +ve   0 to ∞ $u(n)$

$|n| \to -n$ -ve $-\infty$ to -1 $u(-n-1)$

$$x(n) = a^n u(n) + a^{-n} u(-n-1)$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\Omega n} + \sum_{n=1}^{-\infty} a^{-n} e^{-j\Omega n}$$

$n \to -n$

$-n-1$; 0, ∞

$-n$; -1, ∞

n ; 1, $-\infty$

$$\sum_{n=0}^{\infty} (a e^{-j\Omega})^n + \sum_{n=1}^{\infty} a^n e^{j\Omega n}$$

$(a e^{j\Omega})^n$

$$X(e^{j\Omega}) = \frac{1}{1 - ae^{-j\Omega}} + \frac{a e^{j\Omega}}{1 - ae^{j\Omega}}$$

$$\frac{(1 - ae^{j\Omega}) + ae^{j\Omega}(1 - ae^{-j\Omega})}{(1 - ae^{-j\Omega})(1 - ae^{j\Omega})}$$

(6) $x(n) = u(n) - u(n-8)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} (u(n) - u(n-8))\, e^{-j\Omega n}$$

$$= \sum_{n=0}^{7} e^{-j\Omega n}$$

$$\sum_{n=0}^{7} \left(e^{-j\Omega}\right)^n$$

$$X(e^{j\Omega}) = \frac{1 - \left(e^{-j\Omega}\right)^8}{1 - e^{-j\Omega}}$$

$$= \frac{1 - \left(e^{-j\Omega}\right)^8}{1 - (\cos\Omega - j\sin\Omega)}$$

$$\left|X(e^{j\Omega})\right| = \frac{1}{\sqrt{(1-\cos\Omega)^2 + (\sin\Omega)^2}}$$

$$\angle X(e^{j\omega}) = -8\Omega - \tan^{-1}\frac{(\sin\Omega)}{(1-\cos\Omega)}$$

(7) $x(n) = 5\delta(n)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} 5\delta(n)\, e^{-j\Omega n}$$

at $n = 0$

$$= 5$$

(8) $x(n) = 3\delta(n-2)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty} 3\delta(n-2)\, e^{-j\Omega n}$$

at $n = 2$

$= 3 e^{-2j\Omega}$

(9) $x(n) = 4\delta(n-1) + 3\delta(n) + 5\delta(n+2)$

$= \sum_{n=-\infty}^{\infty} 4\delta(n-1) e^{-j\Omega n} + \sum_{n=-\infty}^{\infty} 3\delta(n) e^{-j\Omega n} +$

at $n=1$ at $n=0$

$\sum_{n=-\infty}^{\infty} 5\delta(n+2) e^{-j\Omega n}$

at $n = -2$

$X(e^{j\Omega}) = 4 e^{-j\Omega} + 3 + 5 e^{2j\Omega}$

(10) $x(n) = \{ \underset{\uparrow}{3}, 5, -2 \}$

$= 3\delta(n) + 5\delta(n-1) - 2\delta(n-2)$

$X(e^{j\Omega}) = 3 + 5 e^{-j\Omega} - 2 e^{-2j\Omega}$

Properties of DTFT

$x(n) \xrightarrow{DTFT} X(e^{j\Omega})$

Time shift

$x(n-n_0) \rightarrow e^{-j\Omega n_0} X(e^{j\Omega})$

frequency shift

$e^{j\Omega_0 n} x(n) \rightarrow X(e^{j(\Omega - \Omega_0)})$

Differentiation

$-jn\, x(n) \rightarrow \frac{d}{d\Omega} X(e^{j\Omega})$

Scaling

$$x(an) \rightarrow X\left(e^{\frac{j\Omega}{a}}\right)$$

Convolution

$$x_1(t) * x_2(n) \rightarrow X_1(e^{j\Omega}) \cdot X_2(e^{j\Omega})$$

$$DTFT\left[a^n u(n)\right] = \frac{1}{1 - ae^{-j\Omega}}$$

Problem using property

① $x(n) = 2^n u(n)$

$$DTFT\left[a^n u(n)\right] = \frac{1}{1 - ae^{-j\Omega}}$$

$$DTFT\left[2^n u(n)\right] = \frac{1}{1 - 2e^{-j\Omega}}$$

② $x(n) = 2^{n-3} u(n-3)$

$$DTFT\left[a^n u(n)\right] = \frac{1}{1 - ae^{-j\Omega}}$$

$$DTFT\left[2^n u(n)\right] = \frac{1}{1 - 2e^{-j\Omega}}$$

$$DTFT\left[2^{n-3} u(n-3)\right] = e^{-j\Omega 3} \cdot \frac{1}{1 - 2e^{-j\Omega 2}}$$

③ $x(n) = \left(\frac{1}{2}\right)^n u(n+1)$

$$= \left(\frac{1}{2}\right)^{n+1} \left(\frac{1}{2}\right)^{-1} u(n+1)$$

$$DTFT\left[a^n u(n)\right] = \frac{1}{1 - ae^{-j\Omega}}$$

$$DTFT\left[\left(\frac{1}{2}\right)^n u(n)\right] = \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

$$DTFT\left[\left(\frac{1}{2}\right)^{n+1} u(n+1)\right] = e^{j\Omega}\frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

$$DTFT\left[\left(\frac{1}{2}\right)^{-1}\left(\frac{1}{2}\right)^{n+1} u(n+1)\right] = \frac{2e^{j\Omega}}{1-\frac{1}{2}e^{-j\Omega}}$$

(4) $x(n) = na^n u(n)$

$$DTFT\left[a^n u(n)\right] = \frac{1}{1-ae^{-j\Omega}}$$

$$DTFT\left[-jn\, a^n u(n)\right] = \frac{d}{d\Omega}\left[\frac{1}{1-ae^{-j\Omega}}\right]$$

$$= \frac{(1-ae^{-j\Omega})0 - (1)(jae^{-j\Omega})}{(1-ae^{-j\Omega})^2}$$

$$DTFT\left[\cancel{-j}n\, a^n u(n)\right] = \frac{\cancel{-j}ae^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$

$$DTFT\left[na^n u(n)\right] = \frac{ae^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$

(5) $x(n) = e^{-j2n}(3)^n u(n)$

$$DTFT\left[3^n u(n)\right] = \frac{1}{1-3e^{-j\Omega}}$$

$$DTFT\left[e^{-j2\Omega}(3)^n u(n) \right] = \frac{1}{1-3e^{-j(\Omega+2)}}$$

⑥ $x(n) = \cos\frac{\pi}{3}n(2)^n u(n)$

→

$$= \left[\frac{e^{j\pi/3 n} + e^{-j\pi/3 n}}{2}\right] 2^n u(n)$$

$$x(n) = \frac{1}{2} e^{j\pi/3 n} 2^n u(n) + \frac{1}{2} e^{-j\pi/3 n} 2^n u(n)$$

$$DTFT\left(2^n u(n)\right] = \frac{1}{1-2e^{-j\Omega}}$$

$$DTFT\left[\cancel{\tfrac{1}{2}} e^{j\pi/3 n} 2^n u(n)\right] = \frac{\cancel{\tfrac{1}{2}}}{1-2e^{-j(\Omega-\pi/3)}}$$

$$DTFT\left[\frac{1}{2} e^{j\pi/3 n} 2^n u(n)\right] = \frac{\frac{1}{2}}{1-2e^{-j(\Omega-\pi/3)}}$$

$$DTFT\left[\frac{1}{2} e^{-j\pi/3 n} 2^n u(n)\right] = \frac{1/2}{1-2e^{-j(\Omega+\pi/3)}}$$

$$X(e^{j\Omega}) = \frac{\frac{1}{2}}{1-2e^{-j(\Omega-\pi/3)}} + \frac{\frac{1}{2}}{1-2e^{-j(\Omega+\pi/3)}}$$

⑦ $x(n) = 2j \sin\frac{\pi}{4}n \, (3)^n u(n)$

$$= \cancel{2j}\left[\frac{e^{j\pi/4 n} - e^{-j\pi/4 n}}{\cancel{2j}}\right] 3^n u(n)$$

$$x(n) = 3^n e^{j\pi/4 n} u(n) - 3^n e^{-j\pi/4 n} u(n)$$

$$DTFT\left(3^n u(n)\right) = \frac{1}{1 - 3e^{-j\Omega}}$$

$$DTFT\left[3^n e^{j\pi/4 n} u(n)\right] = \frac{1}{1 - 3e^{-j(\Omega - \pi/4)}}$$

$$DTFT\left[3^n e^{-j\pi/4 n} u(n)\right] = \frac{1}{1 - 3e^{-j(\Omega + \pi/4)}}$$

$$X(e^{j\Omega}) = \frac{1}{1 - 3e^{-j(\Omega - \pi/4)}} - \frac{1}{1 - 3e^{-j(\Omega + \pi/4)}}$$

⑧ Consider signal $x(n)$ and its DTFT $X(e^{j\Omega}) = \frac{1}{1 - ae^{-j\Omega}}$ then find DTFT of the following

ⓐ $x(n+2)$

ⓑ $x(2n)$

ⓒ $x(n) * x(n-1)$

ⓓ $e^{j\pi/2 n} x(n+1)$

→ $DTFT[x(n)] = X(e^{j\Omega})$

ⓐ $DTFT[x(n)] = \frac{1}{1 - ae^{-j\Omega}}$

$DTFT[x(n+2)] = \frac{e^{j\Omega 2}}{1 - ae^{-j\Omega}}$

(b) $x(2n)$

$$DTFT[x(n)] = \frac{1}{1-ae^{-j\Omega}}$$

$$DTFT[x(2n)] = \frac{1}{1-ae^{-j\Omega/2}}$$

(c) $x(n) * x(n-1)$.

$$DTFT[x(n)] = \frac{1}{1-ae^{-j\Omega}}$$

$$DTFT[x(n-1)] = \frac{e^{-j\Omega}}{1-ae^{-j\Omega}}$$

$$X(e^{j\Omega}) = \frac{1}{1-ae^{-j\Omega}} \cdot \frac{e^{-j\Omega}}{1-ae^{-j\Omega}}$$

$$= \frac{e^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$

(d) $e^{j\pi/2 n} x(n+1)$

$$DTFT[x(n+1)] = \frac{e^{j\Omega}}{1-ae^{-j\Omega}}$$

$$DTFT[e^{j\pi/2 n} x(n+1)] = \frac{e^{+j(\Omega-\pi/2)}}{1-ae^{-j(\Omega-\pi/2)}}$$

9) $X(e^{j\Omega}) = \dfrac{e^{j2\Omega}}{1-\frac{1}{3}e^{-j\Omega}}$ find inverse

$IDTFT\left[\dfrac{1}{1-\frac{1}{3}e^{-j\Omega}}\right] = \left(\frac{1}{3}\right)^n u(n)$

$IDTFT\left[\dfrac{e^{j2\Omega}}{1-\frac{1}{3}e^{-j\Omega}}\right] = \left(\frac{1}{3}\right)^{n+2} u(n+2)$

10) $X(e^{j\Omega}) = \dfrac{3}{1-2e^{-j(\Omega-\pi/2)}}$

$IDTFT\left[\dfrac{1}{1-2e^{-j\Omega}}\right] = 2^n u(n)$

$IDTFT\left[\dfrac{1}{1-2e^{-j(\Omega-\pi/2)}}\right] = e^{j\pi/2 n}\, 2^n u(n)$

$IDTFT\left[\dfrac{3}{1-2e^{-j(\Omega-\pi/2)}}\right] = 3e^{j\pi/2 n}\, 2^n u(n)$

11) $X(e^{j\Omega}) = \dfrac{6}{e^{-j2\Omega} - 5e^{-j\Omega} + 6}$ find IDTFT

Put $e^{-j\Omega} = K$.

$= \dfrac{6}{K^2 - 5K + 6} = \dfrac{6}{(K-3)(K-2)}$

$= \dfrac{A}{K-3} + \dfrac{B}{K-2}$

$6 = A(K-2) + B(K-3)$

$k = 2$

$$6 = -B$$

$$\boxed{B = -6}$$

Put $k = 3$

$$\boxed{A = 6}$$

$$= \frac{6}{k-3} + \frac{-6}{k-2}$$

$$= \frac{-6}{3-k} + \frac{6}{2-k}$$

$$= \frac{-6}{3 - e^{-j\Omega}} + \frac{6}{2 - e^{-j\Omega}}$$

by $\frac{1}{3}$ by $\frac{1}{2}$

~~IDTFT~~ $= \frac{-\frac{6}{3}}{1 - \frac{1}{3} e^{-j\Omega}} + \frac{\frac{6}{2}}{1 - \frac{1}{2} e^{-j\Omega}}$

$\downarrow$ IDTFT

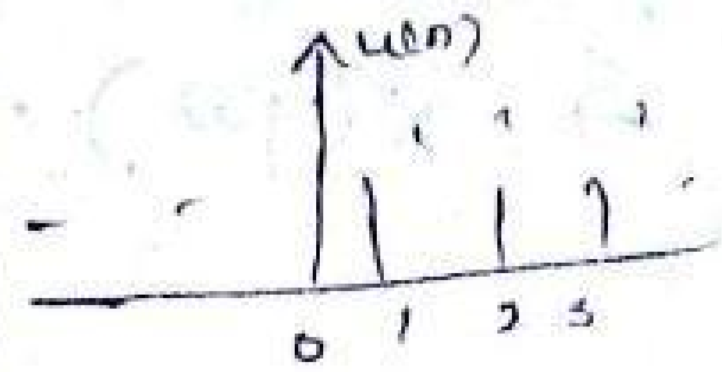
IDTFT $= -\frac{6}{3}\left(\frac{1}{3}\right)^n u(n) + \frac{6}{2}\left(\frac{1}{2}\right)^n u(n)$

$$x(n) = -2\left(\frac{1}{3}\right)^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$$

(12) $X(e^{j\Omega}) = \frac{1}{(1 - ae^{-j\Omega})^2}$ IDTFT = ? using Convolution property

$$X(e^{j\Omega}) = \frac{1}{1 - ae^{-j\Omega}} \cdot \frac{1}{1 - ae^{-j\Omega}}$$

$x(n) = a^n u(n) * a^n u(n)$



$u(3) = 1$

$u(5) = 1$

$u(25) = 1$

$u(-2) = 0$

$x(n) = x_1(n) * x_2(n)$

$$x(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k) \quad a^{n-k} u(n-k)$$

$k \geq 0$ | $n-k \geq 0$, $n \geq k$, $k \leq n$

$$= \sum_{k=0}^{n} a^k a^{n-k}$$

$$= a^k a^n a^{-k}$$

$$= \sum_{k=0}^{n} a^n = a^n \sum_{k=0}^{n} 1 = \underline{a^n(n+1)}$$

CTFT / FT

Continuous & N.P $\quad x(t) \xrightarrow{FT} X(j\omega)$

FT
$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

IFT
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

(1) $x(t) = e^{-at}u(t)$ find Fourier Transform and Calculate its Magnitude & phase.

$$\rightarrow X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at}\, u(t)\, e^{-j\omega t}\, dt$$

$$\int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$\left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty} = \frac{e^{-\infty} - e^{0}}{-(a+j\omega)} = \frac{-1}{-(a+j\omega)}$$

$$\boxed{X(j\omega) = \frac{1}{a+j\omega}}$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\angle X(j\omega) : 0 - \tan^{-1}\left(\frac{\omega}{a}\right)$$

NOTE

$$a^n u(n) \xrightarrow{ZT} \frac{z}{z-a}$$

$$a^n u(n) \xrightarrow{DTFT} \frac{1}{1-a e^{-j\Omega}}$$

$$e^{-at} u(t) \xrightarrow{FT} \frac{1}{a+j\omega}$$

② $x(t) = u(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^\infty e^{-j\omega t}\, dt$$

$$\frac{e^{-j\omega t}}{-j\omega}\Bigg|_0^\infty$$

$$\frac{-1}{j\omega}\left[e^{-\infty} - e^{0}\right]$$

$$x(j\omega) = +\frac{1}{j\omega}$$

$$|x(j\omega)| = \frac{1}{\sqrt{\omega^2}} = \frac{1}{\omega}$$

$$\angle x(j\omega) = 0 - \tan^{-1}\left(\frac{\omega}{0}\right) = -90^\circ$$

③ $x(t) = 2u(t-3)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} 2u(t-3)\, e^{-j\omega t}\, dt \qquad t-3$$

$$= \int_3^\infty 2\, e^{-j\omega t}\, dt$$

$$= 2\,\frac{e^{-j\omega t}}{-j\omega}\Bigg|_3^\infty$$

$$= \frac{-2}{j\omega}\left[e^{-\infty} - e^{-j\omega 3}\right]$$

$$= \frac{-2}{j\omega}\left[-\left(\cos 3\omega - j\sin 3\omega\right)\right]$$

$$= \frac{2}{j\omega}\cos 3\omega - \frac{2}{j\omega}\sin 3\omega \Rightarrow \boxed{\frac{2e^{-3j\omega}}{j\omega}}$$

(4) $x(t) = t u(t)$

$$= \int_{-\infty}^{\infty} t u(t) e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} t \, e^{-j\omega t} dt$$

$$\left[t \frac{e^{-j\omega t}}{-j\omega} - 1 \cdot \frac{e^{-j\omega t}}{-\omega^2} \right|_0^{\infty}$$

$$[0 - 0] - \left[0 - \frac{1}{-\omega^2} \right]$$

$$X(e^{j\omega}) = -\frac{1}{\omega^2}$$

(5) $x(t) = e^{-a|t|}$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} dt$$

$e^{-a|t|} \rightarrow e^{-at}$ (0 to ∞)

$e^{-a|t|} \rightarrow e^{at}$ ($-\infty$ to 0)

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t} dt + \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{e^{(a-j\omega)}}{a-j\omega} \Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)}}{-(a+j\omega)} \Bigg|_{0}^{\infty}$$

$$= \frac{1}{a-j\omega}\left[1-0\right] + \frac{1}{(a+j\omega)}\left[0-1\right]$$

$$= \frac{1}{a-j\omega} + \frac{1}{(a+j\omega)} = \frac{2a}{a^2+\omega^2}$$

(c) $x(t) = A \qquad -\frac{T}{2} \le t \le \frac{T}{2}$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{T/2} A\, e^{-j\omega t}\, dt$$

$$= A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T/2}^{T/2}$$

$$= \frac{A}{-j\omega}\left[e^{-j\omega T/2} - e^{j\omega T/2}\right]$$

$$= 2A\left[\frac{-e^{-j\omega T/2} + e^{j\omega T/2}}{2j\omega}\right]$$

$$x(j\omega) = \frac{2A\sin\frac{T}{2}\omega}{\omega}$$

(7) $x(t) = te^{-at}u(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} t\, e^{-at} e^{-j\omega t} u(t)\, dt$$

$$= \int_{0}^{\infty} t\, e^{-(a+j\omega)t}\, dt$$

$$= t\,\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} - \frac{e^{-(a+j\omega)}}{(a+j\omega)^2}\Bigg|_0^{\infty}$$

$$= [0-0] - \left[0 - \frac{1}{(a+j\omega)^2}\right]$$

$$X(j\omega) = \frac{1}{(a+j\omega)^2}$$

(8) $x(t) = 3\delta(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} 3\delta(t)\, e^{-j\omega t}\, dt$$

at 0

$$3\delta(0)\, e^{-j(0)}$$

$$X(j\omega) = 3$$

9) $x(t) = 2\delta(t-3)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} 2\,\delta(t-3)\, e^{-j\omega t}\, dt$$

at $t = 3$

$$2\,\delta(0)\, e^{-j3\omega}$$

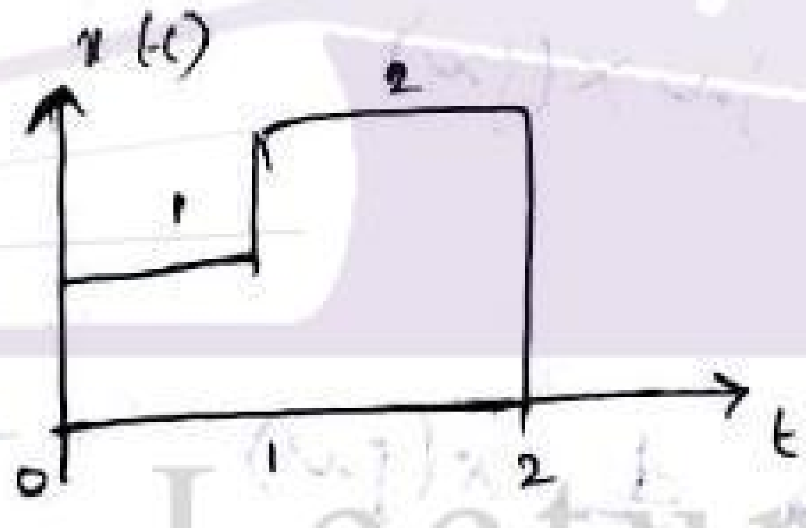
$$X(j\omega) = \underline{2\, e^{-j3\omega}}$$

(10) $x(t) = 2\delta(t+1) - 3\delta(t) + 4\delta(t-2)$

$\downarrow$ F.T

$$X(j\omega) = 2\, e^{j\omega 1} - 3 + 4\, e^{-2\omega j}$$

(11)



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^1 1\, e^{-j\omega t}\, dt + \int_1^2 2\, e^{-j\omega t}\, dt$$

$$\frac{e^{-j\omega t}}{-j\omega}\Big|_0^1 + 2\,\frac{e^{-j\omega t}}{-j\omega}\Big|_1^2$$

$$\frac{1}{-j\omega}\left[e^{-j\omega} - 1\right] + \frac{2}{-j\omega}\left[e^{-j\omega 2} - e^{-j\omega 1}\right]$$

$$-\frac{e^{-j\omega}}{j\omega} + \frac{1}{j\omega} - \frac{2e^{-j\omega 2}}{j\omega} + \frac{2e^{-j\omega 1}}{j\omega}$$

$$\frac{1}{j\omega} - \frac{2e^{-j\omega 2}}{j\omega} + \frac{e^{-j\omega}}{j\omega}$$

$$X(j\omega) = \frac{1 - 2e^{-j\omega 2} + e^{-j\omega}}{j\omega}$$

Properties of FT.

1) Time shift

$$x(t) \rightarrow X(j\omega)$$

$$x(t - t_0) \rightarrow e^{-j\omega t_0} X(j\omega)$$

2) Frequency shift

$$e^{j\omega t} x(t) \rightarrow X j(\omega - \omega_0)$$

3) Time differentiation

$$\frac{d}{dt} x(t) \rightarrow j\omega X(j\omega)$$

4) Frequency differentiation

$$-jt\, x(t) \rightarrow \frac{d}{d\omega} X(j\omega)$$

$$FT[u(t)] = \frac{1}{j\omega}$$

$$FT[e^{-at} u(t)] = \frac{1}{a+j\omega}$$

Problems using properties

(1) $x(t) = 4e^{-2t} u(t)$

$$FT[e^{-2t} u(t)] = \frac{1}{2 + j\omega}$$

$$FT[4e^{-2t} u(t)] = \frac{4}{2 + j\omega}$$

(2) $x(t) = e^{-2(t-3)} u(t-3)$

$$FT\left[e^{-2t} u(t)\right] = \frac{1}{2+j\omega}$$

$$FT\left[e^{-2(t-3)} u(t-3)\right] = \frac{e^{-j\omega 3}}{2+j\omega}$$

(3) $x(t) = e^{j\pi/3 t} e^{-2(t-3)} u(t-3)$

$$FT\left[e^{-2t} u(t)\right] = \frac{1}{2+j\omega}$$

$$FT\left[e^{-2(t-3)} u(t-3)\right] = \frac{e^{-j3\omega}}{2+j\omega}$$

$$FT\left[e^{j\pi/3 t} e^{-2(t-3)} u(t-3)\right] = \frac{e^{-j3(\omega-\pi/3)}}{2+j(\omega-\pi/3)}$$

(4) $x(t) = \cos\frac{\pi}{2}t \, e^{-3t} u(t)$

$$= \left[\frac{e^{j\pi/2 t} + e^{-j\pi/2 t}}{2}\right] e^{-3t} u(t)$$

$$= \frac{1}{2} e^{j\pi/2 t} e^{-3t} u(t) + \frac{1}{2} e^{-j\pi/2 t} e^{-3t} u(t)$$

$$FT\left[e^{-3t} u(t)\right] = \frac{1}{3+j\omega}$$

$$FT\left[e^{j\pi/2 t} e^{-3t} u(t)\right] = \frac{1}{3+j(\omega-\pi/2)}$$

$$FT\left[\frac{1}{2} e^{j\pi/2 t} e^{-3t} u(t)\right] = \frac{1/2}{3+j(\omega-\pi/2)}$$

$$FT\left[e^{-j\pi/2 t} e^{-3t} u(t)\right] = \frac{1}{3+j(\omega+\pi/2)}$$

$$= \frac{\frac{1}{2}}{3+j(\omega-\pi/2)} + \frac{1/2}{3+j(\omega+\pi/2)}$$

$$= \frac{\frac{1}{2}\left[3+j\omega+j\cancel{\pi/2}+3+j\omega-j\cancel{\pi/2}\right)}{\left[3+j(\omega-\pi/2)\right]\left[3+j(\omega+\pi/2)\right]}$$

$$x(j\omega) = \frac{\frac{1}{2}\left[6+2j\omega\right]}{\left[3+j(\omega-\pi/2)\right]\left[3+j(\omega+\pi/2)\right]}$$

(5) $x(t) = te^{-at}u(t)$

$$FT\left[e^{-at}u(t)\right] = \frac{1}{a+j\omega}$$

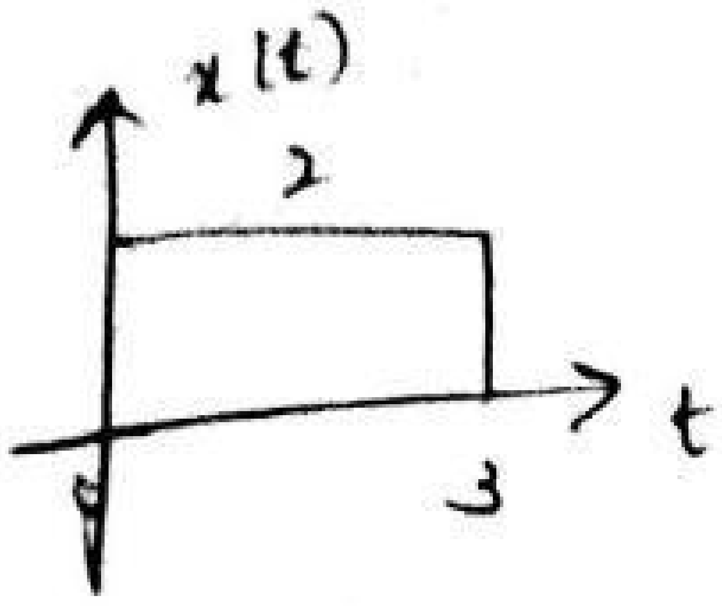
$$FT\left[jte^{-at}u(t)\right] = \frac{d}{d\omega}\left[\frac{1}{a+j\omega}\right]$$

$$= \frac{0-(1)j}{(a+j\omega)^2}$$

$$FT\left[\cancel{-j}te^{-at}u(t)\right] = \frac{\cancel{-j}}{(a+j\omega)^2}$$

$$FT\left[te^{-at}u(t)\right] = \frac{1}{(a+j\omega)^2}$$

(6)



$$x(t) = 2[u(t) + u(t-3)]$$

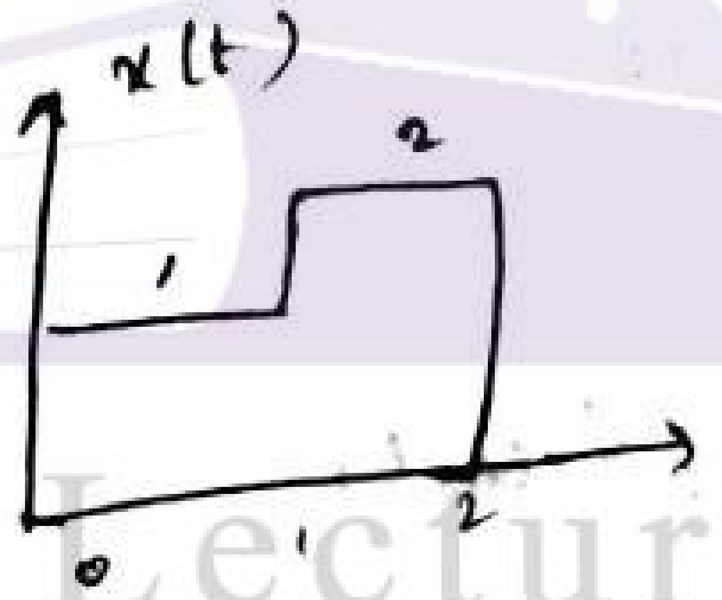
$$FT(u(t)) = \frac{1}{j\omega}$$

$$FT(u(t-3)) = \frac{e^{-j3\omega}}{j\omega}$$

$$FT[2(u(t) - u(t-3)) = 2\left[\frac{1}{j\omega} - \frac{e^{-j3\omega}}{j\omega}\right]$$

$$X(j\omega) = \frac{2(1 - e^{-j3\omega})}{j\omega}$$

(7)



$$x(t) = 1[u(t) - u(t-1)] + 2[u(t-1) - u(t-2)]$$

$\downarrow$ FT

$$= \frac{1}{j\omega} - \frac{e^{-j\omega}}{j\omega} + \frac{2e^{-j\omega}}{j\omega} - \frac{2e^{-j2\omega}}{j\omega}$$

$$= \frac{1 - e^{-j\omega} + 2e^{-j\omega} - 2e^{-j2\omega}}{j\omega}$$

$$= \frac{1 + e^{-j\omega} - 2e^{-j2\omega}}{j\omega}$$

(8) Find IFT

$$X(j\omega) = \frac{e^{-j2\omega}}{3+j\omega}$$

$$IFT\left[\frac{1}{3+j\omega}\right] = e^{-3t}u(t)$$

$$IFT\left[\frac{e^{-j2\omega}}{3+j\omega}\right] = e^{-3(t-2)}u(t-2)$$

(9) $$X(j\omega) = \frac{3}{3+j(\omega-\pi/2)}$$

$$IFT\left[\frac{1}{3+j\omega}\right] = e^{-3t}u(t)$$

$$IFT\left[\frac{1}{3+j(\omega-\pi/2)}\right] = e^{j\pi/2 t}e^{-3t}u(t)$$

$$IFT\left[\frac{3}{3+j(\omega-\pi/2)}\right] = 3e^{j\pi/2 t}e^{-3t}u(t)$$

(10) $$X(j\omega) = \frac{e^{-j4(\omega+\pi/2)}}{2+j(\omega+\pi/2)}$$

$$IFT\left[\frac{1}{2+j\omega}\right] = e^{-2t}u(t)$$

$$IFT\left[\frac{e^{-j4\omega}}{2+j\omega}\right] = e^{-2(t-4)}u(t-4)$$

$$IFT\left[\frac{e^{-j4(\omega+\pi/2)}}{2+j(\omega+\pi/2)}\right] = e^{-j\pi/2 t}e^{-2(t-4)}u(t-4)$$

(11) $x(j\omega) = \dfrac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$

$j\omega = k$

$= \dfrac{-k}{k^2 + 3k + 2}$

$= \dfrac{-k}{(k+1)(k+2)}$

$-k = \dfrac{A}{(k+1)} + \dfrac{B}{(k+2)}$

$-k = A(k+2) + B(k+1)$

Put $k = -2$ Put $k = -1$

$\boxed{1 = A}$

$2 = -B$

$\boxed{B = -2}$

$x(j\omega) = \dfrac{1}{k+1} + \dfrac{-2}{k+2}$

$x(j\omega) = \dfrac{1}{j\omega + 1} + \dfrac{-2}{j\omega + 2}$

$x(j\omega) = \dfrac{1}{1 + j\omega} + \dfrac{-2}{2 + j\omega}$

$IFT\left[\dfrac{1}{1+j\omega}\right] = e^{-t}u(t)$

$IFT\left[\dfrac{-2}{2+j\omega}\right] = -2e^{-2t}u(t)$

$IFT\left[\dfrac{1}{1+j\omega} + \dfrac{-2}{2+j\omega}\right] = x(t) = e^{-t}u(t) - 2e^{-2t}u(t)$

(12) $X(j\omega) = \dfrac{1}{(a+j\omega)^2}$ find IFT using convolution property.

$$= \frac{1}{a+j\omega} \cdot \frac{1}{a+j\omega}$$

$$\Big\downarrow \text{IFT}$$

$$x(t) = e^{-at}u(t) * e^{-at}u(t)$$

$$\boxed{\begin{aligned} x(t) &= x_1(t) * x_2(t) \\ x(t) &= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau \end{aligned}}$$

$$x(t) = \int_{-\infty}^{\infty} e^{-a\tau}\underline{u(\tau)}\, e^{-a(t-\tau)} u(t-\tau)\, d\tau$$

$\tau \geq 0$ $\qquad$ $t - \tau \geq 0$

$t \geq \tau$

$\tau \leq t$

$$= \int_0^t e^{-a\tau} e^{-a(t-\tau)}\, d\tau$$

$$\cancel{e^{-a\tau}}\, e^{-at}\, \cancel{e^{a\tau}}$$

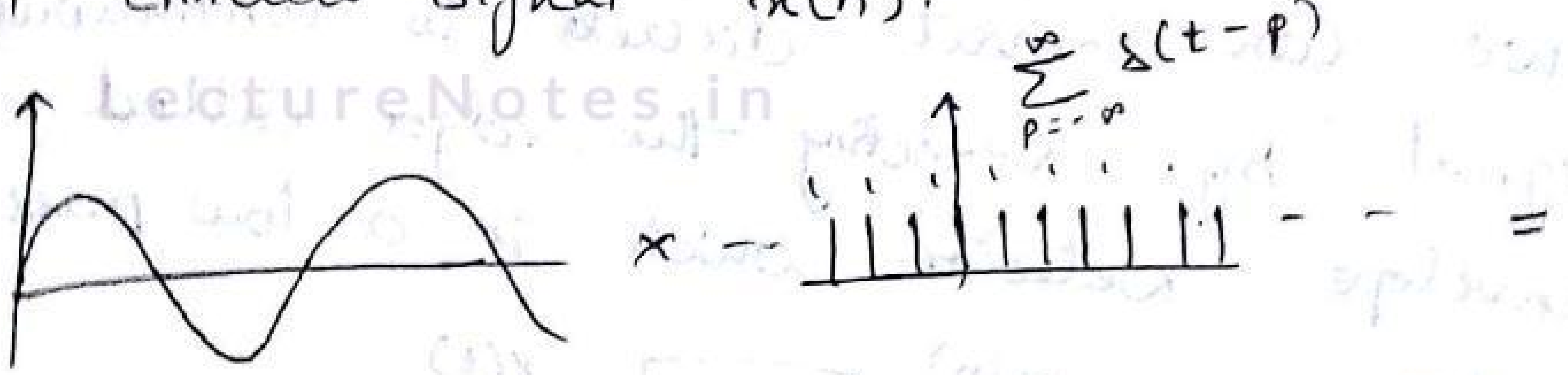
$$\int_0^t e^{-at}\, d\tau$$

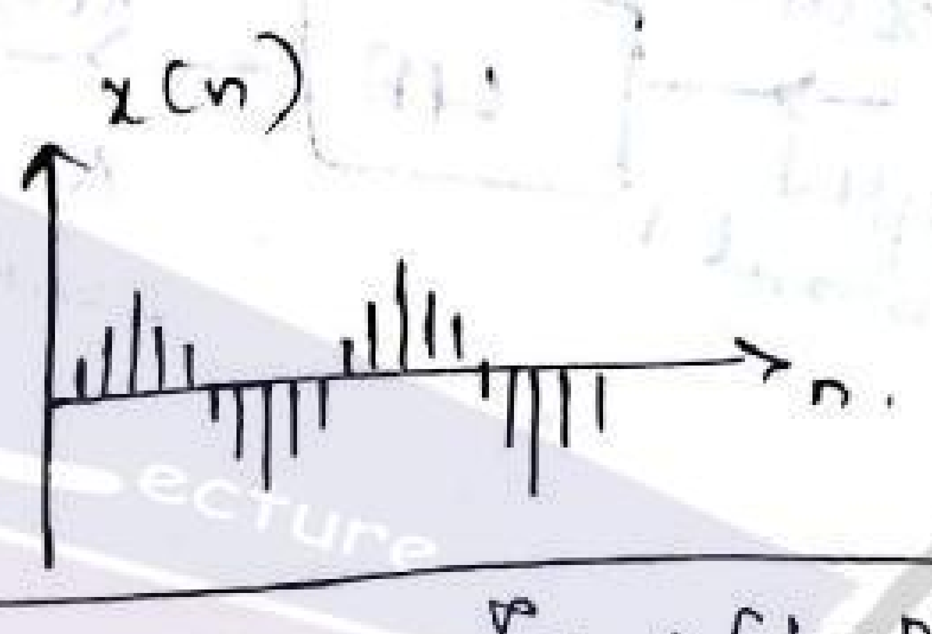
$$e^{-at}\int_0^t d\tau$$

$$= \underline{e^{-at}\, t}$$

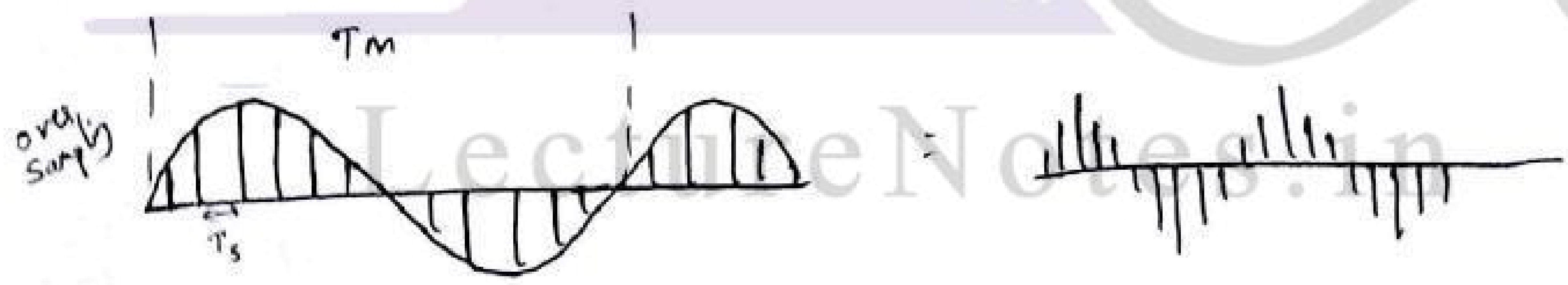
Sampling theorem

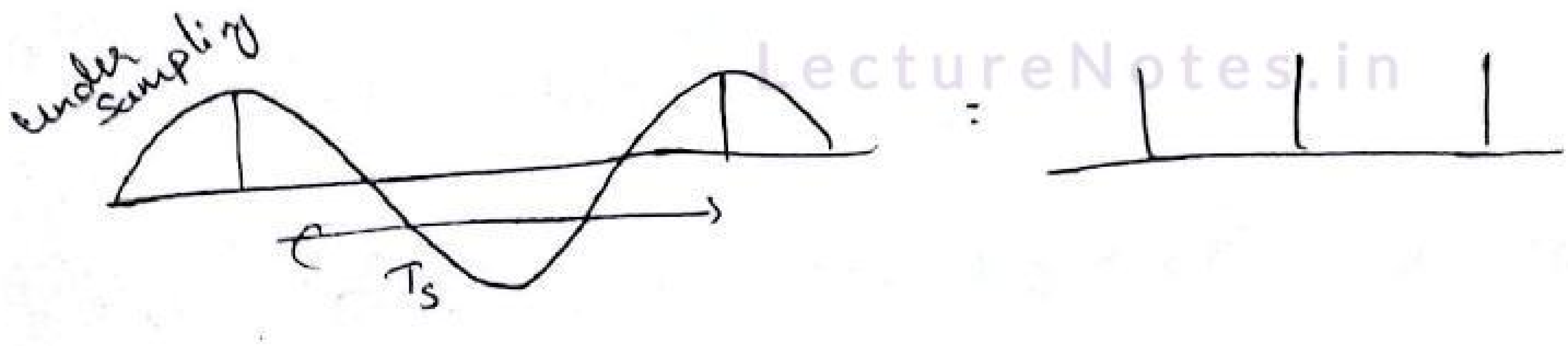
It is a process which converts continuous to discrete signal i.e., Analog to digital signal. Here we multiply continuous signal $x(t)$ with train of impulses which results in discrete signal $x(n)$.

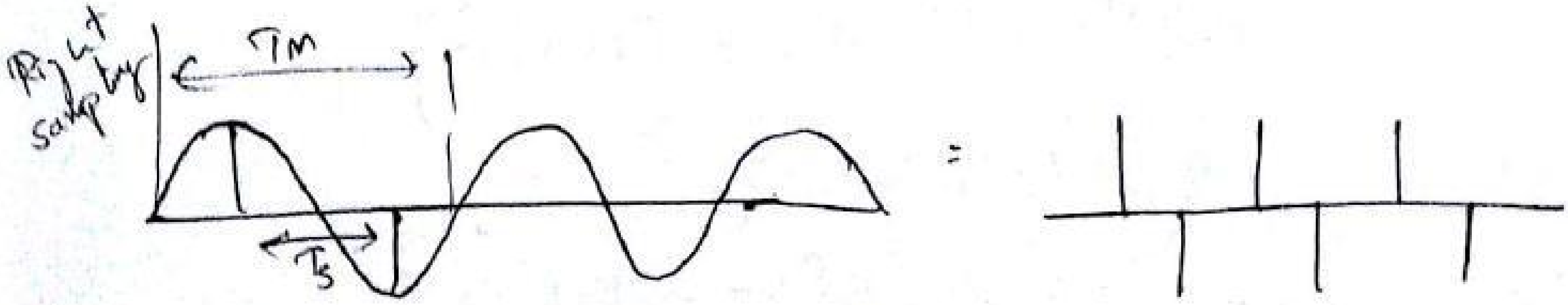




$$\boxed{x(n) = x(t) \times \sum_{p=-\infty}^{\infty} \delta(t-p)}$$







$T_m = 2T_s$

$\frac{1}{f_m} = 2\frac{1}{f_s}$

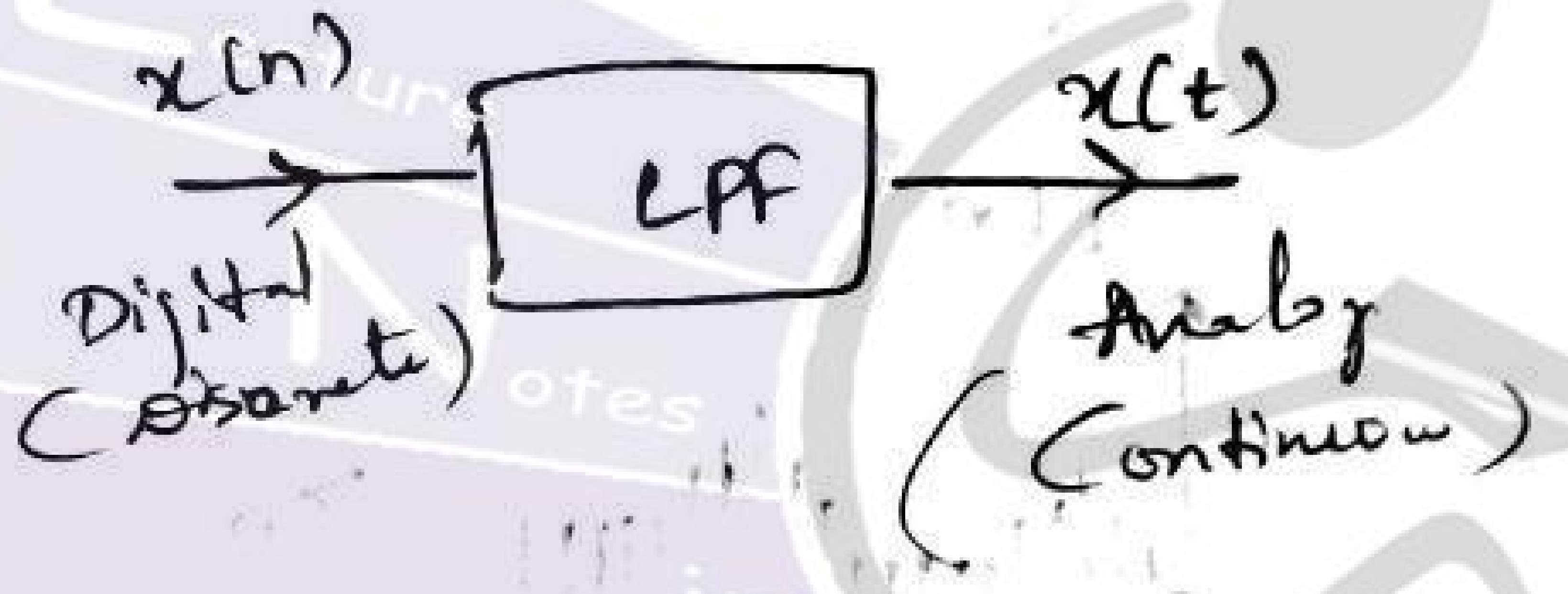
$= \boxed{f_s = 2f_m}$ — Nyquist rate

$f_s \rightarrow$ Sampling frequency

$f_m \rightarrow$ Maximum frequency of the message signal.

Reconstruction of Signal

We can convert discrete to continuous signal by detecting the edges called envelope detector which is a low pass filter.



Module-5

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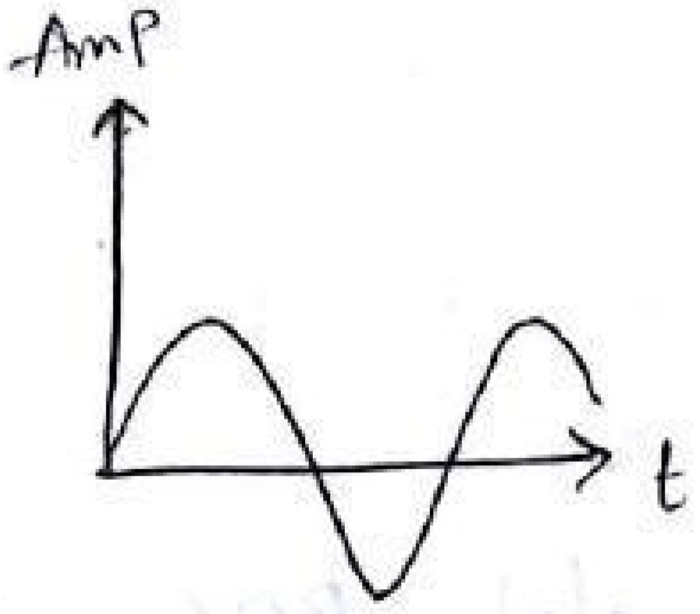
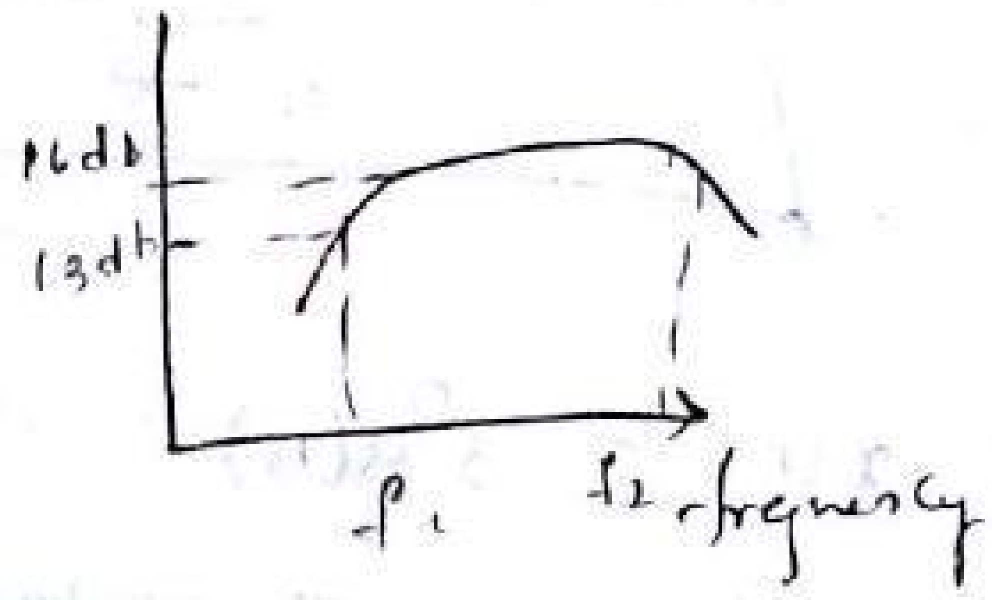
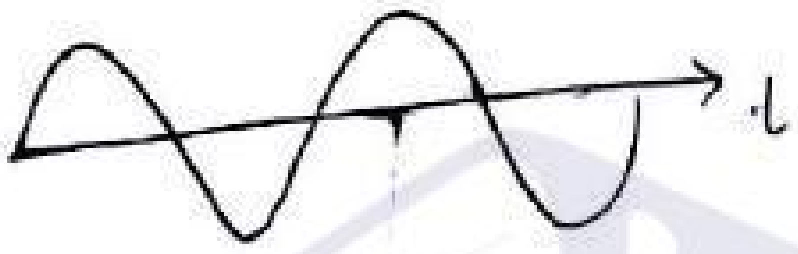
Z - Transform



Time Domain



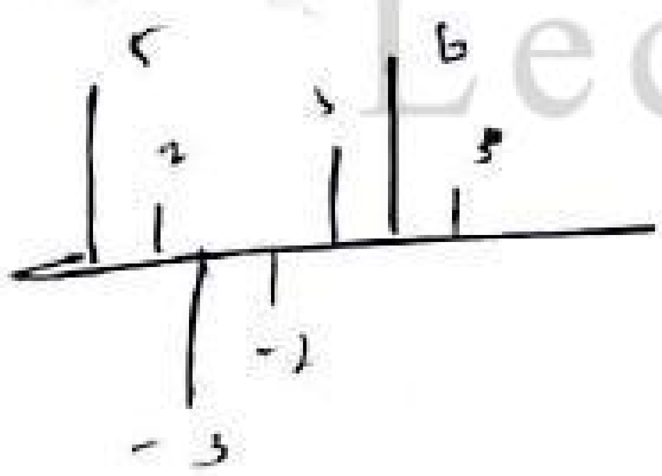
⟶ frequency domain



Continous & periodic — CTFS/FS

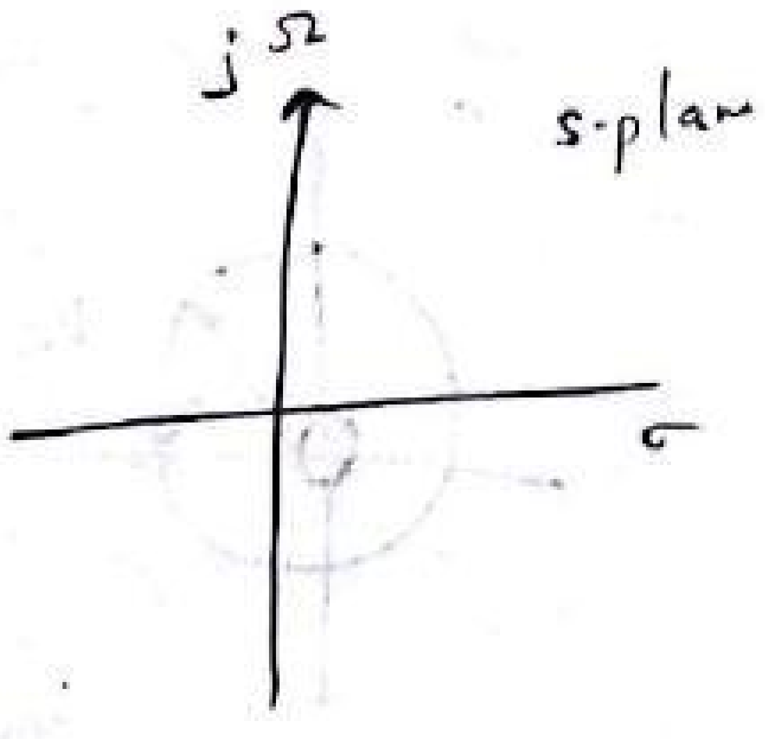
Discrete & periodic — DTFS

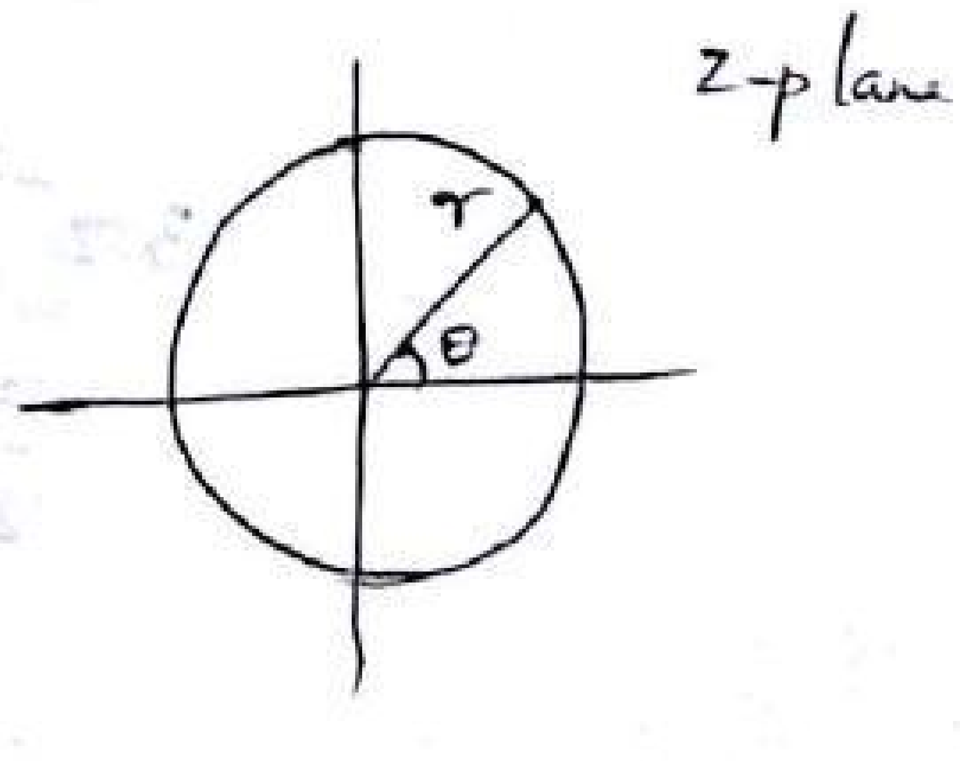
Continuous & Non-periodic — CTFT



Discrete & Non-periodic — DTFT

$$S = \sigma + j\Omega \qquad Z = re^{j\theta} = re^{j\omega}$$





$$x(n) \xrightarrow{Z.T} X(z)$$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}}$$

(1) $x(n) = 3^n u(n)$.

find Z-Transform also plot ROC with poles & zeros

$\rightarrow$ $X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

$= \sum_{n=-\infty}^{\infty} 3^n u(n)\, z^{-n}$

$= \sum_{n=0}^{\infty} 3^n z^{-n}$

$= \sum_{n=0}^{\infty} (3z^{-1})^n$

$$\left\{ \sum_{n=0}^{\infty} a^n = \begin{cases} \frac{1}{1-a} & a<1 \\ \infty & a>1 \end{cases} \right.$$

$X(z) = \dfrac{1}{1-3z^{-1}}$

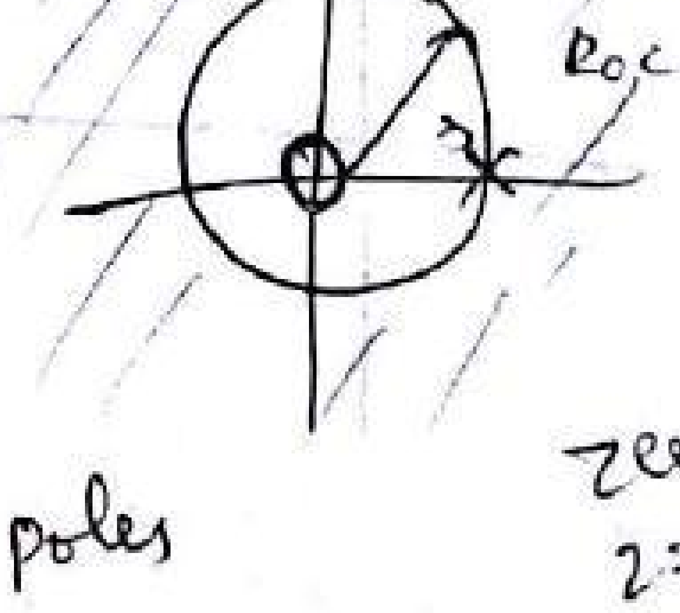
$$\boxed{X(z) = \frac{z}{z-3}}$$

$3z^{-1} < 1$

$\dfrac{3}{z} < 1$

$3 < z$

$\boxed{z > 3}$ ROC



poles

$z-3=0$

$z=3$

zeros

$z=0$

② $x(n) = 2^{n-2} u(n-2)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^{n-2} \underbrace{u(n-2)}_{1 \text{ — amplitude}} z^{-n}$$

$n-2 : 0, \infty$

$n = 2, \infty$

$$= \sum_{n=2}^{\infty} 2^{n-2} z^{-n}$$

$$= \sum_{n=2}^{\infty} 2^{n} 2^{-2} z^{-n}$$

$$= \frac{1}{4} \sum_{n=2}^{\infty} 2^{n} z^{-n}$$

$$= \frac{1}{4} \sum_{n=2}^{n} (2z^{-1})^{n}$$

$$X(z) = \frac{1}{4} \frac{(2z^{-1})^{2}}{1 - 2z^{-1}}$$

$$\left\{ \sum_{n=k}^{\infty} a^{n} = \frac{a^{k}}{1-a} \quad a<1 \right.$$

$$= \infty \quad a>1$$

$$= \frac{1}{\cancel{4}} \frac{\cancel{4} z^{-2}}{1 - 2z^{-1}}$$

$$X(z) = \frac{1}{\cancel{4}} \frac{\cancel{4}}{z^{2} - 2z} = \boxed{\frac{1}{z(z-2)}}$$

ROC

$2z^{-1} < 1$

$\frac{2}{z} < 1$

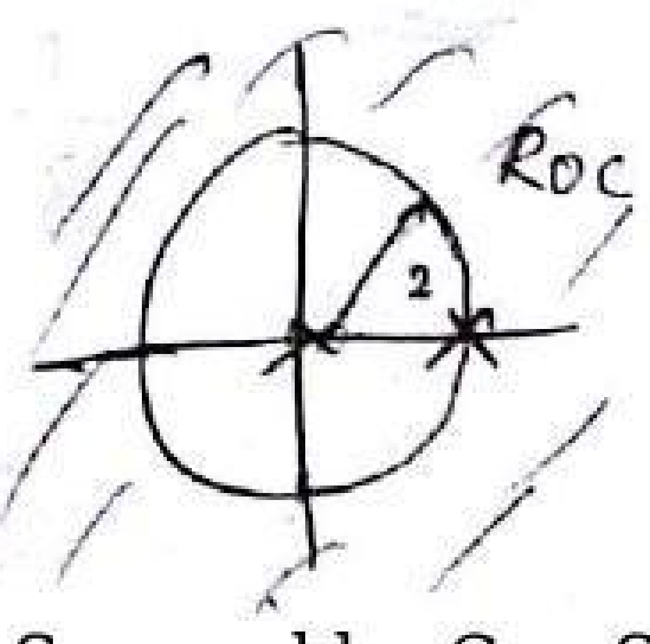
$2 < z$

$\boxed{|z| > 2}$

Poles

$z(z-2) = 0$

$\boxed{\begin{matrix} z = 0 \\ z = 2 \end{matrix}}$



(3) $x(n) = \left(\frac{1}{3}\right)^n u(n+1)$

$\rightarrow$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n \underbrace{u(n+1)}_{1}\, z^{-n}$$

$n+1 : 0, \infty$

$n = -1, \infty$

$$= \sum_{n=-1}^{\infty} \left(\frac{1}{3}\right)^n z^{-n}$$

$$= \sum_{n=-1}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n$$

$$\left\{ \text{WKT} \quad \sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a} \quad ; |a|<1 \right\}$$

$$= \frac{\left[\frac{1}{3} z^{-1}\right]^{-1}}{1 - \frac{1}{3} z^{-1}}$$

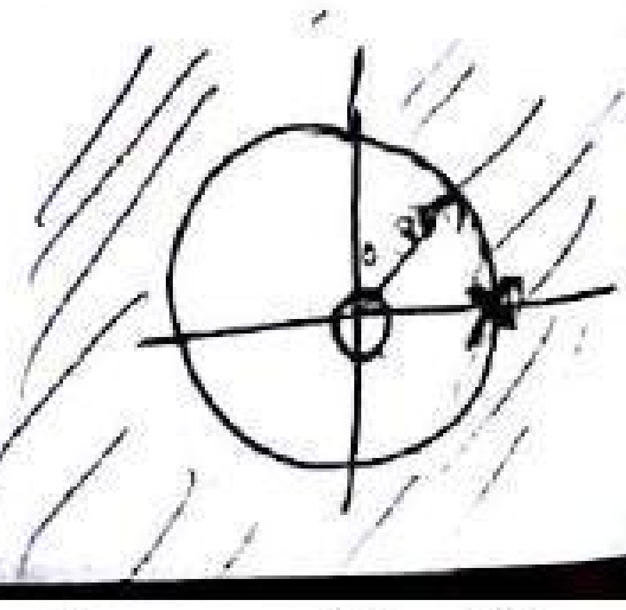
$$= \frac{3z}{1 - \frac{1}{3z}} = \frac{3z}{\frac{3z-1}{3z}} = \boxed{\frac{3z^2}{z - \frac{1}{3}}}$$

ROC

$\frac{1}{3} z^{-1} < 1$

$\frac{1}{3} < z \qquad z > \frac{1}{3}$

$z > 0.33$



Poles

$3z - 1 = 0$

$z = \frac{1}{3}$

zero's

$3z^2 = 0$

$z = 0$

(4) $x(n) = 2^{-n} u(-n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^{-n} u(-n)\, z^{-n}$$

$-n : 0, \infty$

$$= \sum_{n=0}^{-\infty} 2^{-n} z^{-n}$$

$n \to -n$

$$= \sum_{n=0}^{-\infty} \left[(2z)^{-n}\right]^{n} = \sum_{n=0}^{\infty} (2z)^{n}$$

$$\sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a} \quad \text{if } a<1$$

$$\frac{\left[(2z)^{-1}\right]^{0}}{1-(2z)^{-1}} = \frac{1}{1-\frac{z^{-1}}{2}}$$

$$\boxed{= \frac{z}{z-\frac{1}{2}}}$$

ROC

$2z^{-1} < 1 \qquad \frac{2}{z} < 1 \qquad 2 < z$

$z > 2$

$$= \frac{(2z)^0}{1-2z} = \frac{1}{1-2z}$$

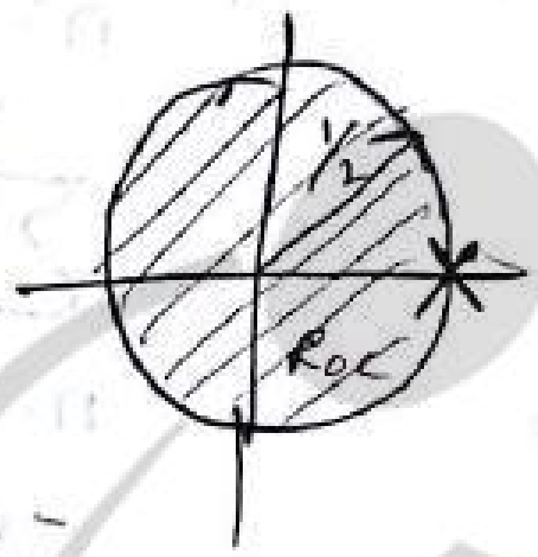
$$= \frac{\frac{1}{2}}{\frac{1}{2} - z} = \boxed{\frac{-\frac{1}{2}}{z-\frac{1}{2}}} \text{ Ans}$$

$u(n+1)$, $u(n-1)$, $u(n)$ } $|z|>$ ... ; $u(-n)$, $u(-n-1)$, $-u(-n-1)$ } in

<u>Roc</u>

$2z < 1$

$$\boxed{|z| < \frac{1}{2}}$$



<u>poles</u>

$z - \frac{1}{2} = 0$

$z = \frac{1}{2}$

(5) $x(n) = u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\sum_{n=-\infty}^{\infty} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n}$$

$$= \sum_{n=0}^{\infty} (z^{-1})^n$$

$$\sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a} \quad \text{if } a<1$$

∞ if $a>1$

$$= \frac{(z^{-1})^0}{1 - z^{-1}} = \frac{1}{1 - z^{-1}}$$

$$= \frac{\cancel{z}}{\frac{1}{\cancel{z}} - 1} \quad \boxed{= \frac{z}{z-1}}$$

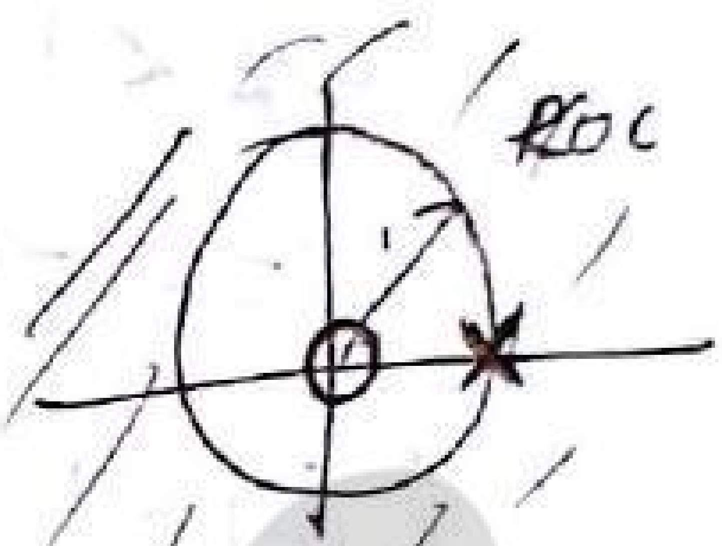
ROC

$$z^{-1} < 1$$

$$\frac{1}{z} < 1$$

$$1 < z$$

$$\boxed{|z| > 1}$$



Poles

$$z - 1 = 0$$

$$\boxed{z = 1}$$

Zeros

$$z = 0$$

(6) $x(n) = -u(-n-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} -1 u(-n-1) z^{-n}$$

$-n-1 = 0, \infty$
$-n = +1, \infty$
$n = -1, -\infty$

$$= \sum_{n=-1}^{-\infty} -1 z^{-n} \longrightarrow \{ n \to -n$$

$$= \sum_{n=1}^{\infty} -z^{n} = -\sum_{n=1}^{\infty} z^{n}$$

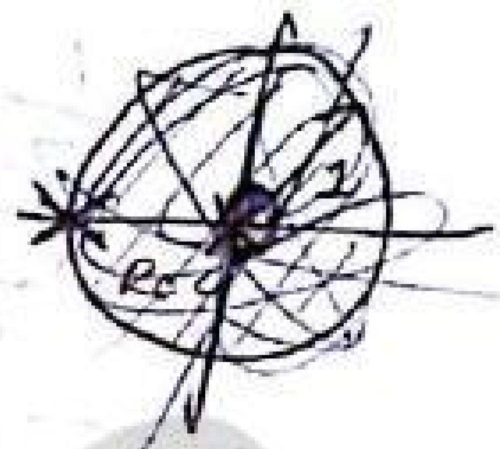
$$= \frac{-[z]^{1}}{1-(z)} = \frac{-z}{1-z} = \boxed{\frac{+z}{z-1}}$$

ROC

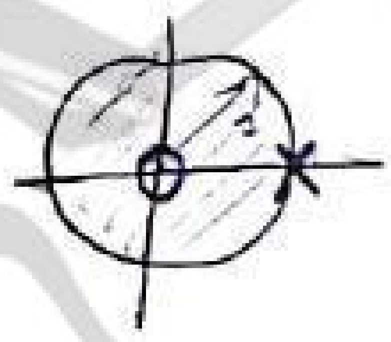
$-z < 1$

$z < -1$

$\boxed{|z| < 1}$



poles	zeros
$z-1=0$	$+z=0$
$z=+1$	$z=0$



(7) $x(n) = 2^n u(n) + (-3)^n u(-n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[2^n u(n) + (-3)^n u(-n)\right] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n} + (-3)^n u(-n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n} + \sum_{n=-\infty}^{\infty} (-3)^n u(-n) z^{-n}$$

$$= \sum_{n=0}^{\infty} 2^n z^{-n} + \sum_{n=0}^{-\infty} (-3)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n + \sum_{n=0}^{\infty} (-3)^{-n} z^n$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n + \sum_{n=0}^{\infty} \cancel{(-3^{-1}z)} \left(-\frac{z}{3}\right)^n$$

$$\frac{(2z^{-1})^0}{1-2z^{-1}} + \frac{\left(-\frac{z}{3}\right)^0}{1 - -\frac{z}{3}}$$

$$= \frac{1}{1-2z^{-1}} + \frac{1}{1+\frac{z}{3}}$$

$$= \frac{z}{z-2} + \frac{3}{3+z}$$

$$\boxed{= \frac{z}{z-2} + \frac{3}{z+3}} = \frac{z(z+3)+3(z-2)}{(z-2)(z+3)}$$

$$\boxed{X(z) = \frac{z^2+6z-6}{(z-2)(z+3)}}$$

ROC

$2z^{-1} < 1$

$\frac{2}{z} < 1$

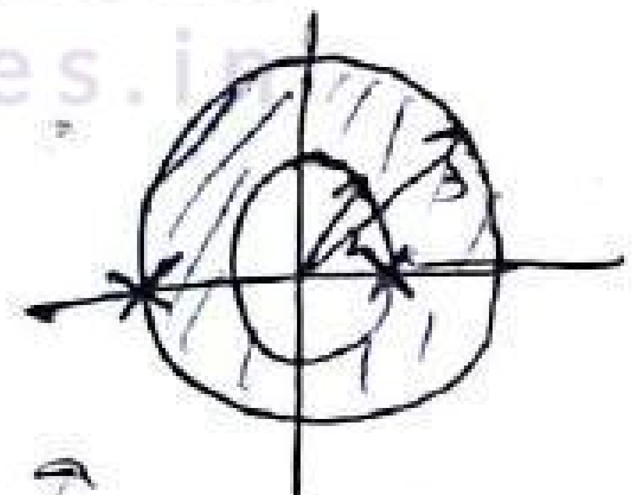
$2 < z$

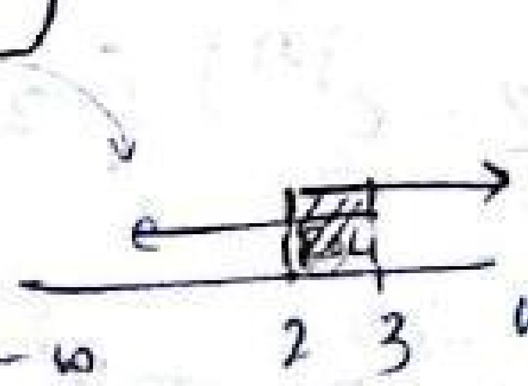
$\boxed{|z| > 2}$

$-\frac{z}{3} < 1$

$z < -3$

$\boxed{|z| < 3}$





poles

$z-2=0$: $z=2$

$z=-3$

8) $x(n) = \cos\Omega n\, u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \qquad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$= \sum_{n=-\infty}^{\infty} \cos\Omega n\, u(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{e^{j\Omega n} + e^{-j\Omega n}}{2}\right] \cdot z^{-n} u(n)$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} e^{j\Omega n} z^{-n} u(n) + e^{-j\Omega n} z^{-n} u(n)$$

$$= \frac{1}{2} \left[\sum_{n=0}^{\infty} \left(e^{j\Omega} z^{-1}\right)^n + \sum_{n=0}^{\infty} \left(e^{-j\Omega} z^{-1}\right)^n\right]$$

$$= \frac{1}{2} \left[\frac{\left(e^{j\Omega} z^{-1}\right)^0}{1 - e^{j\Omega} z^{-1}} + \frac{\left(e^{-j\Omega -1}\right)^0}{1 - e^{-j\Omega} z^{-1}}\right]$$

$$= \frac{1}{2} \left[\frac{1}{1 - e^{j\Omega} z^{-1}} + \frac{1}{1 - e^{-j\Omega} z^{-1}}\right]$$

$$= \frac{1}{2} \left[\frac{z}{z - e^{j\Omega}} + \frac{z}{z - e^{-j\Omega}}\right]$$

$$= \frac{1}{2} \left[\frac{z\left(z - e^{-j\Omega}\right) + z\left(z - e^{j\Omega}\right)}{\left(z - e^{j\Omega}\right)\left(z - e^{-j\Omega}\right)}\right]$$

$$= \frac{1}{2} \left(\frac{z^2 - e^{-j\Omega} z + z^2 - e^{j\Omega} z}{z^2 - e^{-j\Omega} z - e^{j\Omega} z + 1}\right)$$

$$= \frac{1}{2}\left[\frac{\cancel{2}z^2 - \cancel{2}z\left(e^{j\Omega}+e^{-j\Omega}\right)}{z^2 - z\left(e^{j\Omega}+e^{-j\Omega}\right)+1}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}z^2 - \cancel{2}z\cos\Omega}{z^2 - 2z\cos\Omega + 1}\right]$$

$$\boxed{X(z) = \frac{z^2 - z\cos\Omega}{z^2 - 2z\cos\Omega + 1}}$$

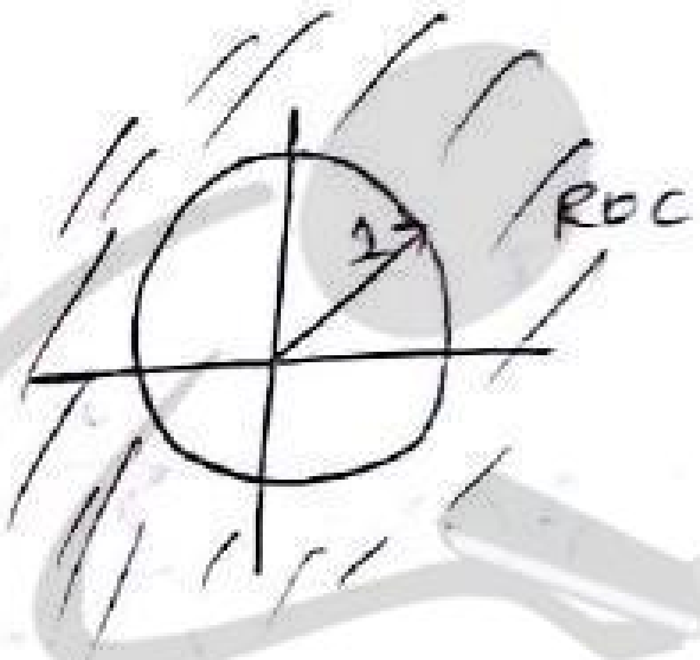
ROC

$e^{j\Omega n} z^{-1} < 1$

$e^{j\Omega} < z$

$z > e^{j\Omega}$

$\boxed{|z| > 1}$



(9) $x(n) = \sin\Omega n\, u(n)$

Ans $X(z) = \frac{z\sin\Omega}{z^2 - 2z\cos\Omega + 1}$

$|z| > 1$

(10) $x(n) = 3\delta(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 3\delta(n) z^{-n}$$

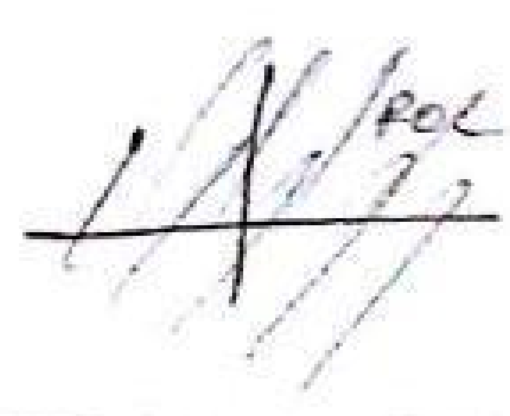
ROC

everywhere in $\delta()$

at $n = 0$

$= \cancel{\Sigma}\ 3\delta(0) z^0$

$\boxed{X(z) = 3}$



(11) $x(n) = 5\delta(n-2)$

$$X(z) = \sum_{n=-\infty}^{\infty} 5\delta(n-2) z^{-n}$$

at $n=2$

$$5\,\delta(0)\,z^{-2}$$

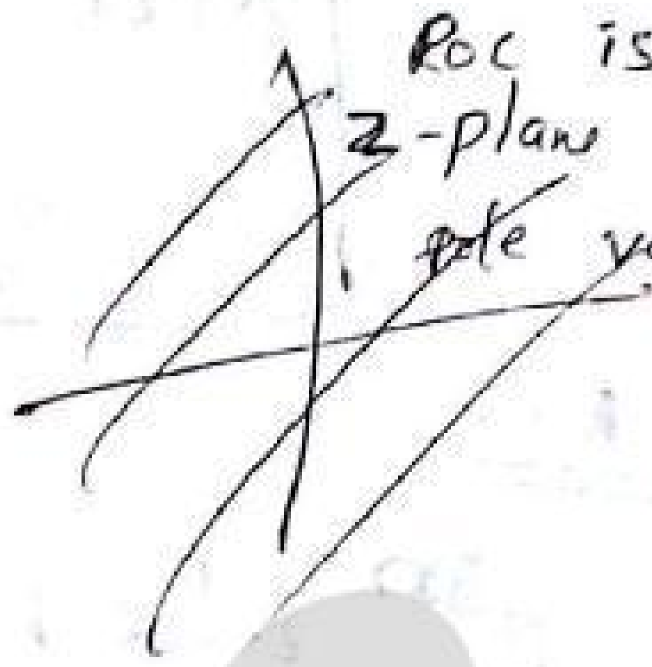
$$\boxed{X(z) = 5z^{-2}}$$

<u>Roc</u> <u>pole</u> $\frac{5}{z^2} = 0$

$z^2 = 0$

$z = 0$

Roc is entire z-plane except pole value.



(12) $x(n) = 6\delta(n+2)$

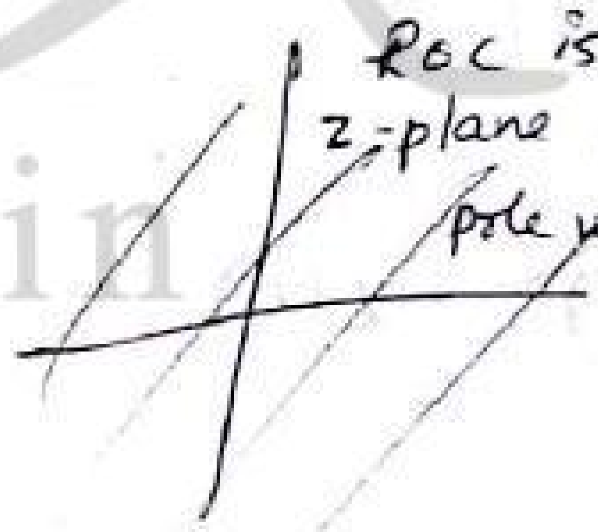
$$X(z) = \sum_{n=-\infty}^{\infty} 6\delta(n+2) z^{-n}$$

at $n=-2$

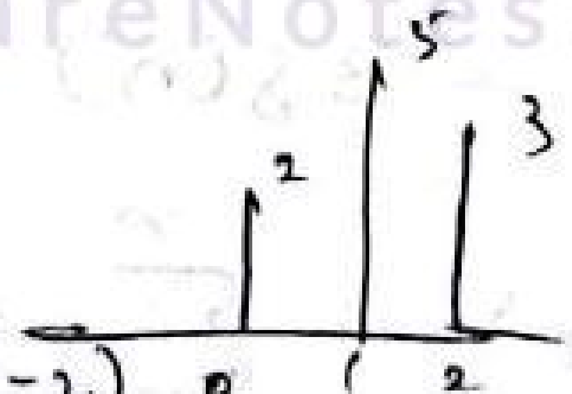
$$6\,\delta(0)\,z^{2}$$

$$\boxed{X(z) = 6z^{2}}$$

Roc is entire z-plane except pole value



(13) $x(n) = \{ \underset{\uparrow}{2}, 5, 3 \}$



$x(n) = 2\delta(n) + 5\delta(n-1) + 3\delta(n-2)$

$$X(z) = \sum_{n=-\infty}^{\infty} \left[2\delta(n) + 5\delta(n-1) + 3\delta(n-2) \right] z^{-n}$$

$$\sum_{n=-\infty}^{\infty} 2\delta(n) z^{-n} + \sum_{n=-\infty}^{\infty} 5\delta(n-1) z^{-n} + \sum_{n=-\infty}^{\infty} 3\delta(n-2)$$

Put $n=0$ $n=1$ $n=2$

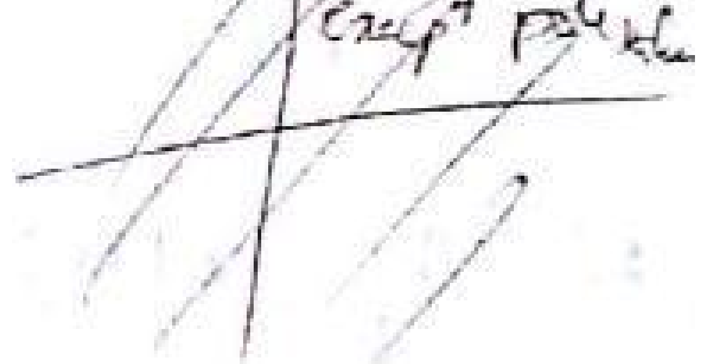
$$X(z) = 2z^{0} + 5z^{-1} + 3z^{-2}$$

$$X(z) = 2 + 5z^{-1} + 3z^{-2}$$

$$X(z) = 2 + \frac{5}{z} + \frac{3}{z^2}$$

$$\boxed{X(z) = \frac{2z^2 + 5z + 3}{z^2}}$$

Roc is entire z-plane except pole value



(14) $x(n) = \{-3, \underset{\uparrow}{2}, 4, 3\}$

$$= -3\delta(n+1) + 2\delta(n) + 4\delta(n-1) + 3\delta(n-2)$$

$$= \sum_{n=-\infty}^{\infty} -3\delta(n+1)z^{-n} + \sum_{n=-\infty}^{\infty} 2\delta(n)z^{-n} + \sum_{n=-\infty}^{\infty} 4\delta(n-1)z^{-n} + \sum_{n=-\infty}^{\infty} 3\delta(n-2)z^{-n}$$

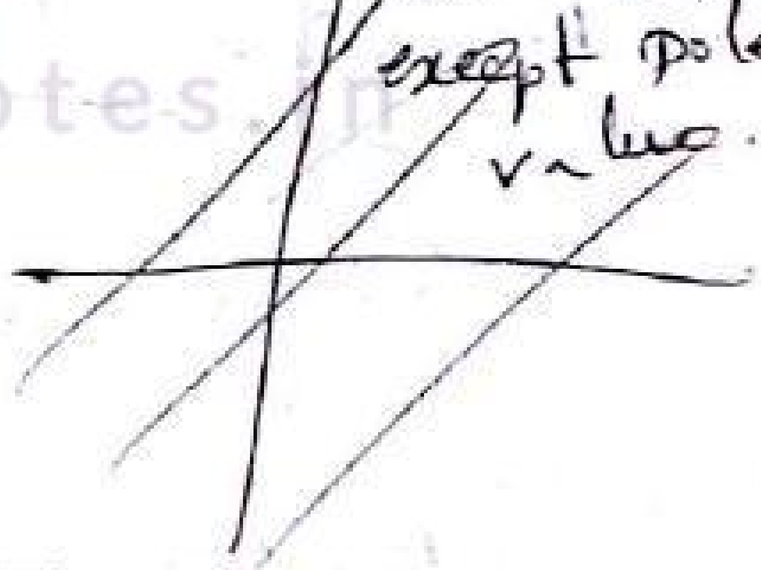
put $n=-1$ $\quad n=0$ $\quad n=1$ $\quad n=2$

$$= -3z + 2z^{0} + 4z^{-1} + 3z^{-2}$$

$$= -3z + 2 + \frac{4}{z} + \frac{3}{z^2}$$

$$\boxed{X(z) = \frac{-3z^3 + 2z^2 + 4z + 3}{z^2}}$$

Roc is entire z-plane except pole value.



6M

Properties of ROC

1) If $x(n)$ is Right sided Sequence then ROC is outside the Circle of Radius r.

i.e., $|z| > r$

2) If $x(n)$ is left sided Sequence then ROC is inside the Circle of Radius 'r'

i.e., $|z| < r$

3) If $x(n)$ is both sided sequence then ROC is Ring b/w radius r_1 & r_2

i.e., $r_1 < |z| < r_2$

4) If $x(n)$ is finite sequence then ROC is entire z-plane.

5) ROC doesnot ~~Contine~~ contain any poles

6) ROC is a Circle Centered about the Origin

Properties of Z-Transform

Refer xerox page no. 5

		ROC
1) $a^n x(n) \rightarrow X\left(\frac{z}{a}\right)$		$\frac{\lvert z\rvert}{a} = R$
2) $x(n-n_0) \rightarrow z^{-n_0} X(z)$		$\lvert z\rvert = R$
3) $x(-n) \rightarrow X\left(\frac{1}{z}\right)$		$\frac{1}{\lvert z\rvert} = R$
4) $n\,(x(n)) \rightarrow -z\frac{d}{dz} X(z)$		$\lvert z\rvert = R$

b) $n(n-1)x(n) = (-1)^2 z^2 \frac{d^2}{dz^2} X(z) \quad |z| = R$

Problems using properties

$z[u(n)] = \frac{z}{z-1} \quad |z| > 1$

$z[-u(-n-1)] = \frac{z}{z-1} \quad |z| < 1$

① $x(n) = 3^n u(n)$

ROC

$z[u(n)] = \frac{z}{z-1} \quad : \quad |z| > 1$

$z[3^n u(n)] = \frac{\frac{z}{3}}{\frac{z}{3} - 1} \quad : \quad \frac{|z|}{3} > 1$

$= \frac{z}{z-3} \quad : \quad \boxed{|z| > 3}$

ROC

~~|z|~~

② $x(n) = 2^{n-2} u(n-2)$

$z[u(n)] = \frac{z}{z-1}$ ROC $|z| > 1$

$z[2^n u(n)] = \frac{\frac{z}{2}}{\frac{z}{2} - 1} = \frac{z}{z-2}$ ROC $\frac{|z|}{2} > 1$

$z[2^{n-2} u(n-2)] = z^{-2} \cdot \frac{z}{z-2}$ ROC $\boxed{|z| > 2}$

$= \frac{z^{-1}}{z-2}$

$= \frac{1}{z(z-2)}$

③ $x(n) = \left(\frac{1}{3}\right)^n u(n+1) \Rightarrow \left(\frac{1}{3}\right)^{-1}\left(\frac{1}{3}\right)^{n+1} u(n+1)$

ROC

$Z[u(n)] = \frac{z}{z-1}$ $\quad |z|>1$

$Z\left[\left(\frac{1}{3}\right)^n u(n)\right] = \frac{\frac{z}{1/3}}{\frac{z}{1/3}-1} \Rightarrow \frac{z}{z-\frac{1}{3}}$ $\quad \frac{|z|}{\frac{1}{3}}>1$

$\boxed{|z|>\frac{1}{3}}$

$Z\left[\left(\frac{1}{3}\right)^{n+1} u(n+1)\right] = z^1 \frac{z}{z-\frac{1}{3}}$

$= \frac{z^2}{z-\frac{1}{3}}$

$Z\left[\left(\frac{1}{3}\right)^{-1}\left(\frac{1}{3}\right)^{n+1} u(n+1)\right] = \frac{3z^2}{z-\frac{1}{3}}$

④ $x(n) = 2^{-n} u(-n)$

ROC

$Z[u(n)] = \frac{z}{z-1}$ $\quad |z|>1$

$Z[2^n u(n)] = \frac{\frac{z}{2}}{\frac{z}{2}-1} = \frac{z}{z-2}$ $\quad \frac{|z|}{2}>1$

$|z|>2$

$Z[2^{-n} u(-n)] = \frac{\frac{1}{z}}{\frac{1}{z}-2}$ $\quad \frac{1}{|z|}>2$

$= \frac{1}{1-2z}$ $\quad \boxed{|z|<\frac{1}{2}}$

$= \frac{1/2}{1/2 - z} \Rightarrow \boxed{\frac{-\frac{1}{2}}{z-1/2}}$

⑤ $x(n) = n\, a^{n-1} u(n)$

$x(n) = n\, a^n a^{-1} u(n)$

ROC

$Z[u(n)] = \frac{z}{z-1}$ $\quad |z| > 1$

$Z[a^n u(n)] = \frac{\frac{z}{a}}{\frac{z}{a} - 1}$ $\quad \frac{|z|}{a} > 1$

$= \frac{z}{z-a}$ $\quad \boxed{|z| > a}$

$Z[n a^n u(n)] = -z \frac{d}{dz}\left(\frac{z}{z-a}\right)$

$= -z\left[\frac{(z-a)1 - z}{(z-a)^2}\right]$

$= -z\left[\frac{z - a - z}{(z-a)^2}\right]$

$= \frac{za}{(z-a)^2}$

$Z[n a^n a^{-1} u(n)] = \frac{1}{a}\frac{za}{(z-a)^2}$

$= \frac{z}{(z-a)^2}$

⑥ $x(n) = n-2\, (3)^{n-2} u(n-2)$

ROC

$Z[u(n)] = \frac{z}{z-1}$ $\quad |z| > 1$

$Z[3^n u(n)] = \frac{\frac{z}{3}}{\frac{z}{3} - 1} = \frac{z}{z-3}$ $\quad \frac{|z|}{3} > 1$

$|z| > 3$

$$z\left[n\,3^{n}\,u(n)\right] = -z\frac{d}{dz}\left(\frac{z}{z-3}\right)$$

$$-z\left[\frac{(z-3)1 - z}{(z-3)^2}\right]$$

$$-z\left[\frac{\cancel{z}-3-\cancel{z}}{(z-3)^2}\right]$$

$$= \frac{3z}{z-3}$$

$$z\left[n-2\;3^{n-2}\,u(n-2)\right] = z^{-2}\,\frac{3z}{z-3}$$

$$\boxed{X(z) = \frac{3}{z(z-3)}}$$

(7) $x(n) = 2^n u(n) + (-3)^n u(-n)$

$= 2^n u(n) + (-\frac{1}{3})^{-n} u(-n)$

$z[u(n)] = \frac{z}{z-1} \quad |z|>1$ $\quad + \quad z[u(n)] = \frac{z}{z-1} \quad |z|>1$

$z\left[2^n u(n)\right] = \frac{z/2}{\frac{z}{2}-1} \quad \frac{|z|}{2}>1$ $\qquad z\left[\left(\frac{-1}{3}\right)^{n} u(n)\right] = \frac{-\frac{z}{3}}{-\frac{z}{3}-\frac{1}{3}} \quad \frac{|z|}{-\frac{1}{3}}$

$= \frac{z/2}{\frac{z-1}{2}} \quad \underline{|z|>2}$ $\qquad = \frac{-z}{-z-\frac{1}{3}} \quad |z|>$

$= \frac{z}{z-2}$ $\qquad z\left[\left(\frac{1}{3}\right)^{-n} u(-n)\right] = \frac{-\frac{1}{z}}{-\frac{1}{z}-3} \quad \frac{1}{|z|}$

$= \frac{-1}{-1-3z}$

$$= \frac{1}{A\left(1+\frac{1}{3}z\right)}$$

$$= \frac{3}{3+z} = \frac{3}{z+3}$$

$$x(z) = \frac{z}{z-2} + \frac{3}{z+3}$$

$$= \frac{z(z+3)+3(z-2)}{(z-2)(z+3)}$$

$$= \frac{z^2+6z-6}{(z-2)(z+3)}$$

(8) $x(n) = n\left(-\frac{1}{2}\right)^n u(n) + \left(\frac{1}{4}\right)^{-n} u(-n)$

ROC

$Z[u(n)] = \frac{z}{z-1}$ $\quad |z| > 1$

$$Z\left[\left(-\frac{1}{2}\right)^n u(n)\right] = \frac{\frac{z}{-1/2}}{\frac{z}{-\frac{1}{2}} - 1}$$

$\frac{|z|}{-\frac{1}{2}} > 1$

$\boxed{|z| = +\frac{1}{2}}$

$$= \frac{z}{z+\frac{1}{2}}$$

$$Z\left[n\left(-\frac{1}{2}\right)^n u(n)\right] = -z\frac{d}{dz}\left(\frac{z}{z+\frac{1}{2}}\right)$$

$$-z\left[\frac{\left(z+\frac{1}{2}\right)\cdot 1 - z\cdot 1}{\left(z+\frac{1}{2}\right)^2}\right]$$

$$= -z\left[\frac{\cancel{z}+\frac{1}{2}-\cancel{z}}{(\;)^2}\right] \Rightarrow \frac{-\frac{z}{2}}{\left(z+\frac{1}{2}\right)^2}$$

ROC

$|z|>1$ $\qquad Z[u(n)] = \frac{z}{z-1}$

$\frac{|z|}{1/4} > 1$ $\qquad Z\left[\left(\frac{-1}{4}\right)^n u(n)\right] = \frac{\frac{z}{1/4}}{\frac{z}{1/4} - 1}$

$|z| > \frac{1}{4}$ $\qquad = \frac{z}{z - \frac{1}{4}}$

$\frac{1}{|z|} > \frac{1}{4}$ $\qquad Z\left[\left(\frac{1}{4}\right)^{-n} u(-n)\right] = \frac{\frac{1}{z}}{\frac{1}{z} - \frac{1}{4}}$

$\cancel{|z| > 4}$

$\boxed{|z| < 4}$ $\qquad = \frac{1}{1 - \frac{1}{4}z}$

$= \frac{4}{4 - z}$

$= \boxed{\frac{-4}{z-4}}$

ROC = $\boxed{\frac{1}{2} < |z| < 4}$

$$\frac{-\frac{z}{2}}{\left(z + \frac{1}{2}\right)^2} \times \frac{-4}{z-4}$$

$$X(z) = \frac{2z}{\left(z+\frac{1}{2}\right)^2 (z-4)}$$

(9) $x(n) = \left(\frac{-1}{3}\right)^n u(n) + \left(\frac{3}{4}\right)^n u(-n-1)$

ROC

$Z[u(n)] = \frac{z}{z-1}$ $\qquad |z| > 1$

$Z\left[\left(-\frac{1}{3}\right)^n u(n)\right] = \frac{\frac{z}{-1/3}}{\frac{z}{\ldots} - 1}$ $\qquad \frac{|z|}{-\frac{1}{3}} > 1$

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$$= \frac{z}{z+\frac{1}{3}} \qquad \boxed{|z| > \frac{1}{3}}$$

1

~~$Z[u(n)] =$~~

$$Z(-u(-n-1)) = \frac{z}{z-1} \qquad |z| < 1$$

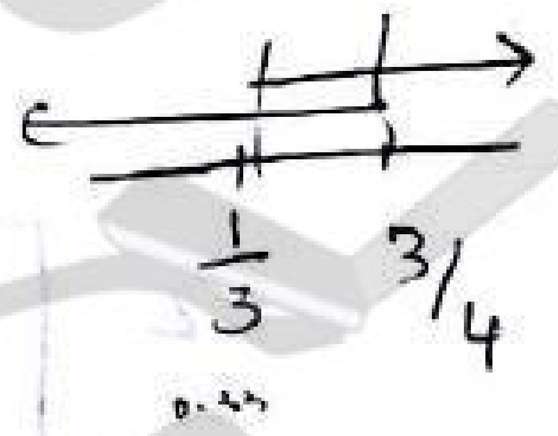
$$\frac{|z|}{3/4} < 1$$

$$Z\left(-\left(\frac{3}{4}\right)^n u(-n-1)\right) = \frac{\frac{z}{3/4}}{\frac{z}{3/4} - 1} \qquad \boxed{|z| < \frac{3}{4}}$$

$$= \frac{z}{z-\frac{3}{4}}$$

ROC

$$Z\left(\left(\frac{3}{4}\right)^n u(-n-1)\right) = \frac{-z}{z-3/4}$$



$$X(z) = \frac{z}{z+\frac{1}{3}} - \frac{z}{z-\frac{3}{4}}$$

ROC

$$\boxed{\frac{1}{3} < |z| < \frac{3}{4}}$$

$$\frac{z\left(z-\frac{3}{4}\right) - z\left(z+\frac{1}{3}\right)}{\left(z+\frac{1}{3}\right)\left(z-\frac{3}{4}\right)}$$

$$= \frac{\cancel{z^2} - \frac{3}{4}z - \cancel{z^2} - \frac{z}{3}}{\left(z+\frac{1}{3}\right)\left(z-\frac{3}{4}\right)}$$

$=$ ~~$3z^2 - \frac{5}{12}z$~~

$$\boxed{X(z) = \frac{-\frac{13}{12}z}{\left(z+\frac{1}{3}\right)\left(z-\frac{3}{4}\right)}}$$

(10) $x(n) = n\left(\frac{1}{2}\right)^n u(n) * \left[\delta(n) + \frac{1}{2}\delta(n-1)\right]$

$Z\left[u(n)\right] = \frac{z}{z-1}$ ROC $|z| > 1$

$Z\left[\left(\frac{1}{2}\right)^n u(n)\right] = \frac{\frac{z}{1/2}}{\frac{z}{1/2} - 1}$ $\frac{|z|}{1/2} > 1$

$\boxed{|z| > \frac{1}{2}}$

$= \frac{z}{z - \frac{1}{2}}$

$Z\left[n\left(\frac{1}{2}\right)^n u(n)\right] = -z\frac{d}{dz}\left(\frac{z}{z-\frac{1}{2}}\right)$

$-z\left[\frac{\left(z-\frac{1}{2}\right)\cdot 1 - z}{\left(z-\frac{1}{2}\right)^2}\right]$

$-z\left[\frac{\cancel{z} - \frac{1}{2} - \cancel{z}}{\left(z-\frac{1}{2}\right)^2}\right]$

$= \frac{\frac{z}{2}}{\left(z-\frac{1}{2}\right)^2} \rightarrow (1)$

$\delta(n) + \frac{1}{2}\delta(n-1)$

$1 + \frac{1}{2}z^{-1} \rightarrow (2)$ ROC entire z-plane

$\frac{\frac{z}{2}}{\left(z-\frac{1}{2}\right)^2} \times \left[1 + \frac{1}{2z}\right]$

Scanned by CamScanner

$$= \frac{\frac{z}{2}}{\left(z-\frac{1}{2}\right)^2} + \frac{\frac{z}{2}}{2z\left(z-\frac{1}{2}\right)^2}$$

ROC

$$\boxed{|z| > \frac{1}{2}}$$

(11) $x(n) = \left(\frac{2}{3}\right)^n u(n) + 2^n u(-n-3)$
$= 2^n u[-(n+3)]$

$Z[u(n)] = \frac{z}{z-1} \quad |z|>1$

$Z[u(n)] = \frac{z}{z-1}, \quad |z|>1$

$Z\left[\left(\frac{2}{3}\right)^n u(n)\right] = \frac{z/\frac{2}{3}}{\frac{z}{2/3}-1} \quad \frac{|z|}{\frac{2}{3}}>1$

$= \boxed{\frac{z}{z-\frac{2}{3}}} \quad \boxed{|z|>\frac{2}{3}}$

$Z[u(-n)] = \frac{\frac{1}{z}}{\frac{1}{z}-1} \quad \frac{|z|}{\frac{1}{z}}>1$

$= \frac{1}{1-z} \quad |z|>\frac{1}{2}$

$Z[u(-(n+3))] = \frac{z^3}{1-z} \quad \frac{1}{|z|}>\frac{1}{2}, \quad 2>|z|$

$Z[2^n u(-(n+3))] = \frac{\left(\frac{z}{2}\right)^3}{1-\frac{z}{2}} \quad \boxed{|z|<2}$

$= \frac{\frac{z^3}{8}}{1-\frac{z}{2}}$

$= \frac{\frac{z^3}{4}}{2-z}$

$= \boxed{\frac{-\frac{z^3}{4}}{z-2}}$

$$\boxed{X(z) = \frac{-\frac{1}{4}z^4}{\left(z-\frac{2}{3}\right)(z-2)}}$$

ROC

$\frac{2}{3} \quad 2$

ROC

$$\boxed{\frac{2}{3} < |z| < 2}$$

(12) $x(n) = n^2 u(n)$

ROC $|z|>1$

$$z[u(n)] = \frac{z}{z-1}$$

$$z[n u(n)] = -z\frac{d}{dz}\frac{z}{z-1}$$

$$= -z\left(\frac{(z-1).1 - z}{(z-1)^2}\right)$$

$$= -z\left[\frac{\cancel{z}-1-\cancel{z}}{(z-1)^2}\right]$$

$$= \frac{z}{(z-1)^2}$$

$$z[n^2 u(n)] = (-)^2 z^2 \frac{d^2}{dz^2}$$

$$= -z^2\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -z^2\left[\frac{(z-1)^2.1 - z\,2(z-1)}{(z-1)^4}\right]$$

$$-z\frac{((z-1) - 2z)}{(z-1)^3}$$

$$= -z^2\left[\frac{z^2+1-\cancel{2z}-2z^2+\cancel{2z}}{(z-1)^4}\right]$$

$$-z\left[\frac{z-1-2z}{(z-1)^3}\right]$$

$$-z^2\left[\frac{1-z^2}{(z-1)^4}\right] = \frac{-z-z^3}{(z-1)^4}$$

$$-z\left[\frac{-1-z}{(z-1)^3}\right]$$

$$= \qquad = -\frac{(z-z^3)}{(z-1)^4}$$

$$\boxed{X(z) = \frac{z+z^2}{(z-1)^3}}$$

(13) $x(n) = \alpha^{|n|}$

$|n| \rightarrow n$ +ve   0 to ∞ $u(n)$

$|n| \rightarrow -n$ -ve $-\infty$ to -1 $u(-n-1)$

$\alpha^{|n|} \rightarrow \alpha^{n}$ 0 to ∞ $u(n)$

$\alpha^{|n|} \rightarrow \alpha^{-n}$ $-\infty$ to -1 $u(-n-1)$

$$x(n) = \alpha^{n} u(n) + \alpha^{-n} u(-n-1)$$

$$x(n) = \alpha^{n} u(n) + \left(\frac{1}{\alpha}\right)^{n} u(-n-1)$$

$$z\left[u(n)\right] = \frac{z}{z-1} \qquad z\left[-u(-n-1)\right] = \frac{z}{z-1} \quad |z| < 1$$

$$z\left[\alpha^{n} u(n)\right] = \frac{z}{z-\alpha} \qquad z\left[-\left(\frac{1}{\alpha}\right)^{n} u(-n-1)\right] = \frac{z}{z-\frac{1}{\alpha}} \quad |z| < \frac{1}{\alpha}$$

$$z\left[\left(\frac{1}{\alpha}\right)^{n} u(-n-1)\right] = \frac{-z}{z-\frac{1}{\alpha}} \quad |z| < \frac{1}{\alpha}$$

$$\frac{z}{z-\alpha} + \frac{-z}{z-\frac{1}{\alpha}}$$

$$\frac{z\left(z-\frac{1}{\alpha}\right) - z\left(z-\alpha\right)}{\left(z-\alpha\right)\left(z-\frac{1}{\alpha}\right)}$$

$$= \frac{\cancel{z^2} - \frac{1}{\alpha}z - \cancel{z^2} + \alpha z}{\left(z-\alpha\right)\left(z-\frac{1}{\alpha}\right)} \qquad \boxed{X(z) = \frac{z\left(\alpha - \frac{1}{\alpha}\right)}{\left(z-\alpha\right)\left(z-\frac{1}{\alpha}\right)}}$$

ROC $(|z|>\alpha) \cap \left(|z| < \frac{1}{\alpha}\right)$

(14) $x(n) = n \sin \frac{\pi}{2} n \, u(n)$

$$z\left[\sin \Omega n \, u(n)\right] = \frac{\cancel{z}\sin\Omega}{z^2 - 2z\cos\Omega + 1} \qquad \text{ROC } \boxed{|z|>1}$$

$$z\left[\sin \frac{\pi}{2} n \, u(n)\right] = \frac{\cancel{z}\sin 90^\circ}{z^2 - 2z\cos 90^\circ + 1}$$

$$= \frac{\cancel{z}}{z^2+1}$$

$$z\left[n \sin \frac{\pi}{2} n \, u(n)\right] = -z \frac{d}{dz}\left(\frac{z}{z^2+1}\right)$$

$$-z\left[\frac{(z^2+1).1 - z\,2z}{(z^2+1)^2}\right]$$

$$-z\left[\frac{z^2+1-2z^2}{(z^2+1)^2}\right]$$

$$-z\left[\frac{1-z^2}{(z^2+1)^2}\right]$$

$$= \frac{-z+z^3}{(z^2+1)^2}$$

$$\boxed{X(z) = \frac{z^3 - z}{(z^2+1)^2}}$$

(15) $x(n) = \left(\frac{1}{3}\right)^n \cos\frac{\pi}{4} n\, u(n)$

ROC $\boxed{|z| > \frac{1}{3}}$

$$Z\left[\cos\Omega n\, u(n)\right] = \frac{z^2 - z\cos\Omega}{z^2 - 2z\cos\Omega + 1}$$

$$Z\left[\cos\frac{\pi}{4} n\, u(n)\right] = \frac{z^2 - z\cos\pi/4}{z^2 - 2z\cos\pi/4 + 1}$$

$$= \frac{z^2 - \frac{z}{\sqrt{2}}}{z^2 - 2z\frac{1}{\sqrt{2}} + 1} = \frac{z^2 - \frac{z}{\sqrt{2}}}{z^2 - 2\sqrt{2} + 1}$$

$$Z\left(\left(\frac{1}{3}\right)^n \cos\pi/4\, n\, u(n)\right) = \frac{\left(\frac{z}{1/3}\right)^2 - \frac{\left(\frac{z}{1/3}\right)}{\sqrt{2}}}{\left(\frac{z}{1/3}\right)^2 - \frac{z}{1/3}\sqrt{2} + 1}$$

$$X(z) = \frac{9z^2 - 3\sqrt{2}z}{9z^2 - 3\sqrt{2}z + 1}$$

4M

(16) $x(n) = \sin\left(\frac{\pi}{4} n + \frac{\pi}{2}\right) u(n-2)$

take $n - 2 = k$

$n = k + 2$

$$= \sin\left(\frac{\pi}{4}(k+2) + \frac{\pi}{2}\right) u(k)$$

$$= \sin\left(\frac{\pi}{4}k + \frac{\pi}{2} + \frac{\pi}{2}\right) u(k)$$

$$= \sin\left(\frac{\pi}{4}k + \pi\right) u(k)$$

$$= \left[\sin\frac{\pi}{4}k\,\underbrace{\cos\pi}_{-1} + \cos\frac{\pi}{4}k\,\underbrace{\sin\pi}_{0}\right]u(k)$$

$$= -\sin\frac{\pi}{4}k\;u(k)$$

$$Z\left[\sin\Omega n\,u(n)\right] = \frac{z\sin\Omega}{z^2 - 2z\cos\Omega + 1}$$

$$Z\left[\sin\frac{\pi}{4}k\,u(k)\right] = \frac{z\sin\pi/4}{z^2 - 2z\cos\pi/4 + 1} \qquad |z| > 1$$

$$Z\left[\underbrace{-\sin\pi/4\;k\;u(k)}_{x(k)}\right] = \frac{-\frac{z}{\sqrt{2}}}{z^2 - \sqrt{2}z + 1}$$

$$Z\left(x(n-2)\right) = z^{-2}\,\frac{-\frac{z}{\sqrt{2}}}{z^2 - \sqrt{2}z + 1}$$

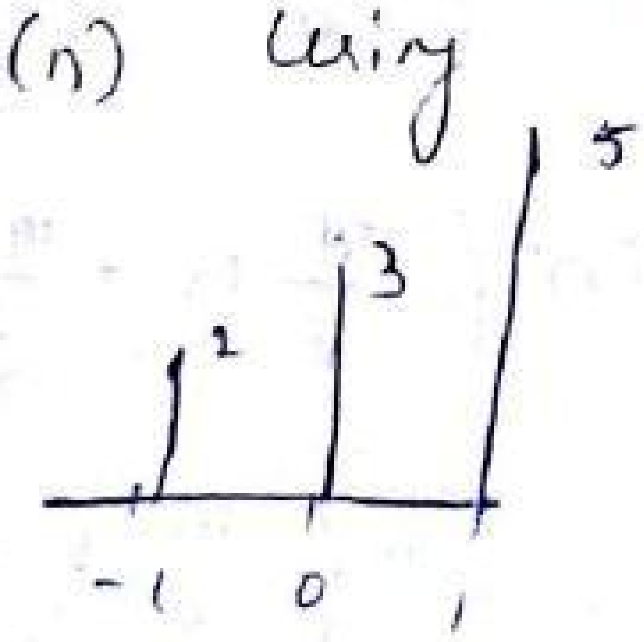
$$\boxed{X(z) = \frac{-\frac{1}{\sqrt{2}}}{z\left(z^2 - \sqrt{2}z + 1\right)}} \qquad |z| > 1$$

(17) $x(n) = \{2, \underset{\uparrow}{3}, 5\}$

$h(n) = \{\underset{\uparrow}{3}, 2\}$

find $y(n) = x(n) + h(n)$ using Z-T

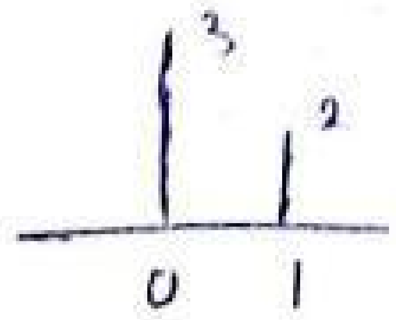
$\Rightarrow$ $x(n) = \{2, \underset{\uparrow}{3}, 5\}$



$$x(n) = 2\delta(n+1) + 3\delta(n) + 5\delta(n-1)$$

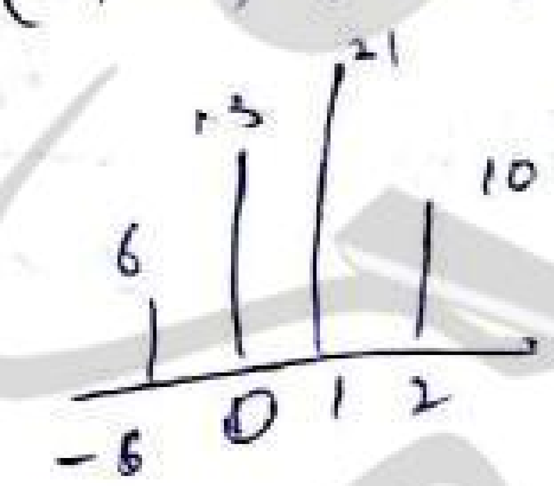
$$X(z) = 2z + 3 + 5z^{-1}$$

$$h(n) = \{\underset{\uparrow}{3}, 2\}$$



$$h(n) = 3\delta(n) + 2\delta(n-1)$$
$$H(z) = 3 + 2z^{-1}$$
$$y(n) = x(n) * h(n)$$

$$Y(z) = X(z) \cdot H(z)$$
$$= (2z + 3 + 5z^{-1}) \cdot (3 + 2z^{-1})$$
$$= 6z + 9 + 15z^{-1} + 4 + 6z^{-1} + 10z^{-2}$$
$$Y(z) = 6z + 13 + 21z^{-1} + 10z^{-2}$$
$$y(n) = 6\delta(n+1) + 13\delta(n) + 21\delta(n-1) + 10\delta(n-2)$$
$$\{6, \underset{\uparrow}{13}, 21, 10\}$$



INVERSE Z-Transform

WKT

$$2^n u(n) \xrightarrow{Z.T} \frac{z}{z-2} \quad |z| > 2$$

$$\frac{1}{2}(-3)^n u(n) \xrightarrow{Z.T} \frac{\frac{1}{2}z}{z+3} \quad |z| > 3$$

$$-3(4)^n u(-n-1) \xrightarrow{Z.T} \frac{3z}{z-4} \quad |z| < 4$$

$$\frac{\frac{1}{3}z}{z-2} \quad |z| > 2 \xrightarrow{I.Z.T} \frac{1}{3}(2)^n u(n)$$

$$\frac{\frac{1}{2}z}{z+3} \quad |z| > 3 \xrightarrow{I.Z.T} \frac{1}{2}(-3)^n u(n)$$

$$\frac{3z}{z-4} \quad |z|<4 \longrightarrow -3(4)^n u(-n-1)$$

$$\frac{-\frac{1}{5}z}{z-2} \quad |z|<2 \longrightarrow \frac{1}{5} 2^n u(-n-1)$$

$\{ n a^{n-1} u(n) \rightarrow \frac{z}{(z-a)^2}$

$$\frac{3z}{(z-2)^2} \quad |z|>2 \longrightarrow 3n\, 2^{n-1} u(n)$$

$$\frac{-5z}{z-4} \quad |z|>6 \longrightarrow -5(4)^n u(n)$$

(1 ×5m)

Inverse Z-Transform using Partial fraction Meth

(1) $X(z) = \frac{5z}{(z-2)(z+4)}$ i) $|z|>4$ find inverse Z-T

ii) $|z|<2$

iii) $2<|z|<4$

$$\frac{X(z)}{z} = \frac{5}{(z-2)(z+4)}$$

$$\frac{5}{(z-2)(z+4)} = \frac{A}{(z-2)} + \frac{B}{(z+4)}$$

$$5 = A(z+4) + B(z-2)$$

put $z=-4$

$$5 = A(0) + B(-6)$$

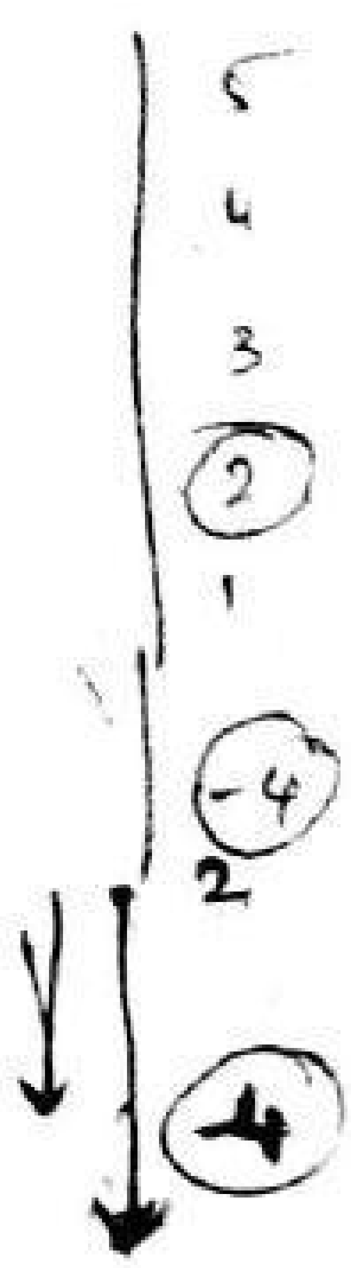
$$\boxed{\frac{-5}{6} = B}$$

Put $z=2$

$$5 = A(6) + B(0)$$

$$\boxed{\frac{5}{6} = A}$$

$$\frac{X(z)}{z} = \frac{\frac{5}{6}}{(z-2)} + \frac{-\frac{5}{6}}{(z+4)}$$



$$X(z) = \frac{\frac{5}{6}z}{(z-2)} + \frac{-\frac{5}{6}z}{(z+4)}$$

i) $x(n) = \frac{5}{6}(2)^n u(n) + \frac{5}{6}(-4)^n u(n)$

ii) $x(n) = -\frac{5}{6}(2)^n(-u-1) + \frac{5}{6}(-4)^n u(-n-1)$

iii) $x(n) = \frac{5}{6}(2)^n u(n) + \frac{5}{6}(-4)^n u(-n-1)$

(2) $X(z) = \dfrac{1 - z^{-1} + z^{-2}}{(1-\frac{1}{2}z^{-1})(1-z^{-1})(1-2z^{-1})}$ $\quad 1 < |z| < 2$

Multiply by z^3

$$= \frac{z^3 - z^2 + z}{(z-\frac{1}{2})(z-1)(z-2)}$$

$$X(z) = \frac{z(z^2 - z + 1)}{(z-\frac{1}{2})(z-1)(z-2)}$$

$$\frac{X(z)}{z} = \frac{z^2 - z + 1}{(z-\frac{1}{2})(z-1)(z-2)}$$

$$z^2 - z + 1 = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-1} + \frac{C}{z-2}$$

$$z^2 - z + 1 = A(z-1)(z-2) + B\left(z-\frac{1}{2}\right)(z-2) + C\left(z-\frac{1}{2}\right)(z-1)$$

Put $z = 1$

$$1 - 1 + 1 = 0 - \frac{1}{2}B + 0$$

$$\boxed{-2 = B}$$

Put $z = 2$

$$4 - 2 + 1 = 0 + 0 + C\left(\frac{3}{2}\right)$$

$$3 = \frac{3}{2}C$$

$$\frac{\cancel{6}^{2}}{\cancel{3}} = C$$

$$\boxed{2 = C}$$

Put $z = \frac{1}{2}$

$$\frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}A$$

$$\frac{3}{4} = \frac{3}{4}A$$

$$\boxed{A = 1}$$

$$\frac{X(z)}{z} = \frac{1}{z-\frac{1}{2}} + \frac{-2}{z-1} + \frac{2}{z-2}$$

$$X(z) = \frac{z}{z-\frac{1}{2}} + \frac{-2z}{z-1} + \frac{2z}{z-2}$$

$$x(n) = \left(\frac{1}{2}\right)^n u(n) - 2(1)^n u(n) \; 2(2)^n(-u-1)$$

(3) $X(z) = \dfrac{z+2}{2z^2 - 7z + 3} \qquad |z| > 3$

$$= \frac{z+2}{2\left(z^2 + \frac{7}{2}z + \frac{3}{2}\right)}$$

$$= \frac{z+2}{2(z-3)\left(z-\frac{1}{2}\right)}$$

$$X(z) = \frac{\frac{1}{2}(z+2)}{(z-3)\left(z-\frac{1}{2}\right)}$$

Mul & Div by z

$$= \frac{\frac{1}{2}\, z\,(z+2)}{z(z-3)\left(z-\frac{1}{2}\right)}$$

$$\frac{X(z)}{z} = \frac{\frac{1}{2}(z+2)}{z(z-3)\left(z-\frac{1}{2}\right)}$$

$$\frac{\frac{1}{2}(z+2)}{z(z-3)\left(z-\frac{1}{2}\right)} = \frac{A}{z} + \frac{B}{(z-3)} + \frac{C}{\left(z-\frac{1}{2}\right)}$$

$$\frac{1}{2}(z+2) = A(z-3)\left(z-\frac{1}{2}\right) + B(z)\left(z-\frac{1}{2}\right) + C(z)(z-3$$

Put $z = 3$.

$$\frac{5}{2} = 0 + \frac{15}{2}B \qquad \boxed{\frac{1}{3} = B}$$

Put $z = \frac{1}{2}$

$$\frac{5}{4} = 0 + 0 + \frac{-5}{\cancel{4}} C$$

$$\boxed{C = -1}$$

Put $z = 0$

$$1 = \frac{3}{2} A$$

$$\boxed{\frac{2}{3} = A}$$

$$\frac{X(z)}{z} = \frac{2/3}{z} + \frac{1/3}{(z-3)} + \frac{-1}{z-\frac{1}{2}}$$

$$X(z) = \frac{\frac{2}{3}\cancel{z}}{\cancel{z}} + \frac{\frac{1}{3}z}{z-3} + \frac{-1z}{\left(z-\frac{1}{2}\right)}$$

3 ↑ up

$\frac{1}{2}$

$$x(n) = \frac{2}{3}\delta(n) + \frac{1}{3}(3)^n u(n) - 1\left(\frac{1}{2}\right)^n u(n)$$

(4) H.W $X(z) = \frac{z+1}{3z^2 - 4z + 1} \quad |z| > 1$

Multiply & Divide by z

$$X(z) = \frac{z(z+1)}{z\, 3\left(z^2 - \frac{4}{3}z + \frac{1}{3}\right)}$$

$$\frac{X(z)}{z} = \frac{z+1}{3z(z-1)\left(z-\frac{1}{3}\right)}$$

$$\frac{1}{3}(z+1) = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-\frac{1}{3}}$$

$\frac{1}{3}(z^2+1) = A(z-1)(z-\frac{1}{3}) + B(z)(z-\frac{1}{3}) + C(z)(z-1)$

$z-1=0$

$z=1$

$\frac{2}{3} = \frac{2}{3}B$

$\boxed{B=1}$

$z=\frac{1}{3}$

$\frac{4}{9} = -\frac{2}{9}C$

$\boxed{-2=C}$

$z=0$

$\frac{1}{3} = \frac{1}{3}A$ $\boxed{A=1}$

$X(z) = \frac{\cancel{z}}{\cancel{z}}1 + \frac{z}{z-1} + \frac{-2z}{z-\frac{1}{3}}$

$\downarrow$ I.Z.T

$x(n) = \delta(n) + (1)^n u(n) - 2(\frac{1}{3})^n u(n)$

(5) $X(z) = \dfrac{2z^3 - 5z^2 + z + 3}{z^2 - 3z + 2}$ $|z| < 1$

Given question is Improper fraction

$$\begin{array}{r|l} & 2z + 1 \\ \hline z^2 - 3z + 2 & 2z^3 - 5z^2 + z + 3 \\ & 2z^3 - 6z^2 + 4z \\ & \underset{(-)}{} \quad \underset{(+)}{} \quad \underset{(-)}{} \\ \hline & \quad z^2 - 3z + 3 \\ & \quad z^2 - 3z + 2 \\ & \quad \underset{(-)}{} \quad \underset{(+)}{} \quad \underset{-}{} \\ \hline & \qquad 1 \end{array}$$

$$X(z) = 2z + 1 + \frac{1}{z^2 - 3z + 2} \rightarrow X'(z)$$

$$X'(z) = \frac{1}{z^2 - 3z + 2}$$

$$X'(z) = \frac{1}{(z-2)(z-1)}$$

$$X'(z) = \frac{z}{z(z-2)(z-1)}$$

$$\frac{X'(z)}{z} = \frac{1}{z(z-2)(z-1)}$$

$$= \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z-1}$$

$$1 = A(z-2)(z-1) + B(z)(z-1) + C(z)(z-2)$$

Put $z=2$

$1 = 0 + 2B + 0$

$$\boxed{B = \frac{1}{2}}$$

Put $z=1$

$1 = 0 + 0 + -C$

$$\boxed{C = -1}$$

Put $z=0$

$1 = 2A$

$$\boxed{A = \frac{1}{2}}$$

$$\frac{x'(z)}{z} = \frac{\frac{1}{2}}{z} + \frac{\frac{1}{2}}{z-2} + \frac{-1}{z-1}$$

$$x'(z) = \frac{\frac{1}{2}z}{z} + \frac{\frac{1}{2}z}{z-2} + \frac{-1z}{z-1}$$

$$x'(z) = \frac{1}{2} + \frac{\frac{1}{2}z}{z-2} + \frac{-z}{z-1}$$

$$x(z) = 2z + \underbrace{1 + \frac{1}{2}}_{\frac{3}{2}} + \frac{\frac{1}{2}z}{z-2} + \frac{-z}{z-1}$$

$$x(n) = 2\delta(n+1) + \frac{3}{2}\delta(n) - \frac{1}{2}(2)^n u(-n-1) + (1)^n u(-n-1)$$

H.W

6

$$X(z) = \frac{z^3 - 3z^2 + 2z + 3}{z^2 - 5z + 6} \quad |z| > 3$$

$$\begin{array}{r|l} & z+2+\frac{1}{z} \\ \hline z^2-5z+6 & z^3-3z^2+2z+3 \\ & z^3-5z^2+6z \\ & -\quad+\quad- \\ \hline & 2z^2-4z+3 \\ & 2z^2-10z+12 \\ & -\quad+\quad- \\ \hline & 6z-9 \\ & \cancel{6z-30} \end{array}$$

$\frac{6}{z}\times 5z$

$\frac{6}{z}\times 6 = \frac{36}{z}$

$$x(z) = z+2+\frac{6z-9}{z^2-5z+6} \quad x'(z)$$

$$\frac{x'(z)}{z} = \frac{z(6z-9)}{z^2-6z+6}$$

5

$$\frac{6z-9}{z^2-6z+6} = \cancel{\frac{A}{z-4.73}}\ \frac{A}{z-4.73} + \frac{B}{z-1.267}$$

$$z(6z-9) = A(z-1.267) + B(z-4.73)$$

put $z = 1.267$

$$-1.771 = -3.463\ B$$

$\boxed{\cancel{B = 0.403}}$ $\boxed{B = 0.511}$

put $z = 4.73$

$$\overset{91.667}{\cancel{14.38}} = 3.463\ A$$

$\boxed{\cancel{A = 5.596}}$ $A = 26.47$

$$x'(z) = \frac{26.47z}{z-4.73} + \frac{0.511z}{z-1.267}$$

$$X(z) = z + 2 + \frac{26.47z}{z - 4.73} + \frac{0.511z}{z - 1.267}$$

$$= \delta(n+1) + 2\delta(n) + 26.47(4.73)^n u(n) + 0.511(1.267)^n u(n)$$

Ans

$$\frac{1}{2}\delta(n) + 3(3)^n u(n) - \frac{3}{2}(2)^n u(n)$$

(7) $x(z) = \dfrac{2z^2+z}{(z-1)(z-\frac{1}{2})^2}$ $\quad |z| > 1$

$$x(z) = \frac{z(2z+1)}{(z-1)(z-\frac{1}{2})^2}$$

$$\frac{x(z)}{z} = \frac{(2z+1)}{(z-1)(z-\frac{1}{2})^2}$$

$$= \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} + \frac{C}{(z-\frac{1}{2})^2}$$

$\therefore$ $2z+1 = A(z-\frac{1}{2})^2 + B(z-1)(z-\frac{1}{2}) + C(z-1)$

Put $z = \frac{1}{2}$

$2 = 0 + 0 = -\frac{1}{2}C$

$\boxed{C = -4}$

Put $z = 1$

$3 = A\frac{1}{4} + 0 + 0$

$\boxed{12 = A}$

Equate the Co-efficients of z^2

$0 = A + B$

$A = -B$

$\boxed{B = -12}$

$$\frac{X(z)}{z} = \frac{12}{z-1} + \frac{-12}{z-\frac{1}{2}} + \frac{-4}{\left(z-\frac{1}{2}\right)^2}$$

$$X(z) = \frac{12z}{z-1} + \frac{-12z}{z-\frac{1}{2}} + \frac{-4z}{\left(z-\frac{1}{2}\right)^2} \quad \left\{ \frac{z}{(z-a)^2} = n a^{n-1} u(n) \right.$$

$$x(n) = 12(1)^n u(n) - 12\left(\frac{1}{2}\right)^n u(n) - \cancel{\frac{-4}{2}\left(\frac{1}{2}\right)^{n-1} u(n)}$$

$$-4 \times n \left(\frac{1}{2}\right)^{n-1} u(n)$$

$$x(n) = 12(1)^n u(n) - 12\left(\frac{1}{2}\right)^n u(n) - \cancel{\left(\frac{1}{2}\right)^{n-1} u(n)}$$

$$-n 8 \left(\frac{1}{2}\right)^{n-1} u(n)$$

$$x(n) = 12(1)^n u(n) - 12\left(\frac{1}{2}\right)^n u(n) - 4n u(n) \left(\frac{1}{2}\right)^{n-1}$$

H.W (8)

$$X(z) = \frac{9}{(z+2)(z-\frac{1}{2})^2} \qquad |z| < \frac{1}{2}$$

Multiply & divide by z

$$\frac{X(z)}{z} = \frac{9z}{z(z+2)(z-\frac{1}{2})^2}$$

$$\frac{X(z)}{z} = \frac{9}{z(z+2)(z-\frac{1}{2})^2}$$

$$9 = \frac{A}{z} + \frac{B}{(z+2)} + \frac{C}{z-\frac{1}{2}} + \frac{D}{(z-\frac{1}{2})^2}$$

$$9 = A(z+2)(z-\tfrac{1}{2})^2 + B(z)(z-\tfrac{1}{2})^2 + C(z)(z+2)(z-\tfrac{1}{2})$$
$$+ D(z)(z+2)$$

Put $z = -2$

$$9 = 4.5B$$
$$\boxed{B = 2}$$

Put $z = \frac{1}{2}$

$$9 = 1.25D$$
$$\boxed{D = 7.2}$$

Put $z = 0$

$$9 = 0.5A$$
$$\boxed{A = 18}$$

Co-efficient of z^3

$$A + B + C = 0$$
$$18 + 2 + C = 0$$
$$\boxed{-20 = C}$$

$$-z\frac{d}{dz}X(z) = -az^{-1}\frac{z}{z-a}$$

inverse transform

$$n\,x(n) = -a\,a^{n-1}u(n-1)$$

$$= -a\,a^{n}a^{-1}u(n-1)$$

$$n\,x(n) = -a^{n}u(n-1)$$

$$\therefore \boxed{x(n) = \frac{-a^{n}u(n-1)}{n}}$$

8M

(12) $X(z) = \dfrac{z^2-2z}{z^2-z+1}$ $|z|>1$

$$X(z) = \frac{z(z-2z)}{\left(z-\left(\frac{1}{2}+\frac{j\sqrt{3}}{2}\right)\right)\left(z-\left(\frac{1}{2}-\frac{j\sqrt{3}}{2}\right)\right)}$$

$$\frac{X(z)}{z} = \frac{(z-2)}{\left(z-\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)\right)\left(z-\left(\frac{1}{2}-\frac{j\sqrt{3}}{2}\right)\right)}$$

$$= \frac{A}{z-\left(\frac{1}{2}+\frac{\sqrt{3}j}{2}\right)} + \frac{B}{z-\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}$$

$$z-2 = A\left(z-\left(\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)\right) + B\left(z-\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)\right)$$

put $z = \frac{1}{2}-\frac{\sqrt{3}}{2}j$

$$\frac{1}{2}-\frac{\sqrt{3}}{2}j-2 = A(0) + B\left(\cancel{\tfrac{1}{2}}-\frac{\sqrt{3}}{2}j-\cancel{\tfrac{1}{2}}-\frac{\sqrt{3}}{2}j\right)$$

$$-\frac{3}{2}-\frac{\sqrt{3}}{2}j = -\sqrt{3}j\,B$$

$$-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}j\right) = -\sqrt{3}j\,B$$

$$\frac{\frac{3}{2}+\frac{\sqrt{3}}{2}j}{\sqrt{3}j} = B$$

$$\boxed{\frac{1}{2}-\frac{\sqrt{3}}{2}j = B}$$

put $z = \frac{1}{2}+\frac{\sqrt{3}}{2}j$

$$\frac{1}{2}+\frac{\sqrt{3}}{2}j-2 = A\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j-\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)+B(0)$$

$$-\frac{3}{2}+\frac{\sqrt{3}}{2}j = \sqrt{3}j\,A$$

$$\frac{-\frac{3}{2}+\frac{\sqrt{3}}{2}j}{\sqrt{3}j} = A$$

$$\boxed{A = \frac{1}{2}+\frac{\sqrt{3}}{2}j}$$

$$X(z) = \frac{1}{2}+\frac{\sqrt{3}}{2}j\,\frac{z}{z-\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)} + \frac{1}{2}-\frac{\sqrt{3}}{2}j\,\frac{z}{z-\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}$$

Inverse Z-T

$$x(n) = \frac{1}{2}+\frac{\sqrt{3}}{2}j\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)^n u(n) + \left(\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)^n u(n)$$

$$\downarrow \qquad\qquad 1^n e^{-j60n}$$

$$(1\angle 60^\circ)^n$$

$$(1\,e^{j60^\circ})^n \Rightarrow 1^n e^{j60n}$$

$$x(n) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right) 1^n \left(\cos 60n + j\sin 60n\right)u(n) + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)1^n$$

$$\left(\cos 60n - j\sin 60n\right)u(n)$$

$$= \left(\frac{1}{2}\cos\frac{\pi}{3}n + \cancel{j\frac{1}{2}\sin\frac{\pi}{3}n} + \cancel{j\frac{\sqrt{3}}{2}\cos\frac{\pi}{3}n} - \frac{\sqrt{3}}{2}\sin\frac{\pi}{3}n\right)u(n) +$$

$$\left(\frac{1}{2}\cos\frac{\pi}{3}n - \cancel{j\frac{1}{2}\sin\frac{\pi}{3}n} - \cancel{j\frac{\sqrt{3}}{2}\cos\frac{\pi}{3}n} - \frac{\sqrt{3}}{2}\sin\frac{\pi}{3}n\right)u(n)$$

$$\cos\frac{\pi}{3}n\,u(n) - \sqrt{3}\sin\frac{\pi}{3}n\,u(n)$$

H.W

(13) $$X(z) = \frac{1+z^{-1}}{1 - z^{-1} + 0.5z^{-2}} \qquad |z| > 1$$

Ans $x(n) = \left(\frac{1}{\sqrt{2}}\right)^n \cos\frac{\pi}{4}n\,u(n) + 3\left(\frac{1}{\sqrt{2}}\right)^n \sin\frac{\pi}{4}u(n)$

Multiply by z^2

$$X(z) = \frac{z^2 + z}{z^2 - z + 0.5}$$

$$X(z) = \frac{z(z+1)}{z^2 - z + 0.5}$$

$$\frac{X(z)}{z} = \frac{z+1}{z^2 - z + 0.5}$$

$$= \frac{z+1}{\left(z - \left(\frac{1}{2} + \frac{1}{2}j\right)\right)\left(z - \left(\frac{1}{2} - \frac{1}{2}j\right)\right)}$$

$$= \frac{A}{z - \left(\frac{1}{2} + \frac{1}{2}j\right)} + \frac{B}{z - \left(\frac{1}{2} - \frac{1}{2}j\right)}$$

$$= A\left(z-\left(\tfrac{1}{2}-\tfrac{1}{2}j\right)\right) + B\left(z-\left(\tfrac{1}{2}+\tfrac{1}{2}j\right)\right)$$

put $z = \frac{1}{2}-\frac{1}{2}j$

$$\frac{1}{2}-\frac{1}{2}j+1 = 0 + B\left(\cancel{\tfrac{1}{2}}-\tfrac{1}{2}j-\cancel{\tfrac{1}{2}}-\tfrac{1}{2}j\right)$$

$$\frac{3}{2}-\frac{1}{2}j = -jB$$

$$\frac{\frac{3}{2}-\frac{1}{2}j}{-j} = B$$

$$\boxed{B = \frac{1}{2}+\frac{3}{2}j}$$

put $z = \frac{1}{2}+\frac{1}{2}j$

$$\frac{1}{2}+\frac{1}{2}j+1 = A\left(\cancel{\tfrac{1}{2}}+\tfrac{1}{2}j-\cancel{\tfrac{1}{2}}+\tfrac{1}{2}j\right)$$

$$\frac{\frac{3}{2}+\frac{1}{2}j}{j} = A$$

$$\boxed{A = \frac{1}{2}-\frac{3}{2}j}$$

$$x(z) = \frac{1}{2}-\frac{3}{2}j\ \frac{z}{z-\left(\frac{1}{2}+\frac{1}{2}j\right)} + \frac{1}{2}+\frac{3}{2}j\ \frac{z}{z-\left(\frac{1}{2}-\frac{1}{2}j\right)}$$

$\downarrow$ I.Z.T

$$x(n) = \left(\tfrac{1}{2}-\tfrac{3}{2}j\right)\left(\tfrac{1}{2}+\tfrac{1}{2}j\right)^n u(n) + \left(\tfrac{1}{2}+\tfrac{3}{2}j\right)\left(\tfrac{1}{2}-\tfrac{1}{2}j\right)^n u(n)$$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$(0.707\angle 45)^n$ $\qquad\qquad$ $(0.707)^n e^{-j45n}$

$(0.707 e^{j45})^n \Rightarrow (0.707)^n e^{j45n}$

$$= \left(\frac{1}{2}-\frac{3}{2}j\right)\left((0.707)^n e^{j45n}\right)u(n) + \left(\frac{1}{2}+\frac{3}{2}j\right)\left((0.707)^n e^{-j45n}\right)u(n)$$

$$= \left(\frac{1}{2}-\frac{3}{2}j\right)(0.707)^n\left(\cos 45n + j\sin 45n\right)u(n) + \left(\frac{1}{2}+\frac{3}{2}j\right)(0.707)^n$$

$$\left(\cos 45n - j\sin 45n\right)u(n)$$

$$= \left(\frac{1}{\sqrt{2}}\right)^n\left[\frac{1}{2}\cos\frac{\pi}{4}n + \frac{1}{2}j\sin\frac{\pi}{4}n - \frac{3}{2}j\cos\frac{\pi}{4}n + \frac{3}{2}\sin\frac{\pi}{4}n\right]u(n)$$

$$+ \left[\frac{1}{2}\cos\frac{\pi}{4}n - \frac{1}{2}j\sin\frac{\pi}{4}n + \frac{3}{2}j\cos\frac{\pi}{4}n + \frac{3}{2}\sin\frac{\pi}{4}n\right]u(n)$$

$$= \left(\frac{1}{\sqrt{2}}\right)^n\left[\cos\frac{\pi}{4}n + 3\sin\frac{\pi}{4}n\right]u(n)$$

$$= \left(\frac{1}{\sqrt{2}}\right)^n\cos\frac{\pi}{4}n\,u(n) + 3\left(\frac{1}{\sqrt{2}}\right)^n\sin\frac{\pi}{4}n\,u(n)$$

H.W

(14) $X(z) = \dfrac{1}{(1+z^{-1})(1-z^{-1})^2} \quad |z|>1$

Multiply by z^3

$$X(z) = \frac{z^3}{(z+1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z+1)(z-1)^2}$$

$$z^2 = \frac{A}{z+1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$z^2 = A(z-1)^2 + B(z+1)(z-1) + C(z+1)$$

put $z=1$

$$1 = 2C \qquad \boxed{C=\frac{1}{2}}$$

$z = -1$

$1 = 4A$

$$\boxed{A = \frac{1}{4}}$$

Co-eff of z^2

$A + B = 1$

$\frac{1}{4} + B = 1$

$B = 1 - \frac{1}{4}$

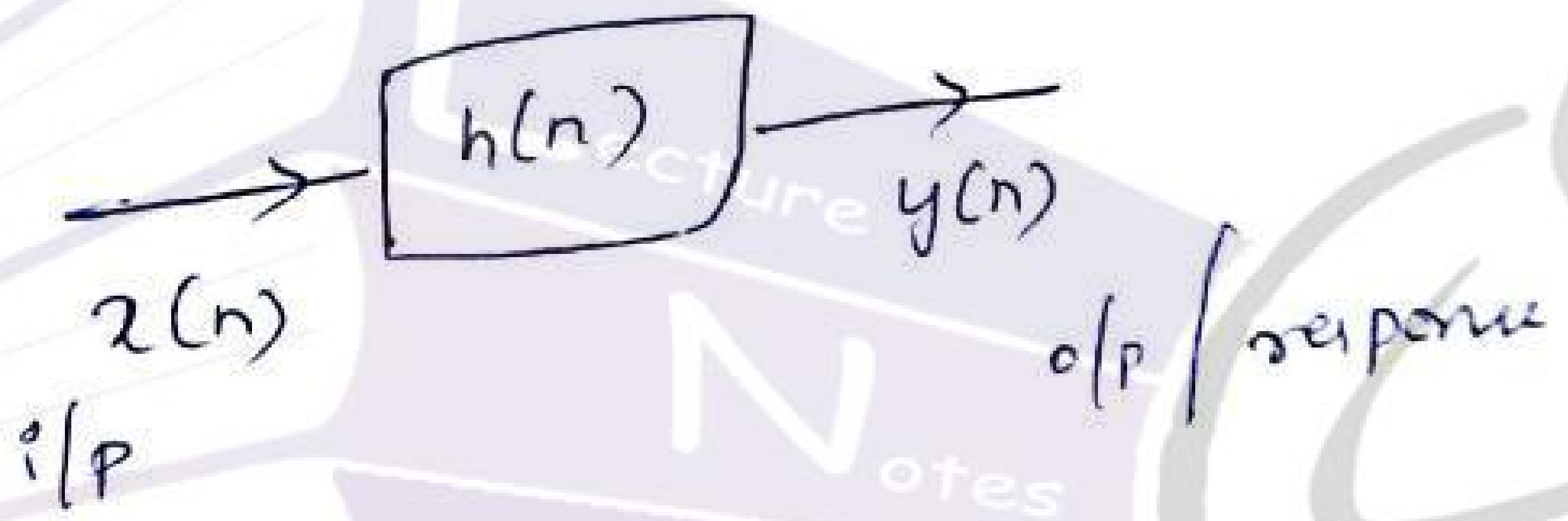
$$\boxed{B = \frac{3}{4}}$$

$$\frac{z}{(z-a)^2} = n a^{n-1} u(n)$$

$$x(z) = \frac{1}{4}\frac{z}{z+1} + \frac{\frac{3}{4}z}{z-1} + \frac{\frac{1}{2}z}{(z-1)^2}$$

$$= \frac{1}{4} 1^n u(n) + \frac{3}{4}(1)^n u(n) + \frac{1}{2} n (1)^{n-1} u(n)$$

(5) TRANSFORM ANALYSIS OF LTI S/m



$$y(n) = x(n) * h(n)$$

$$\downarrow Z.T$$

$$Y(z) = X(z) . H(z)$$

Transfer f^n $\boxed{H(z) = \frac{Y(z)}{X(z)}}$

$$\downarrow I.Z.T$$

$$h(n)$$

$Y(z) = X(z) \cdot H(z)$

$\downarrow$ I.Z.T

$y(n) = ?$

$$\underset{u(n)}{\longrightarrow} \boxed{h(n)} \underset{s(n)}{\longrightarrow}$$

step i/p   step response

$y(n) = x(n) * h(n)$

$s(n) = u(n) * h(n)$

$S(z) = U(z) \cdot H(z)$

$\downarrow$ I.Z.T

$s(n)$

$x(n) \xrightarrow{Z.T} X(z)$

$x(n-1) \longrightarrow z^{-1} X(z)$

$y(n-2) \longrightarrow z^{-2} Y(z)$

$x(n) \rightarrow$ i/p

$y(n) \rightarrow$ o/p / response

$h(n) \rightarrow$ impulse response

$H(z) \rightarrow$ Z-Transform of impulse response, Transfer f^n

$s(n) \rightarrow$ step response

$S(z) \rightarrow$ Z-T of step response

1. For the given difference equation find transfer function.

① $y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n) + \frac{1}{4}x(n-1) - \frac{1}{8}x(n-2)$

$\rightarrow$ $H(z) = ?$

$$Y(z) - z^{-1}Y(z) + \frac{1}{4}z^{-2}Y(z) = X(z) + \frac{1}{4}z^{-1}X(z) - \frac{1}{8}z^{-2}X(z)$$

$$Y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] = X(z)\left[1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$

Multiply & Divide by z^2

$$H(z) = \frac{z^2 + \frac{1}{4}z - \frac{1}{8}}{z^2 - z + \frac{1}{4}}$$

② Find transfer function, impulse Response for causal & Non-Causal s/m also calculate stability of the s/m

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$$

$\rightarrow$ $H(z) = ?$

$$Y(z) - z^{-1}\frac{3}{4}Y(z) + z^{-2}\frac{1}{8}Y(z) = X(z)$$

$$Y(z)\left[1 - z^{-1}\frac{3}{4} + \frac{z^{-2}}{8}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}\frac{3}{4} + \frac{z^{-2}}{8}}$$

$$H(z) = \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}} = \frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

Poles

$z = \frac{1}{2}$

$z = \frac{1}{4}$

Poles value are converging because less than 1. ∴ s/m is stable.

$$\frac{H(z)}{z} = \frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$z = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}}$$

$$z = A\left(z-\frac{1}{4}\right) + B\left(z-\frac{1}{2}\right)$$

put $z = \frac{1}{4}$

$$\frac{1}{4} = 0 + -\frac{1}{4}B$$

$$\boxed{B = -1}$$

$z = \frac{1}{2}$

$$\frac{1}{2} = \frac{1}{4}A$$

$$\boxed{2 = A}$$

$$H(z) = \frac{2z}{z-\frac{1}{2}} + \frac{-z}{z-\frac{1}{4}}$$

Causal

$$h(n) = 2\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$$

Non - Causal

$$h(n) = -2\left(\frac{1}{2}\right)^n u(-n-1) + \left(\frac{1}{4}\right)^n u(-n-1)$$

③ $y(n) - \frac{1}{2}y(n-1) = 2x(n-1)$

find $H(z)$

$h(n)$

$s(n)$

$\rightarrow$ $Y(z) - \frac{1}{2}z^{-1}Y(z) = 2z^{-1}X(z)$

$$Y(z)\left[1 - \frac{1}{2}z^{-1}\right] = 2z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-1}}{\left[1-\frac{1}{2}z^{-1}\right]}$$

$$\boxed{H(z) = \frac{2}{z-\frac{1}{2}}}$$

$$\frac{H(z)}{z} = z^{-1}\frac{2z}{z-\frac{1}{2}}$$

$\downarrow$ I.Z.T

Causal $h(n) = 2\left(\frac{1}{2}\right)^{n-1}u(n-1)$

~~Non - Causal~~

$$\cancel{h(n) = -2\left(\frac{1}{2}\right)^n u(-n-1)}$$

$s(n) = u(n) * h(n)$

$\downarrow Z.T$

$$S(z) = \frac{z}{z-1} \cdot H(z)$$

$$S(z) = \frac{z}{z-1} \cdot \frac{2}{z-\frac{1}{2}}$$

$$\frac{S(z)}{z} = \frac{2}{(z-\frac{1}{2})(z-1)} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}}$$

$$2 = A(z-\tfrac{1}{2}) + B(z-1)$$

$z = \frac{1}{2}$

$$-2 = \frac{1}{2} B$$

$$\boxed{-4 = B}$$

$z = 1$

$$2 = \frac{1}{2} A$$

$$\boxed{A = 4}$$

$$\frac{S(z)}{z} = \frac{4}{z-1} - \frac{4}{z-\frac{1}{2}}$$

$$S(z) = \frac{4z}{z-1} + \frac{-4z}{z-\frac{1}{2}}$$

$\downarrow I.Z.T$

$$s(n) = 4\, 1^n u(n) - 4\left(\tfrac{1}{2}\right)^n u(n)$$

(4) $y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = x(n) - 2x(n-1)$

find if $x(n) = \delta(n) - \frac{1}{3}\delta(n-1)$

$$\rightarrow \quad Y(z) - \frac{5}{6} z^{-1} Y(z) + \frac{1}{6} z^{-2} Y(z) = X(z) - 2z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}\right] = X(z)\left[1 - 2z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1}}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}$$

$$H(z) = \frac{z^2 - 2z}{z^2 - \frac{5}{6} z + \frac{1}{6}}$$

$$H(z) = \frac{z(z-2)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$

$$\frac{H(z)}{z} = \frac{z-2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$

$$z - 2 = \frac{A}{z - \frac{1}{2}} + \frac{B}{z - \frac{1}{3}}$$

$$z - 2 = A\left(z - \frac{1}{3}\right) + B\left(z - \frac{1}{2}\right)$$

$$z = \frac{1}{3}$$

$$-\frac{5}{3} = 0 + -\frac{1}{6} B$$

$$\boxed{10 = B}$$

$$z = \frac{1}{2}$$

$$-\frac{3}{2} = \frac{1}{6} A \qquad \boxed{A = -9}$$

$$H(z) = \frac{-9z}{z-\frac{1}{2}} + \frac{10z}{z-\frac{1}{3}}$$

$$H(z) = -9\left(\frac{1}{2}\right)^n u(n) + 10\left(\frac{1}{3}\right)^n u(n)$$

$$Y(z) = \frac{1-2z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \quad X(z)$$

$$x(n) = \delta(n) - \frac{1}{3}\delta(n-1)$$

$$X(z) = 1 - \frac{1}{3}z^{-1}$$

$$Y(z) = \frac{1-2z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \left(1-\frac{1}{3}z^{-1}\right)$$

$$\times \quad z^2$$

$$Y(z) = \frac{(z-2)\left(z-\frac{1}{3}\right)}{z^2-\frac{5}{6}z+\frac{1}{6}}$$

$$Y(z) = \frac{(z-2)\cancel{\left(z-\frac{1}{3}\right)}}{\left(z-\frac{1}{2}\right)\cancel{\left(z-\frac{1}{3}\right)}}$$

$$Y(z) = \frac{(z-2)}{\left(z-\frac{1}{2}\right)}$$

$$Y(z) = \frac{z}{z-\frac{1}{2}} - \frac{2}{z-\frac{1}{2}}$$

$$Y(z) = \frac{z}{z-\frac{1}{2}} - 2z^{-1}\frac{z}{z-\frac{1}{2}}$$

$$y(n) = \left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{2}\right)^{n-1} u(n-1)$$

(5) $h(n) = \left(\frac{1}{2}\right)^{n-2} u(n-1)$ for the given impulse Response find difference eqn

$$h(n) = \left(\frac{1}{2}\right)^{-1} \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

$$z[u(n)] = \frac{z}{z-1}$$

$$z\left[\frac{1}{2}^n u(n)\right] = \frac{\frac{z}{\frac{1}{2}}}{\frac{z}{\frac{1}{2}} - 1}$$

$$= \frac{z}{z-\frac{1}{2}}$$

$$z\left[\frac{1}{2}^{n-1} u(n-1)\right] = z^{-1} \frac{z}{z-\frac{1}{2}} = \frac{1}{z-\frac{1}{2}}$$

$$z\left[\left(\frac{1}{2}\right)^{-1} \left(\frac{1}{2}\right)^{n-1} u(n)\right] = \frac{2}{z-\frac{1}{2}}$$

$$H(z) = \frac{2}{z-\frac{1}{2}} = \frac{Y(z)}{X(z)}$$

$$= \frac{2z^{-1}}{1 - \frac{1}{2}z^{-1}} = \frac{Y(z)}{X(z)}$$

$$X(z)\, 2z^{-1} = Y(z) - \frac{1}{2}z^{-1}Y(z)$$

$\downarrow$ I.Z.T

$$2x(n-1) = \underline{y(n) - \frac{1}{2}y(n-1)}$$

⑥ $y(n) = x(n) + 0.8x(n-1) + 0.8x(n-2) - 0.49y(n-2)$

find $H(z)$ & plot poles, zeros

→ $y(n) + 0.49y(n-2) = x(n) + 0.8x(n-1) + 0.8x(n-2)$

$\downarrow$ Z.T

$Y(z) + 0.49z^{-2}Y(z) = X(z) + 0.8z^{-1}X(z) + 0.8z^{-2}X(z)$

$Y(z)\left[1 + 0.49z^{-2}\right] = X(z)\left[1 + 0.8z^{-1} + 0.8z^{-2}\right]$

$$\frac{Y(z)}{X(z)} = \frac{1 + 0.8z^{-1} + 0.8z^{-2}}{1 + 0.49z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 + 0.8z + 0.8}{z^2 + 0.49}$$

$H(z)$:

zeros

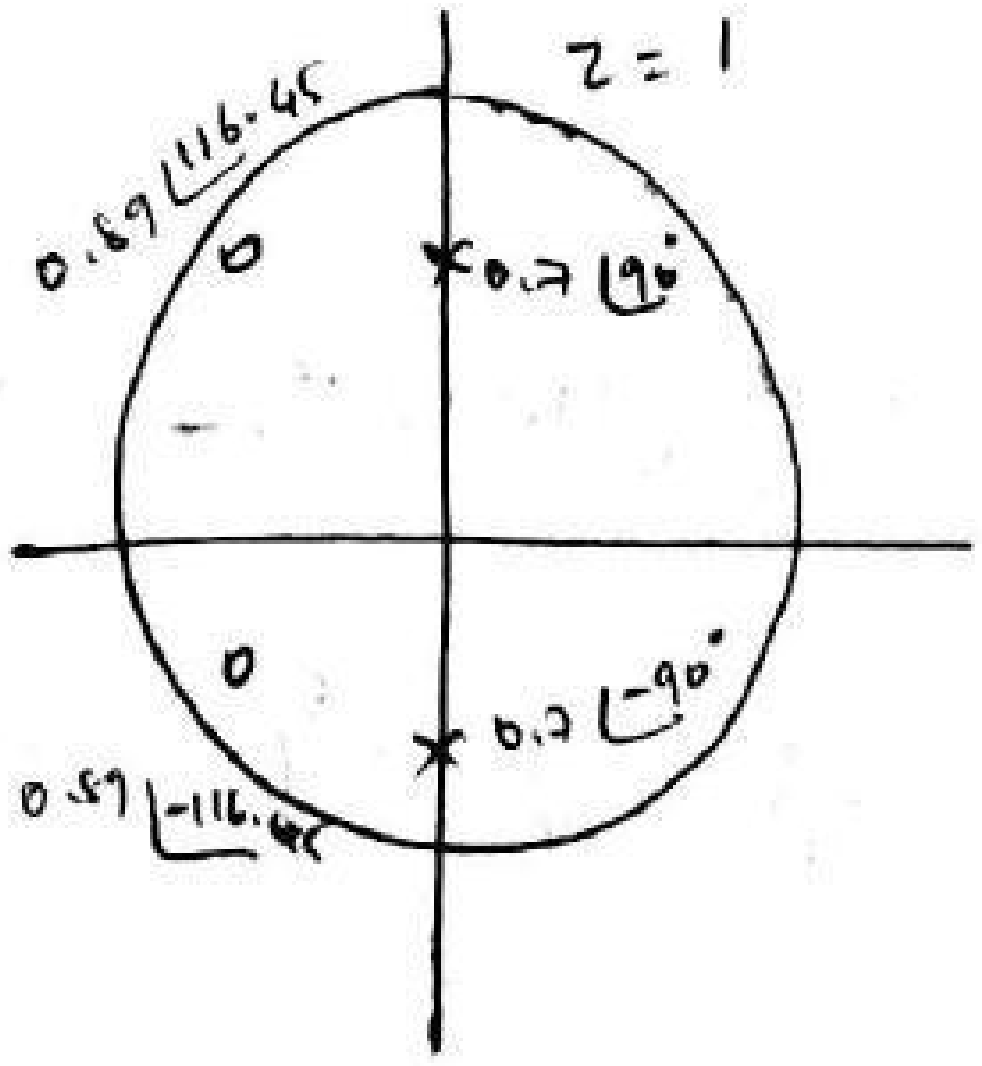
$z = -\frac{2}{5} + j\frac{4}{5} = 0.89 \angle 116$

$z = -\frac{2}{5} - \frac{4j}{5} = 0.89 \angle -116$

poles

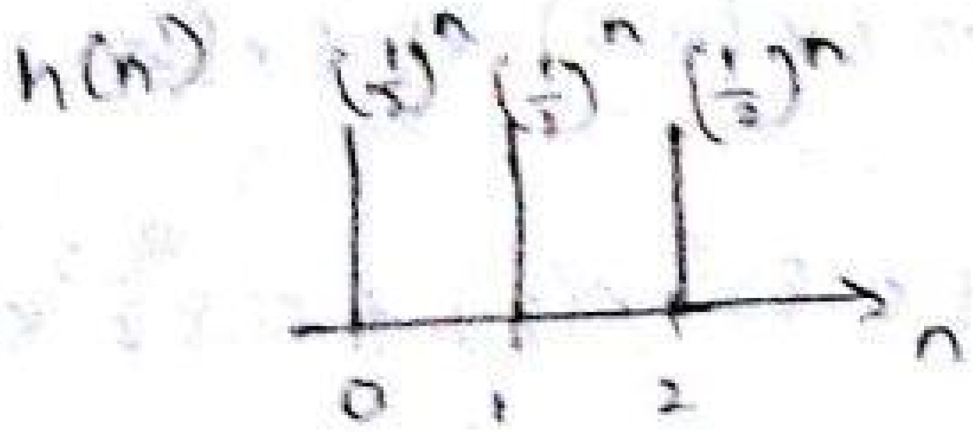
$z = j0.7 = 0.7 \angle 90$

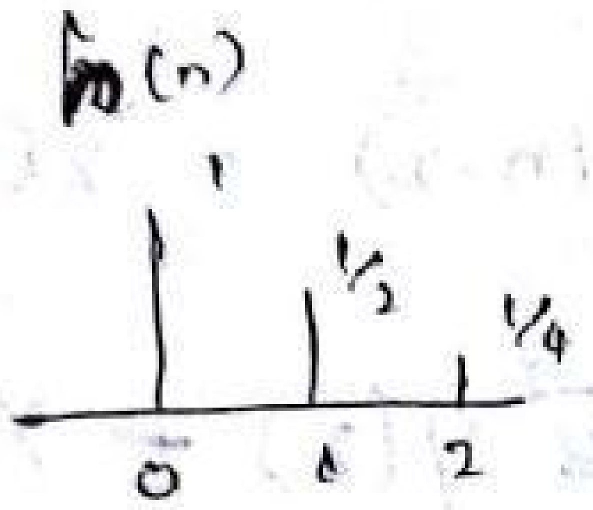
$z = -j0.7 = 0.7 \angle -90°$



(7) $h(n) = \left(\frac{1}{2}\right)^n \quad 0 \le n \le 2$

$x(n) = \delta(n) + \delta(n-1) + 4\,\delta(n-2)$ find $y(n)$





$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$

$H(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$

$X(z) = 1 + z^{-1} + 4z^{-2}$

$y(n) = x(n) * h(n)$

$\downarrow$ Z.T

$Y(z) = X(z) \cdot H(z)$

$= \left[1 + z^{-1} + 4z^{-2}\right] \cdot \left[1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right]$

$= 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{4}z^{-3} + 4z^{-2} + 2z^{-3} + z^{-4}$

$Y(z) = 1 + \frac{3}{2}z^{-1} + \frac{19}{4}z^{-2} + \frac{9}{4}z^{-3} + z^{-4}$

$\downarrow$ I.Z.T

$y(n) = \delta(n) + \frac{3}{2}\delta(n-1) + \frac{19}{4}\delta(n-2) + \frac{9}{4}\delta(n-3) + \delta(n-4)$

$y(n) = \{\underset{\uparrow}{1}, \frac{3}{2}, \frac{19}{4}, \frac{9}{4}, 1\}$

$4 + \frac{1}{2} + \frac{1}{4}$

$\frac{16+2+1}{4} = \frac{19}{4}$

⑧ $x(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2)$

$y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$

find $H(z)$ & $h(n)$ for causal & non-causal s/m.

$\rightarrow$ $X(z) = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}$

$Y(z) = 1 - \frac{3}{4}z^{-1}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{3}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$\times$ by z^2

$$= \frac{z^2 - \frac{3}{4}z}{z^2 + \frac{1}{4}z - \frac{1}{8}}$$

$$H(z) = \frac{z\left(z - \frac{3}{4}\right)}{\left(z - \frac{1}{4}\right)\left(z + \frac{1}{2}\right)}$$

$$\frac{H(z)}{z} = \frac{z - \frac{3}{4}}{\left(z - \frac{1}{4}\right)\left(z + \frac{1}{2}\right)}$$

$$z - \frac{3}{4} = \frac{A}{z - \frac{1}{4}} + \frac{B}{z + \frac{1}{2}}$$

$$z - \frac{3}{4} = A\left(z + \frac{1}{2}\right) + B\left(z - \frac{1}{4}\right)$$

$z = -\frac{1}{2}$ $\qquad$ $z = \frac{1}{4}$

$\frac{-5}{4} = A(0) + -\frac{3}{4}B$ $\qquad$ $-\frac{1}{2} = \frac{3}{4}A$

$\boxed{\frac{5}{3} = B}$ $\qquad$ $\boxed{-\frac{2}{3} = A}$

$$H(z) = \frac{-\frac{2}{3} z}{\left(z - \frac{1}{4}\right)} + \frac{\frac{5}{3} z}{\left(z + \frac{1}{2}\right)}$$

$\downarrow$ I.Z.T

Causal

$$h(n) = -\frac{2}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{5}{3}\left(-\frac{1}{2}\right)^n u(n)$$

Non-Causal

$$h(n) = \frac{2}{3}\left(\frac{1}{4}\right)^n u(-n-1) - \frac{5}{3}\left(-\frac{1}{2}\right)^n u(-n-1)$$

(9) $y(n) - y(n-1) = x(n) + x(n-1)$

find respon for step input

→

$$Y(z) - z^{-1} Y(z) = X(z) + z^{-1} X(z)$$

$$Y(z)\left[1 - z^{-1}\right] = X(z)\left[1 + z^{-1}\right] \qquad \therefore x(n) = u(n)$$

$$Y(z) = X(z) \frac{\left[1 + z^{-1}\right]}{\left[1 - z^{-1}\right]}$$

$$\frac{z}{z-1} \cdot \frac{1 + z^{-1}}{1 - z^{-1}}$$

$$\frac{z}{z-1} \cdot \frac{z+1}{z-1}$$

$$Y(z) = \frac{z(z+1)}{(z-1)^2}$$

$$Y(z) = z \cdot \frac{z}{(z-1)^2} + \frac{z}{(z-1)^2}$$

$\downarrow$ I.Z.T

$$y(n) = n+(1)^{n-1+1} u(n+1) + n(1)^{n-1} u(n)$$

$$y(n) = (n+1)(1)^n u(n+1) + n\, u(n)$$

$$\underline{y(n) = (n+1)\, u(n+1)} + n u(n)$$

(10) $y(n) - 2y(n-1) + 2y(n-2) = x(n) + \frac{1}{2} x(n-1)$

find $H(z)$ & $h(n)$

$\rightarrow$ $Y(z) - 2z^{-1} Y(z) + 2z^{-2} Y(z) = X(z) + \frac{1}{2} z^{-1} X(z)$

$$Y(z)\left[1 - 2z^{-1} + 2z^{-2}\right] = X(z)\left[1 + \frac{1}{2} z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2} z^{-1}}{1 - 2z^{-1} + 2z^{-2}}$$

$$H(z) = \frac{z^2 + \frac{1}{2} z}{z^2 - 2z + 2}$$

$$H(z) = \frac{z\left(z + \frac{1}{2}\right)}{(z-(1+j))(z-(1-j))}$$

$$\frac{H(z)}{z} = \frac{z + \frac{1}{2}}{(z-(1+j))(z-(1-j))}$$

$$z + \frac{1}{2} = \frac{A}{(z-(1+j))} + \frac{B}{(z-(1-j))}$$

$\Rightarrow 1 + \frac{1}{2} = A(z-(1-j)) + B(z-(1+j))$

Put $z = 1+j$

$\frac{3}{2} + j = A(\cancel{1} + j - \cancel{1} + j) + B(0)$

$\frac{3}{2} + j = 2j\, A$

$A = \frac{\frac{3}{2} + j}{2j}$

$\boxed{A = \frac{1}{2} - \frac{3}{4}j}$

B: put $z = 1 - j$

$\frac{3}{2} - j = 0 + B(\cancel{1} - j - \cancel{1} - j)$

$\frac{\frac{3}{2} - j}{-2j} = B$

$\boxed{B = \frac{1}{2} + \frac{3}{4}j}$

$H(z) = \left(\frac{1}{2} - \frac{3}{4}j\right) \frac{z}{(z-(1+j))} + \left(\frac{1}{2} + \frac{3}{4}j\right) \frac{z}{(z-(1-j))}$

$\downarrow$ I.Z.T

$h(n) = \left(\frac{1}{2} - \frac{3}{4}j\right)(1+j)^n u(n) + \left(\frac{1}{2} + \frac{3}{4}j\right)(1-j)^n u(n)$

$(1.41\angle 45)^n$ $(1.41\angle -45)^n$

$(\sqrt{2})^n e^{j45n}$ $(\sqrt{2})^n e^{-45nj}$

$$h(n) = \left(\frac{1}{2} - \frac{3}{4}j\right) (\sqrt{2})^n e^{j\frac{\pi}{4}n} u(n)$$

$$= (\sqrt{2})^n \left[\left(\frac{1}{2} - \frac{3}{4}j\right)\left[\cos\frac{\pi}{4}n + j\sin\frac{\pi}{4}n\right]u(n) \left(\frac{1}{2} + \frac{3}{4}j\right), \left[\cos\frac{\pi}{4}n - j\sin\frac{\pi}{4}n\right]u(n)\right]$$

$$= (\sqrt{2})^n \left\{\left[\frac{1}{2}\cos\frac{\pi}{4}n + \cancel{\frac{1}{2}j\sin\frac{\pi}{4}n} - \cancel{\frac{3}{4}j\cos\frac{\pi}{4}n} + \frac{3}{4}\sin\frac{\pi}{4}\right]u(n)\right.$$

$$\left. + \frac{1}{2}\cos\frac{\pi}{4}n - \cancel{\frac{1}{2}j\sin\frac{\pi}{4}n} + \cancel{\frac{3}{4}j\cos\frac{\pi}{4}n} + \frac{3}{4}\sin\frac{\pi}{4}n\right]u(n)$$

$$= (\sqrt{2})^n \left\{\left[\cos\frac{\pi}{4}n + \frac{3}{2}\sin\frac{\pi}{4}n\right]u(n)\right]$$

$$h(n) = (\sqrt{2})^n \cos\frac{\pi}{4}n\, u(n) + \frac{3}{2}(\sqrt{2})^n \sin\frac{\pi}{4}n\, u(n)$$

(ii) $h(n) =$