13/5/2017

MODULE-II

Line Parameters

Introduction.

Transmission of electric power is done by 3ϕ - 3wire overhead wire. an AC transmission line has resistance inductance and capacitance uniformly distributed all along its length. These are known as constants or Parameters of the transmission line. The Performance of TL depends on these constants or parameters. For instance These constant determine whether the efficiency and voltage regulation of line will be good or poor ∴ A sound concept of these parameters is necessary in order to make the electrical design of the transmission line a technical success. In this module we shall learn or focus our attention on methods of calculating thes parameters

Constants of Transmission Line.

A transmission line has resistance, inductance & Capacitance uniformly distributed all along the length of the line. It is profitable to understand R, L, C Thoroughly

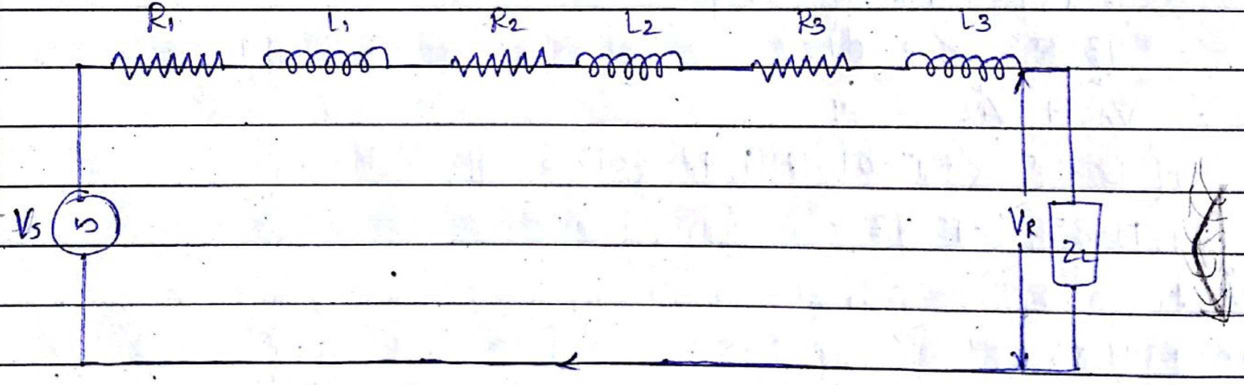


fig 2.1

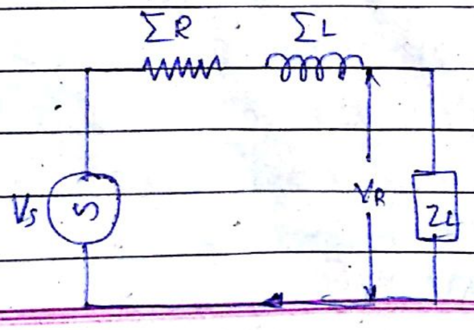


fig - 2.2

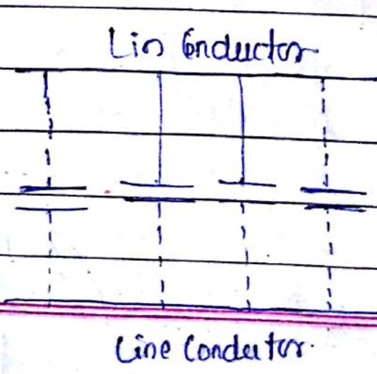


fig - 2.3

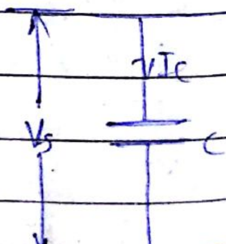


fig 2.4

I) Resistance

It is opposition of Line conductor to current flow. The resistance is distributed all along the length as shown in fig2.1. However the performace cand be analyzed conveinnty if the distributed resistance is considerd as lumped shown in fig-2.2

II) Inductance

When an alternating current flow through a condr, a changing flux is produced which Links the conductor due to this flux Linkages the conductor possesses inductance. Mathematically inductance is defined as the flux linkages per ampere i.e

i,e Inductance $= L = \frac{\psi}{I}$ henry (2.1)

where ψ = flux Linkages in Webber turns
I = Current in Ampers

The inductance is uniformaly distributed along the length of the line as shown in fig-2.1 again for the conveniance of the analysis it can be taken as lumped as shown in fig-2.2

III) Capacitance

We know that any two conductors separated by an insulating material constitute a capacitor. As any two conductors of OH Line are seperated by air which acts as an insulation, there fore capacitance exists between any two over head line conductor. The capacitance between the conductors is the charge / unit potential difference i.e

i,e $C = \frac{Q}{V}$ Farad (2.2)

where q = charge on the line in coulumbs
V = potential difference betn the condrs in Volts

The capacitance is uniformly distributed with whole length of the Line and may be regarded as a uniform series capacitor connected betn the condr shown in fig 2.3. When an ac ~~st~~ t/n is applied on the TL, as the charge on the conductor at any point increases and decreases with increase and decrease of instantaneous value of the voltage betn the conductors at that point (known as charging current). The charging current flows in the transmission line. It affects the voltage drop, efficiency and powerfactor of transmission line

* Resistance of a Transmission Line

The resistance of the TL's conductor is the most important cause of power loss in a TL. The resistance 'R' of line conductor having resistivity 'ρ', length 'l' and area of cross section 'a' is given by

$$R = \frac{\rho l}{a} \quad (2.3)$$

The variation of resistance of metalic conductor with tempreture is practically lenier over the normal range of operation. Suppose R_1 and R_2 are the resistance of of conductor at $t_1°C$ and $t_2°C$ ($t_2 > t_1$) respectivly. If α_1 is the tempreture coefficiant at $t_1°C$ then ~~R2~~

$$R_2 = R_1\left[1 + \alpha_1(t_2 - t_1)\right] \quad (2.4)$$

where $\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1}$ (2.5)

α_0 = tempretere at 0°c

I) In a single phase 2-wire dc line, the total resistance is known as loop resistance. Is equal to the double the resistance of either conductor

II) In case of 3ϕ TL, Resistance per phase is the resistance of the oneconductor

* Skin Effect

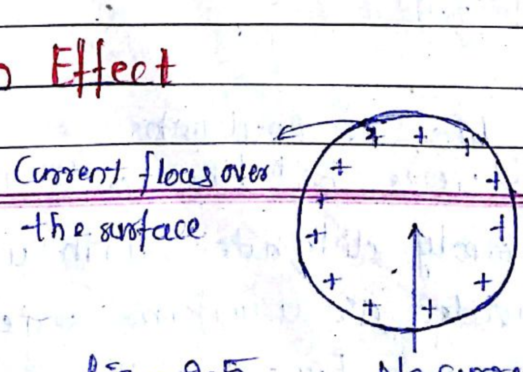


fig - 2.5

When a condr carrying steady direct current (DC) This current is uniformaly distributed whole cross section of the conductor. However an alternating c/n flows through a condr does not distributed uniformly rather it has tendency to concentrate near the surface of conductor as shown in fig-2.5 this is known as Skin effect

" The dencity of alternating current to concentrate ~~to~~ near the surface of conductor is known as skin effect"

15/5/17 ①

* Inductance of Single phase two wire Line

A single phase line consist of ||^el conductors with form a rectangular loop of 1 turn when an alternating current flows through such a loop a changing magnetic flux is setup. the changing flux links the loop and hence the loop possess inductance it may appear that ~~single~~ inductance of 1ϕ line is negligible bcz it consist of loop of 1 turn and the flux path is through air of high reluctance. But as the cross sectional area of the loop is very large (X-sectional) even for small flux dencity, the total flux linking the loop is quite large and hence the line has appreciable inductance

Consider a single phase OH line consisting of two parallel conductors A & B spaced 'd' meters apart as shown in fig-2.6

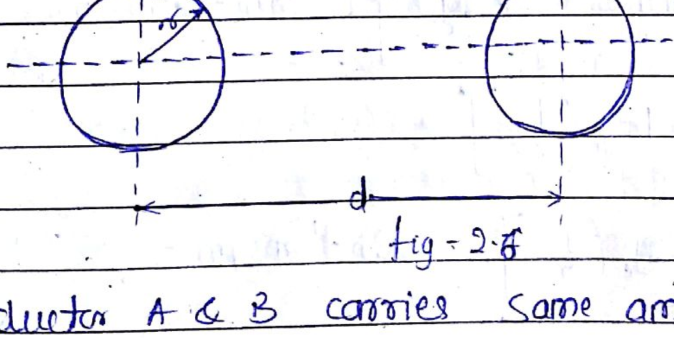


fig - 2.6

Conductor A & B carries same ammount of c/n i,e $(I_A = I_B)$ but, in the opposite direction bcz one forms the return ckt of the other $\therefore I_A + I_B = 0$ —— (2.6)

In order to find the inductor of condr A & B we shall have to consider flux linkages with it ; their will be flux linkages with conductor A due to its own c/n I_A and also due to mutual inductance effect of c/n I_B in the c/n cond^r B

Flux linkages with cond^r A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4}+\int_r^\infty \frac{dx}{x}\right) \quad (2.7)$$

[Seeing article in a 9.4 Pg-207 VKM]

Flux linkage with cond^r A due to I_B $= \frac{\mu_0 I_B}{2\pi}\int_d^\infty \frac{dx}{x}$ — (2.8)

Total flux linkages with cond^r A is

$$\psi_A = (2.7) + (2.8)$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4}+\int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_d^\infty \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left\{ I_A\left[\frac{1}{4}+\int_r^\infty \frac{dx}{x}\right] + I_B\int_d^\infty \frac{dx}{x}\right\}$$

$$= \frac{\mu_0}{2\pi}\left\{\left(\frac{1}{4}+\log_e\infty - \log_e r\right) I_A + \left(\log_e\infty - \log_e d\right) I_B\right\}$$

$$= \frac{\mu_0}{2\pi}\left\{\frac{I_A}{4} + \log_e\infty\,(I_A+I_B) - I_A\log_e r - I_B\log_e d\right\}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A\log_e r - I_B\log_e d\right] \qquad \because (I_A+I_B)=0 \text{ — } (2.6)$$

$I_A+I_B = 0$ or $-I_B = I_A$

$-I_B\log_e d = I_A\log_e d$

$$\therefore \psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A\log_e d - I_A\log_e r\right] \text{ Wb-turns/m}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A\log_e\left(\frac{d}{r}\right)\right]$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \text{ Wb-turns/m} \quad (2.9)$$

Inductance of A; $L_A = \frac{\psi_A}{I_A}$

$$\therefore L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \text{ H/m}$$

$$L_A = \frac{4\pi\times10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \text{ H/m}$$

$$\therefore \boxed{L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e\left(\frac{d}{r}\right)\right]} \quad \text{——— (2.10)}$$

Loop Inductance $= 2L_A$ H/m

$$= 10^{-7}\left[1 + 4\log_e\left(\frac{d}{r}\right)\right] \text{ H/m}$$

$\therefore$ The loop Inductance $= 10^{-7}\left[1 + 4\log_e\left(\frac{d}{r}\right)\right]$ H/m ——— (2.11)

Note that eqn 2.11 is the inductance of 2 wire line and is sometimes called loop inductance. However inductance given by expression ~~2.8~~ & 2.10 is the inductance per condr is equal to Half of the loop inductance.

VVImp

* Inductance of 3φ Over Head line.

Fig below 2.7 shows the ~~two and three~~ A,B&C 3φ condr of 3φ line carrying currents as I_A, I_B and I_C resply.
Let d_1, d_2 and d_3 be the spacings between the condrs as shown.

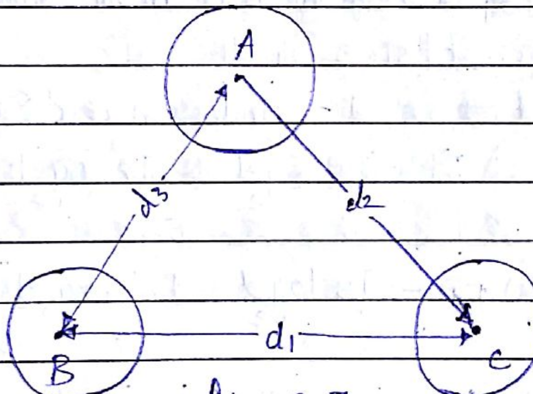


fig - 2.7

Let us further assumed that loads are balanced

i,e $I_A + I_B + I_C = 0$ ——— (2.12)

Consider the flux linkages with condr A there will be flux linkages with condr A due to its own c/n I_A and due to mutual inductance effects of I_B and I_C.

Flux linkages with condr A due to its own c/n is

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^a \frac{dx}{x}\right) \quad \text{——— (2.13)}$$

Flux linkages with condr A due I_B $= \dfrac{\mu_0 I_B}{2\pi}\displaystyle\int_{d_3}^{a} \frac{dx}{x}$ ——— (2.14)

Flux Linkages on Cond^n due I_C = $\frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x}$ — (2.15)

Total flux linkages with cond^r A is

ψ_A = expr (2.13) + Exp (2.14) + Ex (2.15)

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4}+\int_r^{\infty}\frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\int_r^{\infty}\frac{dx}{x}\right)I_A + I_B\int_{d_3}^{\infty}\frac{dx}{x} + I_C\int_{d_2}^{\infty}\frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left\{\frac{I_A}{4} + I_a\left(\log_e\infty - \log_e r\right)\right\} + I_B\left(\log_e\infty - \log_e d_3\right) + I_C\left(\log_e\infty - \log_e d_2\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A\log_e\infty - I_A\log_e r + I_B\log_e\infty - I_B\log_e d_3 + I_C\log_e\infty - I_C\log_e d_2\right]$$

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A - I_B\log_e d_3 - I_C\log_e d_2 + \log_e\infty\left(I_A+I_B+I_C\right)\right]$$

As $I_A+I_B+I_C=0$ | ∵ eq^n 2.12

$$\therefore \psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A - I_B\log_e d_3 - I_C\log_e d_2\right] \quad (2.16)$$

Dec-14

If the 3 conductors A, B & C are placed symmetrical at the corners of equilateral △le

i) Semmetrical Spacing of sides under

$d_1 = d_2 = d_3 = d$ — (2.17) under such cond^{ns} air the flux linkage with cond^r A

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A - I_B\log_e d_3 - I_C\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A - I_B\log_e d - I_C\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A - \left(I_B+I_C\right)\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A - \left(-I_A\right)\log_e d\right]$$ | ∵ $I_A+I_B+I_C=0$, $I_B+I_C=-I_A$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I_A + I_A\log_e d\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4}-\log_e r+\log_e d\right]$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4}+\log_e\left(\frac{d}{r}\right)\right] \text{ Wb turns/m} \quad (2.18)$$ | ∵ $\log a - \log b = \log\frac{a}{b}$

∴ Inductance of conductor A

$$L_A = \frac{\psi_A}{I_A} \quad H/m$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \quad H/m$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \quad H/m$$

$$\boxed{L_A = 10^{-7}\left[0.5 + 2\log_e\left(\frac{d}{r}\right)\right] \quad H/m} \quad \text{——} \quad (2.19)$$

Eqn (2.19) is derived in similar way like (2.17) way the expr for inductance are same for condr B & C

Dec-14 May-16 (7)

ii) Unsymmetrical Spacing

When 3φ line conductors are not equidistance from each other. The condr spacing is said to be unsymmetrical under these conditions the flux linkages & inductance of each phase are not the same. A different inductance in each phase results in unequal voltage drops in the 3 phases even if the I's in the condrs are balanced "In order that vtg drops are equal in all condrs, we generally interchange the positions of the condrs at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of position is known as Transposition". May-16 Dec-14

fig - 2.8 below shows the transposed line

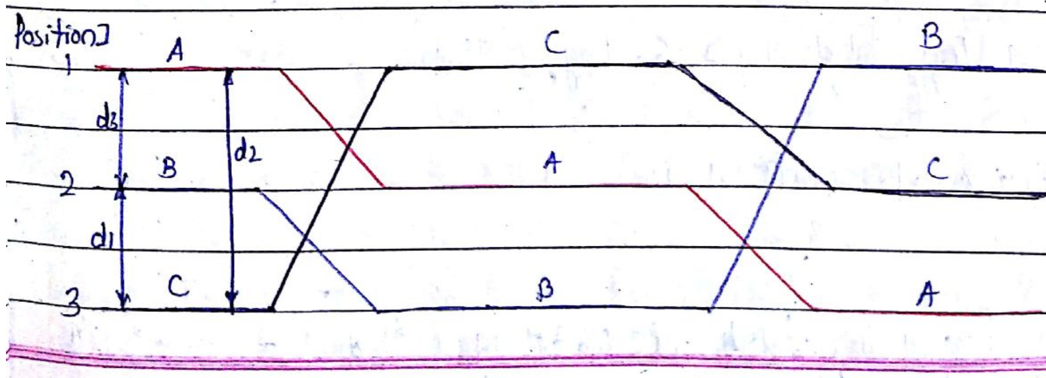


fig - 2.8

Porb

The phase conductors are designated as A, B, C at the positions acupied number 1, 2 & 3. The effect of transposition is that each condr has the same avg inductance. Fig - 2.8 shows a 3ϕ Transposed line has unsymmetrical spacing.

Let us assume further Balanced conditions ,, 16/5/17

i,e $I_A + I_B + I_C = 0$

let us take the C/n $I_A = I(1+j0)$ —— (2.20)

take $I_B = I(-0.5 - j0.866)$ —— (2.21)

& $I_C = I(-0.5 + j0.866)$ —— (2.22)

As prooved above the total flux linkage per meter length of conductor A is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) \cdot I_A - I_B \log_e d_3 - I_C \log_e d_2\right] \quad (2.23) \quad \because eq^n\ 2.16$$

Putting 2.20, 2.21, 2.22 in (2.16) we get

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I - I(-0.5 - j0.866) \log_e d_3 - I(-0.5 + j0.866) \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I}{4} - I \log_e r + 0.5 I \log_e d_3 + j0.866 I \log_e d_3 + 0.5 I \log_e d_2 - j0.866 I \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I}{4} - I \log_e r + 0.5 I\left(\log_e d_3 + \log_e d_2\right) + j0.866 I\left(\log_e d_3 - \log_e d_2\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I}{4} - I \log_e r + I \log_e \sqrt{d_3 d_2} + j0.866 I \log_e (d_3/d_2)\right]$$

$\log a + \log b = \log ab$

$\because \log a - \log b = \log \frac{a}{b}$

$c \log a = \log a^c$

$$= \frac{\mu_0}{2\pi}\left[\frac{I}{4} + I \log_e \frac{\sqrt{d_3 d_2}}{r} + j0.866 I \log_e (d_3/d_2)\right]$$

$$\psi_A = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_3 d_2}}{r} + j0.866 \log_e (d_3/d_2)\right]$$

∴ Inductance of condr A is

$$L_A = \frac{\psi_A}{I_A}$$

$$L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_3 d_2}}{r} + j0.866 \log_e (d_3/d_2)\right] \quad —— (2.24)$$

$$L_A = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_3 d_2}}{r} + j0.866 \log_e (d_3/d_2)\right]$$

$$\therefore L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3 d_2}}{r} + j\,1.732 + \log_e(d_3/d_2)\right] \quad (2.25)\ H/m$$

$|||^{ly}$ rellraimlg (2.25) Inductance of conductor B & C are

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3 d_1}}{r} + j\,1.732 + \log_e\left(\frac{d_1}{d_3}\right)\right] \quad (2.26)$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_1 d_2}}{r} + j\,1.732 + \log_e\left(\frac{d_2}{d_1}\right)\right] \quad (2.27)$$

Inductance of 3ϕ line unsymmetrical spacing

Inductance of each line cond^r is

$$= \frac{1}{3}(L_A + L_B + L_C) = \left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7}\ H/m \quad (2.28) \qquad \because \text{on solving}$$

$$= \left[0.5 + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7}\ H/m \quad (2.29)$$

If we can compare the formula of inductance of an unsymmetrical spaced transposed line with that of symmetrically space line we find that inductance of each cond^r in two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$. The distance d is known as equivelent equilateral spacing for unsymmetrically transposed line.

VVImp

Concept of Self-GMD and Mutual-GMD

The use of self Geometrical Mean Distance (appriviated as GMD) and ge· Geometrical mean distance (Mutual GMD) simply d is the inductance calculation perticulary related to multi cond^r arrangement. The symbols used for these are D_s and D_m resply. we shall breafly discuss these terms

GMR

Self GMD (D_s) :-

In order to have a concept of self GMD (also some times called as Geometrical mean radius : GMR consider the exp^n for inductance per cond^r/mt already derived in expression for inductance of single ϕ 2 wire line. eq^n 2.10

$$\text{Inductance/cond}^r/m \quad L_A = 10^{-7}\left(\frac{1}{2} + 2\log_e\left(\frac{d}{r}\right)\right).$$

$$L_A = 2\times 10^{-7}\left(\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right) \quad (2.30)$$

$$L = \frac{2\times10^{-7}\times1}{4} + 2\times10^{-7}\log_e\left(\frac{d}{r}\right) \quad (2.31)$$

In eqn (2.31) the 1st term $\frac{2\times10^{-7}\times1}{4}$ is the inductance due to flux with in the solid conductor. For many porpouse it is disirable to eliminate this term by the introduction of concept called Self GMD or GMR. If we replace the original solid condr by an equivelent hollow cylinder with extreamly thin walls, The current is confined to the conductor surface & internal condr flux linkage would be almost zero. Consiquently inductance due to internal flux would be zero and the term ~~the~~ $\frac{2\times10^{-7}\times1}{4}$ shall be eliminate. The radius of thos equivelent how cylinder must be sufficiently smaller that the physical raedius of condr to hollow enough addition flux to compancaint for the absence of internal flux linkage. it can be proove mathematically that can be round condr for radius r the self GMD or GMR = 0.7788r, using self GMD eqn (2.31) becomed

$$\text{Inductance/cond}^r\text{/m} = 2\times10^{-7}\log_e\left(\frac{d}{D_s}\right) \quad (2.32)$$

where D_s = GMR or self GMD = $0.7788r$

It may be noted that self GMD of condr depends on the size and shape of the condr and is independent of spacing betn the condrs

19/6/17

Mutual GMD

The Mutual GMD is geomentrical mean-distance between one conductor to other and therefore must be between the largest and smallest such distance. in the fact mutual GMD simply represents the equivelent geometrical spacing

a) The mutual GMD between the two conductors (assuming that spacing between the conductors is large compared to the diameter of each conductor) is equal to the distance between their centers i,e $\boxed{D_m = \text{Spacing between the conductors} = d}$ — (2.33)

b) For single ckt 3ϕ line the mutual GMD is equal to the equivalent equilateral spacing i,e $(d_1 d_2 d_3)^{1/3}$

$$\boxed{D_m = (d_1 d_2 d_3)^{1/3}} \quad \text{--- (2.34)}$$

c) The principle geometrical mean distance can be most profitably employed to 3ϕ double circuit lines.

Consider the conductor arrangement of the double circuit shown in fig 2.9

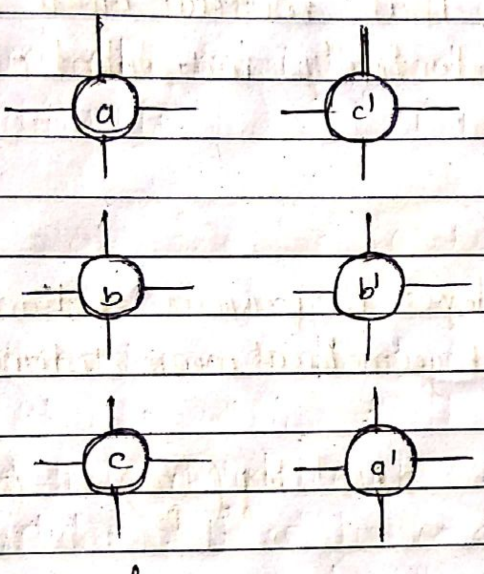


fig - 2.9

Suppose the radius of conductor is r

Self GMD is of conductor $= 0.7788\,r$ --- (2.35)

Self GMD of combination aa' is

$$D_{s1} = (D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4} \quad \text{--- (2.36)}$$

Self GMD of combination bb' is

$$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4} \quad \text{--- (2.37)}$$

Self GMD of combination cc' is

$$D_{s3} = (D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c})^{1/4} \quad \text{--- (2.38)}$$

Equivalent self GMD of one phase

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3} \quad \text{--- (2.39)}$$

The value of D_s is same for all the phases as each conductor has same radius

The mutual GMD between the phases A & B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4} \quad \text{--- (2.40)}$$

Mutual GMD between the phases B & C is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4} \quad \text{--- (2.41)}$$

The mutual GMD between the phases C & A

$$D_{CA} = \left(D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'} \right)^{1/4} \quad (2.42)$$

Equivalent mutual GMD is

$$D_m = \left(D_{AB} \times D_{BC} \times D_{CA} \right)^{1/3} \quad (2.43)$$

Note: D_{aa} or $D_{a'a'}$ means self GMD of the conductor.
$D_{aa'}$ means distance between a and a'

It is worth while to know that mutual GMD depends only upon the spacing and substantially, indipendent of size, shape and orientation of conductor.

Inductance Formulas in GMD

The inductance Formula's developes in previous section can be coveniently expressed in terms of geometrical mean distance

(i) 1-ϕ Line :-

Inductance / conductor / meter $= 2 \times 10^{-7} \cdot \log_e \left(\frac{D_m}{D_s} \right)$ — (2.44)

Where $D_s = 0.7788\, r$ and
D_m = spacing between the conductors = d

(ii) Single circuit 3ϕ - Line :-

Inductance / phase / meter $= 2 \times 10^{-7} \log_e \left(\frac{D_m}{D_s} \right)$ — (2.45)

where $D_s = 0.7788\, r$ and
$D_m = (d_1 d_2 d_3)^{1/3}$

(iii) Double circuit 3ϕ line :

Inductance / phase / m $= 2 \times 10^{-7} \log_e \left(\frac{D_m}{D_s} \right)$ — (2.46)

where $D_s = \left(D_{S1} \times D_{S2} \times D_{S3} \right)^{1/3}$ and
$D_m = \left(D_{AB} \times B_{BC} \times D_{CA} \right)^{1/3}$

Capacitance of 1-φ two wire line

Consider a 1φ OH.TL consisting of two parallel conductors A & B, spaced d m apart in air, suppose that radius of each conductor is 'r' meters.
let their respective charge be +Q & -Q coulombs/m. length

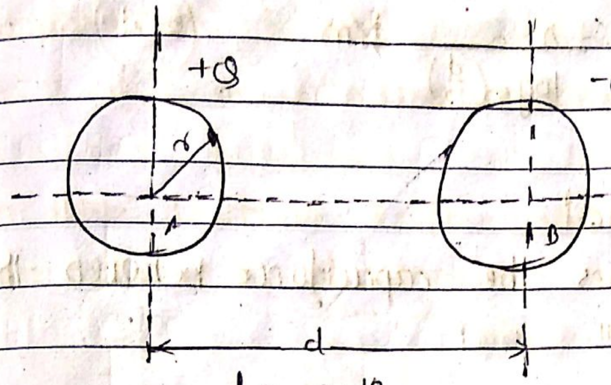


fig - 2 · 10

The total potential difference (pd) between the conductor A and Nutral "infinite plane" is

$$V_A = \int_r^\infty \frac{Q}{2\pi\varepsilon_0 x}dx + \int_d^\infty \frac{-Q}{2\pi\varepsilon_0 x}dx \quad \text{—— (2.47)}$$

∵ Explanation given in 9.9 (i) (ii) Pg - 220, 221 VK-mehta

$$\therefore V_A = \frac{Q}{2\pi\varepsilon_0}\left[\log_e\left(\frac{\infty}{r}\right) - \log_e\left(\frac{\infty}{d}\right)\right] \text{ volts}$$

$$= \frac{Q}{2\pi\varepsilon_0}\left[\log_e\left(\frac{d}{r}\right)\right] \text{ volts} \quad \text{—— (2.48)}$$

IIIly Potential difference betn condr B and nutral "Infinite" plane is

$$V_B = \int_r^\infty \frac{-Q}{2\pi\varepsilon_0}dx + \int_d^\infty \frac{Q}{2\pi\varepsilon_0 x}dx$$

$$= \frac{-Q}{2\pi\varepsilon_0}\left[\log\frac{\infty}{r} - \log\frac{\infty}{d}\right]$$

$$V_B = \frac{-Q}{2\pi\varepsilon_0}\log_e\left(\frac{d}{r}\right) \quad \text{—— (2.49)}$$

Bothe these potentials are wrt same nutral plane since the unlike charges attract each other The P.d between the conductor is

$$V_{AB} = 2V_A$$

$$= 2\frac{Q}{2\pi\varepsilon_0}\log_e\left(\frac{d}{r}\right) \text{ volts} \quad \text{—— (2.50)}$$

$\therefore$ Capacitance $C_{AB} = \dfrac{Q}{V_{AB}}$

$$= \frac{Q}{\frac{2Q}{2\pi \varepsilon_0} \log_e\left(\frac{d}{r}\right)} \quad F/m$$

$$\therefore C_{AB} = \frac{\pi \varepsilon_0}{\log_e\left(\frac{d}{r}\right)} \quad F/m \text{ ——— } (2.51)$$

* Capacitance To Nutral

equation (2.51) Gives the capacitance between the cond^rs of two line see fig - 2.11

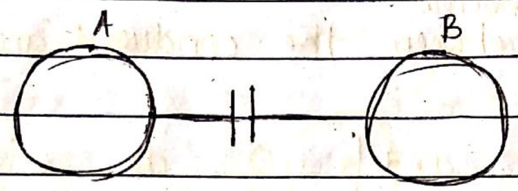


fig - 2.11

Oftetan it is desired to know the capacitance between one of the cond^r and a nutral point between them sence potential of midpoint between the conductors is zero, the P.d between each cond^r and ground or nutral is half the P.d between the conductors. Thus capacitance to ground or capacitance to nutral for the two wire line is twice the line to line capacitance (between the conductors as shown below fig 2.12.

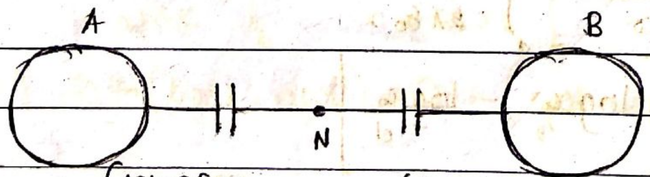


fig - 2.12

$\therefore$ Capacitance to nutral $C_N = C_{AN} = C_{BN}$

$$= 2C_{AB}$$

$$\therefore C_N = \frac{2\pi \varepsilon_0}{\log_e\left(\frac{d}{r}\right)} \quad F/m \text{ ——— } (2.52)$$

You, I may compare eq^n (2.52) To the ~~ind~~ 1 for inductance one difference between ~~cap~~ eq^ns for capacitance & inductance should be noted carefully. The radius in eqn for capacitance the actual outside radius of the conductor and not the GMR of the conductor as the inductance formula. note that eqn (2.52) applies only to the solid ground cond's

Mar-16 VVImp Dec 19

* Capacitance of 3·ϕ Over Head Line (i) [Semmetrical spacing

In a 3ϕ T.L, the capacitance of each conductor is considered instead of capacitance from conductor to conductor. Here again their are two cases that is semmetrical spacing and unsemmetrical spacing

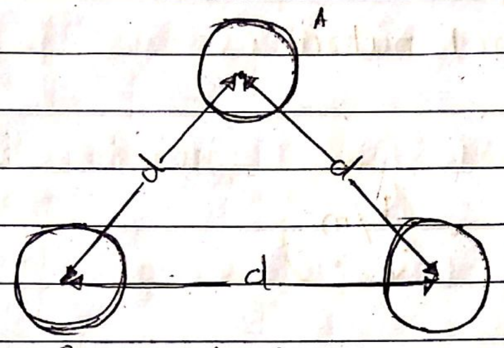


fig 2.13

Fig 2.13 shows 3 conductor A, B & C of 3ϕ OHTL having charges Q_A, Q_B & Q_C per mt length resply let the conductors be equidistant (d-meters) from each other. we shall find the capacitance from line cond^n to the nutral in this semmetrically spaced line reffering to fig 2.13 overall p.d bet^n conductor & infinite nutral plane is gin by (reffering article 9.9 Pg - 220 221 VKM

$$V_A = \int_r^{\infty} \frac{Q_A}{2\pi\varepsilon_0 x} dx + \int_d^{\infty} \frac{Q_B}{2\pi\varepsilon_0 x} dx + \int_d^{\infty} \frac{Q_C}{2\pi\varepsilon_0 x} dx \quad (2.53)$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A(\log\infty - \log r) + Q_B(\log\infty - \log d) + Q_C(\log\infty - \log d)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A\log\infty - Q_A\log r + Q_B\log\infty - Q_B\log d + Q_C\log\infty - Q_C\log d\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[-Q_A\log r - Q_B\log d - Q_C\log d + \log\infty(Q_A+Q_B+Q_C)\right]$$

assuming for Balanced supply $Q_A+Q_B+Q_C=0$

$$= \frac{1}{2\pi\varepsilon_0}\left[(0 - Q_A\log r) + (0 - Q_B\log d) + (0 - Q_C\log d)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A(\log 1 - \log r) + Q_B(\log 1 - \log d) + Q_C(\log 1 - \log d)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left(\frac{1}{r}\right) + Q_B \log_e\left(\frac{1}{d}\right) + Q_C \log_e\left(\frac{1}{d}\right)\right]$$

From standard Balanced supply $Q_A + Q_B + Q_C = 0$

$Q_B + Q_C = -Q_A$

$$\therefore V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left(\frac{1}{r}\right) - Q_A \log\left(\frac{1}{d}\right)\right]$$

$$= \frac{Q_A}{2\pi\varepsilon_0}\left[\log_e\left(\frac{d}{r}\right)\right] \quad (2.54)$$

Capacitor of condr wrt nutral is

$$C_A = \frac{Q_A}{V_A}$$

$$= \frac{Q_A}{\frac{Q_A}{2\pi\varepsilon_0}\log_e\left(\frac{d}{r}\right)} \ F/m$$

$$\boxed{C_A = \frac{2\pi\varepsilon_0}{\log_e\left(\frac{d}{r}\right)}} \ F/m \quad (2.55)$$

Note that this equation is identical to capacitance to nutral for two wire line derived in a similler manner, the expn for capacitance are same for condrs B & C

(ii) Dec-14 Unsemmetrical Spacing (Transposed TL)

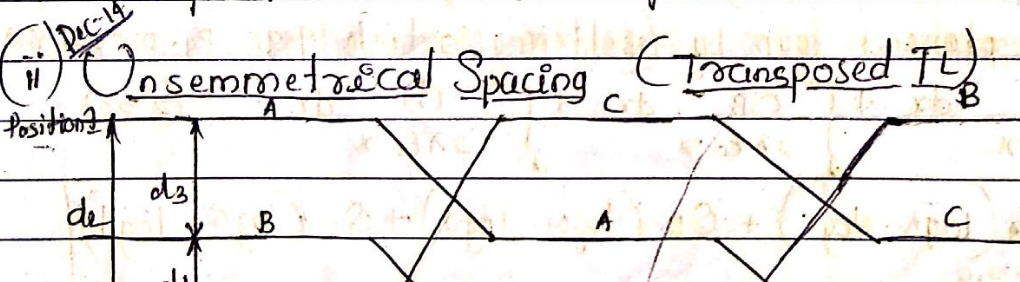


fig - 2.14

fig. 2.14 shows transposed line having unsemmetrical spacing Let us assume balanced condition i.e $Q_A + Q_B + Q_C = 0$ (2.56)

Considering all sections of transposed line for phase A

Potential at 1st position

$$V_1 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e\left(\frac{1}{r}\right) + Q_B \log_e\left(\frac{1}{d_3}\right) + Q_C \log_e\left(\frac{1}{d_2}\right)\right) \quad (2.5$$

Potential at 2nd Position

$$V_2 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e\left(\frac{1}{r}\right) + Q_B \log_e\left(\frac{1}{d_1}\right) + Q_C \log_e\left(\frac{1}{d_3}\right)\right) \quad (2.58)$$

Potential at 3rd Position

$$V_3 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e\left(\frac{1}{r}\right) + Q_B \log_e\left(\frac{1}{d_2}\right) + Q_C \log_e\left(\frac{1}{d_1}\right)\right) \quad (2.59)$$

Average voltage on conductor A is $V_A = \frac{1}{3}(V_1 + V_2 + V_3)$

$$\therefore V_A = \frac{1}{3\times 2\pi\varepsilon_0}\boxed{3Q_A \log\frac{1}{r} + (Q_B+Q_C)\ \log\frac{1}{d_1} + \log\frac{1}{d_2} + \log\frac{1}{d_3}}$$

$$= \frac{1}{3\times 2\pi\varepsilon_0}\boxed{Q_A \log\left(\frac{1}{r}\right)^3 + (Q_B+Q_C)\ \log\frac{1}{d_1 d_2 d_3}}$$

$$= \frac{1}{6\pi\varepsilon_0}\boxed{Q_A \log_e\left(\frac{1}{r^3}\right) - Q_A \log_e\left(\frac{1}{d_1 d_2 d_3}\right)}$$

$\because Q_A + Q_B + Q_C = 0$

$Q_B + Q_C = -Q_A$

$$= \frac{Q_A}{6\pi\varepsilon_0}\boxed{\log\left(\frac{d_1 d_2 d_3}{r^3}\right)}$$

$$= \frac{1}{3}\times\frac{Q_A}{2\pi\varepsilon_0}\log\left(\frac{d_1 d_2 d_3}{r^3}\right)$$

$$= \frac{Q_A}{2\pi\varepsilon_0}\log_e\left(\frac{d_1 d_2 d_3}{r^3}\right)^{1/3}$$

$\because b \log a = \log a^b$

$$V_A = \frac{Q_A}{2\pi\varepsilon_0}\log_e\left(\frac{(d_1 d_2 d_3)^{1/3}}{r}\right) \quad (2.60)$$

$\therefore$ The capacitance from conductor to neutral

$$C_A = \frac{Q_A}{V_A} =$$

$$\boxed{C_A = \frac{2\pi\varepsilon_0}{\log_e\left(\frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right)}\ F/m} \quad (2.61)$$

Problem

① Find the inductance ~~of~~ per 1km of 3Φ Transmission line using 1.24 cm diameters ~~of~~ conductors when these are placed at the corners of an equilateral triangle of each side 2m.

$\log_e = \ln$

$\log_{10} = \log$.

Sol^n

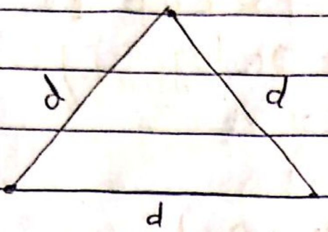


Fig above shows the 3-conductors of 3-ϕ line placed at the corners of equilateral Δ^{le} of each side 2m. Here cond^r spacing d = 2m and cond^r radius $r = \frac{1.24}{2} = 0.62$ cm

Inductance per phase per meter

Inductance /phase / m $= 10^{-7}\left[0.5 + 2\log_e\left(\frac{d}{r}\right)\right]$ From (2.19)

$= 10^{-7}\left[0.5 + 2\log_e\left(\frac{2}{0.0062}\right)\right]$

$= 12.05 \times 10^{-7} \times 1000$

Inductance/ph / Km $= 1.20$ mH

9.4.25

(2) The 3-cond^{r}s 3ϕ line are arranged at the corners of a triangle of sides 2m, 2.5m and 4.5m. Calculate the inductance / km of line when the cond^{r}s are regularly transposed the diameter of each cond^r is 1.24 cm

Sol^n

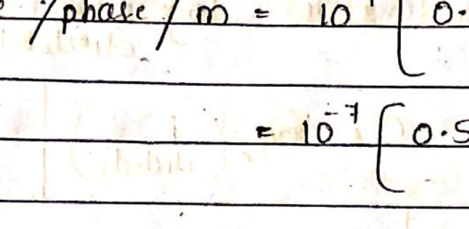


Fig above shows ~~3-ph~~ 3-cond^r of 3-phase line placed at the corners of Δ^{le} of sides $d_1 = 2m$, $d_2 = 2.5m$ & $d_3 = 4.5m$ the conductor radius is $r = \frac{1.24}{2} = 0.62$cm $= 0.0062$m

Equivalent equilateral spacing is Deq ~~or~~ $d_{eq} = \sqrt[3]{d_1 d_2 d_3}$

$= \sqrt[3]{2 \times 2.5 \times 4.5}$

$= 2.82$ m

Inductance / phase / m $= 10^{-7}\left[0.5 + 2\log_e\left(\frac{d_{eq}}{r}\right)\right]$ H

$= 10^{-7}\left[0.5 + 2\ln\left(\frac{2.82}{0.0062}\right)\right]$

$= 12.74 \times 10^{-7} \times 1000$

$\boxed{\text{Inductance / phas / km} = 1.274\,\text{mH}}$

Aug 16 9.5

(3) Calculate the inductance of each conductor in a 3ϕ, 3 wire system when the conductors are arranged in a horizontal plane with spacing such that ~~d_{31}~~ $D_{31} = 4m$, $D_{12} = D_{23} = 2m$ the conductors are transposed and have a diameter of 2.5cm

sol

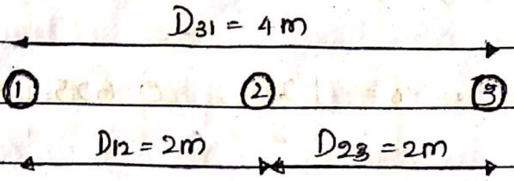


Fig above shows the arrangement of conductors of the 3ϕ line on a horizontal plane. The conductor radius . $r = \frac{2.5}{2} = 1.25$ cm $= 0.0125$m

Equivalent equilateral spacing $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

$= \sqrt[3]{4 \times 2 \times 2}$

$= 2.51$ m

Inductance / phas / m $= 10^{-7}\left[0.5 + 2\log_e\left(\frac{D_{eq}}{r}\right)\right]$ H

$= 10^{-7}\left[0.5 + 2\ln\left(\frac{2.51}{0.0125}\right)\right]$ H

$= 11.11 \times 10^{-7} \times 1000$

$\boxed{\text{Inductance/phase / km} = 1.11\,\text{mH}}$

HW: 9.7 / 217, 218

(4) A 1ϕ TL has two parallel cond's 3mts apart radius of each cond^{r} being 1cm. Calculate the capacitance of line / km given that $\epsilon_0 = 8.85 \times 10^{-12}$ F/m

sol cond^{r} radius $r = 1$ cm $= 0.01$ mt

spacing of cond^{r} $= 3$mts

∴ Capacitance of line is

$$C = \frac{\pi \varepsilon_0}{\log_e\left(\frac{d}{r}\right)} \text{ F/m}$$ From (2.51)

$$= \frac{\pi \times 8.85 \times 10^{-12}}{\ln\left(\frac{3}{0.01}\right)}$$

$$C = 4.876 \times 10^{-12} \times 1000$$

Capacitance /phas/ km = $0.00487 \mu F$

(5) A 3ϕ OH Line has its cond^{rs} arranged at the corners of an equilatral Δ^{le} of 2m side calculate the capacitance of each line cond^{r} per km given that diameter of each cond^{r} is 1.25 cm

Sol^{n} Conductor radius = $r = \frac{1.25}{2} = 0.625$ cm $= 0.00625$

spacing of conductors = d = 2m

Capacitance of line is

Capacitance/ϕ/m $C = \frac{2\pi\varepsilon_0}{\log_e\left(\frac{d}{r}\right)}$ F

$$= \frac{2 \times 3.142 \times 8.85 \times 10^{-12}}{\log_e\left(\frac{2}{0.00625}\right)}$$

$$= 9.644 \times 10^{-12} \times 1000$$

Capacitan/ϕ/km = $0.00964 \mu F$

9.21 258

(6) Calculate the capacitance of a 100 km 3ϕ 50Hz OHTL of 3 cond^{rs} each of diameter 2cm and spaced 2.5m at corners of equilateral Δ^{le}

Sol^{n} Radius of conductor $= r = \frac{2}{2} = 1$ cm $= 0.01$ m

Spacing of cond^{rs} = d = 2.5 m

Capacitance of line $C = \frac{2\pi\varepsilon_0}{\log_e\left(\frac{d}{r}\right)}$ F

Capacitance /ϕ/m = $\frac{2 \times 3.142 \times 8.85 \times 10^{-12}}{\ln\left(\frac{2.5}{0.01}\right)}$

$$= 10.075 \times 10^{-12} \text{ F}$$

Capacitance /ϕ/ km = 10.75 nF

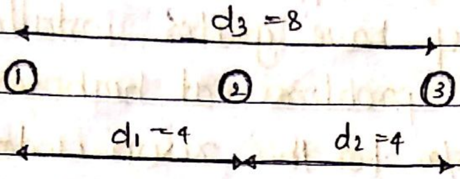
Capacitance / phase / 100 km $= 10.75 \times 10^{-9} \times 100$

$= 1.0075\,\mu F$

Dec-14

(7) A 3ϕ 50 Hz, 132 kV over head line has conductors placed in a horizontal plane 4 ms apart. conductor diameter is 2 cm. if the line length is 100 km calculate the charging C/n / phase assuming complete transposition.

Sol^n



Conductors are arranged horizontally as shown in above fig

radius of conductors $= r = \frac{2}{2} = 0.01\,m$

spacing bet^n the cond^{rs} $= d_{eq} = \sqrt[3]{d_1 d_2 d_3}$

$= \sqrt[3]{4 \times 4 \times 8}$

$= 5.039\,m$

Capacitance / phase / m (to neutral) $= C = \frac{2\pi\varepsilon_0}{\log_e\left(\frac{d_{eq}}{r}\right)}$ F

$= \frac{2 \times 3.142 \times 8.85 \times 10^{-12}}{\ln\left(\frac{5.039}{0.01}\right)}$

$= 8.94 \times 10^{-12}$

Capacitance / phas / km $= C = 8.94 \times 10^{-12} \times 1000$

$= 8.94\,nF$

Capacitance of / phase / 100 km $= C = 8.94 \times 10^{-9} \times 100$

$C_n = \cancel{8.94}\ 0.894\,\mu F$

Capacitive reactance $= X_{C_{ph}} = \frac{1}{2\pi f C}$ or $I_C = \omega C_n V_{ph}$

$= \frac{1}{2 \times 3.142 \times 50 \times 0.894\,\mu}$

$= 3559.76\,\Omega$

Charging C/n / phase $= \frac{V_{ph}}{X_{cph}} = \frac{\cancel{18324}\ 76.21}{3559.76} = \cancel{37}\,A$

$= 21.4088\,A$